Direct Preference for Wealth in Aggregate Household Portfolios*

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Abstract

According to standard theory, wealth should have no intrinsic value. Yet, conventional wisdom, recent theories, and data suggest it might. We verify whether or not households have direct preferences over wealth in selecting assets. The fully structural econometric model focuses on a multivariate Brownian motion in optimal consumption, portfolios and wealth. Using aggregate portfolio data, we find that wealth (i) is directly valued, (ii) reduces marginal utility and (iii) reduces risk aversion, while we reject the HARA, and CRRA restrictions. Consequently, wealth-dependent utility generates a larger IMRS risk, justifying a larger, more predictable risk premium and a lower risk-free rate.

JEL classification: G11, G12.

Keywords: Portfolio choice, Wealth-dependent preferences, Preference for status, Asset pricing, Equity premium, Risk-free rate, Predictability.

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Introduction

Does wealth have intrinsic value to investors? According to standard theory, it shouldn't. Wealth is a mere instrument with which current and future consumer goods are acquired. Utility is defined over these goods only, not over the means through which they are obtained.[1] In this light, wealth can only have an *indirect* effect on portfolio decisions. Given current endowment, consumption paths are chosen optimally; portfolios are a mean to support this optimal path. All the main determinants of asset holdings, including risk aversion or time substitutability, depend on wealth only through its indirect impact on consumption.

Yet, conventional wisdom suggests that net worth might carry additional virtues (see Smith, 1759; Veblen, 1899, for early discussions). Wealth – particularly tangible wealth – encompasses undeniable conspicuous consumption characteristics. People do enjoy nice cars, houses, furniture and jewelry in part for the services they yield, but also for the social status that is attached to their ownership (Robson, 1992; Bakshi and Chen, 1996, refer to this as a "capitalistic spirit"). Alternative theories contend that if (i) non-marketed goods are valuable, (ii) status is a ranking mechanism that determines the allocation of these goods, and (iii) wealth is a metric that determines status, then wealth-dependent utility can be thought of as resulting from these effects. Examples of such non-marketed goods include country club memberships, or invitations to charity events, or even the quality of potential partners in matching games (Cole et al., 1992, 1995; Corneo and Jeanne, 1999).

From a theoretical perspective, direct preference for wealth is an element of state-contingent preferences. These preferences imply that the state – in this case wealth – *directly* affects risk aversion and time substitutability, in addition to its indirect effect discussed earlier. In static terms, this should reinforce the links between wealth and tolerance to risk and/or time substitutability; cross-sectional evidence on portfolios is consistent with this conjecture. For example, it is well known that participation in risky asset markets is mainly limited to a relatively small share of individuals (Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995; Guvenen, 2003b) who appear to have different preferences compared to the average agent. Empirical evidence is consistent with stockholders (i) being wealthier (Poterba, 2000; Aït-Sahalia et al., 2004; Reynard, 2004), and having (ii) a larger elasticity of inter-temporal substitution (Vissing-Jørgensen, 2002; Guvenen, 2003b) and (iii) a lower degree of risk aversion (Mankiw and Zeldes, 1991; Attanasio et al., 2002; Brav et al., 2002).[2]

From a dynamic perspective, state-contingent preferences imply that portfolios will contain dynamic hedging strategies against unfavorable shifts in that state. These strategic portfolios are qualitatively similar to those obtained under time-varying investment sets, and result in time-varying portfolio shares (Cox et al., 1985; Merton, 1990; Lo and Wang, 2001). This time variation in portfolio is a salient feature of the data. Figure 1 plots the aggregate portfolio shares of financial wealth (i.e. cash + stocks + bonds) invested in cash, bonds and stocks for American households and non-profit organizations.[3] To highlight cyclical properties, we also plot the NBER Experimental Recession Index (XRI), a counter-cyclical indicator (right hand side axis). Two observations immediately stand out: (i) portfolio shares of financial wealth are not constant, and (ii) they exhibit strong cyclical patterns. In particular, cash and bonds are counter-cyclical, whereas corporate stocks are pro-cyclical.[4] The formers increase, whereas the latter decreases when there is a high likelihood of recession.

[Insert Figure 1 here]

An intuitively-appealing interpretation is that risk aversion is higher during downturns and that agents react by shifting assets away from risky stocks. This counter-cyclical risk aversion conjecture is confirmed by the pricing literature showing that conditional excess returns on corporate equity are also counter-cyclical: high during recessions and falling during recoveries (Cochrane, 1997; Guvenen, 2003a). From a C-CAPM perspective, because the quantities of consumption risk display poor cyclicality, this implies that the price of that risk is counter-cyclical (Campbell and Cochrane, 1999b; Gordon and St-Amour, 2000, 2004; Melino and Yiang, 2003). Consequently risk aversion is increasing in recessions and decreasing in recoveries.

In our setup, the main element behind these shifts in risk aversion would have to be wealth. Direct impacts of wealth on risk aversion would augment indirect impacts obtained through consumption. Again, this interpretation fits apparently well with the data; compared to consumption, movements in wealth hold more promise in generating corresponding movements in risk aversion. To see this, consider (de-trended) consumption, and financial wealth and (untransformed) consumption shares of wealth plotted in Figure 2, again with the XRI index.[5] The data suggests that (i) wealth is strongly pro-cyclical whereas (ii) consumption has comparatively negligible cyclical component, such that (iii) the consumption share of wealth is strongly counter-cyclical. A high likelihood of a recession is associated with marked declines in wealth, and almost no reduction in consumption, pointing out to considerable smoothing of consumption by agents. Unsurprisingly, the consumption share increases during downturns.

[Insert Figure 2 here]

Based on these observations, our objectives are (i) to develop a model of direct preferences for wealth, (ii) characterize its implications for risk aversion, optimal portfolio and consumption and (iii) to test these predictions using the aggregate portfolio, wealth and consumption data just presented. Finally, although the main emphasis of this paper is in goods space, we (iv) discuss the pricing implications of the model in terms of addressing the main anomalies of the C-CAPM.

In the spirit of the habit literature we focus on time-varying minimum admissible consumption, or (marginal utility) bliss level (Sundaresan, 1989; Constantinides, 1990; Ferson and Constantinides, 1991; Campbell and Cochrane, 1999a). However, contrary to standard habit models, we let bliss be determined by wealth, rather than by lagged consumption. As Munro and Sugden (2003) argue, habit models capture reference-dependent preferences. Past decisions determine the value of a reference point, and deviations about this point determine utility. The reference can be a function of past consumption (as is the case under habit), or endowment (as is the case in this paper). Munro and Sugden (2003, p. 420) contend that the second case is more appropriate in a dynamic framework, and in particular, when buying assets. Wealth-induced movements in bliss are important to the extent that they result in *rotations* (rather than shifts) in the marginal utility schedule.[6] These rotations affect both marginal utility of consumption and consumption risk aversion. In comparison, multiplicatively separable models of direct preferences for wealth (Bakshi and Chen, 1996; Futagami and Shibata, 1998; Gong and Zou, 2002) may only affect the former; additively separable models (Corneo and Jeanne, 2001; Kuznitz and Kandel, 2003; Aït-Sahalia et al., 2004) affect neither.

One possibility is that higher wealth increases what is considered as basic minimum by the agent. In this case, given consumption, a wealthier agent is closer to bliss, has higher marginal utility and is more risk averse. We will refer to this case as a *ratchet* investor. A second possibility is that bliss falls in wealth. In this case, any given consumption is further away from basic minimum as wealth increases. A wealthier agent both gains less marginal satisfaction from a given consumption and is less reluctant to take risk. We will refer to this case as a *blasé* investor.

In the absence of strong priors as to which of the two effects is dominant, we take the model to the data. Standard preference-based approaches to asset markets typically characterize and test the models' asset pricing implications. We depart from these by imposing the full theoretical restrictions from the optimal allocations on a continuous-time econometric model and testing that model over the aggregate household portfolio data plotted in Figures 1–2 instead. Specifically, we characterize the closed-form solutions for optimal consumption, and portfolio in a continuous-time setting. Substituting these rules in the budget constraint yields a multivariate Brownian motion in consumption, asset holdings and wealth. This fully structural econometric model presents estimation challenges as both drifts, diffusions, as well as off-diagonal scedastic elements are (i) constrained by the model and (ii) depending on the state. Fortunately, we show how a suitable identification strategy tackles the first problem while a change of variables addresses the second one.

Our earlier discussion suggests that the *blasé* investor case is more likely given that wealthier agents are less risk averse, that wealth was quite pro-cyclical, and that risk aversion was possibly counter-cyclical. Our empirical findings confirm this intuition. We find that wealth has significantly positive independent value to investors. A consequence is that bliss falls in wealth, as does risk aversion. Moreover, the null hypotheses of alternative preference models, such as iso-elastic (CRRA) or hyperbolic (HARA) preferences are rejected when tested against our wealth-dependent utility alternative.

Because wealth was found to be more volatile and cyclical than consumption, this entails a larger volatility and cyclicality of the marginal utility. From a pricing perspective, this additional co-movement is a welcomed addition to the smooth consumption series in a C-CAPM pricing kernel. We show that the premia entails a second source of risk (in addition to consumption risk), that this risk is the market (a result also found for non-expected utility models, e.g. Duffie and Epstein, 1992), and that its price has the intuitive interpretation of being risk aversion with respect to *wealth*. As market risk is quantitatively more important than consumption risk, a large observed premia can be justified by direct preference over wealth. Our model has therefore a potential to successfully address the equity premium puzzle. Moreover, movements in risk aversion just described are useful to address the predictability puzzle concerning the time variation in conditional excess returns. Even if consumption and wealth covariances are conditionally homoscedastic, counter-cyclical risk aversion will result in counter-cyclical premia. Finally, the large volatility of marginal utility warrants a larger precautionary demand for the risk-free asset, thereby justifying the low risk-free rate.

The rest of this paper is organized as follows. We outline the model, and the closed-form solutions in Section 1. Next, we introduce the empirical methods in Section 2, and present the estimation results in Section 3. We discuss the pricing implications of these results in Section 4, before concluding.

1 Model

This section outlines the model. We subsequently characterize optimal consumption and asset holdings. Finally, we obtain the closed-form expressions for the differential equations governing consumption, asset values and wealth.

