Bribery and Favoritism by Auctioneers in Sealed Bid Auctions

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Abstract

We consider a model of bribery in procurement first-price auctions. The auctioneer cannot award the contract at a price above the winning bid at the auction, but can award the contract to a "dishonest" supplier who submitted a bid above that of the honest supplier. Despite this, bribery does not eliminate the incentive of the dishonest supplier to win the auction outright without a bribe. In fact, bribery may even induce the dishonest supplier to bid more aggressively. We study the allocative distortion caused by bribery in a model of asymmetric bidders. We find that the distortion may actually result in a lower expected price paid by the buyer when the dishonest supplier is "stronger" in that he has a more favorable distribution of costs than the honest supplier. However, the incentives for cost-reducing investment by the suppliers are reduced by bribery and result in a lower level of industry investment than the social optimum.

1. Introduction

Bribery has it antecedents in the practice of reciprocity in early civilizations. In a sense, reciprocity is a form of contract in which a government official is expected to perform some action in return for a payment. In early civilizations, reciprocity was the norm. With time, certain types of reciprocity became socially unacceptable and subject to penalties. This process was driven by both legal and religious forces. The norm of reciprocity first became unacceptable for judges and courts. Later, it became unacceptable for other government officials. The term bribery means an illegal form of reciprocity with government officials.

In the United States, there are two important statutes making bribery a criminal offense. First, there is the federal statute entitled Bribery of Public Officials and Witnesses (18 USC 201). This statute applies to bribery of federal officials. Second, there is a federal statute entitled the Foreign Corrupt Practices Act (15 USC 78). This statute was enacted in 1977 and applies to bribery of foreign officials by American corporations.

In this paper, we will use the term bribery to mean a side-payment to a decision-maker in order to alter a decision on the merits in favor of the person making the side-payment. In particular, the decision-maker is an auctioneer who represents a buyer of some good. The buyer may be viewed as a government and the auctioneer as a government official. Alternatively, the buyer may simply be another corporation and the auctioneer an employee of the corporation. The person who makes a bribe is one of the suppliers willing to produce the good. The favor conferred by the auctioneer is to award the contract for production of the good to the supplier offering the bribe even though this supplier has not won the contract on the merits. Even though the auctioneer cannot alter the price, she can favor the supplier by awarding him the contract at that winning price. In effect, the auctioneer allows the bribing supplier to resubmit his bid.

This type of bribery alters the bidding strategies and distorts the allocation of the contract, but does not supplant the first-price auction. In the model, we examine the implications of bribery when only one supplier can offer bribes to the auctioneer. This asymmetric structure has interesting applications in the international competition among corporations for public projects in various countries. In particular, one might expect that domestic suppliers are in a better position than foreign suppliers to bribe the official auctioneer. The Foreign Corrupt Practices Act has criminal penalties for bribery of foreign officials by U.S. corporations. Thus, U.S. corporations are in a weak position to bribe foreign officials in order to obtain contracts for public projects in their countries. Conversely, the foreign country may have no laws or enforcement preventing bribery of its officials by domestic corporations. Even if no domestic corporations could supply the public project, corporations from differing countries competing for the public project would have differing laws or enforcement with respect to bribery of foreign officials. Until very recently, several European countries allowed their national corporations to have a tax deduction for bribes paid to foreign officials.

The asymmetry in the ability of suppliers to bribe may also arise in the competition for public projects within a country. For instance, large national suppliers may have valuable reputations which would be damaged by a criminal conviction for bribery. Thus, these suppliers may have internal rules prohibiting and punishing bribery. However, smaller local suppliers may not have similar reputations, and may be more willing to risk bribery convictions in order to win contracts.

Finally, the asymmetry assumption may be an appropriate in many private procurement cases. Most corporations have internal rules prohibiting employees from accepting bribes in return for awarding contracts to suppliers. Employees accepting such bribes could be punished by being demoted or fired from their jobs. In addition, the suppliers who paid bribes to employees of the corporation could be blacklisted from future contracts with the corporation. Some suppliers may then prefer to compete without using bribes, while other suppliers may be willing to take the risk of losing future business.

The existence of bribery as defined in this model does not eliminate the incentives for the dishonest supplier to submit meaningful bids in the first-price auction. Indeed, winning the contract by offering the lowest price allows the supplier to avoid paying the bribe. However, the bidding behavior is distorted. We show that the direction of this bidding distortion, whether more or less aggressive, is unclear. However, it is clear that the bidding distortion induces an allocative distortion in which the contract is awarded to the dishonest bidder too often.

An allocative distortion is typical of first-price auctions. However, in contrast to a honest first-price auction, bribery always distorts the allocation in favor of the dishonest bidder, whether he has the most favorable distribution of costs or not. The effect of this distortion on the price paid by the buyer is ambiguous. Depending on the magnitude of the bribe and the asymmetry of the suppliers, we show that the price may actually be lower under bribery. What is more surprising is that this may be the case even when the dishonest supplier has the most favorable distribution of costs.

Finally, we analyze the effects of bribery on the incentives of suppliers to invest in cost reduction. The result is unambiguous in that the first-price auction with bribery results in a lower total investment. However, the magnitude of the bribe determines whether the dishonest supplier invests more or less than the honest supplier.

The paper is structured as follows. In section 2, we characterize the model of bribery in a first-price auction. In section 3, we examine the equilibrium bidding functions for general cost distributions. This section provides some intuition on how the equilibrium bidding functions of the two suppliers would be affected by bribery, but concludes that that there are no general results. In section 4, we define a convenient family of distributions for the costs of the suppliers. This family allows for a tractable solution of the equilibrium bidding functions when the suppliers have differing cost distributions. In section 5, we employ the equilibrium bidding functions from section 4 to examine the allocative distortions caused by bribery, the expected price paid by the buyer, and the expected bribe received by the auctioneer. Section 6 analyzes how bribery affects the incentives for investment in cost reduction. Section 7 discusses certain modeling issues, and section 8 provides some concluding remarks.

2. The Model of Bribery

The buyer has a value v for products with a fixed quantity and quality. The buyer employs an auctioneer to receive bids and award a contract to purchase the products from one of two suppliers using a sealed-bid first-price auction. The contract would normally be awarded to the supplier with the lowest bid at a price equal to that bid. However, the auctioneer can be bribed by one of the suppliers, called the "dishonest" supplier. In return for a bribe, the auctioneer awards the contract to the dishonest supplier by allowing him to revise his bid downward in order to match the lower bid of the "honest" supplier.

We assume that each supplier draws his cost of production c_i, where $i = d,h$, from a distribution $G_i(c)$ with support [0,1] common to both suppliers, and a positive density $g_i(c)$ over this support. The cost c_i is private information for each supplier, but the distribution functions are common knowledge. For simplicity, we also assume that the value of the buyer exceeds the highest possible cost realization ($v > 1$). Finally, we assume that the costs of the suppliers are independently distributed. Thus, we will introduce bribery into an asymmetric independent private value (cost) first-price auction. In general, it is difficult to solve for the equilibrium bid functions in asymmetric first-price auctions. However, the first-price auction becomes more tractable with bribery by one dishonest supplier.

We assume that the auctioneer must run a sealed-bid first-price auction and must award the contract at a price equal to the lowest bid, but not necessarily to the supplier who made the lowest bid. In addition, we assume that only one of the suppliers is dishonest and can offer a bribe to the auctioneer. Thus, if the bid of the dishonest supplier is higher than that of the honest supplier, he can bribe the auctioneer in order to obtain the contract at the price bid by the honest supplier. Let b_d and b_h, be the bid of the dishonest and honest suppliers respectively. If $b_d < b_h$, the contract is assigned to the dishonest supplier at a price b_d. However, if $b_d > b_h$, but $c_d < b_h$, the contract is awarded to the dishonest supplier at a price b_h, but only after paying the auctioneer a fraction α of the surplus $b_h - c_d$. We call α the "percentage bribe". Finally, if $b_h < c_d$ the honest bidder is awarded the contract at a price equal to his bid. Notice that we are implicitly assuming that the auctioneer knows the dishonest supplier's cost. We will comment on this in section 7.

