Simulation Based Inference in Simultaneous Equations

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Abstract

In the context of multivariate regression (MLR) and simultaneous equations (SE), it is well known that commonly employed asymptotic test criteria are seriously biased towards overrejection. In this paper, we propose exact likelihood based tests for possibly nonlinear hypotheses on the coefficients of SE systems. We discuss a number of bounds tests and Monte Carlo simulation based tests. The latter involves maximizing a randomized p-value function over the relevant nuisance parameter space which is done numerically by using a simulated annealing algorithm. We consider limited and full information models, in which case we introduce a multi-equation Anderson-Rubin-type test. Illustrative Monte Carlo experiments show that: (i) bootstrapping standard instrumental variable (IV) based criteria fails to achieve size control, especially (but not exclusively) under near non-identification conditions, and (ii) the tests based on IV estimates do not appear to be boundedly pivotal and so no size-correction may be feasible. By contrast, likelihood ratio based tests work well in the experiments performed.

Key words: Monte Carlo tests, bounds tests, simultaneous equations, nonlinear hypotheses, finite sample tests.

1 Introduction

Econometricians are often confronted with technical difficulties arising from simultaneity when testing parameter restrictions in systems of equations. With few exceptions, the distributions of standard test statistics are known only asymptotically due to feedback from the dependent variables to the explanatory variables. There will obviously be approximation errors when the asymptotic results are applied to samples of moderate size as is frequently the case in simultaneous equations (SE) applications. Although long recognized as a serious issue in statistical inference, finite sample validity has not received the attention it deserves in such contexts. Indeed, tests of parameter significance have almost invariably been based on asymptotic procedures.

Exact procedures have been proposed only for a few highly special cases. Early in the development of econometric theory relating to the SE model, Haavelmo (1947) constructed exact confidence regions for OLS reduced form parameter estimates and corresponding structural parameter estimates. Bartlett (1948) and Anderson and Rubin (1949) proposed exact F-tests for specific classes of hypothesis in the context of a structural equation along with corresponding confidence sets. Maddala (1974) and Dufour and Jasiak (1999) have described finite sample single-equation procedures which can be viewed as extensions of the latter procedures. Some exact specification tests have also been suggested for SE. In particular, Durbin (1957) proposed a bounds test against serial correlation in SE and, more recently, Harvey and Phillips (1980, 1981*a*, 1981*b*, 1989) have suggested tests against serial correlation, heteroskedasticity and structural change in a single structural equation. In both cases, the tests are based on residuals from a regression of the estimated endogenous part of an equation on all exogenous variables. An exact F-test involving reduced form residuals was proposed by Dufour((1987), Section 3) for the hypothesis of independence between the full vector of stochastic explanatory variables and the disturbance term of a structural equation. This procedure generalizes earlier tests suggested by Wu((1973), T_2 statistic) and Hausman((1978), eq. 2.23). From a different standpoint, the finite sample distributions of commonly used estimators and test statistics have also received attention in the literature. For a review of the main findings in this area, the reader may consult Phillips (1983) and Taylor (1983). It is clear from these results that the exact distributions in most cases depend on nuisance parameters. Except for special hypotheses, no work seems presently available that resolves the problem of nuisance parameters in finite samples.

Because of the computational complexity of maximum likelihood methods in SE models, statistical inference has generally been based on instrumental variable (IV) methods. The problems associated with asymptotically valid tests in IV regressions are discussed in Dufour (1997). In particular, it is shown that usual t-type tests, based on common IV estimators, such as two-stage least squares, have significance levels that may deviate arbitrarily from their nominal levels since it is not possible to bound the null distributions of the relevant test statistics to obtain valid inference. This results from identification concerns and is related to the so-called "weak instruments" problem; see, e.g. Nelson and Startz (1990*a*, 1990*b*), Buse (1992), Maddala and Jeong (1992), Angrist and Krueger (1994), Staiger and Stock (1997), Bound, Jaeger and Baker (1995), Hall, Rudebusch and Wilcox (1997), Cragg and Donald (1996), and Wang and Zivot (1998).[1]

With the declining cost of computing, a natural alternative to traditional inference are simulation-based methods such as bootstrapping; for reviews, see Efron (1982), Efron and Tibshirani (1993), Hall (1992), Jeong and Maddala (1993), Vinod (1993), Shao and Tu (1995), Li and Maddala (1996) and Davidson and MacKinnon(1999*a*, 1999*b*, 1999*c*). These surveys suggest that bootstrapping can provide more reliable inference for many problems. In connection with the SE model, examples in which the bootstrap outperforms conventional asymptotics include: Freedman and Peters (1984*a*), Green, Hahn and Rocke (1987), Hu, Lau, Fung and Ulveling (1986), Korajczyk (1985) and Dagget and Freedman (1985). Others however, find that the method leads to little improvement, *e.g.* Freedman and Peters (1984*b*), Park (1985) and Beran and Srivastava (1985). Clearly, there appears to be a conflict in the conclusions regarding the effectiveness of the bootstrap in SE contexts.[2]

This paper addresses these issues and develops alternative simulation based test procedures in limited and full information SE models. Whereas conventional procedures are asymptotically justified, the tests we propose are motivated by finite sample arguments. We focus on likelihood ratio (LR) based statistics. This choice is motivated by the propositions in Dufour (1997) pertaining to LR's *boundedly pivotal* characteristic, *i.e.* the fact that LR admits nuisance-parameter-free bounds. Specifically, Dufour (1997) (Theorem 5.1) provides an exact bound on the null distribution of the LR criterion given general - possibly non-linear[3]- hypotheses on the coefficients of a Gaussian structural equation. In a different (although related) context, namely the multivariate linear regression (MLR) model[4], Dufour and Khalaf (1997) propose a pivotal bound on the null distribution of LR, under general possibly non-linear non-Gaussian hypotheses. Dufour (1997)'s result may be obtained as a special - although non-optimal - case of the latter bound. In the present paper, we extend Dufour and Khalaf (1997)'s bound on the standard LR

[1]For further results relevant to the issue of non-identification, see also Sargan (1983), Phillips (1984, 1985, 1989), Choi and Phillips (1992), McManus, Nankervis and Savin (1994).

[2]In fact, it is well known that bootstrapping may fail to achieve size control when the asymptotic distribution of the underlying test statistic involves nuisance parameters [see Athreya (1987), Basawa, Mallik, McCormick, Reeves and Taylor (1991) and Sriram (1994).

[3]SE LR tests often involve non-linear hypotheses implied by the structure; in connection, see Bekker and Dijkstra (1990) or Byron (1974)

[4]The relationship between the MLR and the SE model is readily seen: when all the predetermined variables of a SE system are strictly exogenous, the reduced form is equivalent to a (restricted) MLR system.

test to the SE context.

The results we obtain on the finite sample bound have two further implications. First, for the specific hypothesis which sets the value of the full vector of endogenous variables coefficients in a limited information framework, we show that Wang and Zivot (1998)'s asymptotic bounds test may be seen as an asymptotic version of the bound we propose here. We use this result to extend the validity of Wang and Zivot (1998)'s bound to the case of general linear hypotheses on structural coefficients. Secondly, for general test problems in the limited information framework, it turns out that our bound is highly related to the Anderson-Rubin test criterion. In this regard, we also propose a multi-equation Anderson-Rubin-type test which also admits a pivotal bound based on the results of Dufour and Khalaf (1997) relating to SURE models. In view of the renewed interest in the Anderson-Rubin test (see Dufour (1997), Dufour and Jasiak (1999), Staiger and Stock (1997) and Wang and Zivot (1998)), extensions to a systems context may prove useful for empirical work.

Although the bounds we introduce here are nuisance-parameter-free, their null distributions may be quite complex. In view of this, we propose, following Dufour and Khalaf (1997), to apply the Monte Carlo (MC) test procedure to obtain a simulation based exact p-value based on the bounds. For further reference, we call the p-value so obtained a BMC p-value. MC test procedures [first proposed by Dwass (1957) and Barnard (1963)] may be viewed as parametric bootstrap tests applied to statistics whose null distribution does not involve nuisance parameters, with however a fundamental additional observation: the associated randomized test procedure can easily be performed to control test size exactly, for a given number of replications. Dufour (1995) extends MC tests to nuisance-parameter-dependent statistics. A randomized procedure called maximized Monte Carlo (MMC) method is specifically proposed which yields provably exact tests (in the sense of level control).[5] Here we propose to combine the BMC test strategy outlined above with an MMC test, which can be run whenever the bounds test is not significant. To understand this strategy, recall that the BMC test is exact in the sense that rejections (at level α) are conclusive. Furthermore, as will become clear form our presentation, the MMC method may be much more expensive (computationally) then the BMC method, which justifies our sequential procedure. We will also show that the MMC algorithm may be written in a way to include a standard parametric bootstrap as a first step. Possibly expensive iterations - to obtain the maximal MC p-value in question which underlies the MMC test - may thus be saved if the bootstrap p-value exceeds α.

It is important, at this stage, to emphasize that the distributional theory which underlies the above procedures holds without imposing regularity conditions on the null hypothesis. In particular, our proposed tests are valid whether identification

[5]For further references regarding MC tests, see Dufour and Kiviet (1996, 1998), Kiviet and Dufour (1997), Dufour, Farhat, Gardiol and Khalaf (1998) and Dufour and Khalaf (1999).

constraints hold or not. Consequently, identification problems are resolved without the need to introduce non-standard, e.g. local-to-zero, asymptotics. Furthermore, although exactness is obtained under parametric assumptions (which are duly defined in the paper), normality is not strictly required.

This paper makes two further contributions relevant to simulation-based tests. First, we show that standard bootstrap-type tests may not be fully successful in the case of LR-based statistics and may fail in the case of Wald IV-based tests. Furthermore, we show that randomization cannot improve the performance of IV-based Wald tests; given the severity of the problem in the presence of identification difficulties, the case is made here for LR tests. To do this, we undertake to explore specifically the identification issue in the context of a small simulation experiment. More precisely, our main findings are: (i) MC methods based on randomization procedures where unknown parameters are replaced by estimators do not achieve size control, and (ii) MMC p-values for IV-based test are always one; in other words, it is does not appear possible to find a non trivial bound on the rejection probabilities, so that standard asymptotic and bootstrap procedures are deemed to fail when applied to such statistics. In contrast, LR-based MMC tests allow one to control the level of the procedure.[6]

The paper is organized as follows. Section 2 develops the notation and definitions. Section 3 presents our general test strategy. Linear hypotheses in the single-equation set-up are considered as a special case in section 4. We also consider another example which extends the Anderson-Rubin test to the multi-equation context. Simulation results are reported in Section 5 and Section 6 concludes the paper.