A Economic environment and preferences

In order to emphasize the role of our alternative preference specification, we consider a complete-markets and representative-agent framework similar that studied by Merton (1971) or by Lucas (1978). The stochastic environment is characterized by continuous information with filtration on $\boldsymbol{Z}_t \in \mathbb{R}^n$, a standard Brownian motion. The investment set consists of n risky securities and one risk-less asset. Denote by $\boldsymbol{\mu}_{p,t} \in \mathbb{R}^n$ and by $\boldsymbol{\sigma}_{p,t} \in \mathbb{R}^{n \times n}$ the adapted drift and diffusion processes for the risky returns, and by $r_t \in \mathbb{R}$ the short rate process. We start by imposing a constant set restriction, i.e. $\boldsymbol{\mu}_{p,t} = \boldsymbol{\mu}_p, \boldsymbol{\sigma}_{i,t} = \boldsymbol{\sigma}_i, \text{and}, r_t = r, \forall i, t$. This assumption is relaxed later when we discuss pricing implications.

The representative agent's objective is to select consumption C_t and portfolio weights $\boldsymbol{v}_t \in \mathbb{R}^n$ so as to maximize VNM utility characterized by direct preference over wealth W_t:

$$\max_{\{C_t, \boldsymbol{v}_t\}_t} \mathrm{E}_0 \int_0^\infty \exp(-\rho t) U(C_t, W_t) dt, \tag{1}$$

subject to

$$dW_t = \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + r]W_t - C_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \tag{2}$$

where E_0 is a conditional expectations operator, and $\rho > 0$ is a subjective discount rate. The agent's within-period utility is given by:

$$U(C, W) = \begin{cases} \frac{(\eta_c C + \eta_0 + \eta_w W)^{1-\gamma}}{1-\gamma}, & \text{if} \gamma \neq 1; \\ \log(\eta_c C + \eta_0 + \eta_w W), & \text{if} \gamma = 1. \end{cases} \tag{3}$$

Utility (3) belongs to the HARA class advocated by Rubinstein (1974), modified to allow for wealth dependence. Following Merton (1990, p. 137), the necessary HARA restrictions are:

$$\eta_c > 0, \quad \eta_c C + \eta_0 + \eta_w W > 0, \quad \gamma \geq 0. \tag{4}$$

These are required to guarantee monotonicity and concavity. To these, we add a further theoretical restriction that bounds below and above the term η_w:

$$-1 < \eta_w / \eta_c < \rho. \tag{5}$$

This condition allows for negative or positive values of the loading of wealth in the utility function, η_w, but limits the size of the effect.[7]

Subject to restrictions (4) and (5), the utility function (3) has interesting properties. The expression $C_{\text{bliss},t} \equiv -(\eta_0 + \eta_w W_t)/\eta_c$ has the interpretation of a fictitious reference, or bliss level of consumption, where bliss is defined with respect to marginal, as opposed to the level of, utility.[8] More precisely, as consumption falls toward bliss, marginal utility goes to infinity, such that $C_{\text{bliss},t}$ is the minimum admissible consumption level. In addition, under WDU, the bliss level changes because of changes in wealth. In contrast, slow-moving habit or durability models let the reference level be a function of past consumption, whether individual, or aggregate. Finally, CRRA utility and HARA utility fix the bliss consumption to 0 and $-\eta_0/\eta_c$ respectively, both state-independent levels.

The marginal utility, and the Arrow-Pratt coefficient of absolute risk aversion (calculated with respect to consumption and wealth) are respectively:

$$U_x = \frac{\eta_x}{(\eta_c C + \eta_0 + \eta_w W)^{\gamma}}, \tag{6}$$

$$R^{ax} \equiv \frac{-U_{xx}}{U_x} = \frac{\gamma \eta_x}{(\eta_c C + \eta_0 + \eta_w W)}, \quad x = c, w. \tag{7}$$

To understand the direct impact of wealth on these variables, consider the effect on marginal utility of consumption following an increase in W. As shown in Figure 3, when $\eta_w < 0$, as wealth increases, so does the minimum level of consumption from $-\eta_w W_0$, to $-\eta_w W_1$. Hence, a negative wealth dependence involves a ratchet effect whereby the bliss consumption level increases in wealth. This leads to a clockwise rotation in the marginal utility schedule from $U_{c,0}$ to $U_{c,1}$, and increases marginal utility from a to b. Put differently, as wealth increases, the agent approaches his reference consumption, and becomes more averse toward consumption risk.

[Insert Figure 3 here]

Next, these movements in marginal utility and risk aversion are reversed for $\eta_w > 0$, as shown in Figure 4. An increase in wealth now reduces minimum admissible consumption from $-\eta_w W_0$, to $-\eta_w W_1$. This causes a counter-clockwise rotation in the marginal utility schedule from $U_{c,0}$ to $U_{c,1}$, and a reduction in the marginal utility of consuming the same level of nondurable good falls from a to b. This effect could be related to blasé behavior; as the investor becomes richer, for a given level of consumption, both marginal utility and consumption risk aversion fall.

`[ Insert Figure 4 here ]`

The utility function (3) can also be thought of as a linear habit model where the habit stock is defined to be a function of current wealth:

$$W_t = \frac{1}{\pi_t} E_t \int_t^{\infty} \pi_s C_s ds, \tag{8}$$

where π_t is a state-price density. In this perspective, instantaneous utility is not only a function of current consumption, but also of the future consumption paths that current wealth could support (see Kuznitz and Kandel, 2003, for a discussion). These paths determine the benchmark through which current consumption scenarios are evaluated.[9] As is shown next, this habit stock interpretation of (3) considerably simplifies the solution to the agent's problem.

B Optimal Consumption and Portfolio Rules

The agent's problem (1) could be solved using standard dynamic programming methods. It turns out however that a simpler alternative is available. We mentioned earlier that the preferences (3) belong to the linear habit class where the habit stock is defined to be current wealth. Schroder and Skiadas (2002) show that closed-form expressions for linear habit models (the primal problem) are conveniently obtained by simple modifications to the standard solutions in models without habit (the dual problem). Their analysis is cast in terms of consumption-based habit, but it can be readily extended to our wealth-based habit setup. First, by appropriately redefining the state-price density, expressions for the dual short rate and dual risk premia can be obtained. Second, these expressions are then substituted back into the known solutions to the dual problem. Third, the solutions to the primal problem are obtained by adding in the wealth-in-the-utility term to the second-step solutions.

In what follows let X_t refer to a variable in the primal problem and let $\hat{X}_t$ refer to its dual problem counterpart. We start by defining the dual variables as follows:

$$\hat{C}_t \equiv C_t + \eta_w/\eta_c W_t, \tag{9}$$

$$\hat{U}(\hat{C}_t) \equiv \frac{(\hat{C}_t + \eta_0/\eta_c)^{1-\gamma}}{1-\gamma} = U(C_t, W_t), \tag{10}$$

where $U(C_t, W_t)$ is as in (3), since expected utility is defined only up to an affine transformation. Next, replace for C_t in budget constraint (2) by using (9) to obtain:

$$\begin{aligned} dW_t &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + r]W_t - \hat{C}_t + \eta_w/\eta_c W_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \\ &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + (r + \eta_w/\eta_c)]W_t - \hat{C}_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \\ &= \{[\boldsymbol{v}_t'(\boldsymbol{\mu}_p - r) + \hat{r}]W_t - \hat{C}_t\}dt + W_t \boldsymbol{v}_t' \boldsymbol{\sigma}_p d\boldsymbol{Z}_t, \end{aligned} \tag{11}$$

where $\hat{r} \equiv r + \eta_w/\eta_c$. Observe that wealth, portfolio, and the risk premia $(\boldsymbol{\mu}_p - r)$ remain unchanged.

Second, let π_t be the (primal) state-price density. The previous analysis suggests that its dual analog must satisfy:

$$\hat{\pi}_t \equiv e^{-(\eta_w/\eta_c)t}\pi_t, \tag{12}$$

from which,

$$d\hat{\pi}_t/\hat{\pi}_t = -(\eta_w/\eta_c)dt + d\pi_t/\pi_t. \tag{13}$$

To see that (12) is the appropriate dual state price density, note that a standard no-arbitrage argument establishes that the risk-free rate and risk premia process for the state-price density $\hat{\pi}_t$ in the dual market must satisfy:

$$\begin{aligned}\hat{r} &= -\mu_{\hat{\pi}}/\hat{\pi}_t & (14)\\ &= (\eta_w/\eta_c) + r & (15)\\ \boldsymbol{\mu}_{\hat{\boldsymbol{p}}} - \hat{r} &= -(1/\hat{\pi}_t)\,\boldsymbol{\sigma}_{\boldsymbol{p}}\boldsymbol{\sigma}'_{\hat{\boldsymbol{\pi}}} & (16)\\ &= \boldsymbol{\mu}_{\boldsymbol{p}} - r, & (17)\end{aligned}$$

as was required under (11).

Hence, the (exogenous) short rate process used by the agent in the dual problem is simply the short rate process in the primal problem plus the wealth dependency parameter; the risk premium process used by the agent is the same in both the dual and the primal problem. Under the iso-morphism result of Schroder and Skiadas (2002), we can:

1. use the known solutions of Merton for the dual problem $(\hat{C}_t, \hat{\boldsymbol{v}}_t)$ as functions of $W_t, \hat{r}, \boldsymbol{\mu}_p - r$,

2. correct the short rate in these solutions using (15),

3. get back the expression for C_t by inverting (9); the expression for $\boldsymbol{v}_t$ is the same as that for $\hat{\boldsymbol{v}}_t$.

Following this iso-morphism approach reveals that the indirect utility $J(W_t)$, the optimal consumption C_t and the value of risky assets $\boldsymbol{V}_t \equiv \boldsymbol{v}_t W_t$ are respectively given by:

$$J(W_t) = \frac{(G + FW_t)^{1-\gamma}}{1-\gamma}, \tag{18}$$

$$\begin{aligned} C_t = {} & \frac{\eta_0}{\eta_c}\left\{\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\rho/(\gamma-1) + 0.5Q/\gamma}{r + \eta_w/\eta_c}\right) - \frac{1}{\gamma}\right\} \\ & + \left\{\left(\frac{\gamma-1}{\gamma}\right)(r + \rho/(\gamma-1) + 0.5Q/\gamma) - \frac{\eta_w}{\eta_c\gamma}\right\} W_t, \end{aligned} \tag{19}$$

$$\boldsymbol{V_t} = \left(\frac{\eta_0/\eta_c}{r + \eta_w/\eta_c}\right)\frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} + \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} W_t, \tag{20}$$

where,

$$F \equiv \eta_c \left\{ \left(\frac{\gamma - 1}{\gamma} \right) \left(r + \frac{\eta_w}{\eta_c} + \frac{\rho}{\gamma - 1} + 0.5Q/\gamma \right) \right\}^{\gamma/(\gamma-1)},$$

$$G \equiv \frac{\eta_0}{\eta_c} \left\{ \left(\frac{\gamma}{\gamma - 1} \right) \frac{1}{\eta_c} \left(\frac{F}{\eta_c} \right)^{-1/\gamma} - \frac{\rho}{F(\gamma - 1)} - \frac{0.5Q}{\gamma F} \right\}^{-1},$$

$$Q \equiv (\boldsymbol{\mu}_p - r)' \boldsymbol{\Sigma}_{pp}^{-1} (\boldsymbol{\mu}_p - r) \geq 0,$$

$$\boldsymbol{\Sigma}_{ij} \equiv \mathrm{E}[\boldsymbol{\sigma}_i d\boldsymbol{Z}_t d\boldsymbol{Z}_t' \boldsymbol{\sigma}_j']$$

It can be shown that these solutions correspond exactly to those obtained using the more traditional dynamic programming approach.

