The Personal-Tax Advantages of Equity*

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Abstract

We compute the value of a firm that pays its cash flows each period through share repurchases in a dynamic environment where personal taxes are paid on realized capital gains and dividends. These results provide a measure of the personal tax advantages of equity financing relative to debt financing, which are often cited as increasing the cost of debt. The initial price of the firm depends on the present value of the taxes paid, which, in turn, depends on the initial price. We solve this valuation problem in closed form in a deterministic setting and numerically in a stochastic setting. We find significant valuation effects from the tax protection afforded by the equity basis. The tax savings are on the order of 40-50% of the taxes paid by the shareholders of firm that distributes cash through dividends, and the cost of capital is reduced by approximately .8 to 1.2 percentage points through the use of repurchases relative to dividends.

1 Introduction

The Modigliani and Miller (1963) model of capital structure choice under taxation still forms the theoretical basis for most pedagogy and practice in modern Finance, despite its obvious empirical and theoretical limitations. This theory predicts a corner solution of all debt financing for all firms, due to the deductibility of interest payments at the corporate level. Such an outcome, however, appears grossly at variance with observed practices and has never been taken seriously as a policy recommendation.

To explain this discrepancy, textbooks have followed researchers in pointing to three considerations that are ignored in the Modigliani and Miller (hereafter, M&M) valuation of the tax shields from debt:

1. Costs of financial distress

2. Redundancy of corporate tax shields

3. Tax advantages to equity at the level of personal taxes.

While considerable research has been devoted to all three of these lines of inquiry, and this research has enriched considerably our qualitative understanding of the tradeoffs involved, it has proved difficult to generate the sort quantitative results that could help researchers evaluate the empirical evidence or help practitioners determine how much debt is too much debt.

With regard to the costs of distress, the M&M model takes as fixed and exogenous the firm's operating policies, assets, and net cash flows. It thus ignores the bankruptcy costs and incentive problems that may distort real choices when the firm is in, or close to, bankruptcy. A great deal of research in the last two decades has been devoted to this issue, and much has been learned about the ways incentive and information problems can influence capital structure. Examples of this work include Jensen and Meckling (1976), Myers (1977), Leland (1998), Parrino and Weisbach (1999) and Moyen (1999). The stylized nature of the models used to capture these tradeoffs, however, makes their quantitative importance difficult to assess. Are these costs sufficiently large to explain the low levels of debt finance firms employ? Or, are they relatively insignificant when compared to the tax benefits of debt, as suggested by the analogy in Miller (1977) to the recipe for "horse and rabbit stew?"

Second, the M&M model surely overstates the tax benefits of debt at the corporate level. The tax shields from interest may, in some states of the world, be redundant, which lowers their expected value.

They may also be risky, which lowers their present value. Models that qualitatively describe the tradeoffs this would impose upon the firm go back to DeAngelo and Masulis (1980), but the dynamic nature of the treatment of tax shields in the tax code has made it difficult to evaluate the quantitative importance of these considerations. Considerable progress has been made on this front recently, using methods based on simulation, in Graham (1996), and especially Graham (1998).

In this paper, we focus on the third factor that investigators have cited in arguing that M&M overstate the tax benefits of debt financing. Debt is tax-disadvantaged at the personal level. Interest payments are taxed as ordinary income. Much of the compensation received by equity holders, on the other hand, comes in the form of capital gains. Miller (1977) argued that this may raise the risk-adjusted cost of debt to the firm, relative to equity, sufficiently to neutralize the tax advantages of debt at the corporate level.

The personal tax advantages of equity are largely attributable to the option to defer capital gains. The value of this option, and thus the cost of equity capital to the firm, depends on the timing of the firm's cash distributions and the way those distributions are split between dividends and share repurchases. In the interest of obtaining tractable expressions, however, researchers who have studied this problem in the past have approached it with static models, making it very difficult to evaluate the quantitative realism or importance of the effects they describe. Miller (1977), for example, simply assumes payments to equity are tax exempt. This is clearly not the case. Even if all cash is distributed through repurchases, investors may have to realize capital gains in order for the distribution to take place. DeAngelo and Masulis (1980) assume a constant, exogenous personal tax rate on equity. Graham (1998) assumes the personal tax rate on equity is a simple linear function of the dividend payout.

Our purpose here is to provide more quantitative guidance as to the magnitude and determinants of the personal tax advantages of equity financing to the firm in a dynamic model. The model follows the original M&M approach of taking as exogenous the firm's pre-tax net operating cash flows. Thus, we abstract from bankruptcy costs and incentive problems. We also ignore the complications at the corporate level studied by Graham (1998). Our model is also partial equilibrium in the sense of taking as exogenous the pricing kernel. Despite these obvious simplifications, this model allows us to pose and answer the following questions: In an environment where the firm can issue debt or equity, and where it can distribute cash through repurchases or dividends, how high is the risk-adjusted cost of equity relative to the risk-adjusted cost of debt? How does this depend on the mix between debt and equity? How does it depend on the firm's dividend policy?

We address these questions, first, in a setting that assumes certain and perpetual cash flows. This allows for closed-form expressions for the firm's value, cost of capital, and the gains to leverage. These expressions can be directly compared to the traditional analyses in M&M, and Miller (1977). The certainty case also illustrates the logic we employ in our numerical analysis of the firm's problem under uncertainty.

Next we describe methods for obtaining numerical solutions to the model when the firm's net cash flows are random, but positive. Using these methods we calculate the value of the firm when all cash flows are distributed through repurchases. Comparisons of this value to the value under full dividend payout provides measures of the effects of the personal tax advantages of equity on the firm's cost of funds.

When net cash flows can be negative, the state space for the valuation problem expands quickly, making it difficult to solve the model numerically. New shares issued to finance negative net cash flows establish new basis values, and the number of shares outstanding at all basis values must be tracked. To evaluate the importance of these effects, we provide approximations by simulating the personal taxes paid assuming the firm's aggregate value evolves exogenously. We then compute the expected present value the taxes paid, and numerically adjust the tax rate in the exogenous value process so that the basis values at which new shares are issued approximately reflects the present value of future personal tax liabilities. For the cases where we have full solutions to the model, this method provides reasonably accurate approximations. We then use this method to evaluate the magnitude of the tax advantages provided by issuing new shares, at higher basis values, as the firm moves through time.

Our results show that the personal-tax advantage of repurchases over dividends or interest payments is substantial. The present value of tax savings are in the range of 40-50 percent of the present value of the personal tax liability that would be incurred using only dividend payouts. The "implicit tax rates" we calculate, rates which yield the same present value for the firm if all distributions were fully taxed, are generally around 60 percent of the tax rate faced by the the firm's shareholders.

To focus directly on the personal-tax consequences of the firm's cash distribution policies, we abstract from a number of considerations that have been important in other research on capital gains taxation.

We ignore liquidity needs or portfolio rebalancing that may lead individual investors to realize gains even if it is disadvantageous from a tax standpoint to do so. The only motive in our model for realizing gains is to facilitate the firm's need to disgorge cash, and the firm repurchases shares at a price that fully compensates the selling shareholder for surrendering the opportunity to continue deferring. In contrast, a large literature in public economics studies the effects taxation based on realization rather than an accrual has on the "effective tax rate" faced by individual investors. (See Poterba (1999) for a recent summary of these results in the context of the measurement of after-tax returns for investors.) For example, Bailey (1969) finds that the option to defer gains reduces their effective taxation by roughly a half, and the opportunity to step up the basis on death reduces it by another half, leading to effective taxation at a rate one-fourth of the statutory rate. This approximation has, in turn, been carried through by many authors, though Balcer and Judd (1987) criticizes this approach. In their dynamic model no constant, proportional, "implicit" rate exists that gives rise to the same investment and consumption behavior as capital gains taxation on a realization basis.

Our results, since they ignore any other motives for realization, can be viewed as estimating the effect of the firm's need to distribute cash on the implicit rate at which capital gains are taxed. To the extent that other motives exist, our results will understate the total tax burden born by equity holders, and hence the advantage of equity repurchases relative to dividends or debt. Similarly, in order to focus purely on the effects of the firm's distribution policies, we assume the rates at which dividends, interest, and capital gains are taxed are identical, and abstract from the distinction between long- and short-term gains. These assumptions will lead us to undervalue the personal-tax benefits of equity financing. We also take the net cash flows of the firm as exogenous. This abstracts from the firm's ability to retain funds to delay taxation at the personal level.

The paper is organized as follows. The next section we evaluate the value the firm and determine its cost of funds under certainty. In Section 3 we provide numerical solutions to the valuation problem under uncertainty. In Section 4 we develop an approximation to the value when there are negative cash flows by simulating the personal tax liability when the dynamics of the firm's value are taken as exogenous. Section 5 provides a brief summary of our conclusions. All proofs are contained in the appendix.

2 The Certainty Case

Before studying environments where closed-form solutions are inaccessible, it is helpful to first understand the simplest cases and examples. We begin, therefore, by evaluating the dynamics of the stock price, number of shares outstanding, firm value, and cost of capital under a certainty model for the firm's net cash flows. We assume these cash flows are constant and perpetual, as in M&M, or that the cash flows grow at a constant rate. We will provide solutions for each of the following cases, in order of complexity.

1. Full, or zero, taxation of all distributions. This covers the cases of full dividend payout, no taxation at the personal level, and taxation of all capital gains whether realized or deferred.

2. Personal taxes paid only when gains are realized.

3. Debt is tax deductible at the corporate level, while personal taxes are paid on capital gains only on realization.

4. Cash flows grow at a constant rate, while personal taxes paid only when gains are realized.

Throughout, we assume that interest, dividends, and realized capital gains are taxed at the same rate at the personal level. For cases 1 and 2, we assume the firm is entirely equity financed. The firm's cost of capital, then, is the cost of equity. The cash flows the firm pays out should be viewed as net cash flow, after corporate taxes. When we consider the third case, we treat corporate taxes explicitly.

The firm can be viewed as simply a sequence of certain, perpetual cash flows, C_t, paid at periodic intervals, indexed by $t = 0, \ldots, \infty$. This cash flow, and the investment and operating activities that produce it, are viewed as exogenous. Thus, we consider only the financial activities of the firm, and ignore the possibility that it may wish to retain funds to shield or defer taxation at the personal level. Again, these simplifying assumptions are in keeping with the benchmark case provided by M&M.