2 Framework

We consider a system of p simultaneous equations of the form

$$YB + X\Gamma = U, \tag{2.1}$$

where $Y = [y_1\ ,\ \dots\ ,\ y_p]$ is an $n \times p$ matrix of observations on p endogenous variables, X is an $n \times k$ matrix of fixed (or strictly exogenous) variables and $U = [u_1\ ,\ \dots\ ,\ u_p] = [U_1\ ,\ \dots\ ,\ U_n]'$ is a matrix of random disturbances. The coefficient matrix B is assumed to be invertible. The equations in (2.1) give the *structural form* of the model. Pre-multiplying both sides by B^{-1} leads to the *reduced form*

$$Y = X\Pi + V, \quad \Pi = -\Gamma B^{-1}, \tag{2.2}$$

or equivalently

$$y = (I_p \otimes X)\pi + v, \tag{2.3}$$

[6]MMC p-values are computed using a simulated annealing (SA) optimization algorithm; for a description of the latter, see Corana, Marchesi, Martini and Ridella (1987) or Goffe, Ferrier and Rogers (1994).

where $y = vec(Y)$, $\pi = vec(\Pi)$, $v = vec(V)$ and $V = [v_1 \ , \ \ldots \ , \ v_p] = [V_1 \ , \ \ldots \ , \ V_n]'$ is the matrix of reduced form disturbances. Further, we suppose the rows $U_1' \ , \ \ldots \ , \ U_n'$ of U satisfy the following distributional assumptions:

$$U_t \sim Jw_t, \quad t = 1 \ , \ \ldots \ , \ n, \tag{2.4}$$

where the vector $w = vec(w_1 \ , \ \ldots \ , \ w_n)$ has a known distribution and J is an unknown non-singular matrix. In particular, this condition will be satisfied when

$$w_t \sim N(0, I_p), \quad t = 1 \ , \ \ldots \ , \ n. \tag{2.5}$$

More generally, when U_t has finite second moments, its covariance matrix will be $var(U_t) = JJ' \equiv \Omega$ and the covariance matrix of V_t will be $Var(V_t) = (B^{-1})'\Omega B^{-1} \equiv \Sigma$. Note that the system's unrestricted reduced form (URF) is an MLR model.

A key feature of SE models is the imposition of identification conditions on the structural coefficients. Usually, these conditions are formulated in terms of zero restrictions on B and Γ. In addition, a normalization constraint is imposed on the endogenous variables coefficients; this is usually achieved by setting the diagonal elements of B equal to one. We can rewrite model (2.1), given exclusion and normalization restrictions as

$$y_i = Y_i\beta_i + X_{1i}\gamma_{1i} + u_i, \quad i = 1 \ , \ \ldots \ , \ p, \tag{2.6}$$

where Y_i and X_{1i} are $n \times m_i$ and $n \times k_i$ matrices which respectively contain the observations on the included endogenous and exogenous variables of the model. If more than m_i variables are excluded from the i-th equation, this equation is said to be *over-identified*.

Many problems are formulated in terms of limited-information (LI) models, comprised by a particular structural equation and the reduced form associated with the included right-hand side endogenous variables such as

$$\begin{aligned} y_i &= Y_i\beta_i + X_{1i}\gamma_{1i} + u_i = Z_i\delta_i + u_i, \\ Y_i &= X_{1i}\Pi_{1i} + X_{2i}\Pi_{2i} + V_i, \end{aligned} \tag{2.7}$$

where $Z_i = [Y_i, X_{1i}]$, $\delta_i = (\beta_i', \gamma_{1i}')'$and X_{2i} refers to the excluded exogenous variables. The associated LI reduced form is

$$\begin{bmatrix} y_i & Y_i \end{bmatrix} = X\Pi_i + \begin{bmatrix} v_i & V_i \end{bmatrix}, \quad \Pi_i = \begin{bmatrix} \pi_{1i} & \Pi_{1i} \\ \pi_{2i} & \Pi_{2i} \end{bmatrix}, \tag{2.8}$$

$$\pi_{1i} = \Pi_{1i}\beta_i + \gamma_{1i}, \quad \pi_{2i} = \Pi_{2i}\beta_i. \tag{2.9}$$

The necessary and sufficient condition for identification follows from the relation $\pi_{2i} = \Pi_{2i}\beta_i$. Indeed β_i is recoverable if and only if

$$rank(\Pi_{2i}) = m_i. \tag{2.10}$$

3 Hypothesis tests on structural coefficients based on reduced forms: the general case

Consider the problem of testing arbitrary restrictions on the structural parameters of model (2.1), under (2.4). Given the transformation that takes the structural system into its reduced form (2.3), namely $\Pi = -\Gamma B^{-1}$, the constraints in question imply nonlinear restrictions on the reduced form parameters. In general, the induced restrictions on Π may be expressed as

$$H_0 : R\pi \in \Delta_0, \tag{3.1}$$

where R is $(r \times kp)$ of rank r and Δ_0 is a non-empty subset of $\Re^r$. This characterization of the hypothesis includes linear restrictions, both within and across equations, and allows for nonlinear as well as inequality constraints. The LR criterion to test H_0 is $n \ln(\Lambda)$,where

$$\Lambda = \frac{|\hat{\Sigma}_0|}{|\hat{\Sigma}|} \ , \tag{3.2}$$

with $\hat{\Sigma}_0$ and $\hat{\Sigma}$ being the restricted and unrestricted ML estimators of Σ in (2.3). In the statistics literature, Λ^{-1} is often called the Wilks criterion.

In this context, the theory and test procedures detailed in Dufour and Khalaf (1997) are directly applicable and may be summarized as follows. To obtain a pivotal bound on the null distribution of Λ, consider restrictions of the form

$$H_0^* : Q\Pi C = D, \tag{3.3}$$

such that $H_0^* \subseteq H_0$, where Q is a $q \times k$ with rank q and C is $p \times c$ with rank c. Linear restrictions that decompose into the latter specific form are called *uniform linear* (UL) in the MLR literature. Let $\Lambda^*(q, c)$ be the reciprocal of the Wilks criterion for testing the latter restrictions. Then, from Dufour and Khalaf (1997) (Theorems 3.1 and 4.1), it is easy to see that: (i) the null distribution of the LR statistic for UL hypothesis involves no nuisance parameters under (2.4) and may easily be obtained by simulation, and (ii) the distribution of Λ is bounded by the distribution of $\Lambda^*(q, c)$. Formally, these observations yield the following basic result.

Theorem 1 *Consider the MLR model (2.1) with (2.4). Let Λ be the statistic defined by (3.2) for testing restrictions which, when written in terms of the reduced form (2.3), take the form (3.1). Further, consider restrictions of the form $Q\Pi C = D$ that satisfy* 3.1 *where Q is $q \times k$ with rank q and C is $p \times c$ with rank c. Let $\Lambda^*(q, c)$ be the inverse of Wilks' criterion for testing the latter restrictions. Then under the null hypothesis, $P[\Lambda \geq \lambda^*(\alpha)] \leq \alpha$, for all $0 \leq \alpha \leq 1$, where $\lambda^*(\alpha)$ is determined such that $P[\Lambda^*(q, c) \geq \lambda^*(\alpha)] = \alpha$.*

The underlying distributional conditions, namely of the (2.4) form, are appreciably less restrictive than those of traditional multivariate analysis of variance which require normal errors. Furthermore, Theorem 1 is valid whether identification is imposed or not. Indeed, the results from Dufour and Khalaf (1997) (specifically Theorem 4.1) which we used here were obtained without imposing any regularity assumption. No further assumptions were needed to extend these results to the SE test context.

For certain specific cases under (2.5), bounding critical points may be obtained from the F distribution. In particular, if $c = 1$,

$$\frac{q[\Lambda^*(q,1) - 1]}{n-k} \sim F(q, n-k). \tag{3.4}$$

Pivotal bounds can be derived in a similar way for the SURE model. This involves rewriting the test problem in terms of the MLR model of which the SURE system under consideration is a restricted form. We will use SURE-type restrictions in the context of the Anderson-Rubin systems test which will be introduced in the next section.

As emphasized above, the bounds implied by Theorem 1 typically involve non-standard distributions[7], however, they can be easily obtained using the BMC method. The procedure may be summarized as follows (see also Appendix B). From the observed data, compute the LR test statistic. Generate N simulated values of the bounding the statistic under the null hypothesis. Then a bounds MC p-value is obtained from the *rank* of the observed value of the test statistic within the set $\Big[$ *observed LR* , *simulated bounding statistics* $\Big]$.

From the description of the BMC procedure, it is easy to see the relationship between MC and parametric bootstrap tests. When the statistics simulated depend on nuisance parameters (say θ), a MC p-value, conditional on θ which we will denote $\widehat{p}_N(LR|\theta)$ may be obtained as follows. From the observed data, compute the LR test statistic. Given θ, generate N simulated values of LR under the null hypothesis. Then the *rank* of the observed LR within the set $\Big[$ *observed LR* , *simulated LRs* $\Big]$ yields $\widehat{p}_N(LR|\theta)$. The (standard) parametric bootstrap corresponds to the case where a consistent estimate of θ (compatible with the null hypothesis), say $\widehat{\theta}$, is used in the latter algorithm. Following the notation set in Dufour and Khalaf (1999), we will refer to $\widehat{p}_N(LR|\widehat{\theta})$ under the name *Local MC* (LMC) p-value. The MMC method involves maximizing $\widehat{p}_N(LR|\theta)$ over all values of θ compatible with the null hypothesis.

At this stage, two points are worth noting. It is evident that for all $0 \leq \alpha \leq 1$ and $\forall\widehat{\theta}$, if the bootstrap p-value exceeds α, then the MMC p-value will also exceed α. This means that non-rejections in the context of LMC tests may be interpreted "exactly", with reference to the MMC test. Furthermore, by construction,

[7] We will show in the next section that an important special case corresponds to the F distribution, using (3.4).

if the BMC p-value is less than α, then we can be sure that the MMC p-value is also less than α. On observing that the BMC procedure avoids the (numerical) complications associated with maximizing $\widehat{p}_N(LR|\theta)$, we recommend the following sequential procedure (with level α). Obtain a BMC p-value first and reject the null hypothesis if the BMC p-value is $\leq \alpha$. If not, obtain an LMC p-value (which corresponds, as argued above, to a standard parametric bootstrap) using the constrained MLE of θ. If the LMC p-value exceeds α, then conclude the test is not significant. Otherwise, run an MMC algorithm. To cut cost, it is not necessary to run the maximization routine to convergence: one may exit and conclude the test is not significant whenever a p-value which exceeds α is reached.

The above test procedure may be applied in a full information, sub-system or single-equation set-ups. We will take up specific single and multi-equation problems in the next section.

4 Hypothesis tests on structural coefficients based on reduced forms: Special Cases

In this section, we focus on specific - yet quite common - test problems: (i) tests of linear constraints on the coefficients of a single structural equation, and (ii) a generalization of the Anderson-Rubin test to the multi-equation context.

4.1 Limited Information tests

To illustrate how the above results may be used, we consider here the problem of testing linear restrictions in a LI framework. For exposition simplicity, we shall restrict attention to hypotheses that set several structural coefficients to specific values. More precisely, we consider in turn hypotheses of the form:

$$H_{01} : \beta_i = \beta_i^0, \tag{4.5}$$

$$H_{02} : \beta_{1i} = \beta_{1i}^0, \tag{4.6}$$

where $\beta_i = (\beta_{1i}', \beta_{2i}')'$ and β_{1i} is $m_{1i} \times 1$, and

$$H_{03} : \beta_{1i} = \beta_{1i}^0, \quad \gamma_{12i} = \gamma_{12i}^0, \tag{4.7}$$

where $\gamma_{1i} = (\gamma_{11i}', \gamma_{12i}')'$ and γ_{12i} is $k_{2i} \times 1$.When the model is identified, (4.5) corresponds to the following restrictions

$$\Pi_{2i}\beta_i^0 = \pi_{2i}, \tag{4.8}$$

or equivalently,

$$S_1 \begin{bmatrix} \pi_{1i} & \Pi_{1i} \\ \pi_{2i} & \Pi_{2i} \end{bmatrix} \begin{bmatrix} 1 \\ -\beta_i^0 \end{bmatrix} = 0, \tag{4.9}$$

where

$$S_1 = \left[O_{(k-k_i,\, k_i)},\; I_{(k-k_i)} \right],$$

and $O_{(s,j)}$ denotes a zero $s \times j$ matrix.