We assume that the percentage bribe α is determined prior to the auction and thus is independent of the outcome of the auction. We also assume that the dishonest supplier knows the value of α prior to bidding. We need not assume that the honest supplier knows the value of α, but only the existence of bribery that would award the contract to the dishonest supplier when $c_d < b_h$.

The ability to bribe the auctioneer does not imply that the dishonest supplier would not attempt to win the contract outright by bidding below the honest supplier. In general, the bribe is a "cost" to the dishonest supplier so he will submit bids that have a positive probability of winning the contract outright.

With this specification of bribery, the buyer obtains some of the benefits of competition in that the equilibrium price is determined by a sealed-bid first-price auction. However, bribery will generally alter the equilibrium bidding functions of both the dishonest and the honest supplier.

3. General Results on the Bidding Functions

In this section, we characterize the equilibrium sealed-bid first-price auction for a given percentage bid assuming general distribution functions for the costs of the honest and dishonest suppliers. We find that the introduction of bribery does not generate unambiguous results for the shift in the equilibrium bidding functions. For example, one cannot conclude that the dishonest supplier bids less aggressively when he has the ability to bribe the auctioneer. Similarly, one cannot conclude that the honest supplier bids more aggressively when he knows that the dishonest supplier can bribe the auctioneer.

Given our assumptions about the bribing process, the honest supplier is effectively bidding against cost of the dishonest supplier. As a result, his bidding strategy is independent of the bidding strategy of the dishonest supplier.[1] Similarly, his bidding strategy is also independent of the percentage bribe a. From the viewpoint of the honest supplier, the bribe is simply a transfer between the dishonest supplier and the auctioneer. Indeed, the honest supplier calculates his bidding strategy by solving the following problem:

(1) $$\max_b \Pi_h[b;c] = (b-c)[1-G_d(b)].$$

The first-order condition of this problem is

(2) $$[1-G_d(b)] - (b-c)g_d(b) = 0,$$

which implies that the optimal bidding function for the honest supplier, $b_h(c)$, is the implicit function defined by [2]

(3) $$b - \frac{[1-G_d(b)]}{g_d(b)} = c.$$

This bidding function of the honest supplier is equivalent to the best take-it-or-leave-it offer that a seller with cost c of producing some object can make to a buyer with random valuation in the interval $[0,1]$ given by the distribution function G_d.

Since the bidding function of the honest supplier is a dominant strategy, the dishonest supplier can take this bidding function as given when calculating its bidding strategy. Thus, the optimal bidding function of the dishonest supplier is obtained by solving the following problem:

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[1] This assumes that the dishonest supplier does not submit bids below his realized costs.

[2] The second order conditions require $G_d(b) - \frac{[1-G_d(b)]}{g_d(b)}$ to be increasing. We will assume this condition to hold for G_i, $i = h,d$.

(4) $$\max_b \Pi_d\left[b;c,b_h(\cdot)\right]=(b-c)\left[1-G_h(b_h^{-1}(b))\right]+\int_{b_h^{-1}(c)}^{b_h^{-1}(b)}(1-\alpha)(b_h(x)-c)g_h(x)dx\,.$$

The first term is the expected profit of the dishonest supplier when he wins the auction outright, and thus pays no bribe. The second term is the expected profit of the dishonest supplier when he loses the auction ($b > b_h$), but bribes the auctioneer because his costs are below the bid of the honest supplier ($c < b_h$). We refer to the difference between the contract price b_h and the cost realization of the dishonest supplier as the "surplus" which can be divided between the dishonest supplier and the auctioneer. In this case, the dishonest supplier retains the fraction ($1 - \alpha$) of this surplus. The first order condition for this problem is

(5) $$\left[1-G_h(b_h^{-1}(b))\right]-\alpha\cdot(b-c)\cdot g_h(b_h^{-1}(b))\frac{db_h^{-1}(b)}{db}=0,$$

which implies that the optimal bidding function of the dishonest supplier, $b_d(c)$, is the implicit function defined by

(6) $$b-\frac{\left[1-G_h(b_h^{-1}(b))\right]}{\alpha\cdot g_h(b_h^{-1}(b))\,\dfrac{db_h^{-1}(b)}{db}}=c,$$

assuming that this solution is interior on the range of bids $[b_h(0),b_h(1)]$ of the honest supplier. The dishonest supplier would never offer a bid below the minimum bid that honest supplier would offer. Thus, if the solution to (6) is below $b_h(0)$, the dishonest supplier also bids $b_h(0)$. Note that $b_h(1) = b_d(1) = 1$, as usual.

We can now examine the effect of bribery on the behavior of the suppliers. One might expect the honest supplier to bid more aggressively in the presence of bribery. The reason is that the honest supplier has to bid below the cost, rather than the bid, of the dishonest supplier. In addition, one might expect the dishonest supplier to bid less aggressively. The reason is that the dishonest supplier need not lose the contract simply because he lost the auction. The dishonest supplier can still obtain the contract by a bribe and would choose to do so when there exists some surplus at the price determined by the bid of the honest supplier. Despite the compelling nature of these two intuitions, neither is true in general. In particular, the equilibrium bidding functions depend on the distribution functions for the costs of the suppliers.

In order to obtain the intuition for this lack of general results, consider the symmetric case, where $G_d = G_h$, and compare the two first-order conditions (2) and (5) above. The first term in each condition is the incentive to raise the bids caused by the increase in revenue on contracts that the supplier wins outright in the auction. For a given cost, common to both suppliers, this first term in (2) for the honest supplier is always less than the corresponding term in (5) for the dishonest supplier. For this reason, the honest supplier has a lower incentive to raise his bid, making him more aggressive (lower bids at any cost realization). This corresponds to the intuition that the honest supplier must bid against the costs of the dishonest supplier, rather that his bid.

The second term in the two first-order conditions is the disincentive to raise the bids caused by the reduced profit on contracts that are lost. The second term for the dishonest supplier is multiplied by a , which is less than 1. For this reason, it would appear that the dishonest supplier has a lower disincentive to raise his bid, making him less aggressive (higher bids at any cost realization). Indeed, the fact that the dishonest supplier can bribe the auctioneer when he loses the auction reduces his disincentive to raise his bid. This corresponds to the intuition that the dishonest supplier has a second chance to obtain the contract through bribery.

However, there is an additional factor in the second term of the first-order condition of the dishonest supplier. In particular, the second term for the dishonest supplier is multiplied by the factor $\frac{db_h^{-1}(b)}{db}$. This factor is the slope of the inverse bidding function of the honest supplier and represents the probability of losing the auction as a consequence of raising his bid. This results in a loss of the percentage a of the surplus. Whenever the margin between bid and cost of the honest supplier, $b_h(c) - c$, is decreasing in c (for example, whenever $g_d(c)$ is nondecreasing), this factor is greater than 1, which is the slope of the effective distribution of bids (costs) by the dishonest supplier facing the honest supplier. As a result, the dishonest supplier experiences a larger increment in the probability of losing the auction outright when he increases his bid marginally. Thus, this factor increases the disincentive of the dishonest supplier to raise his bid, making him more aggressive. The new force works against the two previous forces, and suggests that no general result can be obtained for the relative aggressiveness of the honest and dishonest suppliers in the auction. We provide an example below which illustrates the ambiguity.