Let the value of the firm at time t be V_t, the price per share be p_t, and the number of shares outstanding n_t. When the firm is initially established, n_0 shares are issued. The firm's cash flows will be paid out either as dividends or through repurchases of the shares initially issued. After this date, as the firm's net cash flows are always positive, no new shares are issued.

We ignore consumption or portfolio rebalancing as motives for trade. We also abstract from the distinction between long-term and short-term capital gains. Further, all investors face the same constant after-tax discount rate, $r > 0$. Thus, investors can be viewed as homogeneous, and will not trade privately in equilibrium as long as the price sequence is increasing through time. It will never benefit an individual shareholder realize a capital gain, pay taxes in the current period, and then reset the basis to shield income in the future. Deferring the gain instead provides a tax-free loan from the government. By similar reasoning, a buyer will not be willing to pay what a seller would demand for her shares, unless the buyer was motivated by some other consideration such as desire for consumption or portfolio rebalancing. Since taxes incurred through a transaction will be reflected in the price at which the transaction is executed, if the taxes represent a net loss to the traders, one of them must find the trade unattractive at any price the other finds acceptable. The next lemma establishes this result formally. It will be the basis for our arguments later that we can focus exclusively on the trades between the firm and its initial shareholders. The proof, which is provided in the Appendix, simply relies on the argument that for any feasible realization strategy, the benefits associated with future tax deductions from the step up of the basis will have a lower present value than the taxes that must be paid now to achieve that increase in the basis.

Lemma 1 *Suppose the current price exceeds the initial price at which the shares were issued, $p_t \geq p_o$, and that all current equity claimants hold the shares with a tax basis of p_o. Then a new shareholder would be willing to buy shares at p_t only if that price is less than or equal to the minimum price an existing shareholder would accept for her shares.*

2.1 Benchmark Cases

The next proposition provides formulas for the firm's value and share price dynamics, when there are no taxes or when taxation is symmetric for capital gains and dividends. In keeping with the M&M model, we view the cash flows as constant: $C_t = C$. The algebra leading to these formulas, is detailed in the Appendix. Let p_t^c denote the cum-dividend price, and let p_t^e be the ex-dividend price.

Proposition 1 *When capital gains are taxed each period, whether realized or deferred, and dividends are taxed as ordinary income, the price per share and number of shares obey the following dynamics:*

1. *When cash flows are paid as dividends, the number of shares, cum-dividend price, and ex-dividend price are constant through time.*

$$p_t^c = \frac{1}{n_0}\left(C(1-\tau_p) + \frac{C(1-\tau_p)}{r}\right), \tag{1}$$

$$p_t^e = \frac{1}{n_0}\frac{C(1-\tau_p)}{r}, \tag{2}$$

   and $n_t = n_0$.

2. *When cash flows are used for repurchases, the price per share grows geometrically and the number of shares shrinks geometrically, at the pre-tax rate of interest,* $r/(1-\tau_p)$.

$$p_{t+1} = p_t \left(1 + \frac{r}{1-\tau_p}\right), \tag{3}$$

$$n_{t+1} = \frac{n_t}{1 + \frac{r}{1-\tau_p}}. \tag{4}$$

3. *In both cases the aggregate, ex-distribution value, denoted* V_t*, is constant though time,*

$$V_t = \frac{C}{\frac{r}{1-\tau_p}}. \tag{5}$$

These formulas include the special case of no taxation when $\tau_p = 0$, in which case the number of shares shrinks, and the price grows, at rate r.

The proposition shows that, when capital gains are taxed whether realized or deferred, the decision to distribute cash by dividend or capital gain has no effect on aggregate firm value or aggregate personal tax liability. The shares actually repurchased in the distribution are only partially taxed, due to protection from the basis. In the M&M setting, however, the aggregate capital gain on all outstanding shares is precisely equal to the amount of the distribution, and hence the aggregate tax liability is the same as it would be under full dividend payout.

2.2 Repurchases when Taxes are Paid On Realized Gains Only

When capital gains are not taxed when earned, but rather when realized, there is a wedge between the value of a share to a purchaser, who establishes a new basis at the current price, and the value to a shareholder who holds the share at a lower basis. In our model, there is no motive to trade other than to distribute cash from the firm. Lemma 1 implies that if the price is increasing, the price demanded by an existing shareholder, with an embedded gain, exceeds the price a new, private buyer would be willing to pay. Thus, the firm issues shares only at its founding, and all shareholders hold the shares with the same basis, p_o. We will assume that the price the firm pays for the shares it repurchases fully compensates the shareholder for the loss of the opportunity to defer. Then Lemma 1 implies that this price does, in fact, exceed what a private buyer would be willing to pay. Note that the one point where a new buyer and a seller would view the shares symmetrically is at the initial date, since selling the shares at p_o triggers no tax liability.

We normalize the number of shares originally issued to $n_0 = 1$. At any point we know that

$$C_t = (n_{t-1} - n_t)\, p_t. \tag{6}$$

since the value of the shares repurchased must equal the cash used to repurchase them. Thus, n_t is the number of shares prevailing between the distribution at time t and the distribution at $t+1$.

If an investor sells a share at time t, she would realize a cash flow of

$$p_t(1-\tau_p) + \tau_p p_o$$

and if she holds and sells next period, she realizes a cash flow of

$$p_{t+1}(1-\tau_p) + \tau_p p_o.$$

Equating the present values of these two alternatives gives us a difference equation for the price.

$$p_t(1-\tau_p) + \tau_p p_o = \left(\frac{1}{1+r}\right)\left((1-\tau_p)p_{t+1} + \tau_p p_o\right). \tag{7}$$

Given a price series satisfying this equation at each point in time, a shareholder is, by recursive argument, indifferent between selling shares back to the firm immediately or deferring the gain to an arbitrary point in the future.

The indifference condition, (7), can be rearranged to give

$$p_t = \left(\frac{1}{1+r}\right)\left(p_{t+1} - r\frac{\tau_p}{1-\tau_p}p_o\right). \tag{8}$$

Solving (8) yields the share price

$$p_t = \theta(1+r)^t - \frac{\tau_p}{1-\tau_p}p_o,$$

where θ and the initial value of the equity, p_o, are coefficients to be determined. Evaluating the stock price at time 0, and solving for θ,

$$\theta = \frac{p_o}{1-\tau_p},$$

and so

$$p_t = \frac{p_o}{1-\tau_p}\left((1+r)^t - \tau_p\right). \tag{9}$$

We can now establish the initial value of the firm, p_o. Since buyers and sellers will view the shares symmetrically at this point, this price must simply be the present value of the aggregate after-tax cash flows to the equity holders. The after-tax cash flows accruing to initial equity holders at time t are the after-tax proceeds from the shares repurchased at that time,

$$\begin{aligned}
(n_{t-1}-n_t)\left(p_t(1-\tau_p)+\tau_p p_o\right) &= C_t(1-\tau_p)+(n_{t-1}-n_t)\,\tau_p p_o \\
&= C_t(1-\tau_p)+(n_{t-1}-n_t)\,\tau_p p_t\frac{p_o}{p_t} \\
&= C_t(1-\tau_p)+C_t\tau_p\frac{p_o}{p_t} \\
&= C_t(1-\tau_p)\left(1+\frac{\tau_p}{(1+r)^t-\tau_p}\right) \\
&= C_t(1-\tau_p)\frac{(1+r)^t}{(1+r)^t-\tau_p}.
\end{aligned} \tag{10}$$

The first line uses the constraint that the value of the shares repurchased must equal the cash distributed, equation (6). The third line uses this constraint again, and the fourth line uses the solution for the price, (9).

The limit, as $t \to \infty$, of the right-hand side of (10) is $C_t(1-\tau_p)$. As time passes, the aggregate value of the protection afforded by the initial basis becomes trivially small. The share price rises at close to a geometric rate as shares are extinguished through repurchases. In the limit, therefore, the distribution is fully taxed.

The derivation to this point allows for any deterministic pattern of positive cash flows over time. When these flows are constant, it is simple to compute the initial share price, p_o, as the present value of after–tax flows

$$\begin{aligned}
p_o &= \sum_{s=1}^{\infty}\frac{1}{(1+r)^s}C(1-\tau_p)\frac{(1+r)^s}{(1+r)^s-\tau_p} \\
&= C(1-\tau_p)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}.
\end{aligned} \tag{11}$$

The following proposition summarizes the solution.

Proposition 2 *Suppose that the personal tax rate is τ_p, the after tax discount factor is r and that the firm distributes C each period by repurchasing shares. Then, the initial value of firm is given by*

$$p_0 = C(1-\tau_p)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}. \tag{12}$$

The price at any point in time is given by

$$p_t = C\left((1+r)^t-\tau_p\right)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}. \tag{13}$$

The number of shares outstanding are given by

$$n_{t-1} = \sum_{s=t}^{\infty}\frac{1}{(1+r)^s-\tau_p}\left(\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}\right)^{-1}. \tag{14}$$

The value of the firm is

$$V_t = C\left((1+r)^t-\tau_p\right)\sum_{s=t}^{\infty}\frac{1}{(1+r)^s-\tau_p} \tag{15}$$

and the value of the firm converges to the no tax value of the firm,

$$\lim_{t\to\infty} V_t = \frac{C}{r}+C.$$

A counterintuitive feature of the solution is that the value of firm converges to the no tax value over time, while the after tax cash flows the shareholders receive converges to $(1-\tau_p)C$, the full tax cash flows. At the equilibrium prices, the investor must be indifferent between selling his shares today and paying taxes on the accumulated capital gains, or holding the shares indefinitely. The higher the current price, the higher the price the investor will require to be indifferent between selling today and holding forever. In the limit, as the accumulated capital gains rise, they must receive a price for their holdings equal to what the shares are worth to an investor in an world without taxes.

The price with repurchases is larger than the initial value under dividends, which is

$$\frac{C(1-\tau_p)}{r} = C(1-\tau_p)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s}. \tag{16}$$

The personal-tax advantage of equity, in this setting, is manifest in the fact that the personal tax rate is subtracted in the "discount factors" in equation (12), but not in (16).

