Specialized Human Capital Investment, Growth and Convergence

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Model

Introduction

Over the past decade, human capital has taken center stage as a determinant of economic growth. Models of endogenous or exogenous growth emphasize human capital accumulation as an important engine of growth, if not its primary determinant. Given the dramatic rises in schooling that accompany economic growth, it is hard to dispute the emphasis of these models. However, there have been few attempts to model the pertinent family-level decisions that influence children's schooling . For example, the level and distribution of skills within a family is determined by fertility rates as well as by parents' decisions on investment in human capital and which children should receive these investments. All of these decisions reflect rates of return to various activity. Moreover, these rates of return are influenced by factors that can only be studied in a general equilibrium setting. This paper constructs such a model and attempts to shed light on important features of the development process.

This paper presents a model economy with overlapping generations of families whose offspring receive either skilled or unskilled human capital. Fertility decisions are endogenous, and parents must decide not only how many children to raise, but also the proportions that will receive each type of human capital. Child-rearing requires parental time, which involves sacrificing time at work. Both skilled and unskilled parents are capable of raising both types of children, but skilled parents have a comparative advantage in raising skilled children. Production utilizes inputs of both types of human capital. The parameters of the model are calibrated to match certain crucial features of the world economy under different scenarios, and the model is simulated under these different scenarios. For certain parameter combinations, the results of the model are broadly consistent with the histories of developed economies and the world economy as a whole.

Using this framework, we derive predictions from the model economy related to the following "empirical regularities" of the development process:

1. Human capital ("skill") expands as per capita income rises.
2. The dispersion in educational attainment ("skill level") declines.

3. Fertility rates decline.

4. Life expectancy, and the average age of new labor market entrants both increase.

The first reflects the skill deepening and broadening that accompanies the modernization process. As Becker (1993) notes,

> It is clear that all countries which have managed persistent growth in income have had large increases in the education and training of their labor forces. First, elementary school education becomes universal, then high school education spreads rapidly, and finally, children from middle-income and poorer families begin going to college. (p. 24).

Easterlin (1998) documents the substantial increase in primary enrollment rates for a broad sample of countries from the end of the 19th century to the 1990s. Baier, Dwyer, and Tamura (1999) utilize the data in Mitchell (1993) to document the increasing proportions of the male population that have been exposed to primary, secondary, and tertiary education in an even more comprehensive set of countries. For the U.S., Denison attributes one-fourth of the rise in per capita income from 1929 to 1992 to increases in the average level of schooling.

The decline in the dispersion of enrollment rates is evident from Table 0, which again utilizes data from Mitchell (1993). In order to summarize the data, individual countries are aggregated into continents or regions, although the consistent pattern exhibited is a feature of all of the underlying observations, as well. Although in earlier years the samples are limited by data availability and therefore estimates for these years tend to be dominated by a few developed economies, by 1960, the sample is largely complete. As Table 0 makes clear, commensurate with the rise in average years of male schooling, each region shows a marked decline in the dispersion of exposure to primary, secondary, and tertiary education, as measured by the coefficient of variation. This illustrates the fact that economic growth and industrialization involve upward skill mobility, and "specialization" in skill declines.

Declining fertility rates and the demographic transition are a feature of every development experience. All developed economies past through a period of declining mortality rates, followed by declining fertility.[1] According to Easterlin (1998), the fertility decline starts in the 1950s and 1960s in most developing economies, the exception being sub-Saharan Africa, where fertility rates remain high. While many theoretical models are

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[1]The exception is France, where the two rates declined concurrently. See Chesnais (1985).

able to generate a fertility transition, the model presented here allows us to investigate the relationship between family size and the education opportunities of children. The literature on the effects of "sibship" size on the allocation of time and resources within the family indicates that larger families behave differently than smaller families, and that birth-order effects may be important, as well (see, for example Behrman and Taubman (1986), Lindert (1979) for the modern U.S.). These studies report two major results that are useful in the assessment of our model: first, when families are larger, each child receives less parental investment, and second, parents seem to treat their children systematically differently. In addition, Behrman, et al. (1989) find that schooling is more unequally distributed in larger families in the U.S. And, in a study of Philippine rice villages by Quisumbing (1994), better-endowed parents (in terms of either land or education) treat children more equitably than do poorer parents. These results suggest the need to investigate a general equilibrium model that allows such behavior at the household level. Among other things, it allows investigation changes in intergenerational mobility from skilled to unskilled as an economy develops.

Finally, it is well known that as economies develop, they rely less on the labor of children. For example, according to ILO data (quoted in Doepke (1999)), in Korea in 1960, 1.1% of children aged zero to fifteen were economically active, compared with 4.3% in Brazil. As Korea's economy developed and human capital increased, the use of child labor was nearly eliminated: in 1985, .3% of children between ten and fourteen were active. As is well know, Brazil's development experience has not been so fortuitous and in 1990, 24.3% of ten-to-fourteen-year-olds were still economically active. Overall .... NEED BROADER STATS (ILO data on its way.)

Almost all traditional, non-strategic economic analyses of fertility and the effects of altruistic behavior by parents toward their children imply that identical children will be treated equally.[2] The economic environment faced by parents influences fertility and child-rearing decisions, but frequently, there is no incentive to treat children differently (e.g., Becker, et al. (1990)). Even when this incentive exists, the models are restricted to impose equal treatment of offspring. Historically speaking, this may not be an innocuous restriction. Ample

[2]Exceptions include Mulligan (1997) who studies endogenous parental altruism, and Behrman, Pollak, and Taubman (1995).

evidence exists that children have not been treated equally by parents in many societies.[3] It therefore seems natural to investigate the implication of allowing for this phenomenon in a model of growth and fertility choice.

Without some sort of imperfection in the economic environment, it seems clear that parents would choose to treat all children identically. Of course, different ability levels may encourage different levels of parental transfers in order to maximize productivity. However, other features of the economic environment are equally likely to deliver this outcome. In our model, differential parental transfers can emerge even though children are *ex ante* identical. The particular features of the model that generate this property are a production nonconvexity in the form of increasing returns to scale in skilled human capital accumulation and closed financial markets (so that parents cannot borrow against the earnings of future generations). In such an environment, when parents care about the average utility of their children, it may no longer desirable to equalize transfers or other investments across children, especially at lower incomes. However, as incomes grow, transfers and investment may become more equal.

The results of the model also add something to the ongoing debate about the relative merits of modeling growth as due to endogenous or exogenous forces. Since we use a CES technology, endogenous growth is possible when the elasticity of substitution between the two factors of production – skilled and unskilled labor – is high. When this is not the case, the model will approach a stationary state in the absence of exogenous technical progress. In the simulations, we investigate both scenarios, after calibrating the model to match certain features of the world economy, both past and present. In order to match these features, it is necessary that the model economy exhibit growth in per capita incomes, a property that would not be possible with a low elasticity of substitution. Thus, under this scenario, we assume that unskilled human capital grows exogenously. Since this is the only way that the model can be made to fit the historical data under the low elasticity of substitution assumption, the different results that emerge between the endogenous and exogenous growth scenarios allow us to make claims about the reasonableness of these mechanisms for generating growth.

The next section of the paper formally develops the model economy. In section 3, the model is calibrated

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[3] At its most extreme, this tendency is reflected in the institution of primogeniture, where only the first born son receives inheritance, or, more generally, unigeniture, where inheritance is passed down to a single heir. Primogeniture was codified as law in many European societies and their colonies (see Sadler (2000) for examples), and many great ancient civilizations also adopted the practice among the nobility (see Bergstrom (1995)). As these economies grew and resources expanded, equity in inheritances seems to have become the rule.

and simulated and we compare results from different parameter combinations. Section 4 concludes.

The Model Economy

In this section we present the underlying model of the paper. Initially. there are two types of agents, those with skilled human capital and those with unskilled human capital. The distinction between skilled human capital and unskilled human capital is quite similar to that contained in Becker, Murphy and Tamura, hereafter BMT, (1990). However unlike BMT (1990), unskilled human capital can work with skilled human capital. In fact the wage per unit of unskilled human capital is rising the in the level of skilled human capital available in the economy.

Skilled and unskilled agents have identical preferences. Among skilled agents there can be many types, where a type is given by the level of skilled human capital. Furthermore there are a continuum of agents. The Lebesgue measure of unskilled agents we refer to as the population of the unskilled agents. For each type among the skilled human capital population there are a positive measure of agents.[4]

An agent lives for possibly two periods, young and old. As in Jones (1999) and Tamura (1999b), we introduce mortality into the problem. If a young person lives through the first period, then he or she lives through their old age with probability one. However there exists the possibility that he or she will pass away before they complete their first period of life. This death occurs immediately after the time a parent spends rearing and educating the youth.[5]

While young an individual receives human capital investment, if any, from his or her parent. If he or she survives his or her youth, when an individual is old, he or she chooses own consumption, fertility and the investments in human capital of his or her children. A parent does not care about the human capital levels of his or her children, but rather, only about the incomes of his or her children. Unlike previous work on endogenous fertility and endogenous human capital investment, Tamura (1996,1997,1999ab), an agent can

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[4]In other words, if there were a type of skilled human capital that had zero Lebesgue measure, that type would have no effect on production, or consumption or population. Since this type has no effect on any measurable economic quantity, we ignore them.

[5]Death can occur at two possible ages, if a child receives no human capital investment, it can occur immediately after the rearing time. If a child receives human capital investment, then the death occurs immediately after the education.

specialize his or her human capital investment on a fraction of his or her children. Let superscript s refer to skilled earnings, and superscript u refer to unskilled earnings. We assume that parents care about the average number of adult survivors they have. Finally we assume log preferences for tractability; ignoring individual subscripts for simplicity:

$$a \ln c_t + g \ln\left[b_t - d_t\right] + b\left\{s_t \ln y^s_{t+1} + \left(1 - s_t\right) \ln y^u_{t+1}\right\}, \quad \textbf{(1)}$$

where α, β, γ > 0, $0 \leq s_t \leq 1$, and d_t is the average number of deaths **prior to reaching adulthood** and b_t is the number of births a parent has. We focus on one particular equilibrium. We assume that all parents with the same level of skill choose the same actions. In particular we assume that all identical parents choose the same fraction of children to invest in skilled human capital.[6]

Each individual has the following budget constraint, for the moment we suppress whether earnings are for skilled or unskilled human capital individuals:

$$c_t = y_t\left[1 - b_t\left(q + s_t t_t\right)\right], \quad \textbf{(2)}$$

where 2 >0 is the fraction of time each child takes to rear, s_t is the fraction of children receiving skilled human capital investments, and J_t is the teaching time spent per child receiving skilled human capital. We assume that if a child receives no skilled human capital investments, he or she is endowed with h' units of unskilled human capital. Notice that a parent must spend rearing time and education time on all children, regardless of whether they survive to adulthood.

The skilled human capital accumulation technology has two branches. The first branch is for parents without any skilled human capital. The second branch is for parents with skilled human capital. We assume that the functional form for each is similar to Tamura (1991,1996): **[I CHANGED LAMBDA TO KAPPA HERE]**

$$h^s_{t+1} = \begin{cases} A\bar{h}_t^{j}\left(k h^u_t\right)^{1-j} t_t^{m} & if\ unskilled, \\ A\bar{h}_t^{p}\left(h^s_t\right)^{1-p} t_t^{m} & if\ skilled, \end{cases} \quad \textbf{(3)}$$

$\bar{h}_t = \max\{h^s_t\}$, 0 < φ < π < 1, μ > 0, 1 > κ > 0. The first branch shows that an unskilled parent can

[6]An alternative equilibrium would be one where all identical human capital individuals have the same utility.

produce skilled human capital. Since , $\varphi < \pi$, relative to a skilled parent, an unskilled parent is less able to take advantage of the existing body of knowledge, contained in $\bar{h}_t$. Furthermore even the unskilled parent's existing stock of unskilled human capital, h_t^u, is less productive, $\kappa < 1$, than an equivalent amount of skilled human capital, h_t^s. This indicates that the unskilled parent has a comparative advantage in producing unskilled children, and thus, ceteris paribus, he or she will have more children than a skilled parent. Over time if the body of knowledge in the population rises, then perhaps all unskilled parents will have skilled children. **We assume that unskilled human capital grows exogenously (if at all) over generations at rate, 1.**

The final step in setting up the model is to present production. There is a single consumption good in the economy. It is produced under constant returns to scale in the distribution of human capital. If all human capital levels, skilled and unskilled, were multiplied by , total output of the consumption good would also be multiplied by . There is diminishing returns in each individual type of human capital, that will be elaborated in more detail below. Assume that the two types of human capital, skilled and unskilled are combined using the following CES production technology:

$$
\begin{aligned}
Y_t &= (1-e)^{-\frac{1}{r}}\left\{e\left(N_t h_t^u\right)^r + (1-e)K_t^r\right\}^{\frac{1}{r}}, \\
K_t &= \left(\sum_{j=1}^{M_t} m_{jt} h_{jt}^{\frac{1}{w}}\right)^w, \\
w &> 1 > r, \quad e > 0
\end{aligned}
\tag{4}
$$

where there are N_t unskilled individuals; M_t types of skilled individuals, there are m_{jt} skilled individuals of type j, each with h_{st} units of skilled human capital, and $\sum_{j=1}^{M_t} m_{jt}$ skilled individuals.[7] The technology can be thought of as a standard CES production technology combining labor with a capital input. The important differences are that the capital input comes from the distribution of skilled human capital, and that the capital input demonstrates increasing returns to scale in the number of skilled agents, >1.[8] To see this, assume that all skilled agents are identical with h_t units of skilled human capital, and the population of skilled agents is M, then the capital input is given by:

[7] Thus the Lebesgue measure of unskilled agents is N_t and the ebesgue measure of skilled agents of type j is m_{jt}.