Equation (18) highlights interesting characteristics of the value function. First, we find that $J(W_t)$ is iso-morphic to the instantaneous utility function $U(C_t, W_t)$ in (3). The particular form of wealth dependence that we are considering supposes that the Bernoulli transform is applied to an affine function of wealth. This functional has the property that the value function is also in the HARA class. Note in particular that $\eta_0 = 0$ implies $G = 0$, such that the value function becomes iso-elastic despite the wealth dependence.

Second, following our previous discussion, we can analyze risk aversion using the marginal utility of wealth schedule, $J_{w,t}$, and the distance of an arbitrary wealth level W_t from minimum admissible bliss level. In particular, straightforward manipulations reveal that:

$$W_{bliss} \equiv \frac{-G}{F} = \frac{-\eta_0}{\eta_c r + \eta_w}, \tag{21}$$

$$\frac{-W_t J_{ww,t}}{J_{w,t}} = \frac{\gamma W_t}{W_t - W_{bliss}}. \tag{22}$$

The constant bliss level of wealth (21) can take on negative or positive values depending on the parameters $\eta_i, i = 0, c, w$ and on the interest rate r. In particular, since $\eta_c, r > 0$, $\eta_w < 0$ pushes the bliss level away from zero, $\eta_w > 0$ pushes it toward zero. For $W_{bliss} < 0$, a positive η_w (blasé) moves the bliss asymptote to the right (see Figure 5). Given any wealth level, the agent is closer to bliss, and therefore characterized by a higher degree of absolute risk aversion. A negative η_w (ratchet) decreases absolute risk aversion for the opposite reason. When $W_{bliss} > 0$, movements in bliss are reversed, and we find that a blasé investor has lower absolute risk aversion than a ratchet investor. With respect to relative risk aversion (22), a (negative) positive bliss implies that risk aversion is (pro-) counter-cyclical.

[Insert Figure 5 here]

Next, the optimal rules (19) and (20) are affine in wealth. As for the standard HARA utility, imposing $\eta_0 = 0$ results in the iso-elastic case of both rules being proportional to net worth. Otherwise, wealth dependence affects both the intercept $(C_t, \boldsymbol{V}_t)$ and the slope (C_t) of the closed-form solutions. To isolate these effects, it is useful to resort to our previous analysis of the value function. Note that we can indeed rewrite the optimal rules as:

$$\begin{aligned} C_t &= -\left\{\left(\frac{\gamma-1}{\gamma}\right) W_{bliss}\left[\rho/(\gamma-1)+0.5Q/\gamma\right]+\frac{\eta_0}{\gamma\eta_c}\right\} \\ &\quad +\left\{\left(\frac{\gamma-1}{\gamma}\right)\left[r+\rho/(\gamma-1)+0.5Q/\gamma\right]-\frac{\eta_w}{\gamma\eta_c}\right\} W_t, \qquad (23) \\ (\boldsymbol{\mu}_p - r)'\boldsymbol{V}_t &= \frac{Q}{\gamma}\{-W_{bliss}+W_t\}, \qquad (24) \end{aligned}$$

where W_{bliss} is given by (21). For the rest of this section's analysis, assume that the investor is at least moderately risk averse, i.e $\gamma > 1$.

First, turning to consumption, we obtain the intuitive result that for positive W_{bliss}, minimum consumption, i.e the intercept in (23), is negative (or less positive), and positive (or less negative) otherwise. *Ceteris paribus*, $W_{bliss} > 0$ implies a steeper marginal utility of wealth at the optimum, and consequently, greater J_w risk. The risk-averse investor reacts to this by increasing wealth away from bliss. This is achieved by decreasing consumption and increasing savings. Secondly, regardless of W_{bliss}, a blasé investor always has a lower marginal propensity to consume out of wealth than a ratchet investor. This result is again intuitively appealing. Since a blasé investor positively values status, he always saves more at the margin.

Third, (24) expresses the expected excess return (in \$ terms) on the optimal total wealth portfolio. As usual, higher curvature γ results in more conservative positions. Again, bliss levels of wealth influence the intercept terms. A positive W_{bliss} implies more MU risk at any wealth levels. The risk-averse agent hedges away these risks by selecting more conservative portfolios. Negative bliss values however reduce risk aversion and increase the asset values held in risky assets. Fourth, for reasons discussed earlier, when bliss is negative, $\eta_w < 0$ shifts bliss to the left and decreases risk aversion; the ratchet investor therefore selects a more risky portfolio, the blasé a more conservative one. These positions are reversed for $W_{bliss} > 0$; the blasé investor's portfolio is more risky compared to the ratchet's.

Clearly linearity for the optimal rules (19), and (20) implies that the change in consumption and portfolio are $dC_t = c_w dW_t$, and $d\boldsymbol{V}_t = \boldsymbol{v}_w dW_t$, where $c_w, \boldsymbol{v}_w$ are constants defined by (19) and (20). We can also substitute the solutions in the budget constraint (2) to obtain the closed-form expression for instantaneous changes in wealth. Consequently the instantaneous changes in consumption, the value invested in assets, and wealth are:

$$
\begin{aligned}
dC_t &= \left\{ \left(\frac{\gamma - 1}{\gamma} \right) (r + \rho/(\gamma - 1) + 0.5Q/\gamma) - \frac{1}{\gamma} \frac{\eta_w}{\eta_c} \right\} dW_t, && (25) \\
d\boldsymbol{V}_t &= \frac{\boldsymbol{\Sigma}_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} dW_t, && (26) \\
dW_t &= \left[\frac{\eta_0/\eta_c + (r + \eta_w/\eta_c)W_t}{\gamma(r + \eta_w/\eta_c)} \right] \left\{ \left[\left(\frac{\gamma + 1}{\gamma} \right) 0.5Q + r + \eta_w/\eta_c - \rho \right] dt \right. \\
&\quad \left. + (\boldsymbol{\mu}_p - r)' \boldsymbol{\Sigma}_{pp}^{-1} \boldsymbol{\sigma}_p d\boldsymbol{Z}_t \right\}. && (27)
\end{aligned}
$$

2 Estimation

A Econometric Model

Estimation focuses on the multivariate Brownian motion given by (25)–(27), which can be written as:

$$
\begin{aligned}
dC_t &= c_w dW_t, && (28) \\
d\boldsymbol{V}_t &= \boldsymbol{v}_w dW_t, && (29) \\
dW_t &= [\mu_0 + \mu_w W_t]dt + [\sigma_0 + \sigma_w W_t]dZ_t, && (30)
\end{aligned}
$$

where $c_w, \boldsymbol{v}_w, \mu_0, \mu_w, \sigma_0, \sigma_w$ are constant loadings that depend only on the deep parameters. In principle, estimation of the model could be undertaken in price space or in quantity space. We select the second approach for a number of reasons.

First, the quantity space results impose considerably more theoretical restrictions that are related to the deep parameters on the joint first and second moments. With respect to deep parameters, standard analyses in price space treat the equilibrium quantities in the pricing kernels as exogenous; the theoretical restrictions are imposed on the prices of risk exclusively, with conditional second moments left unrestricted. In comparison, the quantity space analysis produces theoretical restrictions on both first and second moments of changes in consumption, asset holdings and wealth (through the restrictions on $c_w, \boldsymbol{v}_w, \mu_0, \mu_w, \sigma_0, \sigma_w$), while returns are treated as exogenous. In the absence of prior information on η_w in particular, these additional restrictions will be useful in identifying the preference parameters of interest. Second, empirical studies of aggregate optimal consumption and asset holdings are much less frequent than asset pricing studies. We believe that focusing on quantities rather than on returns thus provides another perspective that complements existing results (Lo and Wang, 2001, also argue in favor of using the informational content of quantities more thoroughly).

Transformation One major problem in estimating (28)–(30) is that there exists no closed-form transition density for multi-variate Brownian motions with affine drifts and diffusions. Indeed, analytical expressions for the likelihood function exist only for a limited class of Itô processes (Melino, 1996). Unfortunately, our multi-variate process does not belong to this class. Alternative solutions include discrete (Euler) approximations, and simulating the continuous-time paths between the discretely-sampled data, either through classical (Durham and Gallant, 2002) or through Bayesian (Eraker, 2001) approaches.

Our solution to this problem is different and considerably simpler to implement. It is based on a homoscedasticity-inducing transformation for general Brownian motions. It will be shown that this approach also stationarizes the drift term. Consequently, a standard discretized approximation is appropriate, efficient, and unbiased. In particular, a straightforward application of Itô's lemma reveals the following.

