Money-Back Warranties

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Very Preliminary. Do not cite.

Abstract

We present a model of monopoly provision of money-back warranties. A buyer values an object more than a seller. They are both risk neutral and initially have no private information. The buyer can, however, acquire information and learn his true valuation for the object. Information acquisition is wasteful because it is costly, and because it reduces the gains from trade. The seller uses a warranty to prevent wasteful information acquisition, by offering the buyer part of the benefits that information would have given him, namely the ability not to purchase useless objects.

We compare the optimal contracts when a warranty can be offered and when it cannot. We find that when the cost of information is lower than a threshold, allowing the seller to offer a warranty is welfare improving. However, when the cost of information is higher than the threshold, it is socially optimal to forbid money-back warranties.

1 Introduction

The return of goods is an important part of many economic transactions. Consider for example mail-ordered goods, or electronic and photographic material.[1] Many goods

[1] In the electronics industry, or in other industries that produce and sell high learning products, a large number of returns are not really defective. A high learning product is one that requires user knowledge or expertise for proper operation. Such products often are returned in large numbers but there is really nothing wrong with them, except that the consumer could not figure out how to make them work. See Chapter 2 of Rogers and Tibben-Lembke (1998).

are sold with the option to return them if the buyer is not satisfied. Sometimes the promise is explicit, in which case we have "money-back warranties". Other times, it is implicitly understood that the consumer may return the object for refund or credit.[2]

In the economic literature the term "warranty" usually designates an arrangement where the seller of a product, upon occurrence of a prespecified *contractible* event (malfunctioning of the product), pays a compensation to the buyer. One standard rationale for warranties relies on the seller's incentive to signal his private information about the quality of the product; it is less costly for the seller to offer a warranty when the quality of the good is high rather than low. Other models feature warranties as an additional instrument, beyond the price and the probability of sale, to screen different types of consumers; in such models, risk averse consumers know their own valuation for the object, but the seller does not. Finally, some models present warranties as insurance against the malfunctioning of the product; this explanation relies on consumers being more risk averse than producers.

In this paper we address a different contracting issue, namely the arrangement by which a seller allows the buyer to return the good for a refund, no questions asked. This arrangement does not condition on any commonly observable event, such as "malfunctioning" or "failure". The event that triggers the return is is the buyer's idiosyncratic perception of the quality of the good, which is unobservable to the seller. Such arrangements are common, for instance, in retail sales, where the quality of the good is the enjoyment that the consumer derives from it, and thus may not objectively measurable. Consider as an example the buyer of a book who has the option of returning the book for cash or, as more common in Europe, for credit. Why does the seller offer this option? Risk aversion is implausible, given the small sums involved, and signaling motives on the part of the seller are unlikely. The screening models discussed above do not work either, because the contract does not condition on "failure" of the book. We propose that, in this case, the money-back warranty is a means of preventing the buyer from acquiring information about the book before purchasing it.

To explore this issue, we present a model where buyer and seller are risk neutral

[2] "Generally, customers who believe that an item does not meet their needs, will return it, regardless of whether it functions properly or not. In an interesting example of this behavior, one retailer recently reported the return of two ouija boards. Ouija boards are children toys that, supposedly, allow contact with the spirit world. On one ouija board there was a note describing that it did not work because no matter how hard we tried, we could not get any good answers from the other side. The other ouija board returner said that the reason for return was: too many spirits responded to the ouija board session, and things became too scary. In both cases, the consumers were allowed to return these defective products." Rogers and Tibben-Lembke (1998), p. 19.

and have no private information about the buyer's valuation of the object. Only the buyer can find out his valuation for the object, incurring a cost of information. In this model the seller uses the warranty to discourage wasteful information acquisition on the part of the buyer. Information acquisition is wasteful for two reasons: it is costly, and information that is private to the buyer reduces the gains from trade because consumers with low valuations will not buy.

To understand the role of warranties, consider an object whose value to the seller is zero, and whose value to the buyer is a random variable distributed uniformly between zero and one. Neither the seller nor the buyer know the buyer's true valuation, but the buyer can incur a cost of $3/32$ and learn his true valuation for the object. We now show that, absent a warranty, the best choice for the seller is to set a price of $\sqrt{6/32}$. Acquiring information allows the buyer to not purchase the object if his valuation is below this price. In this event, which has probability $\sqrt{6/32}$, the expected surplus from buying is the expected value of the object minus the price, viz. $-(1/2)\sqrt{6/32}$. The value of information to the buyer is therefore $(1/2)\left(\sqrt{6/32}\right)^2 = 3/32$; this is the amount that the buyer saves if he becomes informed. Thus, at the given price the buyer is willing to forego information acquisition and accept the contract, and the seller makes a profit of $\sqrt{6/32}$. At any higher price the buyer will acquire information, and the seller will earn profits of $1/4 < \sqrt{6/32}$. Consider now the following contract involving a warranty: the seller sets a price of $1/2$, and allows the buyer to return the object in exchange for a restitution of $1/4$.[3] Under this contract, the buyer enjoys negative surplus when his valuation turns out to be less than $1/2$, but he does not return any object which he values at more than $1/4$. When the value is smaller than $1/4$ the buyer returns the object, so the buyer's expected surplus is $(1/4)[-1/4]$. When the value is between $1/4$ and $1/2$ the expected surplus is $(1/4)[(3/8) - 1/2]$. Adding up we obtain the value of information for the buyer, which is exactly $3/32$, hence the buyer is willing to accept this contract without acquiring information. The seller earns $1/2$ when the object is not returned and $1/4$ when it is returned, for a total expected profit of $7/16 > \sqrt{6/32}$. Thus, offering a warranty is profitable for the seller.

As the above example suggests, the seller wants to induce the buyer to purchase without acquiring information. To this end, offering a warranty is preferable to lowering the price. The seller prefers the warranty because it reduces the revenue from only low-valuation types, instead of lowering the price for all types. The buyer regards the

[3] Warranties where the seller gives back the full amount are discussed in Section 5.

warranty as a good substitute for information because it kicks in when his valuation is low, precisely when information would have mattered (the buyer uses information to decide whether or not to buy). In this paper we pose this as a mechanism design problem, and solve for the optimal direct revelation game. We show that at the optimal contract the good is traded more often than would be the case if the buyer was informed. Thus, the optimal contract eliminates the costly information acquisition, and introduces an inefficiency smaller than the inefficiency that would arise if the buyer was informed.

We present a sufficient condition under which the outcome of the optimal revelation game can be implemented as follows: the seller posts a fixed price for the object, and agrees to take back the object and give back part of the price paid—if the buyer so wishes. Under this contract the buyer never gets informed, always purchases the object, and sometimes—when his valuation turns out to be low—returns the object.

We also work out the optimal contract when a warranty is impossible because the object cannot be returned. Objects that cannot be returned are ice-cream cones, and hair cuts. A regulation making it very expensive for sellers to dispose of returned items may also make offering a warranty *de facto* impossible. We solve for the optimal direct revelation mechanism, and we find that the seller can implement that allocation simply by asking for a fixed price. In this case, however, the buyer will sometimes acquire information. To see why, keep in mind that the optimal contract must consider the incentive to acquire information. The only way that a seller can prevent information acquisition in a world where objects cannot be returned is to lower the price, but lowering the price hurts profits. In fact, when the cost of information is very small, it is not profitable for the seller to prevent information acquisition because he would have to lower the price too much. At the optimal contract, the seller anticipates that information will be acquired and asks for the price that is optimal when dealing with an informed agent.