[8] The capita input is a version of Tamura (1992).

$$K_t = M^w h_t \tag{5}$$

This increasing returns in skilled human capital participation will play an important role in determining if unskilled parents have only skilled children.

Since each individual is a set of measure 0, each agent has no control on his or her wage per unit of human capital. Each agent, skilled or unskilled, earns an income that is a product of the wage per unit of human capital and the amount of human capital they have. Earnings for an unskilled worker and a skilled worker of type i are:

$$\begin{aligned}
y_t^u &= w_t^u h_t^u, \\
w_t^u &= (1-e)^{-\frac{1}{r}} \left\{ e\left(N_t h_t^u\right)^r + (1-e) K_t^r \right\}^{\frac{1}{r}-1} e\left(N_t h_t^u\right)^{r-1}, \\
y_{it}^s &= w_{it}^s h_{it}^s, \\
w_{it}^s &= (1-e)^{1-\frac{1}{r}} \left\{ e\left(N_t h_t^u\right)^r + (1-e) K_t^r \right\}^{\frac{1}{r}-1} K_t^{r-\frac{1}{w}} \bar{h}_{it}^{\frac{1}{w}-1}, \\
\bar{h}_{it} &= h_{it} \quad in\, equilibrium
\end{aligned} \tag{6}$$

Since all unskilled agents have the same level of unskilled human capital, all unskilled agents earn the same income. Of the skilled agents, the wage per unit of skilled human capital depends on the amount of skilled human capital an individual has. Since each agent is a set of measure 0, no agent has any control on the wage per unit of human capital. We assume that agents do not form coalitions of measurable size to try and affect the wage per unit human capital of their type. Thus we assume that all agents take all wages in the next period to be independent of their actions.

Unskilled Problem

In this subsection we analyze the unskilled agent's problem. Writing out the problem for the unskilled agent:

$$U\left(h_t^u\right) = \max_{\{n,s,t\}} \left\{ \begin{aligned} & a \ln y_t^u + a \ln\left[1 - b_t\left(q + s_t t_t\right)\right] + g \ln\left[b_t - d_t^u\right] \\ & + b\left\{ s_t\left[\ln w_{t+1}^s + \ln h_{t+1}^s\right] + \left(1 - s_t\right) \ln y_{t+1}^u \right\} \end{aligned} \right\} \tag{7}$$

The first order conditions for optimal choices of fertility, b_t, skilled human capital investment time, $_t$, and share of children receiving skilled human capital investments, s_t, are:

$$\frac{a\left(q + s_t t_t\right)}{1 - b_t\left(q + s_t t_t\right)} = \frac{g}{b_t - d_t^u}$$
$$\frac{a n_t s_t}{1 - b_t\left(q + s_t t_t\right)} = \frac{b m s_t}{t_t} \qquad \textbf{(8)}$$
$$\frac{a n_t t_t}{1 - b_t\left(q + s_t t_t\right)} = b\left\{\ln y_{t+1}^s - \ln y_{t+1}^u\right\}$$

Notice that the effect of expected deaths causes an increase in the marginal benefit of an additional birth. It has no effect on marginal benefit of human capital investment. However because of the timing of youth deaths, it raises the cost of human capital investment. Thus the effect of youth mortality is to increase fertility and decrease investments. Whether the reduction in investment occurs via lowering the share of children receiving investments, and or the lowering of the amount of investment per child receiving investments is unclear.

The first two equations can be solved in terms of the share of children receiving skilled human capital investments, s_t , and the expected number of deaths, d_t^u. The resulting functions are:[9]

$$b_t = \frac{g}{\left(a + g\right)\left(q + s_t t_t\right)}$$
$$t_t = \frac{-\mathrm{B}_t + \sqrt{\mathrm{B}_t^2 - 4\mathrm{A}_t C_t}}{2\mathrm{A}_t} \qquad \textbf{(9)}$$
$$\mathrm{A}_t = \left(a + b m s_t\right) d_t^u s_t$$
$$\mathrm{B}_t = g + a d_t^u q - b m s_t + 2 b m d_t^u q s_t$$
$$C_t = b m q\left(d_t^u q - 1\right)$$

Substituting in for n_t $_t$ into the final equation in (8) and simplifying produces:

$$m = \left[\ln y_{t+1}^s - \ln y_{t+1}^u\right] \qquad \textbf{(10)}$$

Thus the income gap between a skilled child from an unskilled parent and his or her unskilled sibling from the same unskilled parent is a constant percentage, approximately equal to μ. Below we will solve for the specialization rate, s_t , under perfect foresight, but before we do this we must examine the skilled agent's

[9]Except for the addition of investment fraction s_t this is the same result as contained in Tamura (1999b).

problem.

Skilled Problem

In this section we solve the problem facing the typical skilled parent. The appendix shows that if the unskilled parents are investing in a positive fraction of their children, then all skilled parents are investing in all of their children. Furthermore the appendix shows the condition necessary for all skilled parents to invest in their children, even when unskilled parents are producing no skilled children.

Suppose the conditions in the appendix hold so that all skilled parents invest in all of their children. The problem facing a skilled parent becomes:

$$U\left(h_{it}^{s}\right) = \max_{\{n,t\}}\left\{a \ln y_{it}^{s} + a \ln\left[1 - b_{it}\left(q + t_{it}\right)\right] + g \ln\left[b_{it} - d_{t}^{s}\right] + b \ln y_{it+1}^{s}\right\} \tag{11}$$

The first order conditions determining the optimal fertility and human capital investment time for a skilled parent are:

$$\begin{aligned} \frac{a\left(q + t_{it}\right)}{1 - b_{it}\left(q + t_{it}\right)} &= \frac{g}{b_{it} - d_{t}^{s}} \\ \frac{a t_{it}}{1 - b_{it}\left(q + t_{it}\right)} &= \frac{bm}{t_{it}} \end{aligned} \tag{12}$$

Observe that the expected number of deaths facing a skilled parent can differ from the expected number of deaths facing an unskilled parent.[10] As before, the effect of youth mortality is to raise the number of births and lower the amount of investment per child. Since we are focusing on the case where all skilled parents invest in all of their children, we can unambiguously predict that mortality will lower the amount of human capital investments per child.

Solving for fertility and human capital investment time reveals:

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[10]Urban workers had differential mortality compared to rural workers throughout much of human history, see Diamond (1999).

$$
\begin{aligned}
b_{it} &= \frac{g}{(a+g)(q+t_{it})} + \frac{ad_t^s}{a+g} \\
t_{it} &= \frac{-\mathrm{B}_t + \sqrt{\mathrm{B}_t^2 - 4\mathrm{A}_t C_t}}{2\mathrm{A}_t} \\
\mathrm{A}_t &= d_t^s(a+bm) \\
\mathrm{B}_t &= g - bm + (a+2bm)d_t^s q \\
C_t &= bmq(d_t^s q - 1)
\end{aligned} \tag{13}
$$

Since each skilled parent faces the same expected number of deaths, observe that fertility and human capital investment time is independent of the level of skill an individual has. Furthermore notice that if an unskilled parent invests in all his of her children in (9), then he or she will have identical fertility and identical investments per child as a skilled parent only if the mortality is the same. As a consequence of (13) and (9), it is clear that unskilled parents will have higher fertility than skilled parents. Furthermore unskilled parents will invest less per child than a skilled parent invests per child.

Finally since all skilled parents invest the same amount of time per child, all skilled individuals converge to the same human capital level**, as in Tamura (1991)**. Thus the turnpike theorem holds in this model. The ratio of skilled human capital of children of skilled human capital parents i and j is given by:

$$
\frac{h_{it+1}^s}{h_{jt+1}^s} = \frac{A\bar{h}_t^j \left(h_{it}^s\right)^{1-j} t_{it}^m}{A\bar{h}_t^j \left(h_{jt}^s\right)^{1-j} t_{jt}^m} = \left(\frac{h_{it}^s}{h_{jt}^s}\right)^{1-j} \tag{14}
$$

If all unskilled human capital individuals eventually produce only skilled human capital individuals, then all individuals will become identical in the long run.

The long run behavior of the model is given by the solution to (13) when d_t converges to 0. Solving for this case produces the balanced path fertility and human capital investments for skilled parents:

$$
\begin{aligned}
b_t &= \frac{g - bm}{q(a+g)} \\
t_t &= \frac{bmq}{g - bm}
\end{aligned} \tag{15}
$$

It is quite possible for their to exist branches of dynasties that remain unskilled forever. In particular, given the production function, if ≤ 0, then both skilled and unskilled individuals are essential factors of production. If both types are essential then there could be a stationary state of constant flow of new skilled workers, i.e., some children of unskilled parents will forever become skilled, while all skilled parents produce skilled workers. We present both phenomena below in the numerical solutions.

The Appendix presents the sufficient condition for all skilled parents to raise only skilled children. Furthermore Appendix B presents the sufficient condition for endogenous growth. As in Barro and Sala-i-Martin (1995), endogenous growth is only possible if > 0. However > 0 is a necessary condition, but it is not sufficient. Endogenous growth requires that the skilled capital aggregate must grow at a sufficiently fast rate to "pull" the unskilled workers into the skilled population. Essentially the skilled capital aggregate must grow faster than the population growth rate of unskilled workers. In the terminology of the neoclassical growth models, capital accumulation must be more rapid than the population growth rate, i.e., capital deepening must occur.

Numerical Solutions

In this section we detail the numerical solution of the model for various parameter values. We examine several cases. In particular we chose three different values of , = -2.5, -1.25, .525. As we varied , we varied the rate of exogenous technological progress for unskilled workers. We did this in order to maintain a constant average growth rate of income per capita. **The parameters of the simulations were adjusted to match world population in the year 1800 of around 1 billion and in the year 2000 of 6 billion, and per capita income of skilled workers at roughly $30,000. Certain parameters are held constant in the four simulations. The values of the these parameters are α = .395, β = 5.1, γ = .4775, = .25, μ = .05 , A = 2, and π = .5. The values of , , φ, and κ are adjusted in order to maintain the consistency of the data with historical reality under each scenario. The values of these parameters are listed at the top of each panel of Figure 1.**

The number of state variables in this problem varies over time. The economy is characterized by the human capital distribution. It is necessary to know the level of unskilled human capital at time t, h_t^u , and the population of unskilled individuals, N_t and the distribution of skilled human capital at time t, $\left\{h_{jt}^s, m_{jt}\right\}_{j=1}^{M_t}$ where h_{jt}^s is the level of skilled human capital that all individuals in group j has, and m_{jt} is the population skilled group j, and M_t is the number of groups of skilled human capital at time t. Information on all of these variables completely characterizes the economy at time t. The equilibrium we considered was one in which skilled parents only raised skilled children. **Although clearly an approximation, there is ample empricial justification for this assumption. Mobility studies verify that upward mobility is much more likely that downward mobility in industrialized and industrializing countries.**[11] **When combined with the observation that the process of industrialization entails upgrading the skill level of all occupations, we reach the conclusion that downward mobility is a rare occurrence, a conclusion we should note that is consistent with our view that the development process can be reasonable characterized as a process of overcoming nonconvexities in production opportunities while faced with imperfect (or closed) financial markets.**

Under this equilibrium, the actions of the skilled parents are simple to characterize. Their fertility and human capital investment time are identical, no matter the level of skill any skilled parent has. These policy functions are given by (13). Having solved for the policy functions for all skilled parents, the only thing left to solve are the policy functions of the unskilled parents. Recall that all unskilled parents in period t have h_t^u units of unskilled human capital. The growth rate of these units is assumed exogenous and equal to $_u$

1. The policy functions for fertility and human capital investment time, as functions of the share of children receiving investments, are given by (9). Therefore the only numerical problem to be solved is the share of children from unskilled parents that receive investments. The equation determining this is given by (10). Using the results from (13) and (9) and substituting into (10) yields the following implicit function for s_t :

$$m = \ln\left[\frac{1-e}{e} \frac{K_{t+1}^{r-1} \left\{ \sum_{j=1}^{M_{t+1}} m_{jt+1} \left(h_{jt+1}^{s}\right)^{\frac{1}{w}} \right\}^{w-1} \left(A\bar{h}_t^{j} \left(s h_t^{u}\right)^{1-j} \left(t_t^{u}\right)^{m} \right)^{\frac{1}{w}}}{\left(N_t\left(1-s_t\right)\left(b_t^{u}-d_t^{u}\right)\right)^{r-1} s h_t^{u}} \right] \qquad \textbf{(16)}$$

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[11]See, for example, the cross-country study of occupational mobility by Erikson and Goldthorpe (1992). Evidence on earnings and occupation mobility that supports our assumption can be found in Zimmerman (1992) for the U.S. and Chechi, Ichino, and Rustichini (1999) for Italy and the U.S.