Now consider the comparison between the bidding functions with bribery and equilibrium bidding functions if there is no bribery of the auctioneer. The first-order condition for the bidding function of supplier i with an honest auctioneer is

(7) $$\left[1 - G_j(b_j^{-1}(b))\right] - (b-c)\cdot g_j(b_j^{-1}(b))\frac{db_j^{-1}(b)}{db} = 0,$$

where j denotes the other supplier. When we compare this condition to the first-order condition (2) for the honest supplier, we find that the same tradeoffs apply. In particular, the first term representing the incentive to increase the bid would be larger without bribery. However, the second term representing the disincentive to increase the bid will also be larger because the factor $\frac{db_j^{-1}(b)}{db}$ may well be larger than 1. Thus, the total effect may be ambiguous, and it is unclear whether bribery will make the honest supplier more or less aggressive in his bidding.

As an illustration of this ambiguity, we have computed the equilibrium bidding functions for the symmetric case where $G_h(c) = G_d(c) = c^2$. Figure 1 illustrates these bidding functions (the solutions to (2), (5), and (7)). The dashed line represents the bidding function without bribery. The solid lines represent the bidding functions for the honest (darker) and dishonest (lighter) suppliers. With bribery, the bidding functions of the honest and dishonest suppliers cross at approximately $c = .6$. The honest supplier bids more aggressively than the dishonest supplier at low cost realizations, but less aggressively at high cost realizations. In addition, the honest supplier may bid more or less aggressively that he would without bribery. The bidding functions of the honest supplier, with and without bribery, cross at approximately $c = .3$. Bribery induces the honest supplier to bid more aggressively for high cost realizations. However, for low cost realizations, bribery induces the honest supplier to bid less aggressively.

In the remainder of the paper, we will examine these optimal bidding functions for a convenient family of distributions. This family is convenient because it allows us to obtain closed form solutions for the bids, prices, and market shares. Thus, we will be able to address a number of interesting welfare and policy questions.

4. The Bidding Functions for a Convenient Family of Distributions

In this section, we compute the equilibrium behavior and outcomes for a one-parameter family of distribution functions $G(c;t)$ on the costs of the suppliers. For $t > 0$, define $G(c;t) = 1-[1-c]^t$ over the support [0,1]. The corresponding density function is $g(c;t) = t[1-c]^{t-1}$. The parameter t can be interpreted as the number of draws that a supplier would have from a uniform cost distribution. As such, t can be loosely interpreted as capacity because it corresponds to the number of equal size plants owned by a supplier that could be used to produce the good. In section 6 we will assume that t corresponds to an investment that reduces costs.

Let the capacity of the honest supplier be t_h and that of the dishonest supplier be t_d. When $t_i < t_j$, supplier i has a less favorable cost distribution in that there is a lower probability of obtaining a cost below any given c. In other words, $G(c;t_i) < G(c;t_j)$ for all c in the support, so that the distribution for supplier j stochastically dominates that of supplier i.

With this specification for the distribution of costs, we can solve for the equilibrium bidding functions implicitly defined by equations (2) and (5). The equilibrium bidding function of the honest supplier has the following linear form

(8) $$b_h(c) = \frac{1}{1+t_d} + \frac{t_d}{1+t_d}c \ .$$

The equilibrium bidding function of the dishonest supplier is slightly more complicated. When $\alpha \leq t_d/t_h$, the equilibrium bidding function of the dishonest supplier is everywhere above that of the honest supplier and takes the following linear form

(9a) $$b_d(c;\alpha) = \frac{1}{1+\alpha t_h} + \alpha \frac{t_h}{1+\alpha t_h} c \quad , \quad \text{for} \quad \alpha \le t_d / t_h \ .$$

On the other hand, when $\alpha > t_d/t_h$, (9a) is below the bidding function of the honest supplier. However, the bidding function of the dishonest supplier cannot be below the lowest bid $b_h(0)$ of the honest supplier. Otherwise, the dishonest supplier could raise his bid without reducing the probability of winning the auction outright. Thus, the equilibrium bid of the dishonest supplier must equal $b_h(0)$ for low cost realizations. For higher cost realizations, the equilibrium bidding function will take the same form as equation (9a).

(9b) $$b_d(c;\alpha) = \frac{1}{1+\alpha \cdot t_h} + \alpha \cdot \frac{t_h}{1+\alpha \cdot t_h} c \ , \qquad \text{for} \ \alpha > \frac{t_d}{t_h} \ \text{and} \ c > c'$$

$$= \frac{1}{1+t_d} \ , \qquad \text{for} \ \alpha < \frac{t_d}{t_h} \ \text{and} \ c \le c' \ ,$$

where $c' = \dfrac{\alpha \cdot t_h - t_d}{\alpha \cdot t_h (1+t_d)}$. The cutoff cost c' for the equilibrium bidding function is defined as the maximum cost at which the bidding function of the dishonest supplier equal the low bid of the honest supplier, $b_d(c';\alpha) = b_h(0)$.

In a subsequent section, we will compare these bidding functions to the equilibrium bidding functions in auctions without bribery. But as a basic reference point, consider the equilibrium bidding function of a first-price sealed-bid auction without bribery for the symmetric case in which $t = t_h = t_d$. The solution to the first-order condition in (7) is

(10) $$b(c) = \frac{1}{1+t} + \frac{t}{1+t} c \ .$$

The comparison of the symmetric first-price auction with and without bribery is very clear. The bidding function of the honest supplier from (9) is unchanged by the fact that the dishonest supplier can bribe the auctioneer. Thus, he does not bid more aggressively in the presence of bribery. We already know that this is not a general feature of our model and is a consequence of our family of cost distributions.[3]

On the other hand, the bidding function of the dishonest supplier from (9) is everywhere above the bidding function without bribery from (10) for $a < 1$. In other words, as long as the dishonest supplier retains some fraction of the surplus ($b_h - c_d$) after bribing the auctioneer, he will bid less aggressively than the honest supplier. Again, this follows from the symmetry of cost distributions, and is a direct consequence of the equivalence of the bidding behavior for the honest supplier with and without bribery. In this case, the only effect of bribery is that losing the auction is a less attractive alternative for the dishonest supplier.

As the percentage bribe decreases, the dishonest supplier retains more of the surplus, and thus bids less aggressively. In other words, when the percentage bribe is high, the dishonest supplier will bid more aggressively to win the contract outright and avoid the bride. But when the percentage bribe is low, the dishonest supplier can bid higher, knowing that he will still share the surplus on the contracts that he could have won outright with a lower bid.

The extreme case of "favoritism" occurs when $a = 0$. The auctioneer awards the contract to the dishonest supplier whenever some surplus exists and accepts no bribe in return. In this case, the bidding function of the dishonest supplier degenerates to $b_d(c;a) = 1$, the highest possible cost, irrespective of the actual cost realization. In effect, the dishonest supplier does not bid in the auction, and relies on favoritism to obtain the contract whenever it is profitable at the price bid by the honest supplier. As a result, the dishonest supplier will never

[3] What is special here is that $\frac{1-G(b)}{g(b)} = \frac{1-G(b^{-1}(b))}{g(b^{-1}(b))\frac{db^{-1}(b)}{db}}$, where $b^{-1}(b)$ represents the inverse of the symmetric honest bidding function. That is, the hazard rate of the distribution of bids is the same as the hazard rate of the costs generating those bids. Then, the trade-off facing the honest bidder is the same under honest behavior and under bribery.

submit the low bid in the auction, and the bid of the honest supplier will always determine the price.

5. Allocation Distortions and the Distribution of the Surplus

Having characterized the behavior of suppliers under bribery, we can now examine several questions related to efficiency and distribution. The first of these questions is how bribery affects the efficiency of the auction mechanism.

5.1 Bribes and allocation distortions

As we have already mentioned, the sealed-bid auction with bribery awards the contract to the honest supplier whenever his bid is below the cost of the dishonest supplier. Since the bid of the honest supplier is generally above his cost, this implies that our mechanism awards the contract to the dishonest supplier too often. Moreover, when the inverse of the hazard rate, $\frac{[1-G_d(c)]}{g_d(c)}$, is decreasing in c, as in our family of distribution functions, this allocative distortion is greater when the cost of the honest supplier is lower. Indeed, in this case condition (3) implies that $(b\text{-}c)$ is decreasing in c for the honest supplier. That is, when c_h is small, the dishonest supplier can win the contract even if c_d is significantly higher than c_h. When c_h is high, the allocative distortion occurs only when the difference in cost is relatively small.