We ask the question of whether money-back warranties are desirable by comparing the optimal contracts when the object can, or cannot, be returned. In our model, allowing the seller to offer a warranty is not necessarily welfare-improving, because when the buyer returns an object we regard the object as not traded. In fact, when the cost of acquiring information is high relative to the value of the object, forbidding the seller from offering a warranty is the optimal policy. This is because the seller will then find it optimal to lower the price, ensuring trade with probability one and no information acquisition. By contrast, when the cost of information acquisition is low the seller should be allowed to offer a warranty, since otherwise the optimal contract involves information acquisition and limited trade of the object.

1.1 Related Literature

The literature on warranties is vast. All of it, to our knowledge, assumes that there is a contractible event, the "failure", that leads to the warranty being invoked. The textbook model is due to Grossman (1981), where warranties are a means of signalling quality.[4] There are papers where warranties play a screening role; see Matthews and Moore (1987), and the literature cited therein. These models are closer to our, in that private information rests with the buyers. In Heal (1977) warranties play a risk-sharing role. Mann and Wissink (1990) discuss the difference between money-back warranties and replacement warranties; a money-back warranty ensures restitution of (part of) the consumer's purchase price in case of failure, and this terminates the relationship between firm and consumer. Under a replacement warranty, a replaced product carries the same warranty as the original and may itself be replaced if it fails to work. This difference leads to different incentives for firms to produce quality.

Our work contributes to the literature on contracting with precontractual information gathering. This literature has focused on principal-agent relationships where the agent can acquire information about his disutility of production; see Cremer, Khalil and Rochet (1998) for a review. The closest paper to ours is Cremer and Khalil (1992); that paper examines a principal-agent model where before the contract is signed the agent can pay a cost to discover his disutility of production, and after the contract is signed the agent learns his disutility of production at no cost.

2 Model without warranties

A seller has an object which he values at 0. A buyer has value $\theta \geq 0$ for the object.[5] Neither the buyer nor the seller know θ; they regard it as a realization from a random variable distributed according to F with support $[0, 1]$. The buyer can acquire information, i.e. pay c and learn the true value of θ.

The timing is as follows. First, the seller proposes a contract. After observing the contract, the buyer decides whether or not to acquire information. Then the buyer decides whether or not to participate in the contract. This timing captures a situation where information acquisition is a covert activity: the seller cannot observe whether the buyer has acquired information.

[4] See for example its account in Tirole (1995), p. 441-443.

[5] Since the seller values the object more than the buyer, the efficient allocation is for the object always to be traded. Thus, information has no efficiency role in this model. This is discussed further in Section 5.

We denote $\bar{\theta} = E(\theta)$. In what follows we will sometimes need the following assumption:

Assumption 1 *$H(t) := t[1 - F(t)]$ is strictly pseudo-concave, i.e. H has a unique maximum, say t_I, and $H(t)$ is strictly decreasing away from t_I.*

A sufficient condition for Assumption 1 to hold is that the function $\frac{\theta f(\theta)}{1-F(\theta)}$ be strictly increasing in θ; this assumption characterizes the "regular case" in Myerson (1981).

In the next section we characterize the optimal take-it-or-leave-it offer. In the following one we set up the mechanism design problem, and give sufficient conditions under which the optimal contract is indeed a take-it-or-leave-it offer.

2.1 The optimal Tioli

One simple contract is the take-it-or-leave-it offer (Tioli), where the seller asks a price t for the object which the buyer can accept or refuse. To characterize the optimal DRM we need to talk about the value of information for the buyer under a Tioli. The value of information is the increase in utility that the buyer enjoys by virtue of being informed. Consider a Tioli at price t. When $t < \bar{\theta}$, the buyer purchases the object if he is uninformed. Then the value of information is

$$V(t) = \int_0^1 [\max\{0, \theta - t\} - (\theta - t)]\, dF(\theta) = \int_0^t (t - \theta)\, dF(\theta).$$

When $t > \bar{\theta}$, the buyer will not purchase the object if he does not acquire information. Then the value of information is

$$V(t) = \int_0^1 [\max\{0, \theta - t\} - 0]\, dF(\theta) = \int_t^1 (\theta - t)\, dF(\theta).$$

$V(t)$ is continuous at $\bar{\theta}$ and

$$\begin{aligned} V(0) &= 0 \\ V(1) &= 0 \\ V'(t) &= F(t) > 0 \qquad && \text{for } 0 \le t < \bar{\theta} \\ V'(t) &= F(t) - 1 < 0 \qquad && \text{for } \bar{\theta} < t \le 1. \end{aligned}$$

Since $V(t)$ is single-peaked, in general its inverse is double-valued. Denote with $V_-^{-1}(c)$ and $V_+^{-1}(c)$ the smallest and largest value of the inverse. Define $\bar{c} := V\left(\bar{\theta}\right)$,

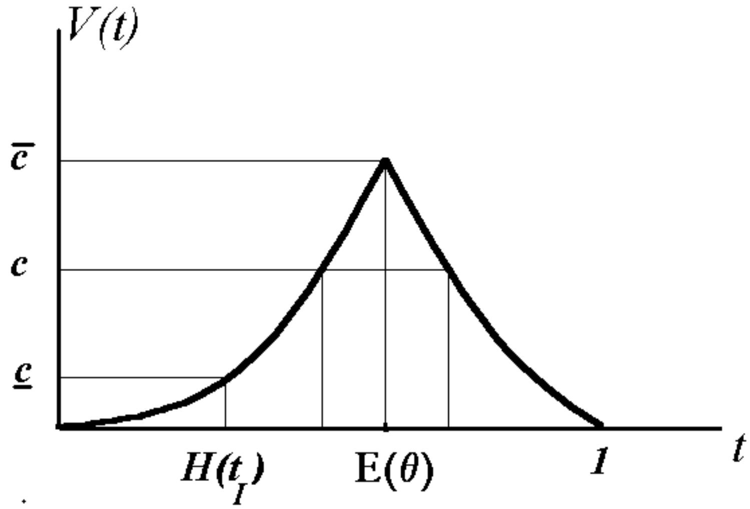

Figure 1: The value of information in a Tioli.

and $\underline{c} := V(H(t_I))$. Because $H(t_I) < \bar{\theta}$, we have $\underline{c} < \bar{c}$.[6] Refer to figure 1. When $c > \bar{c}$ there is no value t for which the buyer acquires information. When $c < \bar{c}$ the buyer acquires information if and only if $V_-^{-1}(c) \leq t \leq V_+^{-1}(c)$.

The following assumption ensures that the optimal price that a seller would ask of an informed buyer is not too large. This is not an overly restrictive assumption.

Assumption 2 $V(t_I) > \underline{c}$.

Proposition 1 *Assume Assumption 2. Then the optimal Tioli has a price* $t^*(c)$ *defined by*

$$t^*(c) = \begin{cases} \bar{\theta} & \textit{for } c > \bar{c} \\ V_-^{-1}(c) & \textit{for } \underline{c} < c < \bar{c} \\ t_I & \textit{for } c < \underline{c}. \end{cases}$$

[6] For any $t < 1$ we have $t[1 - F(t)] = \int_t^1 t dF(\theta) < \int_t^1 \theta dF(\theta) < \int_0^1 \theta dF(\theta) = \bar{\theta}$, and in particular $t_I[1 - F(t_I)] < \bar{\theta}$.

Under this contract, the buyer acquires information if and only if $c < \underline{c}$.

Proof: When $c > \bar{c}$ there is no value t for which the buyer acquires information, thus $t^*(c) = \bar{\theta}$ and $\pi^*(c) = \bar{\theta}$.

