A Bayesian Inference Approach to Testing Mean Reversion in the Swedish Stock Market

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JEL: G10 C11 C15

June 26, 2000

Abstract

In this paper we use a Bayesian approach to test for mean reversion in the Swedish stock market on monthly data 1918-1998. By simply account for the heteroscedasticty of the data with a two state hidden Markov model of normal distributions and taking estimation bias into account via Gibbs sampling we can find no support of mean reversion. This is a contradiction to previous result from Sweden. Our findings suggest that the Swedish stock market can be characterized by two regimes, a tranquil and a volatile, and within the regimes the stock market is random. This finding of randomness is in line with recent evidence for the U.S stock market.

1 Introduction

This paper addresses the question of whether or not the Swedish stock market is determined by random behavior. Previous research by Frennberg and Hansson (1993) concludes this is not the case. However, Berg and Lyhagen (1996) have questioned their findings. Notwithstanding, the evidence of mean reversion via variance ratio, VR, is controversial because the test of the null hypothesis of random walk is only valid under the assumption of constant expected return. The return series from financial markets are well known to exhibit time variation, especially in volatility. Hence, mean reversion might be explained by time-variation, or regime switches, in volatility and taking this aspect into consideration the market might be efficient. Kim et al (1991, 1998a) questions the often used assumption of homoskedastic volatility and argues the significant divergences some times found when using VR statistic might in fact be explained by variance shifts. Their conclusion is that the returns are indeed white noise. Nielsen and Overgaard-Olesen (1999) find weak support of mean reversion when they employ Hidden Markov models and compute variance ratio test on annual Danish stock market data. Malliaropulos and Priestly (1999) utilize a bootstrap approach to test for mean reversion in international stock market data.

This study differs from previous studies on the Swedish stock market in that we employ Bayesian approach to test for mean reversion on standardized excess returns as suggested by Kim et al (1998a). The idea is to capture the time variation in the variance by a two-state Hidden Markov Model, henceforth HMM, of Gaussian mixtures. Thus we assume two regimes: low and high volatility.

Goldfeld and Quandt (1973) introduced the Markov switching models in economics but its application in economics and finance came a decade ago when Hamilton (1989) employed a two-state HMM on GDP data. The drawback with the HMM is that ordinary optimization of the likelihood function can be cumbersome.[1] Albert and Chib (1993) address this problem with a Gibbs sampling approach in order to estimate the two-state HMM suggested by Hamilton (1989).[2] Geman and Geman (1984)'s Bayesian framework of Gibbs sampling is very advantageous. First, we can use prior information in the estimation of the conditional distribution of the parameters, without estimation of a likelihood

[1] Ordinary optimization algorithms often fail to estimate the true HMM correct. Another approach is to employ the simulated annealing, SA, algorithm. This is also a MCMC approach and thus, computer intensive.

[2] Kim et al (1998a, 1998b) extended Albert and Chib's model to a three-state HMM. See the papers by Lunginbuhl and De Vos (1999) and Dueker (1999) for other applications of the Gibbs sampling framework of Albert and Chib (1993).

function. This is an appealing approach as the likelihood function of hidden Markov models can be cumbersome to estimate. Second, all inferences in Gibbs sampling are made from joint distributions of the variates and the unknown parameters of the model. Thus, we are able to account for the parameter uncertainty of the underlying parameters in the model.

In our analysis we find no support of mean reversion in the any of the Nordic stock markets. Our two-state regime switching models of normal distributions suggests that mean reversion if found in the Swedish stock market can be explained by time variation in the volatility and within the regimes the stock markets is random.

The outline of the paper is as follows: In section 2 we describe the underlying assumptions of the variance ratio test. Section 3 presents the data. The methodology is presented in section 4. Section 4.1 describes the two-state regime-switching model. A brief presentation of Bayesian statistics is given in section 4.2. The Gibbs sampler and the prior distributions are specified in section 4.3. The Bayesian re-sampled variance ratio tests are presented in section 4.4. Section 5 presents the results and section 6 concludes the paper.