By truncating the infinite sum in (12), these expressions can be compared quantitatively. Figure 1 plots the present value of taxes paid under repurchases, which is $(C/r) - p_o$ as a fraction of the present value of taxes paid under dividends, $\tau_p C/r$, for a firm with cash flow of 100. The value of the firm, p_o is computed with 500 terms in the summation. The figure plots these ratios against the personal tax rate, using three different values for the after-tax discount rate, 3% (+), 6% (*), and 9% (x). In each case, repurchases save the firm's shareholders 40-50% in the present value of their tax liability.

The firm's cost of equity capital is simply the discount rate that equates the initial value given above to the present value of the pre-personal-tax cash flows. That is, the cost of equity, $\hat{r}$, solves

$$p_o = \frac{C}{\hat{r}}. \tag{17}$$

Figure 2 plots this cost of equity capital, assuming pre-tax return of 6%, for various levels of the tax rate from 0 to 40%. The solid line is the cost of equity capital, in this case where $C = 1$ just $\frac{1}{p_o}$. The dotted line is the cost of capital prevailing when distributions are paid as dividends, $r/(1-\tau_p)$.

2.3 Allowing for Growth

Our formulas for the initial value of the firm generalize in a straightforward manner to the case of constant growth in the cash flows. Up to equation (10), the derivation applies to any positive, deterministic series of net cash flows. If these cash flows grow at a constant rate, g, with an initial cash flow at date t=1 of C, we have, as in equation (11),

$$p_o = C(1-\tau_p)\sum_{s=1}^{\infty}\frac{(1+g)^{s-1}}{(1+r)^s - \tau_p}. \tag{18}$$

Table 1 provides measures of the tax advantage to repurchases versus dividends under different growth rates. In the table, the implict tax rate is the rate that equates the present value of the fully taxed cash flows to the initial value of the firm: τ^* solving $p_o = C(1-\tau^*)/(r-g)$. The percent of PV taxes is computed by calculating the present value of taxes paid as a percentage of the present value of taxes paid under full, proportional taxation. The table shows quite clearly that the advantages of repurchases over dividend payments decrease with the rate of growth in the cash flows, despite the fact that the growth in the firm's cash flows is reflected in the present value that determines the initial basis.

	Growth rates		
	0%	2%	4%
Panel A: Tax rate of 28%			
Implicit tax rate (%)	15.98	18.60	23.84
Percent of PV taxes (%)	57.06	66.41	85.16
Panel B: Tax rate of 35%			
Implicit tax rate (%)	20.58	23.75	29.60
Percent of PV taxes (%)	57.06	66.41	85.16

Table 1: **Tax Advantage to Repurchases Under Certainty:** The implicit tax rate is the rate that, under full taxation, would set the present value of after-tax cash flows equal to their calculated value. The percent of PV taxes is the present value of expected taxes paid, as a percentage of the present value of taxes paid under full taxation. The after-tax discount rate is 6%, and the value of the firm is computed using 200 periods.

2.4 Allowing for Debt Financing

To account for debt financing, and quantify the tax advantage of debt net of personal taxes, we must allow for taxation at the corporate level. Thus, we now interpret C as the firm's earnings before interest and taxes, EBIT, as in M&M. The net after-corporate tax operating cash flow is then $C(1-\tau_c)$, where τ_c is the corporate tax rate. We assume interest expense and dividend payments are constant and perpetual. Let $i = i_t$ be the interest paid by the firm in period t, and let $d = d_t$ be the dividend payment. A firm paying interest of i each period will have $(C-i)(1-\tau_c)$ available to distribute to equity holders. This will be divided between dividends and repurchases. The valuation of dividends and interest payments, which are both fully taxed at the personal level, is straightforward. The value of the firm's debt is the present value of the interest payment, net of personal tax, $i(1-\tau_p)/r$. Similarly, the dividends have value $d(1-\tau_p)/r$. The analysis of the previous section, specifically equation (12), gives the value of the perpetual stream of repurchases as a constant multiple of the cash distributed. Thus, the value of the levered firm is:

$$V_L = [(C-i)(1-\tau_c)-d](1-\tau_p)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}+\frac{d(1-\tau_p)}{r}+\frac{i(1-\tau_p)}{r}. \tag{19}$$

This can be written as the value of an unlevered firm, with all cash distributed through repurchases, as in equation (12), plus the net advantages/disadvantages of dividends and debt,

$$\begin{aligned} V_L &= C(1-\tau_c)(1-\tau_p)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p} + d(1-\tau_p)\left(\frac{1}{r}-\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}\right) \\ &\quad + i(1-\tau_p)\left(\frac{1}{r}-(1-\tau_c)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}\right). \end{aligned} \tag{20}$$

This equation is linear in both i and d, so that dividend policy and capital structure policy will be characterized by corner solutions in this environment. Dividends are dominated. The term multiplying $d(1-\tau_p)$ in equation (20) is unambiguously negative for any $\tau_p > 0$. The relative attractiveness of debt versus equity (with repurchases) is determined by the relative magnitudes of r, τ_p, and τ_c. Assuming $d = 0$, we can rewrite the contribution of debt to the value of the firm as

$$\frac{i(1-\tau_p)}{r}\left(1-r(1-\tau_c)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}\right) = D_L\left(1-r(1-\tau_c)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}\right), \tag{21}$$

where D_L is the value of the debt. The term multiplying D_L is the gain to leverage, and can be instructively compared to the gain to leverage in Miller (1977), which treats the equity flows as fully taxed, but at a preferential rate, τ_s,

$$1-\frac{(1-\tau_c)(1-\tau_s)}{1-\tau_p}.$$

In our setting the flows to equity are not taxed at a preferential rate. Rather the personal-tax advantage to equity comes from the present value of the tax shields from the initial basis.

While providing a quantitative benchmark for the personal tax advantages of equity, this analysis will not, of itself, rationalize interior capital structures at realistic values of the parameters. Our intent is, rather, to gain a better understanding of how large other costs of debt, such as losses in aggregate value associated with financial distress, would have to be to generate interior optima. We can rewrite the gain to leverage, per dollar of debt issued, as follows:

$$\begin{aligned} r\left(\frac{1}{r}-(1-\tau_c)\sum_{s=1}^{\infty}\frac{1}{(1+r)^s-\tau_p}\right) &= r\left(\sum_{s=1}^{\infty}\frac{1}{(1+r)^s}-\frac{(1-\tau_c)}{(1+r)^s-\tau_p}\right) \\ &= r\left(\sum_{s=1}^{\infty}\frac{(1+r)^s\tau_c-\tau_p}{(1+r)^s[(1+r)^s-\tau_p]}\right). \end{aligned} \tag{22}$$

The denominator in each term in this summation is positive. Since $(1+r)^s > 1$, the following lemma follows by inspection of the numerator.

Lemma 2 *When $\tau_p \leq (1+r)\tau_c$ the gain to leverage is positive.*

Inspection of (22) suggests, indeed, that the personal tax rate must be substantially higher than the corporate rate to offset the benefits of deductions at the corporate level. Figure 3 provides a quantitative sense of this. The figure plots the corporate tax rate at which the gain to leverage is zero, for different values of the personal tax rate. For a given personal tax rate, at corporate rates above the curve plotted in the figure, the firm would prefer debt financing over equity financing.

Equation (19) for the value of the firm can also be used to evaluate the effects of capital structure and dividend policy on the firm's overall cost of capital. In a manner analogous to M&M, we ask what discount rate sets the present value of future after-corporate-tax, operating cash flow equal to the value of the firm. We solve for $\hat{r}$ in

$$V_L = \frac{C(1-\tau_c)}{\hat{r}} \tag{23}$$

where V_L is computed using (19). Table 2 reports the results of these computations for a firm with $\tau_c = 34\%$, $r = 6\%$, and two values of the personal tax rate, 28% and 35%.

3 Uncertain Cash Flows

When cash flows are certain, in a setting parallel to that envisioned by M&M, we can solve for the value of the firm under repurchases in closed form. The advantage repurchases provide over dividends is attributable, in our setting, entirely to the tax shield supplied by the initial basis. It takes a mathematically simple form as a reduction in the discount factors attributable to after-tax cash flows (see equations (12) and (16)). We found that this advantage to repurchases is quantitatively substantial for reasonable parameter values, 40-50% of the value of the personal tax liability. It was not, however, sufficiently large to completely offset the advantages at the corporate level of debt that, as assumed by M&M, offers full, non-redundant tax shields at the corporate level.

Our purpose in this section is to evaluate whether these implications hold when cash flows are uncertain. We continue to make assumptions analogous to M&M. We view the firm as a perpetual stream of pre-tax

Interest over EBIT	Dividend Payout Ratio 0%	20%	40%	60%	80%	100%
Panel A: $\tau_p = 28\%$						
0%	7.14	7.35	7.57	7.81	8.06	8.33
20%	6.74	6.89	7.04	7.21	7.38	7.55
40%	6.38	6.48	6.58	6.69	6.80	6.91
60%	6.06	6.12	6.18	6.24	6.30	6.37
80%	5.77	5.79	5.82	5.85	5.87	5.90
100%	5.50	5.50	5.50	5.50	5.50	5.50
Panel B: $\tau_p = 35\%$						
0%	7.55	7.84	8.15	8.48	8.84	9.23
20%	7.21	7.41	7.63	7.86	8.11	8.37
40%	6.89	7.03	7.18	7.33	7.49	7.65
60%	6.60	6.69	6.78	6.87	6.96	7.05
80%	6.34	6.38	6.42	6.46	6.50	6.54
100%	6.09	6.09	6.09	6.09	6.09	6.09

Table 2: **Cost of Capital Calculations:** The body of the table reports the percentage return that equates the value of the firm to present value of after-tax operating cash flows. Rows represent different capital structures, parameterized by the percentage of pre-tax income paid in interest. Columns represent different dividend policies, parameterized by the percentage of after-tax cash flow paid in dividends. The corporate tax rate is assumed to be 34%, the personal rate is 28% or 35%, and the after-tax discount rate is 6%.

net cash flows that are constant in expectation. Thus, when cash flows are paid as dividends, or when all capital gains and losses are taxed symmetrically, whether realized or not, the value of the firm, denoted V_F, is given by

$$V_F = \frac{\hat{C}(1-\tau_p)}{r}$$

where $\hat{C}$ is the expected cash flow under the risk-neutral measure. Let C_t^s denote cash flow in state s at date t. For simplicity, we take the state space to be finite and discrete, and assume net cash flows are independent and identically distributed through time. Denote the risk-neutral probability of cash flow C_t^s as π^s.