When $\underline{c} < c < \bar{c}$, the buyer acquires information if and only if $V_-^{-1}(c) \leq t \leq V_+^{-1}(c)$. Thus, if $t = V_-^{-1}(c)$ then $\pi = V_-^{-1}(c)$; if $t > V_-^{-1}(c)$ then $\pi = H(t) \leq H(t_I) < V_-^{-1}(c)$ (the first inequality holds by definition of t_I, and the second follows from the restriction $\underline{c} < c$). The seller prefers to set $t^*(c) = V_-^{-1}(c)$, whence $\pi^*(c) = t^*(c)$ and the buyer does not acquire information.

When $c < \underline{c}$, if $t = t_I$ the buyer acquires information since by Assumption 2 $t_I < V_+^{-1}(\underline{c})$, and the seller makes $\pi = H(t_I)$. If $t = V_-^{-1}(c)$, then $\pi = V_-^{-1}(c) < H(t_I)$ (the inequality holds because $c < \underline{c}$). The seller prefers to set $t^*(c) = t_I$, whence $\pi^*(c) = H(t_I)$ and the buyer acquires information. ∎

As c increases from zero, at first the buyer's surplus decreases linearly in c; but when c hits $\underline{c}$ the buyer's surplus jumps up by $\underline{c} + F(t_I) E(\theta|\theta < t_I)$.[7] Thus, at $c = \underline{c}$ the buyer appropriates all the welfare gains from sparing the cost c and from selling the object more often

2.2 The optimal mechanism is a Tioli

In this section we study the optimal direct revelation mechanism (DRM). In our context, a DRM is a game where the buyer is asked whether he is informed or not, and his valuation if he is informed. If the buyer reports to be uninformed he is given a probability of getting the object p^u and is forced to pay w^u. If the buyer reports to be informed with type $\widehat{\theta}$ he pays $w\left(\widehat{\theta}\right)$ and is given a probability $p\left(\widehat{\theta}\right)$ of receiving the object. A DRM is described by the list $(p(\theta), w(\theta)) \cup (p^u, w^u)$ of probabilities of getting the object and payments.

We begin by showing that given any DRM where the buyer chooses not to become informed, there is a better mechanism consisting of a Tioli. The Tioli mechanism is better in that it gives the seller the same profit, and it gives the buyer a smaller incentive to acquire information.

Lemma 1 *Consider any direct revelation mechanism where the buyer chooses to stay uninformed. Then there is a take-it-or-leave-it offer (Tioli) which yields at least as great a profit to the seller.*

[7] This is because the seller's profits are continuous in c while welfare is discontinuous because of the shape of the optimal contract.

Proof: See Appendix. ∎

The next theorem guarantees that, when Assumptions 1 and 2 are satisfied, the allocation achieved by the optimal DRM can be implemented through a Tioli.

Theorem 1 *Assume Assumptions 1 and 2. Then the optimal direct revelation mechanism can be implemented through a Tioli.*

Proof: Consider any DRM. If under this mechanism the buyer chooses to stay uninformed, by Lemma 1 the seller can earn a profit at least as great with a Tioli.

Suppose under the DRM the buyer chooses to become informed. Then we show that one of the following two deviations is profitable, depending on the value of c.

Deviation A. switch to a Tioli at t_I.

Deviation B. switch to a Tioli at $\min\left\{V_{-}^{-1}(c), \overline{\theta}\right\}$.

Consider the case $c < V(t_I)$. Then the seller should make deviation A. We must show that, faced with the contract of deviation A, the buyer is willing to acquire information and purchase the object. Then the seller's profits will be $H(t_I)$ which, in view of Assumption 1, are the most that a seller can make when the buyer is informed (see Myerson (1981)): thus this deviation will profitable. The buyer is willing to acquire information because $c < V(t_I)$. We must check that, after the buyer acquires information and accepts the contract, he ends up with a nonnegative surplus. To this end, consider first the case where $t_I > \overline{\theta}$. Rewriting $V(t_I) - c > 0$ we get $\int_{t_I}^{1}(\theta - t_I)\,dF(\theta) - c > 0$, viz. the buyer's expected surplus is positive. Consider next the case where $t_I < \overline{\theta}$. Rewriting this condition we get

$$
\begin{aligned}
0 &< \int_0^{t_I}(\theta - t_I)\,dF(\theta) + \int_{t_I}^{1}(\theta - t_I)\,dF(\theta) \\
&= -V(t_I) + \int_{t_I}^{1}(\theta - t_I)\,dF(\theta) \\
&< -c + \int_{t_I}^{1}(\theta - t_I)\,dF(\theta),
\end{aligned}
$$

where the last inequality follows because $c < V(t_I)$. This proves that the buyer's expected surplus is positive.

Consider the case $c > V(t_I)$. Then the seller should make deviation B. Faced with the contract of deviation B, the buyer does not acquire information and enjoys nonnegative surplus. The seller's profits are $\min\left\{V_{-}^{-1}(c), \overline{\theta}\right\}$ which, in view of Assumption 2, are greater than $H(t_I)$. Since $H(t_I)$ is the most that the seller could be extracting from an informed buyer, deviation B is profitable. ∎

3 Model with warranties

In this section we investigate the optimal direct revelation mechanism in the case where a money-back warranty is possible. For a warranty to be possible, the buyer must be able to experience the object and return it without obliterating its value to himself or the seller. We model this by assuming that the seller can show the buyer the object. If the seller shows the object, the buyer costlessly learns his true valuation. When the equilibrium of the DRM entails the seller showing the object and the object subsequently being not traded with positive probability, we talk about the DRM involving a warranty. We look for the optimal contract, out of all direct revelation mechanisms of the following form:

Stage 1 The buyer is asked whether he is informed. He answers Yes or No.

Stage 2 If the buyer answers Yes, he is asked to reveal his valuation $\widehat{\theta}$ and faced with a schedule $\left(\widetilde{w}\left(\widehat{\theta}\right),\widetilde{p}\left(\widehat{\theta}\right)\right)$ of prices and probabilities of trade. If buyer answers No, he pays a fee b and is shown the object. Then, the buyer is asked to reveal his valuation and faced with a schedule $\left(\widetilde{\widetilde{w}}\left(\widehat{\theta}\right),\widetilde{\widetilde{p}}\left(\widehat{\theta}\right)\right)$.

Of course, the buyer retains the option of acquiring information before participating in this mechanism. However, acquiring information is inefficient since the seller can provide information for free.

Lemma 2 *In the optimal direct revelation mechanism the buyer never acquires information.*

Proof: Take a mechanism, and suppose the buyer acquires information before participating in this mechanism. The seller could change $\left(\widetilde{\widetilde{w}}(\theta),\widetilde{\widetilde{p}}(\theta)\right)$ and make it equal to $\left(\widetilde{w}(\theta),\widetilde{p}(\theta)\right)$, and set $b=c-\varepsilon$. Then the buyer would rather not acquire information, and the seller would improve his revenue by $c-\varepsilon$. Thus, no mechanism that induces a buyer to acquire information can be optimal. ∎

Although at equilibrium the buyer does not acquire information, the possibility of information acquisition affects the shape of the optimal contract. Indeed, the mechanism that maximizes the seller's expected revenue must meet the constraint that the buyer does not acquire information before the contract is signed.

The following lemma, in conjunction with the previous one, implies that the seller does not lose anything by restricting attention to simple contracts where the buyer pays a fee to be shown the object, and then is faced with a schedule $\left(\widetilde{\widetilde{w}}(\theta),\widetilde{\widetilde{p}}(\theta)\right)$.