The formulae for the standard 2SLS and LR-based test statistics for such problems are provided in Appendix A. As pointed out by Davidson and MacKinnon ((1993), chapter 18), the LI model involves a triangular system in which case MLE can be obtained by solving an eigen-value problem. Furthermore, the LI LR statistic may be computed from two characteristic roots; see Kadane (1971), Morimune and Tsukuda (1984), Wang and Zivot (1998) and Oya (1997), among others. Thus, although non-linear restrictions intervene, iterative maximum likelihood algorithms are not needed in practice, a result of interest from the point of vue of randomized tests. We will show next how to derive the bounding statistic in each specific case. Once we obtain the form of the latter statistic, it is then straightforward to implement the sequential procedure described in the previous section.

Conforming with the notation used in Appendix A, let $\hat{\Sigma}_i^{01}$ and $\hat{\Sigma}_i$ be the error covariance LIML estimates imposing and ignoring (4.8), where the latter corresponds to the unrestricted reduced form. Further, let $\hat{\Sigma}_i^{LI}$ denote the LIML error covariance estimate imposing the exclusion restrictions implied by the structure. Observe first that (4.9) is indeed UL, which implies that the statistic $|\hat{\Sigma}_i^{01}|/|\hat{\Sigma}_i|$ is pivotal. Following the notation introduced in the context of Theorem 1 to refer to UL-hypothesis test statistics, let us define

$$\begin{aligned} LR_{01}^{MR} &= n[\ln(\Lambda_{01}^{*}(k-k_i,1))], \\ \Lambda_{01}^{*}(k-k_i,1) &= \frac{|\hat{\Sigma}_i^{01}|}{|\hat{\Sigma}_i|}. \end{aligned} \tag{4.10}$$

Now consider the LIML based LR statistic

$$\begin{aligned} LR_{01}^{LI} &= n[\ln(\Lambda_{01}^{LI})], \\ \Lambda_{01}^{LI} &= \frac{|\hat{\Sigma}_i^{01}|}{|\hat{\Sigma}_i^{LI}|}. \end{aligned} \tag{4.11}$$

Following the arguments of Section 3, we see that the distribution of Λ_{01}^{LI} is bounded by the distribution of $\Lambda_{01}^{*}(k-k_i,1)$.

Whereas $n[\ln(\Lambda_{01}^{LI})]$ has a $\chi^2(m_i)$ asymptotic distribution only under identification assumptions, LR_{01}^{MR} is asymptotically distributed as $\chi^2(k-k_i)$ whether the rank condition holds or not. The asymptotic distribution of the LR_{01}^{LI} statistic is thus bounded by a $\chi^2(k-k_i)$ distribution independently of the conditions for identification. This result was derived under *local-to-zero* asymptotics in Wang and Zivot (1998). Furthermore, exact bounds based on the $F(k-k_i, n-k)$ distribution may also be derived for this problem if the normality assumption (2.5) holds. Indeed, (3.4) implies that

$$\frac{(k-k_i)}{n-k}[\Lambda_{01}^{LI}(k-k_i,1)-1] \sim F(k-k_i, n-k).$$

It is useful to recall that $k-k_i$ is actually the number of "instruments" in the familiar IV-based formulation of the LI model.

The important thing to note regarding the latter bound is that it relates to the well known Anderson-Rubin (AR) statistic. Bartlett (1948) and Anderson and Rubin (1949)Anderson and Rubin (1949) suggested an exact test that can be applied only if the null takes the (4.5) form. The idea behind the test is quite simple. Define $y_i^{\sim} = y_i - Y_i\beta_i^0$. Under the null, the model can be written as $y_i^{\sim} = X_{1i}\gamma_{1i} + u_i$. On the other hand, if the hypothesis is not true, $y_i^{\sim}$ will be a linear function of all the exogenous variables. Thus, the null may be assessed by the F test that the coefficient of the "excluded" regressors is zero in the regression of $y_i^{\sim}$ on all the exogenous variables. It is straightforward to show using the results on UL hypotheses in Dufour and Khalaf (1997) that the AR statistic associated with $\beta_i = \beta_i^0$ corresponds to a monotonic transformation of the LR criterion for testing the UL hypothesis $\Pi_{2i}\beta_i^0 = \pi_{2i}$ against an unrestricted alternative.

Let us now consider the hypothesis (4.6). On partitioning $\Pi_{1i} = [\Pi_{11i}, \Pi_{12i}]$ and $\Pi_{2i} = [\Pi_{21i}, \Pi_{22i}]$ conformably with $\beta_i = (\beta_{1i}', \beta_{2i}')'$ the corresponding reduced form restrictions may be expressed as

$$S_2 \begin{bmatrix} \pi_{1i} & \Pi_{11i} & \Pi_{12i} \\ \pi_{2i} & \Pi_{21i} & \Pi_{22i} \end{bmatrix} \begin{bmatrix} 1 \\ -\beta_{1i}^0 \\ -\beta_{2i} \end{bmatrix} = 0, \tag{4.12}$$

where

$$S_2 = \left[O_{(k-k_i\ ,\ k_i)}\ , I_{(k-k_i)}\right].$$

Let $\hat{\Sigma}_i^{02}$ and $\hat{\Sigma}_i$ be the error covariance LIML estimates imposing and ignoring (4.12), where the latter corresponds, as in the above example, to the unrestricted reduced form. Further, let $\hat{\Sigma}_i^{LI}$ denote the LIML-constrained error covariance estimate. These estimates lead to two LR-based statistics:

$$\begin{aligned} LR_{02}^{MR} &= n[\ln(\Lambda_{02}^{MR})] \\ \Lambda_{01}^{MR} &= \frac{|\hat{\Sigma}_i^{02}|}{|\hat{\Sigma}_i|}, \end{aligned} \tag{4.13}$$

and

$$\begin{aligned} LR_{02}^{LI} &= n[\ln(\Lambda_{02}^{LI})] \\ \Lambda_{01}^{LI} &= \frac{|\hat{\Sigma}_i^{02}|}{|\hat{\Sigma}_i^{LI}|}, \end{aligned} \tag{4.14}$$

The non-linearities in connection with (4.12) stem from the fact that β_{2i} is unknown. However, the special case of (4.12) that corresponds to specific (unknown) values of β_{2i} takes the UL form. Let $\Lambda_{02}^{*}(k-k_i,\ 1)$ denote the reciprocal of the Wilks statistic for testing these UL restrictions against an unrestricted alternative. Then conservative bounds for Λ_{02}^{LI} and Λ_{02}^{MR} can be obtained form the statistic $\Lambda_{02}^{*}(k-k_i, 1)$ or the $F(k-k_i, n-k)$ when applicable. To derive $\Lambda_{02}^{*}(k-k_i, 1)$, any choice for β_{2i} may be used in practice. Indeed, since the statistic's null distribution is nuisance-parameter-free (which justifies its use as a bounding statistic) the value retained for β_{2i} does not matter at all.

To conclude this example, note that our results here have further implications on Wang and Zivot (1998)'s asymptotic bound. Indeed, whereas LR_{02}^{LI} has a $\chi^2(m_{1i})$ asymptotic distribution only if identification holds, $n[\ln(\Lambda_{02}^{*}(k-k_i, 1))]$ is asymptotically distributed as $\chi^2(k-k_i)$ irrespective of the identification status. This implies that the asymptotic distribution of LR_{02}^{LI} is bounded by a $\chi^2(k-k_i)$ distribution independently of the conditions for identification. It follows that Wang and Zivot (1998)'s asymptotic bound is applicable beyond the Anderson-Rubin-type hypothesis. Finally, it is straightforward to see that Wang and Zivot (1998)'s bound is also valid (asymptotically) for the statistic LR_{02}^{MR}, in which case it has the potential of a tighter fit. Of course, the same comment also holds for our exact bound.

Similar results may be derived under (4.7). In this case, the implied reduced form constraints are

$$S_3 \begin{bmatrix} \pi_{1i} & \Pi_{11i} & \Pi_{12i} \\ \pi_{2i} & \Pi_{21i} & \Pi_{22i} \end{bmatrix} \begin{bmatrix} 1 \\ -\beta_{1i}^{0} \\ -\beta_{2i} \end{bmatrix} = \begin{bmatrix} 0 \\ \gamma_{12i}^{0} \\ 0 \end{bmatrix}, \tag{4.15}$$

where

$$S_3 = \left[O_{(k-(k_i-k_{2i}),\ k_i-k_{2i})}, I_{(k-(k_i-k_{2i}))} \right].$$

Thus, conservative bounds for the associated LR-LIML Λ_{03}^{LI} can be obtained from the statistic $\Lambda_{03}^{*}\left((k-(k_i-k_{2i})), 1\right)$ corresponding to the special case of (4.15) where β_{2i} is known, or the $F\left((k-(k_i-k_{2i})), n-k\right)$ when applicable, as previously shown. An asymptotic $\chi^2(k-(k_i-k_{2i}))$ bound may also be considered for the LR statistics, where $k-(k_i-k_{2i})$ represents the effective number of "instruments".

4.2 A multi-equation Anderson-Rubin type test

The AR test has recently received renewed interest. See, for example, Staiger and Stock (1997), Dufour and Jasiak (1999) and Wang and Zivot (1998). However, as it stands, the AR test ignores any restrictions relating to equations other than the ith. Here, we discuss an extension to the multiple equation framework.

Consider, in the context of (2.7) hypotheses of the form

$$H_{04} : \beta_i = \beta_i^{0}, \quad i = 1, \ \ldots \ , \ p. \tag{4.16}$$

Now define $Y^{\sim} = [y_1^{\sim}, ..., y_p^{\sim}]$, where $y_i^{\sim} = y_i - Y_i\beta_i^0,\ i = 1, ..., p$. Under the null hypotheses, the system of equations corresponds to the SURE model

$$y_i^{\sim} = X_{1i}\gamma_{1i} + u_i, \quad i = 1, \ \ldots \ , \ p, \tag{4.17}$$

whereas under the alternative the relevant specification is the MLR model including all the exogenous variables. Thus the problem reduces to testing the underlying SURE exclusion restrictions. Since the test involves the coefficients of different regressors within a MLR model, an exact critical value is not available. Nevertheless, the tests described in Dufour and Khalaf (1997) are applicable and lead to valid inference. Obtaining a bounding statistic is straightforward. The underlying UL hypothesis involves the exclusion of all variables which serve as instruments in any of the p equations.

The test can be readily extended to accommodate additional constraints on the exogenous variables coefficients. Maddala (1974) treats the single equation case. Specifically, consider hypothesis of the form

$$H_{05} : \beta_i = \beta_i^0, \quad \gamma_{11i} = 0_{11i}^0, \tag{4.18}$$

where γ_{11i} is a subset of γ_{1i}. Partition the matrix X_{1i} accordingly and let

$$y_i^{\sim\sim} = y_i - Y_i\beta_i^0 - X_{11i}\gamma_{11i}, \quad i = 1, \ \ldots \ , \ p. \tag{4.19}$$

Then the restricted model becomes the SURE system

$$y_i^{\sim\sim} = X_{12i}\gamma_{12i} + u_i, \quad i = 1, \ \ldots \ , \ p, \tag{4.20}$$

and the test may be carried out as above. Note that the tests are also applicable in a sub-system framework. However, as with the single equation AR test, the requirement is that all structural coefficients pertaining to the right-hand-side endogenous variables be specified under the null.

5 A Simulation study

This section reports an investigation, by simulation, of the performance of the various proposed test procedures. We focus on the LI examples. In each case, we also study 2SLS-based Wald tests, which are routinely computed in empirical practice. The asymptotic and MC test versions of the latter tests are considered. Since a bound is not available for these tests, we focus on the LMC and MMC tests. Unfortunately, our results confirm that IV-based tests realize computational savings at the risk of very poor performance. All the experiments were conducted using Gauss-386i VM version 3.2 and each was based on 1000 replications.