When net cash flows are always positive, we solve numerically for the value of the firm. With positive net cash flows, as under certainty, shares are only issued once, and so the basis is the same for all outstanding shares. If the price always rises through time, as shares are retired, then this basis will always be the initial price. The only state variables on which prices will depend are the current cash flow, C_t, and the number of shares outstanding, n_{t-1}. For the no-tax case, it is simple to verify analytically that the price always rises when net cash flows are positive.[1] While we have not found a proof for the general case, we can verify numerically that along the worst sample paths, where the lowest (but still positive) cash flow is realized, the price increases through time with the repurchases.

While the decision rules for the firm are mechanical in this model, the share price must be determined endogenously. The price a shareholder will demand when the firm offers to repurchase shares will depend on the shareholder's basis, and on the present value of future taxes paid if she decides not to sell her shares back to the firm, which in turn will depend on the future price path. Suppose at date t we enter the period with n_{t-1} shares outstanding. If we know the price next period will be set so that a shareholder with basis p_o will be just indifferent between continuing to hold the shares and selling into the repurchase, then the price the shareholder would demand in the current period, p_t, must satisfy:

$$p_t(1-\tau_p)+\tau_p p_o = \frac{1}{1+r}\left(E_t[p_{t+1}](1-\tau_p)+\tau_p p_o\right). \tag{24}$$

In addition, all cash must be distributed through the repurchase

$$(n_{t-1}-n_t)p_t = C_t. \tag{25}$$

Finally, as there is no distribution at the initial date, $t=0$, the initial price must be set to satisfy

$$p_o = \frac{1}{1+r}\left(E_0[p_1](1-\tau_p)+\tau_p p_o\right) \tag{26}$$

[1] Let n_{t-1} be the number of shares outstanding upon entering period t. The price at date t will satisfy the equalities:

$$p_t = \frac{1}{n_{t-1}}\left(C_t + \sum_{s=1}^{\infty} E_t\left[\frac{C_{t+s}}{(1+r)^s}\right]\right) = \frac{1}{n_t}\left(\sum_{s=1}^{\infty} E_t\left[\frac{C_{t+s}}{(1+r)^s}\right]\right).$$

Therefore,

$$\begin{aligned} p_{t+1}-p_t &= \frac{1}{n_t}\left(C_{t+1} + \sum_{s=1}^{\infty} E_{t+1}\left[\frac{C_{t+1+s}}{(1+r)^s}\right] - \sum_{s=1}^{\infty} E_t\left[\frac{C_{t+s}}{(1+r)^s}\right]\right) \\ &= \frac{1}{n_t}\left(C_{t+1}+\frac{\hat{C}}{r}-\frac{\hat{C}}{r}\right) \\ &= \frac{1}{n_t}C_{t+1}. \end{aligned}$$

or

$$p_o = \frac{E_0[p_1](1-\tau_p)}{1+r-\tau_p}. \tag{27}$$

These three equations, (24), (25), and (27), must be solved to determine the value of the firm, and the paths of the price and the number of shares.

As the firm proceeds through time, the number of shares is shrinking and the price per share is growing, both at roughly geometric rates. Therefore, it is more straightforward numerically to deal with analogous equations for the value of the firm. Examining the value also provides some insight into the dynamics of the tax liability and its impact on prices.

On entering period t, the firm knows the number of shares outstanding, n_{t-1}, and the initial basis, p_o. Once the random cash flow is known, p_t and n_t are determined. We will derive a recursion for the value of the firm, after the current cash flow is known, but before the payment is made. The firm value is given by $n_{t-1}p_t$, before the distribution, and by $n_t p_t$ immediately after. Denote the number of shares repurchased over the period by $\Delta_t \equiv n_{t-1} - n_t$ and so $C_t = \Delta_t p_t$. Then,

$$\begin{aligned} V_t &= n_{t-1}p_t \\ &= (n_t + \Delta_t)p_t \\ &= n_t p_t + C_t \\ &= n_t\left(\frac{1}{1+r}E_t[p_{t+1}] - \frac{r}{1+r}\frac{\tau_p}{1-\tau_p}p_o\right) + C_t \\ &= \frac{1}{1+r}E_t[V_{t+1}] - n_t\frac{r}{1+r}\frac{\tau_p}{1-\tau_p}p_o + C_t, \end{aligned} \tag{28}$$

where the fourth line follows from substituting from the indifference equation for prices, (24), for p_t.

Let $V(c,n)$ be the value of the firm at the beginning of the period, with current cash flow c and current number of shares n, let $\Delta(c,n)$ be the number of shares repurchased over the period, and let n' denote the number of shares next period and c' the cash flow next period. Using this recursive notation, equation (28) can be rewritten as

$$V(c,n) = \frac{1}{1+r}E\left[V(c',n')\big|\,c,n\right] - [n-\Delta(n,c)]\frac{r}{1+r}\frac{\tau_p}{1-\tau_p}p_o + c. \tag{29}$$

The firm's cash constraint can be rearranged to give

$$\Delta(c,n) = \frac{c}{p(c,n)} = \frac{cn}{V(c,n)}, \tag{30}$$

where the second equality follows since $p(c,n) = V(c,n)/n$. Substituting equation (30) into equation (29) yields

$$V(c,n) = \frac{1}{1+r} E\left[V(c',n') \middle| c,n\right] - \left(n - \frac{cn}{V(c,n)}\right) \frac{r}{1+r} \frac{\tau_p}{1-\tau_p} p_o + c. \tag{31}$$

Equation (31) is a functional equation that must be satisfied by $V(c,n)$, and so the solution is a fixed–point to equation (31). Assuming the firm begins with one share, and noting that there is no distribution between $t = 0$ and $t = 1$, then

$$\begin{aligned} E_0[p_1] &= E\left[V(c',1) \mid n = 1\right] \\ &= \sum_s \pi^s V(C^s, 1). \end{aligned} \tag{32}$$

Using this in the expression for p_o, (27), yields

$$p_o = \frac{E\left[V(c',1) \mid n = 1\right](1-\tau_p)}{1+r-\tau_p}. \tag{33}$$

Thus, we can solve for the value of the firm and the initial price by seeking a fixed point, $V(c,n)$, to equations (31) and equation (33).

To solve for this fixed point numerically, we pick a grid for the number of shares, $\{\underline{n}, n_1, n_2, ..., 1\}$, and start with a guess for $E[V(c',n')|c,n]$ for each of these points and for each cash flow outcome. We set $E[V(c',n')|c,n]$ equal to the no tax value for $n < \underline{n}$, since as we show later, the value of the firm must converge to the no–tax value as the number of shares goes to zero. We then guess the initial price, and iterate as follows. Denote the guesses at the i'th iteration as $\psi^i(c,n)$ and p_0^i respectively. Using these on the right-hand side of equation (31), we solve for the firm value, which we denote $V^i(c,n)$. This step involves solving a quadratic equation at each point (c,n). We can then use equation (30) to calculate the number of shares repurchased at each point $\Delta^i(c,n) = cn/V^i(c,n)$. Given these solutions, we compute new guesses for the conditional expectations:

$$\psi^{i+1}(c,n) = \sum_s \pi^s V^i[C^s, n - \Delta^i(n,c)] \tag{34}$$

for each pair (c,n), and use equations (32) and (33) to compute a new guess of the initial price.

$$p_o^{i+1} = \frac{\sum_s \pi^s V^i(C^s, 1)(1-\tau_p)}{1+r-\tau_p} \tag{35}$$

These steps are repeated until we arrive at an approximate fixed point. In practice, this procedure converges relatively quickly.

Table 3 provides numerical results obtained using this procedure for a firm with expected cash flows of 100. In each case there are two possible cash flows, and probability of the higher cash flow is $2/3$. The after-tax discount rate is 6%, and two values for τ_p are used, 28% and 35%.

The table provides various descriptive measures of the effects of personal taxation on the value. The second column gives the tax rate that sets the present value of after-tax cash flows, assuming they were fully taxed, equal to the actual value. That is, we solve

$$p_o = \frac{\hat{C}(1-\tau^*)}{r} \tag{36}$$

for τ^*. In every case, this implicit tax rate is substantially lower than the tax rate on dividends. The third row measures the impact of the tax advantages to equity on the cost of funds to the firm. We find the discount rate, $\hat{r}$, that solves:

$$\frac{\hat{C}}{\hat{r}} = p_o. \tag{37}$$

The difference between this cost of capital and the pre-tax cost of capital under dividends is in every case substantial, between 0.79 and 1.21 percentage points. The final row reports the present value of personal taxes paid as a fraction of the present value of the tax liability under full dividend payout.

From this table we see that, as in the certainty case, the use of repurchases over dividends reduces the present value of the shareholders' personal tax liability by 40% to 50%. There is very little variation in the quantities of interest across cash-flow outcomes of different volatility. This is not surprising, given the nature of the tax liability. Through recursive substitution, and use of the budget constraint, we can rewrite

	Full tax	Cash Flow Outcomes $\{120,60\}$	$\{141,18\}$	No tax
Panel A: Tax rate of 28%				
Value	1,200.00	1,403.73	1,402.01	1,666.67
Implicit tax rate (%)	28.00	15.78	15.88	0.00
Cost of capital (%)	8.33	7.12	7.13	6.00
Percent of PV taxes (%)	100.00	56.34	56.71	0.00
Panel B: Tax rate of 35%				
Value	1,083.33	1,327.71	1,325.45	1,666.67
Implicit tax rate (%)	35.00	20.34	20.47	0.00
Cost of capital (%)	8.33	7.53	7.54	6.00
Percent of PV taxes (%)	100.00	58.11	58.49	0.00

Table 3: **Numerical Measures of Tax Advantage to Equity:** The implicit tax rate is the rate that, under full taxation, would set the present value of after-tax cash flows equal to their calculated value. The cost of capital is the discount rate that sets the present value of the pre-tax, expected cash flows equal to the calculated value. The percent of PV taxes is the present value of expected taxes paid, as a percentage of the present value of taxes paid under full taxation. The tax rate is assumed to be 35%, the expected net cash flow before personal tax is 100, and the after-tax discount rate is 6%.

the equation for the firm's value, (28), as:

$$V_t = \frac{\hat{C}}{r} + C_t - \left(n_t - \sum_{i=1}^{\infty} \frac{E_t\left[\frac{C_{t+i}}{p_{t+i}}\right]}{(1+r)^i} \right) p_o \frac{\tau_p}{1-\tau_p}. \tag{38}$$

Increases in volatility would affect the value through the effects of correlation between C_{t+i} and p_{t+i} on the expectation of their quotient, and through Jensen's inequality. Since the price is growing geometrically, however, these effects become relatively unimportant very quickly. The firm's value is not sufficiently non-linear in the cash flows for the volatility to have a significant effect, when the cash flows are uniformly positive.

