A Theory of Discrimination based on Signalling and Strategic Information Acquisition+

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Abstract

The paper develops a "signalling" based theory of discrimination where workers face different incentives for skill acquisition purely because of their group membership. Workers belonging to the disadvantaged group bear substantial signalling cost. The difference in signalling costs between groups is not due to any unexplained group heterogeneity but discriminatory information policy of the employer. Based on its belief about the group, an employer may not acquire relevant information about the workers of this group, even if such information were costless. It is shown that affirmative action policies can help in the presence of non-convex signalling technology. Factors like co-ordination amongst workers, presence of a 'dynamic' labour market and sub-group formation seem to affect the nature and degree of discrimination.

Key Words: Discrimination, Affirmative Action, Information Acquisition, Signalling.

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1. Introduction

This paper develops a "signalling" based theory of discrimination in which , even when information can be acquired at zero cost, employers choose to remain ignorant about worker skills and may discriminate against some groups. Discrimination in the labour market has been much studied by economists as well as non-economists. While the meaning and exact form of discrimination varies considerably, a common understanding is that a worker belonging to a disadvantaged group earns less than an equally skilled worker belonging to an advantaged group. This could be due to either a discriminatory wage policy where workers are paid different wages for the same job or a discriminatory job assignment policy where each job carries the same wage but a worker from the disadvantage group is assigned, on average, to a job paying less. An important issue to be addressed is how such discriminatory policy can occur and persist in the presence of competition and rational profit maximising employers.

The two main theories of discrimination are taste based theory following Becker (57) and statistical theory following Phelps (72) and Arrow (73). A taste-based theory would argue that employers have a taste for discrimination and are willing to suffer profit loss. But, as critics point out, these employers can be competed away by profit maximising employers and in due course wages of disadvantaged workers would be pushed up in the labour market.

Statistical theory, on the other hand, is based on the imperfect observability of the worker's skill or productivity. It is assumed that the employer can observe an imperfect (noisy) signal of worker's skill. Hence, the wage policy or the job assignment policy would be determined by the (posterior) belief that the employer has about the worker. But this posterior belief depends on the prior belief about the worker's skill. In many cases, group membership is the source of such prior beliefs. Hence, depending on which group a worker belongs to, the employer can have different prior beliefs and consequently different posterior beliefs about the skill level, for the same realisation of the signal. These different prior beliefs can be self fulfilling in many ways. To consider one case (as in Coate and Loury (93)), suppose the skill or productivity of the worker is acquired by an investment choice of the worker. A worker would choose to invest in skill acquisition if the expected benefit outweighs the cost. The employer, given the information about skill types, would assign the skilled worker to skill sensitive but better paying job and would assign the unskilled worker to job paying less and requiring no skill. A wrong assignment would mean production loss for the employer. In the absence of information about the skill types, the employer would conduct a test and observe an imperfect signal of the skill level, which the employer uses to update his prior belief about the worker. Consequently, a worker's prospect of being assigned to the better job depends to some extent on the prior belief held by the employer. Workers in different groups with identical cost (distribution) would face different incentives for skill acquisition depending on the prior beliefs held by the employers. It is then possible that they differ in their acquisitions of skill in such a way that the discriminatory beliefs held by the employer are justified in equilibrium[1].

To some extent one can extend the same criticism to these imperfect signalling models as applied to the taste based models. It is always beneficial to the worker as well as the employer that the skill level is identified accurately. This way the employer can reduce production loss by making the right kind of assignment and the worker can improve the expected benefit with a greater chance of getting the appropriate job or wage. So we would expect to see both employer and the worker striving to improve on the imperfect signal. Suppose, it is the case that the employer can make a small investment and can get a perfect signal of the worker's quality. It would be reasonable to presume that the employer would make the investment. Likewise, if it were possible for the worker to signal his true skill level, then he would possibly do so in many reasonable circumstances where the benefit outweighs the signalling costs. In either case group membership will cease to matter and there will be no basis for discrimination.

We shall show, however, that discrimination is possible even in the presence of such perfect signalling[2]. It turns out that signalling costs would be different for different groups leading in turn to different incentives for the workers. Importantly, signalling costs differ across groups not because of any unexplained heterogeneity but because of discriminatory policy by the employer[3].

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[1] We follow Coate and Loury (93) in the basic set up and denote these self fulfilling beliefs as discriminatory equilibria. Such equilibria also emerge when the degree of imperfections in the signal is thought to be different for different groups. See Phelps (72). For example, it is supposed to be noisier for minority groups.

[2] This is not to suggest that in the real world there is perfect signalling. Rather our paper aims to extend the earlier reasoning.

[3] Spence (74) contains an early suggestion of a similar theory of discrimination where relationship between education and ability is perceived to be different for different groups.

We assume that an employer can observe the skill level at some cost[4]. In that case, a worker's skill level can be revealed either by his own signalling activity or by the information gathering activity of the employer. In the former case, the worker bears the cost where as in the latter case the employer bears the cost. This opens up the possibility of strategic interaction between the worker and the employer. Each would like the other party to incur the cost. Our paper analyses such a strategic interaction. It is shown that discriminatory equilibria can emerge as a consequence.

We add another dimension to the signalling process by assuming that the employer may have a secondary interest in the signalling activity of the worker in addition to the revelation exercise. The cost incurred by the worker could be the source of benefit to the employer. For example, if the worker works hard and produces more to signal his high productivity, then the signalling activity adds to the profit of the employer. So the employer would always prefer the worker to undertake the signalling activity. In some cases the employer bears the cost, as he does not expect the worker to undertake the costly signalling activity. This is where group membership of the worker may matter. Depending on which group the worker belongs to, the employer may have different beliefs about skill level and consequently different strategies- whether to invest in information gathering or not. It is shown that when the employer believes that the proportion of skilled worker is small for the group, the employer would prefer not to acquire information even if it were costless to do so. Hence, workers from the disadvantaged group would have to bear the signalling cost. This reduces the incentives of the workers belonging to this group and consequently a smaller number of workers would choose to invest in skill acquisition. Thus, the discriminatory beliefs of the employer are self-fulfilling[5].

Paradoxically, it is the ability and the willingness of the skilled worker to signal that leads to the discriminatory equilibrium. If it were the case that the skilled worker would always pool with the unskilled worker, then the employer would be better off acquiring information about the skill types. In that event, the skilled worker gets assigned to the better job without having to incur any signalling cost. So the incentive to invest in skill acquisition is the same for all workers irrespective of their group membership. This suggests that any policy intervention to eliminate

[4] The employer can gain access to some monitoring technology by making an investment. We can allow for indirect observability of the skill types as well. In much of the paper this cost will be taken to be zero.

[5] Notice that discrimination takes a different meaning. There is no discriminatory wage policy or job assignment in the equilibrium. A more detailed discussion follows in the next section.

discrimination has to induce direct information acquisition by the employer.

Since it is unlikely that an employer's information acquisition policy can be monitored directly, we consider affirmative action policies designed to achieve equal representation of the workers from the disadvantaged group in the better job. This means that the employer has to assign a certain fraction of workers from this group to the better job irrespective of their skill level. This may improve the earnings of this group but the discriminatory equilibrium is not eliminated. In fact, the incentive of the worker to invest in skill acquisition is reduced and a still lower fraction of the workers choose to acquire skill. But the introductions of non-convexities in the signalling technology- say a fixed cost of signalling- leads to a dramatic change in the result. As an unskilled worker also has a positive probability of getting the better job, the net benefit to a skilled worker from signalling decreases. However, because of the fixed cost the signalling cost does not decrease in a similar fashion[6]. So, for some parameter values the skilled worker does not benefit from signalling. Anticipating this the employer is better off investing in direct information acquisition. This leads to a non-discriminatory equilibrium and the affirmative action policy will be redundant in equilibrium. Clearly, the higher the fixed cost the more likely is such a situation. It would appear that the intervention is more likely to be effective when perceived (not real) group differences are substantial. The larger the difference, the greater is the chance of an unskilled worker being assigned the better job due to the affirmative action policy and the lower is the benefit from signalling by the skilled worker. In a related vein, we also show that co-ordination among workers of a group can make the intervention more effective. A skilled worker may prefer to signal if all other skilled workers are also engaged in signalling, but he may prefer not to do so if none of the other skilled workers signalling. When no one signals, the employer is again forced to invest in information acquisition.