Lemma 3 *Consider any direct revelation mechanism where the buyer does not acquire information. The seller can obtain the same profits using the following contract. The buyer pays a fee* b *and is shown the object; then, the buyer is asked to reveal his valuation and faced with a schedule* $\left(\tilde{\tilde{w}}(\theta), \tilde{\tilde{p}}(\theta)\right)$.

Proof: See Appendix. ∎

Next, we characterize the optimal DRM in terms of the probability that the object is returned. We show that, if $\theta f(\theta)/F(\theta)$ is strictly increasing, the probability that the object is returned is zero for high realizations of θ, and one for low realizations of θ. Thus, the optimal contract is implementable in the following way: The buyer pays a price t and gets property rights over the object. However, the buyer retains the right to return the object to the seller, and get back $t - S$. S is an optimally determined "stocking fee", which the seller gets to keep when the object is returned.

If $\theta f(\theta)/F(\theta)$ is not strictly increasing, at the optimal DRM the probability that the object is returned is decreasing in the realized value of θ, and assumes values of zero, p_1, p_2, or one (where p_1 and/or p_2 may equal one).

Theorem 2 *If* $\theta f(\theta)/F(\theta)$ *is strictly increasing, the optimal probability schedule has one jump. If* $\theta f(\theta)/F(\theta)$ *is not strictly increasing, generically the optimal probability schedule has at most three jumps.*

Proof: We want to characterize the optimal $\left(\tilde{\tilde{w}}(\theta), \tilde{\tilde{p}}(\theta)\right)$ and b. Denote $(w(\theta), p(\theta)) := \left(\tilde{\tilde{w}}(\theta) - b, \tilde{\tilde{p}}(\theta)\right)$, and

$$u_B(\theta) = \theta p(\theta) - w(\theta).$$

Solving for the optimal $(w(\theta), p(\theta))$ is equivalent to solving for the optimal $\left(\tilde{\tilde{w}}(\theta), \tilde{\tilde{p}}(\theta)\right)$ and b.

Without loss of generality we restrict to DRMs that are incentive compatible, i.e. it is optimal for the buyer to announce his true type, and individually rational, i.e. the buyer's expected surplus is nonnegative, $\int_0^1 u_B(\theta)\, dF(\theta) \geq 0$. Finally, it must be the case that the buyer does not gain by acquiring information. A buyer who acquires information and accepts the contract has a surplus of $\int_0^1 \max\{u_B(\theta), 0\}\, dF(\theta) - c$. If it were the case that $\int_0^1 \min\{u_B(\theta), 0\}\, dF(\theta) < -c$, then by acquiring information the buyer could increases his surplus relative to $\int_0^1 u_B(\theta)\, dF(\theta)$, which is nonnegative by individual rationality. Thus, at the optimal contract it must be $\int_0^1 \min\{u_B(\theta), 0\}\, dF(\theta) \geq -c$.

The seller chooses $(w(\theta), p(\theta))$ to maximize exected profits subject to the above constraints. The seller's problem is

$$\max_{w(\theta),p(\theta)} \int_0^1 w(\theta)\, dF(\theta)\; s.t. \begin{cases} \theta p(\theta) - w(\theta) \geq \theta p(\theta') - w(\theta') \text{ for all } \theta, \theta' \; (IC) \\ \int_0^1 u_B(\theta)\, dF(\theta) \geq 0 \; (IR) \\ \int_0^1 \min\{u_B(\theta), 0\}\, dF(\theta) \geq -c \; (VI) \end{cases} \tag{1}$$

Constraint VI says that the value of gathering information prior to signing contract $(w(\theta), p(\theta))$ is lower than the cost of information. The solution to problem 1 is characterized by a Lagrangean. The Lagrangean we write lacks the IC constraints, but we solve it assuming that the schedule $(w(\theta), p(\theta))$ satisfies a local version of IC constraints. From this exercise we obtain necessary conditions for optimality of a schedule. The Lagrangean is

$$\mathcal{L} = \int_0^1 [\theta p(\theta) - u_B(\theta)]\, dF(\theta) + \mu \int_0^1 u_B(\theta)\, dF(\theta) + \lambda \left[\int_0^1 \min\{u_B(\theta), 0\}\, dF(\theta) + c\right].$$

To simplify the Lagrangean, it is useful to derive an expression for $\int_0^y u_B(\theta)\, dF(\theta)$. From the IC constraint we have the well-known expression $u_B(\theta) = u_B(0) + \int_0^\theta p(s)\, ds$ (see Myerson (1981)); then for all $y \geq 0$

$$\begin{aligned} & \int_0^y u_B(\theta)\, dF(\theta) \\ = & \int_0^y \left[u_B(0) + \int_0^\theta p(s)\, ds\right] f(\theta)\, d\theta \\ = & \; u_B(0) F(y) + \int_0^y \int_s^y f(\theta)\, d\theta p(s)\, ds \\ = & \; u_B(0) F(y) + \int_0^y [F(y) - F(s)]\, p(s)\, ds. \end{aligned}$$

Let θ_v denote the value of θ such that $u_B(\theta) > 0$ if and only if $\theta > \theta_v$. By IC $u_B(\theta)$ is nondecreasing, so θ_v is well-defined. Substituting for $\int u_B(\theta)$ the Lagrangean becomes

$$\begin{aligned} \mathcal{L} = & -u_B(0) + \int_0^1 [f(\theta)\theta p(\theta) - (1 - F(\theta))\, p(\theta)]\, d\theta + \mu \left[u_B(0) + \int_0^1 (1 - F(\theta))\, p(\theta)\, d\theta\right] \\ & + \lambda \left[u_B(0) F(\theta_v) + \int_0^{\theta_v} [F(\theta_v) - F(\theta)]\, p(\theta)\, d\theta + c\right] \\ = & \; u_B(0) [-1 + \mu + \lambda F(\theta_v)] \\ & + \int_0^1 \left[f(\theta)\theta - (1 - F(\theta)) + \mu (1 - F(\theta)) + I_{(-\infty,\theta_v]}(\theta)\, \lambda (F(\theta_v) - F(\theta))\right] p(\theta)\, d\theta + \lambda c. \end{aligned}$$

At the optimal contract the seller must not be able to benefit from offering a lump sum transfer, positive or negative, to the buyer. Thus, a necessary condition for optimality is that the derivative of the Lagrangean with respect to $u_B(0)$ equals zero.When we perform the differentiations we take into account the change in θ_v.

$$\begin{aligned} 0 &= \frac{\partial \mathcal{L}}{\partial u_B(0)} = -1+\mu+\lambda F(\theta_v) + u_B(0)\lambda f(\theta_v)\frac{\partial \theta_v}{\partial u_B(0)} + \int_0^{\theta_v} \lambda f(\theta_v)\frac{\partial \theta_v}{\partial u_B(0)} p(\theta)\,d\theta \\ &= -1+\mu+\lambda F(\theta_v)+\lambda f(\theta_v)\frac{\partial \theta_v}{\partial u_B(0)}\left(u_B(0)+\int_0^{\theta_v} p(\theta)\,d\theta\right) \\ &= -1+\mu+\lambda F(\theta_v)+\lambda f(\theta_v)\frac{\partial \theta_v}{\partial u_B(0)} u_B(\theta_v) \\ &= -1+\mu+\lambda F(\theta_v), \end{aligned} \tag{2}$$

where the last equality follows from $u_B(\theta_v)=0$. Using this equality the expression in square brackets inside the integral simplifies to

$$m(\theta) := \begin{cases} f(\theta)\theta + (1-\mu-\lambda)F(\theta) & \text{for } \theta < \theta_v \\ f(\theta)\theta - [1-F(\theta)] + \mu[1-F(\theta)] & \text{for } \theta > \theta_v, \end{cases}$$

and the Lagrangean reads

$$\mathcal{L} = \int_0^1 m(\theta)\,p(\theta)\,d\theta + const.$$

A necessary condition for the function $p^*(\theta)$ to be the solution to problem (1) is that it maximizes the Lagrangean, among all functions $p(\theta)$ with the property that $u_B(0)=u_0$, where u_0 is part of the solution to problem (1).