5.1 Monte Carlo design.

The experiments are based on the LI model (2.7). To simplify the exposition, we henceforth drop the subscript i when referring to (2.7). We consider three endogenous variables ($p = 3$ and $m = 2$) and $k = 3,\ 4,\ 5$ and 6 exogenous variables. In all cases, the structural equation includes only one exogenous variable, the constant regressor. In the following tables, $d = (k-1)-(p-1)$ refers to the *degree of over-identification.* The restrictions tested are of the form (4.5), and (4.6) with $m_1 = 1$. The sample sizes are set to $n = 25,\ 50,\ 100$. The exogenous regressors are independently drawn from the normal distribution, with means zero and unit variances. These were drawn only once. The errors were generated according to a multinormal distribution with mean zero and covariance

$$\Sigma = \begin{bmatrix} 1 & .95 & -.95 \\ .95 & 1 & -1.91 \\ -.95 & -1.91 & 12 \end{bmatrix} \tag{5.1}$$

The other coefficients were

$$\gamma_1 = 1,\ \beta_1 = 10,\ \beta_2 = -1.5,\ \Pi_1 = (1.5, 2)',\ \Pi_2 = \begin{bmatrix} \tilde{\Pi} \\ O_{(k-3,2)} \end{bmatrix}, \tag{5.2}$$

with

$$\tilde{\Pi} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{5.3}$$

The identification problem becomes mores serious as the determinant of $\Pi_2'\Pi_2$ gets closer to zero. In view of this, we also consider:

$$\tilde{\Pi} = \begin{bmatrix} 2 & 1.999 \\ 1.999 & 2 \end{bmatrix},$$
$$\tilde{\Pi} = \begin{bmatrix} .5 & .499 \\ .499 & .5 \end{bmatrix},$$
$$\tilde{\Pi} = \begin{bmatrix} .1 & .099 \\ .099 & .1 \end{bmatrix},$$
$$\tilde{\Pi} = \begin{bmatrix} .01 & .009 \\ .009 & .01 \end{bmatrix},$$

We examine the LR statistics (4.11), (4.14), (4.10) and (4.13). In the Tables which sumarize the results of this experiment, for convenience and clarity, the tests of the form (4.11) and (4.14) are denoted LR_{LIML} and those of the form (4.10) and (4.13) are denoted LR_{OLS}. We also consider Wald statistics based on LIML and 2SLS as defined in Appendix A and denote these statistics Wald_{LIML}

and Wald_{2SLS} respectively. We report the probability of Type I error for the standard asymptotic χ^2 test, and the LMC, MMC and BMC based procedures. The subscripts asy, LMC, MMC and BMC which appear in the subsequent Tables are used to identify these procedures respectively. In the case of the statistic (4.10), the local MC test is denoted PMC to account for the fact that the test is exact since the statistic is pivotal. We have also examined the generalized Wang and Zivot (1998) asymptotic bounds tests to which we refer as BND_z. We perform a power study (by varying the value of $\beta_1$away from the null value of 10) for the tests which size was adequate.

To generate the simulated samples in the LMC case, we consider the restricted LIML estimates of the parameters that are not specified by the null, except for the Wald_{2SLS} statistic. In this case, we use restricted 2SLS estimates for the structural equation and OLS based estimates for reduced form equations which complement the system. From these estimates, sum-of-squared-residuals are constructed which yield the usual estimate covariance estimate.

To ensure the complementarity of the MMC and the bounds procedures, the exact bounds are obtained by simulation (we do not use the F distribution). The Tsionas (1995) Simulated Annealing algorithm was implemented to obtain the maximal p-values. The MC tests in the power study are applied with $N = 19$ and 99 replications. For experiments restricted to size, we use 19 replications, since as is well known, the number of replications has no effect on test size in the context of MC tests. Tables1-5 summarize our findings.

5.2 Results

Although the Monte Carlo experiments were conditional on the selected design,our results show the following.

1. Identification problems severely distort the sizes of standard asymptotic tests. While the evidence of size distortions is notable even in identified models, the problem is far more severe in near-unidentified situations. The results for the Wald test are especially striking: empirical sizes exceeding 80 and 90% were observed! More importantly, increasing the sample size does not correct the problem. This result substantiates so-called "weak instruments" effects. The asymptotic LR behaves more smoothly in the sense that size distortions are not as severe; still some form of size correction is most certainly called for.

2. The performance of the standard bootstrap is disappointing. In general, the empirical sizes of LMC tests exceed 5% in most instances, even in identified models. In particular, bootstrap Wald tests fail completely in near-unidentified conditions.

3. Whether the rank condition for identification is imposed or not, more serious size distortions are observed in over-identified systems. This holds true for asymptotic and bootstrap procedures. While the problems associated with the Wald tests conform to general expectations, it is worth noting that the traditional bootstrap does not completely correct the size of LR tests.

4. In all cases, the Wald tests maximal randomized p-values are always one. This meant that under the null and the alternative, MMC empirical rejections were always zero (this result, for space considerations, is not reported in the Tables). Another Monte Carlo experiment (not reported here) confirms similar results in the context of a quasi-LR statistic based on derived 2SLS reduced form estimates.

5. The bounds tests and the MMC tests achieve size control in all cases. The strategy of resorting to MMC when the bounds test is not conclusive would certainly pay off, for the critical bound is easier to compute. However, it is worth noting that although the MMC are thought to be computationally burdensome, the SA maximization routine was observed to converge quite rapidly irrespective of the number of intervening nuisance parameters.

6. The LIML-LMC performs generally better than the generalized Wang and Zivot (1998) asymptotic bounds tests. Observe however that the LMC test is not exactly size correct, whereas Wang and Zivot (1998)'s tests sizes were not observed to exceed 5%. In situations were size was adequate, the LMC test showed superior power.

7. The performance of the Wald-LIML LMC test may seem acceptable, although the above remark in the case of the MMC p-value also holds in this case. As expected, power losses with respect to the LR test are noted. It is worth noting that since constrained and unconstrained MLE is done analytically, there seems to be arguments in favor of a Wald test if a LIML approach is considered.

The above findings mean that 2SLS-based tests are inappropriate in the weak instrument case and cannot be corrected by bootstrapping. Much more reliable tests will be obtained by applying the proposed LR-based procedures. The usual arguments on computational inconveniences should not be overemphasized. With the increasing availability of more powerful computers and improved software packages, there is less incentive to prefer a procedure on the grounds of execution ease.

5.3 Conclusion

The serious inadequacy of standard asymptotic tests in finite samples is widely observed in the SE context. Here, we have proposed alternative, simulation-based procedures and demonstrated their feasibility in an extensive Monte Carlo experiment. Particular attention was given to the identification problem. By exploiting MC methods and using these in combination with bounds procedures, we have constructed provably exact tests for arbitrary, possibly nonlinear hypotheses on the systems coefficients. We have also investigated the ability of the conventional bootstrap to provide more reliable inference in finite samples. The simulation results show that the latter fails when the simulated statistic is IV-based. In the case of the LR criteria, although the bootstrap did reduce the error in level, it did not achieve size control. In contrast, MMC LR tests perfectly controlled levels. The exact randomized procedures are computer intensive; however, with modern computer facilities, computational costs are no longer a hinderance.

Table 1. Empirical P(Type I error):
Testing a subset of endogenous variables coefficients, LR tests.

$\widetilde{\Pi} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

		LR_{LIML}					LR_{OLS}				
d	n	Asy	LMC	MMC	BMC	BD_Z	Asy	LMC	MMC	BMC	BD_Z
0	25	5.5	5.3	1.7	1.7	3.1	5.5	5.3	1.7	1.7	3.1
	50	5.9	5.3	1.1	1.1	1.6	5.9	5.3	1.1	1.1	1.6
	100	4.9	4.6	1.8	1.7	1.3	4.9	4.6	1.8	1.7	1.3
1	25	7.5	5.3	0.8	0.8	1.3	6.1	3.8	2.3	2.3	3.2
	50	7.9	5.3	0.4	0.2	0.1	4.9	5.2	1.7	1.7	2.2
	100	6.3	5.1	0.8	0.8	0.7	5.8	4.9	2.4	2.5	2.4
2	25	10.2	5.9	0.4	0.4	0.8	5.9	5.3	1.1	1.1	1.6
	50	8.9	5.7	0.8	0.7	0.4	5.8	4.7	2.6	2.6	2.9
	100	6.4	4.5	0.3	0.2	0.4	5.2	5.0	1.6	1.6	2.0
3	25	14.9	6.8	0.6	0.6	0.9	8.2	4.5	2.3	2.3	5.2
	50	9.8	5.0	0.2	0.2	0.1	6.3	3.9	1.9	1.9	3.1
	100	7.4	5.1	0.2	0.1	0.0	4.8	4.5	1.7	1.7	2.7
$\widetilde{\Pi} = \begin{bmatrix} 2 & 1.999 \\ 1.999 & 2 \end{bmatrix}$											
0	25	5.5	6.0	1.1	1.1	1.5	5.5	6.0	1.1	1.1	1.5
	50	5.5	5.9	1.4	1.3	2.0	5.5	5.9	1.4	1.3	2.0
	100	4.7	3.9	2.2	1.8	1.9	4.7	3.9	2.2	1.8	1.9
1	25	14.2	7.1	1.7	1.6	3.0	7.5	4.9	2.3	2.1	4.3
	50	12.7	5.6	1.1	1.1	1.2	5.2	4.5	1.6	1.6	2.0
	100	12.0	6.1	1.5	1.5	1.6	6.1	5.5	2.0	1.9	2.5
2	25	20.0	7.8	1.2	1.1	2.4	7.0	4.4	2.6	2.6	4.2
	50	17.0	6.7	1.8	1.5	1.7	6.4	4.4	2.6	2.6	2.9
	100	15.6	6.1	0.9	0.9	0.9	5.1	4.5	1.6	1.6	2.1
3	25	22.3	8.9	1.4	1.0	2.4	8.8	5.7	3.3	3.3	5.9
	50	23.6	8.6	0.9	0.8	1.4	7.1	4.4	2.5	2.5	4.1
	100	21.0	6.4	1.2	1.0	1.1	5.4	4.1	2.1	2.1	2.2

Table 1. Continued. Empirical P(Type I error):
Testing a subset of endogenous variables coefficients, LR tests.