This allocative distortion does not depend on whether the dishonest supplier has a more favorable cost distribution is (stronger) or less favorable cost distribution is (weaker) than the honest supplier. In particular, the honest supplier may be weaker than the dishonest supplier, and yet bribery discriminates against him. This contrasts with the type of distortion that arises in a first-price auction without bribery, where the weaker supplier bids more aggressively than the stronger supplier for equal cost realizations (see Lebrun 1999).[4]

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[4] Also, see Waehrer (1999).

The type of allocative distortion is also very different under an optimal mechanism (from the buyer's viewpoint) in absence of bribery. Again, an optimal mechanism would discriminate against the stronger bidder. Thus, if the dishonest supplier is the stronger bidder, the optimal mechanism discriminates in the opposite direction than the equilibrium discrimination induced by bribery in the first-price auction. Finally, if the dishonest supplier is the weaker bidder, then both the optimal mechanism and the first-price auction, with or without bribery, discriminate against the stronger honest supplier. [5] However, the optimal mechanism would typically have a wider range of discrimination at high cost realizations of the stronger honest supplier. Exactly the opposite is true in our case. See Figure 2.

To facilitate later results, it is convenient to analyze in some detail the region in which an allocative distortion occurs and the region in which bribes are paid. The bid of the honest supplier is one determinant of these regions. But since the bidding function of the honest supplier is independent of the bidding strategy of the dishonest supplier and of the percentage bribe, we can start by defining the bribery region at a given cost realization of the honest supplier c_h. If the dishonest supplier's costs are very low, his bid will be lower than $b_h(c_h)$, defined by (8), and he will win the contract outright. However, once the costs of the dishonest supplier exceed the level $x(c_h)$ defined by

$$b_d(x(c_h)) = b_h(c_h), \tag{11}$$

then he must bribe the auctioneer in order to obtain the contract. In this case, the price paid by the buyer is $b_h(c_h)$. If $b_d(0) > b_h(0)$, then for low realizations of c_h , $x(c_h) = 0$. Thus, we can express the implicit function defined by (11) as

$$x(c_h) = \frac{t_d(1+\alpha t_h)c_h}{\alpha t_h(1+t_d)} - \frac{t_d - \alpha_h}{\alpha t_h(1+t_d)} \quad \text{for} \quad c_h > c'' = \frac{t_d - \alpha t_h}{t_d(1+\alpha t_h)} > 0, \tag{12}$$

$$x(c_h) = 0 \quad \text{otherwise.}$$

[5] See Myerson, (1981), and McAfee and McMillan, (1989).

For $c_d < x(c_h)$, the dishonest supplier wins the contract outright by submitting the low bid and does not need to bribe the auctioneer. Note that $c'' > 0$ only when $a < t_d/t_h$. And more importantly, note that $c_h > x(c_h)$ for every c_h in this case, and only in this case. Indeed, the bidding function of the honest supplier is everywhere below the bidding function of the dishonest supplier. This will be important for the result in Proposition 1 below. It means that whenever the dishonest supplier obtains the contract with a cost above that of the honest supplier it is because he bribes the auctioneer. In the case where $a > t_d/t_h$, the dishonest supplier sometimes wins outright even when his cost is higher than the cost of the honest supplier.

Finally, bribes (and allocative distortions) will occur only when there is a surplus to be divided between the dishonest supplier and the auctioneer, that is, when the cost of the dishonest supplier is less that the bid of the honest supplier. This means that when the cost of the dishonest supplier exceeds $b_h(c_h)$, the honest supplier is awarded the contract at his low bid (see Figure 3).

5.2 The expected price for the buyer

The special form of discrimination that occurs under bribery raises the question of how this distortion affects the price paid by the buyer. This is our second question. In principle, bribery implies that part of the surplus generated by the transaction accrues to a third party, the auctioneer. But what is the aggregate effect of this loss when combined with the allocative distortion? A first answer to this question can be obtained by comparing the expected price for the buyer in our bribery model with the expected price for the buyer in an efficient mechanism, such as the second price auction with no bribery. Let $Ep_2(t_d, t_h)$ represent the expected price in a (honest) second price auction and $Ep(a; t_d, t_h)$ represent the expected price in our first-price auction with bribery. For our special family of cost distributions, the following proposition provides the answer.

Proposition 1: (i) When $t_d > t_h$, the expected price in a first-price auction with bribery can be below the expected price in a second-price auction without bribery. In particular, there exists a set of percentage bribes $(\overline{a},1]$ such that for any a in this set,

$$Ep_2(t_d, t_h) > Ep(a; t_d, t_h).$$

(ii) When $t_d \leq t_h$, the expected price in a first-price auction with bribery can never be below the expected price in a second-price auction without bribery. In particular,

$$Ep_2(t_d, t_h) < Ep(a; t_d, t_h) \text{ for all } a \leq 1.$$

Proof: See Appendix.

At first glance, this result seems counterintuitive. One would expect that the buyer obtains better deals when discriminating against the stronger supplier, thereby increasing competitive pressure on the stronger supplier. However, the opposite occurs in this model. When the honest supplier is the stronger bidder (case (ii) above), the bribery auction is not better for the buyer than an efficient auction. This is again a special feature of our family of cost distributions. To understand why, consider the price that a supplier expects to receive, conditional on winning, in an honest second-price auction:

$$E\left[c_j \middle| c_j \geq c_i\right] = \frac{1}{1-G_j(c_i)} \int_{c_i}^{1} x \, dG_j(x) = \frac{1+t_j c_i}{1+t_j}. \tag{13}$$

In the case where $i = h$, this coincides with the bid (and thus the expected price conditional on winning) of the honest supplier in our first-price auction with bribery. This is a special feature of our family of distributions. As compared to a second-price auction, our mechanism does not induce "more aggressive behavior" by the honest supplier. In particular, the outcome does not extract a higher price from the buyer when the honest supplier wins. This also explains the result in (ii), which again is special to the family of cost distributions.

We now examine the more interesting and perhaps more surprising result in Proposition 1(i). Note that when $\alpha = 1$, the right hand side in (13) is also the bidding function of the dishonest supplier in our first-price auction with bribery. This means that if both suppliers always paid their bid when winning and if both our auction and the second-price auction allocated the contract in the same way, then the expected price paid by the buyer would also be the same in both auctions. The differences could then be traced to the allocative distortion of our auction and to the divergence between the bid and price received by the dishonest bidder. What is this distortion and what is this divergence? In our case the distortion means that the dishonest supplier will sometimes have a higher cost and still win. Since $c_h > x(c_h)$ for every c_h in our case (i), whenever this distortion occurs the dishonest supplier receives a price equal to the honest supplier's bid, so that the distortion has no consequence for the buyer. However, the divergence extends further. The dishonest supplier sometimes has a cost lower than c_h and higher than $x(c_h)$. In this case, he wins the contract (as under the second-price auction), but only after revising his bid and bribing the auctioneer. In other words, the price is the honest supplier's bid $b_h(c_h)$ instead of the dishonest supplier's bid $b_d(c_d) > (b_d(x(c_h)) = b_h(c_h))$. This is the difference between second-price auction and our bribery auction, and results in a lower price for the buyer when $\alpha = 1$ (and by continuity, for sufficiently large $\alpha < 1$).[6]

There is another issue related to the effects of distortions on the expected price paid by the buyer. As we know, a first-price auction is also a distortionary auction when bidders are asymmetric. In fact, for the family of distributions that we are using, numerical computations indicate that the distortion induced by first-price auctions is beneficial for the buyer.[7] Thus, bribery may not be the driving force of Proposition 1(i), but the use of first-price auction. One

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6 Notice that under the conditions in (ii) $c_h \le x(c_h)$. That means that bribery only occurs when $c_d > c_h$ with no consequence for the price. However, when $x(c_h) > c_d > c_h$ the dishonest supplier gets the contract outright, receiving his bid $b_d(c_d)$ ($< b_d(x(c_h)) = b_h(c_h)$). In our family of cost distributions this reduction in price is outweighed by the fact that, in the case $t_d < t_h$, the bid of the dishonest supplier (equal to $b_h(0)$) is above the price he expects in a second-price auction when he wins with costs below c' as defined in (9b) above.