If $m(\theta)$ is strictly quasimonotone,[8] there is a type θ^* such that $m(\theta)$ is strictly negative for $\theta < \theta^*$ and strictly positive for $\theta > \theta^*$. Then, the function p^* that maximizes $\mathcal{L}$ is $p^*(\theta)=0$ for $\theta<\theta^*$, and $p^*(\theta)=1$ for $\theta>\theta^*$.

A sufficient condition for quasimonotonicity of $m(\theta)$ is that $f(\theta)\theta/F(\theta)$ is strictly increasing. In this case, $m(\theta)$ is strictly quasimonotone on $[0,\theta_v]$. Let us show that $m(\theta)$ is quasimonotone on the entire range of θ. To this end, notice that:

a) $m(\theta_v)$ must be strictly positive, since there must be positive probability of sale at type below θ_v and m is strictly quasimonotone on $[0,\theta_v]$.

b) Because $f(\theta)\theta/F(\theta)$ is increasing, then $f(\theta)\theta/[1-F(\theta)]$ is strictly increasing. Thus, $m(\theta)$ is strictly quasimonotone on $[\theta_v,1]$, and hence strictly positive in view of (a) above.

[8] A function q is strictly quasimonotone if $q(x)=0$ implies $q(x')>0$ for all $x'>x$.

Parts a) and b) together imply that $m(\theta)$ is strictly positive for $\theta > \theta_v$. Then $m(\theta)$ is strictly quasimonotone on $[0,1]$.

If $m(\theta)$ is not quasimonotone, we need to follow a procedure introduced by Myerson (1981). Define $M(\theta) = \int_0^\theta m(s)\,ds$, and let $\overline{M}(\theta)$ denote the convex hull of $M(\theta)$. Because M is differentiable, $\overline{M}$ is differentiable. Let $\overline{m}(\theta)$ denote the derivative of $\overline{M}(\theta)$.

$$\begin{aligned}
\mathcal{L} &= \int_0^1 m(\theta)\,p(\theta)\,d\theta \\
&= \int_0^1 \overline{m}(\theta)\,p(\theta)\,d\theta + \int_0^1 [m(\theta) - \overline{m}(\theta)]\,p(\theta)\,d\theta \\
&= \int_0^1 \overline{m}(\theta)\,p(\theta)\,d\theta + \left[M(\theta) - \overline{M}(\theta)\right] p(\theta)\Big|_{\theta=0}^1 - \int_0^1 \left[M(\theta) - \overline{M}(\theta)\right] dp(\theta) \\
&= \int_0^1 \overline{m}(\theta)\,p(\theta)\,d\theta - \int_0^1 \left[M(\theta) - \overline{M}(\theta)\right] dp(\theta), \qquad (3)
\end{aligned}$$

where the last equality holds because by definition of $\overline{M}(\theta)$ we have $\left[M(\theta) - \overline{M}(\theta)\right] p(\theta)\Big|_{\theta=0}^1 = 0$. We want to characterize the function p^* that maximizes expression (3).

By construction $M(\theta) \geq \overline{M}(\theta)$, so to minimize the second term in expression (3) we would like p^* to be constant. Keeping this in mind, it is easy to see that the optimal p^* must be equal to 1 for values of θ such that $\overline{m}(\theta) > 0$, and equal to zero for values of θ such that $\overline{m}(\theta) > 0$. Since $\overline{m}$ is nondecreasing by construction, there is a $\theta^* < 1$ with the property that $\overline{m}(\theta) > 0$ if and only if $\theta > \theta^*$, and a $\theta_* > 0$ with the property that $\overline{m}(\theta) < 0$ if and only if $\theta < \theta^*$. Thus, $p^*(\theta) = 0$ for $\theta < \theta_*$, and $p^*(\theta) = 1$ for $\theta > \theta^*$.

When $\theta_* < \theta < \theta^*$ the function $\overline{m}(\theta)$ is constant at zero, so $p^*(\theta)$ must be constant unless $M(\theta) - \overline{M}(\theta) = 0$. We now show that the maximum number of points in (θ_*, θ^*) where $M(\theta) = \overline{M}(\theta)$ is one. At such a point, let us call it θ_1, the function m crosses the zero line from below and the following two integral constraints are satisfied: $\int_{\theta_*}^{\theta_1} m(\theta)\,d\theta = 0$, $\int_{\theta_1}^{\theta^*} m(\theta)\,d\theta = 0$. Furthermore, we have a constraint that, at equilibrium, $\mu + \lambda F(\theta_v) - 1 = 0$. To manufacture θ_1 we adjust the three parameters λ, μ, and θ_v in the function m. In general we can manufacture at most one such point, as we have three constraints to meet and three variables to adjust. An additional point $\theta_2 > \theta_1$ would require satisfying one additional integral constraint. ∎

Theorem 2 mentions the condition that $\theta f(\theta)/F(\theta)$ be strictly increasing. Distributions that satisfy the condition are:

- $f(\theta) = const \cdot \theta^{a-1}(1-\theta)^{b-1}$, a Beta with parameters $a = 1/2, b = 1/2$. Then $\theta f(\theta)/F(\theta)$ is increasing (checked with Maple). Notice that when $a = b = 1$, the beta reduces to a uniform. So, the suspicion is that the beta might work for all parameters $a, b < 1$.

- $F_{a,b}(\theta) = (a\theta + b\theta^2)/C$. Then $f_{a,b}(\theta) = (a + 2b\theta)/C$, and

$$\frac{\theta f(\theta)}{F(\theta)} = \frac{a\theta + 2b\theta^2}{a\theta + b\theta^2} \text{ is increasing in } \theta \text{ when } b > 0.$$

If $F^{\alpha}(\theta) = \theta^{\alpha}$ (for $\alpha > 0$), then $\theta f(\theta)/F(\theta)$ is constant. The uniform distribution is a member of this family.

3.1 Characterization and comparative statics

In the next two propositions we characterize the optimal contract. The first proposition says that at the optimal contract the object is traded more often than would be the case if the buyer was informed.

Proposition 2 *Assume Assumption 1. Let θ^* denote the smallest type that keeps the object for sure in equilibrium. Then $\theta^* \leq t_I$.*

Proof: We first notice that $m(\theta^*) = 0$. This follows from our choice of θ^* as the sup of all θ's such that $\overline{m}(\theta) = 0$.