$$\widetilde{\Pi} = \begin{bmatrix} .5 & .499 \\ .499 & .5 \end{bmatrix}$$

d	n	LR_{LIML} Asy	LMC	MMC	BMC	BD_Z	LR_{OLS} Asy	LMC	MMC	BMC	BD_Z
0	25	1.5	2.3	0.4	0.4	0.4	1.5	2.3	0.4	0.4	0.4
	50	1.6	3.4	0.4	0.4	0.4	1.6	3.4	0.4	0.4	0.4
	100	2.7	4.3	0.8	0.7	0.6	2.7	4.3	0.8	0.7	0.6
1	25	2.4	2.0	0.2	0.2	0.1	1.5	1.7	0.3	0.3	0.8
	50	3.4	3.9	0.4	0.4	0.4	1.6	2.6	0.5	0.5	0.6
	100	6.0	5.4	0.5	0.5	0.5	2.8	3.7	7.0	7.0	0.8
2	25	4.3	3.9	0.0	0.0	0.0	1.5	1.2	0.3	0.3	0.2
	50	6.6	5.2	0.3	0.3	0.0	1.6	2.4	0.7	0.7	0.6
	100	7.2	5.4	0.1	0.1	0.2	1.8	2.4	0.4	0.4	0.5
3	25	6.4	3.8	0.0	0.0	0.0	2.2	1.4	0.6	0.6	0.1
	50	6.3	4.6	0.1	0.0	0.1	1.0	0.9	0.1	0.1	0.3
	100	10.9	7.1	0.1	0.1	0.2	1.4	2.1	0.7	0.7	0.7

$$\widetilde{\Pi} = \begin{bmatrix} .1 & .099 \\ .099 & .1 \end{bmatrix}$$

d	n	LR_{LIML} Asy	LMC	MMC	BMC	BD_Z	LR_{OLS} Asy	LMC	MMC	BMC	BD_Z
0	25	1.1	1.4	0.3	0.3	0.2	1.1	1.4	0.3	0.3	0.2
	50	0.5	1.9	0.1	0.1	0.2	0.5	1.9	0.1	0.1	0.2
	100	0.6	1.5	0.2	0.2	0.1	0.6	1.5	0.2	0.2	0.1
1	25	0.6	1.1	0.0	0.0	0.0	0.7	1.0	0.3	0.3	0.4
	50	0.9	1.6	0.0	0.0	0.0	0.2	0.7	0.1	0.1	0.0
	100	1.1	2.4	0.0	0.0	0.0	0.5	1.8	0.0	0.0	0.0
2	25	0.2	2.2	0.0	0.0	0.0	0.4	0.8	0.1	0.1	0.1
	50	2.5	2.7	0.0	0.0	0.0	0.6	0.7	0.0	0.0	0.1
	100	1.5	2.7	0.0	0.0	0.0	0.0	0.9	0.0	0.0	0.0
3	25	1.1	1.4	0.3	0.3	0.2	1.1	1.4	0.3	0.3	0.2
	50	2.4	1.9	0.1	0.0	0.0	0.6	0.9	0.0	0.0	0.2
	100	1.8	2.0	0.1	0.0	0.0	0.5	0.5	0.2	0.2	0.2

Table 1. Continued. Empirical P(Type I error):
Testing a subset of endogenous variables coefficients, LR tests.

$$\widetilde{\Pi} = \begin{bmatrix} .01 & .009 \\ .009 & .01 \end{bmatrix}$$

		LR_{LIML}					LR_{OLS}				
d	n	Asy	MC	MMC	BMC	BD_Z	Asy	LMC	MMC	BMC	BD_Z
0	25	1.1	1.3	0.3	0.3	0.2	1.1	1.3	0.3	0.3	0.2
	50	0.4	1.4	0.1	0.1	0.1	0.4	1.4	0.1	0.1	0.0
	100	0.4	1.3	0.1	0.1	0.0	0.4	1.3	0.1	0.1	0.0
1	25	2.1	1.3	0.1	0.1	0.1	0.6	1.0	0.3	0.3	0.4
	50	1.8	1.3	0.0	0.0	0.0	0.2	0.1	0.0	0.0	0.0
	100	2.5	1.6	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0
2	25	5.4	1.8	0.0	0.0	0.0	0.5	0.8	0.3	0.2	0.2
	50	0.4	1.1	0.0	0.0	0.0	0.4	0.6	0.0	0.0	0.0
	100	1.7	1.7	0.0	0.0	0.0	0.1	0.5	0.0	0.0	0.0
3	25	9.0	2.6	1.0	1.0	0.1	1.4	1.0	0.3	0.3	0.8
	50	5.8	2.0	0.0	0.0	0.0	0.7	0.8	0.0	0.0	0.2
	100	0.4	1.3	0.1	0.1	0.0	0.4	0.5	0.2	0.2	0.1

Table 2. Empirical P(Type I error):
Testing a subset of endogenous variables coefficients, Wald tests.

		Wald - 2SLS		Wald - LIML	
$\widetilde{\Pi} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$					
d	n	Asy	LMC	Asy	LMC
0	25	4.8	4.4	4.8	2.4
	50	4.1	4.5	4.1	4.1
	100	5.0	4.6	5.0	4.9
1	25	8.6	5.8	8.3	3.9
	50	6.4	5.9	6.2	5.1
	100	5.4	4.9	5.5	4.9
2	25	11.0	6.8	9.9	4.3
	50	8.0	5.8	8.5	5.1
	100	7.6	5.9	7.2	4.7
3	25	14.2	8.5	14.3	4.9
	50	10.4	6.0	10.9	4.7
	100	8.1	6.1	7.4	5.0
$\widetilde{\Pi} = \begin{bmatrix} 2 & 1.999 \\ 1.999 & 2 \end{bmatrix}$					
0	25	2.3	2.3	2.3	1.9
	50	2.5	3.4	2.5	3.4
	100	0.7	2.7	0.7	3.4
1	25	8.2	5.3	8.6	3.3
	50	4.6	4.9	5.2	3.0
	100	4.2	4.3	5.1	4.0
2	25	12.6	5.9	13.9	3.1
	50	8.3	5.1	10.4	3.8
	100	7.6	3.7	11.7	3.5
3	25	14.7	7.3	18.7	4.1
	50	13.4	7.9	18.8	4.5
	100	11.6	5.1	17.1	3.7

Table 2. Continued. Empirical P(Type I error):
Testing a subset of endogenous variables coefficients, Wald tests.

d	n	Wald - 2SLS Asy	Wald - 2SLS LMC	Wald - LIML Asy	Wald - LIML LMC
$\widetilde{\Pi} = \begin{bmatrix} .5 & .499 \\ .499 & .5 \end{bmatrix}$					
0	25	4.6	4.2	4.6	1.4
	50	3.9	4.6	3.9	2.0
	100	2.9	3.0	2.9	1.7
1	25	10.9	5.8	6.0	2.0
	50	7.2	5.6	4.8	2.2
	100	6.8	5.2	5.9	2.9
2	25	17.7	11.6	10.5	2.7
	50	13.3	7.4	6.7	2.4
	100	11.0	6.8	8.3	3.1
3	25	22.6	10.2	10.2	2.4
	50	18.3	10.5	10.4	3.4
	100	14.3	7.0	6.3	2.7
$\widetilde{\Pi} = \begin{bmatrix} .1 & .099 \\ .099 & .1 \end{bmatrix}$					
0	25	18.4	9.5	18.4	0.9
	50	11.7	7.6	11.7	0.8
	100	7.2	6.2	7.2	0.8
1	25	50.0	21.2	26.5	0.7
	50	29.0	12.6	14.0	0.7
	100	21.1	9.1	10.8	1.5
2	25	71.6	35.0	30.5	1.2
	50	55.5	21.0	21.8	1.5
	100	41.1	16.5	14.6	1.5
3	25	82.3	44.4	33.4	1.0
	50	72.9	29.9	23.9	1.1
	100	58.1	21.2	17.4	1.3

Table 2. Continued. Empirical P(Type I error):
Testing a subset of endogenous variables coefficients, Wald tests.

$$\widetilde{\Pi} = \begin{bmatrix} .01 & .009 \\ .009 & .01 \end{bmatrix}$$

		Wald - 2SLS		Wald - LIML	
d	n	Asy	MC	Asy	LMC
0	25	58.1	33.5	58.1	0.3
	50	59.2	35.5	59.2	0.4
	100	55.5	31.0	55.5	0.4
1	25	88.9	57.9	75.1	0.4
	50	84.9	49.6	66.8	0.7
	100	85.0	44.8	68.0	0.6
2	25	85.0	44.8	79.7	0.1
	50	55.5	21.0	76.9	0.5
	100	95.3	58.7	74.3	0.6
3	25	99.3	82.3	84.4	1.0
	50	98.9	76.4	81.6	0.6
	100	98.9	70.0	77.8	0.5

Table 3. Empirical P(Type I error):
Testing the full vector of endogenous variables coefficients[8]

		Wald_{2SLS}		LR_{LIML}				LR_{OLS}		Wald_{LIML}		AR
d	n	Asy	LMC	Asy	LMC	MMC	BD_Z	Asy	LMC	Asy	MC	
						$\widetilde{\Pi} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$						
0	25	6.2	4.6	7.7	6.0	7.7	6.2	7.7	6.0	6.2	3.7	5.5
	50	5.3	4.5	6.7	4.5	6.7	5.3	6.7	4.5	5.3	4.2	5.8
	100	5.5	5.9	5.1	4.5	5.1	5.5	5.1	4.5	5.5	5.8	4.2
1	25	9.7	5.1	10.9	5.5	3.1	5.2	8.9	5.3	9.2	3.4	4.8
	50	7.1	5.1	6.8	4.4	2.1	3.5	6.1	4.	6.7	4.1	4.7
	100	6.5	4.8	6.6	4.7	2.2	2.4	6.3	4.3	6.3	4.7	5.3
2	25	11.4	6.2	13.3	6.5	1.6	3.5	8.6	5.0	12.1	4.0	4.6
	50	9.5	5.6	10.1	6.8	2.3	2.5	6.9	5.9	8.9	5.0	4.9
	100	8.2	5.9	6.2	4.1	0.8	1.2	5.2	4.2	7.9	5.6	4.2
3	25	14.8	7.2	16.0	7.5	1.4	2.6	11.4	6.3	15.5	5.0	4.4
	50	11.8	5.4	10.2	4.8	1.2	1.7	7.5	5.2	13.0	4.2	5.6
	100	8.4	6.4	7.4	5.2	0.6	0.2	5.0	4.7	8.0	5.9	4.3
			4.3									
						$\widetilde{\Pi} = \begin{bmatrix} 2 & 1.999 \\ 1.999 & 2 \end{bmatrix}$						
0	25	3.7	3.3	7.7	6.0	6.0	7.7	7.7	6.0	3.7	3.1	5.5
	50	2.5	4.1	6.7	4.5	4.5	6.7	6.7	4.5	2.5	3.9	5.8
	100	2.4	3.5	5.1	4.5	4.5	5.1	5.1	4.5	2.4	3.6	4.2
1	25	8.1	5.0	12.9	5.4	3.8	6.9	8.9	5.3	7.7	2.7	4.8
	50	4.9	3.3	9.7	5.7	3.4	4.3	6.1	4.6	4.4	1.9	4.7
	100	4.4	4.0	11.1	5.5	3.6	4.8	13.3	6.3	4.0	4.1	5.3
2	25	12.8	6.5	18.1	6.6	2.4	4.7	8.6	5.0	11.8	4.1	4.6
	50	9.9	5.2	15.6	7.2	3.8	3.6	6.9	5.9	9.0	3.6	4.9
	100	6.5	4.0	13.2	5.7	2.7	2.5	5.2	4.2	6.0	3.2	4.2
3	25	14.9	6.9	20.7	7.3	2.3	4.1	11.4	6.3	14.8	3.3	4.4
	50	12.1	5.7	20.8	7.3	2.4	3.7	7.5	5.2	14.2	3.6	5.6
	100	9.2	5.0	17.3	6.4	2.2	2.6	5.0	4.7	11.2	3.1	4.3

[8]Empirical rejections for the MMC and the BMC LR_{LIML} test under the null were identical.