Equation (38) also makes clear that the value of the firm must, as time advances, approach the no tax value. The number of shares is steadily decreasing, as cash is distributed through repurchases, so the middle term involving the basis eventually becomes negligible. This implies that

$$\lim_{t\to\infty} V_t = \frac{\hat{C}}{r} + C_t.$$

This may seem curious given what is happening to the after-tax cash flows. They can be written as

$$\Delta_t(p_t(1-\tau_p)+\tau_p p_o) = C_t(1-\tau_p)+(n_{t-1}-n_t)\tau_p p_o. \tag{39}$$

From this expression it is obvious that as the number of shares outstanding approaches zero, the distribution becomes fully taxed. The price at which shares are being repurchased grows, and a smaller fraction of the distribution is shielded by the basis. These outcomes are consistent, however. The firm repurchases shares from its owners at a price that leaves them indifferent between selling, and incurring capital gains tax, and deferring. Since the option to defer indefinitely is always available, the higher the immediate tax liability on realization, the higher a price the shareholders demand until, in the limit, they receive a price equal to what the shares would be worth to new, private buyers in an untaxed world.

4 Uncertain, Negative Net Cash Flows

We next investigate the effect of negative net cash flows on the personal tax liability of the firm's shareholders. When the firm has negative net cash flow, and must issue new shares, these shares will be issued at the prevailing price, which may be considerably higher than the initial price. This, in turn, will create new tax shields, through higher basis shares, and lower the subsequent aggregate tax burden for shareholders.

Unfortunately, a full numerical solution to this problem is computationally unmanageable, because of the path dependencies and expansion of the state space due to the need to track the tax basis at which all outstanding shares are held. In order to evaluate quantitatively the value of personal taxes paid in this situation, we approximate the dynamics of the quantities of interest and estimate their values through simulation. The exercise is similar in spirit to, although considerably less complicated than, the approach employed in Graham (1996). This method can also be easily adapted to accomodate growth in the firm's pre-tax net cash flows.

4.1 The M&M Case of Constant Expected Cash Flows

First, we take the aggregate ex-distribution value of the firm to be fixed and exogenous, at

$$V_F(\tau_o) = \frac{\hat{C}(1-\tau_o)}{r}$$

for a given tax rate τ_o. Using this ex-distribution value, we simulate a sample path for the firm's net cash flows before personal tax under the risk-neutral measure, and create a corresponding sample path for the stock price and number of shares. The the price per share at date t is given by the following laws of motion for the price per share,

$$p_t = \frac{1}{n_t}[C_t(1-\tau_o)+V_F(\tau_o)]. \tag{40}$$

The number of shares obeys

$$n_{t+1} = n_t - \frac{C_t}{p_t} \tag{41}$$

where, if $C_t < 0$, shares will be issued. To determine the taxes paid along the sample path, we assume that if the current price is less than the basis at which any shares are currently held, all those shares have their basis reset at the current price and pay negative taxes of τ_p times the associated capital loss. If shares are repurchased, we assume the highest basis shares are tendered first and repurchased. The only taxes paid along a sample path are the positive taxes associated with gains realized in a repurchase, and the negative taxes paid when the price drops below the level of the basis on shares outstanding, occasioning a capital loss. If the firm experiences a negative cash flow, but the price exceeds all outstanding basis values, new shares are issued but taxes are zero for that date.

Since the probabilities used in drawing cash flows are those associated with the risk neutral measure, we can draw many sample paths, and average the taxes paid across them for at a given date t, to estimate the expected taxes paid under the risk-neutral measure. We then discount this average for t periods to determine a present value of taxes paid in period t. Adding these present values gives the present value of expected taxes paid. Denote this quantity as PVTAX(τ_o). Note that this quantity depends on the tax rate used to compute the exogenous ex-distribution value we assumed initially. For example, if we use τ_p to compute V_F, we will understate the ex-distribution value and, thus, understate the initial basis and overstate subsequent taxes triggered by repurchases.

To minimize these effects, we adjust the tax rate used to compute $V_F(\tau)$. Under full taxation at rate τ the present value of taxes paid is just $\tau\hat{C}/r$. We solve for the tax rate at which this quantity is equal to the estimated present value of expected taxes paid. That is, we find

$$\tau_1 = \frac{PVTAX(\tau_o)r}{\hat{C}}.$$

We then repeat the simulation, calculate $PVTAX(\tau_1)$, and iterate in this way until we find that $\tau_k \approx \tau_{k-1}$. We find the tax rates converge very quickly. In effect, we are iteratively solving the equation

$$\frac{\hat{C}\tau^*}{r} = PVTAX(\tau^*). \tag{42}$$

This approximation will give average taxes paid that are similar in magnitude to what we would expect under a full solution for the equilibrium value of the firm. It will distort somewhat, however, the time path of taxes paid. A check on the accuracy of the method can be made by implementing the approximation in the certainty case, where we have an analytical solution for the equilibrium value, and in the case where cash flows are all positive, where we have a numerical solution for the initial price. This exercise suggests the approximation is quite accurate.

Table 4 provides results on the taxes paid under this approximation for the dynamics of firm value. We assume there are two states for the pre-tax cash flow, which has an expected value of 100 in every period. Thus, the model conforms to the M&M assumption of constant, perpetual expected cash flow. The after-tax discount rate is assumed to be 6%. The table reports results for two tax rates and two levels of volatility for the cash flows. It reports the same measures of the tax benefits of equity with repurchases as previously reported for the cases of certainty and positive cash flows.

The difference between the cost of capital under repurchases and the pre-tax cost of capital under dividends is in every case substantial. The implicit tax rate on equity flows is, again, substantially lower than the personal tax rate, and the difference increases as the personal tax rate increases. The estimated present value of personal taxes paid as a fraction of the present value of the tax liability under full dividend payout varies between 55% and 65%. Over all, the magnitudes in Table 4 are similar to those reported for the certainty case, and for the case where cash flows are strictly positive. The effective tax rate on flows to equity holders is, for example, close to 20% when dividends are taxed at 35%. Thus, it appears that the tax shields afforded by the original basis reduce the personal tax burden quite dramatically, but the equity is also far from tax free as assumed in Miller (1977).

The present value of the tax liability appears to increase somewhat with the volatility of the cash flows. This may seem surprising given that shareholders, who have freedom to defer or realize gains and losses, have an option-like claim on their shares. The shape of the equity holders' tax liability is ambiguous, however. When the firm has a positive cash flow, the after-tax cash flow to equity is given by the right-hand

	Full tax	$\{210,-120\}$	$\{180,-60\}$	No tax
		Cash Flow Outcomes		
Panel A: Tax rate of 28%				
Value	1,200.00	1,392.76	1,402.52	1,666.67
Implicit tax rate (%)	28.00	16.45	15.91	0.00
Cost of capital (%)	8.33	7.18	7.13	6.00
Percent of PV taxes (%)	100.00	58.73	56.81	0.00
Panel B: Tax rate of 35%				
Value	1,083.33	1,315.79	1,324.50	1,666.67
Implicit tax rate (%)	35.00	21.02	20.61	0.00
Cost of capital (%)	8.33	7.60	7.55	6.00
Percent of PV taxes (%)	100.00	60.05	58.83	0.00

Table 4: **Cost of Capital and Implicit Tax Rates with Negative Cash Flows:**The implicit tax rate is that rate that, under full taxation would set the present value of the after–tax cash flows equal to their calculated value. The cost of capital is the discount rate that sets the present value of of the pre–tax, expected cash flows equal to the calculated value. The percent of PV taxes is the present value of the of taxes paid, as a percentage of the the present value of the taxes paid under full taxation. taxes paid assuming fully taxed dividends. Calculations assume an after-tax discount rate of 6%, and that the probability of the higher cash flow is two-thirds. The simulations used to estimate the present value of taxes paid were carried out over 200 periods, with 2,000 sample paths for each case.

side of (39). It is linear in the cash flow, with a slope of $(1-\tau_p)$ and an intercept of $(n_{t-1}-n_t)\tau_p p_o$. If the firm has a negative cash flow, and the price exceeds the basis values of outstanding shares, the after-tax cash flow is just C_t. When the price drops below basis values, there are negative taxes. In the simulations reported in Table 4, negative taxes are only paid 11% to 12% of the time. Consider, therefore, the other two cases. As $n_{t-1}-n_t \to 0$, the after-tax claim of the shareholders becomes $\min\{C_t(1-\tau_p), C_t\}$, which is concave and therefore will have a value that decreases with the volatility of the cash flows. This effect appears to dominate in the numerical results.

Figures 4 and 5 depict the time path of taxes paid. Figure 4 shows the taxes paid in each period, as a fraction of the cash flow, under certainty, using $C_t = 100$, $\tau_p = 35\%$, and other parameters the same as in Table 4. The taxes are small initially because of the tax shield supplied by the original basis. As shares are repurchased, the share price grows, and the tax shield from the basis becomes negligible. Within 50 periods, the flows are being taxed at close to the full 35% rate. Figure 5 shows that with uncertainty and the possibility of negative cash flows a similar pattern emerges. In it we plot the average taxes (under the martingale measure) paid in each period, across sample paths in the simulation reported in the third line of Table 4.

Finally, in Table 5 we provide information about the relative accuracy of our different methods of calculating or estimating the present value of the tax liability. When cash flows are positive, we can compare the approximation in this section to the numerical solutions reported in Section 3. Similarly, for the certainty case, we can compare both the numerical solution and the approximation to the closed-form solution. The table shows that the approximation understates the present value of the tax liability, but this bias is of similar magnitude to the numerical errors associated with the methods from Section 3.