Case $\theta^* < \theta_v$

In this case $m(\theta^*) = 0$ reads

$$f(\theta^*)\theta^* + (1 - \mu - \lambda) F(\theta^*) = 0. \tag{4}$$

Equation (2) in the proof of Theorem 2 gives

$$-1 + \mu + \lambda F(\theta_v) = 0,$$

which can be manipulated to obtain

$$(1 - \mu - \lambda) F(\theta^*) = \lambda F(\theta^*) [F(\theta_v) - 1].$$

Substituting into equation (4) one gets

$$\begin{aligned} f(\theta^*)\theta^* &= \lambda F(\theta^*) [1 - F(\theta_v)] \\ &< \lambda F(\theta_v) [1 - F(\theta^*)] \\ &\leq 1 - F(\theta^*), \end{aligned} \tag{5}$$

where the first inequality follows from $\theta^* < \theta_v$, and the second inequality follows from the fact that, in view of (2), $\lambda F(\theta_v) = 1 - \mu \leq 1$. Thus, the derivative of $H(t)$ evaluated at $t = \theta^*$ is strictly positive. Then the result follows from Assumption 1.

Case $\theta^* > \theta_v$

In this case $m(\theta^*) = 0$ yields immediately

$$1 - F(\theta^*) - f(\theta^*)\theta^* \geq 0.$$

Thus, the derivative of $H(t)$ evaluated at $t = \theta^*$ is positive. Then the result follows from Assumption 1. ∎

The next proposition states that, if at the optimal contract the IR contraint is not binding, and under a condition on the uncertainty about θ, the price in the Tioli-cum-warranty contract is higher than t_I. In addition, the larger c, the more this price is larger than t_I and the higher the stocking fee $S = \theta_v - \theta^*$.

Proposition 3 *Suppose that $\theta f(\theta)/F(\theta)$ is strictly increasing, and that at the optimal mechanism the IR constraint does not bind. Let θ_v denote the type who receives an ex-post utility of 0 in the optimal mechanism. Then*

1. $\theta_v \geq t_I$.
2. *as c increases, θ_v increases and θ^* decreases.*

Proof: See Appendix. ∎

Proposition 4 *Suppose that there is a cost $k > 0$ that the seller incurs on returned items. Suppose that $\theta f(\theta)/F(\theta)$ is strictly increasing, and that at the optimal mechanism the IR constraint does not bind. Then the optimal contract has the same form as in Theorem 2. Denote with θ_{vk} and θ_k^* the types who receive an ex-post utility of zero and the lowest type who keeps the object, respectively, in the optimal contract with k. Then and $\theta_{vk} < \theta_v$ and $\theta_k^* < \theta^*$.*

Proof: See Appendix. ∎

3.2 An example: uniformly distributed valuations.

Consider the case where F is uniform on $[0,1]$. Then $f(\theta)\,\theta/F(\theta)$ is constant and equal to one. In this example, a warranty will be offered for all values of c for which the VI constraint distorts the first-best (for the monopolist) problem.

The optimal contract when a warranty is not possible is:

$$t^*(c) = \begin{cases} 1/2 & \text{for } c \geq 1/8. \quad \text{The buyer will not acquire information.} \\ \sqrt{2c} & \text{for } 1/32 \leq c \leq 1/8. \quad \text{The buyer will not acquire information.} \\ 1/2 & \text{for } 0 \leq c \leq 1/32. \quad \text{The buyer will acquire information.} \end{cases}$$

For $c \geq 1/32$, the seller's profits are $t^*(c)$. For $c < 1/32$ the seller's profits are $1/4$.

Consider now the case where a warranty is possible, concentrating on the nontrivial case where $c < 1/8$. We now show that the optimal contract can be implemented by a Tioli at price $1/2$ together with a warranty with the stocking fee S that solves the equation $c = \int_0^{1/2} |\max\{\theta - 1/2, -S\}|\, d\theta$. To see this, assume the IR constraint is not binding; then, using the fact that $f(\theta)\,\theta/F(\theta)$ is constant, the proof of Proposition 3 Part 1 yields an optimal price $t^{**}(c) = t_I = 1/2$. We now show that when $t^{**}(c) = 1/2$ the IR constraint is never binding. To this end, notice that when $c < 1/8$ the Tioli at price $1/2$ must have a stocking fee S smaller than $1/2$. But such a contract is more advantageous for the uninformed buyer than a simple Tioli with price $1/2$, which leaves the uninformed buyer with zero surplus. Thus, when $c < 1/8$ the IR constraint is not binding and the optimal contract is a Tioli at price $\theta_v = 1/2$ with a warranty with the stocking fee S that solves the equation $c = \int_0^{1/2} |\max\{\theta - 1/2, -S\}|\, d\theta$.

4 Welfare effect of warranties

We discuss the welfare effect of allowing warranties by comparing the optimal contracts when a warranty is possible (Section 3) and when a warranty is not possible (Section 2). Throughout this section we assume that that Assumptions 1 and 2 are verified.

When $c < \underline{c}$, introducing the possibility of a warranty increases efficiency for two reasons. First, it spares the social cost of acquiring information. Second, comparing Propositions 1 and 2 we see that the object is traded more often than would be the case without a warranty. By contrast, when $c > \underline{c}$ allowing the seller to offer a warranty is efficiency-reducing. Indeed, if a warranty is not allowed the seller sets a Tioli price such that the buyer always buys and acquires no information.

We are also interested in the buyer's surplus. When $c > \underline{c}$, the buyer is worse off with a warranty. Indeed, with the warranty the social surplus is lower (the object is not always sold) and the seller's profits are higher. When $c < \underline{c}$ and $f(\theta)\theta/F(\theta)$ is nondecreasing, the buyer is worse off with a warranty, because then the contract with a warranty has a price $\theta_v > t_I$ (see Proposition 3). Thus, with a warranty the buyer's expected surplus is $[1 - F(\theta_v)][\theta_v - E(\theta|\theta > \theta_v)] - c$, while without a warranty it is $[1 - F(t_I)][t_I - E(\theta|\theta > t_I)] - c$.

5 Discussion

We have presented a model of monopoly provision of money-back warranties. In our model, buyer and seller are risk neutral and have no private information about the buyer's valuation for the object. The buyer can, however, acquire information and learn his true valuation for the object. In this setup, we have shown that the optimal contract allows for the object to be returned. Under some conditions the optimal contract can be implemented with a money-back warranty. The seller uses the warranty to prevent wasteful information acquisition, by offering the buyer part of the benefits that information would give him, namely the ability not to purchase useless objects. We have compared the optimal contracts when the object can be returned and when it cannot. We have found that when the cost of information is lower than a threshold, allowing the seller to offer a warranty is welfare improving. However, when the cost of information is higher than the threshold, it is socially optimal to forbid money-back warranties.

In our model the information structure is determined endogenously as part of the equilibrium. Reasoning in terms of information structures affords an insightful perspective on the incentives in this paper. Consider the two polar extremes of

(a) trade with symmetric information, and

(b) trade with the buyer being more informed than the seller.

Since in case (a) the buyer receives zero surplus, the buyer should be prepared to pay something to get from information structure (a) to information structure (b). Acquiring information allows the buyer to do precisely that. The seller, on the other hand, should be prepared to pay something to avoid information structure (b). By choosing the mechanism, the seller has a measure of control over the information structure under which trade takes place. The warranty is an efficient way for the seller to control the buyer's information structure.

In our model information has a purely strategic role, as opposed to an efficiency role. This is because the efficient allocation is for the object always to be traded; information is not necessary to implement the efficient allocation. One could extend the model to allow for the buyer's valuation to be below the seller's. In this case, information assumes an efficiency role because the efficient allocation is to not trade in the event that the seller values the object more than the buyer. As the probability of this event increases, the warranty becomes more efficient. This is because the object is returned when the buyer's valuation is low. An extreme case is when the seller's valuation exceeds the buyer's expected value for the object. In this case, if the buyer is uninformed there are no gains from trade, so the seller prefers the buyer to be informed. Now, a warranty increases efficiency by allowing trade to happen when the buyer's valuation turns out to be high.