2 Variance ratio

The variance ratio test, VR, of Cochrane (1988) has been frequently used as a test of mean reversion. Faust (1992) reports the VR-test is the optimal test for mean reversion. The advantage of the test is that it allows us to study if returns follows a random walk and if this property changes with the investment horizon q. The q period return y_t^q is computed as the q period difference between the log of the monthly index values of the portfolio I_t and I_{t-q}, in our case the Swedish stock market portfolio.

$$y_t^q = I_t - I_{t-q} \tag{1}$$

Let r_q be the monthly return including dividends of the market portfolio. Compounded returns, I_t, are assumed to be a random walk. This implies the arithmetic return being a drift μ plus a white noise term ε_t. In this context the q-month arithmetic return is:

$$y_t^q = q\mu + \varepsilon_t + \ldots + \varepsilon_{t+q} \tag{2}$$

$$y_t^q = \mu + r_{q-1} + \varepsilon_{t+q} \tag{3}$$

The expected q period return is equal to the monthly mean return times the holding period q and the variance of the q period return is q times the variance of monthly returns.

$$E\left[y_t^q\right] = q\mu, \quad Var\left[y_t^q\right] = q\sigma^2 \tag{4}$$

The variance ratio statistic, VR, is defined as:

$$VR(q) = \frac{Var\left[y_t^q\right]}{q \cdot Var\left[y_t^1\right]} \tag{5}$$

$$= 1 \text{ under random walk}$$

In our investigation we have chosen the investment horizon q to range from two to twelve months and yearly up to ten years. This enables us to study the random walk hypothesis both in the short-run and the long-run.

2.1 Data

We use 80 years of monthly Swedish stock market returns including dividends and the Swedish risk-free rate from December 1918 to December 1998. All data are from the Frennberg and Hansson (1998) database. Using these two return series we compute the monthly excess return of the Swedish stock market and subtract the mean of the excess return to get a de-meaned excess return series.

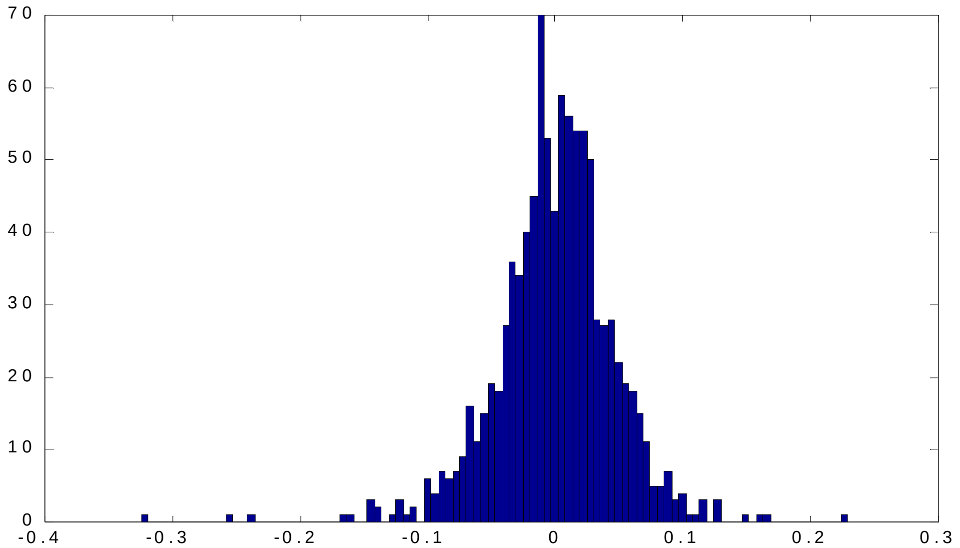

Figure 1: Distribution of de-meaned monthly stock market excess returns 1919-1998.

3 Methodology

3.1 Two-State Hidden Markov Model

Let the monthly de-meaned excess stock returns y_t be described as a two-state hidden Markov model (HMM) of Gaussian mixtures.[3] Where S_t is an unobserved state variable following a Markov process.

$$y_t \sim N\left(0, \sigma_i^2\right) \tag{6}$$

$$\sigma_t^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} \tag{7}$$

$$\Pr\left[S_t = j \mid S_{t-1} = i\right] = p_{ij}, \quad i, j = 1, 2$$

$$\sum_{J=1}^{2} p_{ij} = 1, \quad i = 1, 2$$

The above model is a standard Markov switching model that can be estimated with maximum likelihood (see Hamilton (1994)).

3.2 Bayesian statistic

The fundamental idea behind Bayesian statistic is to condition on the observed data, Y, and regarding the parameters, θ, as random variables. Suppose that $p(\theta)$ is a probability distribution of the parameter θ.

$$p(Y \mid \theta)\, p(\theta) = p(Y, \theta) = p(\theta \mid Y)\, p(Y). \tag{8}$$

The probability distribution of θ conditional on the observed data is expressed by Bayes theorem:

$$p(\theta \mid Y) = \frac{p(Y \mid \theta)\, p(\theta)}{p(Y)}. \tag{9}$$

where $p(\theta)$ is the prior probability density function and describes the information in θ without any knowledge about the data, Y. $p(\theta \mid Y)$ is the posterior probability density