Table 3. Continued. Empirical P(Type I error):
Testing the full vector of endogenous variables coefficients

$\widetilde{\Pi} = \begin{bmatrix} .5 & .499 \\ .499 & .5 \end{bmatrix}$

d	n	$Wald_{2SLS}$ Asy	$Wald_{2SLS}$ LMC	LR_{LIML} Asy	LR_{LIML} LMC	LR_{LIML} MMC	LR_{LIML} BD_Z	LR_{OLS} Asy	LR_{OLS} PMC	$WALD_{LIML}$ Asy	$WALD_{LIML}$ LMC	AR
0	25	5.6	4.0	7.7	6.0	6.0	7.7	7.7	6.0	5.6	2.8	5.5
	50	3.6	4.5	6.7	4.5	4.5	6.7	6.7	4.5	3.6	4.1	5.8
	100	2.7	3.4	5.1	4.5	4.5	5.1	5.1	4.5	2.7	3.0	4.2
1	25	11.9	6.4	12.8	5.4	3.7	6.7	8.5	5.3	8.9	2.7	4.8
	50	6.5	5.2	9.7	5.8	3.4	4.5	6.1	4.6	4.8	3.1	4.7
	100	5.6	4.4	11.1	5.5	3.6	4.8	6.3	4.3	4.1	4.2	5.3
2	25	18.9	10.3	18.0	6.6	2.4	4.7	8.6	5.0	14.2	3.3	4.6
	50	12.1	6.2	15.7	7.3	3.8	3.6	6.9	5.9	10.2	2.6	4.9
	100	9.4	5.0	13.2	5.7	2.7	2.5	5.2	4.2	7.2	2.8	4.2
3	25	23.0	10.2	20.9	7.2	2.4	4.1	11.4	6.3	16.8	3.5	4.4
	50	18.5	8.2	20.9	7.1	2.5	3.7	7.5	5.2	15.8	3.4	5.6
	100	12.4	6.1	17.2	6.4	2.2	2.6	5.0	4.7	12.2	3.6	4.3

$\widetilde{\Pi} = \begin{bmatrix} .1 & .099 \\ .099 & .1 \end{bmatrix}$

d	n	$Wald_{2SLS}$ Asy	$Wald_{2SLS}$ LMC	LR_{LIML} Asy	LR_{LIML} LMC	LR_{LIML} MMC	LR_{LIML} BD_Z	LR_{OLS} Asy	LR_{OLS} PMC	$WALD_{LIML}$ Asy	$WALD_{LIML}$ LMC	AR
0	25	21.7	12.9	7.7	6.0	6.0	7.7	7.7	6.0	21.7	3.0	5.5
	50	13.9	10.4	6.7	4.5	4.5	6.7	6.7	4.5	13.9	2.9	5.8
	100	8.8	6.8	5.1	4.5	4.5	5.1	5.1	4.5	8.8	3.1	4.2
1	25	55.9	28.8	13.0	5.5	3.8	6.8	8.9	5.3	29.9	2.5	4.8
	50	33.7	16.7	9.8	5.7	3.4	4.3	6.1	4.6	16.2	1.8	4.7
	100	23.0	10.8	10.8	5.4	3.6	4.9	6.3	4.3	11.7	2.7	5.3
2	25	75..4	44.8	18.6	6.7	2.6	5.1	8.6	5.0	35.1	2.7	4.6
	50	60.4	26.8	15.7	5.7	3.9	3.6	6.9	4.6	16.2	1.8	4.9
	100	44.1	18.9	13.4	6.0	2.9	2.5	5.2	4.2	16.9	2.4	4.2
3	25	85.5	52.5	21.5	7.5	2.4	4.0	11.4	6.3	37.8	2.6	4.4
	50	77.3	37.3	20.5	7.1	2.4	3.8	7.5	5.2	28.6	2.9	5.6
	100	61.0	22.7	17.4	6.1	2.2	2.5	5.0	4.7	21.5	2.9	4.3

Table 3. Continued. Empirical P(Type I error):
Testing the full vector of endogenous variables coefficients

$$\Pi = \begin{bmatrix} .01 & .009 \\ .009 & .01 \end{bmatrix}$$

d	n	Wald_{2SLS}		LR_{LIML}				LR_{OLS}		Wald_{LIML}		AR
		Asy	LMC	Asy	LMC	MMC	BD_Z	Asy	PMC	Asy	LMC	
0	25	68.7	51.1	7.7	6.0	6.0	7.7	7.7	6.0	68.7	3.7	5.5
	50	68.6	52.0	6.7	4.5	4.5	6.7	6.7	4.5	68.6	3.6	5.8
	100	64.6	44.3	5.1	4.5	4.5	5.1	4.5	4.5	64.6	3.2	4.2
1	25	92.5	72.6	14.3	6.1	4.9	7.6	8.9	5.3	79.0	3.8	4.8
	50	91.1	66.5	10.9	6.0	4.1	4.9	6.1	4.6	73.1	3.9	4.7
	100	90.2	61.3	11.3	5.1	3.7	5.0	6.3	4.3	7.11	3.2	5.3
2	25	98.9	85.3	21.8	6.4	3.1	5.8	8.6	5.0	82.3	2.8	4.6
	50	98.4	79.4	18.1	6.1	4.4	4.6	6.9	4.6	73.1	3.9	4.9
	100	97.5	71.5	14.7	5.4	3.1	2.9	5.2	4.2	76.9	3.2	4.2
3	25	99.6	90.7	26.5	7.7	3.1	5.3	11.4	6.3	84.9	2.5	4.4
	50	99.3	87.2	23.6	6.5	3.0	5.3	7.5	5.2	82.2	3.8	5.6
	100	99.1	81.9	20.7	6.2	2.8	3.0	5.0	4.7	78.5	2.7	4.3

Table 4. Power:
Testing the full vector of endogenous variables coefficients

$H_0 : \beta_{11} = 10$				LR_{LIML}				LR_{OLS}	$Wald_{LIML}$	AR
n	d	β_{11}	N	LMC	MMC	BMC	BD_z	PMC	LMC	
25	0	10.1	19	12.9	12.9	12.9	12.9	12.9	14.2	15.2
			99	14.1	14.1	14.1		14.1	15.0	
		10.2	19	33.4	33.4	33.4	33.4	33.4	32.3	39.3
			99	37.1	37.1	37.1		37.1	34.6	
		10.3	19	59.2	59.2	59.2	59.2	59.2	51.0	67.2
			99	65.8	65.8	65.8		65.8	56.9	
		10.5	19	88.5	88.5	88.5	88.5	88.5	72.8	95.7
			99	94.0	94.0	94.0		94.0	77.6	
		11	19	99.6	99.6	99.6	99.6	99.6	82.7	1.0
			99	1.0	1.0	1.0		1.0	86.0	
	1	10.1	19	9.9	7.1	7.1	11.2	9.5	12.2	10.4
			99	12.2	8.0	8.0		11.4	13.8	
		10.2	19	26.5	19.0	19.0	31.7	23.6	28.0	29.0
			99	30.2	22.8	22.8		27.4	32.6	
		10.3	19	47.9	38.4	38.4	55.8	43.0	44.5	51.5
			99	55.0	44.4	44.3		50.4	51.1	
		10.5	19	82.0	73.7	73.7	89.6	78.3	68.7	86.6
			99	88.6	80.8	80.8		84.8	73.8	
		11	19	98.4	97.7	97.7	99.9	97.9	79.3	99.7
			99	99.9	99.4	99.4		99.6	83.6	
	2	10.1	19	13.5	5.2	5.1	9.3	9.3	11.5	10.7
			99	14.6	5.5	5.4		10.5	12.6	
		10.2	19	31.4	16.4	16.2	29.0	25.3	25.9	29.7
			99	35.9	19.0	18.8		29.4	29.9	
		10.3	19	54.5	34.5	34.3	51.5	46.9	42.3	51.0
			99	59.6	39.2	38.8		48.8	48.9	
		10.5	19	84.3	68.7	68.7	87.6	76.5	65.6	86.6
			99	91.5	77.0	76.8		84.5	69.8	
		11	19	99.1	97.3	97.3	99.8	98.2	77.2	99.6
			99	1.0	99.2	99.2		99.5	80.1	
	3	10.1	19	13.2	4.2	4.2	8.2	8.9	12.4	10.5
			99	15.0	4.7	4.2		9.9	12.6	
		10.2	19	32.2	12.6	12.3	23.1	21.7	28.3	25.7
			99	35.9	14.2	13.7		24.6	30.9	
		10.3	19	56.6	28.4	27.9	46.9	43.4	46.6	48.1
			99	61.4	32.6	30.7		46.5	49.7	
		10.5	19	88.5	66.8	65.4	86.2	76.7	66.2	85.1
			99	93.1	73.8	71.3		84.1	72.3	
		11	19	98.9	96.1	96.0	99.7	97.7	77.4	1
			99	1.0	99.1	99.0		99.6	81.9	

Table 4. Continued. Power:
Testing the full vector of endogenous variables coefficients

$H_0 : \beta_{11} = 10$				LR_{LIML}				LR_{OLS}	$Wald_{LIML}$	AR
n	d	β_{11}	N	LMC	MMC	BMC	BD_z	PMC	LMC	
50	0	10.1	19	23.4	23.4	23.4	27.3	23.4	26.0	24.4
			99	24.3	24.3	24.3		24.3	27.4	
		10.2	19	58.5	58.5	58.5	70.0	58.5	57.7	66.4
			99	65.4	65.4	65.4		65.4	63.9	
		10.3	19	86.5	86.5	86.5	94.3	86.5	80.3	93.1
			99	92.7	92.7	92.7		92.7	85.1	
		10.5	19	99.2	99.2	99.2	99.1	99.2	93.1	99.9
			99	99.9	99.9	99.9		99.9	94.4	
		11	19	1.0	1.0	1.0	1.0	1.0	95.9	1.0
			99	1.0	1.0	1.0		1.0	96.3	
	1	10.1	19	20.5	13.5	13.5	19.0	17.1	24.1	20.5
			99	22.4	15.6	15.6		19.3	25.9	
		10.2	19	59.9	48.0	48.0	59.2	53.2	57.9	60.9
			99	66.9	54.0	54.0		60.2	62.9	
		10.3	19	87.9	81.7	81.7	92.0	84.7	82.1	92.8
			99	93.2	88.4	88.4		90.7	84.8	
		10.5	19	99.5	98.8	98.8	99.8	99.1	92.2	99.8
			99	99.9	99.7	99.7		99.8	94.4	
		11	19	1.0	1.0	1.0	1.0	1.0	95.6	1.0
			99	1.0	1.0	1.0		1.0	96.4	
	2	10.1	19	23.0	11.1	11.1	15.5	18.2	22.0	20.4
			99	24.3	11.9	11.8		20.1	24.1	
		10.2	19	61.3	42.1	42.1	53.1	52.8	56.0	59.8
			99	67.6	46.4	45.8		57.5	59.0	
		10.3	19	88.4	77.0	76.9	88.6	83.1	76.2	89.9
			99	93.2	83.8	83.6		89.1	80.7	
		10.5	19	99.6	98.3	98.3	99.9	98.8	89.7	99.9
			99	99.9	99.6	99.6		99.8	91.6	
		11	19	1.0	1.0	1.0	1.0	1.0	94.3	1.0
			99	1.0	1.0	1.0	1.0	1.0	94.8	
	3	10.1	19	17.7	6.9	6.6	9.7	13.3	18.1	17.1
			99	22.8	7.7	6.9		16.9	22.9	
		10.2	19	54.5	28.0	27.3	38.5	42.3	50.2	48.8
			99	61.8	31.8	30.7		46.2	54.5	
		10.3	19	82.7	61.5	60.1	74.5	73.6	73.4	81.8
			99	90.1	68.7	67.2		79.4	78.8	
		10.5	19	99.0	94.8	94.7	98.6	97.2	89.0	99.0
			99	99.6	98.2	98.0		98.9	92.2	
		11	19	1.0	1.0	1.0	1.0	1.0	85.2	1.0
			99	1.0	1.0	1.0	1.0	1.0	95.3	

Table 4. Power:
Testing the full vector of endogenous variables coefficients

$H_0 : \beta_{11} = 10$				LR_{LIML}				LR_{OLS}	$Wald_{LIML}$	AR
n	d	β_{11}	N	LMC	MMC	BMC	BD_z	PMC	LMC	
100	0	10.1	19	40.3	40.3	40.3	48.3	40.3	43.6	45.9
			99	45.1	45.1	45.1		45.1	49.0	
		10.2	19	90.2	90.2	90.2	95.2	90.2	88.1	94.5
			99	94.7	94.7	94.7		94.7	91.9	
		10.3	19	99.7	99.7	99.7	1.0	99.7	98.0	1.0
			99	99.9	99.9	99.9		99.9	99.4	
		10.5.	19	1.0	1.0	1.0	1.0	1.0	99.7	1.0
			99	1.0	1.0	1.0		1.0	99.9	
		11	19	1.0	1.0	1.0	1.0	1.0	99.9	1.0
			99	1.0	1.0	1.0		1.0	99.9	
	1	10.1	19	37.1	27.8	27.8	33.9	34.2	40.7	39.2
			99	41.4	31.6	31.6		37.9	44.3	
		10.2	19	89.0	82.2	82.2	89.0	85.9	86.9	91.4
			99	93.8	87.0	87.0		90.3	89.9	
		10.3	19	99.6	99.2	99.2	99.7	99.7	97.3	99.8
			99	99.8	99.8	99.8		99.8	98.3	
		10.5	19	1.0	1.0	1.0	1.0	1.0	99.0	1.0
			99	1.0	1.0	1.0		1.0	99.1	
		11	19	1.0	1.0	1.0	1.0	1.0	99.2	1.0
			99	1.0	1.0	1.0		1.0	99.3	
	2	10.1	19	35.1	18.1	18.1	23.2	27.2	37.7	33.6
			99	40.7	19.4	18.9		31.4	41.6	
		10.2	19	86.9	70.5	70.4	81.0	78.6	83.7	87.1
			99	95.1	77.0	76.6		84.3	88.6	
		10.3	19	98.8	97.0	97.0	98.7	97.8	97.1	99.1
			99	99.6	98.4	98.4		98.8	98.9	
		10.5	19	1.0	1.0	1.0	1.0	1.0	99.5	1.0
			99	1.0	1.0	1.0		1.0	99.6	
		11	19	1.0	1.0	1.0	1.0	1.0	99.6	1.0
			99	1.0	1.0	1.0		1.0	99.6	
	3	10.1	19	32.7	11.3	11.0	15.3	24.7	36.7	27.6
			99	38.3	15.6	13.7		27.3	40.6	
		10.2	19	85.2	63.4	62.9	71.6	75.6	82.4	83.0
			99	89.0	70.2			82.0	88.0	
		10.3	19	98.9	93.4	93.0	98.1	96.6	96.4	99.2
			99	99.8	97.5	97.2		98.7	98.4	
		10.5	19	1.0	1.0	1.0	1.0	1.0	99.4	1.0
			99	1.0	1.0	1.0		1.0	99.3	
		11	19	1.0	1.0	1.0	1.0	1.0	99.7	1.0
			99	1.0	1.0	1.0		1.0	99.6	

Table 5. Power:
Testing a subset of endogenous variables coefficients

$H_0 : \beta_{11} = 10$				LR_{LIML}				LR_{OLS}			
n	d	β_{11}	N	LMC	MMC	BMC	BD_z	LMC	MMC	BMC	BD_z
25	0	10.1	19	6.9	3.0	2.2	3.0	6.9	3.0	2.2	3.3
			99	6.9	3.4	1.6		6.9	3.4	1.6	
		10.2	19	9.8	4.5	3.5	5.0	9.8	4.5	3.5	5.0
			99	10.6	5.8	3.5		10.6	5.8	3.5	
		10.3	19	11.8	6.4	5.3	6.4	11.8	6.4	5.3	6.4
			99	13.2	7.7	4.9		13.2	7.7	4.9	
		10.5	19	14.5	8.9	6.8	9.7	14.5	8.9	6.8	9.7
			99	17.1	12.0	6.9		17.1	12.0	6.9	
		11	19	16.8	11.3	8.3	11.2	16.8	11.3	8.3	11.2
			99	19.1	14.9	8.1		19.1	14.9	8.1	
	1	10.1	19	8.5	1.5	1.5	2.0	6.7	2.3	2.3	3.9
			99	8.5	2.8	1.3		6.9	3.4	2.6	
		10.2	19	11.1	2.5	2.5	4.0	9.0	4.1	4.1	6.9
			99	12.9	4.4	2.0		9.3	5.2	3.7	
		10.3	19	13.4	3.7	3.4	6.1	12.0	6.0	5.9	9.1
			99	15.3	7.2	3.5		12.7	8.0	5.6	
		10.5	19	15.9	5.6	5.3	8.9	14.3	8.1	8.1	12.4
			99	18.8	10.3	5.8		16.2	11.2	8.4	
		11	19	18.0	8.4	7.0	11.1	16.2	9.9	9.2	14.8
			99	21.6	10.8	7.8		18.4	13.2	10.5	
	2	10.1	19	7.5	0.6	0.5	1.2	5.6	2.5	2.5	4.3
			99	9.1	1.4	0.7		5.8	2.2	2.1	
		10.2	19	10.6	1.7	1.6	3.3	8.0	4.0	4.0	6.9
			99	10.9	4.5	1.7		8.1	5.5	3.7	
		10.3	19	13.4	2.9	2.9	5.3	10.1	6.2	6.2	10.1
			99	13.6	6.2	2.8		10.6	6.5	5.5	
		10.5	19	15.6	5.4	4.7	7.5	13.0	7.9	7.9	12.9
			99	15.7	7.9	4.6		13.0	9.0	7.4	
		11	19	17.9	6.5	5.6	9.6	15.2	9.8	9.8	16.0
			99	19.4	10.1	5.6		15.3	11.6	9.0	
	3	10.1	19	8.8	1.1	1.1	1.6	5.4	2.7	2.6	6.0
			99	8.7	1.9	0.7		5.3	2.8	2.2	
		10.2	19	11.5	1.9	1.5	2.9	7.7	4.2	4.2	7.9
			99	12.9	3.8	1.3		6.3	3.8	3.1	
		10.3	19	13.9	2.7	2.6	3.7	8.3	4.8	4.8	10.1
			99	15.1	5.4	2.0		8.3	4.8	4.8	
		10.5	19	16.9	3.2	2.9	5.2	10.5	5.7	5.7	12.8
			99	17.7	7.3	2.5		11.4	7.7	6.6	
		11	19	20.4	4.1	3.6	7.6	12.7	7.6	7.6	16.3
			99	22.4	8.7	3.5		13.8	10.4	8.7	

Table 5. Power:
Testing a subset of endogenous variables coefficients

$H_0 : \beta_{11} = 10$				LR_{LIML}				LR_{OLS}			
n	d	β_{11}	N	LMC	MMC	BMC	BD_z	LMC	MMC	BMC	BD_z
50	0	10.1	19	10.3	4.6	4.1	4.3	10.3	4.6	4.1	4.3
			99	10.5	7.5	3.4		10.5	7.5	3.4	
		10.2	19	18.5	10.6	8.6	10.2	18.5	10.6	8.6	10.2
			99	19.8	15.1	7.9		19.8	15.1	7.9	
		10.3	19	25.4	16.4	12.0	15.3	25.4	16.4	12.0	15.3
			99	27.4	21.9	12.4		27.4	21.9	12.4	
		10.5	19	31.9	23.8	18.8	23.6	31.9	23.8	18.8	23.6
			99	35.2	32.2	19.4		35.2	32.2	19.4	
		11	19	38.3	30.2	23.9	29.4	38.3	30.2	23.9	29.4
			99	42.7	37.2	27.0		42.7	37.2	27.0	
	1	10.1	19	10.6	2.6	2.3	2.6	8.3	3.8	3.6	5.2
			99	11.0	4.9	2.5		8.8	5.9	4.0	
		10.2	19	18.7	7.8	6.0	7.8	15.6	8.7	8.6	11.3
			99	19.9	11.9	6.3		17.9	13.8	9.2	
		10.3	19	25.4	11.9	9.7	12.8	21.5	13.7	13.2	17.7
			99	28.8	18.4	10.5		24.7	20.3	15.6	
		10.5	19	34.3	18.8	15.5	19.5	29.4	20.2	19.0	5.6
			99	39.1	27.6	17.0		33.3	28.1	21.7	
		11	19	41.7	25.3	20.7	27.5	37.2	27.0	24.8	33.2
			99	48.2	35.5	24.0		42.7	36.6	29.0	
	2	10.1	19	8.8	1.7	1.1	1.4	6.7	3.5	3.5	4.6
			99	10.3	3.5	1.2		6.9	4.1	3.3	
		10.2	19	15.7	4.6	2.8	4.8	11.8	7.2	7.1	9.7
			99	18.0	7.9	3.5		13.3	9.0	7.0	
		10.3	19	20.4	7.2	5.2	8.2	15.5	9.7	9.6	13.9
			99	23.4	14.0	5.4		17.9	14.1	9.6	
		10.5	19	26.5	11.4	9.1	14.0	21.1	15.2	15.0	19.8
			99	30.3	19.5	10.0		25.3	20.7	15.9	
		11	19	33.0	16.4	13.1	18.2	26.9	20.6	19.5	26.7
			99	37.3	22.9	15.6		31.4	27.1	21.3	
	3	10.1	19	10.3	1.2	0.5	1.1	6.8	2.7	2.7	5.7
			99	11.9	3.4	0.7		7.1	4.5	3.4	
		10.2	19	17.4	3.3	2.2	3.6	12.0	6.9	6.9	11.3
			99	21.4	7.5	2.8		13.8	9.5	7.4	
		10.3	19	23.4	5.5	4.2	6.3	16.0	10.1	10.1	17.4
			99	28.8	12.7	4.8		19.2	14.9	12.0	
		10.5	19	31.6	10.5	8.3	12.3	23.3	15.2	15.2	24.1
			99	37.5	20.2	8.9		26.9	21.5	17.7	
		11	19	41.0	15.9	11.9	18.3	30.6	22.1	21.2	32.8
			99	45.3	27.5	13.6		35.6	29.2	25.3	