The two fundamental ingredients of our analysis are: information acquisition; and the fact that the seller cannot observe the buyer's valuation, which leads to non contractibility of realized quality. Both ingredients are necessary, as the analysis becomes trivial in the absence of any one of them. Indeed, absent information acquisition the analysis would be exactly the same as in Myerson (1981). And if it was possible to contract on the buyer's realized quality, the seller could offer a contract extracting all of the buyer's surplus for any realization; such a contract would leave the buyer with zero surplus state by state, and thus would give no incentives to acquire information.

A salient feature of the optimal contract is that the seller may give back less than the full price if the buyer decides to return the object. Thus, the seller makes money even when the good is ultimately returned. This is not uncommon. Sometimes stocking fees are charged directly, as discount stores of electronic and photographic material often do. The Bookstore of the University of Pennsylvania charges a stocking fee of between 50% and 75% on textbooks returned later than two weeks after the beginning of classes, no matter how good their condition. Stocking fees are built into programs to incent retailers to reduce returns. For instance, in the computer industry "[t]he IBM PC Co.'s seven largest resellers, [...] can qualify for a special allocation of ThinkPads if they accept a 10 percent return penalty and a 10 percent cancellation fee for the order."[9] Of course, there are less direct ways for the seller to earn rents even when the object is returned. For instance, return policies often stipulate a refund not in money, but in credit.

Even in the absence of stocking fees, a money-back warranty may be valuable for the seller. If a warranty imposes on the buyer a cost of returning the object, even one that the seller does not appropriate (for instance, forcing the buyer to call repeatedly

[9] Zarley (1994).

to inquire about the return), the seller can improve on the simple take-it-or-leave-it contract. To see this, consider again the example of Section 3.2. Suppose $c < 1/32$, so the seller's profits without warrranty are $1/4$. Let r denote the cost to the buyer of returning the object, and assume that $r < 2c$. Given a price of $1/2$ with a money-back warranty *without stocking fee*, the buyer accepts the contract without acquiring information, and returns the object whenever $\theta < (1/2) - r$. The seller's profits are $1/2\,(r + 1/2) > 1/4$. Therefore, the seller offers the warranty even if the stocking fee is zero. This shows that, by imposing a wasteful cost on the buyer to return the object, the seller may benefit from a money-back warranty with no stocking fee.

While it is intuitively appealing to imagine that sellers impose wasteful costs on buyers, in this paper we insist on optimal contracts. In an optimal contract there is no role for wasteful costs, as they are dominated by cash transfers to the seller; hence, the prevalence of stocking fees in our model. Indeed, stocking fees can be interpreted as the price that the seller charges for information. Yet, casual empiricism suggests that money-back warranties often involve no stocking fee. Lenient return policies are often ascribed to competition between sellers. This suggests that stocking fees are most likely when the seller has market power. In our analysis we assume a monopolist seller. Extending the analysis to many competing sellers may not be straightforward due to well-known difficulties with multi-principal mechanism design. Is it possible to generate optimal contracts with zero stocking fees in our environment? This is a question for future research.

A Proofs

Proof of Lemma 1.

Proof: Consider any DRM where the buyer does not acquire information and pays a price w^u. We show that a buyer faced with a Tioli at a price w^u does not acquire information, and purchases the object. Without loss of generality we restrict attention to DRMs where the buyer truthfully reveals whether he is informed or not, and his true type if informed. Furthermore, without loss of generality we require the DRM to be individually rational. This means that the uninformed buyer must earn a nonnegative expected surplus, while the surplus of all informed types, not including the information acquisition cost, must be nonnegative. Finally, let us restrict to the nontrivial case where $p^u, w^u > 0$.

By incentive compatibility, an informed buyer must be willing to reveal himself instead of claiming to be uninformed. Thus, as IR is also satisfied,

$$\theta p(\theta) - w(\theta) \geq \max\{0, \theta p^u - w^u\}. \tag{6}$$

The uninformed's utility is

$$\theta p^u - w^u.$$

Thus, under the DRM the value of information is

$$\begin{aligned}\int_0^1 [\theta p(\theta) - w(\theta) - (\theta p^u - w^u)]\, dF(\theta) &\geq \int_0^y [\theta p(\theta) - w(\theta) - (\theta p^u - w^u)]\, dF(\theta) \\ &\geq \int_0^y -(\theta p^u - w^u)\, dF(\theta)\end{aligned}$$

for any $y \geq 0$. Both inequalities follow from (6).

Consider now the contract offering a Tioli with price w^u both to an informed and to an uninformed. Under the Tioli the payoff of an informed is

$$\max\{0, \theta - w^u\}.$$

For an uninformed it is IR to accept the Tioli since his expected utility is $\bar{\theta} - w^u \geq \bar{\theta} p^u - w^u$, and by hypothesis $\bar{\theta} p^u - w^u \geq 0$. The payoff of an uninformed is

$$\theta - w^u.$$

The value of information is therefore

$$\int_0^{w^u} -(\theta - w^u)\, dF(\theta) \leq \int_0^{w^u} -(\theta p^u - w^u)\, dF(\theta).$$

Comparing with the value of information under the DRM we conclude that if it was IC to be uninformed under the DRM, it will also be IC not to become informed under the Tioli. ∎

Proof of Lemma 3

Proof: Without loss of generality we restrict to DRMs that are incentive compatible, i.e. it is optimal for the buyer to announce his true type, and individually rational, i.e. the buyer's expected surplus is nonnegative. Take any DRM characterized by $\left(\widetilde{w}\left(\widehat{\theta}\right),\widetilde{p}\left(\widehat{\theta}\right)\right)$, $\left(\widetilde{\widetilde{w}}\left(\widehat{\theta}\right),\widetilde{\widetilde{p}}\left(\widehat{\theta}\right)\right)$ and b, and suppose that the buyer does not acquire information. We show that the seller can implement the same allocation simply by charging b, showing the buyer the object and then facing the buyer with the schedule $\left(\widetilde{\widetilde{w}}\left(\widehat{\theta}\right),\widetilde{\widetilde{p}}\left(\widehat{\theta}\right)\right)$.

By incentive compatibility, an informed buyer must be willing to reveal himself instead of claiming to be uninformed. Thus, as IR is also satisfied,

$$\theta\widetilde{p}\left(\theta\right)-\widetilde{w}\left(\theta\right)\geq\max\left\{0,\theta\widetilde{\widetilde{p}}\left(\theta\right)-\widetilde{\widetilde{w}}\left(\theta\right)-b\right\}. \tag{7}$$

The uninformed's utility is

$$\theta\widetilde{\widetilde{p}}\left(\theta\right)-\widetilde{\widetilde{w}}\left(\theta\right)-b.$$

Thus, under the DRM the value of information is

$$\begin{aligned}
&\int_0^1\left[\theta\widetilde{p}\left(\theta\right)-\widetilde{w}\left(\theta\right)-\left(\theta\widetilde{\widetilde{p}}\left(\theta\right)-\widetilde{\widetilde{w}}\left(\theta\right)-b\right)\right]dF\left(\theta\right)\\
\geq\ &\int_0^y\left[\theta\widetilde{p}\left(\theta\right)-\widetilde{w}\left(\theta\right)-\left(\theta\widetilde{\widetilde{p}}\left(\theta\right)-\widetilde{\widetilde{w}}\left(\theta\right)-b\right)\right]dF\left(\theta\right)\\
\geq\ &\int_0^y-\left(\theta\widetilde{\widetilde{p}}\left(\theta\right)-\widetilde{\widetilde{w}}\left(\theta\right)-b\right)dF\left(\theta\right)
\end{aligned}$$

for any $y\geq0$. Both inequalities follow from (7).