7 See Marshall and al., (1994).

would then like to know whether bribery could result in a reduction in the expected price for a first-price auction.

Obtaining analytical solutions for the bidding functions and then the expected prices, in an asymmetric first-price auction is a hopeless task. Thus we must rely on numerical solutions in order to provide an answer. Using such computations, we can guarantee that

Proposition 2: The expected price in a first-price auction may increase or decrease as a consequence of bribery.

The result in Proposition 2 can be obtained using numerical computations of first-price auctions. Using the results in Marshall et al. (1994), when $t_d = 4$ and $t_h = 1$, the expected price paid by the buyer is .4943. Alternatively, with bribery and $a = 1$, the expected price is .4958. Thus, bribery increases the expected price paid by the buyer. However, when $t_d = 3$ and $t_h = 2$, the expected prices are .4125 and .4122, respectively. Thus, bribery actually reduces the expected price for sufficiently high values of a.

5.3 The expected bribe

With bribery, the dishonest supplier prefers to retain a larger percentage of the surplus. For any given cost realization c and bid b, the profit function of the dishonest supplier is decreasing in a. Thus, a lower percentage a retained by the auctioneer shifts the profit function of the dishonest supplier upward. The bidding function of the dishonest supplier obviously depends on a, but the envelope theorem implies that a lower percentage bribe will increase the expected profits of the dishonest supplier. Also, the buyer prefers higher values of a, since this increases the dishonest supplier´s aggressiveness without affecting the behavior of the honest supplier. The optimal value of a from the viewpoint of the auctioneer is less clear.

Ex post, the auctioneer would obviously prefer larger percentage bribes. However, larger percentage bribes induce the dishonest supplier to bid lower, and thereby reduce the probability that he needs to bribe in order to obtain the contract. To analyze the solution to this tradeoff, we can use the definitions of the function $x(c_h)$ obtained in subsection 5.1. Using these functions (indexed by a), we can express the expected bribery revenue of the auctioneer in terms of the percentage bribe.

(14) $$EB(a;t_d,t_h) = a\int_0^1\int_{x(c_h;a)}^{b_h(c_h)}[b_h(c_h)-c_d]\,dG_d(c_d)\,dG(c_h).$$

Notice that the derivative of this function with respect to a is,

(14) $$\frac{EB(a;t_d,t_h)}{a}-a\int_0^1\frac{\partial x(c_h;a)}{\partial a}[b_h(x(c_h;a))-x(c_h;a)]g_d(x(c_h;a))dG(c_h)$$

.

The first term is positive. It represents the gain obtained when bribes are still paid. However, the second is negative, and represents the loss from lower probabilities of receiving bribes. The following proposition shows that the tradeoff between these effects always has an interior solution.

Proposition 3. If $t_d < t_h$, then the percentage bribe which maximizes the auctioneer's revenues a^* is smaller than $\frac{t_d}{t_h} < 1$. If $t_d > t_h$ the revenue-maximizing a^* is smaller 1.

<u>Proof:</u> See Appendix.

The optimal a^* for the auctioneer may be very small. If t_d/t_h is small, a^* is clearly small. Indeed, in case where the dishonest supplier is not a serious rival to the honest supplier his behavior without bribery would be very aggressive, with bids very close to cost. This means that the scope for bribes is very limited. If the dishonest supplier bids above the honest supplier it is probably because his cost is above the bid of the honest supplier. In response, the auctioneer prefers a lower a in order to make the dishonest supplier less aggressive. This increases the probability of having surplus to share.

The optimal percentage bribe need not be large even when the cost distributions of the two suppliers are similar. When t_d = t_h = 1, we find that the optimal percentage bribe is approximately 13%.

6. Investment in Cost Reduction

Proposition 1 found that there exist circumstances in which the presence of bribery can actually result in a lower expected price paid by the buyer. This finding relied on the assumption that the cost distribution parameters, t_d and t_h, of the suppliers were fixed. As we mentioned before, we can interpret these parameters as units of capacity, and as such, they could be increased at a cost. In this section, we explore what would happen if the suppliers could choose these cost parameters or capacities before the auction. Bribery affects the expected profits and thus will affect the incentive to invest in capacity prior to the auction. Although it is difficult to infer marginal incentives to invest from the effect of bribery on the expected profits, we should expect that the incentive to invest will be lower for the honest supplier. For the dishonest supplier, the magnitude of the percentage bribe should affect the incentive to invest. We should expect that the incentive to invest will be lower when the dishonest supplier pays a larger percentage of the surplus in the bribe to the auctioneer. However, this intuition could be affected by the shifts in the equilibrium bidding function of the dishonest supplier which arise from changes in the percentage bribe.

Before examining the incentive to invest in the presence of bribery, we first calculate the aggregate investment for a social optimum. Let f be the cost of one unit of capacity. If each unit of capacity is a plant that can produce the good, then f is the fixed cost of constructing the plant. In order to maximize social welfare, the capacities of the suppliers should minimize the sum of the fixed costs and the production costs. The industry fixed costs are simply $f \cdot (t_d + t_h)$. With the family of distribution functions $G(c;t)$, the expected cost of producing the good can be expressed as $(1 + t_d + t_h)^{-1}$. Let the industry capacity be represented by $T = t_d + t_h$. The distribution of capacity among the two suppliers does not affect either the industry fixed costs or the industry production costs. Thus, the social optimum is obtained by minimizing $f \cdot T - (1+T)^{-1}$. The solution is $T^* = f^{-1/2} - 1$. Clearly, f must be less than one.

This efficient level of investment can be attained with second-price auctions. Indeed, assume the suppliers choose capacity in the first stage and then compete in a second-price auction to win the contract in the second stage. For the family of distribution functions $G(c;t)$, the expected profits of the supplier *d* can be expressed as

(16) $$E\Pi_d = \frac{1}{1+t_h} - \frac{1}{1+T} - f\, t_d\,.$$

Supplier *h* has the corresponding expression for expected profits.[8] The first-order condition for each supplier generates a unit-sloped reaction function with the form $t_d + t_h = f^{-1/2} - 1$. Thus, there is a multiplicity of equilibria. However, all the equilibria have the same efficient industry capacity, $t_d + t_h = T^*$. With this background, we can now consider the incentives to invest by the suppliers in the first-price auction with bribery.

The expected profits of the honest supplier are more tractable than the expected profits of the dishonest supplier because the honest supplier only earns profits when his bid is lower that the cost realization of the dishonest supplier. Including the fixed capacity costs, the expected profits of the honest supplier can be expressed as

(17) $$E\Pi_h = \frac{1}{1+t_d}\left(\frac{t_d}{1+t_d}\right)^{t_d} \frac{t_h}{1+T} - f\, t_h\,.$$

The expected profits of the honest supplier are independent of the percentage bribe again. The reaction function of the honest supplier can be obtained from the first-order condition for maximizing the expected profits, given the capacity of the dishonest supplier. This reaction function takes the following form:

(18) $$t_h = \left(\frac{t_d}{1+t_d}\right)^{t_d/2} f^{-1/2} - (1+t_d)$$

It is straightforward to show that this reaction function is downward sloping. Thus, the honest supplier acquires less capacity when the dishonest supplier has a greater capacity. Clearly, the fixed costs f must be sufficiently less than one in order to ensure that the honest supplier acquires some capacity. The reaction function of the honest supplier answers one of the natural questions about bribery. We state the following proposition.

Proposition 4: Bribery results in a lower level of cost-reducing investment for the industry than does the social optimum (or the second-price auction without bribery).

Proof: Upon rearranging the reaction function of the honest supplier, we find that the optimal response of the honest supplier would ensure that the sum of his capacity and that of the dishonest supplier is $\left(\frac{t_d}{1+t_d}\right)^{t_d/2} f^{-1/2} - 1$. Thus, irrespective of the capacity choice of the dishonest supplier, the profit-maximizing response of the honest supplier will prevent the industry from reaching the socially optimal investment. Q.E.D.

This proposition is interesting because it confirms that bribery in this model undermines the aggregate incentive to invest in lower cost technologies. This investment effect is probably more important than the static price effects of bribery. Lower industry investment increases the expected cost of the good. As we pointed out, the expected cost is simply $(1+T)^{-1}$. Depending on the distribution of that capacity among the suppliers, this higher expected cost would generally result in higher prices to the buyer.

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[8] The general derivation of the expected profits of asymmetric suppliers using this family of distributions can be found in Perry and Waehrer (1998).

The expected profits of the dishonest supplier are much more complicated functions of the capacity levels and the percentage bribe. We know that lower percentage bribes result in higher incentives for cost reduction. For any decrease in cost, the dishonest supplier retains a higher surplus. We can illustrate the impact of the percentage bribe on the incentive to invest by the dishonest supplier. Consider the symmetric case in which the two suppliers have a capacity of 1, that is, they each have one plant. After differentiating the expected profits of the dishonest supplier with respect to t_d and simplifying, we find that

(19)
$$\frac{\partial E\Pi_d}{\partial t_d} = \frac{\alpha^2}{18(1+\alpha)} + \frac{(1-\alpha)\left[(19/6) + Ln(1/2)\right]}{24} - f \ .$$

This expression is the marginal return to an additional unit of capacity. It is easy to show that this marginal return is decreasing in α, takes a positive value at $\alpha = 0$, and is negative at $\alpha = 1$.

In order for the unit capacities to be an equilibrium, the values of f and α must be such that first-order conditions for both suppliers are satisfied. This is feasible because the value of α does not affect the first-order condition of the honest supplier. Thus, f can be set to satisfy the first-order condition of the honest supplier ($f = \frac{1}{18}$), while α can then be set to satisfy the first-order condition of the dishonest supplier ($\alpha = \sqrt{1 + \frac{1}{3(13/6 + Ln(1/2))}} - 1$). With these parameters, the social optimum industry capacity is slightly above 3. Thus, we have constructed an equilibrium with identical capacities for the two suppliers, the sum of which is less than the social optimum.

If α is lower, then the dishonest supplier will have an incentive to acquire more than one unit of capacity. The reaction function of the honest supplier in (18) implies that he would then acquire less than one unit of capacity. Thus, a small percentage bribe or favoritism will generate an equilibrium in which the dishonest supplier acquires a larger capacity than the honest supplier. Conversely, a large percentage bribe will generate an equilibrium in which the dishonest supplier acquires a smaller capacity than the honest supplier. Although these results are confined to this example, their implications merit the following summary remark.

Remark: Favoritism and small percentage bribes can encourage the dishonest supplier to invest in cost-reducing capacity more than the honest supplier, rendering this supplier more competitive. Large percentage bribes discourage such investments.

One of the natural applications of this model in international economics is the setting in which the dishonest supplier is the domestic firm with access to the domestic political process. In this situation, the honest supplier is the foreign firm with no such access or with specific legal prohibitions against bribery of foreign officials. The remark implies that favoritism to domestic suppliers can encourage investment in cost reducing capacity. Contrary to some traditional arguments, favoritism creates a dynamic inefficiency in this model, but the dishonest supplier, who is partially protected from competition, may now increase his investment in cost reduction. The same conclusion also applies to bribery environments with low percentage bribes.

7. Discussion of Alternative Modeling of Bribes in Sealed-Bid Auctions

Modeling the bribery process requires making choices as to how game theory can be used to represent a complex bargaining situations with implicit understandings and ill-defined exchanges. In this section, we will discuss the choices we have made to model bribery in auctions and their alternatives.

First, we have tried to avoid a definition of bribery that simply converts an honest auction to supply products to the buyer into an “honest” auction to supply products to the auctioneer who then resells the products to the buyer. This would occur if the auctioneer had enough power to make all parties abide by agreements prior to the auction. Even without assuming such power, the bribe could still take several forms.

In this paper, we assumed that the bribe is a percentage of the surplus between the low bid of the honest supplier and the cost of the dishonest supplier. The percentage bribe is more convenient because bribery will occur whenever the surplus is positive, no matter how small, which simplifies the analysis.[9] This model can be thought of as representing ex post bargaining between the auctioneer and the dishonest supplier about the division of the surplus. The magnitude of the percentage bribe would then represent the bargaining power of the auctioneer. An alternative would be to model the bribe as a fixed amount that the dishonest supplier must pay to the auctioneer irrespective of the low bid by the honest supplier, the cost realization of dishonest supplier, and thus magnitude of the surplus. The justification for such a "fixed bribe" is that the auctioneer may face potential penalties from accepting a bribe and that these penalties are independent of the amount of the bribe. Thus, there is a minimum bribe that warrants bearing the risks of incurring penalties for accepting the bribe. With a fixed bribe, the cost realization of the dishonest supplier must be sufficiently below the bid of the honest supplier in order to generate a surplus that exceeds the minimum fixed bribe acceptable to the auctioneer.

If we assumed that the fixed bribe is β, the honest supplier would become less aggressive. The honest supplier no longer has to underbid the cost of the dishonest supplier, but instead only needs to underbid the cost plus the fixed bribe. This shifts his bidding function upwards. On the other hand, the dishonest bidder faces a new lower bound in his bid. Indeed, the dishonest bidder never bids below the value *b* that solves

$$\frac{1-G_h(b_h^{-1}(b))}{g_h(b_h^{-1}(b))\dfrac{db_h^{-1}(b)}{db}} = \boldsymbol{b}\,. \tag{20}$$

For higher values of the cost, the dishonest supplier faces the same tradeoff as in (5) if we set $\boldsymbol{a}$ = 1, except that the bid of the honest supplier is shifted. This results in less aggressive behavior than in our most aggressive case ($\boldsymbol{a}$ = 1). However, for low values of the fixed bribe *ß* and sufficient asymmetry in favor of the honest supplier (in our family of distributions, $\frac{t_d}{1+t_d} < t_h \boldsymbol{b}$), the new lower bound does not bind and the bidding behavior of both suppliers converges to our

[9] One justification for the percentage bribe is that the auctioneer receives an equity interest in the dishonest supplier in return for awarding the contract to the dishonest supplier at the lower price bid by the honest supplier. Historically, this is not an uncommon form of bribery.

most competitive case (a = 1). Thus, our results should still hold, including the surprising results in Proposition 1 (i).

Another alternative to our model of bribery is to assume that the dishonest supplier can also bribe when he is the high bidder. Indeed, even when the dishonest supplier submits the lowest bid, there is some extra surplus that the auctioneer and the dishonest can appropriate by allowing the dishonest supplier to raise his bid ex post up to the bid of the honest supplier. Notice that this creates greater latitude for the auctioneer. The bribe in our model was seemingly innocuous for the buyer because the auctioneer could not change the price of the contract, only who was awarded the contract. In contrast, it is more serious to allow the auctioneer to manipulate the auction so that the final price could exceed the low bid and equal the high bid. We now discuss the consequences of this higher degree of latitude for the auctioneer.

Assume that, if the honest supplier has bid b_h and the dishonest supplier has bid $b_d < b_h$, the auctioneer would allow the dishonest supplier to revise his bid upward to (almost) b_h in exchange of a payment $a(b_h - b_d)$. This would not change the behavior of the honest supplier. He would still win only when his bid is below the cost of the dishonest supplier. The profit for the dishonest supplier would be $(1-a)(b_h - c)$ if his bid is above that of the honest supplier (but his cost is below this value), and $(1-a)(b_h - b_d) + (b_d - c)$ if his bid is below that of the honest supplier. In other words, the dishonest supplier makes $(1-a)(b_h - c)$ irrespective of how his bid compares to that of the honest supplier, and makes additional $a(b_d - c)$ profits if his bid is below that of the honest supplier (provided his cost is below the bid of the honest supplier). Notice that the first term is independent of his bid, as is the probability of his cost being below the bid of the honest supplier. Then his bid determines whether he makes $a(b_d - c)$ by winning the contract honestly or not. That is the same tradeoff (multiplied by a constant) that the dishonest supplier faces in our original model when $a = 1$. The optimal bid is therefore the same, as is the allocation of the contract. Indeed, the problem faced by the dishonest supplier with this two-way bribing is

$$\max_b \Pi_d\left[b;c,b_h(\cdot)\right]=$$

(21)
$$\int_{b_h^{-1}(b)}^{1}\left[(1-\alpha)b_h(x)+\alpha b-c\right]dG_h(x)+\int_{b_h^{-1}(c)}^{b_h^{-1}(b)}(1-\alpha)(b_h(x)-c)dG_h(x)\equiv,$$

$$\int_{b_h^{-1}(c)}^{1}(1-\alpha)\left(b_h(x)-c\right)dG_h(x)+\alpha(1-G_h(b_h^{-1}(b))(b-c)$$

and since the first term in the right-hand side is a constant in b, (21) is equivalent to (4) when $\alpha=1$. The behavior of the suppliers is then the same as one case of our original model. An important difference, however, is that the price is now always the bid of the honest supplier. Thus, the expected price paid by the buyer would always be higher under bribery.

One other modeling issue that we want to address is our assumption that the auctioneer knows the cost of the dishonest supplier. This assumption may be reasonable in a situation where the dishonest supplier and the auctioneer have a long-run relationship. This is consistent with our differentiation between the honest and dishonest suppliers. Nevertheless, one can criticize this assumption as extreme, and ask what happens if the auctioneer has to infer the cost of the dishonest supplier from the bidding behavior. Thus, assume the auctioneer does not know the cost of the dishonest supplier. If there is still a strictly monotone, pure strategy equilibrium bidding function for the dishonest supplier, then the auctioneer can still infer this cost from the bid. Thus, when bargaining over the bribe, the cost of the dishonest supplier is already common knowledge. Of course, the incentives for the dishonest supplier when deciding his bid are now different. In particular, they include the signaling aspect of his bid. Indeed, in equilibrium, the auctioneer would postulate a bidding function for the dishonest supplier, $b(c)$, and after observing a bid b of the dishonest supplier, ask for a bribe equal to $\alpha(b_h - b^{-1}(b))$. Thus, the problem faced by the dishonest supplier would be to choose the best cost report, that is,

$$\max_z \Pi_d\left[z;c,b_h(\cdot)\right]=$$

(22)
$$(b(z)-c)\left[1-G_h(b_h^{-1}(b(z)))\right]+\int_{b_h^{-1}(z)}^{b_h^{-1}(b(z))}\left[(b_h(x)-c)-\alpha(b_h(x)-z)\right]g_h(x)dx.$$

For $b(c)$ to be the equilibrium bidding function, truthful reporting should be the best choice. This generates the following differential equation as the necessary condition for equilibrium

(23) $$b'(c)\left\{\left[1-G_h(b_h^{-1}(b(c)))\right]-(b(c)-c)\alpha g_h(b_h^{-1}(b(c)))\frac{db_h^{-1}(b(c))}{db}\right\}$$
$$+\alpha\left[G_h(b_h^{-1}(b(c)))-G_h(b_h^{-1}(c))\right]=0.$$

A fully revealing equilibrium should be a strictly monotone solution of this differential equation with $b(1)=1$. For the uniform case (and $\alpha = 1$), this equilibrium exists and is represented in Figure 4 below.

Notice that the new signaling effect, represented by the last term in (23), is positive. When evaluated in the equilibrium bid when costs are known to the auctioneer, the incentive for the dishonest supplier is to raise his bid in order to reduce the bribe paid when a bribe is needed. This is what we should expect. This raises an existence problem because this effect may be sufficiently strong so that every type (or at least high enough types) bids 1. If so, a fully revealing equilibrium would not exist.

7. Concluding remarks

We have analyzed a model of bribery in competitive procurement. One of the suppliers can revise his bid by bribing the auctioneer. Once the bids are submitted, the auctioneer lets this dishonest supplier know what the bid of the other supplier has been. In exchange the auctioneer receives a percentage of the surplus generated. The dishonest supplier still submits meaningful bids since winning the auction outright avoids paying bribe.

We have characterized the behavior of the suppliers in this setting. The chance of bribing and winning the contract even after submitting a losing bid could be expected to induce less aggressive behavior for the dishonest supplier. Similarly, we might expect that the honest supplier would bid more aggressively because he must bid below the cost of the dishonest supplier to eliminate the surplus, prevent bribery, and win the contract. We have shown that this is not true in general. The honest supplier bids against the cost of the dishonest supplier, which may be steeper than his bidding function, the standard target in an honest auction. Similarly, bribery does not necessarily make the dishonest supplier bid less aggressively.

Bribery distorts the allocation in favor of the dishonest supplier. This occurs whether or not the honest supplier's cost distribution stochastically dominates that of the dishonest supplier. This contrasts with what happens in an honest second-price auction, where the allocation is efficient. But it also contrasts with the result in a first-price auction when the honest supplier is the weaker bidder. In a first-price auction, the strong bidder bids less aggressively than the weaker bidder, and the allocative distortion favors the weaker bidder. This is also true for optimal mechanisms from the buyer's viewpoint.

The effect of this distortion and the bribe on the expected price paid by the buyer is ambiguous. As a consequence of bribery, the expected price can be lower compared to both the price in a second-price auction or an honest first-price auction. This can occur even if the dishonest supplier is the stronger bidder, in terms of stochastic dominance of his cost distribution. In fact, for the family of distributions considered, the dishonest supplier must be the stronger bidder in order for a reduction in the expected price to follow from bribery.

Finally, we have analyzed how the relative ex post bargaining power (bribe percentage) of the auctioneer and dishonest supplier affect the payoffs of all parties. The buyer prefers that the auctioneer has all the bargaining power. This makes the dishonest supplier more aggressive in the auction. The dishonest supplier has exactly the opposite interest. Although ex post the auctioneer would prefer to have all the bargaining power, he would choose an intermediate degree of bargaining power ex ante, which maximizes his revenues in light of the equilibrium bidding strategy of the dishonest supplier.

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Appendix

Proof of Proposition 1: Using the results in the previous section, we can calculate the expected price paid by the buyer as

$$(24a)\qquad Ep(\alpha;t_d,t_h)=\frac{1+t_d+t_h}{(1+t_d)(1+t_h)}-\frac{t_d}{(1+t_h)(1+t_d+t_h)}\left[\frac{1+t_d}{t_d}\right]^{t_h}\left[\frac{\alpha\cdot t_h}{1+\alpha\cdot t_h}\right]^{1+t_h}$$

$$\text{for}\quad \alpha\le\frac{t_d}{t_h}\ \text{(Case 1)},$$

$$(24b)\qquad Ep(\alpha,t_d,t_h)=\frac{1}{(1+t_d)}+\frac{t_d}{(1+t_d)(1+t_d+t_h)}\left[\frac{t_d}{1+t_d}\right]^{t_d}\left[\frac{1+\alpha\cdot t_h}{\alpha\cdot t_h}\right]^{t_d}$$

$$\text{for}\quad \alpha\ge\frac{t_d}{t_h}\ \text{(Case 2)}.$$

The expected price in a second-price auction is very easy to calculate for asymmetric cases using this specification for the distribution of costs:

$$(25)\qquad Ep_2(t_d,t_h)=\frac{1+t_d+t_h}{(1+t_d)(1+t_h)}-\frac{t_d\,t_h}{(1+t_d)(1+t_h)(1+t_d+t_h)}.$$

(i) When $t_d > t_h$, expression (24a) for the expected price applies over the entire range of the percentage bribe of $\alpha \le 1$. This expected price is continuous and declines as α increases over this range. At $\alpha = 0$, this expected price with bribery is clearly greater than the expected price under a second-price auction without bribery from (25). However, at $\alpha = 1$, the condition that $t_d > t_h$ implies that this expected price with bribery is lower than the expected price under a second-price auction without bribery. Thus, there exists a $\bar{\alpha}$ such that for $\alpha > \bar{\alpha}$, bribery would result in a lower expected price.

(ii) When $t_d < t_h$, expression (24a) for the expected price applies only for $\alpha \le t_d/t_h$. But under the condition that $t_d < t_h$, this expected price is greater that the expected price under a second-price auction without bribery from (25) at $\alpha = t_d/t_h$, and thus for all $\alpha \le t_d/t_h$.

When $t_d = t_h$, expression (24a) applies for all $\alpha \le 1$, exceeds the expected price without bribery for $\alpha < 1$, and equals the expected price without bribery for $\alpha = 1$. Q.E.D.

<u>Proof of Proposition 3</u>: If $t_d < t_h$, then $x(c_h)$ is always positive. Thus

$$\frac{\partial}{\partial \alpha}\left[\alpha \int_0^1 \int_{x(c_h;\alpha)}^{b_h(c_h)} [b_h(c_h) - c_d] dG_d(c_d)\, dG(c_h)\right] =$$

(26)

$$-\frac{t_d\left(\frac{t_d}{1+t_d}\right)^{td}}{\alpha(1+t_d)^2(1+t_d+t_h)(1+\alpha t_h)}\left(-(1+\alpha t_h)\alpha t_h + \left(\frac{1+\alpha t_h}{\alpha t_h}\right)^{t_d}\left(\alpha t_h(\alpha t_h - t_d) + \alpha t_h + t_d^{\;2}\right)\right)$$

The expression in brackets is concave in αt_h, so we only need to check the sign at $\alpha t_h = t_d$ and $\alpha t_h \rightarrow \infty$. In the first case, the expression takes the value $\left[\left(\frac{\alpha t_h}{1+\alpha t_h}\right)^{\alpha t_h} - 1\right](1+\alpha t_h)\alpha t_h > 0$, and in the second it converges to $\frac{1+t_d}{2} t_d > 0$. Thus, (26) is negative and we can conclude that the expected bribe is decreasing in α in that region.

If $t_d > t_h$, then $x(c_h)$ is zero when for $c_h > \dfrac{t_d - \alpha t_h}{t_d(1+\alpha \cdot t_h)}$. Thus

$$\frac{\partial}{\partial \alpha}\left[\alpha \int_0^1 \int_{x(c_h;\alpha)}^{b_h(c_h)} [b_h(c_h) - c_d] dG_d(c_d)\, dG(c_h)\right]\Bigg|_{\alpha=1} = \frac{\left(\frac{t_h}{1+t_h}\frac{1+t_d}{t_d}\right)^{th}}{(1+t_d)^2(1+t_d+t_h)(1+t_h)^2}$$

$$\left(-(1+t_d)^2(1+3t_h) + (1+t_h)\left(\frac{1+t_h}{t_h}\frac{t_d}{1+t_d}\right)^{t_h}\left((1+t_d)(1+t_d+t_h) + t_h(1+t_h)\left(\frac{t_d}{1+t_d}\right)^{t_d}\right)\right)$$

which is negative if

$$-(1+t_d)^2(1+3t_h) + (1+t_h)\left(\frac{1+t_h}{t_h}\frac{t_d}{1+t_d}\right)^{t_h}\left((1+t_d)(1+t_d+t_h) + t_h(1+t_h)\left(\frac{t_d}{1+t_d}\right)^{t_d}\right) < 0.$$

Notice that $(1+t_h)\left(\frac{1+t_h}{t_h}\frac{t_d}{1+t_d}\right)^{t_h}$ is increasing in t_h, and $\left(\frac{t_d}{1+t_d}\right) < 1$. Thus, the left-hand side above, for $t_h < t_d$, is smaller than

$$-(1+t_d)^2(1+3t_h) + (1+t_h)\big((1+t_d)(1+t_d+t_h) + t_h(1+t_h)\big) < 0.$$

Therefore, $\alpha = 1$ is not the optimal percentage bribe. Q.E.D.

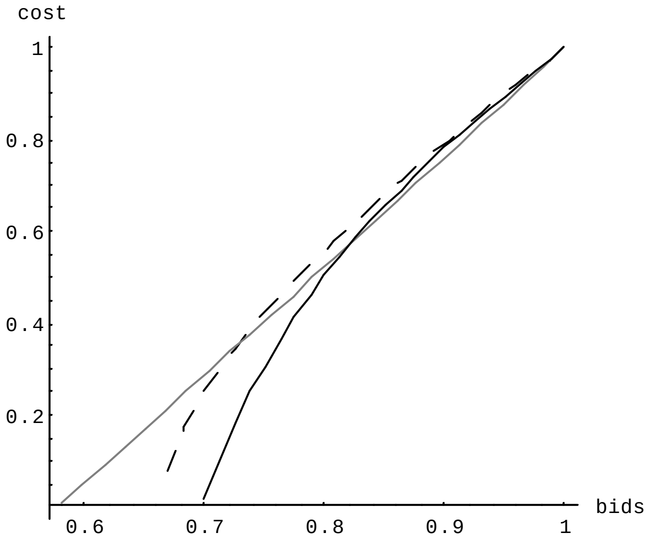

Figure 1

Bidding functions when $G_h(c) = G_d(c) = c^2$

no bribery: dashed curve

honest supplier: darker curve

dishonest supplier: lighter line

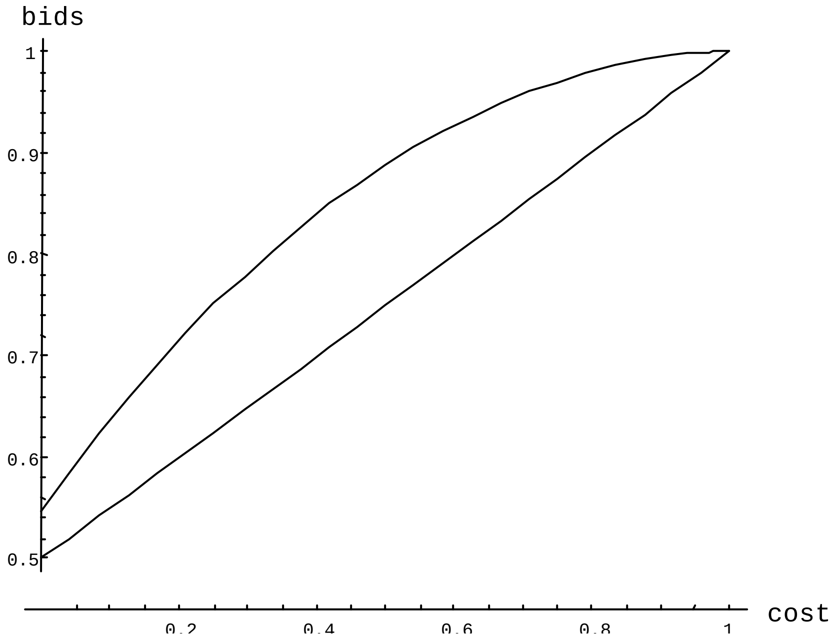

Figure 4

Bidding functions when $F_h(c) = F_d(c) = c$, $\alpha = 1$

Honest bidder: linear curve

Dishonest bidder: concave curve