Table 5. Power:
Testing a subset of endogenous variables coefficients

$H_0 : \beta_{11} = 10$				LR_{LIML}				LR_{OLS}			
n	d	β_{11}	N	LMC	MMC	BMC	BD_z	LMC	MMC	BMC	BD_z
100	0	10.1	19	15.7	7.8	6.3	7.9	15.7	7.8	6.3	7.9
			99	17.1	12.5	7.3		17.1	12.5	7.3	
		10.2	19	33.8	20.8	18.9	22.9	33.8	20.8	18.9	22.9
			99	37.4	32.1	20.4		37.4	32.1	20.4	
		10.3	19	47.6	34.3	29.1	36.6	47.6	34.3	29.1	36.6
			99	52.3	47.7	33.2		52.3	47.7	33.2	
		10.5	19	62.3	50.8	44.4	52.2	62.3	50.8	44.4	52.2
			99	67.7	63.2	48.8		67.7	63.2	48.8	
		11	19	73.6	65.1	57.8	64.2	73.6	65.1	57.8	64.2
			99	78.5	75.0	63.0		78.5	75.0	63.0	
	1	10.1	19	16.9	4.5	4.1	4.7	13.5	6.9	6.7	7.9
			99	16.6	10.4	4.4		14.6	10.7	6.7	
		10.2	19	34.0	14.5	13.0	16.8	29.1	18.6	17.6	23.5
			99	38.9	25.9	15.6		32.6	26.7	21.5	
		10.3	19	49.6	26.2	24.0	28.1	42.4	32.2	30.3	34.6
			99	54.6	44.3	26.0		47.7	39.4	32.1	
		10.5	19	64.0	42.2	37.5	45.6	59.3	48.6	44.7	52.5
			99	69.9	58.9	42.2		63.5	58.1	49.1	
		11	19	73.7	56.5	51.1	60.8	70.9	62.6	58.4	66.7
			99	80.4	68.6	56.1		76.3	72.5	62.5	
	2	10.1	19	17.4	3.0	2.5	3.8	12.9	6.6	6.5	7.9
			99	19.0	12.6	3.3		12.9	9.3	7.5	
		10.2	19	38.5	11.7	10.8	13.0	26.4	18.5	17.7	22.3
			99	42.1	271	10.7		29.7	25.4	19.9	
		10.3	19	52.9	22.3	19.4	24.8	40.7	30.6	29.0	37.4
			99	58.6	42.7	21.5		45.0	40.2	33.6	
		10.5	19	66.5	37.8	33.8	43.0	57.4	47.5	44.6	54.6
			99	70.9	59.6	38.6		62.5	58.3	50.8	
		11	19	77.0	53.2	47.3	58.0	70.9	62.6	50.2	69.0
			99	82.0	70.7	53.2		75.6	71.1	65.1	
	3	10.1	19	15.5	2.5	1.8	2.2	10.9	6.8	6.6	7.4
			99	18.2	8.4	1.7		11.0	8.4	6.7	
		10.2	19	35.4	9.0	6.8	8.7	25.2	178	17.1	21.7
			99	40.6	20.8	7.7		27.1	22.0	19.2	
		10.3	19	49.1	19.3	14.7	17.7	36.6	28.8	27.6	34.5
			99	55.8	34.1	15.3		40.6	36.0	30.6	
		10.5	19	64.3	30.3	25.7	31.8	52.2	44.4	42.8	51.7
			99	72.8	49.5	28.4		59.1	53.1	46.9	
		11	19	74.2	43.2	38.7	48.7	66.8	58.1	56.4	68.7
			99	81.8	64.6	44.4		74.0	69.9	64.0	

6 Appendix A: Standard Limited Information estimators and hypothesis tests

In the context of (2.7), the 2SLS is

$$\begin{aligned} \hat{\delta}_i &= [Z_i' P_i (P_i' P_i)^{-1} P_i' Z_i]^{-1} Z_i' P_i (P_i' P_i)^{-1} P_i' y_i, \\ P_i &= [\ X \quad X(X'X)^{-1} X' Y_i\ ]. \end{aligned} \tag{6.4}$$

Wald-type tests for linear restrictions on structural parameters are routinely associated with 2SLS estimation. For hypotheses of the form $R_i \delta_i = r_i$, where R_i is a known $(q_i \times m_i)$ matrix of rank q_i and r_i consists of known constants, the 2SLS-based Wald test statistic is

$$\begin{aligned} \tau_w &= \frac{1}{s^2}(r_i - R_i \hat{\delta}_i)' - [R_i'(Z_i P_i (P_i' P_i)^{-1} P_i' Z_i)^{-1} R_i]\,(r_i - R_i \hat{\delta}_i), \\ s^2 &= \frac{1}{n}(y_i - Z_i \hat{\delta}_i)'(y_i - Z_i \hat{\delta}_i)'. \end{aligned} \tag{6.5}$$

Imposing identification, the asymptotic null distribution of τ_w is $\chi^2(q)$. For an asymptotic theory conformable with under-identification, see Staiger and Stock (1997).

LIML corresponds to maximizing, imposing (2.9), the likelihood function associated with (2.7)

$$\begin{aligned} \mathcal{L}(y, Y | X_1, X_2) &= -\frac{n(m_i+1)}{2} \ln(2\pi) - \frac{n}{2} \ln |\Sigma_i| - \frac{1}{2} tr \Sigma_i^{-1} D_i' D_i, \\ D &= [\ y_i \quad Y_i\ ] - X \begin{bmatrix} \pi_{i1} & \Pi_{i1} \\ \pi_{i2} & \Pi_{i2} \end{bmatrix}, \end{aligned} \tag{6.6}$$

where Σ_i denotes the relevant error covariance. Numerical maximization may be considered, yet it is well know that an equivalent solution obtains through an eigen-value/eigen-vector problem based on the following determinantal equation

$$\left| [\ y_i \quad Y_i\ ]' M_1 [\ y_i \quad Y_i\ ] - \lambda [\ y_i \quad Y_i\ ]' M [\ y_i \quad Y_i\ ] \right| = 0 \tag{6.7}$$

where

$$\begin{aligned} M &= I - X(X'X)^{-1} X', \\ M_1 &= I - X_1 (X_1' X_1)^{-1} X_1', \end{aligned}$$

and λ refers to the eigen value in question. Indeed, it can be shown (see, for example **?** ((**?**), chapter 18)) that estimator of β_i

$$\widetilde{\beta}_i = \underset{\beta_i}{\text{ARGMIN}} \left\{ \lambda(\beta_i) \right\},$$

where

$$\lambda(\beta_i) = \frac{\begin{bmatrix} 1 \\ -\beta_i \end{bmatrix}' [\ y_i \quad Y_i\ ]' M_1 [\ y_i \quad Y_i\ ] \begin{bmatrix} 1 \\ -\beta_i \end{bmatrix}}{\begin{bmatrix} 1 \\ -\beta_i \end{bmatrix}' [\ y_i \quad Y_i\ ]' M [\ y_i \quad Y_i\ ] \begin{bmatrix} 1 \\ -\beta_i \end{bmatrix}}. \tag{6.8}$$

Formally, the LIML estimator of β and γ_1 is

$$\begin{bmatrix} \widetilde{\beta}_i \\ \widetilde{\gamma}_{1i} \end{bmatrix} = \begin{bmatrix} Y'Y - \widetilde{\lambda}Y'MY & Y'X \\ X'Y & X'X \end{bmatrix}^{-1} \begin{bmatrix} Y' - \widetilde{\lambda}Y'M \\ X' \end{bmatrix} y \tag{6.9}$$

where $\widetilde{\lambda}$ is the smallest root of (**??**), which corresponds to $\lambda(\widetilde{\beta})$. Furthermore, the LIML error covariance estimate is

$$\widetilde{\Sigma}_i = \frac{\begin{bmatrix} y_i & Y_i \end{bmatrix}' M \begin{bmatrix} y_i & Y_i \end{bmatrix}}{n} + \frac{(\widetilde{\lambda} - 1)S}{n}$$

$$S = \frac{\begin{bmatrix} y_i & Y_i \end{bmatrix}' M \begin{bmatrix} y_i & Y_i \end{bmatrix} \begin{bmatrix} 1 \\ -\widetilde{\beta} \end{bmatrix} \left(\begin{bmatrix} y_i & Y_i \end{bmatrix}' M \begin{bmatrix} y_i & Y_i \end{bmatrix} \begin{bmatrix} 1 \\ -\widetilde{\beta} \end{bmatrix} \right)'}{\begin{bmatrix} 1 \\ -\widetilde{\beta} \end{bmatrix}' \begin{bmatrix} y_i & Y_i \end{bmatrix}' M \begin{bmatrix} y_i & Y_i \end{bmatrix} \begin{bmatrix} 1 \\ -\widetilde{\beta} \end{bmatrix}}.$$

The LIML-based LR test statistic is

$$LR = n\left(\ln\left|\widetilde{\Sigma}_i^0\right| - \ln\left|\widetilde{\Sigma}_i\right|\right) = n\left(\ln\left|\widetilde{\lambda}^0\right| - \ln\left|\widetilde{\lambda}\right|\right)$$

where $\widetilde{\Sigma}^0$ is the constrained covariance estimate and $\widetilde{\lambda}^0$ is the constrained minimum of (6.8).

7 Appendix B: Monte Carlo Tests

In Dufour (1995) the finite and large sample theory underlying Monte Carlo tests in the presence of nuisance parameters is discussed. The methodology involved may be summarized as follows.

Let T_0 denote the observed test statistic and suppose its null distribution depends on the unknown parameter θ. Conditional on θ, a Monte Carlo p-value may be obtained asd follows. Generate $i.i.d$ realizations $T_1, ..., T_N$ of T under the null, given θ, and a specified number N of replications. Rank T_j, $j = 1$, ... , N in non-decreasing order and obtain $\hat{p}_N(T_0|\theta)$ where

$$\hat{p}_N(x|\theta) = \frac{NG_N(x) + 1}{N + 1} \ , \tag{7.10}$$

$$G_N(x|\theta) = \frac{1}{N}\sum_{i=1}^{N} I_{[0,\infty)}(T_i - x), \quad I_A(x) = \begin{cases} 1, & if\ x \in A \\ 0, & if\ x \notin A \end{cases} .$$

The LMC test corresponds to the critical region

$$\hat{p}_N(T_0|\widehat{\theta}) \leq \alpha, \quad 0 \leq \alpha \leq 1, \tag{7.11}$$

where $\widehat{\theta}$ is a consistent nusiance parameter estimate compatible with the null hypothesis. Dufour (1995) gives general conditions under which the latter critical region has the correct level asymptotically, $i.e.$ in order to have

$$\lim_{n\to\infty} \left\{ P\left[\hat{p}_N(T_0 \mid \hat{\theta}) \leq \alpha\right] - P\left[\hat{p}_N(T_0 \mid \theta) \leq \alpha\right] \right\} = 0 \ , \tag{7.12}$$

for $0 \leq \alpha \leq 1$ and $plim(\widehat{\theta}) = \theta$. Recall that in our notation, n is the sample size and N refers to the number of MC replications. Note that (7.12) takes the number of replications used explicitly into account; in otherwords, (7.12) holds for a given finite N and the sample size $\rightarrow \infty$. Furthermore, if the statistic is eventually pivotal, the latter p-value will control size exactly. For clarity of exposition, we call the nuisance parameter dependent MC p-value as LMC (conformably with Dufour and Khalaf (1999)) and the pivotal statistics-based p-value as PMC. A bounds MC p-value is obtained replacing $T_1, ..., T_N$ (clearly not T_0) by null realizations of a bounding statistic.

Theoretically exact randomized tests can be obtained as follows. The p-value (7.10) ought to be maximized with respect to the elements of the intervening nuisance parameters. Specifically, it is shown by Dufour (1995) that:

$$P\left(\sup_{\theta\, \in\, M_0} [\hat{p}_N(T_N \mid \theta] \leq \alpha\right) \leq \frac{I[\alpha(N\, +\, 1)]}{N\, +\, 1}, \quad 0 \leq \alpha \leq 1, \tag{7.13}$$

where $I[x]$ is the largest integer less than or equal to x and M_0 refers to the nuisance parameter space under the null. We call the procedure based on (7.13) the maximized MC or MMC test.

In practical applications of MMC tests, a global optimization procedure is needed to obtain the maximal randomized p-value in (7.13). One such procedure, originally proposed by Corana et al. (1987) and later modified by Goffe et al. (1994) is the simulated annealing (SA) algorithm. SA starts from an initial point, say $\hat{\theta}_n$, and sweeps the parameter space at random. An *uphill* step is always accepted while a downhill step may be accepted; the decision is made using the Metropolis criterion. The direction of all moves is determined by probabilistic criteria. As it progresses, SA constantly adjusts the step length so that *downhill* moves are less and less likely to be accepted. In this manner, the algorithm escapes local optima and gradually converges towards the most probable area for optimizing. SA is robust with respect to non-quadratic and even non-continuous surfaces and typically escapes local optima. The procedure is known not to depend on starting values. Most importantly, SA readily handles problems involving a fairly large number of parameters. These procedures are applied in the context of the Monte Carlo experiment reported in Section 5.

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