[3] We have also done estimations using three-state hidden Markov model. The results suggest that a two-state hidden Markov model being more appropriate. The results of the estimations are available on request.

function and gives a description of what is known about θ given the data, Y. Given the data, Y, the conditional probability distribution $p(Y \mid \theta)$ can be seen as a function of the parameters θ. This function is in fact proportional to the likelihood function of θ, $L(Y \mid \theta)$. Let us consider $p(Y)$ as being constant, then we can write the above as

$$p(\theta \mid Y) \propto p(Y \mid \theta)\, p(\theta). \tag{10}$$

This yields the appealing property of the Bayesian approach:

$$p(\theta \mid Y) \propto L(Y \mid \theta)\, p(\theta). \tag{11}$$

The posterior probability density function is proportional to the likelihood function times the prior probability density function. Hence we do not need a specification of the likelihood function to sample from the marginal distributions of the parameters.

3.3 The Gibbs sampler

Gibbs sampling is a special case of the Metropolis (1953) and Hastings (1970) Markov Chain Monte Carlo algorithm, the difference being that in Gibbs sampling we always accept the candidates. Its breakthrough came with the papers by Gefland and Smith (1990) and Gefland et al (1990) which applied the Gibbs sampling framework on various problems. The Gibbs sampler provides the analyst with the tools to sample from the marginal distribution of the parameters of interest. This is an appealing property when faced with cumbersome likelihood functions. The idea behind the algorithm is to sample from the conditional distribution of the parameter space $\{\theta_1, \theta_2, \ldots, \theta_k\}$.

Step 1: Specify arbitrary initial values, $\left(\theta_1^{(0)}, \theta_2^{(0)}, \ldots, \theta_k^{(0)}\right)$, and set $n = 1$.

Step 2: Cycle through the full conditionals by drawing:

(1) $\theta_1^{(n)}$ from $\left[\theta_1 \mid \theta_2^{(n-1)}, \ldots, \theta_k^{(n-1)}\right]$

(2) $\theta_2^{(n)}$ from $\left[\theta_2 \mid \theta_1^{(n)}, \theta_3^{(n-1)}, \ldots, \theta_k^{(n-1)}\right]$

⋮

(k) $\theta_k^{(n)}$ from $\left[\theta_k \mid \theta_1^{(n)}, \ldots, \theta_{k-1}^{(n)}\right]$

Step 3: set $n = n + 1$, and go to step 2.

This cycle is then repeated N times and we obtain the sample values $\left(\theta_1^{(N)}, \theta_2^{(N)}, \ldots, \theta_k^{(N)}\right)$. Where N is set to a large number, in our case N is set to 20.000 iterations.[4] The first M iterations when the chains have not converged are discarded leaving us with a sample of m useful iterations. For a large number m the simulated values $\left(\theta_1^{(M)}, \theta_2^{(M)}, \ldots, \theta_k^{(M)}\right.$ ... $\left.\theta_1^{(N)}, \theta_2^{(N)}, \ldots, \theta_k^{(N)}\right)$ can be treated as an approximate sample from $[\theta_1, \theta_2, \ldots, \theta_k]$(see Tierney (1994)). Now the posterior expectation of the function of the parameters, θ can be estimated by the ergodic average.

$$E\left(f\left(\theta\right)\right) \simeq \frac{1}{m} \sum_{i=M+1}^{N} f\left(\theta^{(i)}\right) \tag{12}$$

3.3.1 Priors and prior distributions

We use conjugate prior distributions and the specification of the prior parameters and their distributions follows from Albert and Chib (1993), Tanner (1996), Kim et al (1998 a), Robert and Casella (1999).[5]

The probabilities for the Markov process to move from one state i at time $t-1$ to state j at time t are called transition probabilities, $p_{ij} = p\left(S_t = j \mid S_{t-1} = i\right)$. The transition probabilities p_{ij} are collected in the transition matrix P, which forms the nucleus of the Markov model. Each row of the transition probability matrix P are generated as random draws from a Dirichlet distribution.[6]

$$\mathsf{P}(i) \sim D\left(u_{i1} + n_{i1}, u_{i2} + n_{i2}\right), \qquad i = 1, 2 \tag{13}$$

where n_{ik}, are the number of transitions from state i to state k. We consider u_{ik}, $i = 1, 2$, $k = 1, 2$, as non-informative priors and set them equal to 1.

[4] This is a computer intensive simulation. Notable is that the increase in CPU power has made this approach feasible. All simulations are done in MATLAB and the estimation time is approximately 6 hours on a standard Intel PII 450 MHz.