The paper explores the possibility of different labour markets or different sectors of a labour market exhibiting different degrees of discrimination. One would expect that a more dynamic labour market with high turnover and public signalling mechanisms (as in academia) would be less discriminatory than another market where workers are stuck to a particular employer for long period and can signal their skill only to the present employer. As shown in the paper, this turns out to be the case in some settings. A high probability of separation from the current employer

[6] Some of the later discussion relies on the presence of this fixed costs. Since most real life production process would involve some fixed or start up cost of one kind or another, we do think the results are of some relevance.

would lead the skilled worker to invest more in the public signal. The net benefit from higher effort (and higher (private) output to the current employer) becomes smaller. So at some stage the worker may invest only in the public signal and if the public signal is not quite deterministic the employer again would make the necessary investment to acquire information about skill types.

Even though racial and gender discrimination have attracted most attention, a general theory ought to address other bases of possible discrimination. For example a particular racial group may further be divided into different subgroups based on class, income, neighbourhoods, language etc. We ask how the complexity of group identity and sub-groups affect the nature of discrimination. To this end we draw attention to a distinction between discrimination based on commonly observed characteristic and commonly unobservable but easily verifiable characteristic. We suppose that while the group's identity could be common knowledge, subgroup's identity need not commonly observable. Subgroup formation can be viewed as a process of making this characteristic common knowledge. We argue that this can be detrimental to the welfare of the group as a whole in some cases.

The main contribution of the paper is two-fold. It supplements the literature on discrimination by showing how discrimination can arise even when there is perfect revelation of worker's skill. The paper is closely related to previous work by Coate and Loury (93), and Milgrom and Oster (86). The structure of the model is similar to the model of negative stereotypes and affirmative action by Coate and Loury (93). Our concept of the disadvantaged group having to incur signalling cost is somewhat similar to the invisibility hypothesis studied by Milgrom and Oster (86). They assume that the members of the disadvantaged group are less visible and hence suffer from lack of promotion which renders visibility. In our model workers from this group are less visible and hence they have to incur the signalling cost. However, in our model, the difference in visibility is not taken as a primitive but is the result of employer's information acquisition policy.

From a general agency perspective, the model shows how information may not be acquired by the employer even if it were costless. This is in contrast to the general presumption that valuable information will always be acquired. The strategic information acquisition result is similar to Cremer (95)[7]. His model focuses on post-contractual work incentives of workers due to an arm's length relationship. Our model, in addition, also looks at how the pre-contractual human capital decisions by workers are affected.

The paper is organised as follows. In section 2, the basic model is presented. The focus is on the strategic information acquisition and signalling by the employer and the worker respectively. Section 3 studies the incentive to acquire skill and shows how discriminatory equilibria can emerge. Section 4 discusses the various extensions of the model. It introduces affirmative action and analyses its effectiveness. In Section 5, the importance of public signals and market turnover is analysed. Section 5 introduces the concept of active and passive discrimination and we briefly discuss the role of sub-groups. The last section concludes with brief comments on possible extensions and applications of the model.

2. The Basic Model:

We consider a scenario where an employer hires a worker to work in two interlinked production processes (markets), X and Y. Outputs in these two markets are denoted by X and Y. They are temporally separated, so that the worker first works in market X and output is realised before production begins in Y. One possible interpretation being that the worker first works in the shop floor (X) before moving on to the management cadre (Y)[8]. A worker can be of two types- skilled and unskilled denoted by s and u respectively. Only the worker knows his type, although as we shall subsequently assume the employer can acquire this information.

2.1 **Technology:**

The two markets are quite different. Output in market X is given by

(A1) $X = X_0 + \alpha\, t\, e, \quad t = s, u$

where α is some fixed parameter with $\alpha t < 1$. s and u also denote the productivity levels with s > u and e is the effort put in by the worker. It is clear that output in this market is not very sensitive to the effort or ability of the worker. To raise output, effort has to increase more than proportionately.

Output in market Y is quite sensitive to the ability or productivity level. This creates the demand for information about the types. We take an extreme case where production process is of two types denoted by y, $y \in \{1, 0\}$. Type 1 production requires that the s-type worker should be assigned to it and there is production loss if u-type worker is assigned. Output does not depend on effort in

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[7] See Cremer and Khalil (93), Cremer, Khalil and Rochet (97) also.

[8] We can interpret it to be a two period relationship as well. But these two periods have to be non-identical. The first period can be also viewed as the probationary phase.

this market. Denoting Y_{t1} as the output when the t-type worker is assigned to y = 1, we assume that

(A2) $Y_{s1} > Y_{u1}$

2.2 **Payoffs:**

Wage rates are fixed and exogenous[9]. In market X, it is simply W_x. In market Y if the worker is assigned y = 1, then wage is W_1, W_0 otherwise. In addition $W_1 > W_0$. Both worker and the employer are risk neutral. Worker's payoff is given by

(A3) $U_t = W_x + E(W_y) - e, \; t = s,u$

where $E(W_y)$ is the expected income depending on the probable job assignment in Y and e is the effort level.

The employer's profit in market X is simply X - W_x. In market Y it depends on the assignment. Let V_{ty} denote the net output realised by the employer when type t worker is assigned job y. So his overall payoff is

(A4) $V = (X - W_x) + (V_{ty})$[10]

We assume that

(A5) $V_{g1} > V_{b0} \geq V_{g0} > V_{b1}$

Both (A3) and (A4) implicitly assume that there is no discounting, which can be introduced without any change in results. (A5) reflects the type-sensitivity of the technology in Y. It also incorporates $W_1 > W_0$. This means there is a positive demand for type information by the employer.

2.3 **Sequence**

The worker and the employer take actions in the following sequence.

1. Employer decides whether to acquire information or not. We consider a scenario where it is possible for the employer to acquire information regarding the type of the worker if he were to make an investment ε. Our focus is on the case where ε is small. The decision will be denoted by I, $I \in \{1, 0\}$.
2. Worker chooses e (effort) in market X, after observing I.
3. Employer realises output X and then chooses the assignment rule $y \in \{1, 0\}$ for the worker in market Y .

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[9] We follow Coate and Loury (93) in focusing on earning differential due to job assignment rather than wage levels. Competition, wage regulation etc. can account for this.

[10] One can consider a more general payoff function $V = \beta (X-W_x) + (1-\beta)(V_{ty} - W_y)$. One can then interpret $\beta = 0$ to be the case where worker's action in X has a purely information revealing role like standard signalling models. Likewise a high β would suggest that the transfer aspect is more important.

4. Payoffs are realised.

The employer has certain belief 0< π <1 that the worker is s-type. At this stage we are assuming that employer can commit to whatever policy chosen at stage 1. For example he can not change it after observing output X even if that might be in his interest. Another interpretation (Cremer 1995) would be to make the employer choose the information or monitoring technology at stage 1. After observing X at stage 3, he decides whether to monitor or not. An efficient monitoring technology would mean the employer could observe the skill of the worker costlessly. But with an inefficient monitoring technology the cost of monitoring is too high so that it is never optimal to monitor irrespective of the outcome in market X.

It will be convenient to label stages 2-4 as game G. If I = 0, then G is a signalling game . On the other hand when I = 1, G is a simple extensive form game with perfect and complete information. We turn to the analysis of this game G.

2.3 Information Acquisition and Signalling:

Case 1: When I = 1, the employer will always make the right assignment resulting in a payoff of

(6) $\pi V_{s1} + (1-\pi) V_{u0} - \varepsilon$

The s-type worker gets W_1 and the u-type gets W_0 in market Y.