Consider now the contract offering both to an informed and to an uninformed buyer to be charged b, be shown the object and then be faced with the schedule $\left(\widetilde{\widetilde{w}}\left(\widehat{\theta}\right),\widetilde{\widetilde{p}}\left(\widehat{\theta}\right)\right)$. Under tthis contract the payoff of an informed is

$$\max\left\{0,\theta\widetilde{\widetilde{p}}\left(\theta\right)-\widetilde{\widetilde{w}}\left(\theta\right)-b\right\}.$$

For an uninformed it is IR to accept this contract by assumption. The payoff of an uninformed is

$$\theta\widetilde{\widetilde{p}}\left(\theta\right)-\widetilde{\widetilde{w}}\left(\theta\right)-b.$$

The value of information is therefore

$$\int_0^{\theta_v} -\left(\theta\tilde{\tilde{p}}(\theta) - \tilde{\tilde{w}}(\theta) - b\right) dF(\theta),$$

which is smaller than the value of information under the DRM. Thus, if it was IC to be uninformed under the DRM, it will also be IC not to become informed under the new contract. Hence, we can replace the DRM with the simple contract above, and the buyer will enter the contract without acquiring information. It is clear then the the seller's profits are unchanged. ∎

Proof of Proposition 3

Proof: **Part 1.**

¿From equation 5, using the assumption that $\frac{f(\theta)\theta}{F(\theta)}$ is nondecreasing we obtain

$$\frac{f(\theta_v)\theta_v}{\lambda F(\theta_v)} \geq [1 - F(\theta_v)].$$

Since the IC constraint does not bind we have $\mu = 0$ in eq. (2), and therefore $\lambda F(\theta_v) = 1$. Substituting into the previous expression,

$$f(\theta_v)\theta_v \geq [1 - F(\theta_v)].$$

Since t_I solves

$$f(t_I)t_I = 1 - F(t_I),$$

and $\frac{f(\theta)\theta}{1-F(\theta)}$ is increasing, the result follows.

Part 2.

When the IR constraint is not binding $\mu = 0$, and θ^* solves $\lambda - 1 = \theta^* f(\theta^*)/F(\theta^*)$. From equation (2) $\lambda = 1/F(\theta_v)$, and substituting into the previous expression we obtain

$$\frac{1}{F(\theta_v)} - 1 = \frac{\theta^* f(\theta^*)}{F(\theta^*)}.$$

Since $\frac{f(\theta)\theta}{F(\theta)}$ is increasing by assumption, this equation requires θ^* to be inversely related to θ_v. So, fix a c and consider the optimal θ^* and θ_v associated to it (the ones solving problem 1). Because the VI constraint holds with equality, we have

$$c = [\theta_v - \theta^*] F(\theta^*) + \int_{\theta^*}^{\theta_v} (\theta_v - \theta) dF(\theta). \tag{8}$$

Take now a $c' > c$, and consider the optimal $\theta^{*\prime}$ and θ_v' associated to c'. Since θ^* is inversely related to θ_v, one of the following two configurations must be true: either $\theta^{*\prime} > \theta^*$ and $\theta_v' < \theta_v$; or $\theta^{*\prime} < \theta^*$ and $\theta_v' > \theta_v$. To conclude the proof, we show that the first configuration is inconsistent with $c' > c$. Pick any $\theta^{*\prime} > \theta^*$ and $\theta_v' < \theta_v$. Equation (8) reads

$$\begin{aligned}
c' &= \left[\theta_v' - \theta^{*\prime}\right] F\left(\theta^{*\prime}\right) + \int_{\theta^{*\prime}}^{\theta_v'} \left(\theta_v' - \theta\right) dF(\theta) \\
&= \left[\theta_v' - \theta^{*\prime}\right] F\left(\theta^{*}\right) + \int_{\theta^{*}}^{\theta^{*\prime}} \left(\theta_v' - \theta^{*\prime}\right) dF(\theta) + \int_{\theta^{*\prime}}^{\theta_v'} \left(\theta_v' - \theta\right) dF(\theta) \\
&\leq \left[\theta_v - \theta^{*}\right] F\left(\theta^{*}\right) + \int_{\theta^{*}}^{\theta^{*\prime}} \left(\theta_v' - \theta\right) dF(\theta) + \int_{\theta^{*\prime}}^{\theta_v'} \left(\theta_v' - \theta\right) dF(\theta) \\
&\leq \left[\theta_v - \theta^{*}\right] F\left(\theta^{*}\right) + \int_{\theta^{*}}^{\theta_v'} \left(\theta_v - \theta\right) dF(\theta) \\
&\leq \left[\theta_v - \theta^{*}\right] F\left(\theta^{*}\right) + \int_{\theta^{*}}^{\theta_v} \left(\theta_v - \theta\right) dF(\theta) = c.
\end{aligned}$$

∎

Proof of Proposition 4

Proof: The Lagrangean in the same as before, except for an added term $\int_0^1 -k\left(1-p(\theta)\right) dF(\theta)$. Thus, the function $m(\theta)$ is the same except for an added term $kf(\theta)$. The same reasoning as in Theorem 2 then yields the optimal contract.

We now show that it must be $\theta_{vk} < \theta_v$ and $\theta_k^* < \theta^*$. Denote

$$m(x|y) = f(x)\,x + \left(1 - \frac{1}{F(y)}\right) F(x).$$

Notice that, since $\frac{f(x)x}{F(x)}$ is nondecreasing, $m(x|y)$is quasimonotone in x for all values of y. Also, $m(x|y)$ is increasing in y for all values of x.

We have that θ^* and θ_v solve $m(\theta^*|\theta_v) = 0$, while θ_k^* and θ_{vk} solve $m(\theta_k^*|\theta_{vk}) + kf(\theta_k^*) = 0$.

By contradiction, suppose it were not true that $\theta_{vk} < \theta_v$ and $\theta_k^* < \theta^*$. We know that θ^* and θ_v satisfy the VI constraint with equality. If $\theta_{vk} > \theta_v$, to satisfy the VI constraint with equality it must be $\theta_k^* > \theta^*$. Similarly, if $\theta_k^* > \theta^*$, to satisfy the VI constraint with equality it must be $\theta_{vk} > \theta_v$. Thus, the only possible configuration is $\theta_{vk} > \theta_v$ and $\theta_k^* > \theta^*$. Now notice that

$$\begin{aligned}
& m\left(\theta_k^*|\theta_{vk}\right) - m\left(\theta^*|\theta_v\right) \\
=\;& \left[m\left(\theta_k^*|\theta_{vk}\right) - m\left(\theta_k^*|\theta_v\right)\right] + \left[m\left(\theta_k^*|\theta_v\right) - m\left(\theta^*|\theta_v\right)\right] > 0,
\end{aligned}$$

where the inequality holds because (a) $m(\theta^*|\theta_v)$ is increasing in θ_v and (b) $m(\theta^*|\theta_v)$ is quasimonotone in θ^* and $m(\theta^*|\theta_v) = 0$. But then $m(\theta_k^*|\theta_{vk}) + kf(\theta_k^*) > m(\theta^*|\theta_v) = 0$, which is a contradiction. Hence the assertion that $\theta_{vk} < \theta_v$ and $\theta_k^* < \theta^*$. ∎

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