	Cash Flow Outcomes				
	$\{100, 100\}$			$\{120, 60\}$	
	Approx.	Numerical	Closed-Form	Approx.	Numerical
Panel A: Tax rate of 28%					
Value	1,407.99	1,404.81	1,400.40	1,407.06	1,403.73
Implicit tax rate (%)	15.52	15.71	15.98	15.58	15.78
Cost of capital (%)	7.10	7.12	7.14	7.11	7.12
Percent of PV taxes (%)	55.43	55.11	57.06	55.63	56.34
Panel B: Tax rate of 35%					
Value	1,336.15	1,328.39	1,323.70	1,336.09	1,327.71
Implicit tax rate (%)	19.83	20.30	20.58	19.83	20.34
Cost of capital (%)	7.48	7.53	7.55	7.48	7.53
Percent of PV taxes (%)	56.66	57.99	58.79	56.67	58.11

Table 5: **Cost of Capital and Implicit Tax Rates with Negative Cash Flows:** The implicit tax rate is that rate that, under full taxation would set the present value of the after–tax cash flows equal to their calculated value. The cost of capital is the discount rate that sets the present value of of the pre–tax, expected cash flows equal to the calculated value. The percent of PV taxes is the present value of the of taxes paid, as a percentage of the the present value of the taxes paid under full taxation. taxes paid assuming fully taxed dividends. Calculations assume an after-tax discount rate of 6%, and that the probability of the higher cash flow is two-thirds. The simulations used to estimate the present value of taxes paid were carried out over 200 periods, with 2,000 sample paths for each case. The column marked Approx. gives results obtained by applying the numerical method described in Section 4, the column marked Numerical gives results from applying the recursive method described in Section 3 and the column marked Closed–Form gives the results from applying Equation 12.

4.2 Allowing for Growth

The approximation developed for calculating the present value of the personal tax liability can be adapted in a straightforward manner to allow for growth in aggregate cash flows. Assume the firm's cash flows follow a geometric random walk with log-normal innovations, combined with a multiplicative shock to determine the sign.

$$C_t = \delta_t \chi_t \tag{43}$$

where

$$\delta_t = \delta_{t-1} \exp(\mu + \frac{1}{2}\sigma^2 + \sigma\varepsilon_t) \tag{44}$$

and χ_t is equal to $+1$ with probability q and -1 with probability $1-q$. Suppose taxation is proportional at rate τ_o. If the pricing kernel is also a geometric random walk with lognormal innovations, and χ_t is independent of the pricing kernel, then the value of this process can be computed (see, for example, Berk, Green and Naik (1999)) as:

$$V_t(\tau_o) = \frac{\delta_t(2q-1)(1-\tau_o)}{1-e^{\mu^*-r}} \tag{45}$$

where μ^* is the risk-neutralized growth rate, which is computed by subtracting from μ a constant that depends on the correlation between the innovations to the pricing kernel and δ_t. We take (45) to be the exogenous process for firm value. We simulate this process, and, using (43) pre-tax cash flows. We compute the shares repurchased, price per share, and taxes paid along each sample path. The are averaged and present valued, to determine a PVTAX(τ_o). We then iterate on the tax rate to a solution to:

$$\text{PVTAX}(\tau^*) = \frac{\delta_t(2q-1)\tau^*}{1-e^{\mu^*-r}} \tag{46}$$

Table 6 reports the results of these calculations. The number of sample paths used here is much higher than in our earlier simulations because of the high variance associated with the random walk. The costs of capital in this table should be interpreted as continuously compounded rates, in line with equation (45).

When contrasted with Table 4, it appears that adding growth has two effects. First, the firm's shareholders are more heavily taxed. The implicit tax rates and the ratio of the present value of taxes paid to the present value of taxes under proportional taxation both are higher in Table 6. This is consistent with the results for the certainty case, and is apparently due to the fact that the growth in the value of the firm and in prices erodes the value of the tax shields from the basis more quickly.

Second, the various measures of the value of the tax liability in Table 6 are decreasing in the volatility of the cash flows, σ. We should interpret increases in σ here as increases in idiosyncratic risk, as we are holding the risk-neutralized growth rate fixed across columns in the table. In contrast, when expected cash flows are perpetual, as in Table 4, the value of the tax liability increased in the volatility. This difference appears to be due to the greater influence of the opportunity to pay negative taxes under the random walk model. Since the cash flows are proportional to the value, which is growing, when negative cash flows occur in this model it is more likely they will lead to large price drops below the basis of outstanding shares, capital losses, and negative taxes. The simulations in Table 6 produce negative taxes 18-19% of the time, for $\sigma = .1$, and 24-25% of the time for $\sigma = .2$. For the constant expected cash flow model from Table 4, the simulations generate negative taxes only 11-12% of the time for all cases. The benefits of these capital losses, in the case of the random walk model, dominate the other sources of concavity in the shareholders after-tax payoff.

		Volatility		
	Full tax	$\sigma = 0.1$	$\sigma = 0.2$	No tax
Panel A: Tax rate of 28%				
Value	1,836.24	2,036.54	2,067.53	2,550.33
Implicit tax rate (%)	28.00	20.15	18.93	0.00
Cost of capital (%)	7.60	7.03	6.96	6.00
Percent of PV taxes (%)	100.00	71.95	67.61	0.00
Panel B: Tax rate of 35%				
Value	1,657.71	1,903.72	1,932.46	2,550.33
Implicit tax rate (%)	35.00	25.35	24.23	0.00
Cost of capital (%)	8.22	7.40	7.31	6.00
Percent of PV taxes (%)	100.00	72.44	69.22	0.00

Table 6: **Cost of Capital and Implicit Tax Rates with Negative Cash Flows and Growth:**The implicit tax rate is that rate that, under full taxation would set the present value of the after-tax cash flows equal to their calculated value. The cost of capital is the discount rate that sets the present value of of the pre-tax, expected cash flows equal to the calculated value. The percent of PV taxes is the present value of the of taxes paid, as a percentage of the the present value of the taxes paid under full taxation. taxes paid assuming fully taxed dividends. Calculations assume an after-tax discount rate of 6%, a risk-neutral growth rate of 2%, and that the probability of the a negative cash flow is two-thirds. The simulations used to estimate the present value of taxes paid were carried out over 100 periods, with 20,000 sample paths for each case.

5 Conclusion

We analyze the advantages of equity financing by comparing the present value of the personal tax liability of equity holders in a firm that distributes all cash through dividends to one that distributes cash through repurchases. Under certainty, where the model admits a closed-form solution, we show this advantage is quantitatively and economically substantial.

The magnitude of the personal-tax advantage to equity under uncertainty is similar to that under certainty. This has been shown using numerical methods to calculate the value of the tax liability when cash flows are certain, and using a Monte Carlo approximation when cash flows are negative, and the firm must issue new shares to finance shortfalls.

The tax advantages of equity at the personal level, in our model, are not sufficient to completely offset the tax advantages of debt at the corporate level in the setting envisioned by M&M, where all payments to debt are fully deductible. The personal tax advantages of equity do seem to be large enough to rationalize interior capital structures when tax shields may be uncertain or redundant, or for plausible levels of bankruptcy costs. When combined with estimates of the marginal tax benefits from debt financing that account for the dynamics and redundant tax shields in corporate taxation, such as those in Graham (1998), our estimates or elaborations of them might provide useful guidance regarding optimal capital structure and the corporate cost of capital.

Appendix

Proofs

Proof of Lemma 1

Let $\{p_t\}_{t=1}^{\infty}$ be a conjectured sequence of equilibrium prices. Consider a shareholder with $\bar{n}$ shares, and basis p_o. Suppose a potential buyer is willing to pay p_t for these shares, and that this buyer's optimal realization strategy would be to sell n^*_{t+s} shares at date $t+s, s = 1, \ldots, \infty$, where $\sum_s n^*_{t+s} = \bar{n}$. Then it must be the case that the price the buyer is willing to pay is less than the present value of after-tax receipts to the buyer at the optimal strategy:

$$\bar{n}p_t \leq \sum_s \frac{n^*_{t+s}}{(1+r)^s}[p_{t+s} - \tau_p(p_{t+s} - p_t)]. \tag{A1}$$

The seller would accept the price p_t only if the proceeds from the sale, net of the taxes paid,

$$\bar{n}[p_t - \tau_p(p_t - p_o)] \tag{A2}$$

exceed the present value of after-tax cash flows from holding the shares and pursuing a realization strategy that is optimal given basis p_o. Denote this value to the seller as $V^s(p_o)$. We will not show that, in fact, it exceeds the after-tax proceeds from sale, (A2), which will establish the result. We know

$$\begin{aligned}
V^s(p_o) &\geq \sum_s \frac{n^*_{t+s}}{(1+r)^s}[p_{t+s} - \tau_p(p_{t+s} - p_o)] \\
&= \sum_s \frac{n^*_{t+s}}{(1+r)^s}[p_{t+s} - \tau_p(p_{t+s} - p_t)] - \sum_s \frac{n^*_{t+s}}{(1+r)^s}\tau_p(p_t - p_o) \\
&\geq \bar{n}p_t - \sum_s \frac{n^*_{t+s}}{(1+r)^s}\tau_p(p_t - p_o) \\
&> \bar{n}p_t - \sum_s n^*_{t+s}\tau_p(p_t - p_o) \\
&= \bar{n}p_t - \bar{n}\tau_p(p_t - p_o).
\end{aligned} \tag{A3}$$

The first line follows from the fact that the value to the seller under the seller's optimal realization strategy must be at least as high as under the buyer's optimal strategy. The third line reflects the fact that the value under the buyer's optimal strategy exceeds the current, conjectured price by (A1). The fourth line follows since the conjectured price exceeds the basis, and thus the taxes paid today exceed their present value if deferred. This inequality is strict if the interest rate is positive. The last line is the after-tax proceeds from sale, which establishes the result.■