Lemma 1 *Let $X_t \in \{C_t, \boldsymbol{V}_t\}$ be defined as follows:*

$$X_t = x_0 + x_w W_t, \tag{31}$$

$$dW_t = (\mu_0 + \mu_w W_t)dt + (\sigma_0 + \sigma_w W_t)dZ_t, \tag{32}$$

where $\boldsymbol{x}, \boldsymbol{\mu}, \boldsymbol{\sigma}$ are constants defined in (19) *and* (20)*, and in* (27)*, and consider the following transformation:*

$$\tilde{X}_t = \frac{\log[x_w\sigma_0 + \sigma_w(X_t - x_0)]}{\sigma_w}, \tag{33}$$

Then, $\tilde{X}_t$ has constant drift and diffusion given by:

$$d\tilde{X}_t = \left[\frac{\mu_w}{\sigma_w} - 0.5\sigma_w\right] dt + dZ_t. \tag{34}$$

Proof. First, (31) and (32) reveal that:

$$\begin{aligned} dX_t &= [x_w\mu_0 + \mu_w(X_t - x_0)]dt + [x_w\sigma_0 + \sigma_w(X_t - x_0)]dZ_t & (35) \\ &= \mu(X_t)dt + \sigma(X_t)dZ_t. & (36) \end{aligned}$$

Next, by Itô's lemma, we have for $\tilde{X}_t = \tilde{X}(X_t)$:

$$d\tilde{X}_{j,t} = \left[\mu(X_t)\tilde{X}'(X_t) + 0.5\sigma(X_t)^2\tilde{X}''(X_t)\right] dt + \sigma(X_t)\tilde{X}'(X_t)dZ_t \tag{37}$$

Observe that $\mu_0/\mu_w = \sigma_0/\sigma_w$ to substitute in (37) to obtain (34). ∎

The transformation (33) requires that its first derivative with respect to the Itô process X_t is the inverse of the diffusion. It is usually introduced in order to stationarize the diffusion (Shoji and Ozaki, 1998; Aït-Sahalia, 2002; Durham and Gallant, 2002). In our case, both drift and diffusion are affine and have intercept and slope coefficients that are closely inter-related. Consequently the theoretical restrictions implied by the model are such that the transformation *also* stationarizes the drift term. This is fortunate since the resulting transformed model is easily estimated by maximum likelihood. In particular, the discretization of (34):

$$\Delta \tilde{X}_t = \left[\frac{\mu_w}{\sigma_w} - 0.5\sigma_w\right] + \epsilon_t \tag{38}$$

where ϵ_t is a standard Gaussian term, can be consistently and efficiently estimated by MLE (e.g. Gourieroux and Jasiak, 2001, pp. 287–288).

Likelihood function The optimal rules in (19) and (20) take the moments of the returns' distribution $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$, as well as the risk-free rate r as given. These moments could be estimated in an external round, using a two-step method, and substituted back into the optimal rules to obtain the predicted rules. Instead, we perform a single-step procedure and incorporate the mean and covariance matrix of the risky returns into the calculation of the likelihood function.[10] This approach has the advantage of factoring in the parametric uncertainty concerning $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$ into the calculation of the standard errors of the deep parameters. Specifically, denote by $\tilde{\boldsymbol{X}}_t \equiv [\tilde{C}_t, \tilde{\boldsymbol{V}}_t, \tilde{W}_t]'$ the $n+2$ vector of transformed variables, the model to be estimated is the following:

$$\begin{pmatrix} \Delta \tilde{\boldsymbol{X}}_t \\ \Delta \boldsymbol{P}_t / \boldsymbol{P}_{t-1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}_x \\ \boldsymbol{\mu}_p \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon}_x \\ \boldsymbol{\epsilon}_p \end{pmatrix}, \quad \begin{pmatrix} \boldsymbol{\epsilon}_x \\ \boldsymbol{\epsilon}_p \end{pmatrix}, \sim \text{N.I.D.} \left(\begin{pmatrix} \boldsymbol{0}_x \\ \boldsymbol{0}_p \end{pmatrix}, \begin{pmatrix} \boldsymbol{I}_x & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_{pp} \end{pmatrix} \right), \tag{39}$$

where $\boldsymbol{\mu}_x$ is given by (38), and $\boldsymbol{I}_x$ is an $n+2$ identity matrix.

First, in accordance with the maintained assumption of the model, all the innovations are Gaussian. Second, as mentioned earlier, the transformation in Lemma 1 implies that the quantities innovations are standardized white noise. Third, consistent with the model, the covariance matrix is block diagonal, i.e. we impose the absence of cross-correlations between innovations in quantities and returns. Any potential covariance between the two is fully taken into account in the closed-form solutions; allowing for additional correlations is not theoretically justified.

With these elements in mind, the contributions to the likelihood function (with constant term omitted) are given by:

$$f_t = -0.5 \log[\det(\boldsymbol{\Sigma})] + \log[\det(\boldsymbol{K}_t)] - 0.5\boldsymbol{\epsilon}_t' \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}_t \tag{40}$$

where $\boldsymbol{\Sigma}$ is defined implicitly in (39), while $\boldsymbol{K}_t \equiv \text{Diag}([K_{c,t}, \boldsymbol{K}_{v,t}, K_{w,t}, 1, \ldots 1])$ and $K_{x,t} = 1/[x_w\sigma_0 + \sigma_w(X_t - x_0)]$ is a Jacobian correction term associated with the transformation (38). The parameter vector is then $\boldsymbol{\theta} \equiv \{\gamma, \rho, \eta_0, \eta_c, \eta_w, \boldsymbol{\mu}_p, \boldsymbol{Q}_{pp}\}$, where $\boldsymbol{Q}_{pp} \equiv \text{Chol}(\boldsymbol{\Sigma}_{pp})$ is the n-dimensional triangular Cholesky root of the returns covariance matrix.

Hypothesis tests It will be recalled that theoretical restrictions for HARA and WDU utility are necessary to guarantee that marginal utility is non-negative. In particular, for both models, restriction (4) is required for monotone preferences, whereas for WDU utility, (5) verifies that the agent has a positive effective discount rate.

We also consider two benchmarks in assessing the performance of the WDU model. As mentioned earlier, CRRA utility is obtained by imposing that $\eta_0, \eta_w = 0$, whereas HARA utility imposes $\eta_w = 0$. To the extent that it has been studied extensively in asset pricing models, CRRA utility constitutes a natural benchmark. HARA utility, although less popular, has the advantage of optimal rules which are not proportional to wealth (see the previous discussion). Both the theoretical restrictions and the model selection tests will be performed and discussed below.

B Data

Our data set consists of post-war U.S. quarterly observations on aggregate consumption, asset holdings and corresponding returns indices. The time period covered ranges from 1952:II to 2000:IV, for a total of 195 observations. All quantities are expressed in real, per-capita terms, where the aggregate price index is taken to be the implicit GDP deflator. Similarly, all returns are converted in real terms by subtracting the inflation index.

Consumption The consumption series is the aggregate expenditure on Non-Durables and Services. The source of the data is the Bureau of Economic Analysis NIPA series. This series has been used in most asset pricing studies.

Assets The aggregate portfolio holdings are defined as follows:

$$\begin{aligned} \boldsymbol{V}_t &= [V_{0,t}, V_{1,t}, V_{2,t}] \\ &= [\text{Deposits, Bonds, Stocks}]. \end{aligned}$$

Each asset holdings are obtained from the Flow of Funds Accounts made available by the Board of Governors of the Federal Reserve (Table L.100). They correspond to the level values of asset holdings by households and non-profit organizations (see also Lettau and Ludvigson, 2003). More precisely the individual assets (mnemonic) and financial wealth are:

- Deposits (FL15400005): Includes foreign, checkable, time, savings deposits and money market fund shares.
- Bonds (FL153061005): U.S. government securities (Treasury and Agency).
- Stocks (FL153064105): Corporate equities directly held by households.
- Wealth: Deposits + Bonds + Stocks ($W_t = V_{0,t} + V_{1,t} + V_{2,t}$).

Deposits will thus be taken to represent the risk-free asset, whereas both long-term government bonds and corporate equity are proxies for the risky assets.

The choice of specific portfolio holdings was dictated by a number of practical elements. First, these assets correspond to some of the largest asset holdings for U.S. households, and their returns have been studied extensively in the asset pricing literature, thus providing useful benchmarks for our analysis. In particular, we are interested in verifying whether the pricing anomalies associated with cash and stock returns have dual analogs in the quantity space. Second and related, these assets have corresponding returns series. Those returns are required to evaluate the distributional parameters $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$ that are used to compute the theoretical asset holdings. Other portfolio holdings such as pension and life insurance reserves are also important in relative size. However, no clear returns indices are available for these assets.[11]

Our definition of wealth has been used in theoretical models of portfolio choice (e.g. Campbell et al., 2003). Its main advantage is that wealth is thus observable and the definition provides more structure on the econometric model since one of the theoretical asset holding is defined residually.[12] However, the definition is narrow in the sense that it abstracts from tangible (real) and human wealth. Unfortunately, real returns indices on durable goods are difficult to evaluate, and these assets were omitted from our selected holdings series $\boldsymbol{V}_t$. Moreover, human wealth is not observable, whether in levels or in rates of returns and thus also eliminated.

Table 1 reports the sample moments for the consumption and asset holdings in percentage of wealth (those series were plotted in Figures 1–2). A first observation is that the shares of wealth allocated to consumption, deposits and stocks are roughly of the same order of magnitude, and similarly volatile. Bonds on the other hand represent a much lower share of wealth and are smoother.

[Insert Table 1 here]

Returns We follow Campbell et al. (2003) in constructing the returns series that correspond to our assets definition. The return on cash is taken to be the real return on 3-months Treasury Bills. The return on bonds is proxied by the real return on 5-years T-Bills. Finally, stock returns are evaluated as the value-weighted returns on the NYSE, NASDAQ and AMEX markets. Bond and stock returns were obtained from the CRSP data file. Again, the inflation series is computed from the GDP deflator.

Table 2 presents sample moments of the real returns. These series have been widely discussed in the asset pricing literature, so we only briefly outline their main features. First, we observe that both bonds and stocks warrant a positive premia. Equity returns however are clearly larger, and much more volatile. Next, we find that both cash and bonds as well as bonds and stocks are positively, and similarly correlated. Cash and stocks on the other hand display no covariance.

[Insert Table 2 here]

3 Results

A Estimation details

Identifiability The theoretical model in (25)–(27) presents some important challenges for identification. Indeed, the parameters are often expressed as ratios of one another which usually results in poor identifiability. These problems affect both the HARA and WDU models, but not the CRRA model. As is well known, utility is defined only up to an affine transformation such that the parameter η_c plays no role in the optimal rules when $\eta_0, \eta_w = 0$. In preliminary estimation rounds, we experimented with numerous identification strategies which we briefly discuss.