[5] See also Gilks et al (1996) "Markov Chain Monte Carlo in Practice" and Tanner (1996) "Tools for Statistical Inference: Methods for the Exploration of Posterior Distributions and Likelihood Functions" as well as Robert and Casella (1999) "Monte Carlo Statistical Methods".

[6] The Dirichlet density function has the property it can assume a large number of various shapes in the sample space [0, 1]. An other property of the multivariate dirichlet distribution is that the sampled probabilities sum to unity. This makes the Dirichlet distribution family very suitable in representing any experiments on multivariate continuous random variables in the [0, 1] space. See also Mittelhammer: Mathematical Statistics for Business and Economics.

In order to satisfy the constraint, $\sigma_1^2 < \sigma_2^2$, we need to first generate σ_1^2 and re-define σ_2^2 conditional on σ_1^2.

$$\sigma_2^2 = \sigma_1^2 (1 + h) \tag{14}$$

where $h > 0$. Where σ_1^2 and $\overline{h} = (1 + h)$ are random draws from the inverse-gamma, IG, distribution family.[7]

$$Y_{1t} = \frac{y_t}{\sqrt{\sigma_1^2 (1 + S_{2t} h)}} \tag{15}$$

$$\left[\sigma_1^2 \mid \widetilde{Y}_{1T}, \widetilde{S}_T, \widetilde{\theta}_{j \neq \sigma_1^2}\right] \sim IG\left(\frac{v_1 + T}{2}, \frac{\delta_1 + \sum_{t=1}^{T} Y_{1t}^2}{2}\right), \tag{16}$$

$$Y_{2t} = \frac{y_t}{\sqrt{\sigma_1^2}} \tag{17}$$

We define N_2 as the number of times state 2 occurs $N_2 = \{t : S_t = 2\}$ and T_2 is the sum of the elements in N_2.

$$\left[\overline{h} \mid \widetilde{Y}_{2T}, \widetilde{S}_T, \widetilde{\theta}_{j \neq h}\right] \sim IG\left(\frac{v_2 + T_2}{2}, \frac{\delta_2 + \sum_{t=1}^{N_2} Y_{2t}^2}{2}\right) I_{[\overline{h}>1]}, \tag{18}$$

We use non-informative priors and set v_1, v_2, δ_1, and δ_2 equal 1.

3.3.2 Missing data simmulation

We regard the states as missing data. Thus, we cannot observe the states. However, we can compute the probability of a given observation y_t belongs to state i, $i = 1, 2$, and from this information construct forecast probabilities of which state i, $i = 1, 2$, observation y_{t+1} belongs to. The probabilities are computed for all observations y_t, $t = 1...T$, via the local updating algorithm of Robert (1993).[8] This is repeated for every Gibbs run. The local updating algorithm is a forward algorithm in which each state is simulated from the full conditional $(1 \leqslant i \leqslant k)$. Thus we have utilized the fact that this is a first order Markov chain as the distribution only depends on the value of two neighboring states.

[7] A random sample from the inverse gamma is the reciprocal of a draw of a random number from the gamma distribution. The (inverse) gamma density function is employed as a prior distribution as it enables the researcher to sample nonnegative real numbers. See also Mittelhammer (1995).

[8] We have also made runs using the forward-backward algorithm. See pages 690-693 in Hamilton (1994) "Time Series Econometrics".

$$p\left(S_1 = i \mid S_2, ..., \mathrm{P}\right) \propto \rho_i p_{iS_2} f\left(x_1 \mid 0, \sigma\right)$$

$$p\left(S_1 = i \mid ..., S_{t-1}, S_{t+1}, ..., \mathrm{P}\right) \propto p_{S_{t-1}i} p_{iS_{t+1}} f\left(x_j \mid 0, \sigma\right), \quad (1 < t < T)$$

$$p\left(S_T = i \mid ..., S_{T-1}, \mathrm{P}\right) \propto p_{S_{T-1}i} f\left(x_n \mid 0, \sigma\right)$$

Where $(\rho_i, ..., \rho_k)$ is the stationary distribution of the transition matrix P and $f(\cdot \mid 0, \sigma)$ denotes the density of the normal distribution. Thus, the ρ_i's are computed from the transition matrix at each iteration of the Gibbs sampler. Using the probabilities from the local updating algorithm we generate the two states $S = 1, 2$, from a two point distribution. The states are generated by drawing random numbers from a uniform distribution. We set the state $S_t = 1$; if the generated number is less or equal to $p_1/(p_1 + p_2)$. If it is greater than $p_1/(p_1 + p_2)$, we set $S_t = 2$. This is repeated for all observations $t = 1...T$.

3.4 A Bayesian approach to variance ratio test

Remember our basic assumption that y_t is heteroskedastic de-meaned return with variance $\sigma_t^2(\theta)$ which can be described by a mixture of two normal distributions $y_t \sim N(0, \sigma_i^2)$. $\theta = \{\sigma_1^2, \sigma_2^2, S_t, p_{11}, p_{22}\}$ is a parameter vector describing the dynamics of $\sigma_t^2(\theta)$. The following two re-sampled based variance ratio tests have been suggested by Kim et al (1998a). At the end of each run of the Gibbs sampling algorithm the following procedure is computed:

Step 1: We divide the monthly returns y_t by the standard deviation σ_t in order to get the standardized returns y_t^*.

Step 2: Scramble the standardized returns y_t^* to yield a new randomized vector y_t^{r*}.

Step 3: Create a new series of de-standardized randomized monthly returns y_t^r by scaling the randomized-standardized returns y_t^{r*} by the standard deviation σ_t.

We now have four return series, first the original returns y_t, second the standardized original returns y_t^*, third a randomized standardized returns y_t^{r*} and fourth a randomized de-standardized returns y_t^r. Next we calculate the q-month variance ratio for the four return series. The significance levels of the VR are estimated as the fraction of VR for the artificial returns that fall below the VR of the original historical returns. Thus we will have two tests for every q-month horizon. First, a test based on original returns. Second, a test based on standardized returns. At the end of the Gibbs sampling we will have 20.000 realizations of each of the two tests for each of the 20 q-month test horizons. An advantage with our Bayesian approach is that we are able to account for the parameter uncertainty in θ as well as the effect of the randomization.

4 Results

4.1 Bayesian inference on parameter estimates

The convergence of the Gibbs sampler or burn in time is determined via monitoring techniques. The convergence of the Gibbs sampler or burn in time has been determined by running several Gibbs sequences and by using different values of the priors. This is done in order to reveal possible slow mixing of the Markov chain. We monitor all parameters of the Gibbs sequence, figure 2, and the burn in time based on the worst scenario, the parameter with the slowest mixing. The mixing, being based on the average value versus the number of iterations, the transition probability p_{11} can been seen in figure 2. The variance parameters converge quickly, but the transition probabilities exhibits slow convergence. Thus the burn in time is based on the latter and m is set to 8.000 iterations, leaving 12.000 Gibbs sequences from which to make statistical inference.

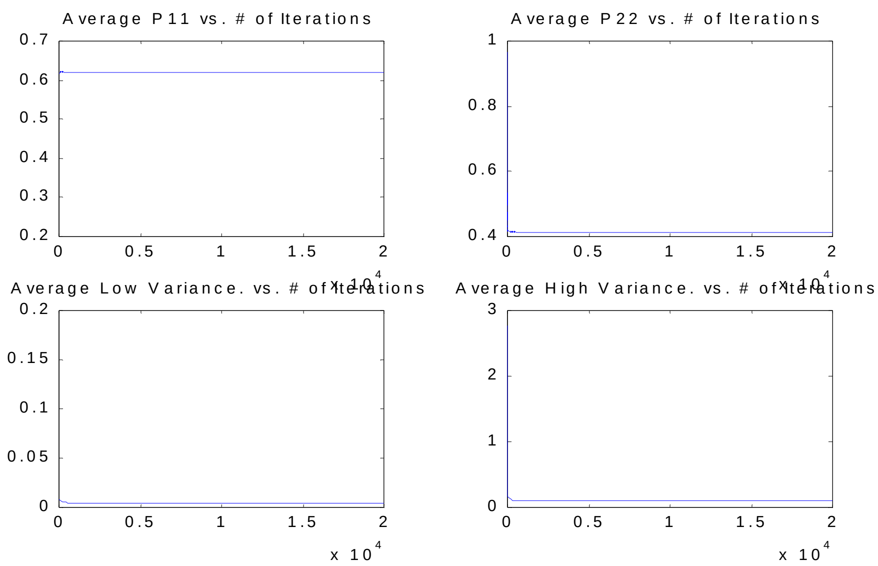

Figure2: Ergodic avererage of estimated parameters vs. # itererations

The stability of the states is quite clear from figure 3. The graph in figure 3 is called assignment map and plot the assignment of the states as gray levels against the iterations, black for state 1 and white for state 2 (see Robert and Mengersen (1998)).[9] Our Gibbs sampler is able to find stable allocations for the data set. Thus, we have quite clear allocation of the low volatility state and a bit blurred picture of the allocations to the high volatility state. This is also confirmed by figure 4, the probabilities of a specific observations being allocated to state 1.

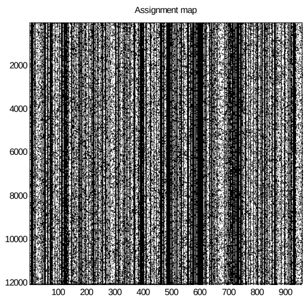

Figure 3: Assignment map

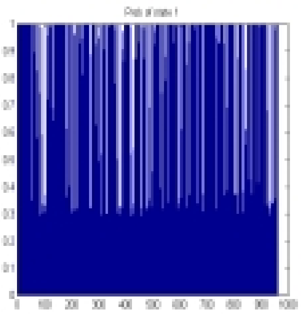

Figure 4: Probability of low volatility per observation for Sweden

The mean, median and the 2.5 upper and lower percentiles of the posterior distribution of the transition probabilities are presented in table 1. Given that we are in regime S we can compute the duration of the regime by $1/\left(1-p_{ij}\right)$ conditional on $i = j$.[10] The last column in table 1 shows the persistence or duration of a state. The expected duration of the states is 2.6 months and 1.7 months for state 1 and state 2. Thus, we seem to catch the heteroscedasticty by switching between regimes with different volatility.

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[9] Robert and Megersen refer to allocation maps. In recent literature (Bilio, Motfort and Robert (1999)) the word assignment maps are used instead of allocation maps.

[10] For proof see Kim and Nelson (1999) pages 71-72

Table 1: Transition probabilities.

Parameter	Posterior mean	median	duration
p_{11}	0.6210 [0.6209, 0.6210]	0.6210	2.6385
p_{22}	0.4131 [0.4130, 0.4132]	0.4130	1.7036

Comment: 2.5 and 97.5 percentiles within brackets

Table 2: Volatlity

Parameter	Posterior mean	median
σ_1	7.9542 [6.6787, 9.6309]	7.8756
σ_2	36.6216 [30.7490, 44.3412]	36.2594

Comment: 2.5 and 97.5 percentiles within brackets

The volatility of Swedish stock market excess return has two regimes, one with a low and one with high standard deviation. The mean, median and the 2.5% upper and lower percentiles of the conditional distributions of the estimated standard deviation parameters are presented in table 2. There is a significant difference in the variance between the two states with 8.0% and 36.6% volatility for state 1 and state 2. The posterior distributions of the volatility parameters are presented in figure 5.

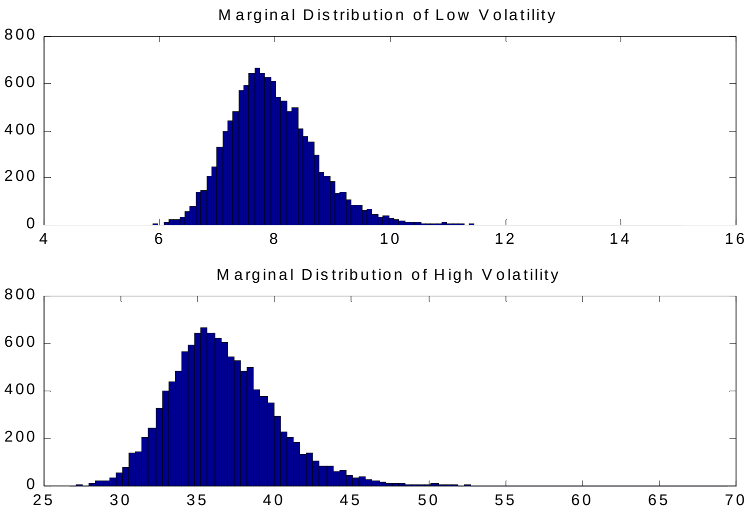

Figure 5: Posterior distribution of low and high volatility for Sweden

4.2 Variance Ratios

We will exemplify the sampled distributions of the different variance ratios using histograms of the results from the five-year horizon, $q = 60$ months.

Figure 6 shows the distributions of the variance ratio test computed for the five-year horizon on the randomized standardized returns and randomized de-standardized returns and the standardized original returns. The mean, median and 95% interval of the variance ratios for all twenty investment horizons is presented in table 3 and table 4.

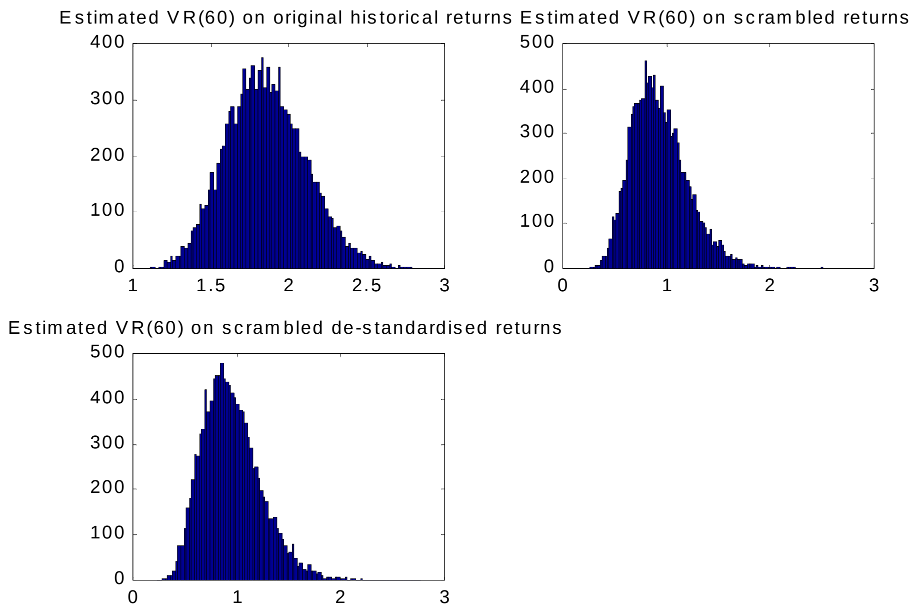

Figure 6: Conditional distribution of 5-year VR for Sweden

The probability values of the VR decrease as the horizon q increase. This is expected as the randomization of the returns leads to flatter posterior distributions of the VR as the investment horizon q increases. The maximum and minimum values of the original VR are $(VR(q)1.62342)$ at 24 months and $(VR(q) = 0.7628)$ at 108 months. This is an unexpected result especially as the high VR occur at 12, 24, and 36 months. Thus, it justifies our approach of utilizing computations of monthly VR with short-run horizons of $2 - 12$ months and long-run horizons of 1 to 10 years. A general result is that the p-values from the standardized returns are lower then the p-values computed from the VR test of the original returns. Our lowest p-value is 0.4125 at 96 months horizon for standardized returns to be compared with the p-value of 0.8449 for original returns. Our highest p-values are all from the short run horizons and the p-values decay with the investment horizon.

However, we cannot reject the null hypothesis of random walk for any of horizon q and any this result is robust to VR computed on standardized or de-standardized returns.

Frennberg and Hansson (1993) finds support of mean reversion in the Swedish stock market and the mean reversion to increase with the length of the investment horizon. This they conclude indicates that the risk in the Swedish stock market decrease with the holding period. Our analysis offsets their result. By simply account for the heteroscedasticty of the data and taking estimation bias into account we can find no support of mean reversion. On the contrary the Swedish stock market can be characterized by two regimes, a tranquil and a volatile, and within the regimes the stock market is random. This finding is in line with what Kim et al (1998a) finds for the U.S. stock market 1926-1986. Thus, accounting for time-variation in volatility and estimation bias improves the variance ratio test.[11]

[11] We have successfully employed this methodology on the Swedish, Norwegian Danish and Finnish stock market data during 1947-1998. Again we find that a tranquil and a volatile regime can describe the volatility. We find no support of mean reversion in these stock markets when we account for the time variation in volatility. This version of the paper is available upon request.

Table 3: Variance ratios

Investment horizon,	Variance ratio VR(q)		
q (months)	Original	Scrambled	Prob. Value
2	1.1635	1.0011 [0.9392, 1.0676]	0.9998
3	1.2024	1.0011 [0.9092, 1.1001]	0.9994
4	1.2369	1.0011 [0.8850, 1.1280]	0.9994
5	1.2547	1.0014 [0.8685, 1.1500]	0.9987
6	1.2652	1.0014 [0.8496, 1.1679]	0.9974
7	1.2971	1.0014 [0.8331, 1.1876]	0.9976
8	1.3235	1.0012 [0.8187, 1.2061]	0.9978
9	1.3504	1.0008 [0.8059, 1.2215]	0.9981
10	1.3888	1.0005 [0.7948, 1.2351]	0.9983
11	1.4347	1.0003 [0.7833, 1.2488]	0.9986
12	1.4844	1.0002 [0.7727, 1.2645]	0.9992
24	1.6234	0.9934 [0.6774, 1.3833]	0.9977
36	1.5316	0.9822 [0.6037, 1.4671]	0.9841
48	1.4284	0.9676 [0.5453, 1.5337]	0.9507
60	1.2687	0.9512 [0.5011, 1.5924]	0.8686
72	1.0926	0.9352 [0.4572, 1.6375]	0.7325
84	0.9013	0.9180 [0.4213, 1.6840]	0.5361
96	0.7794	0.9024 [0.3866, 1.7349]	0.4126
108	0.7628	0.8880 [0.3549, 1.7687]	0.4225
120	0.7648	0.8751 [0.3287, 1.8014]	0.4485

Comment: 2.5 and 97.5 percentiles within brackets

Table 4: Variance ratios

Investment horizon, q (months)	Variance ratio VR(q) Standardised	Scrambled standardised	Prob. Value
2	1.1792 [1.1403, 1.2182]	0.9986 [0.9360, 1.0618]	1.0000
3	1.2469 [1.1775, 1.3169]	0.9974 [0.9056, 1.0937]	1.0000
4	1.3025 [1.2030, 1.4031]	0.9960 [0.8827, 1.1169]	0.9999
5	1.3417 [1.2185, 1.4698]	0.9948 [0.8647, 1.1380]	0.9998
6	1.3709 [1.2242, 1.5251]	0.9935 [0.8469, 1.1543]	0.9996
7	1.4183 [1.2517, 1.5934]	0.9922 [0.8310, 1.1718]	0.9997
8	1.4669 [1.2799, 1.6625]	0.9910 [0.8171, 1.1876]	0.9996
9	1.5126 [1.3061, 1.7272]	0.9898 [0.8066, 1.2023]	0.9997
10	1.5667 [1.3428, 1.7992]	0.9886 [0.7955, 1.2142]	0.9999
11	1.6279 [1.3918, 1.8770]	0.9875 [0.7824, 1.2282]	1.0000
12	1.6907 [1.4404, 1.9540]	0.9863 [0.7707, 1.2421]	1.0000
24	2.0037 [1.6134, 2.4181]	0.9731 [0.6678, 1.3501]	0.9998
36	2.0489 [1.5883, 2.5479]	0.9609 [0.5993, 1.4346]	0.9990
48	2.0097 [1.5269, 2.5484]	0.9480 [0.5422, 1.5043]	0.9970
60	1.8537 [1.3860, 2.3900]	0.9344 [0.4892, 1.5589]	0.9877
72	1.6442 [1.1890, 2.1873]	0.9205 [0.4522, 1.6095]	0.9601
84	1.4437 [0.9822, 2.0016]	0.9060 [0.4157, 1.6521]	0.8992
96	1.3213 [0.8601, 1.8903]	0.8914 [0.3836, 1.6950]	0.8449
108	1.3087 [0.8413, 1.8856]	0.8765 [0.3541, 1.7394]	0.8406
120	1.3103 [0.8349, 1.8962]	0.8617 [0.3292, 1.7732]	0.8433

Comment: 2.5 and 97.5 percentiles within brackets

5 Conclusion

This paper addresses the question if the Swedish stock market is subject to mean reversion. Previous studies find support of mean reversion in the Swedish stock market and the mean reversion to increase with the length of the investment horizon. However the result of these studies are controversial as they ignore the assumption of constant expected return. Resent research have found that heteroscedasticity seriously affects the probability of the variance ratio test to reject the null hypothesis of random walk.

We model the well-known heteroscedasticity of the stock market returns with a two state hidden Markov model of normal mixtures. The model is estimated with Bayesian approach of Gibbs sampling, a computer intensive Markov chain Monte Carlo method. Our two state hidden Markov model is clearly specified along with the priors and prior distributions employed in the Gibbs sampler. Further we use the information at each run of Gibbs sampler to compute variance ratios test on standardized as well as de-standardized returns.

Our analysis finds no support for mean reversion and we cannot reject the null hypothesis of random walk for any of the investment horizons. This result is robust to variance ratios computed on standardized or de-standardized returns. Our two state regime switching models of normal distributions captures the variance as a tranquil and a volatile state and suggests that mean reversion if found in the Nordic stock markets can be explained by time variation in the volatility. Within the regimes the market is random.

Acknowledgments

This paper is to be presented at the 8:th World Congress of the Econometric Society in Seattle, August 2000.

I am indebted to professor Tobias Rydén and Birger Nilsson for valuable comments and suggestions on various aspects of the paper. I would like to thank the participants at the Washington Area Finance Association Spring Meeting 2000, European Finance Management Association Annual Meeting 2000 and the seminar participants at the Department of Economics at Lund University. Financial support from Jan Wallander & Tom Hedelius' foundation, Stiftelsen för främjande av ekonomisk forskning vid Lunds Universitet, Crafoordska foundation and The Royal Swedish Academy of Science is gratefully acknowledged.

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