Let $\left(b_t^s, t_t^s\right)$ be the optimal choice of fertility and investment time of the skilled parents, given by (13), and $\left(b_t^u, t_t^u\right)$ be the choice of fertility and human capital investments of unskilled parents, as a function of the share of children receiving investments (given by (9)), then

$$
\begin{aligned}
m = {} & \ln\left[\frac{1-e}{e}\right] + \frac{1}{w}\ln\left[A\bar{h}_t^j\left(s h_t^u\right)^{1-j}\left(t_t^u\right)^m\right] - \ln\left[\left(N_t\left(1-s_t\right)\left(b_t^u - d_t^u\right)\right)^{r-1} s h_t^u\right] \\
& + \left(wr-1\right)\ln\left[\sum_{j=1}^{M_t} m_{jt}\left(b_t^s - d_t^s\right)\left(A\bar{h}_t^p\left[h_{jt}^s\right]^{1-p}\left(t_t^s\right)^m\right)^{\frac{1}{w}} + N_t s_t\left(b_t^u - d_t^u\right)\left(A\bar{h}_t^j\left(s h_t^u\right)^{1-j}\left(t_t^u\right)^m\right)^{\frac{1}{w}}\right]
\end{aligned}
\quad \textbf{(17)}
$$

can be solved numerically to determine the optimal share of unskilled children receiving investments. Once s_t is determined it is a simple matter to use (9) to solve for the choice of fertility and investments, $\left(b_t^s, t_t^s\right)$ and $\left(b_t^u, t_t^u\right)$. When these policy functions are determined it is a simple matter to update the state variables in the economy,

$$
\begin{aligned}
h_{t+1}^u &= s h_t^u \\
N_{t+1} &= N_t\left(b_t - d_t^u\right)\left(1 - s_t\right) \\
\left\{h_{jt+1}^s, m_{jt+1}\right\}_{j=1}^{M_t} &= \left\{A\bar{h}_t^p\left(h_{jt}^s\right)^{1-p}\left(t_t^s\right)^m, m_{jt}\left(b_t^s - d_t^s\right)\right\}_{j=1}^{M_t} \\
h_{j+1t+1}^s &= A\bar{h}_t^j\left(h_{jt}^s\right)^{1-j}\left(t_t^u\right)^m \\
m_{j+1t+1} &= N_t\left(b_t^u - d_t^u\right)s_t \\
M_{t+1} &= \begin{cases} M_t & if\ s_t = 0 \\ M_t + 1 & if\ s_t > 0 \end{cases}
\end{aligned}
\quad \textbf{(18)}
$$

The first line of (18) updates the unskilled human capital level. If there is exogenous technological progress, then the unskilled human capital level rises. The second line of (18) updates the number of unskilled individuals in the population. The unskilled population rises or falls if the gross growth rate of population, n_t rises enough to offset those that become skilled s_t. The third line of (18) is the law of motion of all skilled children from skilled parents. Since all skilled parents choose the same fertility and the same amount of time to invest in their children, the law of motion for this group is quite simple. Observe that the existence of the human capital spillover in the accumulation technology induces human capital convergence. The fourth line of (18) gives the human capital of the first generation of skilled children from unskilled parents. The number of new skilled workers is given by the fifth line of (18). Finally the evolution of the number of skilled human capital types is given by the last line of (18).

The four panels of Table 1 contain the results from these solutions. Each panel covers one of the four different parameter sets. The first column gives the simulated year. The second column lists the average income in the world. The third column contains the average income of the skilled population, where we average over all individuals who received skilled human capital from their parent. The fourth column contains the average income of the unskilled population. The final three columns are the population of skilled individuals, the population of unskilled individuals and world population.

The four different panels of Table 1 produce markedly different results. In the first panel, = .55, income growth occurs for both types of individuals. Income grows for the skilled because population is rising and because skilled human capital is rising. For unskilled individuals, their income rises because of the rising capital contribution in the economy provided by the skilled workers. Notice that the population of unskilled workers peaks in 1960 with a population of 3580 million. By 2000 the unskilled population is reduced to 2510 million, and a minority of the world's 6030 million population. World population growth slows dramatically as the world becomes more and more skill dominated. The dip in per capita income of the skilled that occurs in 2000 arises because of the initial creation of skilled individuals by unskilled parents. The old skilled population in 1960 is 48 million. In 2000 the old skilled population is 51 million original skilled descendants and 820 million newly skilled. They are less skilled than their skilled counterparts; the original skilled workers have 1.12 units of skilled capital and the newly skilled have only .000556 units of skilled capital. However by the next generation the skill levels of their descendants are 2.1 and .046 and the skill levels of their grandchildren are 3.95 and .59. So from .05 percent of incomes, the next generation rises to 2.2 percent and the grandchildren are at 14.9 percent. The great-grandchildren have 7.4 and 2.9 units of skilled capital and the relative income is 39 percent.

This parameterization produces compete exodus of unskilled workers into the skilled worker category. When this occurs the model becomes like Tamura (1999ab). Long run balanced growth per capita income growth is given by:

$$\frac{y_{t+1}}{y_t} = A\left(\frac{bmq}{g - bm}\right)^m \left(\frac{g - bm}{q\left[a + g\right]}\right)^{w-1} \quad \textbf{(19)}$$

This is the long run growth rate in this economy, **but this rate will not be reached until the year 2360.**

Observe that the world population growth slows tremendously after 2000. By 2200, the world population grew by only 2.6 percent over the previous 40 years. Thus while population growth is positive, the model indicates that world population growth rates never return to the level attained over the 1800 to 2000 period. Essentially the entire world enters into a demographic transition, moving from unskilled to skilled. Since the entire world becomes skilled, it is obvious that the share of the economic pie produced by the skilled population is 100 percent.

The next panel, = .25 illustrates **a scenario with continuous** rapid world population growth. Population grows at a 1.18 percent annual rate from 2160 to 2200, compared with the .07 percent annual rate in the previous solution. **Notice that the annual world population growth is above the rate of population growth from 1800 to 2000 of .84 percent. [REWORD]** Thus the world never enters into a demographic transition and the population of unskilled workers rises perpetually. In the long run the model produces the interesting result that half the world's population is skilled and the other half remains unskilled. This stationary distribution is interesting because it requires that in every generation each unskilled parent invests in a constant proportion of his or her children. Since population growth is faster in this model, average per capita income growth is slower than per capita income growth of the skilled workers. Thus the relative earnings of skilled workers rises compared to unskilled workers. Unskilled worker income growth is completely driven by the rising level of capital input provided by the skilled workers. The share of world output produced by skilled workers rises from about 36 percent to about 100 percent by the end of the solution in year 7800. This is obvious since they are half the world's population, but their average income grows at a faster rate than the poor.

In both Table 1a and Table 1b, the solutions are able to match the world population in 1800 and 2000 as well as the average income in the world in 2000. However they have different predictions concerning the long run population growth rate in the world. By year 7800, the end of the solution, = .55 produces an annualized population growth rate of .05 percent. For = .25, the annualized population growth rate is .96 percent.

The final two panels of Table 1 provide the results when < 0. In each of these cases the value of implies that both factors of production are essential. Hence it is not possible for all unskilled workers to produce only skilled progeny. However the cases produce two different population scenarios. For = -.5, the world initially engages in a demographic transition, and almost all children of unskilled parents becomes skilled. Interestingly the world in 1800 is about 50 percent unskilled. In 1840, only 3.7 percent of the population is unskilled. However this undershoots the long term fraction of the world's population that will remain unskilled, which asymptotes to 100 percent! Thus the model predicts that the Malthusian portion of the world, where Malthusian implies low skill and high population growth, becomes the entire world.

Table 1c has a different prediction in terms of income. Per capita income growth occurs for both the skilled and the unskilled. However, not surprisingly, the growth rate of income of the skilled exceeds the growth rate of the unskilled. Unskilled worker income rises because the capital component provided by the skilled workers is rising and because there is exogenous growth among the unskilled workers. The share of the world output produced by the skilled falls from 92 percent in 1800 to 0 by the end of the solution in year 7800.

Table 1d presents the results for = -1.7. Notice that world population is growing without bound as in the case for = .25. Here however world population grows at a faster rate than for = .25, because the unskilled share of the population is rising and not constant at half. In fact the unskilled population essentially becomes the entire world population. The share of the world output produced by the skilled population rises initially from 54 percent in 1800 to 65 percent in 1920. After 2400 the share of output produced by the skilled population falls below 80 percent and trends downward forever. There are occasional blips upward when unskilled parents produce skilled children.

The population histories produced by = -.5 and = -1.7 vary greatly from the case = .55. For =

-.5 predicts population growth at tremendous rates. From a value of 6130 million in 2000, population reaches 181000 million by 2200. Population growth averages almost 2 percent annually at the end of the solution in year 7800. With = -1.7, the long run population growth rate is similar to that obtained under = -.5. By the end of the sample, population growth averages almost 2 percent annually.

Table 2 produces the time series for the relative importance of the skilled population both in terms of relative economic production and relative population. The skilled share of the world's population attains three possible values for these 4 cases. In the first case, = .55, the model shows that all unskilled eventually choose to raise only skilled children by year 2600. For = .25, the model produces something like a stationary skilled share of the world's population. This stationary value appears to be 50 percent. For both negative values of , the skilled share of the world's population goes to 0 by the end of the solution. For > 0, the skilled share of GDP goes to 100 percent by the end of the solution. For = .55, 2600 the entire population is skilled. For < 0, the skilled share of GDP goes to 0 by the end of the sample (actually to 2.2 percent in the case of = -1.7).

Table 3 provides more information from these solutions. The four panels contain information concerning the average years of schooling in the population, the average total fertility rate in the world, as well as the total fertility rates for skilled and unskilled parents, and life expectancy of skilled, unskilled and the world as a whole. Recall that each period is 40 years, so that the maximum life expectation is 80. To calculate life expectation for the progeny of parents, we use the following values for skilled parents and unskilled parents:

$$life\,\mathrm{exp}\,ectation_t = \frac{80\left(b_t^s - d_t^s\right) + 40\left(q + t_t^s\right)d_t^s}{b_t^s} \; if\ skilled$$

$$life\,\mathrm{exp}\,ectation_t = \frac{80\left(b_t^u - d_t^u\right) + s_t 40\left(q + t_t^u\right)d_t^u + \left(1 - s_t\right)40 q d_t^u}{b_t^u} \; if\ unskilled$$

There are interesting differences between the four sets of solutions. In the cases where < 0, the long run stationary world total fertility rate, TFR, is around 4.33 children. This accounts for the rapid population growth. While life expectancy hits its theoretical maximum in all four of the cases, when < 0, age at entry into the labor force, essentially education + 6 or 40(+), is 10. Thus by the measures of primary schooling, the typical worker has only 4 years of schooling. In contrast, for > 0, the world TFR falls well below 4.33, at 2.04 for = .55 and 3.09 for = .25. In both of these cases the world TFR is an average of the total fertility rates of each type.[12] Life expectancy rises to it theoretical maximum, but the second difference is in the age at entry into the labor force. Observe that for > 0, the age at entry is significantly above 10. It rises to 21.5 for = .55 and 15 for = .25. Thus in the case of = .55, the average individual is a college graduate, whereas for = .25, the average individual completes primary school or about 9 years of schooling.