Proof of Proposition 1

With dividends, we can now write the value process as follows. At the ex-distribution date:

$$V_t^e = \frac{C(1-\tau_p)}{r}.$$

At the cum-distribution point it is:

$$V_t^c = C(1-\tau_p) + \frac{C(1-\tau_p)}{r}.$$

Thus, the personal taxation enters the valuation proportionally. The stock price is just the above values divided by the constant number of shares, so that the price drops with each distribution by the per-share, after-tax value of the distribution:

$$p_t^c - p_t^e = \frac{C(1-\tau_p)}{n_0}. \tag{A4}$$

When all capital gains, realized or unrealized, are taxed each period, the share price grows, and the number of shares shrink, at the pre-tax discount rate of $r/(1-\tau_p)$. At the beginning of each tax period, the shareholder's basis is the current price. This basis is independent whether the shareholder is a buyer, establishing a new basis, or a seller. Thus, any shareholder will be indifferent between receiving p_t for her shares, or holding them for one more period to receive value of

$$\frac{1}{1+r}(p_{t+1}(1-\tau_p) + \tau_p p_t).$$

Equating this quantity to p_t gives a difference equation for the price,

$$p_{t+1} = p_t \left(1 + \frac{r}{1-\tau_p}\right).$$

If aggregate ex-distribution equity value is constant through time, then $p_t n_t = p_{t+1} n_{t+1}$, and the number of shares must evolve as

$$n_{t+1} = \frac{n_t}{1 + \frac{r}{1-\tau_p}}.$$

This aggregate value will be

$$V_t = \frac{C(1-\tau_p)}{r}$$

if we can show that, under the above laws of motion for the price and the number of shares, the aggregate taxes paid by the equity holders under repurchase do, in fact, equal $C\tau_p$. The aggregate tax liability at time t is given by

$$\begin{aligned} n_{t-1}\tau_p(p_t - p_{t-1}) &= \tau_p \left[n_{t-1} p_{t-1} \left(1 + \frac{r}{1-\tau_p}\right) - n_{t-1} p_{t-1} \right] \\ &= \tau_p n_{t-1} p_{t-1} \frac{r}{1-\tau_p}. \end{aligned} \tag{A5}$$

At date $t-1$ the cash distributed in a repurchase must be equal in value to the shares repurchased, so

$$\begin{aligned} C &= p_{t-1}(n_{t-2} - n_{t-1}) \\ &= p_{t-1} \left[n_{t-1} \left(1 + \frac{r}{1-\tau_p}\right) - n_{t-1} \right] \\ &= p_{t-1} n_{t-1} \frac{r}{1-\tau_p}. \end{aligned} \tag{A6}$$

Comparing (A5) to (A6) we see that the aggregate tax liability of the equity holders is, indeed, equal to $\tau_p C$.■

Proof of Proposition 2

The initial value of the firm, equation (12) follows by taking the present value of the cash flows in equation (10) as in the text. Equation (13) follows by substituting equation (12) into the solution for the price at time t, equation (9). Substituting equation (13) into the cash constraint, equation (6) and solving for the number of shares repurchased at time time t gives

$$\begin{aligned} n_{t-1} - n_t &= \frac{C}{p_t} \\ &= \frac{C}{C[(1+r)^t - \tau_p]\left(\sum_{s=1}^{\infty} \frac{1}{(1+r)^s - \tau_p}\right)} \\ &= \frac{1}{(1+r)^t - \tau_p}\left(\sum_{s=1}^{\infty} \frac{1}{(1+r)^s - \tau_p}\right)^{-1}. \end{aligned} \tag{A7}$$

The number of shares remaining at time t is given by $n_{t-1} = \sum_{s=t}^{\infty}(n_{t-1} - n_t)$, and using equation (A7), this is equal to

$$n_{t-1} = \sum_{s=t}^{\infty} \frac{1}{(1+r)^s - \tau_p}\left(\sum_{s=1}^{\infty} \frac{1}{(1+r)^s - \tau_p}\right)^{-1}$$

thus proving equation (14). Equation (15) follows from multiplying equations (14) and (15). To take the limit, multiply and divide the value of the firm by $1/(1+r)^{t-1}$ to get

$$\begin{aligned} V_t &= C\left[(1+r)^t - \tau_p\right]\sum_{s=t}^{\infty} \frac{1}{(1+r)^s - \tau_p} \\ &= C\left[(1+r) - \tau_p/(1+r)^{t-1}\right]\sum_{s=1}^{\infty} \frac{1}{(1+r)^s - \tau_p/(1+r)^{t-1}}. \end{aligned}$$

Taking the limit as $t \to \infty$,

$$\begin{aligned} \lim_{t\to\infty} V_t &= \lim_{t\to\infty} C\left[(1+r) - \tau_p/(1+r)^{t-1}\right]\sum_{s=1}^{\infty} \frac{1}{(1+r)^s - \tau_p/(1+r)^{t-1}} \\ &= C(1+r)\sum_{s=1}^{\infty} \frac{1}{(1+r)^s} \\ &= \frac{C}{r} + C. \end{aligned}$$

∎

Proof of Lemma 2 This follows from inspection of equation (22). ∎

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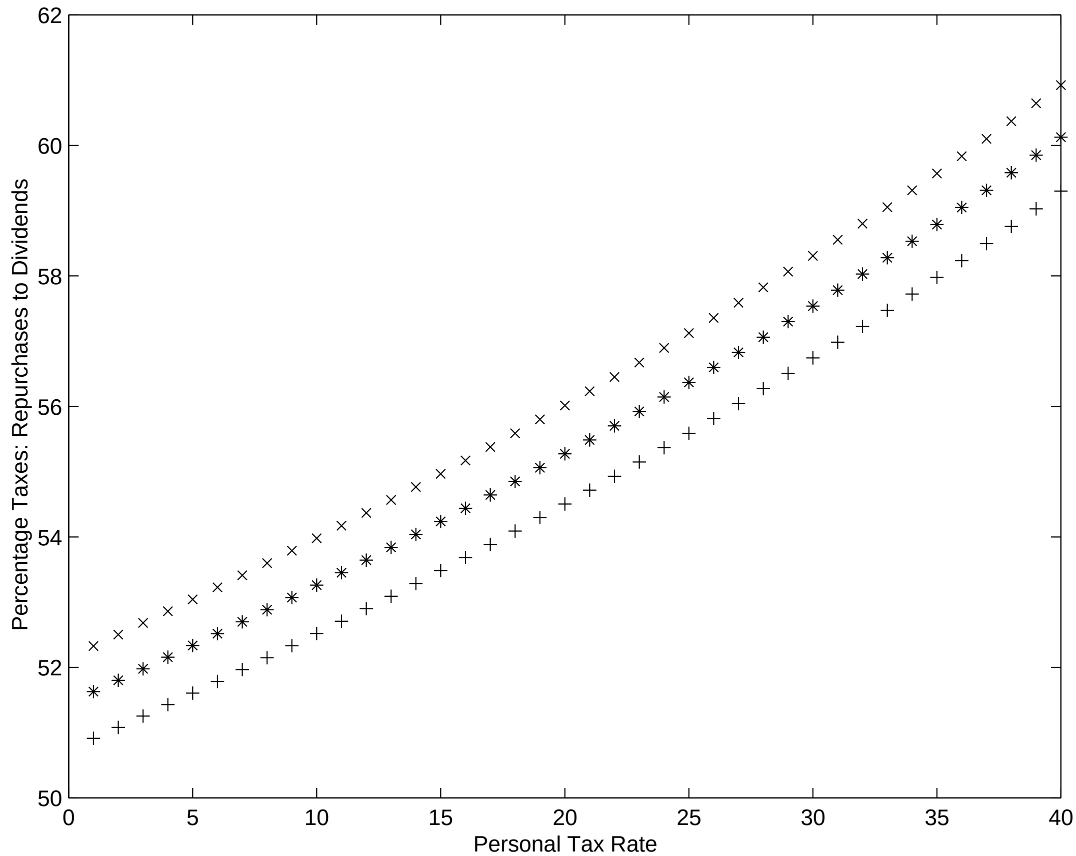

Figure 1: Present value of taxes paid, as a fraction of the value of the tax liability under full taxation of all equity flows, plotted against the personal tax rate. The three curves are associated with different values of the pre–tax discount rate, 3% (+), 6% (*) and 9% (x).

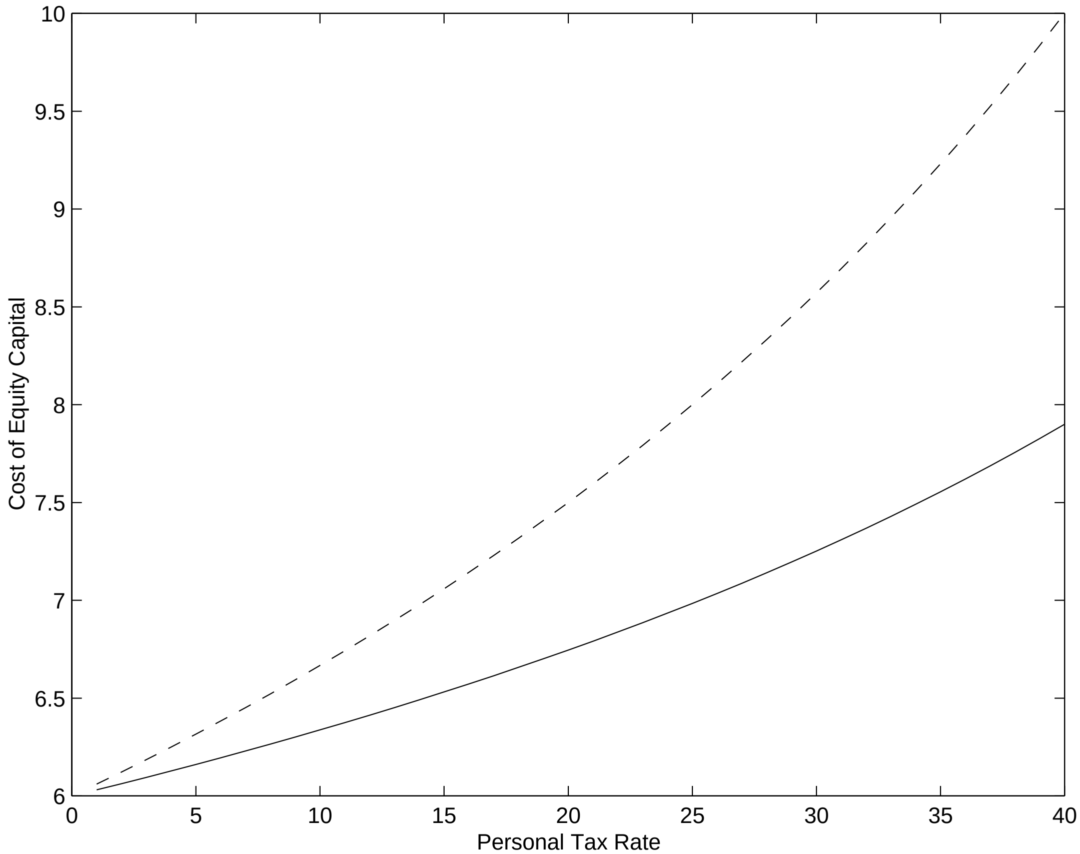

Figure 2: The cost of equity capital under certainty assuming an after–tax return of 6%, plotted against the personal tax rate. The solid line is the cost of equity capital $\frac{C}{p_0}$. The dashed line is the cost of capital prevailing when distributions are paid as dividends, $\frac{r}{1-\tau_p}$.

