Evolving Cityscapes: Agglomeration and Specialization with Mobile Labor and Vertical Linkages

Souleymane Coulibaly[a,b,*]

[a] *DEEP-HEC, University of Lausanne, Switzerland*

[b] *TEAM, University of Paris 1 Panthéon-Sorbonne, France*

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Abstract

In "new economic geography" models, spatial concentration typically arises either because of worker mobility or because of vertical linkages among firms. We examine a setup that combines those two approaches in conjunction with local congestion costs. We find that, as trade costs are lowered, the spatial concentration of total activity ("agglomeration") follows an inverse u-shaped evolution, while the degree of specialization of locations increases. The evolution of spatial configurations accommodated by this model is consistent with changes in sectorial employment patterns within US metropolitan areas over the 1850-1990 period.

J.E.L. classification: F12, R12, R15

Key words: agglomeration, specialization, congestion cost, input-output linkages

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*BFSH1 1015 Lausanne (Switzerland), Tel: +41 (0)21 692 3474, Fax: +41 (0)21 692 3365, e-mail: souleymane.coulibaly@unil.ch

1 Introduction

Imagine a city with two districts (a central city and its suburbs) and two industries with increasing returns to scale at a stage where transport costs between the two districts are prohibitive. Each industry will be evenly spread between the two locations to supply local demand at low cost, and thus the internal geography of the city will be made of two diversified districts. Imagine then that transport costs decrease, and that firms and their workers are allowed to move. Firms will tend to concentrate in a single location (the central city) to save on fixed costs, workers will move to this central city, but this will in turn crowd this location, so that some firms may in fact be better off locating in the suburb. In a such simple model, it is hard to say what will be the concentration pattern of the two industries within the two locations. It is further hard to say how this internal geography might evolve with falling trade costs and what will be its impact on the relative size of the two locations as they become more and more integrated.

Abdel-Rahman and Anas [2] consider this issue to primary importance in the latest Handbook of urban and regional economics. A glance at the economic geography literature indicates two types of outcome. Abdel-Rahman [1] proposes two configurations: a specialized configuration, where each location receives only one industry, and a diversified configuration, where each location hosts both industries. These configurations are determined by interactions between returns to scale and transport costs: when returns to scale are high, firms have an incentive to concentrate in one location, hence specialization; when transport costs prevail more, firms spread between locations to supply local demand at low cost, hence diversification. Duranton and Puga [4] propose a configuration where diversified and specialized locations coexist. The diversified locations are locations where innovating firms locate to develop their ideal production process and then switch to specialized locations for mass production.

Our framework can accommodate these two outcomes: when trade costs are high, firms spread evenly between the two locations to supply local demand at low cost, leading to diversified locations; at intermediate trade costs, agglomeration effects interact with congestion costs and transport costs to shape a diversified core and a specialized periphery; and at sufficiently low trade costs, each industry concentrates in one location, leading specialized locations. Patterns of urban evolution in the United States are consistent with such an outcome. The census micro data collected and harmonized by the Minnesota Population Center based on random samples of the American population drawn from fourteen federal censuses between 1850 and 2000 allow us to extract some salient facts.[1]

[1] www.ipums.org

See the reference section for authors details.

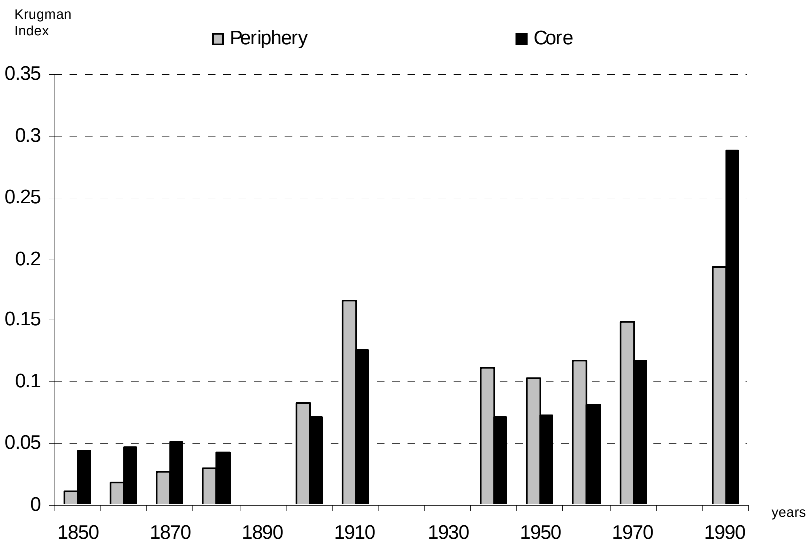

Figure 1: Specialization in central and peripheral US metropolitan districts.

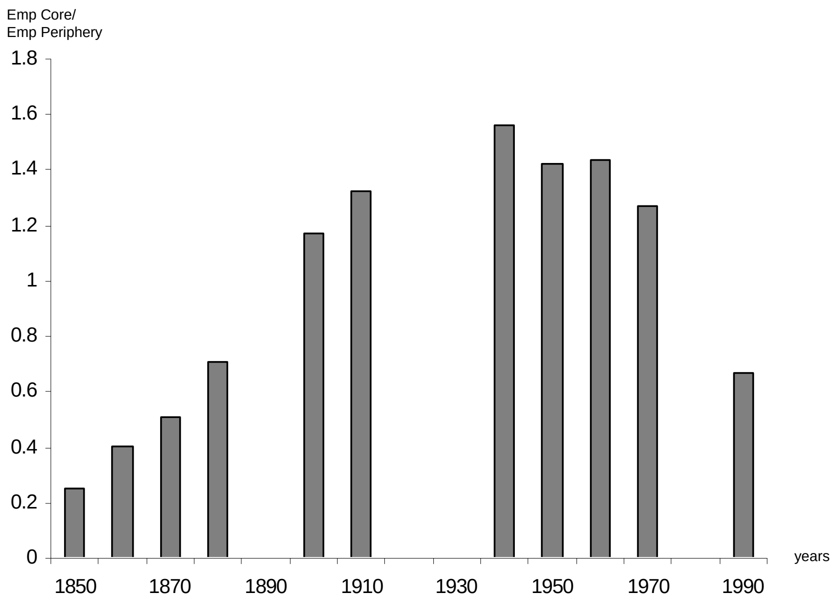

Figure 2: Relative employment in central and peripheral US metropolitan districts.

We focus on tradable goods and services and construct four aggregated industries: durable manufactured goods, non-durable manufactured goods, financial services and business services.[2] Figure 1 computes the Krugman bilateral specialization index in US city centers and suburbs which represent our two locations.[3] We observe an increasing specialization between the core (city centers) and the periphery (suburbs) over the period 1850-1990. Closer inspection of the data reveals that city centers have been specializing in financial and business services while suburbs are specializing in manufactures. Figure 2 shows the evolution of relative employment. It seems that central districts first received more workers until a peak around 1940, and then started losing employment relative to the suburbs.

These results correspond to the following spatial evolution: as integration proceeds within a city, specialization increases monotonically, in parallel with a non-monotonic agglomeration trend where center first gains and then loses workers. The aim of this paper is to build a model that reproduces this outcome. We analyze what we believe is a parsimonious model for the purpose at hand, building on well known analytical tools of the new economic geography. The model features both interregionally mobile labor and input-output linkages, thus combining the main locational forces of the "core-periphery" model initially developed by Krugman [7] and the "vertical linkages" model of Krugman and Venables [8].[4] In addition, our model has two imperfectly competitive sectors that differ in the intensity of their vertical linkages, and we add an exogenous congestion cost.[5]

We study our model in terms of its prediction in three dimensions of the spatial economy:

- the spatial distribution of aggregate activity, which we refer to as "agglomeration",
- the sectorial composition of locations, which we refer to as "specialization", and
- the tendency of different industries to locate in the same or in different locations, which we refer to as "co-location".

[2] Financial services include security and commodity brokerage and investment, insurance and real estate. Business services include advertising, accounting, auditing and book-keeping services.

[3] For most of the years, the IPUMS sample includes 1,000 individuals. We dropped years using a different sample size for the sake of coherence. This is why some years are missing in Figure 1 and 2.

[4] Puga [11] has developed a model that encompasses both the core-periphery and the vertical-linkages models as special cases. Our model differs from that of Puga by two asymmetries we introduce: the two sectors have different intensities of intra-industry linkages, and labor is sector-specific, that is each sector uses a specific type of labor so that workers can move between regions but not between sectors. This, arguably, makes our model more suitable to the analysis of relatively small-scale spatial reallocations such as those occurring inside individual metropolitan areas.

[5] Multi-sector models with vertical linkages have been developed by Venables [13] and Fujita, Krugman and Venables [6] in chapter 16. Our framework differs from theirs by the two asymmetries described in the previous footnote.

The qualitative predictions of the new economic geography models can be categorized into three types. The first type features dispersion and no specialization at high transports costs, agglomeration and specialization at intermediate transport costs, and finally agglomeration and specialization again at low transports costs (Krugman [7], Krugman and Venables [9] and Puga [11]). Models of the second type feature dispersion and specialization at high transport costs, agglomeration and specialization at intermediate transports costs and finally "redispersion" and "despecialization" at low transport costs (Krugman and Venables [8], Venables [13] and Puga [11]). The third type appears in chapter 16 of Fujita, Krugman and Venables [6] where there is dispersion at any transports costs associated with the following specialization pattern: no specialization at high transport costs, specialization at intermediate transport costs and "despecialization" at low transport costs.

Simulations of our model suggest a simple but striking evolution of the two-location economy as trade costs are gradually reduced. We find that, at early stages of integration, when trade cost are still very high, industries tend to split evenly between the locations, so that sectors co-locate within each location, and there is no specialization. When trade costs fall to some intermediate level, a core-periphery distinction emerges among the two locations: the strong-linkages industry partially, then totally, clusters in one location (the center) which also receives some weak-linkages industry firms. As trade costs keep decreasing, the weak-linkages firms located in the center start relocating to the periphery. Finally, once trade costs have fallen sufficiently low, locations completely specialize, and industries no longer co-locate.

The paper proceeds as follows: we present the building blocks of our model in Section 2; Section 3 reports simulation results that characterize the equilibrium configurations for changing trade costs and variations in other key parameters. We summarize the qualitative behavior of the model in Section 4, and Section 5 concludes.

2 The model

Our basic setup is as follows. We consider a two-location two-industry model. Trade between the two locations are of "iceberg" type τ, such that for each unit of a good shipped from location 1, only $1/\tau$ unit arrives in location 2 ($\tau > 1$). The economy consists of two monopolistically competitive industries producing differentiated goods x and y under increasing returns to scale. Each variety of each differentiated good is produced by a unique firm. For a differentiated good m ($m = x, y$), the number of varieties produced (and thus the number of firms located) in location r ($r = 1, 2$) is denoted $n_{m,r}$. Labor is sector specific, that is, there are $x-$type workers and y-type workers. These workers can move between regions but not between sectors.[6] We assume intra-industry input-output linkages, with stronger linkages in industry y than in industry x. There are no inter-industry linkages, so that the interaction between sectors is only through

[6] This assumption is empirically realistic (see for instance Miller, 1984, and Flinn, 1986).

general equilibrium effects. All workers are also consumers, and we write that $\lambda_{m,1}$ workers of industry m are located in region 1 and $(1-\lambda_{m,1})$ are located in region 2, with $0 \leq \lambda_{m,1} \leq 1$. Finally, we assume congestion costs within each region: as the number of firms in a region increases, the real wage of that region decreases by a factor δ. This is the easiest way to introduce significant congestion costs so as to counterbalance the two agglomeration forces (workers mobility and input-output linkages). These congestion costs can be thought of in a number of ways, such as the opportunity costs of commuting, environmental degradation or costs of immobile factors such as land. In Appendix 1, we describe a model that endogenises the congestion costs by including an agricultural sector with immobile workers.[7] The resulting equilibrium equations are similar, but it turns out that the congestion costs induced by the immobile agricultural sector are never sufficient to yield the spatial evolution we seek to reproduce in this paper.

2.1 Consumers

Let us focus on location 1 (the corresponding results for location 2 are analogously derived). All consumers are identical, and they consume all the varieties produced in the economy. They share the following Cobb-Douglas utility:

$$U = x^{\mu} y^{1-\mu}, \tag{1}$$

where $0<\mu<1$. Hence, consumers spend a share μ of their income on good x and $1-\mu$ on good y. x and y are Dixit-Stiglitz composites of varieties i:

$$x = \left(\int_0^{n_{x,1}} x_{i,1}^{(\sigma-1)/\sigma} di + \int_0^{n_{x,2}} x_{i,2}^{(\sigma-1)/\sigma} di \right)^{\sigma/(\sigma-1)}, \tag{2}$$

$$y = \left(\int_0^{n_{x,1}} x_{i,1}^{(\sigma-1)/\sigma} di + \int_0^{n_{x,2}} x_{i,2}^{(\sigma-1)/\sigma} di \right)^{\sigma/(\sigma-1)}. \tag{3}$$

The elasticity of substitution σ (with $\sigma > 1$) is assumed to be constant and identical for all the varieties of the two goods. Solving the consumer maximization problem yields the following price indices (see Appendix 2 for the derivations):

$$G_{m,1} = \left[n_{m,1} p_{m,1}^{1-\sigma} + n_{m,2} \left(\tau p_{m,2} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{4}$$

$$G_{m,2} = \left[n_{m,1} \left(\tau p_{m,1} \right)^{1-\sigma} + n_{m,2} p_{m,2}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{5}$$

where $p_{m,r}$ is the equilibrium price of all varieties of good m in location r. We can also derive the demand function for each variety in each location:

[7] Since we do not have a traditional sector, we have to find a numeraire. We cannot use a differentiated good as numeraire because markups vary with the intensity of returns to scale. Since nominal wages are simply set in each labor market, we thus use the wage of industry y in location 2, which will be always defined in the model, as the numeraire.

$$Q_{m,1} = E_{m,1} p_{m,1}^{-\sigma} G_{m,1}^{\sigma-1} + E_{m,2} p_{m,1}^{-\sigma} \tau^{1-\sigma} G_{m,2}^{\sigma-1}, \tag{6}$$

$$Q_{m,2} = E_{m,1} p_{m,2}^{-\sigma} \tau^{1-\sigma} G_{m,1}^{\sigma-1} + E_{m,2} p_{m,2}^{-\sigma} G_{m,2}^{\sigma-1}, \tag{7}$$

where $Q_{m,r}$ denotes the quantity of a given variety of good m produced in location r, and $E_{m,r}$ is the is the total expenditure on this variety in location r.

2.2 Producers

We assume that all firms share an identical production technology involving a fixed input F_m, which can differ between industries, and a unique constant marginal input γ. Both inputs are expressed in terms of a composite $Z_{m,r}$. Following Fujita *et al.* [6] in chapter 14, we assume that for each location this composite input can be expressed, up to a constant threshold, as $Z_{m,r} = l_{m,r}^{1-\alpha_m} \Psi_{m,r}^{\alpha_m}$, where $l_{m,r}$ denotes the quantity of labor, $\Psi_{m,r}$ is a CES composite of intermediate good for industry m in location r including all the varieties of good m, and α_m represents the share of intermediate inputs in the total production requirement for good m. The optimal combination of the composite inputs is derived in Appendix 3. Following standard simplifying practice, we assume that the substitution elasticity among varieties in the composite input Ψ equals the substitution elasticity in consumers' utility function, σ. Importantly, we impose that $\alpha_x < \alpha_y$, so that intermediate inputs have a lower weight in the production technology of industry x than in that of industry y.

A firm's total cost is $F_m + \gamma Q_{m,r}$, where $Q_{m,r}$ is the quantity produced. Profits of a firm in industry m and location r are:

$$\pi_{m,r} = p_{m,r} Q_{m,r} - w_{m,r}^{1-\alpha_m} G_{m,r}^{\alpha_m} \left(F_m + \gamma Q_{m,r}\right) \tag{8}$$

Firms with monopoly power set marginal revenue equal to marginal cost, where $MR = p_{m,r}\left(1 - \frac{1}{\varepsilon}\right)$, ε being the price elasticity of demand. Since, in monopolistic competition, $\varepsilon \to \sigma$ as $n \to \infty$, firms' optimization implies that:

$$p_{m,r}\left(1 - \frac{1}{\sigma}\right) = \gamma w_{m,r}^{1-\alpha_m} G_{m,r}^{\alpha_m}. \tag{9}$$

With free entry and exit in all industries, profits are driven to zero in equilibrium. Substituting (9) in (8) at the zero-profit equilibrium yields the optimal level of firm output $Q_m^* = F_m\left(\sigma - 1\right)/\gamma$ and the associated optimal input $Z_m^* = F_m + \gamma Q_m^* = F_m \sigma$. Since firms make zero profits in this scenario, their wage bill must be proportional to the total value of production, in accordance with the labor share of inputs, and hence $w_{m,r}\lambda_{m,r} = \left(1 - \alpha_m\right) n_{m,r} p_{m,r} Q_m^*$.

2.3 Normalizations and equilibrium

We can make some normalizations that simplify the model without loss of generality. First, following Fujita *et al.* [6], we impose that the marginal input requirement equals the constant markup, that is $\gamma = (\sigma - 1)/\sigma$, which implies that:

$$p_{m,r} = w_{m,r}^{1-\alpha_m} G_{m,r}^{\alpha_m}. \tag{10}$$

We can also choose the fixed input requirement F_m such that the equilibrium firm scale becomes $Q_m^* = (1-\alpha_m)^{-1}$. The value of a location's wage bill in each of these industries now simplifies to $w_{m,r}\lambda_{m,r} = n_{m,r}p_{m,r}$. Combining this equation with equations (4), (5) and (10) leads to the following expressions for the sectorial price indices in the two locations:

$$G_{m,1}^{1-\sigma} = \lambda_{m,1} w_{m,1}^{1-\sigma(1-\alpha_m)} G_{m,1}^{-\alpha_m\sigma} + \lambda_{m,2} w_{m,2}^{1-\sigma(1-\alpha_m)} G_{m,2}^{-\alpha_m\sigma} \tau^{1-\sigma}, \tag{11}$$

$$G_{m,2}^{1-\sigma} = \lambda_{m,1} w_{m,1}^{1-\sigma(1-\alpha_m)} G_{m,1}^{-\alpha_m\sigma} \tau^{1-\sigma} + \lambda_{m,2} w_{m,2}^{1-\sigma(1-\alpha_m)} G_{m,2}^{-\alpha_m\sigma}. \tag{12}$$

It is evident that the price index of an industry in a given location depends on the industry's wage rate in the location as well as on the price index of that sector in the other location. We can derive the wages associated with the optimal level of production using equations (6) and (7):

$$Q_{m,1} = E_{m,1} p_{m,1}^{-\sigma} G_{m,1}^{\sigma-1} + E_{m,2} p_{m,1}^{-\sigma} \tau^{1-\sigma} G_{m,2}^{\sigma-1} = Q_{m,1}^* = (1-\alpha_m)^{-1}, \tag{13}$$

$$Q_{m,2} = E_{m,1} p_{m,2}^{-\sigma} G_{m,1}^{\sigma-1} + E_{m,2} p_{m,2}^{-\sigma} \tau^{1-\sigma} G_{m,2}^{\sigma-1} = Q_{m,2}^* = (1-\alpha_m)^{-1}, \tag{14}$$

which we can re-write as:

$$\frac{p_{m,1}^{\sigma}}{1-\alpha_m} = E_{m,1} G_{m,1}^{\sigma-1} + E_{m,2} \tau^{1-\sigma} G_{m,2}^{\sigma-1}, \tag{15}$$

$$\frac{p_{m,2}^{\sigma}}{1-\alpha_m} = E_{m,1} G_{m,1}^{\sigma-1} + E_{m,2} \tau^{1-\sigma} G_{m,2}^{\sigma-1}. \tag{16}$$

Using the pricing rule (10) we obtain the following wage equations:

$$\left[w_{m,1}^{1-\alpha_m} G_{m,1}^{\alpha_m}\right]^{\sigma} = (1-\alpha_m)\left[E_{m,1} G_{m,1}^{\sigma-1} + E_{m,2} \tau^{1-\sigma} G_{m,2}^{\sigma-1}\right], \tag{17}$$

$$\left[w_{m,2}^{1-\alpha_m} G_{m,2}^{\alpha_m}\right]^{\sigma} = (1-\alpha_m)\left[E_{m,1} G_{m,1}^{\sigma-1} \tau^{1-\sigma} + E_{m,2} G_{m,2}^{\sigma-1}\right]. \tag{18}$$

Wages in the two sectors are linked through expenditures E, which take into account both final and intermediate consumption. At the zero-profit equilibrium, wages constitute the only source of income. Combining equation (1) with the optimal shares of the composite inputs (derived in Appendix 3), we can derive the following expenditure equations:

$$E_{m,r} = \mu_m \left(\lambda_{x,r} w_{x,r} + \lambda_{y,r} w_{y,r}\right) + \frac{\alpha_m}{1-\alpha_m} \lambda_{m,r} w_{m,r}, \tag{19}$$

where μ_m is μ for industry x and $1-\mu$ for industry y. The last step is to define the real wage equations. We assume that there are congestion costs, such that the real wage falls with the number of workers in a location. Specifically, we postulate the following real wage equation:

$$\omega_{m,r} = \frac{w_{m,r} \left(\lambda_{x,r} + \lambda_{y,r}\right)^{-\delta}}{G_{x,r}^{\mu} G_{y,r}^{1-\mu}}, \delta > 0, \tag{20}$$

where the exponent δ represents the real-wage reducing impact of congestion in each location. The full model consists of the sixteen non-linear equations (11), (12), (17), (18), (19) and (20) for $r = 1, 2$ and $m = x, y$. For a given allocation of labor between industries and locations $\lambda_{m,r}$, these equations define the short-run equilibrium, that is the market clearing price indices and wages. In the long run, sectorial labor moves between locations in response to real wage differences.[8]

We can summarize this model by describing the locational forces at work. There are two agglomeration forces: forward and backward linkages. These forces are due to the fact that firms tend to locate close to both the final and intermediate goods big markets. There are two dispersion forces: the market crowding effect (within each sector) and the congestion cost (within as well as across sectors). We now explore how these forces combine to shape the internal geography of our two-location economy as economic integration proceeds.

3 Numerical analysis

We are interested in the model's predictions regarding agglomeration, specialization and co-location at different levels of trade costs. Our definition of these location features is in terms of numbers of workers (rather than in, say, output values). Since the equilibrium equations derived in the previous section are highly non-linear, the model is not analytically tractable, and we have to resort to numerical analysis to explore equilibria.[9]

In the following, we will describe the equilibrium regime for two sets of parameters: α_x=0.40, α_y=0.45 , 0.65$\leq \delta \leq$0.70 and α_x=0.43, α_y=0.47 , 0.68$\leq \delta \leq$0.80 that allow consistent solutions and yield interesting results. We assume reasonable returns to scale ($\sigma = 4$) and an equal budget share for the two goods ($\mu = 1/2$). These two assumptions ensure the results not to depend on asymmetries (namely higher returns to scale or higher consumption share for one of the two goods) other than the two we assume in this paper: different intensity of intra-industry linkages, and inter-industry labor immobility. For these parameter combinations, the model accommodates four types of equilibria:

[8] We imply the usual *ad hoc* migration dynamics whereby the flow of migrants is a linear function of the real wage difference between the two locations (see Baldwin *et al.*, 2003, ch. 2, for a thorough discussion).

[9] All of the numerical computations were done using the software GAMS.

1. both industries are evenly spread between the two locations,
2. the two industries are each completely concentrated in a different location,
3. industry y is completely concentrated in one location and industry x is unevenly spread between the two locations,
4. industry x is evenly spread while industry y's share is higher in one location.

In terms of *agglomeration*, the distribution of aggregate labor (and hence activity) across locations, regimes 1 and 2 represent perfect dispersion, and regimes 3 and 4 represent partial agglomeration. In terms of *specialization*, locations' relative industry shares, regime 1 is completely diversified, regime 2 is completely specialized and regimes 3 and 4 are incompletely specialized. Finally, regarding *co-location*, the two industries are perfectly co-located in regime 1, partially co-located in regimes 3 and 4 and perfectly separated in regime 2.

3.1 Sustainability of completely specialized equilibria

As a first step, we explore the conditions under which a completely specialized equilibrium (where workers of each industry are completely concentrated in one location) is sustainable. Henceforth we assume that, if complete specialization applies, industry x clusters in location 1 and industry y in location 2, so that $\lambda_{x,1} = \lambda_{y,2} = 1$ and $\lambda_{x,2} = \lambda_{y,1} = 0$. For these values of $\lambda_{m,r}$, the congestion cost parameter δ does not matter in the real wage equations (20) and combining equations (11)-(12) and (20) yields the following relations between nominal and real wages:

$$\frac{\omega_{x,2}}{\omega_{x,1}} = \tau^{1-2\mu}\frac{w_{x,2}}{w_{x,1}}, \tag{21}$$

$$\frac{\omega_{y,1}}{\omega_{y,2}} = \tau^{2\mu-1}\frac{w_{y,1}}{w_{y,2}}. \tag{22}$$

Using the previous conditions on $\lambda_{m,r}$, equations (11)-(12) and (19) simplify, and we can substitute them into the wage equations to obtain the following expressions:

$$\frac{w_{x,1}^{(1-\alpha_x)\sigma}}{1-\alpha_x} = \left(\mu + \frac{\alpha_x}{1-\alpha_x}\right) w_{x,1}^{\sigma(1-\alpha_x)} + \mu w_{y,2} w_{x,1}^{\sigma(1-\alpha_x)-1}, \tag{23}$$

$$\frac{w_{x,2}^{(1-\alpha_x)\sigma}}{1-\alpha_x} = \left(\mu + \frac{\alpha_x}{1-\alpha_x}\right) w_{x,1}^{\sigma(1-\alpha_x)}\tau^{1-\sigma-\alpha_x\sigma} + \mu w_{y,2} w_{x,1}^{\sigma(1-\alpha_x)-1}\tau^{\sigma-\alpha_x\sigma-1}, \tag{24}$$

$$\frac{w_{y,1}^{(1-\alpha_y)\sigma}}{1-\alpha_y} = (1-\mu)\, w_{x,1} w_{y,2}^{\sigma(1-\alpha_y)-1} \tau^{\sigma-\alpha_y\sigma-1} + \begin{pmatrix} 1-\mu \\ +\frac{\alpha_y}{1-\alpha_y} \end{pmatrix} w_{y,2}^{\sigma(1-\alpha_y)} \tau^{1-\sigma-\alpha_y\sigma}, \tag{25}$$

$$\frac{w_{y,2}^{(1-\alpha_y)\sigma}}{1-\alpha_y} = (1-\mu)\, w_{x,1} w_{y,2}^{\sigma(1-\alpha_y)-1} + \left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right) w_{y,2}^{\sigma(1-\alpha_y)}. \tag{26}$$

We want to track how $\omega_{m,r}/\omega_{m,s}$ evolves with falling trade costs. We focus on the two sets of parameters described above and simultaneously solve equations (23)-(35) for different levels of trade costs. We can then compute the relevant relative real wages using equations (21) and (22) and plot them in Figure 3 (and Figure 17 in Appendix 8).

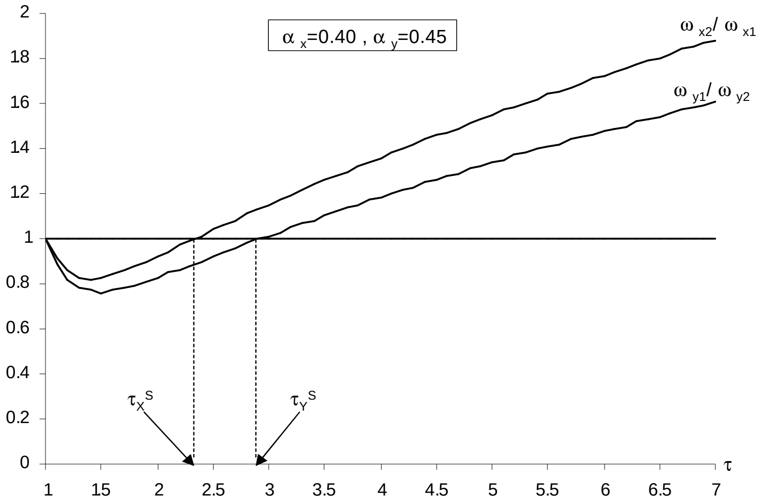

Figure 3: Sustain points of the two industries.

The sustain points are $\tau_x^S \simeq 2.3$ and $\tau_y^S \simeq 2.9$ for α_x=0.40, α_y=0.45 ($\tau_x^S \simeq 2.6$ and $\tau_y^S \simeq 3.3$ for α_x=0.43, α_y=0.47). Figure 3 shows that the two industries are completely concentrated in different locations at sufficiently low trade costs. For intermediate trade costs, complete concentration of the x industry in location 1 is no longer sustainable while industry y remains clustered in location 2. In this intermediate range, because of the existence of the congestion costs, industry x will not necessarily spread evenly across the two locations - an issue we will explore later on. For high trade costs, the agglomeration of the strong input-output industry is not sustainable either, and neither of the two industries is completely concentrated in one location.

3.2 Incompletely specialized equilibria

3.2.1 Sustainability of concentration of one industry

So far, we have consciously neglected a relevant but complicating fact: for $\tau > \tau_x^S$ (see Figure 3 above), $\lambda_{x,1}$ is no longer equal to 1, and thus we cannot retain only equations (23)-(26) to analyze the sustainability of the complete-concentration equilibrium of industry y. In fact, for $\tau > \tau_x^S$, the simulations underlying Figure 3 imply that we neglect the impact of congestion costs. In order to take into account congestion costs, we use the full expressions for the price index, expenditures and nominal and real wages, with the condition that $\lambda_{y,2} = 1$. This condition simplifies the industry y price index, but we now have an additional variable, $\lambda_{x,1}$. To close the model, we use the fact that workers in industry x migrate between locations 1 and 2 until the real wage in the two locations is equalized. This yields a system of nine equations with nine unknowns described in Appendix 4. The next step is to solve these nine non-linear equations numerically for different values of trade and congestion costs. Unfortunately, we cannot solve the equation system for our baseline parameters. But we can obtain some results by setting $\alpha_x = 0.25$ and $\alpha_y = 0.5$ rather than $\alpha_x = 0.40$ and $\alpha_y = 0.45$, with $\mu = 0.5$.

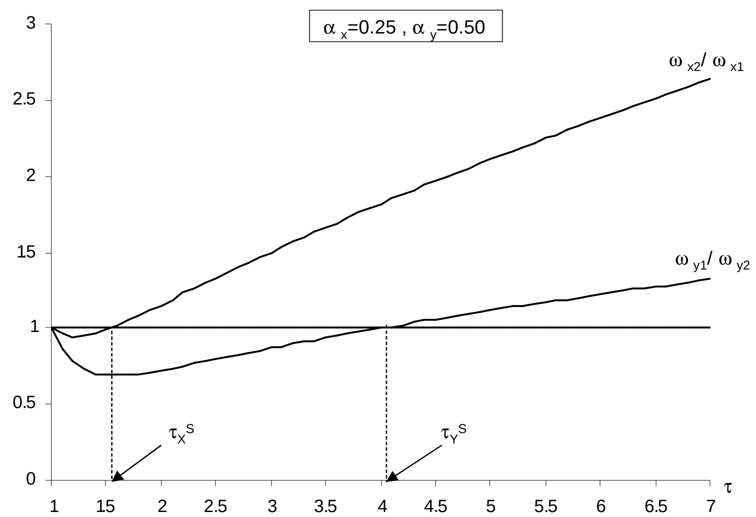

Figure 4: Sustain points ignoring congestion costs.

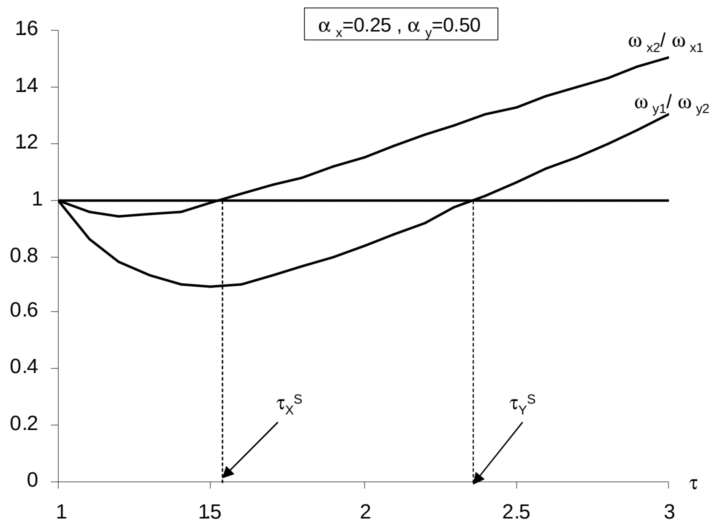

Figure 5: Sustain points including congestion costs.

Before correction (Figure 4) the sustain points was $\tau_x^S \simeq 1.54$ and $\tau_y^S \simeq 4$, and after correction (Figure 5) the values are $\tau_x^S \simeq 1.54$ and $\tau_y^S \simeq 2.3$. The lower value of τ_x^S in industry y is consistent, since the deviation of industry x from full concentration in location 1 induces a higher population size in location 2, and thus δ matters more in reducing concentration forces in industry y.

In sum, the analysis of sustain points suggests that economic integration will favor the concentration of industries in different locations, and the sector with stronger input-output linkages will become concentrated "earlier" than that with weaker input-output linkages.

3.2.2 Location of the dispersed industry

Figure 3 has shown an incomplete-specialization range of trade costs for which the concentration of the weak input-output industry in location 1 is not sustainable while the strong input-output linkages industry remains clustered in location 2. Now, we examine what happens to the non-concentrated industry in this parameter range.

In the incomplete-specialization range, we have that $\lambda_{y,1} = 0$ and $\lambda_{y,2} = 1$, i.e. industry y remains concentrated in location 2. We choose the wage of this industry in location 2 as the numeraire, setting $w_{y,2} = 1$. The complete concentration of industry y in location 2 is sustainable as long as the real wage in this location is higher than that in location 1 ($\omega_{y,2} > \omega_{y,1}$). These conditions simplify the equilibrium equations (11),(12), (17), (18), (19) and (20), as presented in Appendix 5.

The analysis consists of simultaneously solving these nine non-linear equations for different values of trade costs. We then compute the real wage differential in industry x, $\omega_{x,1} - \omega_{x,2}$ and plot it against $\lambda_{x,1}$. Figures 6-8 (and 18-20 in Appendix 8) plot the computed real wage differentials against labor shares in industry x for our baseline parameter combinations, that is α_x=0.40, α_y=0.45, 0.65≤ δ ≤0.70 and α_x=0.43, α_y=0.47, 0.68≤ δ ≤0.80.

The simulations lead to a consistent set of qualitative results:

- when trade costs are very low, the real wage gap is in favor of location 1 ($\omega_{x,1} - \omega_{x,2} > 0$), inciting industry x's workers to locate in location 1. Hence, we observe full concentration of industry x in this location;
- when trade costs increase, the real wage gap is negative for high values of $\lambda_{x,1}$ and positive for low values, so that we have a stable partial equilibrium $\lambda^*_{x,1}$;
- $\lambda^*_{x,1}$ decreases as trade costs increase and increases as congestion costs increase.

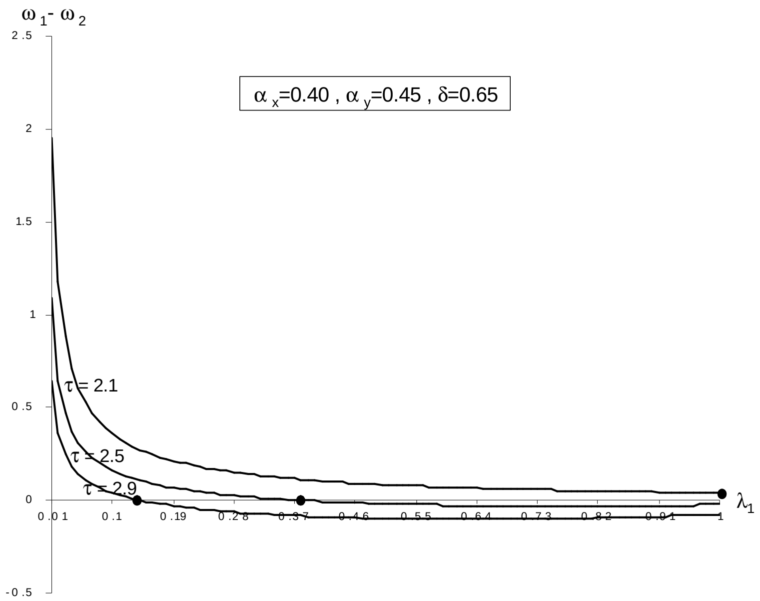

Figure 6: Real wage differential in industry x.

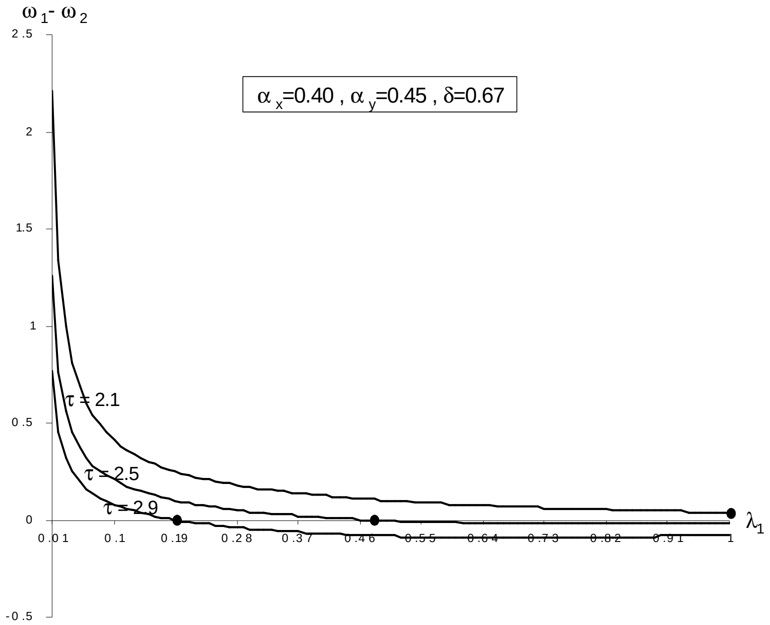

Figure 7: Real wage differential in industry x.

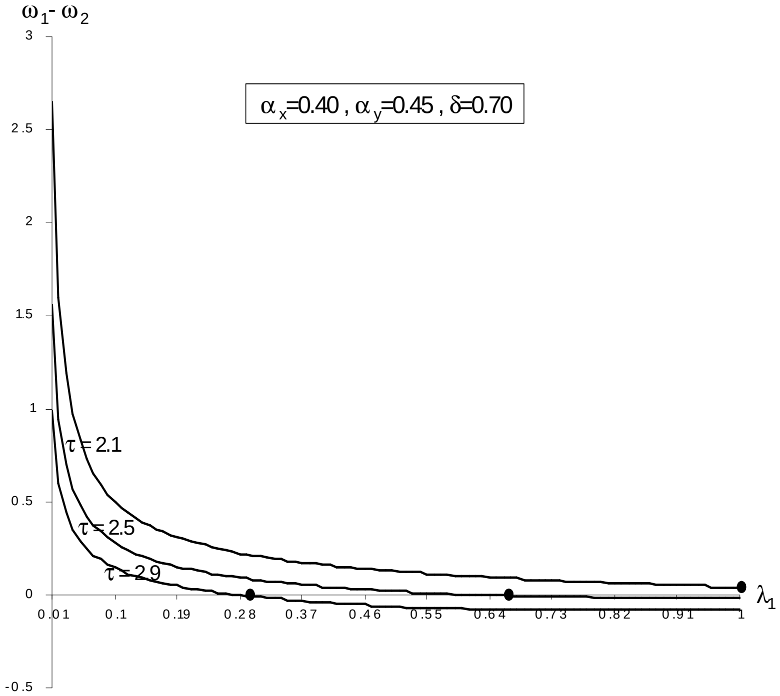

Figure 8: Real wage differential in industry x.

The first result reflects the sustain point analysis: when trade costs are low, firms can supply both markets at low cost, and because of vertical linkages they have an incentive to agglomerate in one location (location 1 for x-firms and location 2 for y-firms). If trade costs are higher, it becomes costly to supply remote consumers, and some x-firms will relocate to location 2, and the x-firms share in location 1 will be $\lambda^*_{x,1}$ rather than 1. It is obvious that the share of relocating x-firms in location 2 $(1 - \lambda^*_{x,1})$ will be higher the higher the trade costs. On the other hand, since the size of location 2 increases because of the relocation of some x-firms, congestion costs increase. These higher congestion costs will attenuate the incentive of x-firms to relocate in location 2, yielding a stable partial concentration of x-firms in location 2.

The pattern of equilibria is slightly different for α_x=0.43, α_y=0.47, 0.68$\leq$ δ $\leq$0.80 (see Figures 18-20 in Appendix 8), especially for relatively low congestion costs and trade costs. For these values, we obtain three equilibria: one full stable concentration, one partial unstable concentration and one partial stable concentration of industry x, instead of only one full stable concentration. For higher trade and congestion costs, the results are the same and are summarized in the following proposition.

Proposition 1 *At high trade costs, location 2 receives all y-firms and more than half of x-firms; as trade costs fall, the weak-linkages industry moves away from the strong-linkages industry, and location 1's share in x-firms increases. Once trade costs are low enough, the weak-linkages industry completely concentrates in location 1 and the strong-linkages industry totally concentrates in location 2.*

3.3 Stability of the symmetric equilibrium

Now we turn to the stability analysis of the perfectly dispersed equilibrium, where both industries are spread evenly across the two locations ($\lambda_{x,1} = \lambda_{x,2} = \lambda_{y,1} = \lambda_{y,2} = 0.5$). As stated above, our model differs from existing economic geography models (e.g. those mentioned in the introduction) in two key ways: we assume an asymmetry in the intensity of intra-industry linkages (industry x is assumed to have lower input-output linkages) and two types of labor, specific to each industry (workers can move between locations but only within the same industry). These two asymmetries substantially affect the usual perfectly dispersed equilibrium when trade costs are very high.

We are interested in the following question: "starting from a perfectly dispersed equilibrium, how does a reallocation of labor between locations affect relative real wages?" If relative wages change in favor of the location that receives the labor inflow, then the initial configuration was not a stable equilibrium. Conversely, if relative wages change in favor of the location from which labor has migrated, then the initial configuration was a stable equilibrium.

One specificity of our model is that we have two state variables: the weak-linkage industry labor allocation (λ_x) and the strong-linkages industry labor allocation (λ_y). This increases the complexity of the perfectly dispersed equilibrium. To make the model tractable, we refer to the two assumptions $d\lambda_x/d\lambda_y =$

$d\omega_x/d\lambda_y$ and $d\lambda_y/d\lambda_x = d\omega_y/d\lambda_x$, which means that a reallocation of labor in a given industry affects labor in the other industry through the variation induced in the real wage of this latter industry.[10]

We are interested in the variation in real wages due to labor reallocation $d\omega_x/d\lambda_x$ ($d\omega_y/d\lambda_y$). A positive value in this this variation suggests that labor reallocation implies a real wage gain, hence the perfectly dispersed equilibrium breaks. To solve the model, we focus on price indices, expenditures and nominal and real wages equations. The perfectly dispersed equilibrium implies that $G_{x,1} = G_{x,2} = G_x$, $G_{y,1} = G_{y,2} = G_y$, $w_{x,1} = w_{x,2} = w_x$, $w_{y,1} = w_{y,2} = w_y$. First, we have to evaluate the symmetric equilibrium values of the variables and then totally differentiate the system formed by price indices, expenditures and nominal and real wages.[11] These steps are described in detail in Appendices 6a and 6b. Our model is symmetric in the sense that $d\lambda_{x,1}$ = -$d\lambda_{x,2}$ = $d\lambda_x$, $d\lambda_{y,1}$ = -$d\lambda_{y,2}$ = $d\lambda_y$, $dw_{x,1}$ = -$dw_{x,2}$ = dw_x, $dw_{y,1}$ = -$dw_{y,2}$ = dw_y, $dG_{x,1}$ = -$dG_{x,2}$ = dG_x, $dG_{y,1}$ = -$dG_{y,2}$ = dG_y, $d\omega_{x,1}$ = -$d\omega_{x,2}$ = $d\omega_x$, $d\omega_{y,1}$ = -$d\omega_{y,2}$ = $d\omega_y$. At the perfectly dispersed equilibrium, we find that:

$$w_{x,1}= w_{x,2}= (1-\alpha_x)\,/\,(1-\alpha_y)$$
$$w_{y,1} = w_{y,2}=1$$
$$G_{x,1}= G_{x,2}= (1-\alpha_x)\,/\,(1-\alpha_y)\left[\left(1+\tau^{1-\sigma}\right)/2\right]^{1/[1-\sigma(1-\alpha_x)]}$$
$$G_{y,1} = G_{y,2}= \left[\left(1+\tau^{1-\sigma}\right)/2\right]^{1/[1-\sigma(1-\alpha_y)]}$$

Total differentiation around the perfectly dispersed equilibrium yields ten equations of interest: the derivative of G_m with respect to λ_m, the derivative of w_m with respect to λ_x and λ_y and the derivative of ω_m with respect to λ_x and λ_y.[12] Using the same benchmark parameter values as in the sustain point analysis, we can simultaneously solve these equations for different levels of trade and congestion costs. This allows us to plot $d\omega_x/d\lambda_x$ ($d\omega_y/d\lambda_y$) against τ for our baseline parameter values α_x=0.40, α_y=0.45 , 0.65$\leq \delta \leq$0.70 (and α_x=0.43, α_y=0.47 , 0.68$\leq \delta \leq$0.80 in Appendix 8). As long as these derivatives are negative, indicating that migration of workers to the other location reduces their real wage, perfect dispersion is a stable equilibrium.

[10] This assumption is an ad-hoc way to link migration to the real wage of the two industries. An alternative approach would be to set different levels of λ_x and λ_y exogenously and to combine the final effects, but this would increase the complexity of the simulations beyond the scope of this paper.

[11] The linear approximation to the function $y = f(x)$ around x^* and $y^* = f(x^*)$ involves computing $dy = \sum_{j=1}^{n} \frac{\partial f}{\partial x_j}(x^*)\, dx_j$. This is derived in Appendix 6b.

[12] Note that the derivative of G_x (G_y)with respect to λ_y (λ_x) is zero with the chosen functional forms.

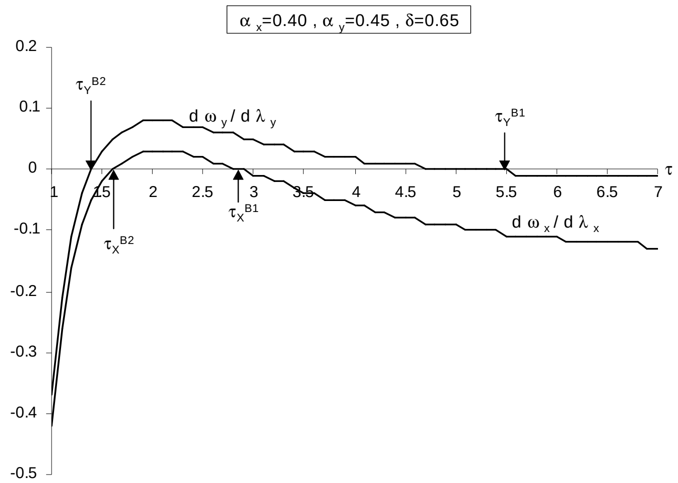

Figure 9: Stability of the perfectly dispersed equilibrium.

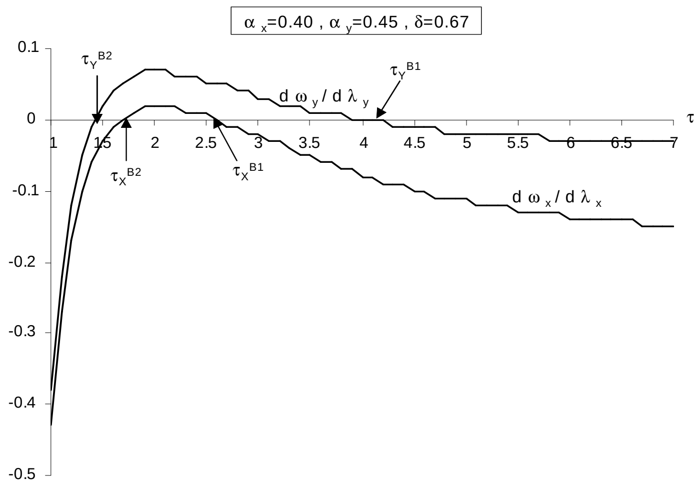

Figure 10: Stability of the perfectly dispersed equilibrium.

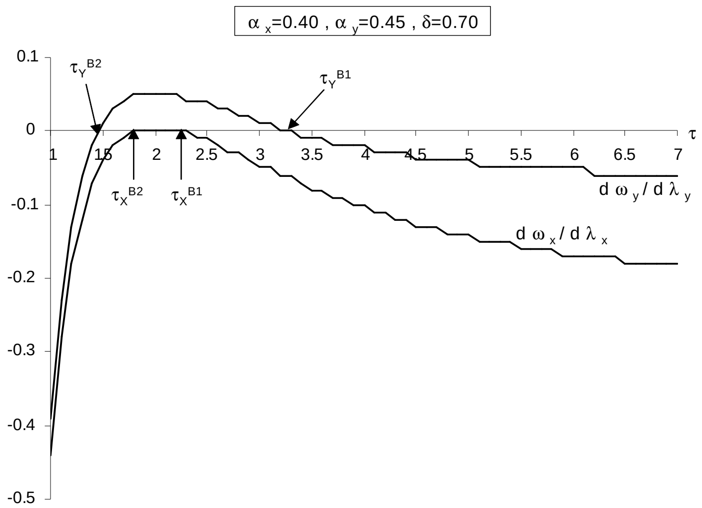

Figure 11: Stability of the perfectly dispersed equilibrium.

Figures 9-11 (Figures 21-23 in Appendix 8) plot $d\omega_x/d\lambda_x$ and $d\omega_x/d\lambda_y$ for different parameters values. We find that, when trade costs are very high, the two industries split evenly between the two locations to supply local consumers at low cost. Dispersion forces are stronger than agglomeration forces. As trade costs decrease, we reach a first break point τ_y^{B1} (equal to 5.5, 4.2 and 3.3 for δ=0.65, 0.67 and 0.70 respectively) at which the strong-linkages industry deviates from the symmetric equilibrium to concentrate in the center. Because of congestion costs, industry y will concentrate only partially until the sustain point τ_y^S, which is the trade cost level for which full concentration becomes sustainable. To get this partial agglomeration, we would have to solve the complete equations system defining the equilibrium (the four price index equations, the four expenditure equations, the four nominal wage equations and the four real wage equations). This system turns out to have no numerical solution. However, the existence of congestion costs helps us to infer that the strong-linkages industry will partially agglomerate in location 2, and that share will increase until full concentration as trade costs keep on decreasing.

The symmetric equilibrium in industry x breaks at a lower trade cost τ_x^{B1} (equal to 2.9, 2.6 and 2.3 for $\delta = 0.65$, 0.67 and 0.70 respectively) and the simulations indicate that $\tau_y^S > \tau_x^{B1}$, which suggests that when industry x deviates from the perfectly dispersed equilibrium, the results obtained in Section 3.2.2 describes x-firms' relocation behavior.

As trade costs keep decreasing, we reach a "reverse break point" first in industry x, τ_x^{B2} (equal to 1.6, 1.7 and 1.8 for $\delta = 0.65$, 0.67 and 0.70 respectively) and then in industry y, τ_y^{B2} (equal to 1.4, 1.5 and 1.5 for $\delta = 0.65$, 0.67 and 0.70 respectively). Our simulations indicate that τ_x^{B1} and τ_y^{B1} decrease as congestion costs increase, and that τ_x^{B2} and τ_y^{B2} increase as congestion costs increase. This means that, at very low level of trade costs, the perfectly dispersed equilibrium is a possible outcome.

We can summarize the break point analysis in the following proposition:

Proposition 2 *Starting from a symmetric equilibrium at high or low trade costs, the strong-linkages industry has a higher incentive to deviate from this equilibrium to exploit agglomeration externalities.*

3.4 The bifurcation diagram

In this section, we sum up the previous findings on firms location as trade and congestion costs vary. Our simulation results for the various specialized and dispersed configurations suggest a coherent pattern. At high trade costs, the two industries spread evenly between the two locations. As trade costs fall, the strong-linkages industry deviates first from the symmetric equilibrium and partially concentrates in one location. As trade costs decrease further, agglomeration forces matter more, and we end up with a full concentration of industry y in location 2. Meanwhile, the weak-linkages industry remains spread evenly between the two locations until a critical level of trade costs, $\tau_x^{B1}<\tau_y^S$, below which this industry partially concentrates in location 2. As trade costs keep on decreasing, agglomeration forces also matter more in industry x, and we end up with a full concentration of industry x in location 1. For very low trade costs, we can have a full concentration of each industry in one location, or an even spread of the two industries between the two locations.

Figure 12 illustrates the typical spatial evolution generated by our model. The bold lines represent industry y (with strong linkages) and the fine lines represents industry x (with weak linkages). τ_m^B and τ_m^S represent the break point and the sustain point of industry m respectively . The dashed bold line represents our inferred pattern for industry y when it deviates from the perfectly dispersed equilibrium and is not yet totally concentrated in location 2.[13]

[13] To have an estimation of what is going on for this trade costs range, we have to solve the whole equilibrium equations which is numerically infeasible.

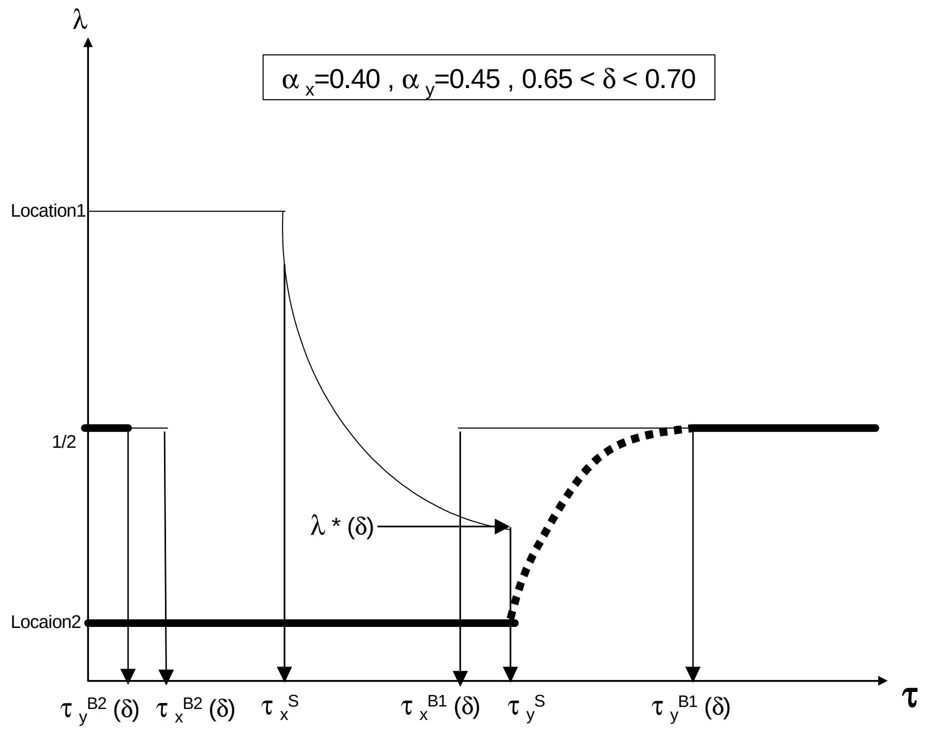

Figure 12: Bifurcation diagram.

Figure 12 reveals a rich pattern of firms location for trade costs in the range $\left[\tau_x^S; \tau_y^{B1}\right]$, and this differentiates our model from existing economic geography models cited in the introduction. We can distinguish three phases:

1. the weak-linkages industry is evenly spread between the two locations, while the strong-linkages industry partially concentrates in location 2;

2. the weak-linkages industry is still evenly spread between the two locations, while the strong-linkages industry is totally concentrated in location 2;

3. the strong-linkages industry remains concentrated in location 2, while the weak-linkages industry partially concentrates in location 1.

These phases correspond to the following internal geography of our economy:

- Phase1: location 1 receives half of industry x and less than half of industry y, while location 2 receives half of industry x and more than half of industry y. Location 2 is thus bigger than location 1 in terms of number of workers.

- Phase2: location 1 receives half of industry x and no y-firms, while location 2 receives half of industry x and all of the y-firms. Location 2 is again bigger than location 1.
- Phase 3: location 1 receives more than half of industry x and no y-firms, while location 2 receives less than half of industry x and all y-firms. Location 1's size increases and location 2's size decreases.

In addition to these three phases, we have the cases of even spread of the two industries at high trade costs and full agglomeration of each industry in one location at low trade costs.

Until now, we have focused on a set of baseline parameters. In the next section, we test the robustness of our findings with respect to departures from these baseline parameter values.

3.5 Robustness

One of the challenges of this paper is to retrieve relevant information from sixteen strongly non-linear equations representing price indices, wages, real wages and expenditures for the two locations and industries. We used different parameter combinations to analyze specialized equilibria and symmetric equilibria. It appeared that for intermediate values of α_x and α_y with α_x closer to α_y, and intermediate values of congestion costs, the break point analysis is identical to that obtained with the baseline parameter combinations.

3.5.1 Sustainability of completely specialized equilibria

The sustain point analysis can be reproduced for a wide range of parameters. For an overview of the impact of these parameters on $\omega_{m,r}/\omega_{m,s}$, we organized the simulation in two ways. First, we set $\alpha_x = 0.25$, $\alpha_y = 0.5$ and let μ vary from 0.1 to 0.9. Secondly, we set $\mu = 0.5$ and let α_x and α_y vary from 0.1 to 0.9, with $\alpha_x < \alpha_y$. Figures 13 and 14 summarize the findings, with a base scenario where $\mu = 0.5$, $\alpha_x = 0.25$ and $\alpha_y = 0.5$. For this base scenario, we find $\tau_x^S \approx 1.55$ and $\tau_y^S \approx 4$. The completely specialized equilibria are stable as long as the relative real wage curves are below 1.

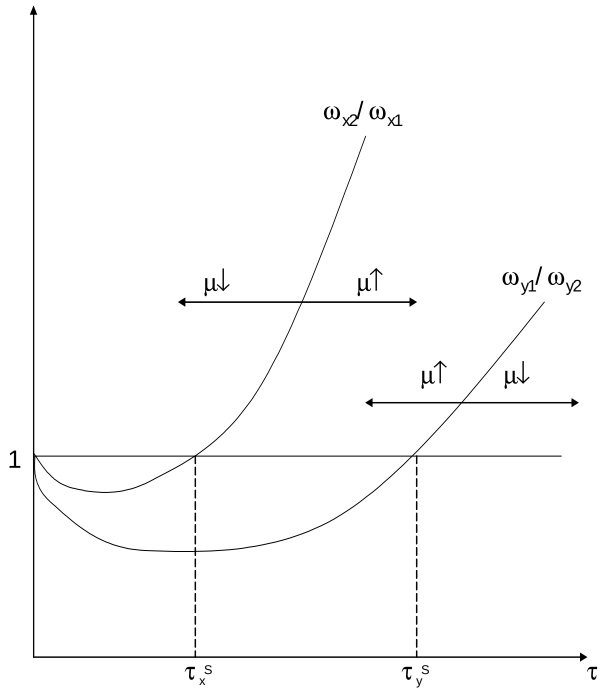

Figure 13: Relative real wage when μ varies.

Figure 13 illustrates how the sustain point analysis varies when the expenditure share (μ) of each of the two goods varies. When consumers prefer the y good (which has a high intermediates share in production) more strongly ($\mu < 0.5$), the real wage curve of this industry moves to the right, and this industry remains clustered in location 2 for higher value of trade costs. Conversely, the complete concentration of the weak input-output linkages industry breaks for lower trade costs. The reverse pattern holds when consumers shift expenditure towards the x good (weak input-output linkages, $\mu > 0.5$). The simulations show that for $\mu^* = 0.6$, the complete-concentration equilibria break at the same trade cost ($\tau^* \approx 2.35$) for the two industries. For $\mu > \mu^*$, the concentration of the y industry breaks before that of the x industry as trade costs increase. We can show analytically that for $\mu < \mu^*$, the concentration of the weak input-output linkages industry implies that of the strong input-output linkages (see Appendix 7 for the proof). Conversely, for $\mu > \mu^*$, the concentration of the strong input-output linkages industry implies that with the weak input-output linkages.

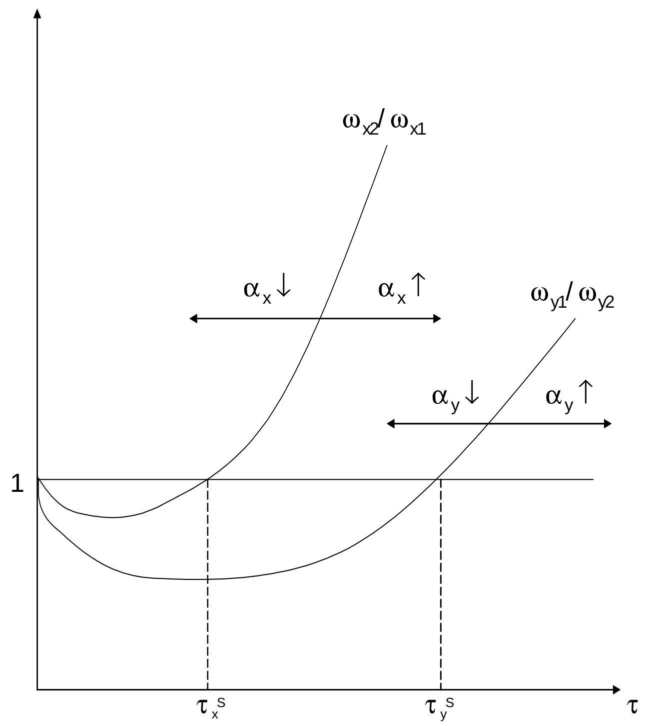

Figure 14: Relative real wages when α_x and α_y vary.

Figure 14 illustrates how these results vary when the intensities of the input-output linkages (α_x and α_y) are changed. We continue to assume that industry y uses more intermediate goods in its production process than industry x ($\alpha_x < \alpha_y$), but within each industry we can consider different levels of linkages, and the simulation shows that when this intra-industry linkage increases, the real wage curve of the two industries moves to the right and therefore the industries remain clustered for higher value of trade costs.

These simulations show that final expenditure shares and intensity of intermediate inputs act as substitutable concentration forces in this model: a higher expenditure share or a higher intensity of intra-industry linkages reinforce concentration.

3.5.2 Incomplete specialization

The analysis of incompletely specialized equilibria, where industry y is totally concentrated in location 2 while industry x is spread unevenly between locations, yield similar results for a wide range of parameter different to the baseline parameters. At high trade costs, location 2 receives all the y-firms and more than half of x-firms. As trade costs fall, the weak-linkages industry moves away from the strong-linkages industry, and location 1's share in x-firms increases. Once trade costs are low enough, the weak-linkages industry completely concentrates in location 1 and the strong-linkages industry completely concentrates in location 2.

3.5.3 Stability of the symmetric equilibrium

The simulations yield various configurations depending on the intensity of intra-industry linkages and congestion costs. We can summarize these results with the following four scenarios:

- For any intensity of intra-industry linkages and congestion cost and for very low trade costs, the symmetric equilibrium is always stable for the two industries.
- $d\omega_m/d\lambda_m > 0$ for higher trade costs, hence the symmetric equilibrium is never stable.
- $d\omega_m/d\lambda_m < 0$ for any level of trade costs, hence the symmetric equilibrium is always stable.
- $d\omega_y/d\lambda_y < 0$ for higher trade costs while $d\omega_x/d\lambda_x < 0$ for any level of trade costs: the symmetric equilibrium is never stable in the strong-linkages industry while it is always stable in the weak-linkages industry.

3.5.4 Adding a traditional sector

Appendix 1 describes the model when featuring an agricultural sector. This approach allows us to use the agricultural good as the numeraire. Most results obtained are qualitatively identical but the congestion costs induced by the immobile agricultural workers are never sufficient to yield the partial agglomeration of the x-industry that we obtained in section 3.2.2.

This exercise reveals that our results showing increasing specialization associated with a bell-shape agglomeration pattern are not due to our modelling of congestion costs. However, our formulation of congestion costs appears to be useful in yielding partial agglomeration (which of course is in line with empirical observations).

4 Agglomeration, specialization and co-location

Simulations of our model yield a rich set of locational predictions that are summarized in the bifurcation diagram of Figure 12. The behavior of our model becomes even clearer when we illustrate the equilibria of our model separately in terms of agglomeration, specialization and co-location. The following graphs focus on location 2 which we assume to constitute the central location.

4.1 Agglomeration

We define *agglomeration* in terms of the locational allocation of total labor. The typical configuration of equilibrium agglomeration levels at different levels of trade costs within location 2 is represented in Figure 15.

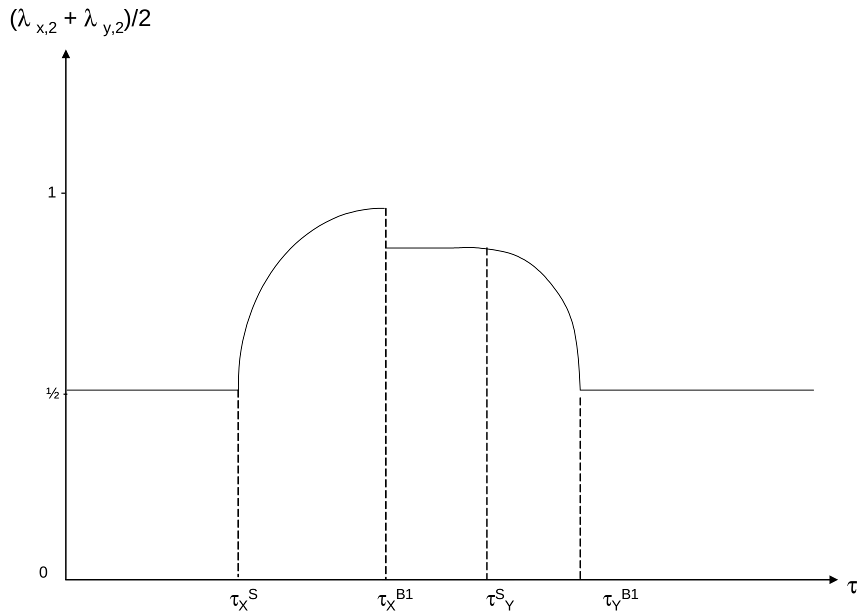

Figure 15: Comparative statics of agglomeration.

We find that agglomeration follows a bell-shape trajectory as trade costs are lowered. Total labor (and hence aggregate activity) is evenly spread between the two locations when trade costs are high and low. At intermediate trade costs, location 2's size increases while location 1's size decreases. As trade costs keep decreasing, the size of location 2 starts decreasing until perfect dispersion corresponding to two equalized locations' sizes.

4.2 Specialization and co-location

Specialization is defined in our model using the Herfindahl index, H=$(\lambda_{x,2}$ / $(\lambda_{x,2}+\lambda_{y,2}))^2$+$(\lambda_{y,2}$ / $(\lambda_{x,2}+\lambda_{y,2}))^2$, with 0.5$\leq$ H $\leq$1. This index is traced for different levels of trade costs in Figure 16.

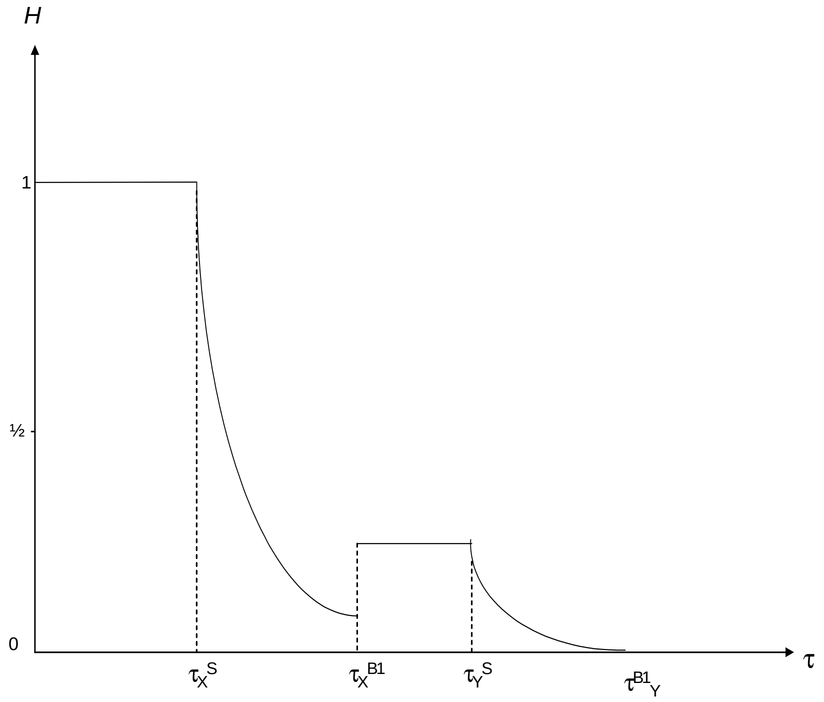

Figure 16: Comparative statics of specialization.

Figure 16 shows an increasing specialization of location 2 as trade costs are lowered. We have no specialization at high trade costs, since the two industries are evenly distributed between the two locations. As trade costs are lowered, some y-firms in location 1 relocate to location 2, hence increasing specialization in this location. As trade costs keep decreasing, the x-firms located in location 2 start relocating in location 1 and the specialization of location 2 in y-firms is reinforced while that of location 1 in x-firms is also reinforced. At low trade costs, the strong-linkages industry (y) is totally agglomerated in location 2 while the weak-linkages industry is totally concentrated in location 1; hence a perfect industrial specialization of each location. The decrease in specialization observed for τ_x^{B1} is due to the fact that when x-firms deviate from the symmetric equilibrium, they relocate to location 2 and thus reduce the relative share of this location in y-firms. But this gap is progressively corrected by the relocation of x-firms to location 1 because of higher congestion costs in the crowded central city.

Figure 16 also gives insights on the behavior of our model in terms of sectorial *co-location*. There is a decreasing trajectory of co-location in location 2 as trade costs are lowered. The two industries are perfectly co-located at high trade costs when the two industries are evenly distributed between the two locations. As trade costs decrease, some y-firms leave location 1 to relocate to location 2, reducing the relative share of industry x in location 2. As trade decrease further, the x-firms located in location 2 start relocating to location 1 and further reduce the degree of co-location of the two industries in location 2. At low levels of trade costs, there is no more co-location since each industry is clustered in one location.

Our results thus appear to depart from the other Dixit-Stiglitz-Krugman models: at high trade costs we have dispersion associated with no specialization and perfect co-location, at intermediate transport costs we have partial agglomeration, partial specialization and partial co-location, and finally at low transport costs, we have “redispersion” associated with perfect specialization and no co-location. This locational evolution is consistent with the stylized facts on US city centers and suburbs over the period 1850-1990 described in the introduction.

5 Conclusion

We have tracked locational equilibria in an integrating economy consisting of two locations, using a new economic geography model with two industries, two industry-specific interregionally mobile production factors and exogenous locational congestion costs. We assume that the two industries have different intensities of intra-industry linkages, and workers are allowed to move between regions but not between sectors. These assumptions make our model more suitable to the analysis of relatively small-scale spatial reallocations such as those occurring inside individual metropolitan areas, or regions within a country. We found that, at early stages of integration, industries tend to evenly split between locations so that sectors co-locate within each location with no specialization. When trade costs fall to an intermediate level, a core-periphery distinction emerges among the two locations: the strong-linkages industry partially then totally clusters in one location (the core) which also receives weak-linkages industry firms. As trade costs further decrease, the weak-linkages firms located in location 2 relocate to location 1 until full agglomeration. Finally, once trade costs have fallen sufficiently low, locations completely specialize, and industries no longer colocate. However, at those advanced levels of integration, the peripheral location recaptures activity from the core, so that the overall degree of agglomeration is reduced. The threshold values of trade cost, as well as the uniqueness or multiplicity of equilibria in certain parameter ranges, depend on the calibration of the model, in particular with respect to the expenditure shares of the two industries and to the importance of locational congestion costs. Our model accommodates the locational patterns of Abdel-Rahman [1] and Duranton and Puga [4]: we have first diversified locations when trade costs are high, at intermediate trade costs specialized and diversified locations coexist, and at low trade costs we have only specialized locations. Our results thus reproduce the stylized facts on US metropolitan employment patterns over the period 1850-1990.

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APPENDIX

Appendix 1: The Model with a Traditional Sector

In addition to industry x and y, we consider an agricultural sector producing a homogenous good under constant returns to scale. This good is traded costlessly and used as numeraire. There are agricultural-type workers who are immobile and we assume an initial even distribution of these workers between the two locations, that is $\lambda_{a,1} = \lambda_{a,2} = 1/2$.

Let us focus on a single location. All consumers are identical, and they consume all the goods produced in this location. They share the following Cobb-Douglas utility:

$$U = a^{\delta} x^{\mu} y^{1-\mu-\delta} \tag{27}$$

where $0 < \delta, \mu < 1$. This function means that consumers spend a share δ of their income on the agricultural good, a share μ on good x and $1-\mu-\delta$ on good y.

All the results derived in the paper are the same except for expenditures and real wage equations. Wages in the two monopolistic sectors are linked through expenditures E, which take into account both final and intermediate consumption. At the zero-profit equilibrium, wages constitute the only source of income. Equation (1) suggests that all consumers spend a share δ, μ and $1-\mu-\delta$ on the a, x and y goods respectively. Combining this with the condition of optimal share of the composite inputs (derived in Appendix 2), we can derive the following expenditure equations:

$$E_{m,r} = \mu_m \left(\lambda_{x,r} w_{x,r} + \lambda_{y,r} w_{y,r} + \lambda_{a,r}\right) + \frac{\alpha_m}{1-\alpha_m} \lambda_{m,r} w_{m,r} \tag{28}$$

where μ_m is μ for industry x and $1-\mu-\delta$ for industry y. The last step is to redefine the real wage equations, which are now only nominal wages deflated by the cost of living in a given location:

$$\omega_{m,r} = \frac{w_{m,r}}{G_{x,r}^{\mu} G_{y,r}^{1-\mu-\delta}}. \tag{29}$$

The sustain point analysis is conducted as in section 3.1 and yields similar results. Equations (21)-(35) are changed to:

$$\frac{\omega_{x,2}}{\omega_{x,1}} = \tau^{1-2\mu-\delta} \frac{w_{x,2}}{w_{x,1}} \tag{30}$$

$$\frac{\omega_{y,1}}{\omega_{y,2}} = \tau^{2\mu+\delta-1} \frac{w_{y,1}}{w_{y,2}} \tag{31}$$

$$\frac{w_{x,1}^{(1-\alpha_x)\sigma}}{1-\alpha_x} = \left(\mu + \frac{\alpha_x}{1-\alpha_x}\right) w_{x,1}^{\sigma(1-\alpha_x)} + \mu \left(1 + w_{y,2}\right) w_{x,1}^{\sigma(1-\alpha_x)-1} \tag{32}$$

$$\frac{w_{x,2}^{(1-\alpha_x)\sigma}}{1-\alpha_x} = \left(\mu + \frac{\alpha_x}{1-\alpha_x}\right) w_{x,1}^{\sigma(1-\alpha_x)} \tau^{1-\sigma-\alpha_x\sigma} + \mu w_{x,1}^{\sigma-\alpha_x\sigma-1} \left(\begin{array}{c} \frac{\tau^{1-\sigma-\alpha_x\sigma}+\tau^{\sigma-\alpha_x\sigma-1}}{2} \\ +w_{y,2}\tau^{\sigma-\alpha_x\sigma-1} \end{array} \right) \tag{33}$$

$$\frac{w_{y,1}^{(1-\alpha_y)\sigma}}{1-\alpha_y} = (1-\mu-\delta)\, w_{y,2}^{\sigma(1-\alpha_y)-1} \left(\frac{\begin{array}{c} w_{x,1}\tau^{\sigma-\alpha_y\sigma-1} + \\ \tau^{1-\sigma-\alpha_y\sigma} \\ +\tau^{\sigma-\alpha_y\sigma-1} \end{array}}{2} \right) + \left(\begin{array}{c} 1-\mu-\delta \\ +\frac{\alpha_y}{1-\alpha_y} \end{array} \right) w_{y,2}^{\sigma(1-\alpha_y)} \tau^{1-\sigma-\alpha_y\sigma} \tag{34}$$

$$\frac{w_{y,2}^{(1-\alpha_y)\sigma}}{1-\alpha_y} = (1-\mu-\delta)\,(1+w_{x,1})\, w_{y,2}^{\sigma(1-\alpha_y)-1} + \left(1-\mu-\delta+\frac{\alpha_y}{1-\alpha_y}\right) w_{y,2}^{\sigma(1-\alpha_y)} \tag{35}$$

The key difference comes from the real wage analysis. Indeed, for the range of trade costs that exists when the weak-linkages industry's complete agglomeration is no longer sustainable while the strong-linkages industry remains totally agglomerated in location 2, the real wage differential $\omega_{x,1} - \omega_{x,2}$ is always negative. This indicates that when industry x deviates from the completely agglomerated equilibrium, it collapses in location 2 where industry is still totally agglomerated. Hence, congestion costs induced by adding an agricultural sector appear to be not sufficiently high to yield a partial agglomeration of industry x as we obtain in the paper.

The break point analysis is also conducted as in section 3.3 but the results are slightly different: there is no reverse break point as in the case developed in the paper. Equations characterizing the break in Appendix 5b are now:

$$\frac{dG_x}{d\lambda_x} = Z\left(W^{\frac{1}{1-\sigma(1-\alpha)}}\left(\frac{2(1-\alpha)}{(1-\sigma)(1-\beta)} + \frac{1+\alpha\sigma-\sigma}{1-\sigma}\frac{dw_x}{d\lambda_x}\right) - \frac{\alpha\sigma}{1-\sigma}\frac{dG_x}{d\lambda_x}\right)$$

$$\frac{dG_y}{d\lambda_y} = Z\left(W^{\frac{1}{1-\sigma(1-\beta)}}\left(\frac{2}{1-\sigma} + \frac{1+\beta\sigma-\sigma}{1-\sigma}\frac{dw_y}{d\lambda_y}\right) - \frac{\beta\sigma}{1-\sigma}\frac{dG_y}{d\lambda_y}\right)$$

$$\frac{dw_x}{d\lambda_x} = \Theta^{\frac{1}{(1-\alpha)\sigma}} \left(\begin{array}{c} \Theta^{1-\sigma(1-\alpha)} Z\left(\frac{2}{\sigma}\frac{(1-\alpha)\mu+\alpha}{1-\beta} + \frac{(1-\alpha)\mu+\alpha}{(1-\alpha)\sigma}\frac{dw_x}{d\lambda_x} + \frac{\mu}{\sigma}\frac{dw_y}{d\lambda_x}\right) \\ +\frac{\sigma-\alpha\sigma-1-(\sigma+\alpha\sigma-1)\tau^{1-\sigma}}{2(1-\alpha)\sigma} W^{-\frac{2-(1-\alpha)\sigma}{1-(1-\alpha)\sigma}} \frac{dG_x}{d\lambda_x} \end{array} \right)$$

$$\frac{dw_x}{d\lambda_y} = \Theta^{\frac{1}{(1-\alpha)\sigma}+1-\sigma(1-\alpha)} Z\left(\frac{2\mu}{\sigma} + \frac{(1-\alpha)\mu+\alpha}{(1-\alpha)\sigma}\frac{dw_x}{d\lambda_y} + \frac{\mu}{\sigma}\frac{dw_y}{d\lambda_y}\right)$$

$$\frac{dw_y}{d\lambda_x} = \Phi^{\frac{1}{(1-\beta)\sigma}} \left(\Phi^{1-\sigma(1-\beta)} Z\left(\frac{2}{\sigma}\frac{(1-\alpha)}{1-\beta} + \frac{(1-\mu-\delta)}{\sigma}\frac{dw_x}{d\lambda_x} + \frac{(1-\mu-\delta)(1-\beta)+\beta}{\sigma}\frac{dw_y}{d\lambda_x}\right)\right)$$

$$\frac{dw_y}{d\lambda_y} = \Phi^{\frac{1}{(1-\beta)\sigma}} \left(\begin{array}{c} \Phi^{1-\sigma(1-\beta)} Z\left(\frac{2((1-\mu-\delta)(1-\beta)+\beta)}{(1-\beta)\sigma} + \frac{(1-\mu-\delta)}{\sigma}\frac{dw_x}{d\lambda_y} + \frac{(1-\mu-\delta)(1-\beta)+\beta}{\sigma}\frac{dw_y}{d\lambda_y}\right) \\ +\frac{\sigma-\beta\sigma-1-(\sigma+\beta\sigma-1)\tau^{1-\sigma}}{2(1-\beta)\sigma} W^{-\frac{2-(1-\beta)\sigma}{1-(1-\beta)\sigma}} \frac{dG_y}{d\lambda_y} \end{array} \right)$$

$$\frac{d\omega_x}{d\lambda_x} = W^{\Gamma}\left(\frac{1-\beta}{1-\alpha}\right)^{-\mu}\left(\frac{dw_x}{d\lambda_x} - \mu W^{-\frac{1}{1-\sigma+\alpha\sigma}}\frac{dG_x}{d\lambda_x}\right)$$

$$\frac{d\omega_y}{d\lambda_y} = \left(\frac{1-\alpha}{1-\beta}\right)^{-\mu} W^{\Gamma}\left(W^{-\frac{1}{1-\sigma+\alpha\sigma}}\frac{dw_y}{d\lambda_y} - (1-\mu-\delta)\, W^{-\frac{1}{1-\sigma(1-\beta)}}\frac{dG_y}{d\lambda_y}\right)$$

with:

$$\Gamma = -\frac{\mu}{1-\sigma+\alpha\sigma} - \frac{1-\mu-\delta}{1-\sigma+\beta\sigma}$$
$$\Theta = (1-\alpha)\mu + \alpha + 2\mu(1-\beta)$$
$$\Phi = (1-\beta)(1-\mu-\delta) + \beta + 2(1-\mu-\delta)(1-\alpha)$$
$$Z = \frac{1-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}, W = \frac{1+\tau^{1-\sigma}}{2}.$$

Notice that since the real wage $\omega_{m,r}$ does not depend on $\lambda_{m,r}$, $\frac{d\omega_x}{d\lambda_y} = 0$ and $\frac{d\omega_y}{d\lambda_x} = 0$.

Finally, adding a traditional sector yields the same qualitative results but the congestion costs induced by this sector with immobile workers are not sufficient to induce a partial agglomeration of industry x.

Appendix 2: Derivation of the Price Index

The efficient consumption of each variety of good m ($m = x, y$) is the solution to the following minimization problem (which is in fact the dual of the utility maximization problem commonly considered):

$$Min\ \left\{\int_0^{n_{m,1}} m_{i,1}p_{m,1}di + \int_0^{n_{m,2}} m_{i,2}p_{m,2}\tau di\right\}$$
$$s.t.\ m = \left(\int_0^{n_{m,1}} m_{i,1}^{\frac{\sigma-1}{\sigma}} di + \int_0^{n_{m,2}} m_{i,2}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}.$$

$p_{m,r}$ is the price of good m in location r (knowing that all the varieties of a good have the same price). The Lagrangian for this problem is then:

$$\pounds = \int_0^{n_{m,1}} m_{i,1}p_{m,1}di + \int_0^{n_{m,2}} m_{i,2}p_{m,2}\tau di +$$
$$G_m \left\{ m - \left(\int_0^{n_{m,1}} m_{i,1}^{\frac{\sigma-1}{\sigma}} di + \int_0^{n_{m,2}} m_{i,2}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \right\}.$$

G_m, the Lagrange multiplier of the problem, represents the price index of the composite good m. The first order conditions of this problem are:

$$p_{m,1} = G_m m_{i,1}^{-\frac{1}{\sigma}} m^{\frac{1}{\sigma-1}}$$
$$\tau p_{m,2} = G_m m_{i,2}^{-\frac{1}{\sigma}} m^{\frac{1}{\sigma-1}}.$$

These expressions can be re-written as:

$$p_{m,1}^{1-\sigma} m = G_m^{1-\sigma} m_{i,1}^{\frac{\sigma-1}{\sigma}}$$
$$(\tau p_{m,2})^{1-\sigma} m = G_m^{1-\sigma} m_{i,2}^{\frac{\sigma-1}{\sigma}}.$$

Summing over i and combining these two relations yields the expression of the price index for each of the two goods m.

Appendix 3: Deriving the Optimal Input Allocation Rule

In each industry, the optimal way to combine labor and the composite intermediate good follows from the cost minimization problems for $m = \{x, y\}$:

$$
\begin{gathered}
Min\left\{w_{m,r}l_{m,,r} + G_{m,r}\Psi_{m,r}\right\} \\
s.t.\, Z_{m,r} = l_{m,r}^{1-\alpha_m}\Psi_{m,r}^{\alpha_m}.
\end{gathered}
$$

$w_{m,r}$ is the nominal wage rate in industry m and location r, and we can derive the optimal input allocation rule:

$$
\frac{\Psi_{m,r}}{l_{m,r}} = \frac{\alpha_m}{1-\alpha_m}\frac{w_{m,r}}{G_{m,r}} \tag{36}
$$

Appendix 4: System of Equations Solving for the Agglomerated Equilibrium of Industry y

Note: Recall that $\lambda_{x,2} = 1 - \lambda_{x,1}$.

$$
\begin{aligned}
G_{x,1}^{1-\sigma} &= \lambda_{x,1}w_{x,1}^{1-\sigma(1-\alpha_x)}G_{x,1}^{-\alpha_x\sigma} + \lambda_{x,2}w_{x,2}^{1-\sigma(1-\alpha_x)}G_{x,2}^{-\alpha_x\sigma}\tau^{1-\sigma} \\
G_{x,2}^{1-\sigma} &= \lambda_{x,1}w_{x,1}^{1-\sigma(1-\alpha_x)}G_{x,1}^{-\alpha_x\sigma}\tau^{1-\sigma} + \lambda_{x,2}w_{x,2}^{1-\sigma(1-\alpha_x)}G_{x,2}^{-\alpha_x\sigma} \\
w_{x,1}^{(1-\alpha_x)\sigma} &= (1-\alpha_x)\begin{pmatrix} \left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,1}w_{x,1}G_{x,1}^{\sigma-\alpha_x\sigma-1} + \\ \left(\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,2}w_{x,2} + \mu w_{y,2}\right)\tau^{1-\sigma}G_{x,1}^{-\alpha_x\sigma}G_{x,2}^{\sigma-1} \end{pmatrix} \\
w_{x,2}^{(1-\alpha_x)\sigma} &= (1-\alpha_x)\begin{pmatrix} \left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,1}w_{x,1}G_{x,1}^{\sigma-1}G_{x,2}^{-\alpha_x\sigma}\tau^{1-\sigma} + \\ \left(\left(\mu + \frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,2}w_{x,2} + \mu w_{y,2}\right)G_{x,2}^{\sigma-\alpha_x\sigma-1} \end{pmatrix} \\
w_{y,1}^{(1-\alpha_y)\sigma} &= (1-\alpha_y)\begin{pmatrix} (1-\mu)\,\lambda_{x,1}w_{x,1}\tau^{\sigma-\alpha_y\sigma-1}w_{y,2}^{\sigma(1-\alpha_y)-1} + \\ \left(\mu\lambda_{x,2}w_{x,2} + \left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right)w_{y,2}\right)w_{y,2}^{\sigma(1-\alpha_y)-1}\tau^{1-\sigma-\alpha_y\sigma} \end{pmatrix} \\
w_{y,2}^{(1-\alpha_y)\sigma} &= (1-\alpha_y)\begin{pmatrix} (1-\mu)\,\lambda_{x,1}w_{x,1}w_{y,2}^{\sigma(1-\alpha_y)-1} + \\ \left(\mu\lambda_{x,2}w_{x,2} + \left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right)w_{y,2}\right)w_{y,2}^{\sigma(1-\alpha_y)-1} \end{pmatrix} \\
\frac{\omega_{x,2}}{\omega_{x,1}} &= \frac{w_{x,2}(\lambda_{x,2}+1)^{-\delta}}{w_{x,1}\lambda_{x,1}^{-\delta}}\frac{G_{x,1}^{\mu}\tau^{1-\mu}}{G_{x,2}^{\mu}} \\
\frac{\omega_{y,1}}{\omega_{y,2}} &= \frac{w_{y,1}\lambda_{x,1}^{-\delta}}{w_{y,2}(\lambda_{x,2}+1)^{-\delta}}\frac{G_{x,2}^{\mu}}{G_{x,1}^{\mu}\tau^{1-\mu}} \\
w_{x,1}\lambda_{x,1}^{-\delta}G_{x,2}^{\mu} &= w_{x,2}\left(1+\lambda_{x,2}\right)^{-\delta}G_{x,1}^{\mu}\tau^{1-\mu}
\end{aligned}
$$

Appendix 5: System of Equations Solving the Real Wage Differential in the Weak-Input-Output Linkages Industry

Notice that in these equations, $\lambda = \lambda_{x,2}$

$$
\begin{aligned}
G_{x,1}^{1-\sigma} &= (1-\lambda)\, w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma} + \lambda w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma}\tau^{1-\sigma} \\
G_{x,2}^{1-\sigma} &= (1-\lambda)\, w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma}\tau^{1-\sigma} + \lambda w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma} \\
w_{x,1}^{(1-\alpha_x)\sigma} G_{x,1}^{\alpha_x\sigma} &= \left[\begin{array}{c} (\mu(1-\alpha_x)+\alpha_x)(1-\lambda)\, w_{x,1} G_{x,1}^{\sigma-1} + \\ ((\mu(1-\alpha_x)+\alpha_x)\lambda w_{x,2} + \mu(1-\alpha_x))\, G_{x,2}^{\sigma-1}\tau^{1-\sigma} \end{array}\right] \\
w_{x,2}^{(1-\alpha_x)\sigma} G_{x,2}^{\alpha_x\sigma} &= \left[\begin{array}{c} (\mu(1-\alpha_x)+\alpha_x)(1-\lambda)\, w_{x,1} G_{x,1}^{\sigma-1}\tau^{1-\sigma} \\ +((\mu(1-\alpha_x)+\alpha_x)\lambda w_{x,2} + \mu(1-\alpha_x))\, G_{x,2}^{\sigma-1} \end{array}\right] \\
w_{y,1}^{(1-\alpha_y)\sigma}\tau^{\alpha_y\sigma} &= \left[\begin{array}{c} \mu(1-\alpha_y)(1-\lambda)\, w_{x,1}\tau^{\sigma-1} + \\ (\lambda(1-\mu)(1-\alpha_y)\, w_{x,2} + (1-\mu)(1-\alpha_y)+\alpha_y)\,\tau^{1-\sigma} \end{array}\right] \\
\omega_{x,1} G_{x,1}^{\mu}\tau^{1-\mu} &= w_{x,1}(1-\lambda)^{-\delta} \\
\omega_{x,2} G_{x,2}^{\mu} &= w_{x,2}(1+\lambda)^{-\delta} \\
\omega_{y,1} G_{x,1}^{\mu}\tau^{1-\mu} &= w_{y,1}(1-\lambda)^{-\delta} \\
\omega_{y,2} G_{x,2}^{\mu} &= (1+\lambda)^{-\delta}
\end{aligned}
$$

Appendix 6a: Stability Analysis of the Dispersed Equilibrium

If we substitute the expenditures equations in the wage equations, the expressions we have to totally differentiate are the following:

$$
\begin{aligned}
G_{x,1} &= \left(\lambda_{x,1} w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma} + \lambda_{x,2} w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma}\tau^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
G_{x,2} &= \left(\lambda_{x,1} w_{x,1}^{1-\sigma(1-\alpha_x)} G_{x,1}^{-\alpha_x\sigma}\tau^{1-\sigma} + \lambda_{x,2} w_{x,2}^{1-\sigma(1-\alpha_x)} G_{x,2}^{-\alpha_x\sigma}\right)^{\frac{1}{1-\sigma}} \\
G_{y,1} &= \left(\lambda_{y,1} w_{y,1}^{1-\sigma(1-\alpha_y)} G_{y,1}^{-\alpha_y\sigma} + \lambda_{y,2} w_{y,2}^{1-\sigma(1-\alpha_y)} G_{y,2}^{-\alpha_y\sigma}\tau^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
G_{y,2} &= \left(\lambda_{y,1} w_{y,1}^{1-\sigma(1-\alpha_y)} G_{y,1}^{-\alpha_y\sigma}\tau^{1-\sigma} + \lambda_{y,2} w_{y,2}^{1-\sigma(1-\alpha_y)} G_{y,2}^{-\alpha_y\sigma}\right)^{\frac{1}{1-\sigma}}
\end{aligned}
$$

$$w_{x,1}=(1-\alpha_x)^{\frac{1}{(1-\alpha_x)\sigma}}\left(\begin{array}{c}\left[\begin{array}{c}\left(\mu+\frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,1}w_{x,1}\\+\mu\lambda_{y,1}w_{y,1}\end{array}\right]G_{x,1}^{\sigma-\alpha_x\sigma-1}+\\\left[\begin{array}{c}\left(\mu+\frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,2}w_{x,2}\\+\mu\lambda_{y,2}w_{y,2}\end{array}\right]G_{x,1}^{-\alpha_x\sigma}G_{x,2}^{\sigma-1}\tau^{1-\sigma}\end{array}\right)^{\frac{1}{(1-\alpha_x)\sigma}}$$

$$w_{x,2}=(1-\alpha_x)^{\frac{1}{(1-\alpha_x)\sigma}}\left(\begin{array}{c}\left[\begin{array}{c}\left(\mu+\frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,1}w_{x,1}\\+\mu\lambda_{y,1}w_{y,1}\end{array}\right]G_{x,1}^{\sigma-1}G_{x,2}^{-\alpha_x\sigma}\tau^{1-\sigma}+\\\left[\begin{array}{c}\left(\mu+\frac{\alpha_x}{1-\alpha_x}\right)\lambda_{x,2}w_{x,2}\\+\mu\lambda_{y,2}w_{y,2}\end{array}\right]G_{x,2}^{\sigma-\alpha_x\sigma-1}\end{array}\right)^{\frac{1}{(1-\alpha_x)\sigma}}$$

$$w_{y,1}=(1-\alpha_y)^{\frac{1}{(1-\alpha_y)\sigma}}\left(\begin{array}{c}\left[\begin{array}{c}(1-\mu)\lambda_{x,1}w_{x,1}+\\\left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right)\lambda_{y,1}w_{y,1}\end{array}\right]G_{y,1}^{\sigma-\alpha_y\sigma-1}+\\\left[\begin{array}{c}(1-\mu)\lambda_{x,2}w_{x,2}+\\\left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right)\lambda_{y,2}w_{y,2}\end{array}\right]G_{y,1}^{-\alpha_y\sigma}G_{y,2}^{\sigma-1}\tau^{1-\sigma}\end{array}\right)^{\frac{1}{(1-\alpha_y)\sigma}}$$

$$w_{y,2}=(1-\alpha_y)^{\frac{1}{(1-\alpha_y)\sigma}}\left(\begin{array}{c}\left[\begin{array}{c}(1-\mu)\lambda_{x,1}w_{x,1}+\\\left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right)\lambda_{y,1}w_{y,1}\end{array}\right]G_{y,1}^{\sigma-1}G_{y,2}^{-\alpha_y\sigma}\tau^{1-\sigma}+\\\left[\begin{array}{c}(1-\mu)\lambda_{x,2}w_{x,2}+\\\left(1-\mu+\frac{\alpha_y}{1-\alpha_y}\right)\lambda_{y,2}w_{y,2}\end{array}\right]G_{y,2}^{\sigma-\alpha_y\sigma-1}\end{array}\right)^{\frac{1}{(1-\alpha_y)\sigma}}$$

$$\omega_{x,1}=\frac{w_{x,1}(\lambda_{x,1}+\lambda_{y,1})^{-\delta}}{G_{x,1}^{\mu}G_{y,1}^{1-\mu}}$$

$$\omega_{x,2}=\frac{w_{x,2}(\lambda_{x,2}+\lambda_{y,2})^{-\delta}}{G_{x,2}^{\mu}G_{y,2}^{1-\mu}}$$

$$\omega_{y,1}=\frac{w_{y,1}(\lambda_{y,1}+\lambda_{x,1})^{-\delta}}{G_{x,1}^{\mu}G_{y,1}^{1-\mu}}$$

$$\omega_{y,2}=\frac{w_{y,2}(\lambda_{x,2}+\lambda_{y,2})^{-\delta}}{G_{x,2}^{\mu}G_{y,2}^{1-\mu}}$$

Appendix 6b: Price Index and Nominal Wages at the Symmetric Equilibrium

Let consider the optimal input allocation rules:

$$\frac{X_r^I}{\lambda_{x,r}}=\frac{\alpha_x}{1-\alpha_x}\frac{w_{x,r}}{G_{x,r}}\tag{37}$$

$$\frac{Y_r^I}{\lambda_{y,r}}=\frac{\alpha_y}{1-\alpha_y}\frac{w_{y,r}}{G_{y,r}}\tag{38}$$

Dividing these two relations yields:

$$\frac{w_{x,r}}{w_{y,r}}=\frac{X_r^I}{Y_r^I}\frac{G_{x,r}}{G_{y,r}}\frac{\alpha_y\left(1-\alpha_y\right)}{\alpha_x\left(1-\alpha_y\right)}\tag{39}$$

Then, let consider the optimal final consumption demand:

$$X_r^F = \mu \frac{Income}{G_{x,r}} \tag{40}$$

$$Y_r^F = (1-\mu) \frac{Income}{G_{y,r}} \tag{41}$$

Dividing these two relations yields:

$$\frac{G_{x,r}}{G_{y,r}} = \frac{\mu}{1-\mu} \frac{Y_r^F}{X_r^F} \tag{42}$$

Combining (39) and (42) yields:

$$\frac{w_{x,r}}{w_{y,r}} = \frac{\mu}{1-\mu} \frac{\alpha_y (1-\alpha_x)}{\alpha_x (1-\alpha_y)} \left(\frac{X_r^I}{Y_r^I} \frac{Y_r^F}{X_r^F} \right) \tag{43}$$

Since we consider the symmetric equilibrium, we can assume the following conditions:

$X_r^I = \alpha_x K_1$, $Y_r^I = \alpha_y K_1$, $X_r^F = \mu K_2$, $Y_r^F = (1-\mu) K_2$ where K_1 and K_2 are constants. With these conditions, (43) becomes:

$$w_{x,r} = \frac{1-\alpha_x}{1-\alpha_y} w_{y,r} \tag{44}$$

If we take as numeraire the wage in industry y in location 2, we have following values for the symmetric equilibrium:

$$w_{x,1} = w_{x,2} = \tfrac{1-\alpha_x}{1-\alpha_y} \text{ , } w_{y,1} = w_{y,2} = 1 \text{ ,}$$
$$G_{x,1} = G_{x,2} = \tfrac{1-\alpha_x}{1-\alpha_y} \left(\tfrac{1+\tau^{1-\sigma}}{2} \right)^{\frac{1}{1-\sigma(1-\alpha_x)}} \text{ ,}$$
$$G_{y,1} = G_{y,2} = \left(\tfrac{1+\tau^{1-\sigma}}{2} \right)^{\frac{1}{1-\sigma(1-\alpha_y)}}$$

Using the total differentiation expression, we get the following expressions:

$$
\begin{aligned}
\frac{dG_x}{d\lambda_x} &= Z\left(W^{\frac{1}{1-\sigma(1-\alpha_x)}}\left(\frac{2(1-\alpha_x)}{(1-\sigma)(1-\alpha_y)}+\frac{1+\alpha_x\sigma-\sigma}{1-\sigma}\frac{dw_x}{d\lambda_x}\right)-\frac{\alpha_x\sigma}{1-\sigma}\frac{dG_x}{d\lambda_x}\right)\\
\frac{dG_y}{d\lambda_y} &= Z\left(W^{\frac{1}{1-\sigma(1-\alpha_y)}}\left(\frac{2}{1-\sigma}+\frac{1+\alpha_y\sigma-\sigma}{1-\sigma}\frac{dw_y}{d\lambda_y}\right)-\frac{\alpha_y\sigma}{1-\sigma}\frac{dG_y}{d\lambda_y}\right)\\
\frac{dw_x}{d\lambda_x} &= \Theta^{\frac{1}{(1-\alpha_x)\sigma}}\left(\begin{array}{c}\Theta^{1-\sigma(1-\alpha_x)}Z\left(\begin{array}{c}\frac{2}{\sigma}\frac{(1-\alpha_x)\mu+\alpha_x}{1-\alpha_y}+\frac{2(1-\mu)}{\sigma}\frac{d\omega_y}{d\lambda_x}\\+\frac{(1-\alpha_x)\mu+\alpha_x}{(1-\alpha_x)\sigma}\frac{dw_x}{d\lambda_x}+\frac{\mu}{\sigma}\frac{dw_y}{d\lambda_x}\end{array}\right)\\+\frac{\sigma-\alpha_x\sigma-1-(\sigma+\alpha_x\sigma-1)\tau^{1-\sigma}}{2(1-\alpha_x)\sigma}W^{-\frac{2-(1-\alpha_x)\sigma}{1-(1-\alpha_x)\sigma}}\frac{dG_x}{d\lambda_x}\end{array}\right)\\
\frac{dw_x}{d\lambda_y} &= \Theta^{\frac{1}{(1-\alpha_x)\sigma}-\sigma(1-\alpha_x)+1}Z\left(\frac{2}{\sigma}\frac{(1-\alpha_x)\mu+\alpha_x}{1-\alpha_y}\frac{d\omega_x}{d\lambda_y}+\frac{2(1-\mu)}{\sigma}+\frac{(1-\alpha_x)\mu+\alpha_x}{(1-\alpha_x)\sigma}\frac{dw_x}{d\lambda_y}+\frac{\mu}{\sigma}\frac{dw_y}{d\lambda_y}\right)\\
\frac{dw_y}{d\lambda_x} &= \Phi^{\frac{1}{(1-\alpha_y)\sigma}-\sigma(1-\alpha_y)+1}Z\left(\begin{array}{c}\frac{2}{\sigma}\frac{(1-\alpha_x)}{1-\alpha_y}+\frac{2((1-\mu)(1-\alpha_y)+\alpha_y)}{(1-\alpha_y)\sigma}\frac{d\omega_y}{d\lambda_x}\\+\frac{(1-\mu)}{\sigma}\frac{dw_x}{d\lambda_x}+\frac{(1-\mu)(1-\alpha_y)+\alpha_y}{\sigma}\frac{dw_y}{d\lambda_x}\end{array}\right)\\
\frac{dw_y}{d\lambda_y} &= \Phi^{\frac{1}{(1-\alpha_y)\sigma}}\left(\begin{array}{c}\Phi^{1-\sigma(1-\alpha_y)}Z\left(\begin{array}{c}\frac{2}{\sigma}\frac{(1-\alpha_x)}{1-\alpha_y}\frac{d\omega_x}{d\lambda_y}+\frac{2((1-\mu)(1-\alpha_y)+\alpha_y)}{(1-\alpha_y)\sigma}\\+\frac{(1-\mu)}{\sigma}\frac{dw_x}{d\lambda_y}+\frac{(1-\mu)(1-\alpha_y)+\alpha_y}{\sigma}\frac{dw_y}{d\lambda_y}\end{array}\right)\\+\frac{\sigma-\alpha_y\sigma-1-(\sigma+\alpha_y\sigma-1)\tau^{1-\sigma}}{2(1-\alpha_y)\sigma}W^{-\frac{2-(1-\alpha_y)\sigma}{1-(1-\alpha_y)\sigma}}\frac{dG_y}{d\lambda_y}\end{array}\right)\\
\frac{d\omega_x}{d\lambda_x} &= \left(\frac{1-\alpha_x}{1-\alpha_y}\right)^{1-\mu}W^{\Gamma}\left(\begin{array}{c}-\delta\left(1+\frac{d\omega_y}{d\lambda_x}\right)+\frac{1-\alpha_y}{1-\alpha_x}\frac{dw_x}{d\lambda_x}-\mu\frac{1-\alpha_y}{1-\alpha_x}(W)^{-\frac{1}{1-\sigma(1-\alpha_x)}}\frac{dG_x}{d\lambda_x}\\-(1-\mu)W^{-\frac{1}{1-\sigma(1-\alpha_y)}}\frac{dG_y}{d\lambda_x}\end{array}\right)\\
\frac{d\omega_x}{d\lambda_y} &= \left(\frac{1-\alpha_x}{1-\alpha_y}\right)^{1-\mu}W^{\Gamma}\left(-\delta\left(\frac{d\omega_x}{d\lambda_y}+1\right)+\frac{1-\alpha_y}{1-\alpha_x}\frac{dw_x}{d\lambda_y}-(1-\mu)(W)^{-\frac{1}{1-\sigma(1-\alpha_y)}}\frac{dG_y}{d\lambda_y}\right)\\
\frac{d\omega_y}{d\lambda_x} &= \left(\frac{1-\alpha_x}{1-\alpha_y}\right)^{-\mu}W^{\Gamma}\left(\begin{array}{c}-\delta\left(1+\frac{d\omega_y}{d\lambda_x}\right)+\frac{dw_y}{d\lambda_x}-\mu\frac{1-\alpha_y}{1-\alpha_x}(W)^{-\frac{1}{1-\sigma(1-\alpha_x)}}\frac{dG_x}{d\lambda_x}\\-(1-\mu)W^{-\frac{1}{1-\sigma(1-\alpha_y)}}\frac{dG_y}{d\lambda_x}\end{array}\right)\\
\frac{d\omega_y}{d\lambda_y} &= \left(\frac{1-\alpha_x}{1-\alpha_y}\right)^{-\mu}W^{\Gamma}\left(-\delta\left(\frac{d\omega_x}{d\lambda_y}+1\right)+\frac{dw_y}{d\lambda_y}-(1-\mu)W^{-\frac{1}{1-\sigma(1-\alpha_y)}}\frac{dG_y}{d\lambda_y}\right).
\end{aligned}
$$

We set:

$$
\begin{aligned}
\Gamma &= \frac{\mu\sigma(\alpha_x-\alpha_y)+\sigma-\alpha_x\sigma-1}{(1-\sigma+\alpha_x\sigma)(1-\sigma+\alpha_y\sigma)}\\
\Theta &= (1-\alpha_x)\mu+\alpha_x+\mu(1-\alpha_y)\\
\Phi &= (1-\alpha_y)(1-\mu)+\alpha_y+(1-\mu)(1-\alpha_x)\\
Z &= \frac{1-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\\
W &= \frac{1+\tau^{1-\sigma}}{2}
\end{aligned}
$$

Appendix 7

The condition $\omega_{x,1} > \omega_{x,2}$ translates to $w_{x,1} > \tau^{1-2\mu}w_{x,2}$ using equation (21), and this latter condition combined with equations (23) and (24) implies that:

$$
w_{x,1} > \frac{\mu(1-\alpha_x)}{\mu+(1-\mu)\alpha_x}\frac{\tau^{2(1-\mu)\sigma(1-\alpha_x)-1}-1}{1-\tau^{1-2\mu\sigma(1-\alpha_x)}}w_{y,2}. \tag{45}
$$

Equation (35) implies that $w_{x,1} = w_{y,2}$ and the condition found above becomes:

$$\mu\left(1-\alpha_x\right)\tau^{2(1-\mu)\sigma(1-\alpha_x)-1} + \left[\mu + (1-\mu)\,\alpha_x\right]\tau^{1-2\mu\sigma(1-\alpha_x)} < \alpha_x + 2\mu\left(1-\alpha_x\right) \tag{46}$$

which is equivalent to:

$$\mu\left(1-\alpha_x\right)\tau^{2(1-\mu)\sigma(1-\alpha_x)-1} + \left[\mu + (1-\mu)\,\alpha_x\right]\tau^{1-2\mu\sigma(1-\alpha_x)} \leq 1 \tag{47}$$

for $\mu \leq 1/2$. For $w_{x,1} = w_{y,2}$, equation (25) yields:

$$\left(\frac{\omega_{y,1}}{\omega_{y,2}}\right)^{(1-\alpha_y)\sigma} = \\ (1-\mu)\left(1-\alpha_y\right)\tau^{2\mu\sigma(1-\alpha_y)-1} + \left[(1-\mu)\left(1-\alpha_y\right)+\alpha_y\right]\tau^{1-2(1-\mu)\sigma(1-\alpha_y)}.$$

Hence, for $\mu \leq 1/2$ and $\alpha_x < \alpha_y$, (46) implies that $\omega_{y,2} > \omega_{y,1}$. That is, the complete concentration of the weak input-output industry in location 1 implies the complete concentration of the strong input-output linkages industry in location 2.

Appendix 8

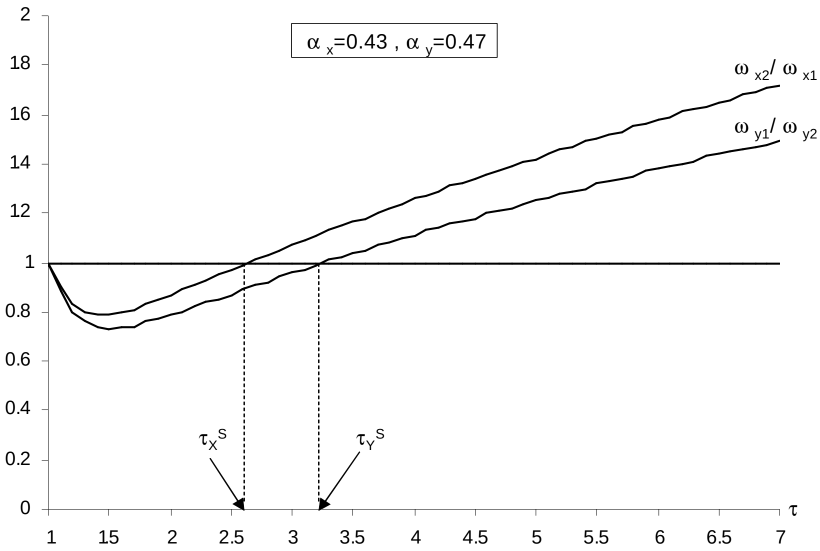

Figure 17: Sustain points of the two industries.

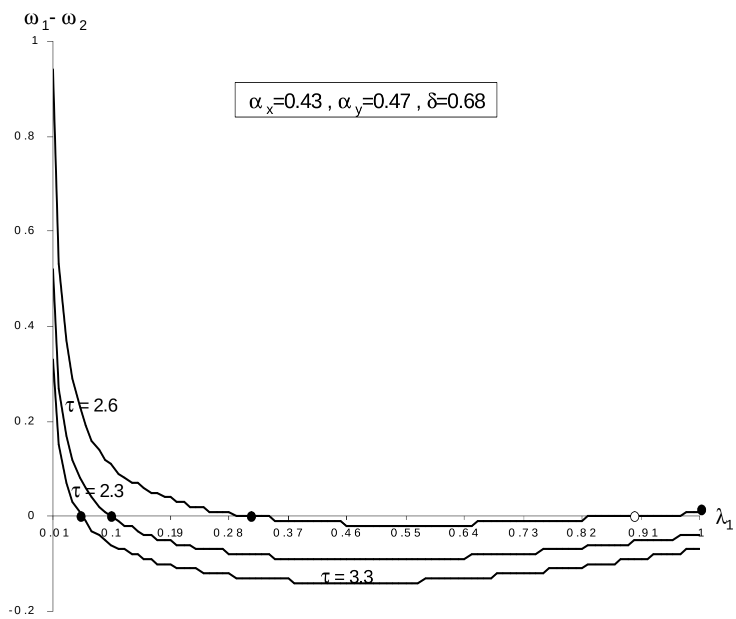

Figure 18: Real wage differential in industry x.

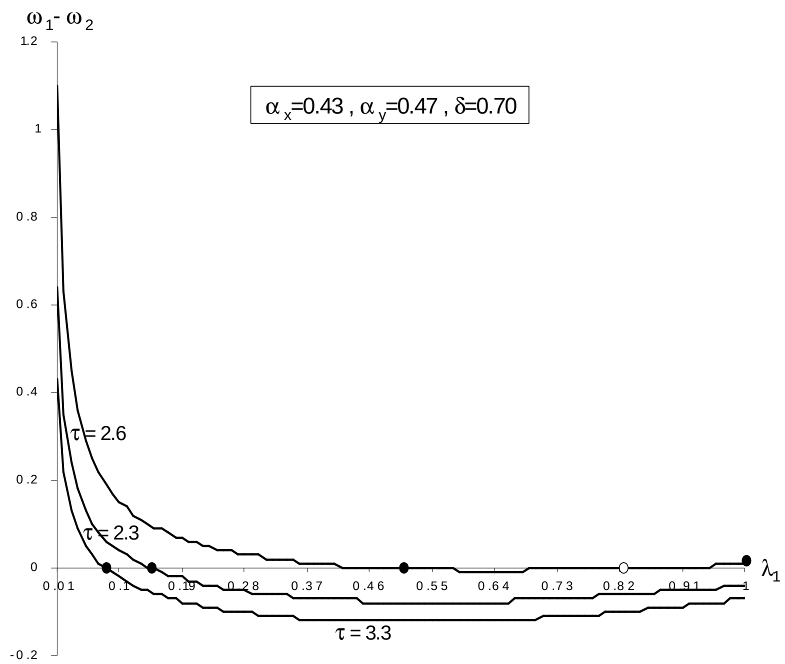

Figure 19: Real wage differential in industry x.

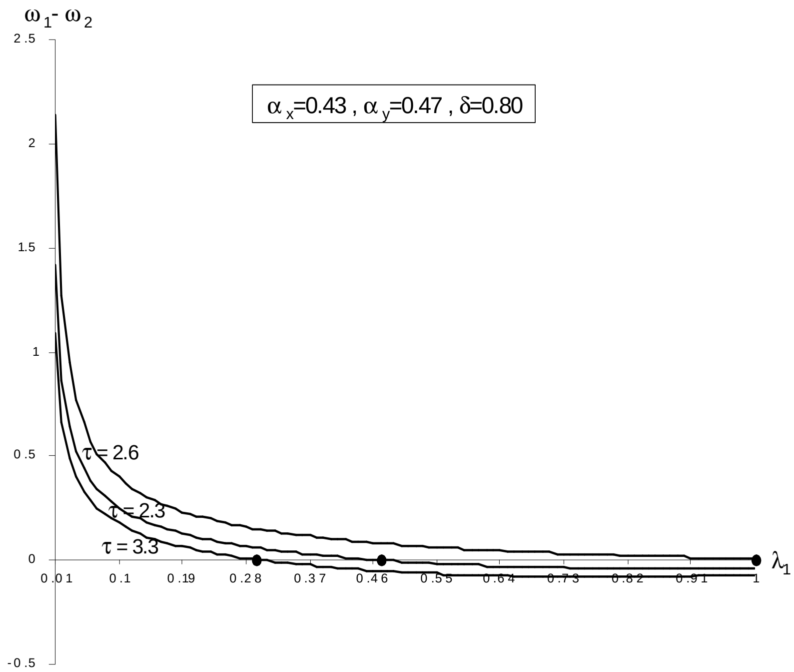

Figure 20: Real wage differential in industry x.

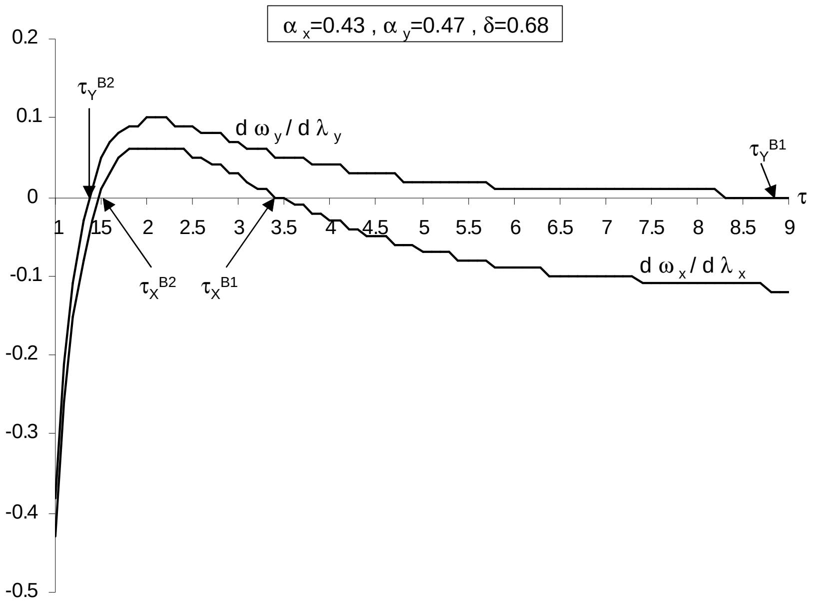

Figure 21: Stability of the perfectly dispersed equilibrium.

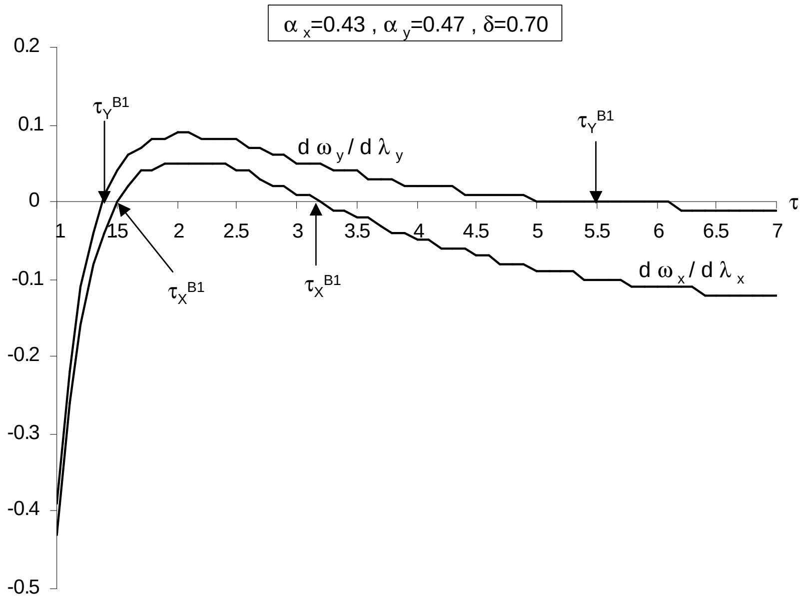

Figure 22: Stability of the perfectly dispersed equilibrium.

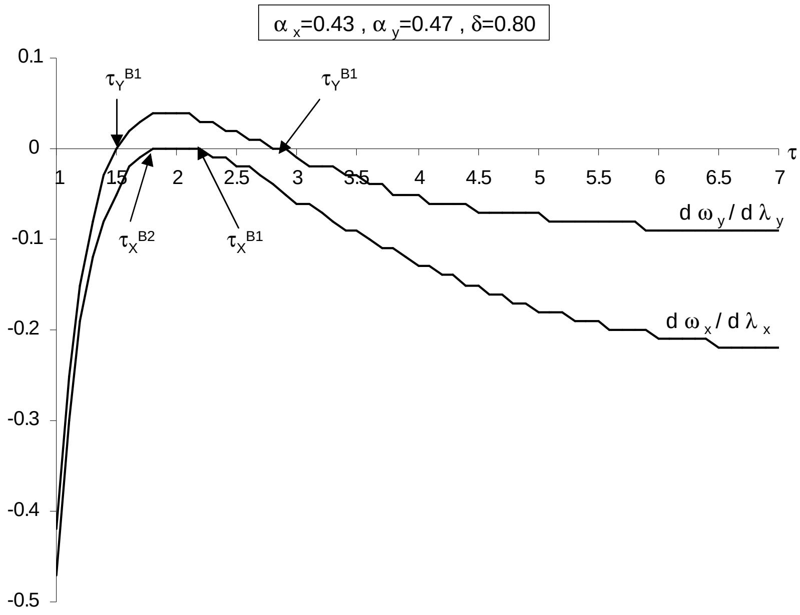

Figure 23: Stability of the perfectly dispersed equilibrium.

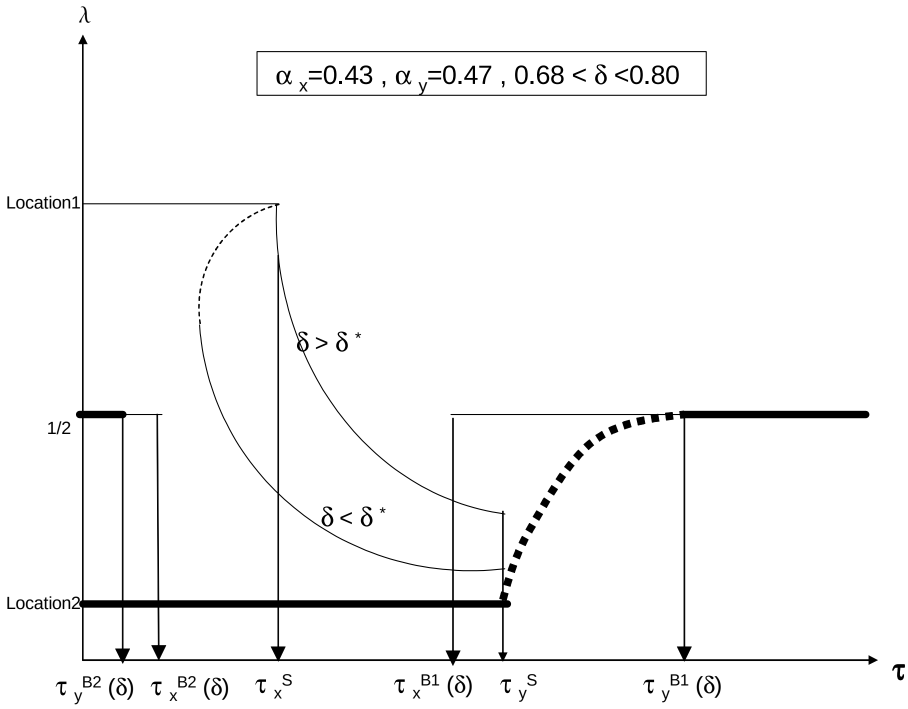

Figure 24: Bifurcation diagram.