This difference in total fertility rates and the age at entry into the labor force appears to be the most useful way in which to compare these solutions with the world history. Table 4 contains the information from Baier, Dwyer and Tamura (1999). The data shows the fraction of the world labor force that has had no education, the fraction of workers with some exposure to primary education (but no secondary schooling), the fraction of workers with some exposure to secondary education (but no higher education) and the fraction of workers with some exposure to higher education. The underlying data come from B. R. Mitchell (1993). While the data is not completely smooth, this is mostly due to the incorporation of more countries over time. The sample of countries is complete by 1960. Over that shorter period, there has been a monotonic decline in the fraction of workers with no education at all. There has been a very slight decline in the fraction of workers with only exposure to some primary education, from 42 percent to 40 percent. There has been practically a doubling of the worker type with some secondary schooling, from 18.6 percent to 36.2 percent. Finally there has been an explosion in the share of workers with some higher education exposure, 2.2 percent to 14 percent. The calculated average age at finishing education if one calculates the average years of schooling plus 6 to this sample, is given in the final column of Table 4. We assigned 0 years of schooling to those with no education exposure, 3 years of schooling for those with some primary schooling, 10 years of schooling for those with some secondary schooling and 15 years for those with some higher education. Since the 1850 data only contains the United States and the United Kingdom, and these countries lead the way in universal education, see Goldin (19xx), it is clear that the average age of entry into the labor force has risen by at least 100 percent over the past 150 years. This suggests that a model of rising numbers of unskilled workers raising skilled children better fits the data.

Figures

In this section we present some results of the numerical solutions in graphic form. We present the share

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[12]Where in the case of = .55, there is only one type, skilled workers.

of the population that are skilled for the four parameter sets. We also present the share of GDP that is produced by the skilled population. Also included are graphs containing log of average per capita income, log of average skilled per capita income, log of unskilled income.

Figure 1 shows the evolution of the share of total world population that is skilled under the four different scenarios. This is the same inormation that was summarized for forty-year intervals in Table 2. When   = .55, this share initially decreases, but then rises rapidly, the economy converging to a sitiuation where all workers are skilled. Recall that under this scenario, unskilled human capital is not growing over time. When combined with the fact that unskilled labor is non-essential to production, we arrive at the conclusion that one day in the future, all workers will be skilled. Under the scenario   = .25, however, the skilled share of the world's population approaches a value near .5. Under the other two scenerios, the skilled share of world population approaches zero. When   = -.5, this variable first increases, then decreases monotonically. When   = -1.70, there is some flucutation during certain time periods, but a monotonic decline for approximately 1500 years. As can been seen in Figure 6, the periods of fluctuation and monotonicity in the graph in Figure 1 coincide with fluctuations in the rate of transition from skilled to unskilled (e.g., the fraction of children of unskilled parents who receive skilled human capital investment. Thus, these are periods where labor-force fluctuations brought on by parental decisions cause fluctuations in other per-capita magnitutes.

Figure 2 plots the skilled share of world GDP, extending the information in Table 3. Here, the distinction between the cases of   > 0 and   < 0 are striking. In the former case, the share of output produced by skilled workers monotonically approaches one, although at a slightly lower rate with the lower value of .
When   < 0, on the other hand, skilled workers eventually account for all of the output produced. When   = .05, the decline in the skilled share of output is monotonic, whereas when   = -1.70, this share increases for two generations, falls monotonically for almost 3000 years, then oscillates as it converges to zero. As we will notice below, these oscillations are due to those in the share of unskilled children who receive skilled human capital.

Figure 3 plots the natural log of average per capita income. The anomolous result that log per capita incomes are higher when   = .25 compared with log per capita incomes when   = .55 arises because the population under the first case is much larger than the population in the second case. The average level of skilled human capital is lower in the former case, but this is compensated for by a more rapidly growing skilled population. In both of these cases, growth is purely endogenous, and in the limit, the growth of unskilled and average earnings is due to growth in the stock of skilled human capital. When < 0, in contast, growth is due to a combination of this effect and exogenous growth in unskilled human capital. In fact, without the latter, the economy would approach a stationary state. We can see from the figure that the values of the parameters that were chosen in order to match certain features of the world economy at certain dates imply that the cases with endogenous growth result in higher per capita incomes, if only from 2040 onwards when = .25 (see Table 1).

Figure 4 shows the natural log of average skilled per capita income. Since the output share of skilled workers approaches unity so rapidly when > 0, it should come as no surprise that the series for these cases should mimic the series for overall average income (Figure 3). When < 0, the series eventually oscillate as they increase. These oscillations do not appear in Figure 3 because the skilled share of output is negligable (Figure 1). Figure 4 plots this series for unskilled workers. Note that when = .55, the series "disappears" around 3800. This is due to the fact that there are no more skilled workers in the economy, their measure having converged to zero. Here, too, unskilled workers seem to make out better when > 0 than when < 0 and unskilled human capital grows exogenously.

Figure 6 shows the fraction of children from unskilled parents who receive skilled human capital investment. In effect, it is a child's probability of upward mobility. When > 0, this probablity remains positive, converging to one when = .55 and seemingly converging to approximately .23 when = .25. When < 0, the probablity converges to zero. The case of = .55 is perhaps most interesting. Here, in the initial generations, there is no mobility. Then, the value jumps to almost .75 within two generations, declines steadily and converges to approximately .55. Then, it jumps discontinuously to one and remains there for the duration. In this case, the industrial revolution is also a mobility revolution, and as the economy matures, mobility levels off. The presence of the second mobility revolution predicted by the model is intruiging. Once again, the series oscillates for certain time periods when = -1.70. These oscillations are due to fluctuations in the returns to skilled human capital, as can be seen from Figure 4.

Figure 7 shows a measure of inequality of income, the coefficient of variation of per capita income. In two of the cases, a Kuznets-curve pattern emerges, with incquality initially increasing, then converging to zero. This occurs both when = .55 and when = -.50. When = -1.70, a similar pattern emerges, albeit with higher inequality embroidered with the fluctuations that have become familiar for this case by now. For the cases with < 0, inequality converges to zero because all the world's workers are eventually unskilled (see Figure), and unskilled agents share equal earnings. When > 0, skilled income has a tendency to converge, (see equation (18)). The effects of this tendency are most apparent when = .55. In this case, we have already seen that the share of the population that is skilled rapidly approaches one. Since the entire population is skilled and skilled incomes converge, inequality approaches zero. When = .25, there continues to be mobility from skilled to unskilled. Even though skilled earnings converge, the different skill levels and earnings of these "new entrants" into the skilled labor force keep imply persistent (and even increasing) income inequality.

Figure 8 plots the coefficient of variation of skilled per capita income. Again, since in the limit, the entire population is skilled when > 0, these series merely mimic those in Figure 7. With < 0, inequality in skilled per capita income increases and eventually begins to fluctuate, these flucutations become progressively more severe when = -.50.

Table 0: Average Years of Schooling and Dispersion of Schooling Exposure By Continent

	Europe		North America & Caribbean		South America		Asia & Oceana		North Africa & Middle East		Africa (exc North)	
	H	CV	H	CV	H	CV	H	CV	H	CV	H	CV
1860	1.23	2.35										
1870	1.79	1.61										
1880	2.19	1.25	7.17	0.16								
1890	2.68	0.96	7.63	0.13								
1900	3.51	0.66	7.91	0.13								
1910	3.87	0.66	6.69	0.74	0.63	3.51	0.64	4.22				
1920	4.38	0.54	7.42	0.38	1.54	2.03	1.19	2.97				
1930	4.32	0.68	8.11	0.29	2.20	1.61	1.61	2.50				
1940	4.99	0.48	8.58	0.39	2.01	1.59	6.42	0.41				
1950	5.39	0.46	9.10	0.39	2.94	1.11	2.64	1.81	1.16	2.64		
1960	5.99	0.43	9.73	0.33	3.41	0.95	3.31	1.32	1.88	1.72		
1970	6.76	0.40	10.28	0.34	4.58	0.66	4.38	0.93	3.19	1.05	3.83	0.85
1980	8.22	0.39	11.13	0.29	5.81	0.53	5.44	0.70	5.12	0.66	4.25	0.67
1990	9.45	0.38	11.78	0.29	7.65	0.35	6.46	0.56	6.73	0.45	5.64	0.47
2000	10.43	0.35	12.28	0.27	8.49	0.30	7.11	0.48	7.86	0.39	5.73	0.49

H – average years of schooling

CV – coefficient of variation in male exposure to primary, secondary, and tertiary education.

Source: Baier, Dwyer, and Tamura (1999) and

Table 1a: Average Income, Average Skilled Income and Unskilled Income (in 1990 dollars)
Skilled, Unskilled and Total Population (in millions)
= .55, = 2.16026901, = 1, φ = .35, κ = 1.25e-14

Year	Average Income	Average Skilled Income	Unskilled Income	Skilled Population	Unskilled Population	Total Population
1800	2577	5736	2126	80	712	792
1840	2814	9733	2168	82	1100	1180
1880	3058	16717	2221	84	1730	1820
1920	3369	29560	2299	91	2810	2900
1960	3830	55114	2422	919	3580	4500
2000	6070	11490	3504	3520	2510	6030
2040	57077	67376	16550	6000	931	6930
2080	949269	1015316	93624	7010	380	7400
2120	8037638	8292725	351305	7510	187	7700
2160	3.78e+07	3.85e+07	944836	7830	104	7930
2200	1.23e+08	1.24e+08	2063004	8080	62.4	8140

Table 1b: Average Skilled and Unskilled Earnings and Population
= .25, = 1.75926901, = 1, φ = .35, κ = 5.85e-10

Year	Average Income	Avg Skilled Income	Unskilled Income	Skilled Population	Unskilled Population	Total Population
1800	551	2250	388	246	880	1126
1840	847	1232	668	412	1116	1528
1880	1452	3188	918	484	1636	2120
1920	2297	5821	1280	674	2314	2988
1960	3729	10128	1848	978	3278	4252
2000	6366	18286	2772	1484	4564	6040
2040	10814	30056	4221	2292	6340	8640
2080	19753	54446	6751	3588	9040	12640
2120	38422	104067	11347	5900	13220	19120
2160	78650	203419	20008	10140	19840	29980

2200	170838	419718	37028	17460	30600	48000

Table 1c: Average Income, Average Skilled Income and Unskilled Income (in 1990 dollars)
Skilled, Unskilled and Total Population (in millions)
= -.5, = 1.9, = 1.7448301, φ = .159, κ = 3.5e-6

Year	Average Income	Avg Skilled Income	Unskilled Income	Skilled Population	Unskilled Population	Total Population
1800	3	489	.2	162	708	870
1840	241	247	238	321	860	1180
1880	572	629	554	330	1330	1660
1920	1132	1606	1033	352	2100	2450
1960	2085	4076	1800	404	3370	3780
2000	3704	9702	3071	661	5470	6130
2040	6709	13222	5861	1180	9630	10800
2080	12599	24239	11228	2020	18800	20900
2120	23756	45306	21551	3520	38700	42300
2160	44557	87037	40938	6080	80900	87000
2200	82817	166415	76933	10600	170000	181000

Table 1d: Average Income, Average Skilled Income and Unskilled Income (in 1990 dollars)
Skilled, Unskilled and Total Population (in millions)
= -1.7, = 1.1, = 1.8788301, φ = .25, κ = 2.5e-3

Year	Average Income	Avg Skilled Income	Unskilled Income	Skilled Population	Unskilled Population	Total Population
1800	151	794	77	60.2	659	719
1840	215	2242	62	61.5	1020	1080
1880	288	5117	44	256	1310	1570
1920	425	1494	80	474	1830	2300
1960	887	4186	160	519	3080	3600
2000	2045	13709	433	546	5560	6110
2040	3607	42916	591	2030	8600	10600
2080	5500	17799	1169	4500	14100	18600
2120	10892	39583	2278	5600	29000	34600

2160	23730	157633	4513	10200	56400	66700
2200	43379	213837	9004	24400	102000	126000