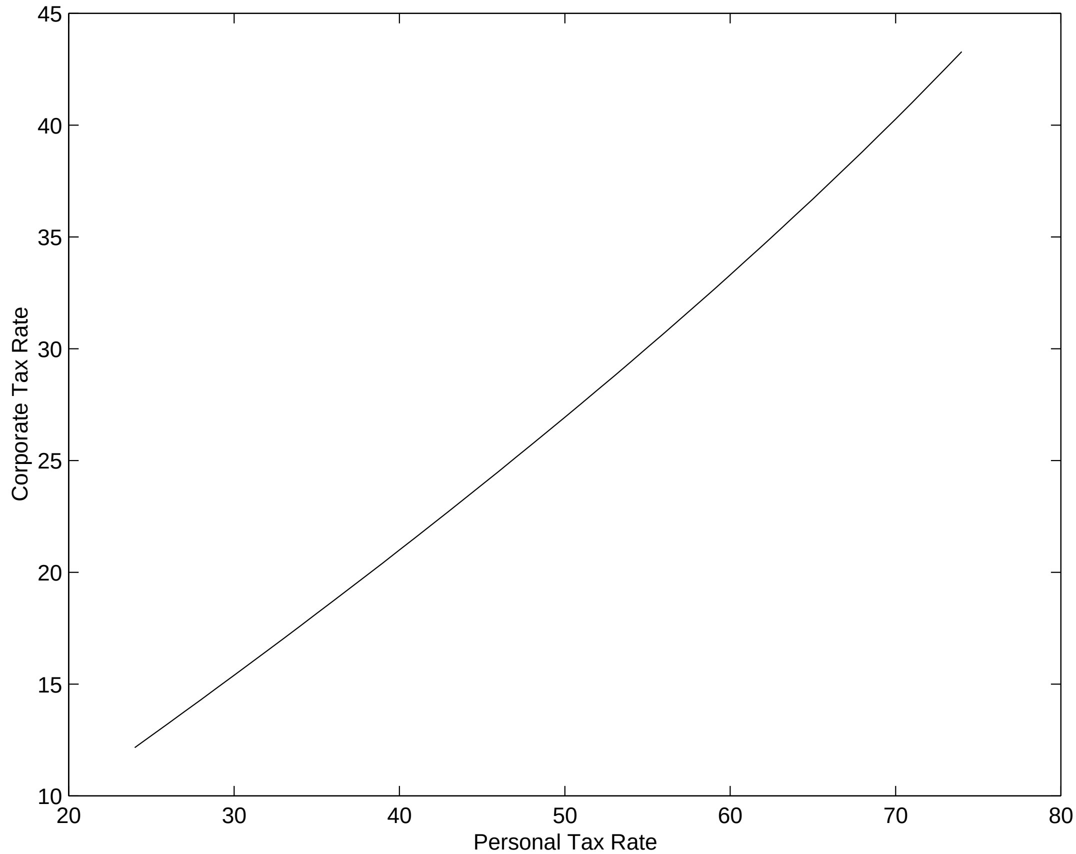

Figure 3: Corporate tax rates at which the firm is indifferent between debt and equity financing, assuming that all distributions to equity are made through repurchases. At a given personal tax rate, for corporate tax rates above the curve plotted in the figure, the firm would prefer debt to equity financing.

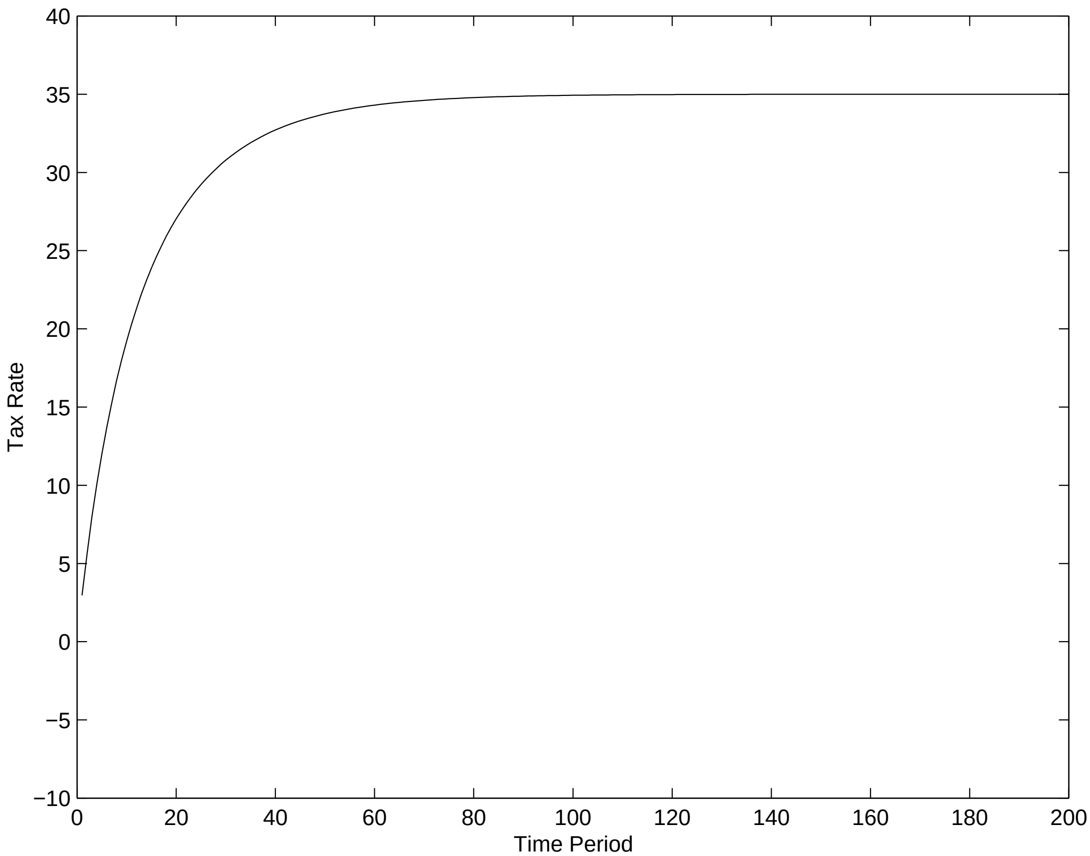

Figure 4: Taxes paid each period as a fraction of the pre–tax cash flow under certainty.

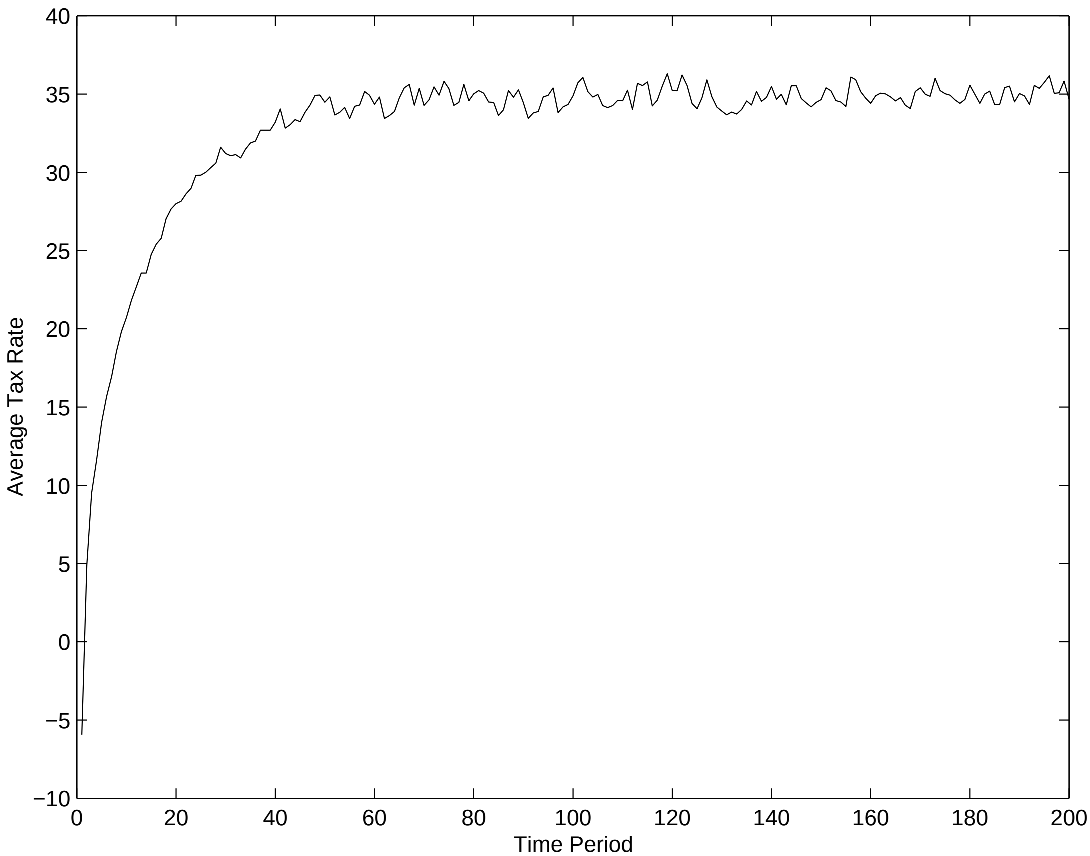

Figure 5: Taxes paid each period as a fraction of the pre–tax cash flow with uncertain cash flows.