A first approach was to fix the subjective discount rate ρ to a realistic value, and to let η_c be flexible. We found that both HARA and WDU models were then poorly identifiable; results were highly dependent on starting values, and convergence problems were noticed. Second, we let ρ be flexible, and fixed η_c. Whereas HARA utility was well identified and yielded realistic ρ estimates, the WDU model was not. In particular, we found that we could not identify ρ and η_w separately. Nonetheless, the effective discount rate $\rho - \eta_w/\eta_c$ was uniquely identifiable, and realistic. Finally, we fixed both ρ, η_c, and found this approach to be the most satisfactory. Both models were then clearly identified, with robustness to starting values and rapid convergence. We found that the curvature parameter γ was completely independent from the choice of calibration for η_c. Moreover, changing η_c resulted in changing the estimated η_0, η_w in the same proportions, such that the T-statistics were always unaffected by the calibrated value of η_c. This again indicates that although the ratios $\eta_0/\eta_c, \eta_w/\eta_c$ are well identified, the separate parameters are not. We therefore fix $\rho = (1 + 0.035)^{1/4} - 1$, a realistic value, and follow the asset pricing literature in arbitrarily imposing $\eta_c = 1$ to estimate γ, η_0, η_w. The vector of free parameters is then $\{\gamma, \eta_0, \eta_w, \boldsymbol{\mu}_p, \boldsymbol{Q}_{pp}\}$.

B Parameter estimates

Table 3 presents the estimated parameters for model (39). Panel A imposes the CRRA restrictions that $\eta_0 = \eta_w = 0$; Panel B imposes the HARA restriction that $\eta_w = 0$. Panel C relaxes these restrictions altogether for the WDU model.

[Insert Table 3 here]

Theoretical restrictions First, regarding the monotonicity restriction, CRRA utility trivially respects non-negative marginal utility. In the case of HARA and WDU, this condition needs to be verified. We test that monotonicity is always maintained by evaluating (4) at the minimum consumption and wealth levels:

$$\eta_c \min(C_t) + \eta_0 + \eta_w \min(W_t) > 0$$

Since $\eta_c \equiv 1$ and η_w is estimated positive, this approach is sufficient to guarantee monotonicity throughout our sample. For HARA utility, the statistic (standard error) is 6,474 (0.35); for WDU, it is 6,6157 (87.95). We thus conclude that monotonicity condition (4) is verified for both HARA and WDU.

Second, we verify that the effective discount rate for WDU preferences is non-negative as in (5). Since $\eta_c \equiv 1$, this is obtained by testing

$$H_0 : \rho - \eta_w = 0,$$

against the alternative of negative discounting. Evaluated at our parameter estimates in Panel C, the effective discount rate is -0.0099 (0.0121), a negative but low value that is not statistically significant, such that the null is not rejected. We therefore conclude that all three models satisfy the theoretical restrictions and proceed with the analysis of the point estimates.

Individual estimates The estimates for the curvature parameter γ in Table 3 are positive, significant and realistic for all three preference specifications. Indeed, it is widely recognized that this parameter should be positive, but less than 10 for iso-elastic utility (e.g. Mehra and Prescott, 1985). Moreover, the point estimates are lower for WDU. Whether or not this translates into a lower level of risk aversion for these functionals will be addressed below.

Next, we find that the bliss parameter η_0 is negative and very significant for HARA utility, and even more negative, but less significant under WDU. This implies that the reference consumption level is positive under HARA preferences. Under WDU, the fictitious bliss level ranges between -100 and -800 and remains negative throughout. Third, the wealth dependence parameter η_w is positive, and significant, thereby rejecting the null of HARA utility when tested against the WDU alternative. Our results are therefore consistent with a statistically significant blasé behavior with respect to financial wealth. A test of the joint CRRA restrictions ($\eta_0 = \eta_w = 0$) reveals that the null of iso-elastic preferences is strongly rejected when tested against either HARA or WDU.

C Relative Risk Aversion Estimates

Figure 6 plots the risk aversion estimates for the three utility functions. Panel A plots the consumption risk aversion, $-C_t U_{cc,t}/U_{c,t}$, panel B the wealth risk aversion $-W_t U_{ww,t}/U_{w,t}$, and panel C the indirect utility function risk aversion $-W_t J_{ww,t}/J_{w,t}$. The dotted line corresponds to CRRA utility, the dashed line to HARA, and the thick, solid line to WDU preferences.

[Insert Figure 6 here]

We find in Panel A that CRRA utility generates the highest, and WDU the lowest level of consumption risk aversion. Moreover, consumption risk aversion under HARA is almost flat compared to that obtained under WDU, i.e. HARA generates no perceptible cyclical variation in attitudes towards consumption risk. Clearly, in panel B, the wealth risk aversion index is zero for both CRRA and HARA. The level for WDU is positive, generally lower, and less volatile compared to consumption risk aversion.

A reduced-form interpretation of the representative agent's risk aversion can be obtained from the indirect utility function J_t, and its corresponding relative risk aversion index $-W_t J_{ww,t}/J_{w,t}$ in (22). This variable is plotted in panel C. Because the indirect utility is iso-morphic to the instantaneous utility, the CRRA function has a constant index equal to γ. For most of our sample, this level is lower than that obtained under HARA and WDU. Note finally that the risk aversion under WDU is lower than that obtained under HARA, with parallel, although more volatile, time paths.

We can explore the issue of cyclical movements of attitudes toward risk by comparing risk aversion series with indices of the state of the economy. For that purpose, we use the University of Michigan Consumer Confidence Index, a pro-cyclical subjective state measure, which we plot against the various measures of risk aversion obtained under WDU. Panel A of Figure 7 plots the consumption risk aversion against the confidence index. Clear counter-cyclical patterns emerge. Risk aversion is initially decreasing until the late 60's, when the confidence index is stable. Then, risk aversion increases as the index falls in the early and mid 70's. The gradual recovery in consumer sentiment is associated with a smooth decline in consumption risk aversion.

[Insert Figure 7 here]

The correspondence between attitudes toward risk and consumer confidence is even more striking for wealth risk aversion in panel B. Pro-cyclical movements in wealth risk aversion mimic almost exactly those in confidence, particularly up until the mid 70's. After that period, the gradual increase in confidence is associated with a smooth increase in wealth risk aversion.

We therefore find strong counter-cyclical movements in consumption risk aversion, and strong pro-cyclical movements in wealth risk aversion. To verify which one of those two conflicting influences dominates, we plot the indirect risk aversion (22) against the confidence index in panel C. Again, a counter-cyclical movement clearly emerges. To understand this result, our estimates reveal that the bliss level of wealth (21) is $W_{bliss} = 4126.1$, a positive value, whereas wealth in our sample ranges between 11 thousand and 48 thousand \$. As wealth increases above bliss, movements in marginal utility are reduced and risk aversion falls. This accords with our previous discussion of the value function in (18) that for positive bliss, a blasé investor has lower and counter-cyclical risk aversion.

4 Discussion: Implications for Asset Returns

Our empirical results obtained from the estimation of optimal consumption and portfolio rules using only financial assets and financial wealth can be summarized as follows:

1. The intercept parameter η_0 is non-zero and significant.

2. The wealth-dependence term η_w is positive and significant.

3. The curvature parameter γ is positive, realistic, and lower under WDU.

4. Risk aversion is counter-cyclical.

Because there is a relative paucity of empirical results in goods space for these models, it is difficult to establish whether or not our findings make sense on a comparative basis. Nonetheless, we can use the implied theoretical restrictions in price space to obtain further perspective on our goods-space results.

We consequently consider the implications of our model and of our results for asset returns. For that purpose, we relax the assumption that the investment set is constant. Again, we can resort to the isomorphism result of Schroder and Skiadas (2002) to map the result from the dual problem to the primal problem. A standard argument establishes that equilibrium state-price deflator in the dual market is given by the marginal utility of consumption:

$$\hat{\pi}_t = e^{-\rho t}\hat{U}_{\hat{c},t}. \tag{41}$$

Based on this we can:

1. compute the process for $d\hat{\pi}_t/\hat{\pi}_t$ using dual utility (10) and dual consumption (9),

2. compute the dual short rate and risk premia processes $\hat{r}_t, \boldsymbol{\mu_{\hat{p},t}} - \hat{r}_t$ using (14) and (16),

3. map those dual expressions back into their primal counterparts $r_t, \boldsymbol{\mu_{p,t}} - r_t$ by inverting (15) and (17).

These calculations reveal that the risk premia is:

$$\boldsymbol{\mu}_{p,t} - r_t = R_t^{rc}\mathrm{Cov}_t\left(\frac{dC_t}{C_t}, \frac{d\boldsymbol{P}_t}{\boldsymbol{P}_t}\right) + R_t^{rw}\mathrm{Cov}_t\left(\frac{dW_t}{W_t}, \frac{d\boldsymbol{P}_t}{\boldsymbol{P}_t}\right). \tag{42}$$

The risk-free rate is given by:

$$r_t = \rho - \eta_w/\eta_c + R_t^{rc}\mathrm{E}_t\left(\frac{dC_t}{C_t}\right) + R_t^{rw}\mathrm{E}_t\left(\frac{dW_t}{W_t}\right)$$
$$- 0.5\left(\frac{\gamma+1}{\gamma}\right) \times \left[(R_t^{rc})^2\mathrm{Var}_t\left(\frac{dC_t}{C_t}\right) + 2R_t^{rc}R_t^{rw}\mathrm{Cov}_t\left(\frac{dC_t}{C_t}, \frac{dW_t}{W_t}\right) + (R_t^{rw})^2\mathrm{Var}_t\left(\frac{dW_t}{W_t}\right)\right] \quad (43)$$

where:

$$R_t^{rx} \equiv \frac{-X_t U_{xx,t}}{U_{x,t}} = \left(\frac{\gamma\eta_x X_t}{\eta_c C_t + \eta_0 + \eta_w W_t}\right), \ X_t \in \{C_t, W_t\}$$

are the consumption and wealth relative risk aversion indices.

Risk premia The premia (42) is a two-factor pricing model, with the C-CAPM consumption beta supplemented by the CAPM total wealth return beta. In particular, (7) reveals that the quantity of consumption risk, i.e. $\mathrm{Cov}_t\left(dC_t/C_t, d\boldsymbol{P}_t/\boldsymbol{P}_t\right)$, is priced by the Arrow-Pratt risk aversion level, measured with respect to *consumption*, i.e. R_t^{rc}. Similarly, the quantity of the market risk, i.e. $\mathrm{Cov}_t\left(dW_t/W_t, d\boldsymbol{P}_t/\boldsymbol{P}_t\right)$, is priced by the corresponding Arrow-Pratt risk aversion, measured with respect to *wealth*, i.e. R_t^{rw}. The model can thus be interpreted as a linear combination of a static CAPM ($\eta_c = 0$), and a standard C-CAPM ($\eta_w = 0$), where the weights depend on the relative contributions of consumption and wealth to the agent's utility.