Case 2: Now suppose I = 0. If both types choose same effort level then the employer assigns the worker to $y \in \{1, 0\}$ based on expected payoff. For example if π is the prior belief that the worker is s-type , then the expected payoff from assigning him to y = 1 is given by

(7) $\pi V_{s1} + (1-\pi) V_{u1}$

Similarly one can find out the payoff from assigning him to y = 0. Clearly for $\pi < \pi_c$, the employer would assign the worker to y=0, where π_c is given by

(8) $\pi_c V_{g1} + (1-\pi_c) V_{b1} = \pi_c V_{g0} + (1-\pi_c) V_{b0}$.

or, $\pi_c = (V_{b0} - V_{b1}) / [(V_{g1} - V_{g0})+ (V_{b0} - V_{b1})]$

Let us consider the case when I = 0, and $\pi < \pi_c$. The worker knows that in the normal course, both types would be assigned y = 0 and the wage would be W_0. But the s-type worker can signal his type by producing in excess in market X. Since, the marginal cost of production (in excess of X_0) is lower for the s-type than the u-type, it is possible for the s-type to separate from the u-type.

Define output level X* such that

(9) $X^* = X_0 + \alpha b (W_1 - W_0)$

X* is the output level such that the b-type would be indifferent between producing X_0

and receiving a wage of W_0 in market Y , and producing X* and receiving an wage W_1 in market Y. Recall that wage in the market X is same irrespective of the output level[11]. Using (A1) and s > u, we have

(10) $W_x + W_0 < W_x + W_1 - (X^* - X_0)/\alpha s$

or, $\alpha s(W_1 - W_0) > (X^* - X_0)$

This means that when faced with an uninformed employer the s-type would in fact produce an output level X* to signal his type. This is confirmed by the following proposition. We shall consider sequential equilibria of this game G. (e^*, y^* , μ) constitutes an equilibrium of the game if the strategy pair satisfies sequential rationality given belief μ and the belief system μ is consistent with the strategies and Bayes' rule whenever applicable. The same definition applies to the case when I = 1, except that $\mu \in \{1, 0\}$ in this case.

Proposition 1:

Consider G with I = 0 and $\pi < \pi_c$. There is an equilibrium where the s-type produces X* and u-type produces X_0. The employer assigns the worker with output $X \geq X^*$ to y = 1 and y=0 otherwise. The out of equilibrium belief held by the employer is given by μ (s, X < X*) = 0. Output levels lower than X* are believed to have come from the u-type only.

Proof: The proof is standard and hence is omitted. The appendix contains a brief sketch[12].

This has an interesting implication for the incentive of the employer to make investment in information gathering. Notice that the unique outcome in G is completely determined by employer's choice of I. When I = 1, it is given by (6) and when I = 0, it is given by the outcome defined in Proposition 1. The next proposition shows that employer's policy I is in turn determined by the prior belief π.

Proposition 2:

If $0< \pi < \pi_c$, given the payoffs, the employer will always choose I = 0 (even if $\varepsilon = 0$). But when $\pi > \pi_c$, for small values of ε , the employer will choose I = 1.

Proof:

If $\pi < \pi_c$ and I = 1, then s-type will also choose an output level X_0 as wage does not depend on the output in this market. But when I = 0, the s-type would be

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[11] This is not essential. One can introduce bonus payment where bonus = $\delta(X - X_0)$, but the bonus does not completely compensate for the effort $\delta u<1$.

[12] One can use refinement notions (see Cho & Sobel (90)) to show that this equilibrium is unique.

forced to choose a higher output level X*. If they pool with the u-type, the employer's posterior belief is same as π and given this belief the employer would always choose to assign the worker to job y = 0 as

$\pi V_{s1} + (1-\pi) V_{u1} < \pi V_{s0} + (1-\pi) V_{u0}$, for $\pi < \pi_c$.

When the prior belief of the worker being good type is higher, $1> \pi > \pi_c$, for sufficiently small $\varepsilon > 0$,

$\pi V_{s1} + (1-\pi) V_{u0} - \varepsilon > \pi V_{s1} + (1-\pi) V_{u1} > \pi V_{s0} + (1-\pi) V_{u0}$

Since the s-type knows that it would get a higher wage irrespective of the outcome in market X because of the second part of the inequality, he would never engage in costly signalling activity. **[]**

As evident from the above proposition, unless ε is very high it does not play any role in our model, because the employer chooses I = 0 for some groups even if $\varepsilon = 0$. To avoid repetition and extra notation we shall take ε to be zero for the rest of the paper[13].. All results will hold good for small ε. First best is attainable in our model when $\varepsilon = 0$. In this outcome, the s-type also chooses a zero effort level and the employer finds out the true type. But because of strategic reasons, in certain cases it will never be achieved. The incentive not to acquire information is very similar to Cremer's (95) model of arm's length relationship. In his model, the employer prefers not to acquire information about the worker's type, because it increases the incentive of the worker to improve performance in the first period and the employer can induce him to choose higher effort at lower cost. But, in his model, a worker's type is due to random factors and not due to prior action by the worker. So one can not examine how the employer's information policy would affect the incentive of the worker while choosing his type. This is what we are going to examine in the next section.

3. Discriminatory Equilibria:

We consider a scenario where groups of workers are assigned to an employer. All employers are alike, so the analysis will focus on a single employer. As mentioned earlier the worker can make certain investment (at cost c) and become a s-type. This investment decision will be denoted by m , $m \in \{1,0\}$. A worker will choose to be a s-type (m = 1) if the net gain from being a s-type is greater than the cost c. The net gain depends on how the game G, as described in the previous section, is played. We shall consider two racial groups *b* and *w*. Both

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[13] All results are true when $\varepsilon > 0$ but small. Small costs are to be interpreted in a relative fashion, compared to the profit of the employer and the earnings of the worker.

groups have the same distribution of the cost c. We assume that

(A6) c is distributed according to some distribution function F over the interval [0 C]. For simplicity and ease of computation we restrict attention to uniform F.

The employer does not observe the investment decision and has beliefs about the likelihood of a particular group worker being of s-type. These are given by π_b and π_w. Throughout, high and low refer to π being greater than or less than π_c.

An equilibrium will be denoted by a strategy pair (m_j*(c), I_j*), j = b, w such that

(1) worker's expected payoff is maximised given his own c and employer's I*

(2) employer's payoff is maximised given his beliefs π_j

(3) and π_j is consistent with worker's strategy m_j* and the cost distribution F.

For convenience we shall use an equivalent notion and denote an equilibrium to be a pair of self fulfilling beliefs (π_b*, π_w*), where π_j* is uniquely determined by m_j* and F()

$$\pi_j^* = F(c^*), \text{ such that } m_j^* (c \leq c^*, I_j^*) = 1 \text{ and } m_j^* (c > c^*, I_j^*) = 0.$$

Groups are ex-ante identical with exactly same F. We can have a discriminatory equilibrium where employer's belief -low π_a and high π_h - is fulfilled in equilibrium. Before we formally state it we need a couple of more conditions. We assume that

(A7) $s \leq s_0$

where $s_0 = u\Delta W / (\Delta W - \pi_c C)$

Since s would affect the signalling cost, this condition guarantees that these costs are not non-substantial and they matter. Lastly

(A8) $C > \Delta W > \pi_c C$

This condition assumes that wage differential is neither too high nor too low relative to the cost distribution.

Proposition 3: Given assumptions A(1) - A(8), there exists a discriminatory equilibrium (π_b*, π_w*) with π_b*< π_w*.

Proof: Given the linear payoff structure and the uniform distribution, we can construct such an equilibrium easily. Let the prior belief be such that $\pi_b < \pi_c < \pi_w$. For group *b*, by propositions 1 and 2, the s-type worker will chose effort $e^* = (X^*-X_0)/\alpha s$ to signal his type. The net expected payoff to a worker from making an investment c is given by (using (9) and $\Delta W \equiv W_1 - W_0$)

$$\Delta W - e^* = \Delta W\ (1-u/s)$$

So, a worker with $c \leq \Delta W$ (1-u/s) will choose m =1. This implies that for this group a fraction $\pi_b^* = \Delta W$ (1-u/s)/ C will be s-type. Given (A7), it is clear that $\pi_b^* < \pi_c$.

On the other hand, $\pi_w > \pi_c$. By proposition 2, $I_w^* = 1$ and the net expected payoff to a s-type worker is simply ΔW. A worker with $c \leq \Delta W$ will make the investment implying $\pi_w^* = \Delta W/C$. Using (A8), it is clear that $\pi_h^* > \pi_c$. []

This is not the unique equilibrium. There is another equilibrium where the employer believes both groups to be identical and in equilibrium they will be identical.

Example 1: Let s = 3, u = 1, $\alpha = 1/6$, $V_{s1} = 20$, $V_{u0} = 10$, $V_{s0} = 8$, $V_{u1} = 4$, $W_1 = 10$, $W_0 = 4$ and normalise $X_0 = 0$. It can be checked that all the assumptions are satisfied. The separating level of output is X* = 1 and the u-type would have to put in an effort level $e_u = 6$. Both groups *b* and *w* have the same uniform distribution of c over the interval [0 16] . As stated in the previous discussion, the employer believes that $\pi_b < \pi_c < \pi_w$. In this example $\pi_c = 1/3$. It can be checked that for group *b*, only 1/4th of the total number of workers will choose m = 1 and for group *w*, 3/8th of the total number of workers will choose m = 1. Hence $\pi_b = 1/4 < 1/3 < 3/8 = \pi_w$ is a discriminatory equilibrium.

This is quite similar to the discrimination model of Coate and Loury (93). In their model, the employer has different prior beliefs for the two racial groups. Both groups face a test and the employer observes an imperfect signal of their type. Since the signal is imperfect, the posterior belief of the employer is determined to some extent by the prior. So the group facing an adverse prior has a smaller chance of being assigned y=1. This reduces the incentive for this group to invest and consequently, a smaller number of workers choose m =1. This, in turn, justifies the low prior associated with this group. In our model, a perfect but costly signal is present (the s-type can choose higher effort in market X to reveal its type). As seen in the previous discussion, employer's decision to acquire information is strategic and depends on the prior belief. He chooses not to acquire the information for the group with a low π. So, like the Coate & Loury model, the group with adverse prior faces reduced incentives because it is this group which would have to bear the signalling cost. Reduced incentives lead to a smaller fraction choosing m = 1, justifying the adverse prior[14].

There may be some disagreement regarding whether this equilibrium constitutes a case of discrimination or not. Notice that all s-type workers get the same wage in equilibrium irrespective of their group memberships. A s-type worker

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[14] When ε is large, one can not find pure strategy equilibrium for group *h*. But with mixed strategies similar discriminatory equilibrium can be sustained.

is always assigned y = 1 and this job carries a fixed wage. Since the disadvantaged group (group *b*) has fewer members making the investment in skill, the average income of this group is lower. However, we shall consider it a form of discrimination as prior to making the investment choice both groups are identical[15] and it is the information acquisition policy of the employer that distorts group *b*'s incentive and discourages some group *b* workers from making the investment.

Before we turn to certain policy interventions, it is worth pointing out that improvements in productivity in the form of a rise in s can eliminate the discriminatory equilibrium. This will reduce the signalling cost incurred by the s-type worker. A subsidy on the investment cost c for the disadvantaged group will have a similar impact. Moreover, such changes need not be permanent. In our model , a single shock would imply that the discriminatory beliefs can not be sustained and the only sustainable beliefs by the employer would be the non-discriminatory one. Moreover the extent of changes in s or subsidy on c need not be large to guarantee a non-discriminatory equilibrium $\pi_b^* = \pi_w^*$, irrespective of how high π_w^* may be. If $\pi_b > \pi_c$ because of interventions, the only possible equilibrium is with $\pi_b^* = \pi_w^*$.

4. Affirmative Action

In this section we examine how affirmative action[16] policies will fare in our model context. Affirmative action (denoted AA) policies can be result oriented as well as process oriented. For example, equal opportunity laws would be process oriented and proportional representation of different groups in better jobs would be result oriented. Since it is unlikely that government can directly influence the information policy of the employer, we will consider policy related to outcomes. In our context, affirmative action will require the employer to achieve proportional representation of both groups in the better job. Throughout the paper affirmative action would mean the following condition

(AA) $\lambda_b = \lambda_w$, where λ is the fraction of the group assigned to job y = 1.

As a result, the employer will have to either assign more workers from the

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[15] We feel skills gap is an important aspect of discrimination. In our model, the gap is purely as a result of wage incentives. However, it could also be a reflection of other social and cultural factors (geographic segregation, social norms etc) which have racial dimensions. See Loury (98),, Akerlof (97).

[16] Affirmative actions refer to policies aimed at combating differences between groups in earnings and employment and undoing unfair group discrimination. See Welch (76), Lundberg (91), Coate & Loury (93), Kim-Sau (98), Benoit (99) among others, for various models of affirmative action.

group with adverse prior to y= 1 or assign less workers from the other group to y = 1. We shall rule out the second possibility by assuming that the disadvantaged group is a relatively small minority[17]. This means the employer would assign some u-type workers to the better job rather than wrongly assign a large number of s-type worker to the worse job. Even though the case where affirmative action policies affect employer's assignment of workers from the advantaged group can have some interesting implications, we have chosen not to focus on them.

4.1 Discriminatory Equilibria and Affirmative Action

Introduction of (AA) does not mean that the incentives to members of the group will improve or the discriminatory equilibrium will cease to exist. In fact, the incentive to invest can go down for the group. Consider Example 1 discussed earlier. In equilibrium, 1/4 of the group *b* workers and 3/8 of the group *w* workers would be assigned the better job y=1. This however, violates condition AA. Hence this particular discriminatory equilibrium is not feasible any longer. But that does not necessarily imply that there is no other discriminatory equilibrium satisfying the constraint AA. It can be shown that $\pi_b \approx 3/16$[18], $\pi_w = 3/8$, $I_b = 0$ and $I_w = 1$ constitute an equilibrium. The s-type workers from group *b* signal their type by choosing a higher level of effort and are assigned y =1. The employer assigns group *b* worker with output $X \geq X^*$ to y=1 and in addition assigns some more workers (chosen at random) with $X = X_0$ also to y = 1. Now a worker has a small chance (3/13) of being assigned y = 1 and getting the higher wage W_1 even without incurring the cost c. Both are represented equally in the better job y = 1. But notice that the fraction of s-type workers in group *b* has decreased from 1/4 to 3/16. The incentive to choose m =1 decreases because of the positive probability of a u-type worker being assigned y = 1. The level of effort (cost of signalling) required to signal one's type also decreases but it is less than the fall in the net benefit from signalling. The group with the adverse prior gets `patronised'[19] but there is no improvement in their skill level. We can summarise this in the following proposition.

Proposition 4 : Given any initial discriminatory equilibrium (π_b^*, π_w^*), there always exists , under affirmative action, another discriminatory equilibrium (π_b^{**},π_w^{**}) with $\pi_b^{**} < \pi_b^*$ and $\pi_h^{**} = \pi_h^*$.

Proof: Let (π_b^{**},π_w^{**}) be the equilibrium under AA. It is obvious that $\pi_w^{**} = \pi_w^*$ as

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[17] If $V_{s1} - V_{s0} > V_{u0} - V_{u1}$, then the second possibility is ruled out irrespective of the group sizes.

[18] The value of π_a is 0.194. The details are in the proof to the proposition 4.

[19] Coate and Loury (93) use this term to describe such equilibria. This patronisation critique has been much discussed in the recent literature.

employer's policy towards this group remains unchanged given our assumption of group *b* being a small minority.

Assume that the employer's information policy towards group *b* also remains unchanged. We shall show later that this is indeed the case. Since this is a discriminatory equilibrium, the u-types from group *b* would be assigned to y = 1 with a positive probability (ρ*). Using AA it can be seen that

(11) $\rho^* = (\pi_w^* - \pi_b^{**}) / (1-\pi_b^{**})$

So the expected payoff to a u-type worker from this group would be

(12) $U_u = \rho^* W_1 + (1-\rho^*) W_0$

Let e** be the effort level chosen by the s-type worker to separate himself in the new equilibrium. Given (A1 & 9) it can be seen that

(13) $e^{**} = \Delta W(1-\rho^*) u/s$

Since the s-type will be assigned to the job y= 1 with certainty , the s-type receives a net payoff given by

(14) $U_s = W_1 - \Delta W(1-\rho^*) u/s$

Hence a worker with $c \leq U_s - U_u$ will find it worthwhile to make the investment and choose m =1. Since $U_s - U_u = \Delta W(1-\rho^*)(1- u/s)$, consistency requires that

(15) $\Delta W(1-\rho^*)(1- u/s) = \pi_b^{**} C$

Using the value of ρ* and the relation $\pi_w^* = \Delta W/C$ in the above equation we get a quadratic equation with the roots given by

(16) $\pi_b^{**} = [1 \pm \{ 1- 4\pi_w (1-\pi_w) (1-u/s)\}^{1/2}] / 2$

The existence of real roots is guaranteed by (1-u/s) <1 and $4\pi_w (1-\pi_w) \leq 1$ (note that max. $\pi_w (1-\pi_w) = 1/4$).

Let π^l and π^s denote the larger and the smaller root respectively. It can be shown that $\pi^l > \pi_w$ and hence can not be a candidate for our equilibrium. On the other hand it can be verified that

$$0 < \pi^s < \pi_b^* .$$

Hence ($\pi_b^{**} = \pi^s$, π_w^*) constitute a discriminatory equilibrium. Since $\pi^s < \pi_b^*$, given this belief employer's choice of I = 0 as assumed earlier is also optimal. **[]**

4.2 Non-convex signalling technology

However, there are cases where the introduction of affirmative action would eliminate any discriminatory equilibrium. One such case is when there is some sort of increasing returns in the production of X. To see this , let output X be given by

(A1') $X = X_0 + \alpha\, t\, e \quad e \geq e_0$, t = s, u

$\quad = X_0 \quad e < e_0$.

This means that there is a fixed cost associated with signalling. We have seen that under affirmative action policy the gain from signalling decreases for the s-type in the separating equilibrium. Given the fixed cost component , in certain cases the s-type would be better off in a pooling situation. In the absence of signalling , the employer is better off choosing information policy I = 1 for the group *b* as well[20]. The only equilibrium consistent with this is where both groups have the same π and are represented equally in the better job. It is interesting to note that in equilibrium the affirmative action policy is redundant or the equal representation constraint is non-binding for the employer.

Proposition 5: With the signalling technology given by (A1'), introduction of affirmative action leads to the non-discriminatory equilibrium for sufficiently large e_0.

Proof: The details of the proof are given in the appendix. It can be shown that there is no discriminatory equilibrium with $\pi_b^{**} < \pi_w^{**}$ if the following condition is satisfied.

(17) $(e_0 /C) > \pi_w (1-\pi_w)$

As discussed earlier, following the introduction of AA[21], the net benefit of signalling goes down but the signalling cost is bounded below by e_0. Hence the s-types do not signal. In the absence of signallling, the employer is better off acquiring information to avoid wrong assignment and production loss. []

However, in the absence of signalling a trivial equilibrium can emerge where $\pi_b = 0$. If the s-type is going to pool with the b-type , then there is no reason why any worker would invest in being a s-type. So we can have another equilibrium where $\pi_a^* = 0$ and I = 0. But it can be argued that this is not very robust. If we allow for the possibility that a worker becomes a s-type without any investment with a small probability γ, then I = 0 is not optimal for the employer given that it is cost less Moreover, if workers face some uncertainty about the implementation of the affirmative action policy while making their investment decisions, then π_a will be bounded away from zero. We shall discuss a situation of this sort in the next section to show how intra-group interactions may matter.

Example 2: Same as Example 1, except that $e_0 = 4$.

So the initial discriminatory equilibrium is given by $\pi_b^* = 1/8$ and $\pi_w^* = 3/8$.

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[20] Most models analyse affirmative actions in the context where equal treatment is desirable. Benoit (99) models a situation where equal treatment is not enough to correct income inequality and affirmative action program is necessary to overcome existing group differences.

[21] Even in the absence of AA, a sufficiently large e_0 can upset the discriminatory equilibrium. But this is same as assuming the absence of signalling possibilities for the worker.

With the introduction of the policy $\lambda_b = \lambda_w$,in any discriminatory equilibrium with $\pi_b^{**} < \pi_w^*$ and I = 0, the u-type will be assigned y = 1 with probability $\rho = (3/8 - \pi_b^*)/(1-\pi_b^*)$. So in a separating equilibrium (with s-type signalling) the expected payoff to the s-type will be 6 (given $e_0 = 4$). For $\pi_b = 1/8$, $\rho = 2/7$ and the u-type's expected payoff is 40/7. The net gain to the s-type is 2/7, which mean $\pi_b << 1/8$. But for lower π_b the payoff to the u-type increases and that of the s-type stays same. At some point , the s-type will be better off pooling with the u-type. Given that there is no signalling, the employer is better off choosing I = 1 as long as $\pi > 0$. When I =1, the only possible equilibrium is with $\pi_b^* = 3/8$.

4.2.1 *The extent of discrimination* :

The condition guaranteeing non-existence of discriminatory equilibria can be interpreted in various ways. One possible interpretation of condition would be that affirmative action generally would succeed when the groups are sufficiently apart in terms of the employer's beliefs. Notice that (17) can be re-written to imply

$$\pi_w^* - \pi_b^* > \pi_w^* (1-\pi_w^*)$$

Since $\pi_b^{**} < \pi_b^*$, whenever a discriminatory equilibrium exists, a large $(\pi_w^* - \pi_b^*)$ implies that in the new equilibrium ρ^* is going to be large as well. It is only then that the u-types have a very high probability of being chosen for the better job and the s-type has no incentive to signal. In the absence of any signalling the employer prefers to gather information.

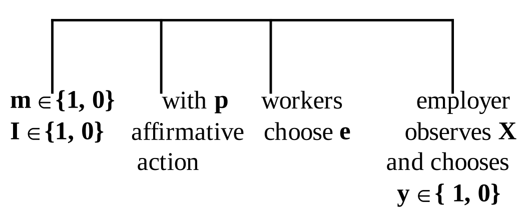
We can modify the previous example to illustrate this point. Consider two scenarios. In one, let $e_0 = 3$, s = 6, u = 1 and everything else same as the previous examples. The initial discriminatory equilibrium is (3/16 , 3/8). With affirmative action the new discriminatory equilibrium is (1/16, 3/8). In the second scenario, let everything remain same except the support of c. Let C be 12. So the initial equilibrium is (1/4, 1/2). The fraction of s-types in the group *w* has gone up and so also the distance between the two groups. Now with affirmative action, the only equilibrium is (1/2, 1/2).

4.2.2 *Intra-group Interaction and Co-ordinated Action*:

Interaction among members of the disadvantaged group generally receive little attention in models of discrimination. However, it may matter significantly in determining the effectiveness of affirmative action policies. To see this, we allow sub groups of workers to co-ordinate at stage 2, before choosing output in market X. It is more likely that interaction among members takes place at this stage (once they are matched to an employer) rather than at the beginning. We shall assume that these subgroups can not write binding agreements, but can communicate freely and the type information is known to all members of the group. So , for example , all the s-types belonging to this group can form a subgroup and co-ordinate their action.

Such co-ordination is not going to make a difference to the analysis of the model in the absence of affirmative action. But the introduction of proportional representation policy makes a difference. Suppose there are n_s s-type workers in this group of n workers. Since the AA policy is common knowledge, these workers know that the employer would have to assign a certain number (m) of workers to the better job. When the s-types signal they are assigned the better job with certainty and the remaining $(m-n_{s})$ workers are drawn from the rest of the group. But if all the s-type workers do not signal, then the employer will draw all m workers from the group at random. In fact this can be an equilibrium when n_s is not too small. Given that all $(n_s - 1)$ workers are not signalling , a particular s-type worker will not benefit from deviating from a no-signalling action if the signalling cost outweighs the benefit of increased chance of getting the better job. Below we use an example to illustrate this point. If the s-type workers do not signal, the employer anticipating this ,will choose to acquire information. The only equilibrium will then be the non-discriminatory one.

To illustrate this point we use a slightly different model of affirmative action. We introduce some uncertainty about the implementation of this policy. The workers as well as the employer , while choosing their actions in period 1, believe that affirmative action policy will be implemented with probability p , where 0< p< 1. The sequence of events is given below.



It is a generalisation of the model in the previous section. The assumption of non-deterministic affirmative action can be justified on the grounds that these policies may be subject to regular reviews and may not be in place always. Moreover, even if the laws exist, the government’s commitment can not be taken for granted.

Example 3:

Consider a case where $\Delta W = 10$, $C = 12$, $u = 1$, $s = 4$, $e_0 = 2$. We shall show that a discriminatory equilibrium exists in the absence co-ordination among workers but this outcome fails to be an equilibrium with co-ordination by s-type workers from group *b.*

Let net outputs V be such that $\pi_c = 5/7$. In the absence of any affirmative action policy we have a discriminatory equilibrium given by $\pi_b^* = 5/8$ and $\pi_w^* = 5/6$. Now consider a scenario where with probability p = 9/37 affirmative action may be introduced. So there are two subgames where the workers would choose effort in market X. Let us label them as A and NA, depending on whether affirmative action is to be implemented or not. We claim that there is a discriminatory equilibrium with $\pi_b^{**} = 1/2$ and $\pi_w^{**} = 5/6$.

To verify the claim notice that in subgame A, if all s-types signal , then the expected payoff to a s-type worker is 10 - 2 = 8. The expected payoff to a u-type worker is 20/3, as he will assigned to the better job with probability (5/6 - 1/2) / (1/2) = 2/3. Given that all other s-type workers are signalling, any one s-type worker will not find it worthwhile to deviate and pool with the u-type. In the subgame NA, again all s-types will signal by choosing effort $(\Delta W/s) = 2.5$. No u-type will be assigned to the better job. So the total expected payoff of a s-type worker is p(8) + (1-p) 7.5, where as that of a u-type worker is 20p/3. Hence assuming that I =0 for this group, a worker with $c \leq p(8) + (1-p)\, 7.5 - 20p/3$ will choose to be a s-type. Given that c is distributed uniformly over the interval [0 12] and p = 9/37, this means that the fraction of s-type worker for this group will be 1/2.

However, this is not an equilibrium if all the s-types could co-ordinate. If none of the s-type worker signals in subgame A, then the expected payoff to a s-type worker in that subgame will be 50/6. A s-type worker does not benefit from deviating and signalling as 8 < 50/6. So we have a complete pooling situation in subgame A. Now the employer knows that with some probability p there will be pooling. Since the employer has to assign a fraction 5/6 of these to the better job, lack of any information would mean less s-types and more b-types in the better job. With probability 1/6 a s-type will not be assigned to the better job and a b-type's probability of being assigned the better job goes up by 1/6. This would lead to a production loss in Y. The expected loss in net output would be $\Delta Y = (V_{s1} - V_{s0})/6 + (V_{u0} - V_{u1})/6$. On the other hand, if the employer chooses to acquire information this loss ΔY will be avoided but there will be no signalling in subgame NA as well.

The employer will loose the extra output $(X^* - X_0)/2$ (as $\pi_b = 1/2$). So from the employer's point of view the difference in payoff would be given by

$$V(I = 1) - V(I=0) = p((V_{s1} - V_{s0})/6 + (V_{u0} - V_{u1})/6) - (1-p)(X^* - X_0)/2$$

The employer will be better off acquiring information if this term is positive. This will be true for a range of values. One such possibility is given by $V_{s1} = 15$, $V_{s0} = 10 = V_{u0}$, $V_{u1} = 4.5$ and $\alpha < 2/3$. It can be checked that $\pi_c = 5/7$ as assumed earlier. Given α, $(X^* - X_0)$ will always be less than 20/3. It can be verified that $V(I=1) > V(I=0)$.

But if the employer chooses I = 1 for this group as well, then the only possible outcome is the non-discriminatory equilibrium $\pi_b^{**} = \pi_w^{**} = 5/6$. This example shows that affirmative action policy need not be effective in itself unless there some co-ordination amongst members of the disadvantaged group. It is also interesting note that even though the affirmative action policy is likely to be implemented with a small probability, it turns out to be effective[22].

4.3 Public Signals and Turnover:

Even though our analysis was conducted in terms of a single employer, it is consistent with the presence of several employers each with similar beliefs. Information is assumed to flow freely between employers. Output X is publicly observable and so is the information collected by the employer. One can argue that employer's information on the workers is hard information and can be used by the worker while switching employers. One can use a model of this kind to justify the fixed wage structure assumed earlier. But our purpose here is different. We are interested in asking how the results will change if we allow X to be a private signal-not observed by other employers. Since wages are given and the employer will always prefer to assign the s-type worker to the better job the private nature of the signal does not matter. But if there is a positive probability that worker gets separated from the employer after the production stage X, then it matters.

As a first step let us assume that X is the only signalling variable. The worker gets separated from the current employer (after stage 2) with an exogenous probability μ. X can not be observed publicly, but the hard information collected by the employer can be used by the worker and other employers. This means group *w* is not affected by this positive turnover. For group *b*, the situation is quite different. Recall that for this group $\pi < \pi_c$, and this is the common belief. So even if a worker

[22] However, it must be pointed out that the example is quite special. We do need $e_0 > 0$ and $p < 1$ to construct an example where discriminatory equilibria exist without co-ordination but fail to exist with co-ordination amongst workers. A full formal model will have to address some aspects of simultaneous signalling by more than one worker.

chooses high output to signal his type, he will be assigned y = 0 by the new employer. So the payoff to the s-type from signalling would be given by

(20) $U_s = (1-\mu) W_1 + \mu W_0 - e^*$

where e* is the equilibrium separating effort level. Note that e* will be also low compared to the previous case depending on μ. Assuming output X is given by (A1), the separating output level is given by $\alpha u(1-\mu)\Delta W$, the s-type will have to put in $e^* = (1-\mu)\Delta W(u/s)$. The net benefit to a s-type worker will be given by

(21) $U_s – U_u = (1-\mu) \Delta W (1-u/s)$.

This means the fraction of s-type workers will decrease as μ increases. This is quite similar to the observation by Milgrom and Oster (86), that the disadvantaged group will benefit from greater bonding and lower mobility of workers.

The effect of μ can go the other way also. As in the previous section, suppose output is given by (A1') rather than A1. Then clearly as μ increases, the benefit of signalling falls but the cost of signalling is bounded below by e_0. So for μ exceeding some critical value the s-type will choose to pool. One can use the arguments from the previous section to show that this will result in the non-discriminatory equilibrium.

A more interesting case emerges when workers have access to other signals- namely a public signal Z. Suppose the worker can affect the value of Z by putting effort f. Like e, f is also chosen prior to the separation with the employer. Both X and Z are realised at the same time. Z does not yield any direct benefit to the employer. We shall assume that f enters the utility function in a symmetrical fashion. However, unlike the private signal X, Z has a stochastic nature. Otherwise there is no reason why Z would not be chosen by the worker in the first place. We assume that

(A9) Z takes two values $Z = Z_s$ and Z = 0. For the s-type $Z = Z_s$ with probability q(f), where q(f) is increasing in f, concave and 0< q(f) <1. For the u-type Z = 0 with certainty.

The utility to a s-type worker is given by

(22) $U = \mu (q(f) W_1 + (1-q(f))W_0) + (1-\mu) W_1 - e - f$ for f > o and $e \geq e^*$,

where e* is the separating level of effort.

Note that the worker is assigned to y = 0 when Z = 0 is realised. Given that the initial prior belief about this group π_b is less than π_c , the posterior belief will also be less than π_c. This justifies the assignment y= 0 . Moreover we would like to keep the analysis of the model exactly same as in the absence of any turnover. The following assumption guarantees this

(A10) $e^* \leq f^* = \text{argmax}\{q(f) W_1 + (1-q(f))W_0\}$

Now we can state the following proposition assuming A1' and A(9) - A(10) to hold.

Proposition 6 : Given any discriminatory equilibrium with zero turnover, there exists a μ_0 such that for $\mu > \mu_0$ the only equilibrium is the non-discriminatory one.

Proof: Let $\mu = 0$. Then using (A10) one can show that the s-type worker will use only the signal X and put in effort e*, where $e^* = \Delta W (1\text{-}1/s)$. He will not use the public signal as not only does he have to incur greater cost but the probability of getting the higher wage is also less, $q(f^*) < 1$. So the analysis of the model is same as before and we have a discriminatory equilibrium ($\pi_b^* < \pi_w^*$).

However, if $\mu > 0$, the worker will choose $f > 0$. His decision to use the public signal is independent of whether he is using X or not , although the optimal value $f^*(\mu)$ will depend on whether e* is positive. Assuming that the worker continues to use X , the separating level of effort now will be $(1-\mu)\ \Delta Wu/s$. Given our assumptions on q(f), it is easy to see that

$$\mu\ (q(f)\ W_1 + (1-q(f))W_0) + (1-\mu)\ W_1 - e^* - f^*(\mu) > (1-\mu)\ W_1 - e^*$$

Since use of Z is inevitable, the private signal now plays a secondary role. The benefit of this signal conditional on the use the public signal Z will be given by $(1-q(f))(1-\mu)\ \Delta W$. This value is falling as μ rises and as a consequence f rises. On the other hand the cost of the private signal also falls but is bounded below by e_0 . One can use continuity arguments to show that the signalling cost will be greater than the benefit for some values of $\mu \geq \mu_0$. For this value of μ, the s-type worker will use only the public signal.

But if the s-type worker uses the public signal, employer does not benefit directly. Of course, the employer can use the information provided by Z to assign all those with $Z = Z_s$ to the better job y =1. But since $q(f) < 1$, some good types will be assigned to the job y = 0 leading to a production loss in market Y. The total expected production loss will be $\pi_b\ (V_{s1} - V_{s0})\ (1-\mu)\ (1-q(f)) > 0$. Given this the employer will prefer to acquire information at stage 1 to avoid the production loss.

Once the employer sets I = 1, workers have no incentive to use the public signal. So both groups *b* and *w* face exactly the same incentives and in equilibrium $\pi_b{}^* = \pi_w{}^*$. []

Example 4:

Let s = 3, u = 1, $e_0 = 3/2$, $\alpha = 1/6$, $W_1 = 10$, $W_0 = 4$ and $\pi_c = 1/3$, $q(f) = \sqrt{f}/2$, $f \in [0\ \ 4)$. When $\mu = 0$, s-type worker would choose $e^* = 2$ as $U(e^* = 2, f = 0) = 10 - 2$

= 8 > U(e* = 2, f>0) and U(e* =2, f = 0) > U(e =0, f* =9/4) = 25/4. When $\mu = 1/4$, the worker would choose e* = 3/2 and f* = 9/64, e* = 3/2 is the separating level of effort for X and given that with probability ¼ he will face another employer f* = 9/64 is the optimal level of effort for Z. This is his optimal choice since

U(e* = 3/2, f* = 9/64)= 7.2 > U(e* = 3/2, f= 0) = 7 > U(e = 0, f>0)

However, when $\mu = 2/3$, he is better off focussing only on Z. Note that the effort required to separate in X is still 3/2 as $e^* \geq e_0$. Hence

U (e* = 3/2, f =0) = 4.5 < U(e* = 3/2, f* = 1) = 5.5 < U(e = 0, f* = 9/4) = 25/4

So the worker will choose only the public signal.

This would suggest that a dynamic labour market with high turnover and worker's ability to signal to their prospective employers would be less susceptible to discrimination. A more traditional labour market where workers are locked into a particular employer and there is no public signalling mechanism would exhibit greater discrimination. It must be noted , however, that the above model is quite special and even though public signals play an important role, there is no signalling in equilibrium. This is due to our assumption that the information acquired by the employer is hard information and is freely available to others. Without this assumption, the incentives of the group *w* workers will also change and we may see some public signalling in equilibrium.

5. Passive Discrimination

Given that information plays a major role in our model of discrimination, one can make a distinction between discrimination based on a *commonly verifiable* but *not directly observable* characteristic and discrimination based on *directly observable* characteristic. Discrimination based on socio-political background and associations (caste, religion, affiliations etc.) belongs to the former category. Gender and racial discrimination belong to the second category. In the former context, if the employer wishes to discriminate then he can seek information about this variable. In the case of discrimination based on directly observable characteristic, we can ask whether the employer would actively seek information regarding this characteristic if it were not observable. Discrimination of either type will be called a case of *active discrimination* if either the employer is actively seeking or willing to seek information on group identifying characteristic.

If the employer is following discriminatory policy *because* of direct observability of some characteristic and will not do so if the characteristic were unobservable (but easily verifiable), then we shall call it a case of *passive* *discrimination.* Passive discrimination means that if the group characteristic is not common knowledge then the employer will follow non-discriminatory policy. The employer is not going to seek information on the group characteristic and then follow a discriminatory policy. In our model, discrimination can be passive in certain cases.

Let us consider a particular racial group which can be divided into two sub-groups (group I and II). Subgrouping can be on the basis of class or other socio-political affiliations[23]. We also assume that these two groups have different distributions of c so that group II has a higher fraction π_{II} of s-type workers. Let π_I and π_{II} be such that employer will follow a policy of I = 1 for group II and I = 0 for group I, when group identity is common knowledge. In other words, $\pi_I < \pi_c$ and $\pi_{II} > \pi_c$.

Now, let us suppose that group information is not common knowledge and in the sequence of actions the employer chooses whether to ask for the group information (Y) or not (N), before stage 1.

First consider the case where group sizes are such that[24] the population average of a s-type is $\pi_p < \pi_c$. In that case, if the employer chooses Y then this case is exactly like our previous model of section 3. But if it chooses N, then the employer can only have a common information policy for all, which in this case would be I = 0. In the continuation game, since $\pi_p < \pi_c$, a policy of I = 0 implies that all the s-type would be forced to choose e* to signal their type. Since signalling takes place with certainty the employer is clearly better off with a policy of N and I = 0.

On the other hand, if $\pi_p > \pi_c$ then in the absence of any subgrouping the employer would have to choose I = 1 for the whole group. But the existence of subgruoping allows the employer to seek such information and follow a discriminatory policy on the basis of the sub-group identity. Formally, the employer would choose Y and then I = 0 for group I and I = 1 for group II. The group I workers will have to then incur the signalling cost and face a reduced incentive.

The welfare implications of these sub-grouping can go either way as the previous two paragraphs show. Because of our assumptions of costless (or near costless) information and small value of α, social welfare is maximised when the

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[23] Such bifurcation within racial group is an increasing phenomenon. See Wilson (80) for an evaluation of their importance.

[24] Assume that for group I, c is distributed over [0 24] and for group II, [0 12]. Let the probability of a worker belonging to gr. I be 5/6. The probability of a group 1 worker being s-type is 1/4, where as

employer follows a uniform policy of I = 1 and groups do not have to engage in costly signalling. But this does not mean that any discriminatory policy is dominated by a uniform policy in welfare terms. For example, a policy of I = 0 for all is dominated by a discriminatory policy where at least one group does not have to signal. So sub-group formation has a positive impact on group's welfare when $\pi_p < \pi_c$ and this impact is reversed when $\pi_p > \pi_c$. This suggests that if the elite (groupII) amongst the disadvantaged group are sizeable to influence the overall policy towards the whole group, then formation of an elite sub-group within the group works to the detriment of the whole group.

This also has interesting general implications for group formations and their sizes in cases where groups are formed in a conscious fashion (clubs, associations etc.)[25]. A group, which is likely to receive favourable treatment (group II) in a discriminatory policy, has to worry about its size too. Even with a high π, a small size may not be enough to ensure benefits of a discriminatory policy.

6. Conclusion

We have shown that there are situations where employers may prefer not to acquire information directly even if they are costless purely because of strategic reasons. By committing not to acquire direct information about the skill type of the worker, an employer induces the skilled worker to signal his type by engaging in costly production enhancement (to the benefit of the employer). However, this non-acquisition policy by the employer can work only when the prior probability of the worker being skilled is not too high. When the prior belief about a worker being good is high the employer is better off acquiring the information and the worker as consequence does not have to engage in costly signalling. Hence the prior beliefs of the employer play an important role in shaping the incentives of the workers. We have shown how discriminatory beliefs about two ex-ante homogenous groups can lead to a discriminatory equilibrium. The paper's contribution to the literature on discrimination lies in the demonstration of fact that such equilibria exist even when there is free information available about the skill types.

The nature of the signalling technology plays an important role in determining the effectiveness of affirmative action policies and we do believe that most real life production process would have a fixed cost. So the analysis of section 4 is quite relevant. Co-ordination amongst worker of the disadvantaged group deserve some attention. It is not clear whether it can yield an empirically testable

for group II it is 1/2. For the population as a whole $\pi_p = 7/24 < 1/3$.

proposition, but casual empiricism would suggest that it has a role to play in any discriminatory situation. Our analysis also shows that the issue of sub-groups or within group bifurcation is quite important in shaping the discriminatory policy towards the whole group. Despite the oversimplified structure, our model has shown that more dynamic labour markets will have less discrimination. This again would seem to weigh well with the real world.

At a more general level, this paper has two features, which are not standard in agency models. It is normally believed that information is valuable, so free information will always be acquired. In our model, in some sense information is still valuable but the employers would prefer not to acquire it.[26] This is not surprising as in game contexts players can sometimes do better by remaining uninformed . Agency models where similar strategic considerations are present can share the same result. The second point relates to the nature of signalling. Many models share the feature of costly signals , but do not address the possibility of the this cost being someone else's benefit. We believe, this possibility can arise in many social situations. This may be important in welfare terms because in such cases socially wasteful signalling can take place[27]. Our model is an extreme case with costless information and full separation. Hence too much should not be made out of the result but this issue deserves more attention than it has received.

The model has many restrictive features. Information has an all or nothing character. Moreover, the model is not consistent with high information cost (large ε). One can however address these two issues in a more general set up where the quality of information would depend on the amount of investment made by the employer and this investment cost varies. Our model captures one end of the spectrum where full information is available at no cost.

Lastly, the model has many other potential areas of applications. In the study of organisational structures and incentives, direct information acquisition becomes a function of the given organisational structure rather than the employer's strategy. For example, in small firm information flows easily, so a good type worker need not have to engage in costly signalling[28]. A large firm on the other hand would resemble

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[25] See Basu (89) on related issues like the optimal size of associations.

[26] In equilibrium there is no loss of information, good types are always assigned the right job.

[27] Note that even if the signalling variable does not enhance productivity, it need not be socially wasteful if there is no other way to screen the good types. But in our model since information can be obtained rather cheaply, even though work effort (the signalling variable) enhances production, it is socially wasteful.

[28] The influence cost model of Milgrom and Roberts (88) does address similar issues. See Cremer (95) also.

a market where the information related to skill type of the workers would be difficult to acquire.

One could address similar issues in the context of law and economics. our model can be adapted to study the question of the 'burden of proof' and its implications for incentives of the individuals. Of course, legal scholars would point towards the irrelevance of group information[29] and the presence of a universal law of " presumed innocent unless proved otherwise". But in many semi-judicial set up these two features might be absent and one can look at how different rules affect the compliance incentives .

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[29] See Tirole (96) for an interesting model of group reputation where group belonging does affect compliance standard. Historically, in 19th century India, certain tribes used to be classified as criminal tribes. A member of such a tribe would be presumed guilty unless he can prove his innocence.

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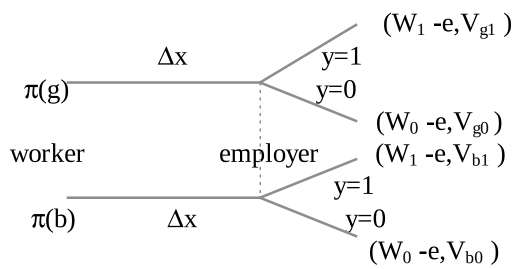
Appendix:

Proof of Proposition 1



The Signalling Game

Proof: It is easy to verify that the strategies and belief described in the proposition constitute an equilibrium. Given employer's strategy, good type worker is better off playing his equilibrium strategy because,

$$U = w + W_0 < w - (X^*-X_0)/\delta g + W_1$$

From condition D6, it is clear that the bad type has no incentive to deviate either. Likewise, given the beliefs as specified by μ, employer's strategy is also optimal. In addition it can be shown that the belief structure do satisfy the divinity criterion.

Consider any deviation to $X<X^*$. Let $\rho = \rho(1,X;\mu)$ denote the probability that the employer would assign the worker to job 1. The good type would deviate to such an output level iff

$$\rho W_1 + (1-\rho) W_0 - (X-X_0)/\delta g > W_1 -(X^*-X_0)/\delta g$$

or $$\rho > 1- (X^*-X) /\delta g(W_1-W_0)$$

Let P_g be the set of such ρ such that the good type find it worth deviating.

Similarly, the bad type would consider deviating to X iff

$$\rho W_1 + (1-\rho) W_0 - (X-X_0)/\delta b > W_0$$

or, $\rho > 1- (X^* - X)/\delta b(W_1-W_0)$

This would give us the set P_b , the set of ρ such that the bad type would deviate.

Since b<g, it can be easily seen that $P_g \subseteq P_b$. This would mean for any possible response of the employer if the good type finds it worth deviating so does the bad type. This justifies the out of equilibrium belief held by the employer. It can also be verified that this is the only equilibrium with such a property.

Proof of Proposition 5:

Let (π_b^*,π_w^*) be the initial equilibrium. Depending on the value of e_0 , the s-type chooses either e_0 or e* as defined in Proposition 3, $e^* = \Delta Wu/s$. Let (π_b^{**},π_w^{**}) be the new equilibrium under affirmative action. For the proposition to hold it must be the case that $e_0 > e^{**}$, where e** is the effort put in by the s-type in the new equilibrium (as defined by 13). So we have to consider two cases.

Case I: $e_0 \geq e^*$.

By (15), it must be the case that,

(17) $W_1 - e_0 - W_0 -\rho^* \Delta W = \pi_b^{**} C$

$(1-\rho^*)\Delta W - e_0 = \pi_b^{**} C$

or, $\Delta W (1-\pi_w^*) - e_0 (1- \pi_b^{**}) = \pi_b^{**} (1-\pi_b^{**})C$,

Solving this we get,

(18) $\pi_b^{**} = [d \pm \sqrt{\{d^2 - 4(1-\pi_w^*)\pi_w^* + 4e_0 /C\}}] / 2$

where $d = 1- (e_0/C)$.

Like before, it can be shown that real roots exist. The larger of the two roots is greater than π_w , so it can not be part of a discriminatory equilibrium. So we focus on the smaller of the two roots.

For an equilibrium to exist it must be the case that $\pi_b^{**} > 0$. But using (18), it can be shown that $\pi_b^{**} < 0$ iff

(19) $(e_0 /C) > \pi_w (1-\pi_w)$

So, a large e_0 with $e_0 > \pi_w^* (1-\pi_w^*)C$ would mean there is no π_b^{**} satisfying $\pi_w^* > \pi_b^{**} > 0$.

Case II: $e_0 < e^*$. This case is exactly similar the previous case except that we have to see whether condition (19) guarantees that there is no other $\pi_b^{**} > 0$ such that

$e^{**} > e_0$. Suppose such a π_b^{**} exists and $e^{**} > e_0$. Then it must be true that

$$[(1-\pi_w)\, \Delta W\, b \,/\, (1-\pi_b^{**})\, s] > e_0$$

where π_b^{**} is given by (16). After some manipulation it can be shown that the above inequality is true iff

$$((1-\pi_w)\, \pi_w\,)\, b) \,/\, ((1-\pi_b^{**})s) > (e_0/C).$$

However, given (19) it must be the case that

$$((1-\pi_b^{**})s) < b$$

or, $((1-\pi_w)\, \pi_w\,) > b/s$, using the value of π_b^{**} from (16)

or, $e_0/C > b/s$ using (19)

Since $C > \Delta W$, this condition implies that $e_0 > \Delta W/s$, which is violated by our assumption that $e_0 < e^{*} = \Delta W\, b\, /s$.

Hence , condition (19) implies that there is no discriminatory equilibrium with $\pi_b^{**} > 0$ under affirmative action. []