Table 2: Relative Importance of Skilled and Unskilled

	= .55		= .25		= -.50		= -1.7	
Year	share of pop.	share of GDP	share of pop.	share of GDP	share of pop.	share of GDP	share of pop.	share of GDP
1800	.125	.278	.083	.351	.005	.920	.103	.544
1840	.0854	.295	.322	.462	.320	.328	.070	.731
1880	.0578	.316	.239	.517	.237	.261	.048	.855
1920	.039	.344	.221	.567	.172	.244	.244	.858
1960	.027	.385	.227	.617	.125	.244	.181	.852
2000	.321	.608	.232	.665	.095	.250	.121	.814
2040	.797	.941	.255	.709	.115	.227	.071	.848
2080	.928	.993	.272	.751	.105	.203	.260	.843
2120	.968	.999	.292	.791	.093	.177	.231	.839
2160	.983	1	.319	.827	.078	.153	.125	.834
2200	.990	1	.349	.859	.066	.132	.168	.827
2600	1	1	.436	.987	.010	.025	.132	.764
3000	1	1	.472	.999	.001	.004	.099	.683
3400	1	1	.490	1	0	.001	.069	.584
3800	1	1	.500	1	0	0	.047	.476
4200	1	1	.504	1	0	0	.030	.367
4600	1	1	.506	1	0	0	.054	.263
5000	1	1	.507	1	0	0	.008	.159
7000	1	1	.502	1	0	0	.004	.022

Table 3a: Average Education, Life Expectancy and Total Fertility Rates
= .55, = 2.16026901, = 1, φ = .35, κ = 1.25e-14

Year	education + 6	skilled life exp.	unskilled life exp.	life exp.	skilled TFR	unskilled TFR	average TFR
1800	11.25	44.11	48.95	48.35	4.36	5.45	5.36
1840	10.85	44.15	49.63	49.16	4.27	5.43	5.36
1880	10.58	45.29	50.10	49.82	4.01	5.38	5.33
1920	10.39	48.60	51.15	51.05	3.39	5.28	5.23
1960	10.27	57.47	53.51	53.61	2.40	4.60	4.54
2000	11.37	73.60	56.80	57.12	2.04	3.45	2.67
2040	16.05	79.96	64.92	72.29	2.04	2.86	2.11
2080	19.98	80	76.70	79.62	2.04	2.82	2.07
2120	20.88	80	79.95	80	2.04	2.91	2.06
2160	21.17	80	80	80	2.04	2.97	2.05
2200	21.30	80	80	80	2.04	3.00	2.05
2600	21.46	80	80	80	2.04	3.05	2.04
3000	21.46	80	80	80	2.04	3.05	2.04
3400	21.46	80	80	80	2.04	3.05	2.04
3800	21.46	80	80	80	2.04	2.04	2.04
4200	21.46	80	80	80	2.04	2.04	2.04
4600	21.46	80	80	80	2.04	2.04	2.04
5000	21.46	80	80	80	2.04	2.04	2.04
7000	21.46	80	80	80	2.04	2.04	2.04

Table 3b: Average Education, Life Expectancy and Total Fertility Rates

= .25, = 1.75926901, = 1, φ = .35, κ = 5.85e-10

Year	education + 6	skilled life exp.	unskilled life exp.	life exp.	skilled TFR	unskilled TFR	average TFR
1800	10.83	44.21	47.30	46.96	4.37	5.11	5.05
1840	11.01	43.98	47.76	47.52	4.31	5.45	5.22
1880	12.02	44.78	49.55	48.43	4.12	5.37	5.18
1920	10.68	47.14	50.71	50.31	3.64	5.34	5.07
1960	10.87	53.73	50.96	51.29	2.71	5.30	4.87
2000	11.39	68.24	51.61	53.71	2.06	5.20	4.67
2040	12.04	79.58	52.58	55.84	2.04	5.09	4.51
2080	12.29	80	54.79	58.10	2.04	4.89	4.31
2120	12.48	80	58.54	61.45	2.04	4.60	4.04
2160	12.72	80	64.32	66.51	2.04	4.26	3.73
2200	13.06	80	71.65	72.87	2.04	4.01	3.50
2600	14.18	80	80	80	2.04	3.88	3.25
3000	14.59	80	80	80	2.04	3.85	3.16
3400	14.80	80	80	80	2.04	3.84	3.12
3800	14.91	80	80	80	2.04	3.83	3.10
4200	14.96	80	80	80	2.04	3.83	3.09
4600	14.98	80	80	80	2.04	3.83	3.09
5000	14.99	80	80	80	2.04	3.83	3.08
7000	14.94	80	80	80	2.04	3.83	3.09

Table 3c: Average Education, Life Expectancy and Total Fertility Rates

= -.5, = 1.9, = 1.7448301, $\varphi = .159$, $\kappa = 3.5e\text{-}6$

Year	education + 6	skilled life exp.	unskilled life exp.	life exp.	skilled TFR	unskilled TFR	average TFR
1800	10.05	43.64	46.87	46.84	4.37	5.06	5.06
1840	11.00	43.98	47.52	47.50	4.31	5.45	5.18
1880	12.35	44.78	49.64	48.31	4.12	5.43	5.21
1920	11.70	47.14	50.12	49.21	3.64	5.38	5.17
1960	11.24	53.73	51.20	50.70	2.71	5.27	5.04
2000	10.69	68.24	54.82	55.63	2.06	4.97	4.79
2040	10.87	79.57	59.66	60.41	2.04	4.65	4.47
2080	10.92	80	68.94	69.35	2.04	4.35	4.22
2120	10.82	80	77.76	77.82	2.04	4.3	4.19
2160	10.70	80	79.96	79.96	2.04	4.31	4.22
2200	10.59	80	80	80	2.04	4.32	4.25
2600	10.09	80	80	80	2.04	4.37	4.36
3000	10.01	80	80	80	2.04	4.38	4.38
3400	10	80	80	80	2.04	4.38	4.38
3800	10	80	80	80	2.04	4.38	4.38
4200	10	80	80	80	2.04	4.38	4.38
4600	10	80	80	80	2.04	4.38	4.38
5000	10	80	80	80	2.04	4.38	4.38
7000	10	80	80	80	2.04	4.38	4.38

Table 3d: Average Education, Life Expectancy and Total Fertility Rates

= -1.7, = 1.1, = 1.8788301, φ = .25, κ = 2.5e-3

Year	education + 6	skilled life exp.	unskilled life exp.	life exp.	skilled TFR	unskilled TFR	average TFR
1800	11.03	43.53	48.56	48.04	4.37	5.46	5.38
1840	10.70	43.98	49.38	49.00	4.15	5.44	5.38
1880	10.48	46.84	49.87	49.73	3.70	5.11	5.04
1920	10.83	52.92	49.61	49.73	3.07	5.31	4.95
1960	11.58	62.43	52.94	52.94	2.43	5.16	4.87
2000	11.05	73.04	56.64	57.25	2.10	4.93	4.76
2040	10.61	78.99	62.68	63.32	2.04	4.22	4.12
2080	11.52	79.97	70.12	70.39	2.04	4.26	3.90
2120	12.12	80	76.89	77.17	2.04	4.39	4.10
2160	11.40	80	79.58	79.61	2.04	4.12	3.97
2200	11.32	80	79.99	79.99	2.04	4.08	3.87
2600	11.15	80	80	80	2.04	4.21	4.05
3000	10.85	80	80	80	2.04	4.27	4.14
3400	10.59	80	80	80	2.04	4.30	4.22
3800	10.40	80	80	80	2.04	4.33	4.27
4200	10.26	80	80	80	2.04	4.34	4.30
4600	10.32	80	80	80	2.04	4.38	4.32
5000	10.08	80	80	80	2.04	4.37	4.36
7000	10.02	80	80	80	2.04	4.38	4.37

Table 4: Historical Education Exposure

Year	no education	some primary schooling	some secondary schooling	some higher education	average age at end of schooling
1850	.725	.271	.003	.001	6.86
1860	.677	.317	.005	.001	7.02
1870	.572	.419	.007	.001	7.34
1880	.510	.478	.011	.002	7.57
1890	.444	.533	.020	.003	7.84
1900	.297	.665	.033	.005	8.40
1910	.586	.384	.027	.004	8.02
1920	.512	.437	.046	.005	7.85
1930	.484	.439	.069	.008	8.13
1940	.201	.626	.154	.018	9.69
1950	.407	.439	.137	.016	8.93
1960	.374	.419	.186	.022	9.45
1970	.297	.420	.248	.035	10.26
1980	.231	.387	.303	.079	11.38
1990	.144	.400	.335	.121	12.36
2000	.099	.399	.362	.140	12.92

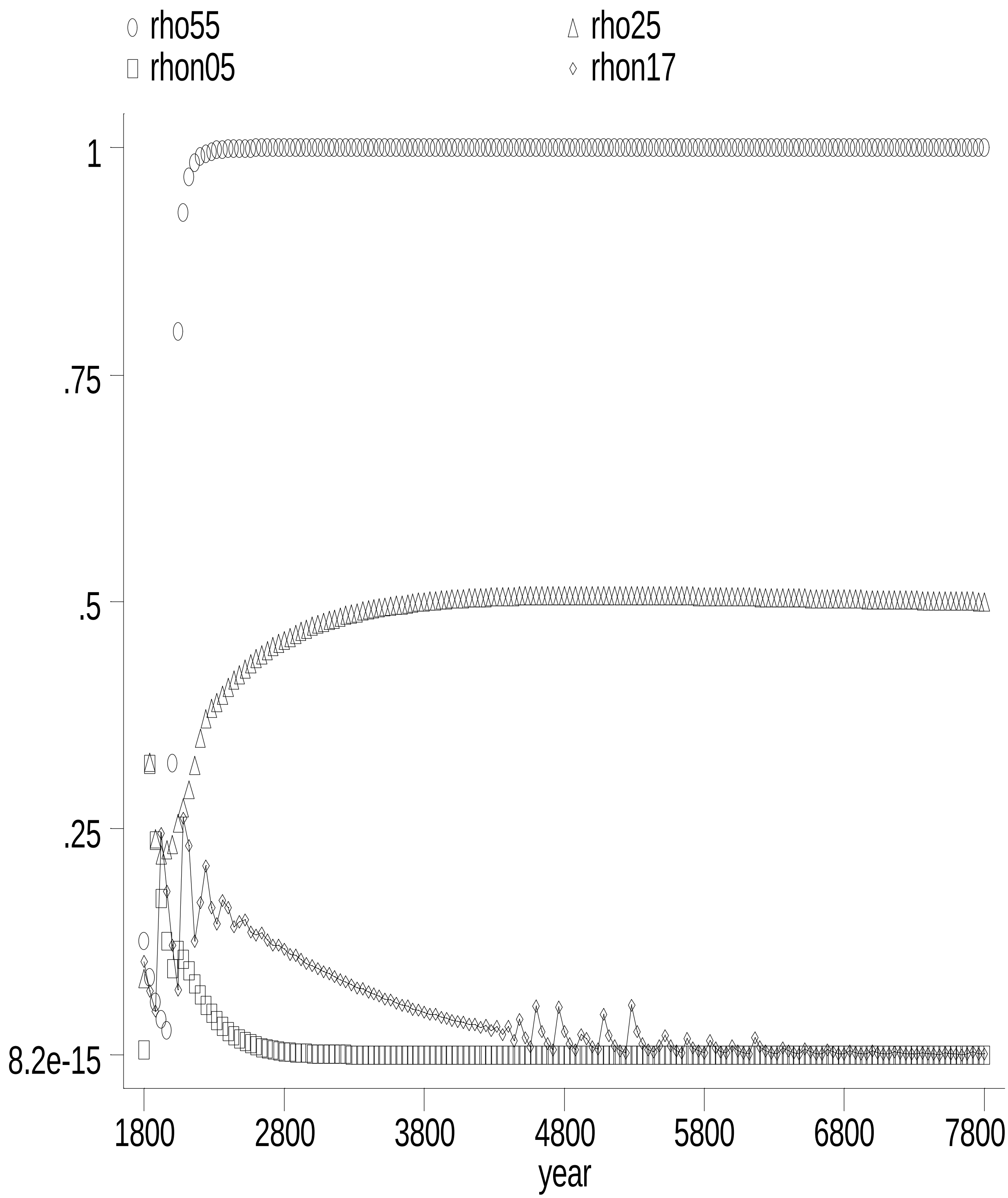

Figure 1: Skilled share of the world population   = .55,   = .25,   = -.50,   = -1.70