Duffie and Epstein (1992) also obtain a two-factor model, although their model is derived under non-expected utility, rather than VNM preferences. In addition, the relative weights depend on the distance between risk aversion, and the inverse of the elasticity of inter-temporal substitution. Hence, an expected-utility maximizer (risk aversion inversely equal to elasticity of inter-temporal substitution) does not price market risk. In contrast, our agent maximizes expected utility but, for $\eta_w \neq 0$, nonetheless values market risks. Moreover, the relative weights under WDU reflect the importance of consumption versus wealth risk aversion. This is fortunate to the extent that it provides an intuitively appealing interpretation where each risk is being priced by its corresponding risk aversion measure.

Our estimates indicate that optimal consumption is not proportional to wealth (finding 1). This has important consequences for the pricing equations. To see this consider the case where $\eta_0 = 0$ in (19).

Then, the consumption/wealth ratio is constant, and the growth rates on consumption and wealth are equal: $dC_t/C_t = dW_t/W_t$. Consequently, so are the covariance terms. Substitute in the premium (42) to obtain that:

$$\begin{aligned} \boldsymbol{\mu}_{p,t} - r_t|_{\eta_0=0} &= \left(\frac{\gamma\eta_c C_t}{\eta_c C_t + \eta_w W_t} + \frac{\gamma\eta_w W_t}{\eta_c C_t + \eta_w W_t} \right) \mathrm{Cov}_t \left(\frac{dC_t}{C_t}, \frac{d\boldsymbol{P}_t}{\boldsymbol{P}_t} \right), \\ &= \gamma \mathrm{Cov}_t \left(\frac{dC_t}{C_t}, \frac{d\boldsymbol{P}_t}{\boldsymbol{P}_t} \right), \end{aligned}$$

which is simply the standard C-CAPM with CRRA preferences, in which wealth dependence plays no role. Hence, our finding 1 that $\eta_0 \neq 0$ is important to allow for wealth dependence to impact asset returns. In this perspective, our unequivocal rejection of the CRRA model can be interpreted as the dual in goods space of its empirical anomalies in price space.

Second, the presence of a second source of IMRS risk is a welcomed addition in finding a solution for the equity risk premium puzzle. Finding 2 establishes that $\eta_w > 0$ such that the price of the market risk, R_t^{rw}, is positive. This result is consistent with the multi-factor empirical literature which finds that market risk is positively valued by the market (Chen et al., 1986; Ferson and Harvey, 1991). If the quantity of market risk is also positive, then a high equity premia need not be explained by consumption risk alone. This is also confirmed in our data set. Table 4 establishes that the total wealth risk of corporate stocks is much larger (by a ratio of 91:1) than consumption risk. A consequence of estimating $\eta_w > 0$ is that this larger market risk can justify the high observed premia at a lower level of risk aversion. This is consistent with our finding 3 that the curvature parameter γ, and the risk aversion estimates in general, are lower under WDU. These results parallel those of Aït-Sahalia et al. (2004) who derive the pricing kernels under direct preference for luxury goods (measured essentially by durables, and thus a component of total wealth). Imposing positive marginal utility of luxury goods (equivalent in our case to $\eta_w > 0$), they find that luxuries, who are mainly owned by rich people, are more covariant with returns. Consequently, a larger quantity of risk, and a lower risk aversion justify the observed premia.

[Insert Table 4 here]

Third, note that the prices of both risks will in general be time-varying. Our results indicate that relative risk aversion with respect to consumption (wealth) was counter- (pro-) cyclical, with the overall indirect utility risk aversion being counter-cyclical (finding 4). This result would be consistent with the predictability puzzle whereby the conditional premia are observed to fall during booms, and pick up during recessions (Cochrane, 1997; Guvenen, 2003a). In the absence of strong conditional heteroscedasticity effects in the quantities of consumption or market risks, predictability would be explained in our model by cyclical movements in risk aversion. Similar counter-cyclical properties are obtained by Yogo (2003). Resorting to direct preferences for durables, he explains the predictability of expected stock returns through cross effects in marginal utility over durables and non-durables. During a recession, durables consumption typically falls. For sufficiently high elasticity of substitution between durables and non-durables, the marginal utility of non-durables increases. Consequently, the required premia for holding the risky asset also increases during troughs. In our case, as discussed earlier, $\eta_w > 0$ yields a similar increase in the marginal utility of consumption when wealth falls during recessions.

Risk-free rate As is well known, the risk-free rate puzzle is a by-product of the equity premium puzzle (Weil, 1989; Kocherlakota, 1996). A high risk aversion implies a low elasticity of inter-temporal substitution, and a high risk-free rate to induce savings. We have already mentioned that wealth dependence result in lower curvature indices (finding 3), thereby potentially addressing the risk-free rate puzzle.

Nonetheless, it is interesting to study the impact of WDU for the predicted risk-free rate. As in the standard case, the risk-free rate (43) captures a first-order and a second-order effect reflecting the mean and the variance of the IMRS. In our model however, marginal utility depends on movements in both consumption and in wealth.

Our empirical findings would be consistent with a low risk-free for a number of reasons. First, the effective discount rate in (43) is now $\rho - \eta_w/\eta_c$; a positive η_w consistent with blasé behavior (finding 2) reduces it and consequently helps in reproducing the low observed r_t. Second, a low r_t is achieved if the second-order effect on IMRS is stronger than the first-order one. More precisely, allowing for wealth dependence affects both the mean (through the conditional mean terms for consumption and wealth growth) and the variance of the IMRS (through their conditional variance and covariance terms).

In particular, regardless of the sign of η_w, the variance of innovations in wealth enters negatively and reduces the risk-free rate. Table 4 shows that the volatility of wealth growth is more than 60 times larger than that of consumption growth (Lettau and Ludvigson, 2003, p. 2, also find that measured wealth growth is much more volatile than consumption growth over short horizons). This effect should therefore be important towards reducing the predicted rates. Moreover, the sample moments indicate that consumption growth is positively correlated with wealth growth. Since η_w was estimated to be positive this covariance in (43) tends to reduce further the predicted rate. Note however that $\eta_w > 0$ implies that the mean growth rate of wealth affects positively the predicted risk-free rate. Since the empirical moments in Table 4 indicate that mean consumption and wealth growth rates are roughly equal, this first-order effect could be important in increasing the predicted rate.

Internal vs external WDU Our WDU model has been derived under an *internal* wealth-dependence effect, whereby the agent's *own* wealth affects his utility. In contrast, Campbell and Cochrane (1999a) consider an *external* habit where bliss is determined by the *other agents'* consumption levels. It seems relevant therefore to ask how our results are modified if we substitute the aggregate wealth level, say $\bar{W}_t$, instead of the personal wealth W_t in the preferences (3).

It can be shown that the premium (42) is unaffected when wealth preferences are external. Since aggregate wealth is beyond the agent's control in the latter case, external wealth is simply an exogenous state variable that conditions preferences. Following Cox et al. (1985), if this variable is valued by the investor, it is priced, if in addition it covaries with other assets, then it warrants a premium.[13] However, under external WDU, the risk-free rate (43) is modified compared to internal wealth preferences. In particular, the use of Envelope theorem under internal wealth preferences implies that the effective discount rate is $\rho - \eta_w/\eta_c$; under external wealth preferences, this rate is simply ρ. The agent internalizes the fact that he can (partially) control future wealth, and therefore future bliss. Consequently, at the optimum, the agent's subjective rate of time preference is affected. A ratchet investor ($\eta_w < 0$) is more impatient since higher future wealth raises bliss and its associated marginal utility risk; a blasé investor ($\eta_w > 0$) is more patient for the opposite reasons.

To conclude, our wealth-dependent framework has the theoretical potential to successfully address the three main pricing anomalies of the C-CAPM. Our empirical findings in goods space are consistent with a WDU explanation of empirical asset returns puzzles. Whether or not similar estimation results in price space are obtained will require further analysis which we leave on the research agenda.

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Notes

[1]Static problems define utility over terminal wealth, but implicitly assume that this wealth is entirely consumed.

[2]Clearly the two observations are linked to one another under VNM preferences.

[3]The data is obtained from the Flow of Funds, is made available by the Board of Governors of the Federal Reserve Bank, and is discussed in further details below.

[4]Our cyclical patterns are robust to the choice of the wealth series in computing consumption and portfolio shares. Replacing 'Financial wealth' by the more comprehensive 'Net worth' (i.e. including tangible assets, mutual funds, pension plans, ... net of liabilities) in the denominator in order to derive the consumption and portfolio shares has no qualitative incidence on the patterns identified in Figure 1.

[5]The de-trended series are measured as deviations from a quadratic deterministic trend for log consumption and log wealth respectively.

[6]See also Falato (2003), or Barberis et al. (2001) for wealth-dependent utility models allowing for rotations in the marginal utility schedules.

[7]The theoretical restriction (5) stems from the financial problem we are analyzing. Consider the discrete-time analog of maximizing (1) subject to (2). First-order and Envelope conditions yield the following:

$$U_{c,t} = \exp(-\rho)\mathrm{E}_t\{[U_{c,t+1} + U_{w,t+1}]R_{i,t+1}\},$$

or,

$$1 = \exp(-\rho)(1 + \eta_w/\eta_c)\mathrm{E}_t\left\{\left(\frac{\eta_c C_{t+1} + \eta_0 + \eta_w W_{t+1}}{\eta_c C_t + \eta_0 + \eta_w W_t}\right)^{-\gamma} R_{i,t+1}\right\}.$$

This Euler equation has a familiar representation, with the exception that the subjective discount factor is now modified to allow for wealth dependence. It is reasonable to expect that the *effective* discount factor, $\exp(-\rho)(1+\eta_w/\eta_c) \in (0,1)$. Restriction (5) follows immediately.

[8]The bliss level is fictitious in the sense of being a subjective, possibly negative, reference level.

[9]In comparison, typical habit models define the benchmark habit stock in terms of cumulated *lagged* consumption (e.g. Constantinides, 1990, p. 522):

$$U = U(C_t, X_t), \text{ where } X_t \equiv \int_0^t \exp[-a(t-s)]C_s ds + \exp(-at)X_0.$$

[10]Following standard practices, the risk-free rate r is calibrated to its mean value.

[11]For example, pension reserves are typically invested differently by fund managers whether they are defined benefit or defined contribution. Finding a unique pricing index for this series in the absence of detailed information on the funds' composition is impractical.