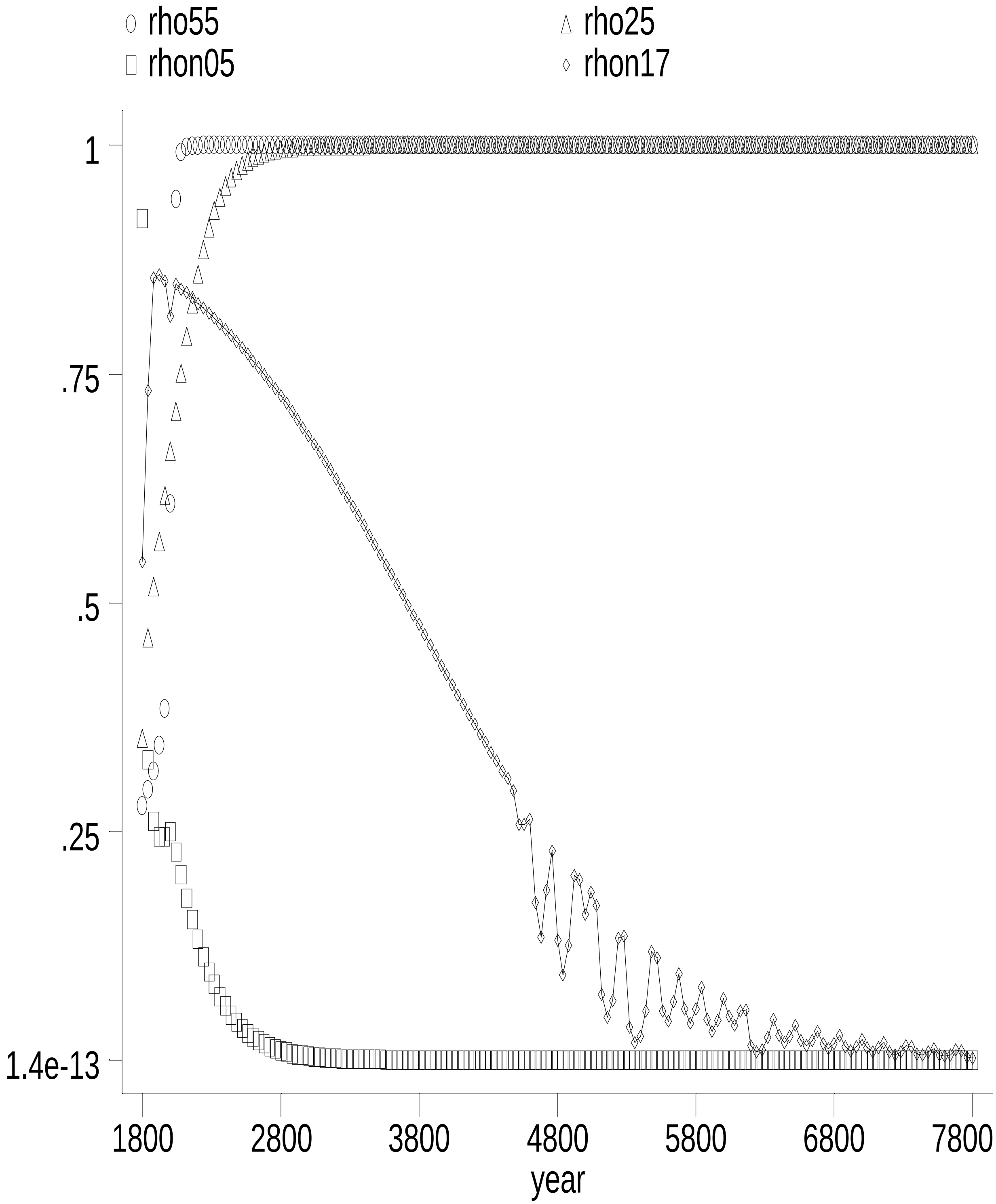

Figure 2: Skilled share of world GDP, = .55, = .25, = -.50, = -1.70

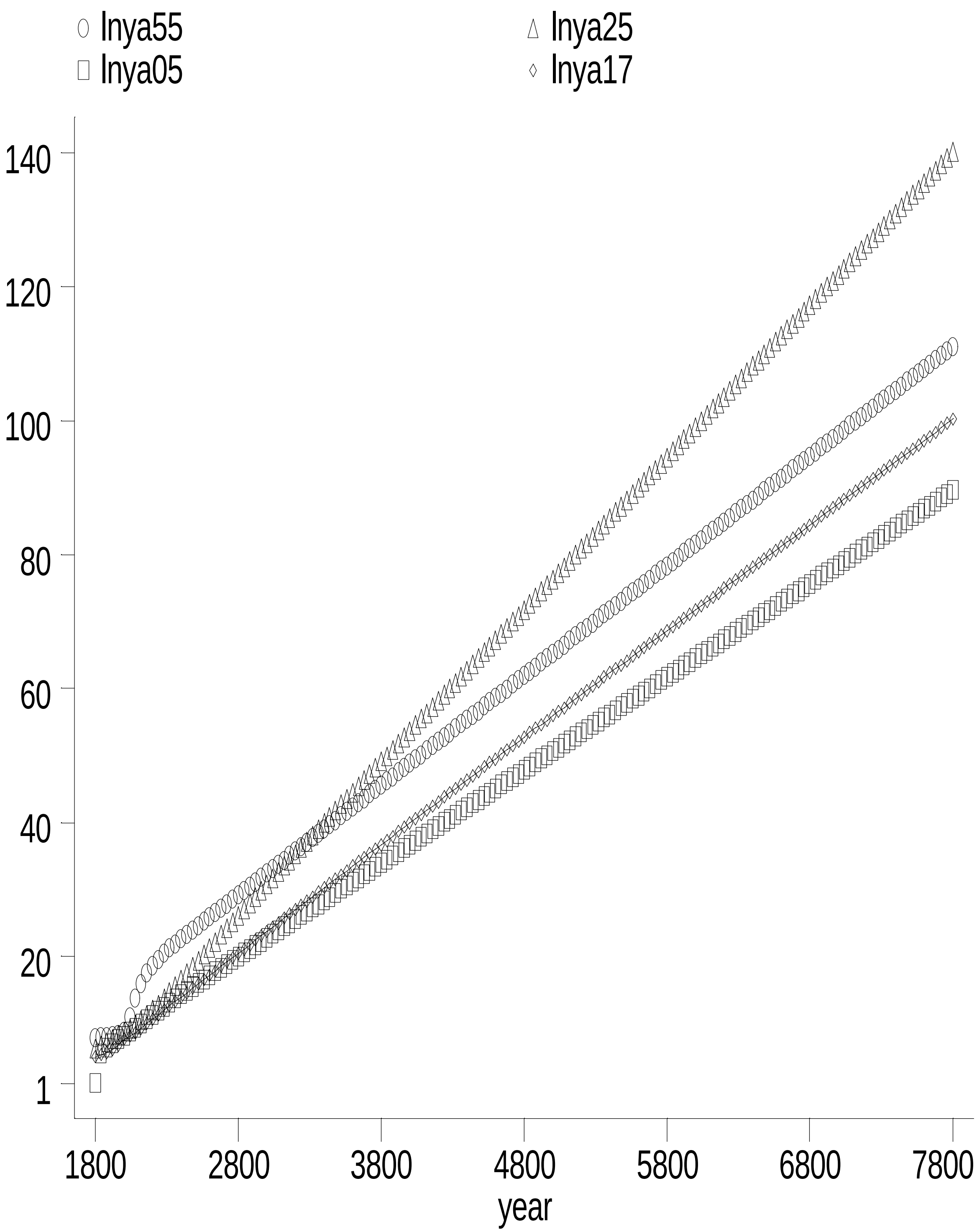

Figure 3: Time series of log average per capita income, = .55, = .25, = -.50, = -1.70

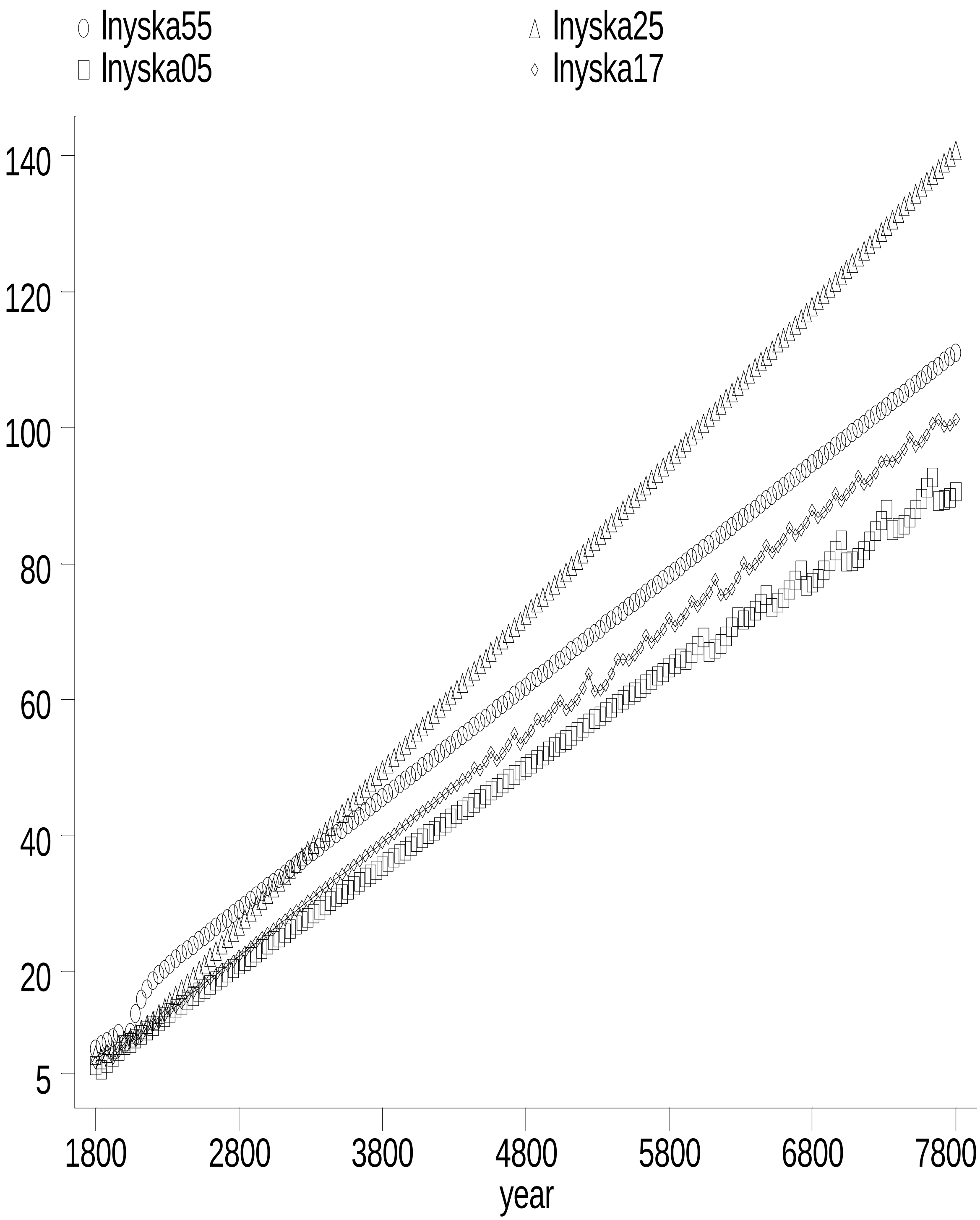

Figure 4: Time series of log average skilled per capita income, = .55, = .25, = -.50, = -1.70

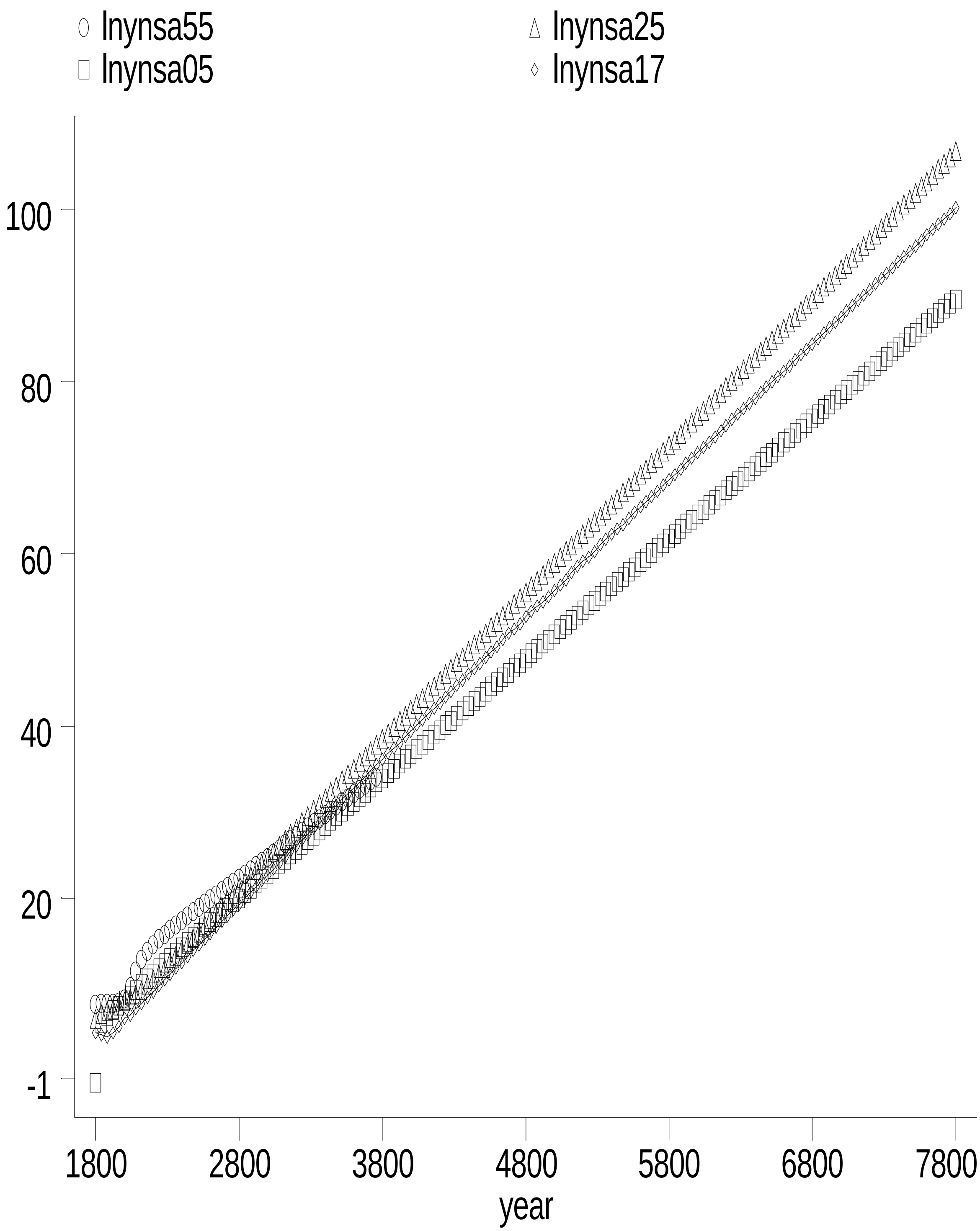

Figure 5: Time series of log of unskilled per capita income, = .55, = .25, = -.50, = -1.70

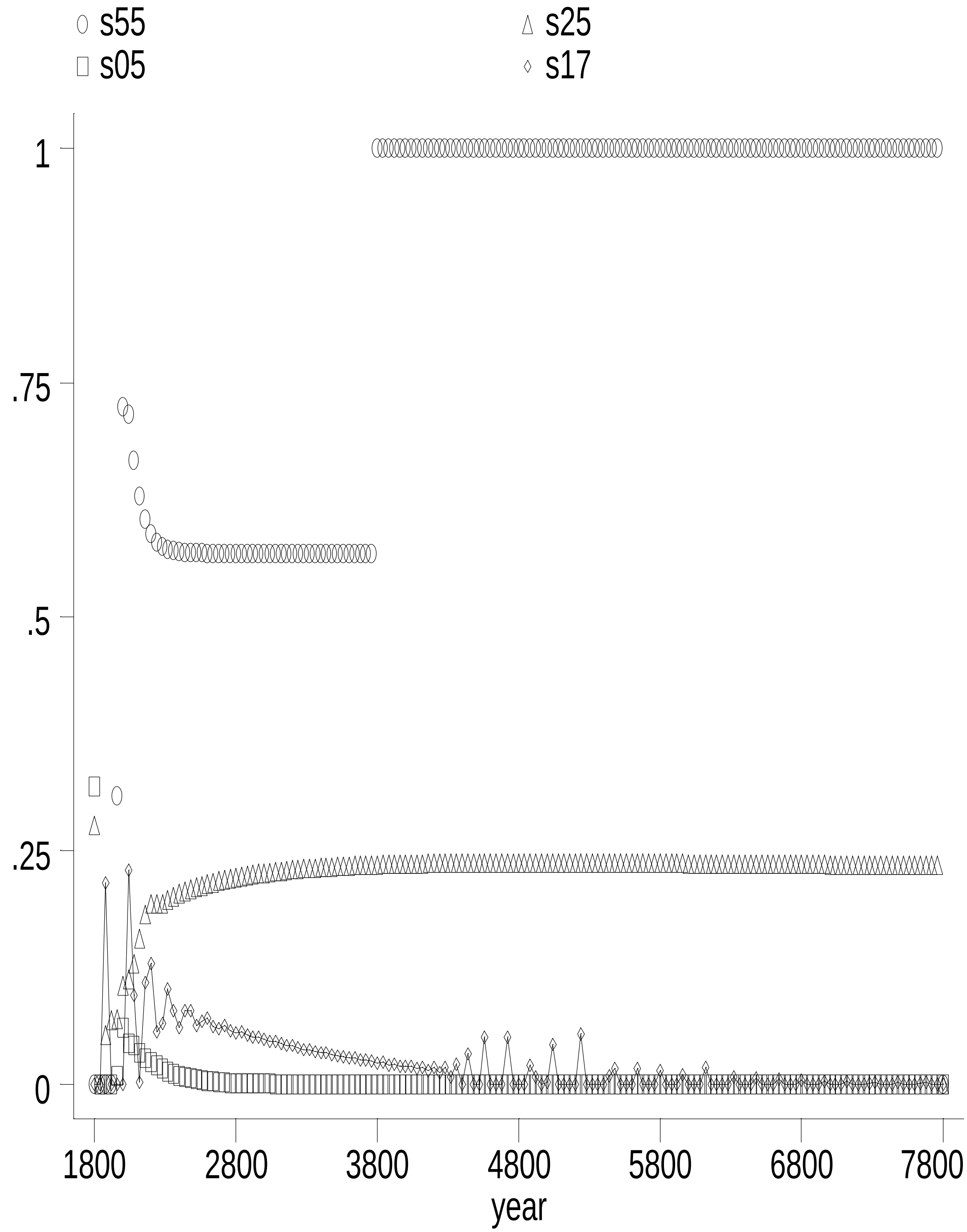

Figure 6: Fraction of children from unskilled parents receiving skilled human capital investments
= .55, = .25, = -.50, = -1.70

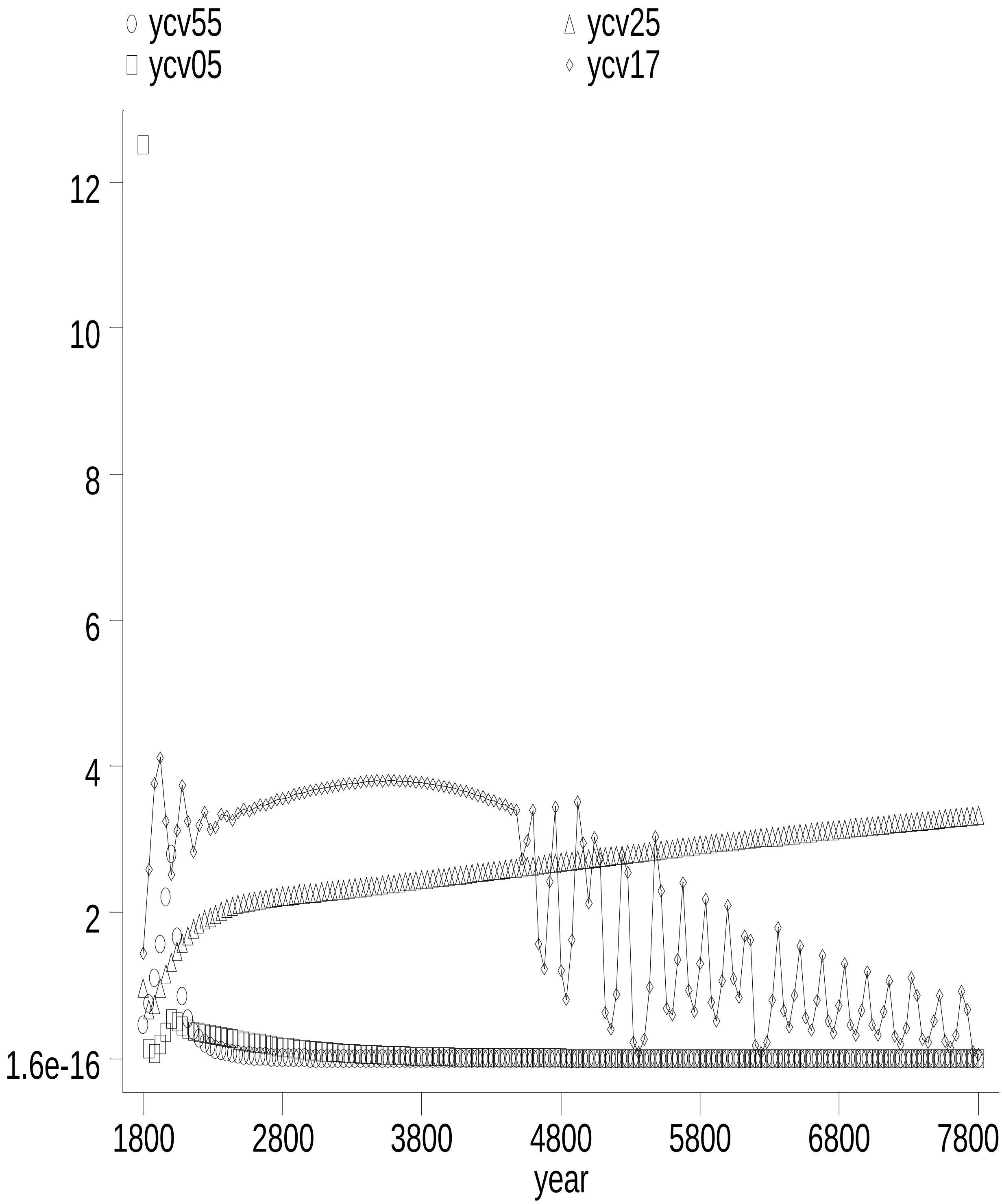

Figure 7: Time series of coefficient of variation of per capita income, = .55, = .25, = -.50, = -1.70

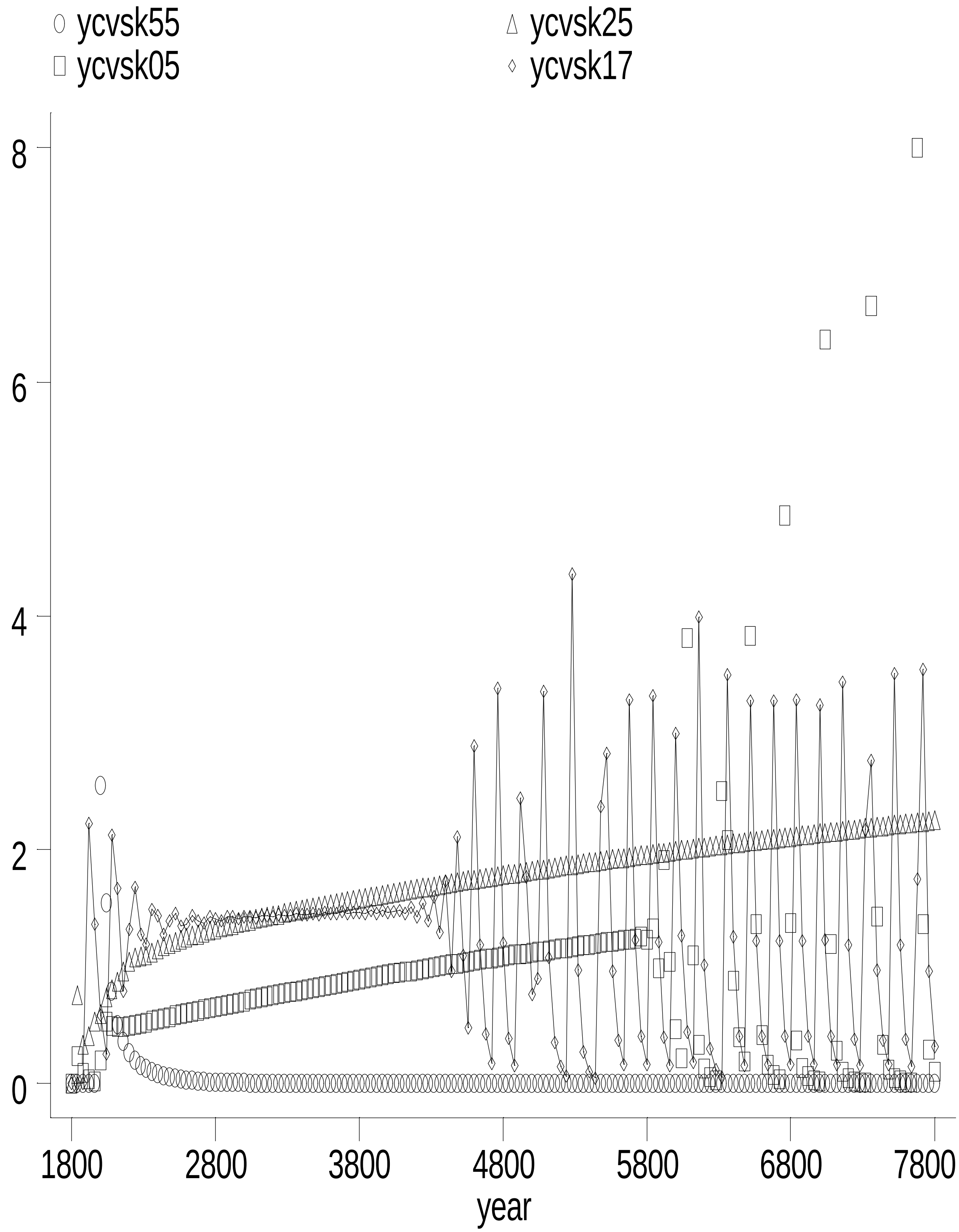

Figure 8: Time series of coefficient of variation of skilled per capita income,
= .55, = .25, = -.50, = -1.70

Appendix

In this appendix we present the sufficient conditions that simplify the skilled parent's problem. Suppose that all unskilled parents raise skilled children. Since a skilled parent has higher productivity in human capital investment than an unskilled parent, clearly his or her children will earn more than the newly skilled children from an unskilled parent. Therefore if all unskilled parents invest in all of their children, then all skilled parents invest in all of their children.

Now consider the case where only a fraction of children of unskilled parents receive human capital investments. The unskilled parents fertility and human capital investment time are:

$$
\begin{aligned}
\frac{a\left(q + s_t t_t\right)}{1 - b_t\left(q + s_t t_t\right)} &= \frac{g}{b_t - d_t} \\
\frac{a b_t s_t}{1 - b_t\left(q + s_t t_t\right)} &= \frac{bms_t}{t_t} \\
\frac{a b_t t_t}{1 - b_t\left(q + s_t t_t\right)} &= b\left\{\ln y_{t+1}^{s} - \ln y_{t+1}^{u}\right\}
\end{aligned}
\qquad \textbf{(A1)}
$$

The first two equations can be solved in terms of the share of children receiving skilled human capital investments. The resulting functions are:

$$
\begin{aligned}
b_t &= \frac{g}{(a+g)\left(q + s_t t_t\right)} + \frac{a d_t}{a+g} \\
t_t &= \frac{-\mathrm{B}_t + \sqrt{\mathrm{B}_t^2 - 4\mathrm{A}_t C_t}}{2\mathrm{A}_t} \\
\mathrm{A}_t &= \left(a + bms_t\right) d_t^u s_t \\
\mathrm{B}_t &= g + a d_t^u q - bms_t + 2bmd_t^u q s_t \\
C_t &= bmq\left(d_t^u q - 1\right)
\end{aligned}
\qquad \textbf{(A2)}
$$

Substituting in for n_t $_t$ into the final equation in (A2) and simplifying produces:

$$-\ m + \left[\ln y^{s}_{t+1} - \ln y^{u}_{t+1}\right] \geq 0 \qquad \textbf{(A3)}$$

where if (A3) holds as an equality, then there is an interior solution for $1 > s_t > 0$. However if (A3) holds as a strict inequality, then $s_{it} = 1$.[13]

Assume that all unskilled parents choose to invest in a fraction of their children, but not all of their children, thus (A3) holds as a strict equality.

Now examine the problem facing a skilled parent. If he or she invests in a fraction of his or her children, the first order conditions are given in (A1). If he or she chooses to invest in all of his or her children, then (A3) holds as a strict inequality. A skilled parent compares the utility from investing in all of his or her children with the utility from investing in only a fraction of his or her children. This produces:

$$\begin{aligned}
&U(s=1) - U(s<1) \\
&= a \ln\left[\frac{1 - b^{s}_{t}\left(q + t^{s}_{t}\right)}{1 - b^{s}_{t}\big|_{s_t<1}\left(q + s_t t^{s}_{t}\big|_{s_t<1}\right)}\right] + g \ln\left[\frac{b^{s}_{t} - d^{s}_{t}}{b^{s}_{t}\big|_{s_t<1} - d^{s}_{t}}\right] \\
&+ b \ln\left[y^{s}_{t+1}\big|_{s=1}\right] - b\left\{s_t \ln\left[y^{s}_{t+1}\big|_{s<1}\right] + \left(1 - s_t\right)\ln y^{u}_{t+1}\right\} \qquad \textbf{(A4)} \\
&> a \ln\left[\frac{1 - b^{s}_{t}\left(q + t^{s}_{t}\right)}{1 - b^{s}_{t} q}\right] + g \ln\left[\frac{b^{s}_{t} - d^{s}_{t}}{b_t\big|_{s_t=0}}\right] + b \ln\left[y^{s}_{t+1}\big|_{s=1}\right] - b \ln y^{u}_{t+1} - bm
\end{aligned}$$

The third line comes from (A3) with equality if a skilled parent invests in only a fraction of his or her children.[14] The fourth line comes from replace s=0 in the denominator of the first term of the third line and s=1 in the third term of the third line. Thus a sufficient condition for skilled parents choosing to invest in all of their children is if the fourth line of (A4) is greater than or equal to 0. The earnings differential between a skilled child of a skilled parent (who chooses only skilled children) and the unskilled can be written as:

[13] The left hand side of (A3) will be strictly less than 0 if s_t=0.

[14] Obviously if the left hand side of (A3) is negative, then the skilled parent will not choose to invest in any of his or her children.

$$\ln\left[\frac{(1-e)K_{t+1}^{r-\frac{1}{w}}\left(A\bar{h}_t^{p}h_{it}^{1-p}\left[\frac{bmq}{g-bm}\right]^{m}\right)^{\frac{1}{w}}}{e\left(N_{t+1}^{r-1}\left[h_{t+1}^{u}\right]^{r}\right)}\right] \qquad \textbf{(A5)}$$

Therefore the condition ensuring that skilled parents only raise skilled children becomes:

$$g\ \ln\left[\frac{g-bm}{g}\right]+b\ln\left[\frac{(1-e)K_{t+1}^{r-\frac{1}{w}}\left(A\bar{h}_t^{p}h_{it}^{1-p}\left[\frac{bmq}{g-bm}\right]^{m}\right)^{\frac{1}{w}}}{e\left(N_{t+1}^{r-1}\left[h_{t+1}^{u}\right]^{r}\right)}\right]-bm>0 \qquad \textbf{(A6)}$$

During the numerical solutions we verify that (A6) holds for each generation.

Existence of Endogenous Growth

In this section of the appendix we examine the condition for endogenous growth. If ≤ 0, then endogenous growth is impossible for all agents. Since unskilled agents are not investing it would take exogenous increases in their productivity to raise their earnings. Now consider the case or > 0. First examine the case where all unskilled agents become skilled. Skilled earnings rise across generations if:

$$\frac{y_{it+1}^{s}}{y_{it}^{s}}=\frac{K_{t+1}^{1-\frac{1}{w}}h_{it+1}^{\frac{1}{w}}}{K_{t}^{1-\frac{1}{w}}h_{it}^{\frac{1}{w}}}>1$$

Given the results in the paper that all skilled agents become identical in their human capital, assume that all skilled agents are identical. Replacing this assumption into () produces:

$$\left(\frac{M_{t+1}}{M_t}\right)^{w-1}\frac{h_{t+1}}{h_t}$$
$$= n^{w-1}At^{\,m}$$
$$= \left(\frac{g - bm}{q\left[a + g\right]}\right)^{w-1}\left(\frac{qbm}{g - bm}\right)^{m} A > 1$$

Thus with population growth, $\frac{g - bm}{q\left(a + g\right)}$ >1, and human capital accumulation $A\left(\frac{qbm}{g - bm}\right)^{m}$ >1,

endogenous growth will occur in a world of skilled agents.

Now consider the case that there always exists unskilled agents. Assume that all unskilled agents invest in some of their children, but not all of their children. Assume that the share receiving investments is constant and equal to s. Assume further that skilled parents increase the level of skilled human capital of their children, and that skilled parents have more than 1 child. These two restrictions are:

$$\frac{h^s_{it+1}}{h^s_{it}} \approx A\left(\frac{qbm}{g - bm}\right)^{m} > 1$$
$$n_t = \frac{g - bm}{q\left[a + g\right]} > 1$$

Earnings growth for skilled and unskilled individuals requires:

$$\frac{y^s_{it+1}}{y^s_{it}} = \left\{ \frac{e\left(N_t(1-s)\dfrac{g - bms}{q\left[a+g\right]}h^u \right)^r + (1-e)K^r_{t+1}}{e\left(N_t h^u\right)^r + (1-e)K^r_t} \right\}^{\frac{1}{r}-1} \left(\frac{K_{t+1}}{K_t}\right)^{r-\frac{1}{w}} \left(\frac{h^s_{it+1}}{h^s_{it}}\right)^{\frac{1}{w}}$$

$$\frac{y^u_{it+1}}{y^u_{it}} = \left\{ \frac{e\left(N_t(1-s)\dfrac{g - bms}{q\left[a+g\right]}h^u \right)^r + (1-e)K^r_{t+1}}{e\left(N_t h^u\right)^r + (1-e)K^r_t} \right\}^{\frac{1}{r}-1} \left(\frac{N_t}{N_t(1-s)\dfrac{g - bms}{q\left[a+g\right]}} \right)^{1-r}$$

Let 8_N and 8_K be the gross growth rates of the unskilled population, N, and the skilled resources, K. Then the first term on the right hand side of each equation in () can be written as:

$$\left\{ \frac{e\left(N_t\, l_N h^u\right)^r + (1-e)\left(l_K K_t\right)^r}{e\left(N_t h^u\right)^r + (1-e)K^r_t} \right\}^{\frac{1}{r}-1}$$

Define =max$\{8_N, 8_K\}$. Then the first term approaches the constant value l^{1-r}. We can rewrite the growth rates of income of skilled and unskilled individuals as:

$$\frac{y_{it+1}^{s}}{y_{it}^{s}} = l^{1-r}\, l_{K}^{r-\frac{1}{w}} \left(\frac{h_{it+1}^{s}}{h_{it}^{s}}\right)^{\frac{1}{w}} > l^{1-r}\, n^{wr-1}\, l_{h}^{r}$$

$$\frac{y_{it+1}^{u}}{y_{it}^{u}} = l^{1-r}\, l_{N}^{r-1}$$

If skilled parents have more than 1 child and increase skilled human capital across the generations, then skilled earnings grow. The second equation is clearly the growth bottleneck. If =max{8_N, 8_K}=8_N, then it is clear that endogenous growth cannot occur for incomes of the unskilled. Thus the restriction for endogenous growth to occur becomes:

$$l_{K} > l_{N} = (1-s)\frac{g - bms}{q[a+g]}$$

The term after the equal sign is decreasing in s, so the upper bound on $_{N}$ is found by letting s approach zero, yielding

$$l_{N} = \frac{g}{q[a+g]}$$

A sufficient condition for endogenous growth is:

$$\left(\frac{g - bm}{q[a+g]}\right)^{w} A\left(\frac{qbm}{g - bm}\right)^{m} > \frac{g}{q[a+g]}$$

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