[12]In particular, (20) reveals that, for the risk-free asset:

$$V_{0,t} = v_{00} + v_{w,0}W_t \tag{44}$$

where,

$$v_{00} = -v_{10} - v_{20}, \quad v_{w0} = 1 - v_{1w} - v_{2w}. \tag{45}$$

[13]Clearly, modifying the model so that the agent's bliss now depend on the difference between own and aggregate wealth, i.e. $\eta_w(W_t - \bar{W}_t)$ would eliminate the market risk factor in the premia (42). In equilibrium, a representative agent's wealth is also the aggregate wealth: $W_t = \bar{W}_t$.

Tables

Table 1: Sample moments: Shares of wealth

	mean	std	correlation			
Consumption	0.525	0.096	1.000	0.907	-0.233	-0.947
Deposits	0.473	0.123		1.000	-0.519	-0.970
Bonds	0.088	0.031			1.000	0.295
Stocks	0.440	0.110				1.000

Table 2: Sample moments: Real Returns

	mean	std	correlation		
Deposits	0.017	0.022	1.000	0.236	0.005
Bonds	0.036	0.130		1.000	0.212
Stocks	0.137	0.342			1.000

Table 3: Parameter Estimates

Param.	Estim.	T-stat.	Estim.	T-stat.	Estim.	T-stat.
	A. CRRA		B. HARA		C. WDU	
γ	6.187	4.822	5.799	2.747	5.341	2.937
η_0	0	—	-17.319	-49.462	-93.815	-3.943
η_w	0	—	0	—	0.019	3.215
μ_1	0.007	3.329	0.007	2.395	0.008	4.061
μ_2	0.026	7.602	0.028	6.129	0.023	3.846
$Q(1,1)$	0.031	19.369	0.031	19.685	0.030	19.894
$Q(1,2)$	0.017	2.573	0.017	3.008	0.016	2.935
$Q(2,2)$	0.080	19.073	0.079	19.914	0.080	20.204

Note: Estimated model (39). Assets: [Deposits, Bonds, Stocks], Wealth: $W_t = V_{0,t} + V_{1,t} + V_{2,t}$. Sample period: 1952:II–2000:IV. Fixed parameters $\rho = (1+0.035)^{1/4} - 1$, and $\eta_c = 1$. $\boldsymbol{\mu}_p$ are the drift parameters, $\boldsymbol{Q}_{pp} \equiv \text{Chol}(\boldsymbol{\Sigma}_{pp})$ is the Cholesky root of the covariance matrix of the returns process.

Table 4: Sample moments: Consumption, wealth growth and stock returns

	mean	std	covariances		
Consumption growth	0.02356	0.02079	0.00043	0.00051	0.00051
Wealth growth	0.03658	0.16291		0.02654	0.04701
Stock returns	0.13811	0.34218			0.11709

Figure Legends

Figure 1 Shares of wealth: Cash, bonds and stocks.

Figure 2 De-trended log wealth and log consumption and untransformed consumption share.

Figure 3 Effects of increase in wealth on marginal utility, Ratchet Investors ($\eta_w < 0$).

Figure 4 Effects of increase in wealth on marginal utility, Blasé Investors ($\eta_w > 0$).

Figure 5 Marginal utility of wealth at the optimum.

Figure 6 Arrow-Pratt Measures of Risk Aversion.

Figure 7 Risk Aversion and University of Michigan Consumer Confidence Index.

Figures

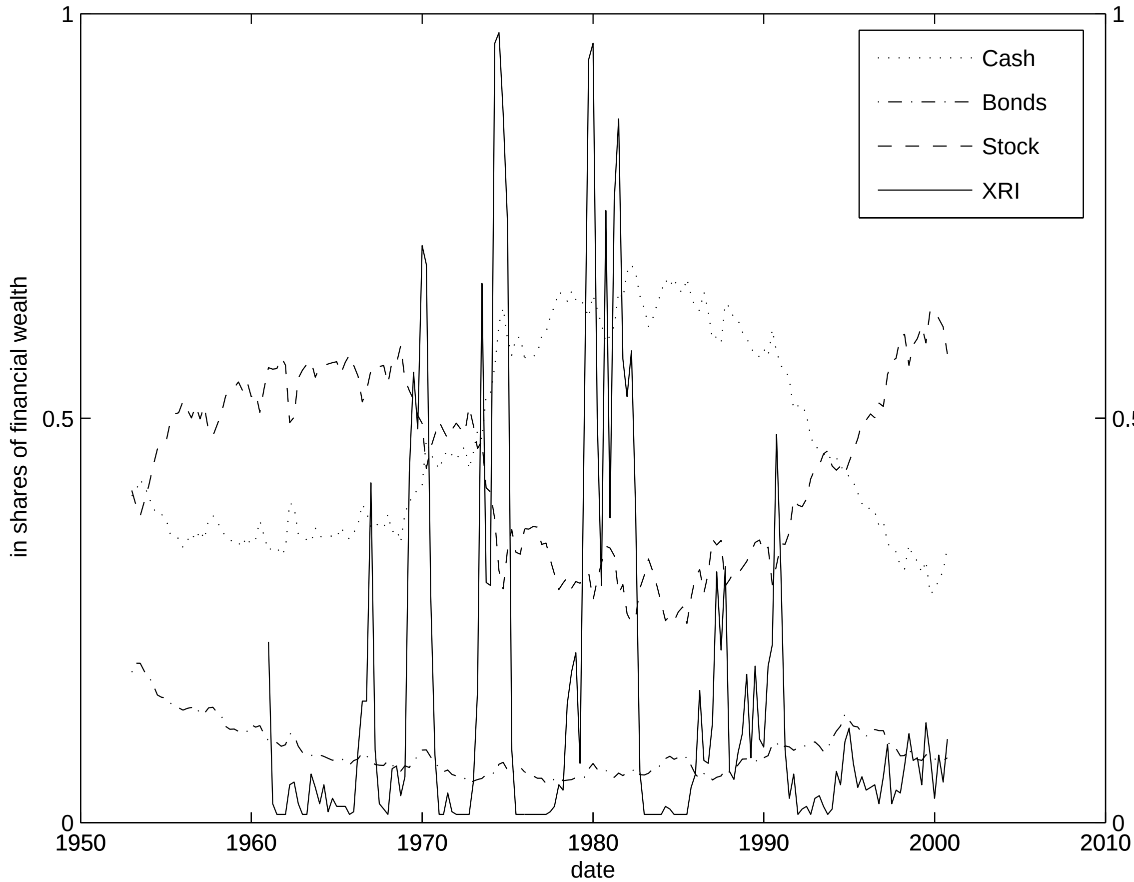

Figure 1:

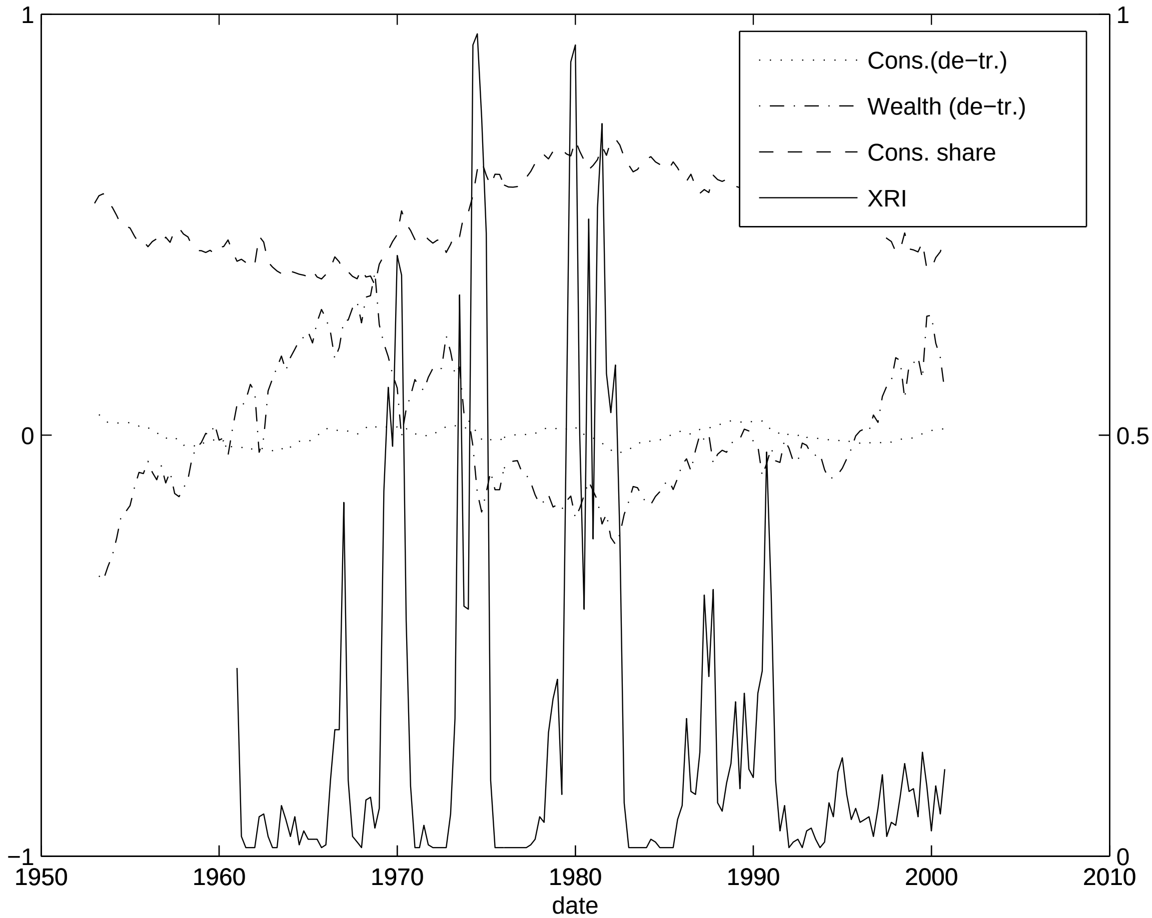

Figure 2:

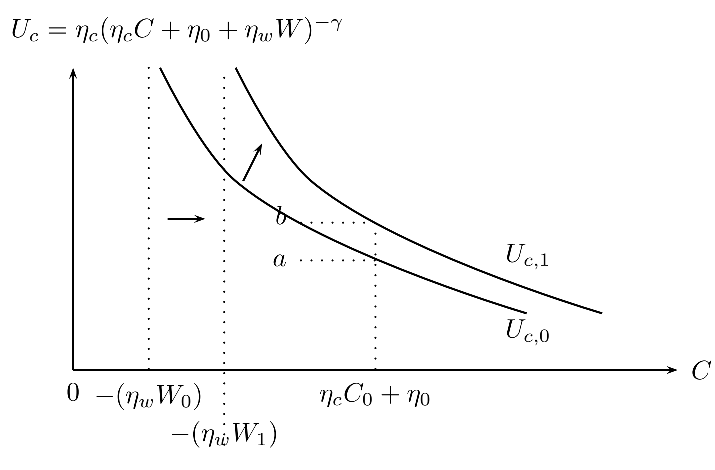

Figure 3:

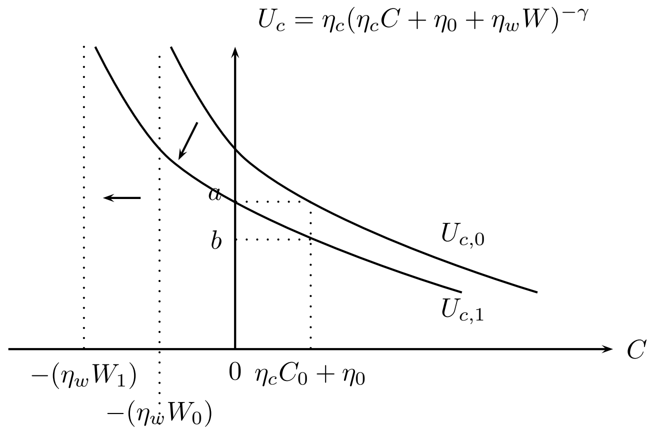

Figure 4:

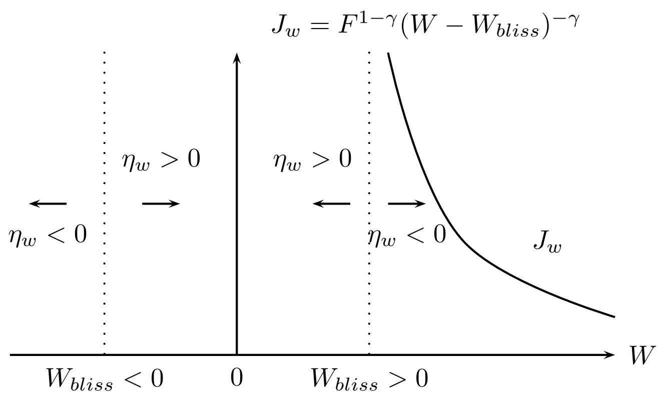

Figure 5:

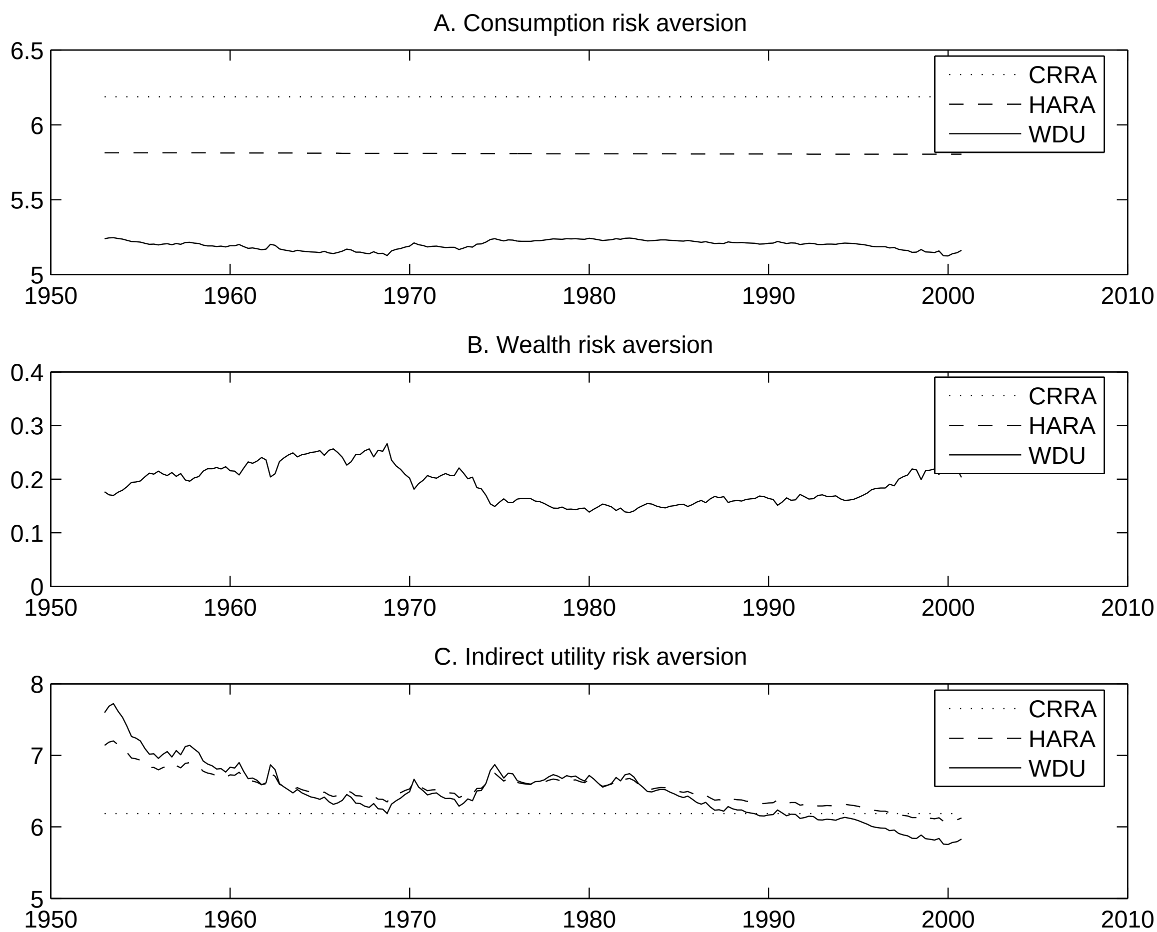
A. Consumption risk aversion
6.5
6
5.5
5
1950
1960
1970
1980
1990
2000
2010
CRRA
HARA
WDU
B. Wealth risk aversion
0.4
0.3
0.2
0.1
0
1950
1960
1970
1980
1990
2000
2010
CRRA
HARA
WDU
C. Indirect utility risk aversion
8
7
6
5
1950
1960
1970
1980
1990
2000
2010
CRRA
HARA
WDU

Figure 6:

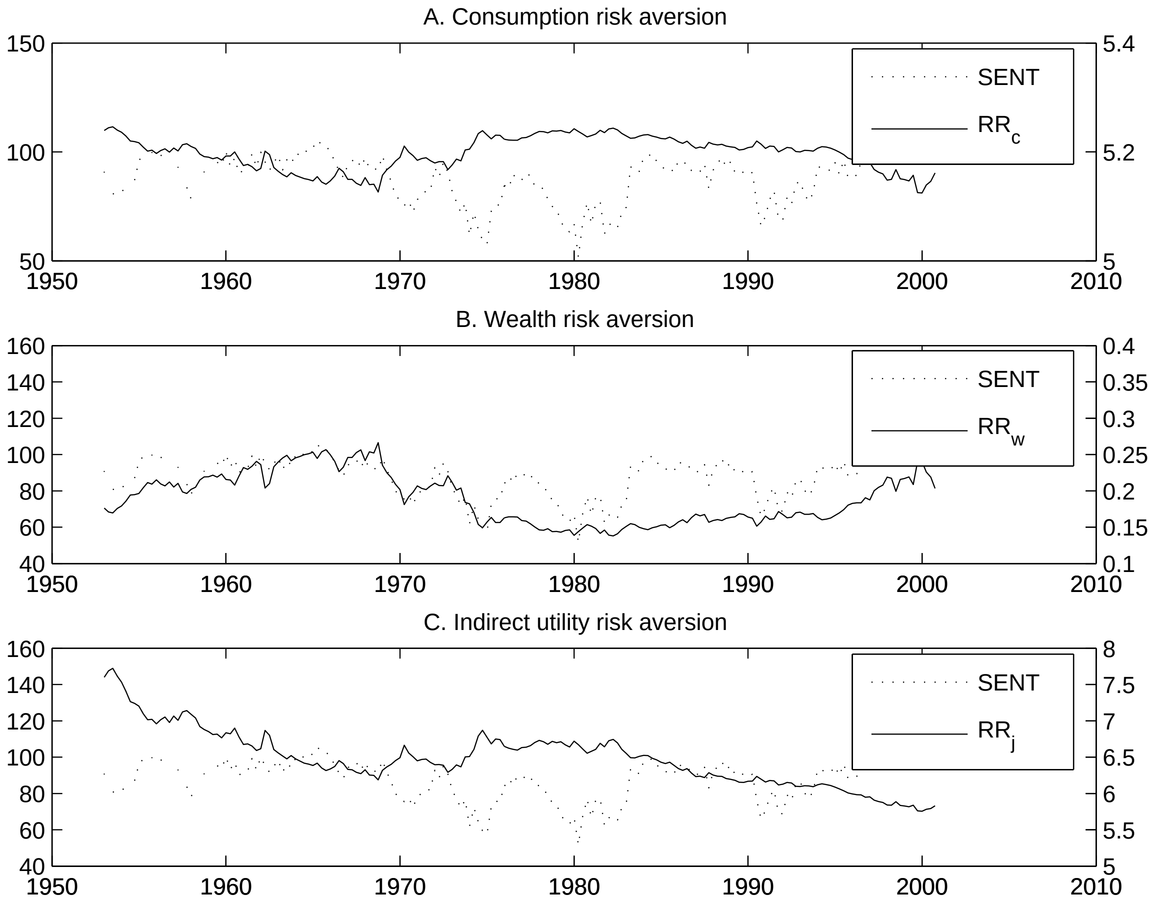
A. Consumption risk aversion
150
100
50
5.4
5.2
5
1950
1960
1970
1980
1990
2000
2010
SENT
RR_c
B. Wealth risk aversion
160
140
120
100
80
60
40
0.4
0.35
0.3
0.25
0.2
0.15
0.1
1950
1960
1970
1980
1990
2000
2010
SENT
RR_w
C. Indirect utility risk aversion
160
140
120
100
80
60
40
8
7.5
7
6.5
6
5.5
5
1950
1960
1970
1980
1990
2000
2010
SENT
RR_j

Figure 7: