Cash breeds Success: The Role of Financing Constraints in Patent Races*

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Abstract

This paper studies the impact of cash constraints on equilibrium winning probabilities in a patent race between an incumbent and an entrant. We develop a model where cash-constrained firms finance their R&D expenditures with an investor who cannot verify their effort. In equilibrium, the incumbent faces better prospects of winning the race the less cash-constrained he is and the more cash-constrained the entrant is. We use NBER evidence from pharmaceutical patents awarded between 1975 and 1999 in the US, patent citations, and COMPUSTAT and fit probabilistic regressions of the predicted equilibrium winning probabilities on measures of the incumbent's and potential entrants' financial wealth. The empirical findings support our theoretical predictions.

Keywords: Patent Race, incumbent, entrant, financial constraints, empirical estimation.

JEL Classification: G24, G32, L13

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1 Introduction

Do a firm's financing constraints affect its decisions to pursue innovation? Since Steven Fazzari's, Glenn Hubbard's, and Bruce Petersen's seminal paper, economists have found that financing matters through various channels for total firm level investment in R&D (Fazzari, Hubbard and Petersen, 1988) For example, Brownyn Hall shows that the source of financing matters (Hall, 1992) and Charles Himmelberg and Bruce Petersen show that internal finance predicts R&D expenditures of small high tech firms (Himmelberg and Petersen, 1994). But do a firm's financing constraints also affect its rivals' decisions to pursue innovations?

To our surprise, the role of financing constraints in patent races has not yet been studied, neither theoretically nor empirically. Theorists have studied thoroughly how firms' incentives to engage in R&D depend on technological standing and market structure. Jennifer Reinganum shows that incumbent firms have less incentives to innovate than entrants in a stochastic setup because additional investments in R&D speed up the cannibalization of their current monopoly profits (Reinganum, 1983). Opposing this view, Richard Gilbert and David Newbery show that incumbents can preempt entrants from racing for incremental innovations if the player who spends most is guaranteed to innovate with probability one (Gilbert and Newbery, 1982). In this paper, we incorporate financing constraints explicitly into the model of Reinganum and test the model's predictions empirically.

In our model, entrepreneurs will finance their R&D expenditures partly from internal funds, and partly using external sources depending on the amount of cash they have. The probability of making the discovery at a point in time depends on the effort exerted by the entrepreneur, which cannot be verified by the investor. Thus, in equilibrium, finance is costly for the entrepreneur and the marginal cost of innovative activity is increasing in the fraction of outside funds to the total investment, very much following the logic proposed by Michael Jensen and William Meckling (Jensen and Meckling, 1976). The increase in the marginal cost of innovating shifts a player's best response function in the patent race monotonically, which in turn results in a monotonic change in the equilibrium R&D expenditures. The practical upshot is that in a setting of strategic interactions, financial standing power is a source of comparative advantage. It is this prediction that we test in our empirical investigation.

We face three major empirical challenges. First, we need data on financial standing *and* patent awards, but existing data sets typically contain information on *either* finance *or* patents only. Therefore, we construct a data set that combines both. We use the NBER Patent Citations Data File developed by Bronwyn Hall, Adam Jaffe and Manuel Trajtenberg (Hall, Jaffe and Trajtenberg, 2002), which records all utility patents granted in the United States between 1963 and 1999 and links every patent granted after 1975 to all the patents it cites and to the CUSIP code of the assignee as it appears in COMPUSTAT. We merge the patent records with COMPUSTAT to obtain the winners' and losers' financial data before the patent was awarded.[1] Second, we need to identify in the data which firms were incumbents and which firms were entrants to every race. Since a patent must cite the prior technology it builds on, we consider the owners of the patents for the cited technologies as incumbents to the race. Third, we need to be sure that patents are a good measure of innovative success. Therefore, we focus on the drug industry, where patents are crucial to reap the returns to R&D investment (see Levin et al., 1987, and Cohen, Nelson and Walsh, 2000).[2]

Our model links the probability that the winner of the race is either an incumbent or an entrant to the underlying characteristics of the race, e.g., the firms' financial resources, the value of the prize and the value of prior innovations. To test our predictions we fit logistic regressions of the fraction of incumbent-won races on these variables. We find that a firm's probability of winning a race is increasing -on average- in its stock of cash and decreasing in its rivals' stocks of cash. The predicted impacts are not only statistically significant but also economically meaningful: differences in stocks of cash imply large differences in the probability of winning. Our results are robust to different definitions of incumbency.

This paper is related to several strands of the literature but novel in its focus and comprehensiveness. The literature has devoted some attention to the commitment effects of financial structure on generic strategies in oligopolistic product market games. A capital structure choice that is observed by rivals can influence a firm's aggressiveness in the product market (see James Brander and Tracy Lewis, 1986; Vojislav Maksimovic, 1988, and Julio Rotemberg and David Scharfstein, 1990; Drew Fudenberg and Jean Tirole, 1986; and Patrick

[1] Our focus on COMPUSTAT makes us restrict attention only to publicly traded firms. Usually these firms are relatively wealthy, so financing constraints should bind less for them. Thus, if we find that the predictions of our model are verified for a set of less cash-constrained firms like those in COMPUSTAT, they should also be satisifed in the set of smaller, private firms.

[2] It is widely acknowledged that firms in many other industries use other mechanisms to protect the competitive advantages of R&D (e.g., superior marketing, customer service, client switching costs) and in such industries patent records do not represent well their innovations and the races for them. We have limited ourselves to the study of patents in the pharmaceutical industry because we rely on patent data to measure success in a race. However, our method can be applied in a straight forward way to the study of any race in any industry provided that a satisfactory measure of success is available.

Bolton and Scharfstein, 1990). Judith A. Chevalier tests these predictions empirically (Chevalier, 1995). We depart from this literature in two respects. First, we assume that financing choices are not observable to rivals in our paper, so that the commitment effects of financing choices play no role. We believe that our assumption is appropriate to analyze the interaction between large firms, where rivals find it difficult to disentangle the financing of individual projects from the overall financing of the concern. Second, we focus on a different comparative statics exercise. Instead of varying the capital structure, we vary the financing *need* of firms.

Our empirical investigation explores a game theoretic setup with a comprehensive data base. Only few studies share these two features. Richard Blundell, Rachel Griffith and John Van Reenen study the relationship between market share and innovation using a panel of British pharmaceutical firms. They find that firms with more market dominance innovate more often (Blundell, Griffith and Van Reenen, 1999), consistent with Gilbert and Newbery's "efficiency effect". In contrast to their study we incorporate financing explicitly into ours and show that financing matters even if we control for size effects.

Iain Cockburn and Rebecca Henderson (Cockburn and Henderson, 1994) address whether or not R&D investments are strategic. Gathering detailed data at the individual project level for ten of the largest firms in the pharmaceutical industry, they find that research investments are only weakly correlated across firms. However, as they acknowledge, their study may miss correlations between investments of smaller potential entrants and the large firms by focusing only on the large players.[3] We trade off the detail of project level data for a more comprehensive data base and show that the winning probabilities of firms are significantly affected by other firms' characteristics. Moreover, we include measures of the player's financial wealth in the empirical analysis.

Josh Lerner (Lerner, 1997) does find that strategic variables explain the decision of firms to innovate. Lerner finds that the leaders in the disk drive industry between 1971 and 1988 were less likely to improve their disk drive density than the laggards.[4] Lerner's approach owes much of its elegance to the fact that

[3] The authors mention that the firms they sample account for approximately 25 to 30% of the worldwide sales and R&D of the Ethical Drugs Industry and claim that these firms are not markedly unrepresentative of the industry in terms of size, or of technical and commercial performance.

[4] Note that this result is diametrically opposed to the results of Blundel, Griffiths and Van Reenen (Blundell, et all., 1999): technology laggards have more incentives to innovate because, unlike leaders, their innovative efforts do not cannibalize profits from "shelving" current innovations.

the distance to the maximum disk drive in the industry measures the strategic interaction appropriately. Also, he focuses on an industry where not only the first but any firm that innovates is awarded the prize so he can treat observation errors independently across firms. We cannot rely on such assumptions in the pharmaceutical industry and are forced to take a more detailed view. Our approach identifies strategic behavior from the outcome of races where the winner takes all and finds results consistent with Lerner's.[5]

The remainder of this article is organized as follows. The next Section of the paper develops the model. It derives the comparative statics on the probability that a given firm wins the race conditional on its financial resources. Section 3 describes the data and the construction of our proxies for incumbency and the value of innovations. Section 4 develops the econometric model and presents the estimation results. The final section summarizes our results.[6]

2 Theory

We consider the financing of research in a version of Jennifer Reinganum's model (Reinganum, 1983). There are two firms: an incumbent, I, and an entrant, E. The incumbent produces and sells the "state-of-the-art" product. The firms can enter a research race for a higher quality product. We model the uncertain success in this research race as the outcome of a Poisson process. The state-of-the-art product and the innovation are protected by patents of infinite length. The sales of the incumbent's product yield a flow profit of π to the incumbent. If the incumbent innovates, sales of the new (and possibly also of the old) product yield a profit $\overline{\pi}_I$ to him. If the entrant innovates, he obtains flow profits of $\overline{\pi}_E$ and the incumbent obtains flow profits $\underline{\pi}_I$. This formulation allows for drastic and non-drastic innovations.

If a firm enters the research race, it has to spend once and for all a fixed cost F. Once this cost is sunk it can exert a flow of effort a_h for $h = E, I$. If a firm spends effort a_h its instantaneous probability of innovation is a_h^α, where $\alpha < 1$. The non-pecuniary cost of effort is equal to a_h. Firms have limited financial resources, W_h. If $W_h < F$ the firm needs outside funds to finance the fixed cost.

[5] Another advantage of our approach is that we do not have to control for technological opportunity. Since we focus on races that have actually occured and been won by someone, our observations are conditional on there being a technological opportunity to explore.

[6] To avoid duplication in the paper, some proofs and tables have been relegated to a supplementary appendix which can be downloaded from http://www.hec.unil.ch/dszalay.

We assume that many investors compete in Bertrand fashion for the right to finance a firm's investment. They make take-it-or-leave-it offers to firms and then firms decide whether or not to accept the contract. A firm with $W_h < F$ that rejects its contract cannot innovate, i.e., has probability of innovation equal to zero for all a_h. After the firm has accepted a contract, it chooses its research intensity a_h. Contracts between investors and a firm are not observable to other investors and the other firm.

We assume that contracts are not observable to third parties in order to rule out commitment effects of finance. That is, we adopt the simultaneous move assumption from Jennifer Reinganum's paper and take Nash-Equilibrium as our equilibrium concept. We do not consider sequential (Stackelberg) games where one firm can observe the financing of the other firm before it chooses its research intensity.[7]

We begin our analysis with the derivation of firms' best responses. We first discuss the entrants optimal choice of research intensity for a given research intensity of the incumbent. Afterwards, we repeat this analysis for the incumbent. In each of these discussions we begin with a characterization of optimal contracts. Then we characterize the firm's research intensity that results from accepting an optimal contract.

2.1 The Entrants' Problem

2.1.1 The Financing of the Entrant

The Poisson nature of research implies that there are two classes of positive probability events: either the incumbent innovates before the entrant or vice-versa. Within these classes, events differ only in the time of innovation. We consider stationary contracts where the repayment conditions depend on who wins the race but not on when he wins. Thus, the model has essentially two outcomes. We place no further restrictions on the form of contracts. Contracts with any arbitrarily complex time-dependent repayments (in the sense of the length of time elapsed since the arrival of the innovation) have a simple equivalent representation where the entrant commits to repay a constant share, s, of profits from the time of innovation to infinity. Since everybody is risk-neutral, all that matters is the present value of the repayment stream.

If the incumbent wins the race the entrant's profits are zero. Therefore, the entrant can repay the finance he has obtained only if he wins the race. The initial payment of $F - W_E$ and the share of profits s completely

[7] Our main results are not affected by this modeling choice.

describe all relevant information of financial contracts.

Let $V_E(W_E, a_I, s)$ denote the value of the entrant's claim of future profits for given values of wealth, the incumbent's research, and the investor's repayment shares. The entrant's problem is to accept or reject a contract offered by the investor and to choose his research effort conditional on accepting. We solve this problem by backward induction. The second stage of the entrant's problem can be described by the following asset equation:

$$rV_E(W_E, a_I, s) = \max_{a_E} a_E^\alpha \left((1-s) V_E^+ - V_E(W_E, a_I, s)\right) - a_I^\alpha V_E(W_E, a_I, s) - a_E, \tag{1}$$

where r is the risk-free interest rate and $V_E^+ \equiv \frac{\overline{\pi}_E}{r}$, i.e., the net present value of the perpetual flow of profits, $\overline{\pi}_E$, starting at the time of innovation. We assume that $V_E^+ > F$. In a short interval of time between t and $t + dt$ the entrant innovates with probability $a_E^\alpha dt$ and the incumbent innovates with probability $a_I^\alpha dt$. In case the entrant innovates, he receives a share $(1-s)$ of all future profits and thus a claim that is worth $(1-s) V_E^+$ as of the time of innovation. If either the entrant or the incumbent innovates, the entrant loses the value of its current claim, $V_E(W_E, a_I, s)$. The flow cost of research during the small interval of time is $a_E dt$.

The maximization problem on the right hand side of (1) is strictly concave in a_E. Let $a_E(s)$ denote a solution to this problem. The first-order condition,

$$\alpha (a_E(s))^{\alpha-1} \left((1-s) V_E^+ - V_E(W_E, a_I, s)\right) = 1, \tag{2}$$

is necessary and sufficient for the unique optimal choice of $a_E(s)$ induced by the contract $\{F - W_E, s\}$. We observe that the entrant's choice of effort is distorted downwards relative to the first-best whenever $s > 0$. The entrant is reluctant to exert the efficient amount of effort because he receives less than the social value of the innovation.

We can multiply both sides of condition (2) by $a_E(s)$ and obtain the condition

$$\alpha (a_E(s))^{\alpha} \left((1-s) V_E^+ - V_E(W_E, a_I, s)\right) = a_E(s). \tag{3}$$

If we substitute condition (3) into the asset equation (1) we can solve for the value of the claim to the entrant

$$V_E\left(W_E, a_I, s\right) = (1-s)\frac{(1-\alpha)\left(a_E\left(s\right)\right)^{\alpha} V_E^+}{(1-\alpha)\left(a_E\left(s\right)\right)^{\alpha} + a_I^{\alpha} + r}. \tag{4}$$

With perfect competition in the investors market, the equilibrium contract maximizes $V_E\left(W_E, a_I, s\right)$ subject to the constraint that the investor breaks even, i.e.,

$$s\frac{\left(a_E\left(s\right)\right)^{\alpha} V_E^+}{\left(a_E\left(s\right)\right)^{\alpha} + a_I^{\alpha} + r} = F - W_E. \tag{5}$$

We let s^* denote an optimal contract. We now give conditions for the existence of an optimal contract.

Lemma 1 *For all $W_E \geq 0$ and F there exists $\overline{a}_I$ such that a unique optimal contract exists if and only if $a_I \leq \overline{a}_I\left(V_E^+, W_E\right)$. $\overline{a}_I\left(V_E^+, W_E\right)$ is nondecreasing in both its arguments. It is strictly increasing in V_E^+ whenever $\overline{a}_I > 0$. It is strictly increasing in W_E whenever $F > W_E$ and $\overline{a}_I > 0$.*

The proof of Lemma 1 is somewhat lengthy and therefore relegated to the Appendix. The intuition for the result is straightforward. The higher the research effort chosen by the incumbent, the smaller the expected value of the prize for a given effort level by the entrant. As a result, the value of the investor's claim is decreasing in a_I for fixed s, and the investor requires a larger share of profits the higher is a_I. But an increase in s decreases the entrant's incentive to provide effort. Eventually, that is for large enough a_I, these discouraging effects are so strong that an optimal contract ceases to exist. On the other hand, an increase in the value of the race, V_E^+, balances these effects, so that the higher is the value of the race, the larger the critical level of the incumbent's effort $\overline{a}_I$ that chokes off the entrant's innovative efforts. Likewise, the higher is the entrant's wealth, the smaller is the amount of money needed from the investor and the less discouraging is an increase in the incumbent's effort.

We now investigate whether or not the entrant will accept the contract. Accepting is optimal if and only if

$$V_E\left(W_E, a_I, s^*\right) - \min\left\{F, W_E\right\} \geq 0. \tag{6}$$

where $\min\{F, W_E\} = W_E$ if and only if the entrant is financially constrained. The entrant accepts the optimal contract if and only if the project generates a nonnegative net present value to him, accounting for agency costs due to asymmetric information.

Lemma 2 *Suppose V_E^+ is sufficiently large so that the entrant engages in research for $a_I = 0$. Then, for all $W_E \geq 0$ and F, there exists $\overline{\overline{a}}_I$ such that the entrant accepts the optimal contract if and only if $a_I \leq \overline{\overline{a}}_I\left(V_E^+, W_E\right)$. $\overline{\overline{a}}_I$ is nondecreasing in both its arguments. $\overline{\overline{a}}_I$ is strictly increasing in V_E^+ whenever $\overline{\overline{a}}_I > 0$, and strictly increasing in W_E whenever both $\overline{\overline{a}}_I > 0$ and $F > W_E$.*

Proof. Substituting conditions (4) and (5) into condition (6) we obtain

$$\left(\frac{\hat{a}_E^{*\alpha}V_E^+ - (F - W_E)\left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)}{(1-\alpha)\,\hat{a}_E^{*\alpha} + a_I^\alpha + r}\right)(1-\alpha) \geq W_E. \tag{7}$$

where $\hat{a}_E^*$ is the effort level induced by the optimal contract. $\hat{a}_E^*$ is defined by the condition

$$\alpha\left(\hat{a}_E^{*\alpha}V_E^+ - \left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)(F - W_E)\right)\left(a_I^\alpha + r\right) - \hat{a}_E^*\left((1-\alpha)\,\hat{a}_E^{*\alpha} + a_I^\alpha + r\right) = 0 \tag{8}$$

Using condition (8) we can simplify condition (7) and obtain

$$\frac{\hat{a}_E^*}{a_I^\alpha + r} \geq \frac{\alpha}{1-\alpha}W_E. \tag{9}$$

Differentiating with respect to a_I^α in condition (9), we find that the left-hand side of this inequality is a decreasing (non-decreasing) function of a_I^α if and only if $\frac{d\hat{a}_E^*}{da_I^\alpha} < \frac{\hat{a}_E^*}{a_I^\alpha + r}$ $\left(\frac{d\hat{a}_E^*}{da_I^\alpha} \geq \frac{\hat{a}_E^*}{a_I^\alpha + r}\right)$. Applying the implicit function theorem to condition (8), we obtain

$$\frac{d\hat{a}_E^*}{da_I^\alpha} = \frac{\left(-\alpha\left(F - W_E\right)\left(a_I^\alpha + r\right) + \alpha\left(\hat{a}_E^{*\alpha}V_E^+ - \left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)(F - W_E)\right) - \hat{a}_E^*\right)}{-\left(\alpha^2\hat{a}_E^{*\alpha-1}\left(V_E^+ - (F - W_E)\right)\left(a_I^\alpha + r\right) - \left(\left(1-\alpha^2\right)\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)\right)} \tag{10}$$

The denominator of this expression is negative, because $\hat{a}_E^* > \overline{a}_E \equiv \arg\max_{a_E}\left(A\left(a_E;\cdot\right) - B\left(a_E;\cdot\right)\right)$ implies that $\frac{\partial}{\partial a_E}\left(A\left(a_E;\cdot\right) - B\left(a_E;\cdot\right)\right)\big|_{a_E = \hat{a}_E^*} < 0$. (see the proof of lemma 1.) Using condition (8) (and some straightforward manipulations) to simplify expression (10) we obtain

$$\frac{d\hat{a}_E^*}{da_I^\alpha} = \frac{\hat{a}_E^*}{a_I^\alpha + r}\underbrace{\frac{1}{\alpha}\left(1 - \frac{(1-\alpha)\left((1-\alpha)\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)}{\left(-\alpha^2 \frac{\left(a_I^\alpha + r\right)^2}{\hat{a}_E^*}(F - W_E) + (1-\alpha)\left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)\right)}\right)}_{\equiv \Gamma\left(\hat{a}_E^*\right)} \tag{11}$$

Straightforward algebra shows that $\Gamma\left(\hat{a}_E^*\right) < 1$, which implies that $\frac{d\hat{a}_E^*}{da_I^\alpha} < \frac{\hat{a}_E^*}{a_I^\alpha + r}$. In turn this implies that $\frac{d}{da_I^\alpha}\left(\frac{\hat{a}_E^*}{a_I^\alpha + r}\right) < 0$. Consequently, there is $\overline{\overline{a}}_I\left(V_E^+, W_E\right)$ uniquely defined by the condition

$$\left.\frac{\hat{a}_E^*\left(a_I; \cdot\right)}{a_I^\alpha + r}\right|_{a_I = \overline{\overline{a}}_I} = \frac{\alpha}{1-\alpha} W_E$$

such that condition (9) is satisfied if and only if $a_I \leq \overline{\overline{a}}_I\left(V_E^+, W_E\right)$. Since $\frac{\hat{a}_E^*}{a_I^\alpha + r}$ is decreasing in a_I^α and

$$\frac{d\hat{a}_E^*}{dV_E^+} = \frac{\alpha \hat{a}_E^{*\alpha}\left(a_I^\alpha + r\right)}{-\alpha^2 \frac{\left(a_I^\alpha + r\right)^2}{\hat{a}_E^*}(F - W_E) + (1-\alpha)\left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)} > 0$$

(which follows again from applying the implicit function theorem to condition (8)), $\overline{\overline{a}}_I\left(V_E^+, W_E\right)$ is increasing in V_E^+. Finally, we observe that

$$\frac{\frac{d\hat{a}_E^*}{dW_E}}{a_I^\alpha + r} = \frac{\alpha\left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)}{-\alpha^2 \frac{\left(a_I^\alpha + r\right)^2}{\hat{a}_E^*}(F - W_E) + (1-\alpha)\left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)} > \frac{\alpha}{1-\alpha}$$

(By simple algebra, this condition is equivalent to $0 > -\alpha^2 \frac{\left(a_I^\alpha + r\right)^2}{\hat{a}_E^*}(F - W_E)$, which is obviously true). That is, an increase in W_E increases the left-hand side of condition (9) by more than it increases the right-hand side. Hence, $\overline{\overline{a}}_I\left(V_E^+, W_E\right)$ is increasing in W_E. ■

The logic of the argument is rather simple. The value of the prize that goes to the entrant is a strictly decreasing function of the incumbent's level of research effort. As a result, the entrant is willing to engage in research if and only if the opponent's effort is not too high. Conversely, for a given a_I, the value of the entrant's claim is the higher the higher is V_E^+. As a result, the critical level of the incumbent's research effort that chokes off the entrant's research incentives is a non-decreasing function of V_E^+. Similarly, an increase in wealth increases the net value of the entrant's claim by reducing agency costs of contracting. Moreover, this reduction of agency costs outweighs the increase in the investment cost to the entrant. As a result, the critical level of incumbent effort that chokes off the entrant's incentive to enter the research race is again an increasing function of W_E.

We are now ready to characterize the implications of optimal contracting on the research race.

2.1.2 The Entrant's Best-Response Function in the Patent Race

Let $b_E\left(a_I; W_E, V_E^+\right)$ denote the solution to the entrant's problem as a function of a_I if he has wealth W_E and the value of the prize is V_E^+, i.e., the best response function. The best response function has the following properties:

Proposition 1 *i)* $b_E\left(a_I; W_E, V_E^+\right) > 0$ *for all* $a_I \leq \min\left\{\overline{a}_I, \overline{\overline{a}}_I\right\}$ *and* $b_E\left(a_I; W_E, V_E^+\right) = 0$ *else;*
ii) $\frac{db_E\left(a_I; W_E, V_E^+\right)}{dW_E} \geq 0$ *with strict inequality whenever* $F > W_E$ *and* $a_I < \min\left\{\overline{a}_I, \overline{\overline{a}}_I\right\}$, *otherwise* $\frac{db_E\left(a_I; W_E, V_E^+\right)}{dW_E} = 0$;
iii) $\frac{db_E\left(a_I; W_E, V_E^+\right)}{dV_E^+} \geq 0$ *with strict inequality whenever* $a_I < \min\left\{\overline{a}_I, \overline{\overline{a}}_I\right\}$.
iv) $b_E\left(a_I; W_E, V_E^+\right)$ *is quasi-concave in* a_I. *For* W_E *close to* F, $b_E\left(a_I; W_E, V_E^+\right)$ *is increasing in* a_I *for all* $a_I \leq \min\left\{\overline{a}_I, \overline{\overline{a}}_I\right\}$. *For* W_E *close to zero,* $b_E\left(a_I; W_E, V_E^+\right)$ *is single-peaked and decreasing in* a_I *at* $a_I = \min\left\{\overline{a}_I, \overline{\overline{a}}_I\right\}$.

Proof. Property i) is a direct corollary to Lemmas 1 and 2. ii) follows from the implicit function theorem applied to condition (8),

$$\frac{d\hat{a}_E^*}{dW_E} = \frac{\alpha\left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)\left(a_I^\alpha + r\right)}{-\alpha^2 \frac{\left(a_I^\alpha + r\right)^2}{\hat{a}_E^*}\left(F - W_E\right) + (1-\alpha)\left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)} > 0$$

Moreover, since both $\overline{a}_I$ and $\overline{\overline{a}}_I$ are nondecreasing in W_E, $\min\left\{\overline{a}_I, \overline{\overline{a}}_I\right\}$ is nondecreasing in W_E (and strictly increasing if $\min\left\{\overline{a}_I, \overline{\overline{a}}_I\right\} > 0$ and $F > W_E$.) By the same logic we prove iii): we have

$$\frac{d\hat{a}_E^*}{dV_E^+} = \frac{\alpha \hat{a}_E^{*\alpha}\left(a_I^\alpha + r\right)}{-\alpha^2 \frac{\left(a_I^\alpha + r\right)^2}{\hat{a}_E^*}\left(F - W_E\right) + (1-\alpha)\left(\hat{a}_E^{*\alpha} + a_I^\alpha + r\right)} > 0.$$

To prove iv) we begin by showing that $\hat{a}_E^*$ is a quasi-concave function of a_I^α which is either decreasing everywhere or first increasing then decreasing. Recall from above that

$$\frac{d\hat{a}_E^*}{da_I^\alpha} = \frac{\left(-\alpha\left(F-W_E\right)\left(a_I^\alpha+r\right)+\alpha\left(\hat{a}_E^{*\alpha}V_E^+-\left(\hat{a}_E^{*\alpha}+a_I^\alpha+r\right)\left(F-W_E\right)\right)-\hat{a}_E^*\right)}{-\alpha^2\hat{a}_E^{*\alpha-1}\left(V_E^+-\left(F-W_E\right)\right)\left(a_I^\alpha+r\right)+\left(\left(1-\alpha^2\right)\hat{a}_E^{*\alpha}+a_I^\alpha+r\right)} \equiv \frac{X}{Y}$$

Differentiating once more and using $\frac{d^2\hat{a}_E^*}{\left(da_I^\alpha\right)^2} = \frac{\frac{dX}{da_I^\alpha}-\frac{dY}{da_I^\alpha}\frac{d\hat{a}_E^*}{da_I^\alpha}}{Y}$, we obtain

$$\frac{d^2\hat{a}_E^*}{\left(da_I^\alpha\right)^2} = \frac{1}{Y}\left[\begin{array}{c} -2\alpha\left(F-W_E\right)+2\left\{\alpha^2\hat{a}_E^{*\alpha-1}\left(V_E^+-\left(F-W_E\right)\right)-1\right\}\frac{d\hat{a}_E^*}{da_I^\alpha} \\ -\left[-\alpha^2\left(\alpha-1\right)\hat{a}_E^{*\alpha-2}\left(V_E^+-\left(F-W_E\right)\right)\left(a_I^\alpha+r\right)\frac{d\hat{a}_E^*}{da_I^\alpha}++\left(1-\alpha^2\right)\alpha\hat{a}_E^{*\alpha-1}\frac{d\hat{a}_E^*}{da_I^\alpha}\right]\frac{d\hat{a}_E^*}{da_I^\alpha}\end{array}\right]$$

Except for the term $\{\cdot\}$ all the terms in this expression are strictly negative for all $\hat{a}_E^*$. The term in $\{\cdot\}$ becomes negative for $\hat{a}_E^*$ sufficiently large. Consequently, $\frac{d^2\hat{a}_E^*}{\left(da_I^\alpha\right)^2} < 0$ for $\hat{a}_E^*$ large enough. Observe in addition, that $\hat{a}_E^*$ has at most one extremum. The reason is that at all points where $\frac{d\hat{a}_E^*}{da_I^\alpha} = 0$ we also have $\frac{d^2\hat{a}_E^*}{\left(da_I^\alpha\right)^2} = \frac{-2\alpha(F-W_E)}{Y} < 0$. Consequently, $\hat{a}_E^*$ can have at most one extremum, which is maximum. This implies in particular that $\hat{a}_E^*$ is quasi-concave in a_I^α.

Using condition (11) it is easy to show that $\frac{d\hat{a}_E^*}{da_I^\alpha} \gtreqless 0$ if and only if $\hat{a}_E^* \gtreqless \left(\frac{\alpha}{1-\alpha}\left(a_I^\alpha+r\right)^2\left(F-W_E\right)\right)^{\frac{1}{\alpha+1}}$. Define the convex function $\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right)$ as follows

$$\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right) \equiv \left(\frac{\alpha}{1-\alpha}\left(a_I^\alpha+r\right)^2\left(F-W_E\right)\right)^{\frac{1}{\alpha+1}} \tag{12}$$

For V_E^+ sufficiently high, we have $\hat{a}_E^*\left(a_I^\alpha\right)\big|_{a_I^\alpha=0} > \widetilde{\widetilde{a}}_E\left(a_I^\alpha\right)\Big|_{a_I^\alpha=0}$. Since $\hat{a}_E^*\left(a_I^\alpha\right)$ gets concave for a_I^α large and $\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right)$ is convex, the two functions must eventually intersect. To the right of the intersection, $\hat{a}_E^*\left(a_I^\alpha\right)$ is decreasing and $\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right)$ continues to increase. Hence, $\hat{a}_E^*\left(a_I^\alpha\right)$ and $\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right)$ cannot intersect more than once.

Finally, we prove that the best response may change its slope. Recall from the proof of Lemma 1 that an optimal contract exists whenever $\hat{a}_E^*\left(a_I^\alpha\right) \geq \tilde{a}_E\left(a_I^\alpha\right)$, where the function $\tilde{a}_E\left(a_I^\alpha\right)$ is defined implicitly by equation (26). Finally, from Lemma 2, an optimal contract is accepted for $\hat{a}_E^*\left(a_I^\alpha\right) \geq a_I''\left(a_I^\alpha\right)$ where the function a_I'' is defined as $a_I''\left(a_I^\alpha\right) \equiv \frac{\alpha}{1-\alpha}W_E\left(a_I^\alpha+r\right)$. It is easy to show that $\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right) \geq \tilde{a}_E\left(a_I^\alpha\right)\forall a_I^\alpha$. If W_E is close to F, then both $\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right)$ and $\tilde{a}_E\left(a_I^\alpha\right)$ take values close to zero for all a_I^α. As a result, the intersection of $\hat{a}_E^*\left(a_I^\alpha\right)$ with $a_I''\left(a_I^\alpha\right)$ is to the left of the intersections of $\hat{a}_E^*\left(a_I^\alpha\right)$ with $\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right)$ and $\tilde{a}_E\left(a_I^\alpha\right)$. If on the other hand, W_E is close to zero, then $a_I''\left(a_I^\alpha\right)$ takes values close to zero for all a_I, and $\hat{a}_E^*\left(a_I^\alpha\right)$ intersects with $\widetilde{\widetilde{a}}_E\left(a_I^\alpha\right)$ and $\tilde{a}_E\left(a_I^\alpha\right)$ before it intersects with $a_I''\left(a_I^\alpha\right)$. ■

From Lemmas 1 and 2 it follows directly that the entrant exerts a strictly positive amount of research effort if and only if the research effort chosen by the incumbent is not too large. The higher the value of the race, i.e., the higher V_E^+, the higher is the entrant's research effort. If $W_E \geq F$ then the financing constraints are slack and an increase in W_E has no effect whatsoever on the entrant's best response. The best-response function in this case coincides with the one in Reinganum's model. If $W_E < F$ then the financing constraints bind. The larger $F - W_E$, the larger the repayment share to the investor and the smaller the entrant's effort choice. Intuitively, an increase in $F - W_E$ increases the agency costs of finance and increases the entrant's marginal costs of innovative activity.

It is interesting to note that financing may change the nature of strategic interaction in a local sense. In particular, it does so if the entrant's wealth is close to zero. In that case, for low levels of the incumbent's innovative activity, an increase in the incumbent's research effort increases the entrant's incentive to increase his research effort. That is, in that region research efforts are strategic complements in the sense of Jeremy Bulow, John Geanakoplos and Paul Klemperer (Bulow, Geanakoplos and Klemperer, 1985). This is the standard case which also arises in Jennifer Reinganum's model without financing constraints. However, as the level of the incumbent's research effort is increased, the expected value of the entrant's claim decreases. In addition, the share of profits that must be given to the investor to make him break even increases as the incumbent's research effort increases. As a result, the entrant's marginal incentives to increase his effort are eventually decreased and the nature of strategic interaction changes to strategic substitutes. This effect will not occur if the entrant needs to raise only a small amount of finance from the investor, i.e., if W_E is close to F.

2.2 The incumbent's problem

Consider now the incumbent's problem. Let $V_I^+ \equiv \frac{\overline{\pi}_I}{r}$ denote the net present value of the incumbent's firm if it wins the race and let $V_I^- \equiv \frac{\underline{\pi}_I}{r} \geq 0$ denote the value of the incumbent firm if the entrant wins the race. If $V_I^- > 0$ then the innovation is non drastic. Finally, recall that π is the flow profit of the firm if it uses its current technology. As it is standard, we assume that $V_I^- \leq \frac{\pi}{r}$.

2.2.1 The Financing of the Incumbent

A contract between the incumbent firm and an investor specifies the initial investment $F - W_I$ and a repayment scheme. We again restrict attention to stationary contracts in the sense that the repayment scheme does not depend on the date of the innovation. Any contract of this type, whatever complex repayment structure it may have, can be written in equivalent form in terms of repayment shares in the different contingencies. Let these shares be $(\sigma^-, \sigma^+, \sigma)$, corresponding to the investor's share in the profits when the entrant innovates, the incumbent innovates, and when no one innovates, respectively. Let $\boldsymbol{\sigma} = (\sigma^-, \sigma^+, \sigma)$ denote the vector of repayment shares, and let $V_I(a_E, W_I, \boldsymbol{\sigma})$ denote the value of the incumbent's claim to the ongoing firm before any innovation has occurred. For brevity we shall write $V_I(\cdot)$ for $V_I(a_E, W_I, \boldsymbol{\sigma})$.

To characterize optimal contracts we proceed again in two steps. First, we characterize the best contracts that can be offered to the incumbent conditional on engaging in research. Second, we investigate whether the incumbent will indeed find it optimal to engage in research.

With financing, the asset equation takes the form

$$rV_I(\cdot) = \max_{a_I} a_I^\alpha \left(\left(1-\sigma^+\right) V_I^+ - V_I(\cdot)\right) + a_E^\alpha \left(\left(1-\sigma^-\right) V_I^- - V_I(\cdot)\right) + (1-\sigma)\pi - a_I. \tag{13}$$

The difference to the entrant's asset equation is that the incumbent receives flow profits π as long as no innovation occurs and that the value of the incumbent's firm if the entrant wins the race, V_I^-, may be positive. Since the right-hand-side of the asset equation is strictly concave in a_I, a solution to the incumbent's problem must satisfy the first-order condition

$$\alpha a_I(\boldsymbol{\sigma})^{\alpha-1}\left((1-\sigma^+)V_I^+ - V_I(\cdot)\right) = 1. \tag{14}$$

Multiplying condition (14) on both sides by $a_I(\boldsymbol{\sigma})$ and substituting the resulting expression into (13) we solve for the value of the incumbents claim

$$V_I(a_E, W_I, \boldsymbol{\sigma}) = \frac{(1-\alpha)\,a_I(\boldsymbol{\sigma})^{\alpha}(1-\sigma^+)V_I^+ + a_E^{\alpha}(1-\sigma^-)V_I^- + (1-\sigma)\pi}{(1-\alpha)\,a_I(\boldsymbol{\sigma})^{\alpha} + a_E^{\alpha} + r}. \tag{15}$$

In addition, investors must break even. Formally, it must be true that

$$\frac{a_I(\boldsymbol{\sigma})^{\alpha}\sigma^+V_I^+ + a_E^{\alpha}\sigma^-V_I^- + \sigma\pi}{a_I(\boldsymbol{\sigma})^{\alpha} + a_E^{\alpha} + r} = F - W_I. \tag{16}$$

An optimal contract maximizes (15) subject to (16) and (14). It is interesting to note that financing does not always involve a loss of efficiency for the incumbent. It is sometimes possible to implement the first-best outcome even if the incumbent needs to raise cash from outside investors, i.e., even if $W_I < F$.[8]

Lemma 3 *There exists* $\overline{a}_E^{FB} \equiv \overline{a}_E^{FB}\left(V_I^-, W_I, \pi\right)$ *such that a contract that implements the first-best outcome exists if and only if* $a_E \leq \overline{a}_E^{FB}$. $\overline{a}_E^{FB}$ *is strictly positive for* $\frac{\pi}{r} > F - W_I$ *and bounded for* $F - W_I > V_I^-$. $F - W_I \in \left(V_I^-, \frac{\pi}{r}\right)$, $\overline{a}_E^{FB}$ *weakly increasing in its arguments, and strictly increasing whenever* $\overline{a}_E^{FB} > 0$.

The incumbent firm can pledge its current profits to finance its current research expenditures. If the current profits are relatively large relative to the size of the investment, then a first-best financing contract is feasible for low research efforts of the entrant. The exact condition that we derive in the appendix states that the first-best outcome is implementable if and only if $W_I + \frac{a_E^{\alpha}V_I^- + \pi}{a_E^{\alpha}+r} \geq F$. $\frac{a_E^{\alpha}V_I^-}{a_E^{\alpha}+r}$ is the expected present value of the incumbent's firm if the entrant innovates and $\frac{\pi}{a_E^{\alpha}+r}$ is the net present value of the incumbent's current stream of profits. These values are independent of the incumbent's own effort. As a result, these values can be pledged without creating any moral hazard problems with respect to the choice of effort. The higher

[8] The proofs of our results in this section follow the same logic as those for the entrant's problem. To avoid repetition, we have relegated these proofs to a supplementary appendix, which can be downloaded from http://www.hec.unil.ch/dszalay.

the research activity of the entrant, the higher the likelihood that the incumbent loses his current profits, and thus the smaller the value of pledgeable profits. As a result, first-best financing becomes eventually impossible for a large enough research activity of the entrant.

Next, we investigate whether or not the incumbent will accept the optimal contract offer that implements the first-best outcome. In view of our previous lemma, this question is relevant when $F - W_I \leq V_I^-$, because in that case, first-best financing is feasible for all a_E. The incumbent chooses to enter the research race if and only if

$$V_I\left(a_E\right) \geq F + \frac{a_E^{\alpha} V_I^- + \pi}{a_E^{\alpha} + r}. \tag{17}$$

i.e., if and only if the net surplus of the project is larger than the value of profits and wealth the incumbent obtains if he does not do any research at all.

Lemma 4 *For V_I^+ sufficiently large there exists $\overline{\overline{a}}_E^{FB} \equiv \overline{\overline{a}}_E^{FB}\left(V_I^+, V_I^-, \pi\right)$ such that the incumbent accepts a contract that implements the first-best outcome if and only if $a_E \leq \overline{\overline{a}}_E^{FB}$. $\overline{\overline{a}}_E^{FB}$ is increasing in V_I^+ and decreasing in V_I^- and π.*

The net value of engaging in research versus not doing so depends in a quite complex way on the entrant's research effort. However, if the race is sufficiently valuable, the incumbent's participation region is convex. While it is important to understand the case of first-best financing, the case is not very rich in terms of comparative statics. In particular, the incumbent's cash plays (by definition of first-best) no role. More interesting in this respect, is the case of a financially constrained incumbent, which we now address. The following Lemmas are essentially identical to Lemmas 1 and 2 in the analysis of the entrant's problem, and we state them without further comment.

Lemma 5 *For all $W_I \geq 0$ and F there exists $\overline{a}_E \equiv \overline{a}_E\left(V_I^+, W_I, V_I^-, \pi\right)$ such that a unique optimal contract exists if and only if $a_E \leq \overline{a}_E$. $\overline{a}_E$ is nondecreasing in all its arguments. It is strictly increasing in V_I^+, V_I^-, and π whenever $\overline{a}_E > 0$. It is strictly increasing in W_I whenever $F > W_I$ and $\overline{a}_E > 0$.*

Lemma 6 *Suppose V_I^+ is sufficiently large so that the incumbent engages in research for $a_E = 0$. Then, for all $W_I \geq 0$ and F, there exists $\overline{\overline{a}}_E > 0$ such that the entrant accepts the optimal contract if and only* *if* $a_E \leq \overline{\overline{a}}_E\left(V_I^+, W_I; \cdot\right)$. $\overline{\overline{a}}_I$ *is nondecreasing in* V_I^+ *and* W_I. $\overline{\overline{a}}_I$ *is strictly increasing in* V_E^+ *and strictly increasing in* W_I *whenever* $F > W_I + \frac{a_E^\alpha V_I^- + \pi}{a_E^\alpha + r}$.

2.2.2 The Incumbent's Best-Response Function

We are now ready to state the effects of financing on the incumbent's best response function in the patent race. These effects are essentially isomorphic to those in the entrant's case; they differ only in the feasibility conditions of first-best.

Proposition 2 *i)* $b_I\left(a_E; W_I, V_I^+\right) > 0$ *for all* $a_E \leq \min\left\{\overline{a}_E, \overline{\overline{a}}_E\right\}$ *and* $b_I\left(a_E; W_I, V_I^+\right) = 0$ *else;*
ii) $\frac{db_I\left(a_E; W_I, V_I^+\right)}{dW_I} \geq 0$ *with strict inequality whenever* $F > W_I + \frac{a_E^\alpha V_I^- + \pi}{a_E^\alpha + r}$ *and* $a_E < \min\left\{\overline{a}_E, \overline{\overline{a}}_E\right\}$, *otherwise* $\frac{db_I\left(a_E; W_I, V_I^+\right)}{dW_I} = 0$;
iii) $\frac{db_I\left(a_E; W_I, V_I^+\right)}{dV_I^+} \geq 0$ *with strict inequality whenever* $a_E < \min\left\{\overline{a}_E, \overline{\overline{a}}_E\right\}$.
iv) $b_I\left(a_E; W_I, V_I^+\right)$ *is quasi-concave in* a_E. *For* W_I *close to* $F - \frac{a_E^\alpha V_I^- + \pi}{a_E^\alpha + r}$, $b_I\left(a_E; W_I, V_I^+\right)$ *is increasing in* a_E *for all* $a_E \leq \min\left\{\overline{a}_E, \overline{\overline{a}}_E\right\}$. *For* W_I *close to zero,* $b_I\left(a_E; W_I, V_I^+\right)$ *is single-peaked and decreasing in* a_E *at* $a_E = \min\left\{\overline{a}_E, \overline{\overline{a}}_E\right\}$.

The proof is analogous to the one in the entrant's case and therefore omitted.

2.3 Equilibrium and Comparative statics

Propositions 1 and 2 state that the R&D race is a game with strategic complements, as defined by Jeremy Bulow, John Geanakoplos and Paul Klemperer (Bulow, Geanakoplos and Klemperer, 1985) on the domain where best responses are strictly positive when both players are not too financially constrained. It has best response functions that are continuous and increasing in the other firm's research intensity until they drop to zero, because either no optimal contract exists or the firm does not accept its contract anymore. The best response functions are non-decreasing in own wealth, and strictly increasing in own wealth in the case of second-best. Provided that V_I^+ and V_E^+ are large relative to F, the game has an equilibrium for all W_I and W_E.[9]

[9] While the case of very financially constrained players is interesting from a theoretical perspective, it does not seem to be relevant for our empirical analysis. Our empirical investigation below uses firms that are in COMPUSTAT, i.e., publicly traded. These firms' assets should be reasonably large relative to the fixed cost of an R&D race.

We establish formally the main properties of our equilibrium.

Proposition 3 *For V_I^+ and V_E^+ large relative to F, V_I^- and $\frac{\pi}{r}$, the R&D race between the cash constrained incumbent and the entrant has a Nash equilibrium. For W_I and W_E sufficiently close to F and α sufficiently small and V_I^+ and V_E^+ sufficiently large, the race has a unique equilibrium, $\{a_I^*, a_E^*\}$.*

Proof. Existence of equilibrium follows from arguments similar to Reinganum (Reinganum, 1985). Therefore, we omit a formal proof here. For uniqueness, observe that best response functions are increasing wherever they are positive for W_I and W_E sufficiently close to F. Thus, it suffices to have the best-responses concave. We have

$$\frac{d^2\hat{a}_E^*}{da_I^2} = \frac{d^2\hat{a}_E^*}{\left(da_I^\alpha\right)^2}\alpha^2 a_I^{2\alpha-2} + \frac{d\hat{a}_E^*}{da_I^\alpha}\alpha\left(\alpha-1\right)a_I^{\alpha-2}.$$

So $\frac{d^2\hat{a}_E^*}{da_I^2} < 0$ iff $\frac{d^2\hat{a}_E^*}{\left(da_I^\alpha\right)^2}\alpha a_I^\alpha + \frac{d\hat{a}_E^*}{da_I^\alpha}\left(\alpha-1\right) < 0$. Clearly, that is satisfied if $\frac{d^2\hat{a}_E^*}{\left(da_I^\alpha\right)^2} < 0$. When $\frac{d^2\hat{a}_E^*}{\left(da_I^\alpha\right)^2} \geq 0$, then $\frac{d^2\hat{a}_E^*}{da_I^2} < 0$ if

$$\frac{2\left\{\alpha^2\hat{a}_E^{*\alpha-1}\left(V_E^+ - \left(F - W_E\right)\right) - 1\right\}\frac{d\hat{a}_E^*}{da_I^\alpha}}{Y}\alpha a_I^\alpha + \frac{d\hat{a}_E^*}{da_I^\alpha}\left(\alpha-1\right) < 0,$$

(see the proof of proposition 1 for the definition of Y) which is equivalent to

$$\left\{\alpha^2\hat{a}_E^{*\alpha-1}\left(V_E^+ - \left(F - W_E\right)\right) - 1\right\}\left(2\frac{\alpha}{1-\alpha}\frac{a_I^\alpha}{a_I^\alpha + r} + 1\right) < \frac{\left(1-\alpha^2\right)\hat{a}_E^{*\alpha}}{a_I^\alpha + r}.$$

For α close to zero that condition is implied by the condition for existence of a contract. (see the proof of lemma 1.) By continuity, $\frac{d^2\hat{a}_E^*}{da_I^2} < 0$ for small α. An analogous proof applies to the best response function of the incumbent. Finally, a concave increasing and a convex increasing function intersect at most once. ∎

Proposition 4 *The probability that the incumbent wins the race is increasing in W_I and decreasing in W_E whenever $\frac{db_E(a_I^*;\cdot)}{da_I} \leq 1$ and $\frac{db_I(a_E^*;\cdot)}{da_E} \leq 1$.*

The probability that the incumbent wins is $\frac{a_I^*}{a_I^*+a_E^*}$. An increase in W_I directly increases a_I^* and also increases a_E^* because the entrant's best response function is increasing. However, the second effect is smaller

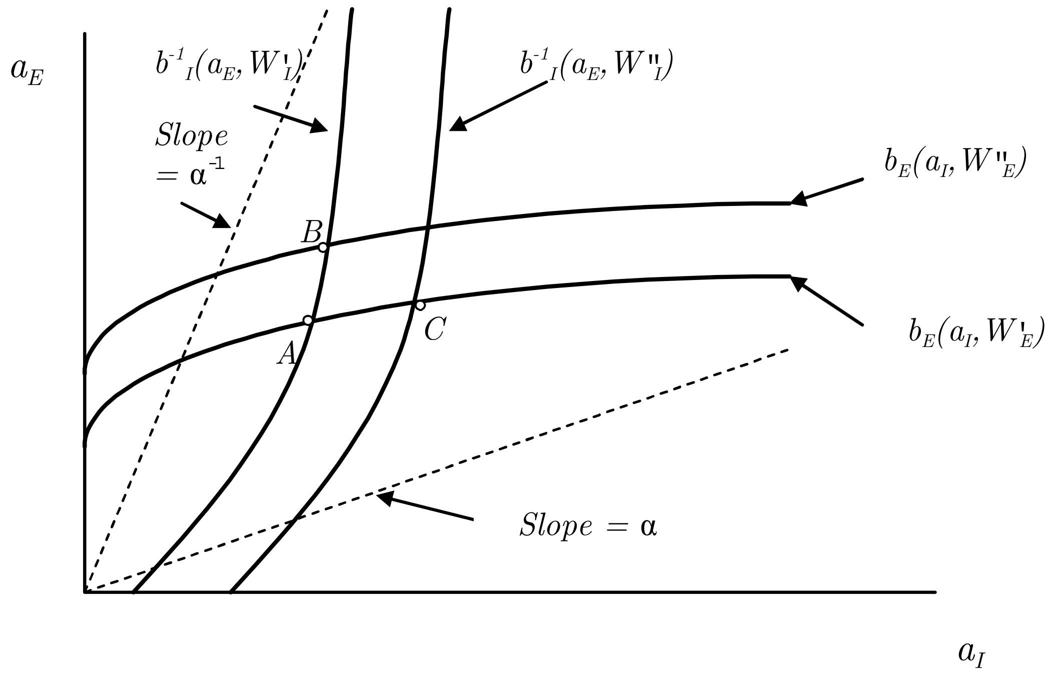

Figure 1: Inside a cone defined by the functions $a_E = \alpha a_I$ and $a_E = \frac{1}{\alpha} a_I$ the best response functions have slopes smaller than one. An increase in the entrant's wealth from W'_E to W''_E shifts the entrant's best response function up. The equilibrium changes from point A to point B. Likewise, an increase in the incumbent's wealth from W'_I to W''_I shifts the incumbent's best response function outwards and changes the equilibrium from point A to C.

when the best response functions are not too steep around the equilibrium. In particular, the slopes of the best response functions are smaller than one whenever the equilibrium is not too asymmetric, in the sense that a_I^* and a_E^* do not differ by more than a factor α. To see this, observe that $\frac{db_E(a_I;\cdot)}{da_I} = \alpha \frac{\hat{a}_E^*}{a_I} \frac{a_I^\alpha}{a_I^\alpha + r} \Gamma\left(\hat{a}_E^*\right),$ where $\Gamma\left(\hat{a}_E^*\right)$ is defined in (11). Since both $\frac{a_I^\alpha}{a_I^\alpha + r} < 1$ and $\Gamma\left(\hat{a}_E^*\right) < 1$ (see the proof of lemma 2), we have $\frac{db_E(a_I;\cdot)}{da_I} < 1$ for all $\hat{a}_E^* < \frac{a_I}{\alpha}$. By the same reasoning $\frac{db_I(a_E;\cdot)}{da_E}$ for $\hat{a}_I^* < \frac{a_E}{\alpha}$. We illustrate these findings in Figure 1.

The effects of the remaining parameters on the equilibrium research efforts are ambiguous. Anything that causes $\overline{\pi}_E$ to increase (say an increase in demand) will also increase $\overline{\pi}_I$. As a result both reaction functions are shifted upwards by an increase in the value of the patent race as measured by V_E^+ and V_I^+ and the effect on the equilibrium efforts is unclear. Increases in $\underline{\pi}_I$ and π have two effects. On the one hand it may become feasible to write first-best contracts so that the incumbent's best response function shifts up. On the other hand, an increase in operating profits makes the incumbent reluctant to destroy these profits, so that he reduces his research efforts and his best response function shifts downwards.

We now proceed to investigate whether the predictions of our game are verified empirically.

3 The Data

3.1 Data set Construction

We use two sources of data. The first source is the NBER Patent Citations Data File developed by Bronwyn Hall, Adam Jaffe and Manuel Trajtenberg (Hall, Jaffe and Trajtenberg, 2002), which collects information of all utility patents granted in the United States between 1963 and 1999. We can identify the technological category of the patents, the dates they were awarded and the assignee in the database. Each patent awarded after 1975 is linked to all the patents it cites and the assignee names in the patent records are matched to the name of the company as it appears in COMPUSTAT, our second source of data. We get from COMPUSTAT the financial information of the patent assignees whose stock is publicly traded in the U.S.

We regard each patent award in the data as the outcome of a race. This implies that the NBER Patent Citations Data File is of use to us only for industries that rely heavily on patent protection as a way of appropriating the returns of R&D. It is well recognized that patenting is crucial to protect the competitive advantages of R&D in the pharmaceutical industry (see the survey conducted by Richard Levin, Alvin Klevorick, Richard Nelson and Sidney Winter (Levin et al., 1987), and its follow-up by Wesley Cohen, Richard Nelson, and John Walsh (Cohen, Nelson and Walsh, 2000).[10] Thus, we restrict our sample to patents in the technological category 3, i.e., Drugs and Medical, and the subcategories 31, 33 and 39: Drugs, Biotechnology, and Miscellaneous Drugs, respectively.

Then we classify patent assignees as either incumbents or entrants to a race.[11] We define incumbency such that we exploit the wealth of data in the patent databases as comprehensively as possible. We first find

[10] Firms in many other industries use rather superior marketing, customer service or improved product characteristics instead of patents.

[11] In our model, an incumbent is the player that is currently profiting from the existing technology, while an entrant is not. It is difficult to construct an equivalent empirical measure, unless a dataset is constructed specifically for this purpose. Josh Lerner (Lerner, 1997), for example, collects a data base of disk drive manufacturers, from the industry's annual reports. Hence he is able to observe the disk drive characteristics that each firms sells, and when innovators market higher disk drive densities. As far as we know, this is the only study that takes a step towards defining incumbency at the firm level.

all the citations made by each pharmaceutical patent in the NBER data base. Then we record the assignees of the cited patents and the dates at which the cited patents were assigned. We then say that a patent was won by an incumbent if the assignee also owned at least one of the cited patent that is not "too old".

By "too old" we mean that some patents might not have value any more to the holder and thus not be relevant for his decision to develop a new product. Since it is difficult to assess when a patent has no incumbency value anymore to the potential innovator, we use several measures of incumbency. Thus, the winner of a patent is said to be an incumbent if he owns at least one cited patent that is at most one year old, or at most 2 years old, or at most 3, 4, 5, 10 or 20 years old. The last is the most generous possible definition of an incumbent, since property rights extend for 20 years at most. All of our empirical tests will be performed for all these seven definitions of incumbency.[12]

We believe that the citations are a good measure of the previously existing technology over which the citing patent is built because it is the legal obligation of the applicant to cite all the prior art of the innovations he claims. In fact, the patent examiner, who must be a specialist in the field, examines these citations and decides which ones to be included finally in the award.

3.2 Data Description

The NBER data set has 121,204 patents in the subcategories 31, 33 and 39 between 1975 and 1999. We are able to classify 91,656 of these. The remaining patents are lost using our definition of incumbency due to missing observations in the assignee names of citing or cited patents. This problem is particularly acute for the older patents.

Table 1 summarizes the results of applying our definition of incumbency. Under the most generous definition of incumbency, a patent is won by an entrant if the assignee owns none of the citations or if the citations it owns are older than 20 years. In that case, 65.11% of all classifiable patents between 1975 and 1999 were awarded to entrants. A more restrictive definition of an incumbent, e.g., the youngest citation is older than 5 years, implies a larger percentage of patents won by entrants: 73.81%. Not surprisingly, the

[12] We have also repeated our empirical tests for the cases where incumbency is defined as having cited your own patents that are up to 25 or 30 years old. Due to patent law, we should not expect 25 or 30 year old patents to have any incumbency value. However, we believe that repeating the exercise through these other definitions of incumbency can make more clear that incumbency matters and our empirical approach to define is relevant. We will comment these results later in the paper.

percentage of entrant-won patents decreases in time. To a large extent this is due to the fact that we expect to have lost proportionally more incumbent won patents in the earlier years: entrant won patents with few young citations can always be classified. Moreover, even using the most generous definition of incumbency, almost two thirds of the patents are won by entrants.[13]

4 Econometric Analysis

4.1 A Logit Approach

Let $\lambda_{ih}^* = (a_{ih}^*)^\alpha$ denote the equilibrium hazard rate of firm $h \in \{E, I\}$ in race i. The Nash Equilibrium of our model can be written as

$$\begin{aligned}
\lambda_{iI}^* &= \lambda_I^*(W_{iI}, W_{iE}, V_{iE}^+, V_{iI}^+, \pi_i, S_{iI}, S_{iE}) = \lambda_I^*(\mathbf{X}_{iI}, \mathbf{X}_{iE}; \boldsymbol{\beta}_I) = \lambda_I^*(\mathbf{X}_i; \boldsymbol{\beta}_I), \\
\lambda_{iE}^* &= \lambda_E^*(W_{iI}, W_{iE}, V_{iE}^+, V_{iI}^+, \pi_i, S_{iI}, S_{iE}) = \lambda_E^*(\mathbf{X}_{iI}, \mathbf{X}_{iE}; \boldsymbol{\beta}_E) = \lambda_E^*(\mathbf{X}_i; \boldsymbol{\beta}_E),
\end{aligned}$$

where W_{iI} and W_{iE} are measures of financial wealth, V_E^+ and V_I^+ measure the values of the new patent to the winner, π_i measures the value of the patent that is replaced, and S_{iI} and S_{iE} are vectors of other variables we us as empirical controls. $\boldsymbol{\beta}_I$ and $\boldsymbol{\beta}_E$ are the parameter vectors associated to the exogenous variables. The incumbent's equilibrium winning probability is

$$\Pr(\text{race } i \text{ is won by the incumbent}) = \int_0^\infty e^{-(\lambda_{iI}^* + \lambda_{iE}^*)t} \lambda_{iI}^* dt = \frac{\lambda_{iI}^*(\mathbf{X}_i)}{\lambda_{iI}^*(\mathbf{X}_i) + \lambda_{iE}^*(\mathbf{X}_i)} = \frac{\frac{\lambda_I^*(\mathbf{X}_i)}{\lambda_E^*(\mathbf{X}_i)}}{\frac{\lambda_I^*(\mathbf{X}_i)}{\lambda_E^*(\mathbf{X}_i)} + 1}$$

If we approximate the hazard rates with exponential functions of a linear index of the parameters, i.e. if we take $\lambda_{ih} \approx \exp(\mathbf{X}_i \beta_h)$, then we we can write $\frac{\lambda_{iI}^*}{\lambda_{iE}^*} \approx \exp(\mathbf{X}_i \boldsymbol{\beta}_I - \mathbf{X}_i \beta_E) = \exp(\mathbf{X}_i(\beta_I - \beta_E)) \equiv \exp(\mathbf{X}_i \beta)$. Notice that the hazard rate ratio depends only on the difference $\beta_I - \beta_E$ and not on each parameter individually. Henceforth we write β for this difference. The incumbent's equilibrium winning probability

[13] While this observation is only preliminary, it is consistent with the predictions of Reinganum (Reinganum, 1983) and the results of Lerner (Lerner, 1997): all other things constant, the incumbent will have less incentives than the entrant to invest more heavily in research and develop the next innovation.

simplifies to

$$\Pr(\text{race } i \text{ won by the incumbent}) = \frac{\exp(\mathbf{X}_i\boldsymbol{\beta})}{1+\exp(\mathbf{X}_i\boldsymbol{\beta})} \tag{18}$$

Since the expression on the right hand side of condition (18) is nothing but the well-known logit formula, this suggest that we might want to interpret our model in the sense that firms "submit" their exogenous variables at the beginning of the race and "nature" picks the incumbent with probability

$$\Pr(I_i = 1) = \Pr(\beta_0 + \beta_1 W_I + \beta_2 W_E + \mathbf{c}\boldsymbol{\alpha} + \varepsilon_i \geq 0), \tag{19}$$

where $I_i = 1$ if the winner of patent i is an incumbent, and 0 otherwise. W_I and W_E are measures of the incumbent's and entrant's financial resources, respectively, the vector $\mathbf{c}$ includes the control variables $V_{iE}^+, V_{iI}^+, \pi_i, S_{iI}, S_{iE}$, and $\boldsymbol{\alpha}$ is the vector of their associated parameters. The error term, ε_i, represents the randomness in the choice of nature. If we assume that the error term follows the Weibull, conditions (19) and (18) become equivalent.[14] Hence, we can test our model with a logit regression.

The comparative statics of our model regarding the effect of financial constraints to the racing behavior are that β_1 should be significantly different from zero and positive, while β_2 should be significantly different from zero and negative. The strength of this test is that both variables determine the outcome of the race jointly and this is precisely how this is implemented empirically. We also test the role of strategic interactions of this game through the regressors included in $\mathbf{c}$. Observe, as we noted earlier, that our estimation identifies the vector of parameters $\boldsymbol{\beta}$ (but not β_I and β_E) from the variation in the observed outcome of the races.

[14] It should be noted that the equivalence holds only for the Weibull. Some authors prefer to use the normal distribution for the error term, but this would not link directly the probabilities in the theoretical model to those in the econometric model. We compute but do not report here parameter estimates using the normality assumption. As it often happens, the estimates we obtained in both cases are extremely similar, most likely being different only because of the difference in the variances that scale the parameters under each distributional assumption (see, for example, Davidson and MacKinnon, 2004, Chapter 11).

4.2 Specification

4.2.1 Cash: W_E and W_I.

We use the level of cash holdings (COMPUSTAT item 36) by the firm as our measure for financial wealth, W. The amount held as cash is precisely what the firm can use to finance R&D without requiring external finance, which is more costly. However, a firm may not pledge the whole stock of cash to one race if it engages in many races simultaneously. Since we cannot observe the number of races that the firm was engaged in at a certain point in time, we use the value of total assets as a proxy, assuming that the two measures are positively correlated. Then, we normalize the value of cash holdings by the amount of total assets to proxy for the amount of cash that a firm has per race. Since firms choose how much to spend in the race based on their availability of cash before the race is won we use three different lags of W: one, two and three years before the patent is awarded.

The estimation of the parameters in (19) poses one major challenge: while we can observe directly in COMPUSTAT W_E when an entrant wins and W_I when an incumbent wins, we have to find reasonable proxies for W_E when an incumbent wins the race and for W_I when an entrant wins. We assume that, when an incumbent wins a patent, any firm in the industry that had no cited patents is a potential entrant. W_E is proxied by the average of the normalized value of cash holdings in the given time period over all the firms without cited patents in the same four-digit Standard Industry Classification Code (SIC).

In case of W_I the problem could in principle be solved in the same way as for W_E. When a patent is won by an entrant, the financial wealth of incumbents could be proxied by the average of W over all firms with cited patents. Unfortunately, this solution proves unfruitful *ex post*. We find very few matches when we merge the CUSIP codes of the assignees of patents cited by entrant-won patents in the NBER Patents Citations Database with COMPUSTAT. In fact, in regressions that we do not report here, almost all the observations on entrant-won patents were lost due to the unobservability of W_I : less than 2% of the usable sample corresponded to patents won by entrants. The estimates from such regressions are clearly not to be trusted, since the percentage of entrant-won patents in the population exceeds 65%.

To overcome this problem we proxy the incumbent's financial wealth with a measure that would bias the results against our maintained hypothesis. Our hypothesis is that β_1 should be significantly different from zero and positive. If the downward-biased estimate is still significantly different from zero and positive then so should also be an unbiased estimate. One way to bias the estimate downward is to proxy W_I for entrant-won races with the maximum value of cash holdings to total assets in the same SIC code for the given time period. In other words, we use the highest cash to assets ratio, i.e., the firm richest in cash per race, in the industry to proxy for the incumbent's financial wealth when entrants win. Intuitively, this would bias downward the maximum-likelihood estimator of β_1 because it would associate a failure to win the race by an incumbent with levels of financial resources that are, by definition, higher than those of any of the actual incumbents.[15]

4.2.2 The market value of the award: V_E^+ and V_I^+.

Our model does not give an unambiguous prediction of the effects of V_E^+ and V_I^+, respectively. However, the outcome of the race clearly depends on these measures and it is necessary to include them as control variables in all our empirical specifications.

Measures of the value of each patent are not easy to come by. For example, the value of intangibles is not disaggregated to the patent level in COMPUSTAT. However, Bronwyn Hall, Adam Jaffe and Manuel Trajtenberg have shown recently that the market value of a patent can be approximated well with the number of citations that a patent receives (Hall, Jaffe and Trajtenberg, 2004). While the number of citations has been used traditionally as a measure of the *social* value of a patent (see, for example, Trajtenberg, 1990), Hall et al. have used the NBER Patent Citations data to show that an extra citation per patent boosts the firm's market value by 3% on average. Thus, we use the number of citations that a patent received in its whole lifetime to measure V_E^+ and V_I^+. We use one measure for both because we cannot tell how each player profits differently from each patent. However, the number of citations may be a good proxy for both because increases in the number of citations are most likely indicative of increases in both V_E^+ and V_I^+. Notice too that the strategic behavior in the race depends on the ex ante expectations of these values not on their realizations. Thus, what we use is only a proxy.

Table 2 summarizes this measure. Panel A shows the average number of citations received by patents

[15] We have illustrated the downward bias on the maximum likelihood estimates in a previous version, which is available upon request.

won by entrants and incumbents for seven definitions of incumbency. Incumbent-won patents appear to receive on average more citations, i.e., are more valuable. This may be because, all other things constant, incumbents need stronger incentives than entrants to innovate due to the cannibalization effect. Panel B tests if the difference of the means is significantly different from zero. When incumbency is defined as owning at least one cited patent that is not older than 5 years, we can reject strongly that patents won by entrants are, on average, more valuable than patents won by incumbents. However, this measure may be inaccurate because of differences across time in the propensity of applicants and reviewers to include citations, and by the natural truncation in the count of citations made by the more recent patents. Thus, we rescale the numbers of citations by the average number of citations received by a patent in each grant year and technological category (these factors are provided by Hall, et al., 2002). As shown in Panel C, for all definitions of incumbency, we reject strongly that patents won by entrants are more valuable.

4.2.3 The market value of cited patents: π.

Our measure for π is the average number of citations received by the cited patents. We distinguish between cited patents that are less than one year old, between 1 and 2 years old, 2 and 3, and so forth, up to between 10 and 20 year-old citations. In all cases, we rescale these counts by the average number of citations received in the technological group in the particular grant year. As discussed above, the theoretical effect of π is ambiguous. Thus, the net effect of π on the probability that the incumbent wins is an empirical issue.

4.2.4 Patenting Experience and Firm Size

We include the average number of patents accumulated by the incumbents and the entrants to the date of the award of the patent, in the same patent class, to control for the effectiveness of the player's obtaining patents. We would expect that players who have accumulated more patents in the past in the same class would be more experienced in the patenting process and thus be more likely to obtain a new patent, *ceteris paribus*. We control also for the average size of the incumbents and entrants. We expect the size to capture other unobservable variables, and that larger firms would be more likely to win given races all other things constant. For example, size might capture some variation in the effectiveness of R&D, that is not accounted for by the previous patenting experience of the firm. We use in all regressions year dummies as further controls. These should capture exogenous aggregate changes in financing conditions or additional changes in procedures in the US Patent Office.

4.2.5 The Error term

Finally, since we work with a large cross-section of patents, the error term, ε, could be heteroskedastic. For every specification we compute the maximum likelihood estimates of the logit under the assumption that the error is homoskedastic. We use these estimates to perform the BRMR specification test suggested by Davidson and MacKinnon (2004), where the alternative hypothesis is that $Var(\varepsilon_i) = \exp(\mathbf{Z}_i\boldsymbol{\theta})$. In $\mathbf{Z}_i$ we include all the exogenous characteristics that describe the race i : the citations received by the patent and the average number of citations received by all the patents cited by patent i.[16]

4.3 Results

We estimate the parameters of (19) when we use one, two or three year lags of the value of cash holdings normalized by total assets. Table 4 shows these estimates when consider balance sheet data two years before the award of the patent. Tables ii and iii in the Web Supplement show the estimates for one and three year lags. Each column in these tables is for one of the definitions of an incumbent.[17]

4.3.1 Base Specification

The results shown in the first column of Table 4 are very consistent with the predictions of the model. In this case we use the most generous definition of an incumbent, which is when patents that are up to twenty years old still make the firm an incumbent to the race.

[16] This test is performed by fitting the model

$$\widehat{V}_i^{-\frac{1}{2}}(I_i - \frac{\exp(\mathbf{X}_i\widehat{\boldsymbol{\beta}})}{\exp(\mathbf{X}_i\widehat{\boldsymbol{\beta}})+1}) = \widehat{V}_i^{-\frac{1}{2}} \frac{\exp(\mathbf{X}_i\widehat{\boldsymbol{\beta}})}{(\exp(\mathbf{X}_i\widehat{\boldsymbol{\beta}})+1)^2}\mathbf{X}_i\mathbf{b} + \widehat{V}_i^{-\frac{1}{2}} \frac{\exp(\mathbf{X}_i\widehat{\boldsymbol{\beta}})}{(\exp(\mathbf{X}_i\widehat{\boldsymbol{\beta}})+1)^2}(-\mathbf{X}_i\widehat{\boldsymbol{\beta}})\mathbf{Z}_i\mathbf{c}+u,$$

where $\widehat{V}_i^{-\frac{1}{2}}$ and $\widehat{\boldsymbol{\beta}}$ are the maximum likelihood estimates of the error variance and slope parameters, respectively, of the homoskedastic logit model, i.e., $\boldsymbol{\theta} = \mathbf{0}$. Under the null hypothesis, the explained sum of squares of this regression is asymptotically distributed as $\chi^2(r)$, where r is the dimension of $\mathbf{Z}$.

[17] In all cases we also computed probit estimates. The results are not reported here but are extremely similar. The statistical inference from probit estimates is not different than from logit.

Cash: The estimates of β_1 and β_2 are highly statistically significant and have the sign predicted by our model: the incumbent's cash to total assets ratio parameter has a positive sign and the entrant's cash to total assets ratio parameter has a negative sign. We interpret the value and discuss the economic significance of these estimates and most others in Section 4.3.3 below.

Size: The size of incumbents, measured by the total book value of assets one year before the race has a positive sign, whereas the size of entrants has a negative sign. Both estimates are significantly different from zero. All other things constant, larger entrants or incumbents are more likely to win than smaller ones.[18]

Number of Patents: As expected, the more patents the incumbent has accumulated, the more likely it is that he wins the race (a positive and significant estimate). The same is true for entrants but the coefficient of the entrant's accumulated patents is much larger than that of the incumbent.

Value of Race: The estimate of the parameter associated to the market value of the patent raced for, as proxied by the number of citations it receives is positive and significantly different from zero in the first column. A higher expected value of the patent shifts right both best-response functions, so *ex-ante* the magnitude of this effect cannot be assessed (ultimately, it depends on the slope of both reaction functions, which cannot be identified with this data).

Age of Cited Patents: Table 4 shows also the role of old cited patents on the incumbent's incentives to innovate. In the case of cited patents that are between 3 and 20 years old, the higher π the smaller the probability that the winner is an incumbent. All of the associated coefficients are negative and significantly different from zero, to the 0.01 level. However, the cited patents that are less than two years old *increase* the probability that the winner is an incumbent the more valuable they are. Thus, incumbents with recent patents of high value are able to patent more within the next two years of these awards. This may happen because subsequent related innovations follow more easily from a race won recently by the same firm. The more valuable the patent, the more incentives the incumbent will have to obtain similar patents soon. After two years this effect seems to disappear and the value of cited patents operates through the replacement effect of innovation.

Further tests: The number of usable observations is significantly smaller than the total sample size.

[18] We have also used the value of total plant and equipment as a size control. The results are virtually unchanged, and thus not reported here.

After the merge with COMPUSTAT, only 5,143 patents of 91,656 are usable, at most. This is inevitable, and to a large extent expected since a large number of patents are awarded to universities, or privately traded firms. However, the proportion of entrant-won races in the sample used for estimation is not too much different from the same proportion over the whole sample. Finally, note that homoskedasticity cannot be rejected.

Definition of Incumbent: It is interesting to compare the estimates across columns in Table 4. From left to right, we report the estimates for narrower definitions of incumbency. If incumbency is defined as the winner also having cited patents that are up to 5, or 10 years old then we have a fit of the model that is very consistent with the theory and with the results of the first column. The estimates for β_1 and β_2 are robust to narrowing down the definition of incumbency to 10, 5 or even 4 years. As predicted by the model, the richer in cash is the incumbent (entrant), the more likely it is that the incumbent (entrant) wins the race, in all cases. The estimates decrease in absolute value. This is clear for β_1 because for broader definitions of incumbency we have to approximate the losing incumbent's cash resources with those of the cash-richest firm in its industry in fewer cases. The estimate of β_2 is strongest also when we account for the effect of 10 year-old or 20 year-old citations, predicting a more powerful effect of cash balances on the chances of winning the race.

In the second and third columns the estimated effect of total assets value in the probability of winning is similar to the first column. As before, more experience in patenting makes either type significantly more likely to win.

As we narrow further the incumbency definition to 2 years the estimate for the incumbents assets suggests that smaller incumbents are more likely to win *ceteris paribus*. This estimate may be a result of the downward bias that we impose on our tests or also that the correct definition of incumbency is between 4 and 20 years. Also, the estimated effect of patenting experiences by the entrant weakens for narrower definitions.

Note that in the last two columns we can only include as controls the average number of citations of patents cited that are at most three or two years old. This may explain why the effect of cash is smaller in these columns too, although the estimates remain consistent with the theory.

4.3.2 Other lags for cash holdings

When we use one-year or three-year rather than one-year lags for the measures of financial wealth and the size of the firm the estimates tell a story that is, at least qualitatively, similar to the previous case. With three year lags the fit that is most consistent with the theory occurs with the 5, 10 or 20-year-old definitions of incumbency. Cash constraints have the effect predicted by our theory: a cash-richer incumbent or entrant is more likely to win. The magnitude of the cash coefficients is very similar with three-year lags, and smaller in absolute value when using one year lags. With three year lags the coefficients of accumulated patents by entrants or incumbent are still close to each other, and the effect of changes in the value of cited patents is as larger than before.

With one year lags we can use more observations and we can to match the patent data with COMPUSTAT for proportionally more entrant won patents. Thus, the downward bias on the coefficients associated to the incumbent's variables (cash to assets ratio or total assets value) is larger. However, we note that the Pseudo R-Squared coefficients are smallest for this case. In fact, the last two columns show that more experience by the entrants is associated on average with a smaller probability of winning. Tables ii and iii in the Web Supplement show the results.

We conclude from this analysis that the data suggests that the empirical model is correctly specified when we use a definition of incumbency between 5 and 20 years, and in those cases the results are as predicted by the theory. The results are generally robust, but most consistent with the theory when we use two or three year lags for the balance sheet data.

4.3.3 Discussion of Economic Significance

We have shown above that the cash availability of an incumbent or an entrant one, two or three years before a patent is awarded has an effect on the outcome of the race that is statistically different from zero. To see whether or not this effect is also economically significant we compute first the marginal effect of a change in the cash availability on the probability that the winner is the incumbent. These effects are reported in Table 5. We compute the average change in the probability that the incumbent wins with respect to a change in the value of cash available by US$1 million when all other variables take their median value and remain constant and using the coefficients for the benchmark specifications (Table 4 below, and Table iii in the Web Supplement). We see that the increase (decrease) in the probability that the entrant (incumbent) wins the race is on average between 0.00128 and 0.00214. To have a better sense of this estimate in the sample of firms used here we compute the difference between the predicted probabilities that the winner is an incumbent in a race where the entrant firm is in the 9th and in the 1st deciles of the sample distribution of cash divided by assets. We call this difference $\Delta P_{1\to 9}$. We find that cash has an economically significant effect: *ceteris paribus*, an entrant firm in the 9th decile of the cash to assets distribution is more likely to win the race than one in the 1st decile by a difference in probability between 0.29 and almost 0.4.

The marginal effect for incumbents' wealth is smaller, and this is not surprising because of the downward bias on β_1 and the possibility that first-best contracting is feasible for the incumbent for some races. Nevertheless, the difference in the predicted probabilities of an incumbent winner at the 9th and 1st deciles of cash is significant, i.e., $\Delta P_{1\to 9}$ is between 0.45 and 0.54.

Table 5 shows too that each accumulated patent matters much more to entrants than to incumbents. Having an additional patent increases on average the probability that the incumbent wins by 0.002, whereas it increases the probability that the entrant wins by at least 0.021. Since incumbents on average have about twice more patents than entrants in this sample, this result may be indicative of diminishing returns in patenting experience. In this table we see too that the largest effect of the won citations on the probability that an incumbent wins is by those that are at most one year old.

4.3.4 Further Specifications

So far, our proxy for the player's cash per race for a patent has been the amount of cash (lagged one, two or three years) divided by the total value of assets. Thus, we have assumed that the size of the firm, i.e., assets, approximates well the number of races that the firm chooses to be in. To test the appropriateness of this proxy we augment the specification to allow for the interaction between cash divided by assets and the number of citations received by the patent. If a firm engages simultaneously in different races any additional cash made available would be spent in the most profitable races so as to equate the marginal profit in every race. Thus, the firm would try to be less cash constrained in more profitable races and there, the probability of success should be less sensitive to cash. Given that the number of citations received is itself a measure of the value of a patent, $\overline{\pi}$, then if we estimate the model

$$\Pr(I_i = 1) = \Pr(\beta_0 + \beta_1 W_I + \beta_2 W_E + \beta_3 \overline{\pi} * W_I + \beta_4 \overline{\pi} * W_E + \mathbf{c}\boldsymbol{\alpha} + \varepsilon_i \geq 0), \tag{20}$$

we would expect β_3 to be negative and β_4 to be positive. Table 6 shows the estimates of the parameters in (20). For parsimony, we report here only the estimates using cash and assets lagged two or three years and for the three broadest definitions of incumbency (20, 10 or 5 years). These cases showed the best fit for the benchmark estimation.

When cash and assets are lagged two years (first three columns) the estimates are similar with respect to the specification without the interactions. The estimates for $\beta_1, \beta_2, \beta_3$, and β_4 have the expected sign in all the columns. The estimate for β_4 is not significant at the 95% level for the 20-year definition of incumbency but all others are at the 99% level. Note that by augmenting the specification to allow for interactions, the estimated direct effect of cash appears seems to be bigger as the absolute values of $\widehat{\beta}_1$ and $\widehat{\beta}_2$ have increased. In the next section we interpret these values in terms of their effect in the probabilities of winning the race.

All the estimates associated to patent counts are all smaller, in absolute values. The average number of patents accumulated by entrants is still statistically different from zero and it increases on average the probability that an entrant wins. However, the value of the coefficient is much smaller. The last three columns show the estimates using the three-year lags for the balance sheet variables and show basically similar results as the previous specification, but with smaller estimates for the patent counts.

We argued previously that the total cash balance might not capture well the role of cash availability in a patent race when players are financially constrained because each firm might be racing simultaneously for many patents. All of our results above show that the model's predictions are consistent with the data when we divide cash by assets. In the Appendix we report the estimates of the parameters in (20), but in this case cash is not divided by total assets (see Table iv, Web Supplement). As we expected, several estimates are no longer consistent with the theoretical predictions. The total number of patents accumulated by entrants have now a positive effect on the probability that the incumbent wins and the incumbent's size has a negative effect. The signs of the estimates of β_2 and β_4 are the opposite too.

Panel B shows the marginal effects and $\Delta P_{1\to 9}$ for the model with interactions between cash and $\overline{\pi}$. Here the effect of cash is economically significant too, as $\Delta P_{1\to 9}$ is between 0.46 and 0.62 for the entrant, and between 0.4 and 0.6 for the incumbent.

4.4 Further Robustness Checks

We argued above that changes in the player's cash availability have unambiguous effects on the equilibrium probabilities of winning the race, and thus are testable, when the value of the race is high enough. It remains to be checked that the data set we use includes mostly races that satisfy these conditions. It is not possible to tell ex-ante what are the values of the boundaries after which the game is one of strategic complementarity. To see if there is a reason for concern, we estimated our model with the patents in the sample that received more citations than the median. The results with the upper half of the sample are qualitatively and quantitatively similar than for the whole sample. We omit these results here for parsimony.[19]

We estimated our model too defining incumbency with patents up to 25 or 30 years old. In these cases, the fit was very poor. We take this as positive news because patents *expire* after 20 years. Thus, there is little room for concern that our incumbency index is capturing something else.

We chose to define the entrant as an average firm in the same industrial segment (4 digit SIC code) as the incumbent in cases when the winner of the race was the incumbent. As an alternative we defined an entrant as any firm that had also patented in the same subclasses as the patent raced for but with no citations by it. Under this definition, any firm that has patented in the same subclass in the past, but is not necessarily in the same industry, is assumed to have raced for any new patents in that subclass. This approach resulted unfeasible: for most patents, the set of firms that had obtained patents in the same subclass that were not cited was either small or often empty. Moreover, very few of these could be matched with COMPUSTAT to obtain their financial information.

Table v of the Web Supplement compares the observed entrant firms (the winners) with the firms without cited patents in the same industry (the assumed losers). Winning entrants are about five times larger than

[19] These results are available upon request.

losers in terms of assets (Panel A) but only twice as much of cash, on average (Panel B). Losing entrants do have some patenting experience though. While they have about six times less patents than winning entrants, they have on average accumulated over 18 patents in the subcategories 31, 33 and 39 by the time of the award. Thus, we believe that the firms we have picked to represent entrant are not foreign to any race for a patent within their industry segment.

5 Conclusions

This paper provides a way to understand the role of financing constraints in innovation. It incorporates the contracting problem into a race between an incumbent and an entrant. Our theoretical model shows that wealthier firms are more likely to innovate and our empirical findings support this claim.

We study sequences of races but not the evolution of particular firms within the industry. An interesting question for future research is how the financing constraints of firms evolve over time as they accumulate patents and how this affects the dynamics of industry structure. We pursue these questions in ongoing research.

References

Blundell, Richard, Rachel Griffith, and John Van Reenen, "Market Share, Market Value and Innovation in a Panel of British Manufacturing Firms," *Review of Economic Studies*, 1999, *66*, 529–554.

Bolton, Patrick and David Scharfstein, "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review*, 1990, *80* (1), 93–106.

Brander, James and Tracy Lewis, "Oligopoly and Financial Structure: The Limited Liability Effect," *American Economic Review*, 1986, *76* (5), 956–970.

Bulow, Jeremy, John Geanakoplos, and Paul Klemperer, "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, 1985, *93* (3), 488–511.

Chevalier, Judith, "Capital Structure and Product-Market Competition: Empirical Evidence from the Supermarket Industry," *American Economic Review*, 1995, *85* (3), 415–435.

Cockburn, Iain and Rebecca Henderson, "Racing to Invest? The Dynamics of Competition in Ethical Drug Discovery," *Journal of Economics and Management Strategy*, 1994, *3*, 481–519.

Cohen, Wesley, Richard Nelson, and John Walsh, "Protecting Their Intellectual Assets: Appropriability Conditions and Why U.S. Manufacturing Firms Patent (or Not)," 2000. Working Paper No. 7552, NBER.

Davidson, Russell and James MacKinnon, *Estimation and Inference in Econometrics*, New York, NY: Oxford University Press, 2004.

Fazzari, Steven, Glenn Hubbard, and Bruce Peterson, "Financing Constraints and Corporate Investment," *Brookings Papers on Economic Activity*, 1988, *1*, 141–195.

Fudenberg, Drew and Jean Tirole, "A 'Signal-Jamming' Theory of Predation," *RAND Journal of Economics*, 1986, *17* (3), 366–376.

Gale, Douglas and Martin Hellwig, "Incentive Compatible Debt Contracts: The One Period Problem," *Review of Economic Studies*, 1985, *52*, 647–664.

Gilbert, Richard and David Newbery, "Preemptive Patenting and the Persistence of Monopoly," *American Economic Review*, 1982, *72*, 514–526.

___ **and** ___ , "Uncertain Innovation and the Persistence of Monopoly: Comment," *American Economic Review*, 1984., *74*, 238–242.

Hall, Bronwyn, "Investment and Research and Development at the Firm Level: Does the Source of Financing Matter?," 1992. Working Paper No. 4096, National Bureau of Economic Research.

___ **, Adam Jaffe, and Manuel Trajtenberg**, "The NBER Patent-Citations Data File: Lessons, Insights and Methodological Tools," in Adam Jaffe and Manuel Trajtenberg, eds., *Patent, Citations and Innovations: A Window on the Knowledge Economy*, MIT Press, 2002, chapter 13, pp. 403–459.

——, ——, and ——, "Market Value and Patent Citations," *RAND Journal of Economics*, 2004, *forthcoming.*

Himmelberg, Charles and Bruce Petersen, "R and D and Internal Finance: A Panel Study of Small Firms in High-Tech Industries," *Review of Economics and Statistics*, 1994, *76*, 38–51.

Jensen, Michael and William Meckling, "Theory of the Firm: Managerial Behaviour, Agency Costs and Capital Structure," *Journal of Financial Economics*, 1976, *3*, 305–360.

Lerner, Josh, "An Empirical Exploration of a Technology Race," *RAND Journal of Economics*, 1997, *28* (2), 228–247.

Levin, Richard, Alvin Kelvorick, Richard Nelson, and Sidney Winter, "Appropriating the Returns from Industrial Research and Development," *Brookings Papers on Economic Activity*, 1987, pp. 783–831.

Maksimovic, Vojislav, "Capital Structure in Repeated Oligopolies," *RAND Journal of Economics*, 1988, *19* (3), 389–407.

Mayer, Colin, "New Issues in Corporate Finance," *European Economic Review*, 1988, *32*, 1167–1188.

Milgrom, Paul and John Roberts, "Comparing Equilibria," *American Economic Review*, 1994, *84*, 441–459.

Reinganum, Jennifer, "Uncertain Innovation and the Persistence of Monopoly," *American Economic Review*, 1983, *73*, 741–748.

——, "Uncertain Innovation and the Persistence of Monopoly: A Reply," *American Economic Review*, 1984, *74*, 243–246.

——, "Innovation and Industry Evolution," *Quarterly Journal of Economics*, 1985, *100* (1), 81–99.

Rotemberg, Julio and David Scharfstein, "Shareholder-Value Maximization and Product Market Competition," *Review of Financial Studies*, 1990, *3* (3), 367–391.

Trajtenberg, Manuel, *Economic Analysis of Product Innovation - The Case of CT Scanners*, Cambridge, MA: Harvard University Press, 1990.

Appendix 1: Proofs

Proof of Lemma 1. An optimal contract is the solution to the problem

$$\max_{s} V_E\left(W_E, a_I, s\right) \qquad (21)$$
$$s.t.\ s\frac{\left(a_E\left(s\right)\right)^{\alpha} V_E^{+}}{\left(a_E\left(s\right)\right)^{\alpha}+a_I^{\alpha}+r}=F-W_E$$

where $a_E(s)$ is defined by (2). It is easy to show that $\frac{dV_E(W_E,a_I,s)}{ds}<0$ for all s. Thus, the solution, when it exists, is the smallest s that satisfies equations (5) and (3). It is convenient to represent the solution by the research effort induced by the contract. Let $\hat{a}_E \equiv \hat{a}_E\left(a_I, F, W_E, V_E^{+}\right)$ denote an effort level induced by an incentive compatible contract where the investor breaks even and let $\hat{a}_E^{*}$ denote the effort level induced by the optimal, incentive compatible break-even contract. Combining (5) and (3) we observe that $\hat{a}_E$ satisfies the condition

$$\underbrace{\alpha\left(\hat{a}_E^{\alpha}V_E^{+}-\left(\hat{a}_E^{\alpha}+a_I^{\alpha}+r\right)\left(F-W_E\right)\right)\left(a_I^{\alpha}+r\right)}_{:=A\left(a_E,a_I,F,W_E,V_E^{+}\right)\big|_{a_E=\hat{a}_E}}=\underbrace{\hat{a}_E\left(\left(1-\alpha\right)\hat{a}_E^{\alpha}+a_I^{\alpha}+r\right)}_{:=B\left(a_E,a_I\right)\big|_{a_E=\hat{a}_E}}. \qquad (22)$$

By straightforward algebra and calculus, the functions[20] $A(a_E;\cdot)$ and $B(a_E;\cdot)$ as defined in condition (22) have the following properties. Whenever $V_E^{+}-F\geq 0$ and $W_E\geq 0$, with at least one strict inequality, then $A(a_E;\cdot)$ is increasing concave in a_E and satisfies $A(a_E;\cdot)|_{a_E=0}\leq 0$ and $\lim_{a_E\to\infty}A(a_E;\cdot)=\infty$. $A\left(a_E,a_I,F,W_E,V_E^{+}\right)$ is increasing in V_E^{+} and decreasing in $F-W_E$ for all a_E and a_I. $B(a_E;\cdot)$ is increasing convex in a_E and satisfies $B(a_E,\cdot)|_{a_E=0}=0$ and $\lim_{a_E\to\infty}B(a_E,\cdot)=\infty$. Thus, the function

$$A\left(a_E;\cdot\right)-B\left(a_E;\cdot\right)=\alpha\left(a_E^{\alpha}V_E^{+}-\left(a_E^{\alpha}+a_I^{\alpha}+r\right)\left(F-W_E\right)\right)\left(a_I^{\alpha}+r\right)-a_E\left(\left(1-\alpha\right)a_E^{\alpha}+a_I^{\alpha}+r\right) \qquad (23)$$

is strictly concave in a_E. An optimal contract exists if and only if $\max_{a_E}\left(A(a_E;\cdot)-B(a_E;\cdot)\right)\geq 0$. Let $\overline{a}_E\equiv\arg\max_{a_E}\left(A(a_E;\cdot)-B(a_E;\cdot)\right)$.[21] By straightforward calulus we find that $\overline{a}_E$ satisfies the first-order

[20] Throughout the paper we shall use the $\cdot$-notation to represent parameters that are kept constant during the discussion at hand.
[21] Clearly, $\overline{a}_E$ exists and is unique.

condition

$$\alpha^2 \left(V_E^+ - (F - W_E)\right)(a_I^\alpha + r) = \left(1-\alpha^2\right)\overline{a}_E + \overline{a}_E^{1-\alpha}(a_I^\alpha + r) \tag{24}$$

Substituting (24) into (23) we find

$$A(\overline{a}_E;\cdot) - B(\overline{a}_E;\cdot) = \frac{1-\alpha}{\alpha}\overline{a}_E\left(\overline{a}_E^\alpha + a_I^\alpha + r\right) - \alpha\left(a_I^\alpha + r\right)^2 (F - W_E)$$

Thus, an optimal contract exists if and only if

$$\overline{a}_E^{1+\alpha} + \overline{a}_E(a_I^\alpha + r) \geq \frac{\alpha^2}{1-\alpha}(a_I^\alpha + r)^2 (F - W_E) \tag{25}$$

Define $\tilde{a}_E$ (uniquely) by condition (25), stated as an equality

$$\frac{\alpha^2}{1-\alpha}(a_I^\alpha + r)(F - W_E) = \frac{1}{a_I^\alpha + r}\tilde{a}_E^{1+\alpha} + \tilde{a}_E \tag{26}$$

To prove our lemma, we show that i) $\overline{a}_E(a_I^\alpha;\cdot)$ is an increasing and concave function of a_I^α; ii) $\tilde{a}_E(a_I^\alpha;\cdot)$ is an increasing and convex function of a_I^α; and iii) if $\tilde{a}_E(0;\cdot) > \overline{a}_E(0;\cdot)$ then the slope of $\tilde{a}_E(a_I^\alpha;\cdot)$ with respect to a_I^α is for all a_I larger than the slope of $\overline{a}_E(a_I^\alpha;\cdot)$, which implies that $\tilde{a}_E(a_I^\alpha;\cdot) > \overline{a}_E(a_I^\alpha;\cdot)$ for all a_I^α.

i) $\overline{a}_E(a_I^\alpha;\cdot)$ is increasing and concave: Applying the implicit function theorem to condition (24), and using condition (24) to simplify we obtain

$$\frac{d\overline{a}_E}{da_I^\alpha} = \frac{\alpha^2\left(V_E^+ - (F - W_E)\right) - \overline{a}_E^{1-\alpha}}{(1-\alpha^2) + (1-\alpha)\overline{a}_E^{-\alpha}(a_I^\alpha + r)} = \frac{\overline{a}_E}{(a_I^\alpha + r)}\frac{\left(1-\alpha^2\right)}{(1-\alpha^2) + (1-\alpha)\frac{a_I^\alpha + r}{\overline{a}_E^\alpha}} > 0.$$

Differentiating another time we find that

$$\begin{aligned}\frac{d^2\overline{a}_E}{da_I^{\alpha 2}} &= \frac{\frac{d\overline{a}_E}{da_I^\alpha}\left(a_I^\alpha+r\right)-\overline{a}_E}{\left(a_I^\alpha+r\right)^2}\frac{\left(1-\alpha^2\right)}{\left(1-\alpha^2\right)+\left(1-\alpha\right)\frac{a_I^\alpha+r}{\overline{a}_E^\alpha}} \\ &\quad -\frac{\overline{a}_E}{a_I^\alpha+r}\frac{\left(1-\alpha^2\right)\left(1-\alpha\right)\frac{\overline{a}_E^\alpha-\alpha\overline{a}_E^{\alpha-1}\frac{d\overline{a}_E}{da_I^\alpha}\left(a_I^\alpha+r\right)}{\left(\overline{a}_E^\alpha\right)^2}}{\left(\left(1-\alpha^2\right)+\left(1-\alpha\right)\frac{a_I^\alpha+r}{\overline{a}_E^\alpha}\right)^2}\end{aligned}$$

Since $\frac{\left(1-\alpha^2\right)}{\left(1-\alpha^2\right)+\left(1-\alpha\right)\frac{a_I^\alpha+r}{\overline{a}_E^\alpha}}<1$, we have $\frac{d\overline{a}_E}{da_I^\alpha}<\frac{\overline{a}_E}{a_I^\alpha+r}$, which implies that $\frac{d^2\overline{a}_E}{da_I^{\alpha 2}}<0$.

ii) $\tilde{a}_E\left(a_I^\alpha;\cdot\right)$ is increasing and convex: Proceeding analogously we note that

$$\frac{d\tilde{a}_E}{da_I^\alpha}=\frac{\frac{\alpha^2}{1-\alpha}2\left(a_I^\alpha+r\right)\left(F-W_E\right)-\tilde{a}_E}{\left(1+\alpha\right)\tilde{a}_E^\alpha+\left(a_I^\alpha+r\right)}=\frac{\tilde{a}_E}{a_I^\alpha+r}\left(1+\frac{\left(1-\alpha\right)\tilde{a}_E^\alpha}{\left(1+\alpha\right)\tilde{a}_E^\alpha+a_I^\alpha+r}\right)>0$$

and

$$\begin{aligned}\frac{d^2\tilde{a}_E}{da_I^{\alpha 2}} &= \frac{\frac{d\tilde{a}_E}{da_I^\alpha}\left(a_I^\alpha+r\right)-\tilde{a}_E}{\left(a_I^\alpha+r\right)^2}\left(1+\frac{\left(1-\alpha\right)\tilde{a}_E^\alpha}{\left(1+\alpha\right)\tilde{a}_E^\alpha+a_I^\alpha+r}\right) \\ &\quad +\frac{\tilde{a}_E}{a_I^\alpha+r}\frac{\alpha\left(1-\alpha\right)\tilde{a}_E^{\alpha-1}\left(a_I^\alpha+r\right)-\left(1-\alpha\right)\tilde{a}_E^\alpha}{\left(\left(1+\alpha\right)\tilde{a}_E^\alpha+a_I^\alpha+r\right)^2}\end{aligned}$$

Since $1+\frac{(1-\alpha)\tilde{a}_E^\alpha}{(1+\alpha)\tilde{a}_E^\alpha+a_I^\alpha+r}>1$ we observe that $\frac{d\tilde{a}_E}{da_I^\alpha}>\frac{\tilde{a}_E}{a_I^\alpha+r}$. Moreover,

$$\begin{aligned}&\frac{\frac{d\tilde{a}_E}{da_I^\alpha}\left(a_I^\alpha+r\right)-\tilde{a}_E}{\left(a_I^\alpha+r\right)^2}-\frac{\tilde{a}_E}{a_I^\alpha+r}\frac{\left(1-\alpha\right)\tilde{a}_E^\alpha}{\left(\left(1+\alpha\right)\tilde{a}_E^\alpha+a_I^\alpha+r\right)^2} \\ =\ & \tilde{a}_E\frac{\left(1-\alpha\right)\tilde{a}_E^\alpha}{\left(\left(1+\alpha\right)\tilde{a}_E^\alpha+a_I^\alpha+r\right)\left(a_I^\alpha+r\right)^2}-\tilde{a}_E\frac{\left(1-\alpha\right)\tilde{a}_E^\alpha}{\left(a_I^\alpha+r\right)\left(\left(1+\alpha\right)\tilde{a}_E^\alpha+a_I^\alpha+r\right)^2}>0\end{aligned}$$

As a result, we can state $\frac{d^2\tilde{a}_E}{da_I^{\alpha 2}}>0$.

iii) if $\tilde{a}_E\left(0;\cdot\right)>\overline{a}_E\left(0;\cdot\right)$ then $\tilde{a}_E\left(a_I^\alpha;\cdot\right)>\overline{a}_E\left(a_I^\alpha;\cdot\right)$ for all a_I : Comparing the slopes of the functions we find that $\tilde{a}_E>\overline{a}_E$ implies that

$$\frac{d\tilde{a}_E}{da_I^\alpha}=\frac{\tilde{a}_E}{a_I^\alpha+r}\left(1+\frac{\left(1-\alpha\right)\tilde{a}_E^\alpha}{\left(1+\alpha\right)\tilde{a}_E^\alpha+a_I^\alpha+r}\right)>\frac{\overline{a}_E}{\left(a_I^\alpha+r\right)}\frac{\left(1-\alpha^2\right)}{\left(1-\alpha^2\right)+\left(1-\alpha\right)\frac{a_I^\alpha+r}{\overline{a}_E^\alpha}}=\frac{d\overline{a}_E}{da_I^\alpha}$$

Therefore, the functions intersect if and only if $\tilde{a}_E\left(0;\cdot\right)\leq\overline{a}_E\left(0;\cdot\right)$.

From i) through iii), it follows that there is a unique $\overline{a}_I\left(V_E^+, W_E\right)$ such that an optimal contract exists for all $a_I \leq \overline{a}_I\left(V_E^+, W_E\right)$. $\overline{a}_I$ is nondecreasing in V_E^+ and strictly increasing in V_E^+ whenever $\overline{a}_I > 0$. This follows from applying the implicit function theorem to condition (24), which shows that

$$\frac{d\overline{a}_E}{dV_E^+} = \frac{\alpha^2\left(a_I^\alpha + r\right)}{\left(1-\alpha^2\right) + \left(1-\alpha\right)\overline{a}_E^{-\alpha}\left(a_I^\alpha + r\right)} > 0,$$

and observing on the other hand that $\tilde{a}_E$ is independent of V_E^+. Finally, $\overline{a}_I\left(V_E^+, W_E\right)$ is nondecreasing in W_E and strictly increasing if $\overline{a}_I > 0$ and $F > W_E$. Applying the same logic as before we find

$$\frac{d\overline{a}_E}{dW_E} = \frac{\alpha^2\left(a_I^\alpha + r\right)}{\left(\left(1-\alpha^2\right) + \left(1-\alpha\right)\frac{a_I^\alpha + r}{\overline{a}_E^\alpha}\right)} > -\frac{\frac{\alpha^2}{1-\alpha}\left(a_I^\alpha + r\right)}{\frac{1+\alpha}{a_I^\alpha + r}\tilde{a}_E^\alpha + 1} = \frac{d\tilde{a}_E}{dW_E}$$

for all a_I which proves the result. ■

Table 1: Percentage of Patents Won by Entrants in the Drugs and Medical Category, each Year for Different Definitions of Incumbency

	Percentage of patents awarded to an entrant in a year						
	Winner of the race is an entrant if						
Year patent	youngest own citations is older than:						
was awarded	20 years	10 years	5 years	4 years	3 years	2 years	1 years
1975	76.26	76.26	78.58	81.61	85.53	90.41	98.32
1980	63.83	64.31	70.85	75.36	81.27	89.58	99.23
1985	63.94	65.48	73.61	78.35	83.00	89.55	97.15
1990	66.86	69.32	75.59	78.71	83.60	89.63	97.57
1995	62.94	65.42	72.88	76.53	81.89	89.72	97.96
1999	65.26	67.19	74.00	77.26	81.91	89.46	97.72
1975-1999	65.11	67.03	73.81	77.24	82.10	89.32	97.59

The percentages shown above are computed over 91,656 patents awarded between 1975 and 1999 in the US that are found in the NBER Patent Citations Data Files, in the technological category 3 (Drugs and Medical), subcategories 31,33 and 39.

Table 2: Comparison of the Citations Received by Patents Won by Entrants and by Incumbents in the Drugs and medical Technological Category, for Different Definitions of Incumbency

Panel A: Average citations received by patents

	Winner of the race is an entrant if						
Patents	youngest own citations is older than:						
awarded to:	20 years	10 years	5 years	4 years	3 years	2 years	1 years
Entrants (μ_E)	4.29	4.25	4.18	4.18	4.18	4.18	4.21
Incumbents (μ_I)	4.10	4.16	4.32	4.37	4.40	4.53	4.99

Panel B: Difference of means test, assuming unequal variances.
Alternative hypothesis: $\mu_E - \mu_I > 0$

	Winner of the race is an entrant if						
	youngest own citations is older than:						
	20 years	10 years	5 years	4 years	3 years	2 years	1 years
T statistic	3.103	1.336	-2.014	-2.563	-2.615	-3.370	-3.215
P-value	0.999	0.909	0.022	0.005	0.005	0.000	0.001

Panel C: Difference of means test, assuming unequal variances, and using citations re-scaled by the average number of citations by grant year in the same technological field.
Alternative hypothesis: $\mu_E - \mu_I > 0$.

	Winner of the race is an entrant if						
	youngest own citations is older than:						
	20 years	10 years	5 years	4 years	3 years	2 years	1 year
T statistic	-2.120	-2.857	-2.418	-2.970	-3.279	-3.948	-2.778
P-value	0.017	0.002	0.008	0.002	0.001	0.000	0.002

The statistics shown above are computed over 91,656 patents awarded between 1975 and 1999 in the US that are found in the NBER Patent Citations Data Files, in the technological category 3 (Drugs and Medical), subcategories 31,33 and 39.

The factors for re-scaling are provided by Hall, et al., (2002).

Table 3: Summary Statistics

Variables	Winner is an entrant if youngest own citations is older than: 20 years N	20 years Mean	20 years Std. Dev.	10 years N	10 years Mean	10 years Std. Dev.	5 years N	5 years Mean	5 years Std. Dev.
1. Balance Sheet Items, ($ Billion), lagged two years.									
Incumbent's assets	17,592	10.795	12.820	17,614	11.036	12.929	17,733	11.644	13.336
Entrant's assets	16,909	4.275	7.107	16,887	4.559	7.331	16,768	5.248	7.870
Incumbent's cash	11,641	1.544	2.655	12,122	1.607	2.709	13,402	1.731	2.807
Entrant's cash	11,972	0.180	0.203	11,491	0.184	0.209	10,211	0.200	0.220
2. Patenting Experience									
Incumbent's average accumulated patents	83,212	213.65	293.66	82,708	211.64	291.72	80,882	211.24	289.15
Entrant's average accumulated patents	68,759	94.20	205.70	69,940	100.92	214.91	74,431	114.59	230.99

3. Average number of citations received by the cited patents that are[a]:	N	Mean	Std. Dev.
less than 1 year old	91,656	0.59	8.11
between 1 and 2 years old	91,656	6.67	53.21
between 2 and 3 years old	91,656	24.98	240.95
between 3 and 4 years old	91,656	42.37	319.13
between 4 and 5 years old	91,656	65.93	571.29
between 5 and 10 years old	91,656	345.32	2,203.93
between 10 and 20 years old	91,656	559.89	6,751.38
Total citations received by the patent[a]	91,656	0.75	2.53

The sample includes all patents awarded between 1975 and 1999 in the US that are found in the NBER Patent Citations Data Files, in the technological category 3 (Drugs and Medical), subcategories 31,33 and 39, for which the assignee is found in COMPUSTAT.

[a] All counts of number of citations are re-scaled by the factors provided by Hall, et al., (2002).

Table 4: Parameter Estimates for Logit Regressions of the Probability that the Winner is an Incumbent on Incumbent's and Entrants' Measures of Financial Resources (I)

The dependent variable is the incumbent/entrant index, which equals 1 if the patent was awarded to an incumbent, and zero if it was awarded to an entrant.

Regressors	Winner is an entrant if youngest own citations is older than:					
	20 years	10 years	5 years	4 years	3 years	2 years
Incumbent's cash, divided by	12.702***	13.457***	12.561***	10.944***	9.591***	7.142***
total assets, lagged two years	(1.360)	(1.246)	(1.024)	(0.963)	(0.932)	(0.963)
Entrant's cash, divided by	-10.689***	-10.460***	-9.615***	-9.450***	-8.612***	-7.837***
total assets, lagged two years	(1.425)	(1.308)	(0.987)	(0.938)	(0.894)	(0.902)
Incumbent's total assets	0.089***	0.086***	0.061***	0.040**	0.019	-0.121***
($ Million), lagged two years	(0.023)	(0.021)	(0.018)	(0.018)	(0.020)	(0.028)
Entrants' total assets	-0.618***	0.665***	-0.567***	-0.538***	−0.529***	-0.626***
($ Million), lagged two years	(0.072)	(0.066)	(0.055)	(0.052)	(0.053)	(0.073)
Incumbent's average	0.823e-2***	0.863e-2***	0.896e-2***	0.888e-2***	0.887e-2***	0.935e-2***
accumulated patents	(0.083e-2)	(0.071e-2)	(0.064e-2)	(0.061e-2)	(0.061e-2)	(0.065e-2)
Entrant's average	-0.133***	-0.128***	-0.111***	-0.103***	-0.098***	-0.088***
accumulated patents	(0.009)	(0.008)	(0.006)	(0.006)	(0.006)	(0.006)
Total citations received by	0.140***	-0.217***	-0.087*	0.005	0.118***	-0.387***
the patent[a]	(0.036)	(0.046)	(0.042)	(0.050)	(0.034)	(0.058)
Average number of citations received by the cited patents that are[a]:						
less than 1 year old	0.906***	0.771***	0.723***	0.626***	0.678***	1.015***
	(0.219)	(0.202)	(0.169)	(0.150)	(0.140)	(0.160)
between 1 and 2 years old	0.022	-0.014	-0.056	-0.051*	-0.035	0.030
	(0.057)	(0.047)	(0.035)	(0.031)	(0.028)	(0.026)
between 2 and 3 years old	-0.107***	-0.137***	-0.123***	-0.139***	-0.170***	NA
	(0.033)	(0.027)	(0.023)	(0.022)	(0.022)	
between 3 and 4 years old	-0.139***	-0.151***	-0.149***	-0.179***	NA	NA
	(0.026)	(0.022)	(0.020)	(0.019)		

(continues)

Table 4: continued.

The dependent variable is the incumbent/entrant index, which equals 1 if the patent was awarded to an incumbent, and zero if it was awarded to an entrant.						
Regressors	Winner is an entrant if youngest own citations is older than:					
and regression statistics	20 years	10 years	5 years	4 years	3 years	2 years
(continued)						
Average number of citations received by the cited patents that are[a]:						
between 4 and 5 years old	-0.187***	-0.210***	-0.240***	NA	NA	NA
	(0.027)	(0.023)	(0.024)			
between 5 and 10 years old	-0.328***	-0.389***	NA	NA	NA	NA
	(0.026)	(0.025)				
between 10 and 20 years old	-0.634***	NA	NA	NA	NA	NA
	(0.050)					
Year dummies[b]	Yes	Yes	Yes	Yes	Yes	Yes
Number of observations	5,143	5,119	4,965	4,871	4,726	4,514
Likelihood ratio (χ^2)[c]	5,887.93	5,569.17	4779.44	4,380.99	3,853.60	2,973.80
P-value	0.000	0.000	0.000	0.000	0.000	0.000
Pseudo R^2	0.826	0.785	0.707	0.678	0.655	0.642
BRMR test of Heteroskedasticity (χ^2)[d]	0.31	0.93	0.90	0.57	0.37	0.73
P-value	0.999	0.996	0.989	0.989	0.985	0.867
Proportion of entrant won patents in full sample[e]	0.651	0.670	0.738	0.772	0.821	0.893
Proportion of entrant won patents in estimation sample	0.492	0.509	0.579	0.622	0.686	0.790

The sample includes all patents awarded between 1975 and 1999 in the US that are found in the NBER Patent Citations Data Files, in the technological category 3 (Drugs and Medical), subcategories 31,33 and 39, for which the assignee is found in COMPUSTAT. The estimates are obtained by maximum likelihood, from a logit regression of the probability that the winner of the race is an incumbent, on the regressors shown above. Estimates of the standard errors are shown below the parameter estimate, in parenthesis. Those followed by *** are significant to the 0.01 level, by ** to the 0.05 level, and by * to the 0.1 level.

[a] All counts of number of citations are re-scaled by the factors provided by Hall, et al., (2002).

[b] A dummy for 24 of the 25 years in the sample. Equals one when the observation corresponds to that year.

[c] The null hypothesis is that all the parameters in the model are equal to zero.

[d] The null hypothesis is that the model is homoskedastic. The model for hesteroskedasticity specifies the variance of the logit error term as an exponential function of the citations received by the patent and by the average of its cited patents of different ages.

[e] The total number of patents in the sample before the match with COMPUSTAT is 91,656.

Table 5: Marginal Effects of Explanatory Variables on the Probability that the Incumbent Wins and their Economic Significance

All estimates using the results reported in Tables 4 and ??.
All estimates are computed at the sample median of all variables, unless noted.

Variables	[1]Lagged two years			[1]Lagged three years		
	Winner is an entrant if youngest own citations is older than:					
	20 years	10 years	5 years	20 years	10 years	5 years
Incumbent's cash / assets[1]	0.808e-3	0.829e-3	0.614e-3	0.753e-3	0.781e-3	0.483e-3
$\Delta P_{(1\rightarrow 9)}$ [a]	0.5181	0.5426	0.4497	0.4767	0.4977	0.3588
Entrants' cash /assets[1]	-0.209e-2	-0.198e-2	-0.148e-2	-0.214e-2	-0.205e-2	-0.128e-2
$\Delta P_{(1\rightarrow 9)}$	-0.3966	-0.3882	-0.3333	-0.3782	-0.3879	-0.2903
Incumbent's average accumulated patents	0.002	0.002	0.002	0.002	0.002	0.002
Entrant's average accumulated patents	-0.032	-0.031	-0.023	-0.021	-0.026	-0.024
Total citations received by the patent[b]	0.034	0.029	0.001	0.025	0.027	0.002
Average number of citations received by the cited patents that are[b]:						
less than 1 year old	0.220	0.187	0.152	0.155	0.139	0.205
between 1 and 2 years old	0.005	-0.003	-0.012	0.003	-0.001	-0.013
between 2 and 3 years old	-0.026	-0.033	-0.026	-0.021	-0.032	-0.034
between 3 and 4 years old	-0.220	-0.037	-0.031	-0.022	-0.031	-0.035
between 4 and 5 years old	-0.034	-0.051	-0.050	-0.030	-0.043	-0.055
between 5 and 10 years old	-0.045	-0.094	NA	-0.051	-0.087	NA
between 10 and 20 years old	-0.080	NA	NA	-0.104	NA	NA

The sample includes all patents awarded between 1975 and 1999 in the US that are found in the NBER Patent Citations Data Files, in the technological category 3 (Drugs and Medical), subcategories 31,33 and 39, for which the assignee is found in COMPUSTAT.

[a] $\Delta P_{(1\rightarrow 9)}$ is the predicted average difference between the probability that the incumbent wins when the incumbent's or the entrant's cash/assets correspond to the 9th and the 1st decile of their distributions.

[b] All counts of number of citations are re-scaled by the factors provided by Hall, et al., (2002).

Table 6: Parameter Estimates for Logit Regressions of the Probability that the Winner is an Incumbent on Incumbent's and Entrants' Measures of Financial Resources (II)

The dependent variable is the incumbent/entrant index, which equals 1 if the patent was awarded to an incumbent, and zero if it was awarded to an entrant.

Regressors	[1]Lagged two years			[1]Lagged three years		
	Winner is an entrant if youngest own citations is older than:					
	20 years	10 years	5 years	20 years	10 years	5 years
Incumbent's cash, divided by	17.540***	18.104***	15.649***	16.305***	15.740***	12.372***
total assets[1]	(1.274)	(1.086)	(0.937)	(1.156)	(1.011)	(0.867)
Incumbent's cash interacted	-2.224**	-2.251***	-2.176***	-2.414***	-1.862***	-1.400***
with the number of citations	(0.958)	(0.547)	(0.341)	(0.429)	(0.375)	(0.329)
Entrant's cash, divided by	-18.905***	-19.657***	-18.486***	-19.408***	-19.561***	-17.984***
total assets[1];	(1.375)	(1.037)	(1.607)	(1.204)	(1.012)	(0.905)
Entrant's cash interacted	0.384	1.109**	1.040***	1.041*	1.040***	1.181***
with the number of citations	(1.164)	(0.455)	(0.264)	(0.580)	(0.355)	(0.304)
Incumbent's total assets	0.138***	0.101***	0.014	-0.056***	0.032	0.104***
($ Million)[1]	(0.022)	(0.022)	(0.019)	(0.018)	(0.022)	(0.024)
Entrants' total assets	-1.212***	-1.263***	-1.204***	-1.322***	-1.334***	-1.288***
($ Million)[1]	(0.073)	(0.066)	(0.057)	(0.058)	(0.067)	(0.076)
Incumbent's average	0.404e-2***	0.482e-2***	0.471e-2***	0.409e-2***	0.456e-2***	0.427e-2***
accumulated patents	(0.046e-2)	(0.040e-2)	(0.036e-2)	(0.042e-2)	(0.036e-2)	(0.032e-2)
Entrant's average	-0.179e-3***	-0.105e-3***	0.008e-3	-0.179e-3***	-0.087e-3*	0.039e-3
accumulated patents	(0.063e-3)	(0.055e-3)	(0.050e-3)	(0.056e-3)	(0.048e-3)	(0.042e-3)
Total citations received by	0.727***	-0.489***	0.209***	0.683***	0.121***	0.140**
the patent[a]	(0.147)	(0.081)	(0.054)	(0.094)	(0.034)	(0.055)
Average number of citations received by the cited patents that are[a]:						
less than 1 year old	1.074***	0.715***	0.706***	1.068***	0.664***	0.740***
	(0.180)	(0.145)	(0.120)	(0.172)	(0.145)	(0.127)
between 1 and 2 years old	0.077	0.030	0.024	0.035	0.026	0.006
	(0.051)	(0.036)	(0.028)	(0.048)	(0.036)	(0.028)
between 2 and 3 years old	-0.076***	-0.110***	-0.081***	-0.086***	-0.122***	-0.110***
	(0.028)	(0.022)	(0.020)	(0.025)	(0.020)	(0.019)

(continues

Table 6: continued.

Panel A: the dependent variable is the incumbent/entrant index, which equals 1 if the patent was awarded to an incumbent, and zero if it was awarded to an entrant.

Regressors and regression statistics	[1]Lagged two years: Winner is an entrant if youngest own citations is older than: 20 years	10 years	5 years	[1]Lagged three years: 20 years	10 years	5 years
Average number of citations received by the cited patents that are[a]:						
between 3 and 4 years old	-0.080***	-0.103***	-0.105***	-0.106***	-0.077***	-0.108***
	(0.021)	(0.018)	(0.017)	(0.018)	(0.020)	(0.017)
between 4 and 5 years old	-0.134***	-0.186***	-0.249***	-0.129***	-0.172***	-0.234***
	(0.023)	(0.022)	(0.022)	(0.021)	(0.020)	(0.021)
between 5 and 10 years old	-0.328***	-0.382***	NA	-0.349***	-0.383***	NA
	(0.022)	(0.020)		(0.022)	(0.020)	
between 10 and 20 years old	-0.907***	NA	NA	-0.836***	NA	NA
	(0.050)			(0.045)		
Year dummies[b]	Yes	Yes	Yes	Yes	Yes	Yes
Number of observations	5,143	5,119	4,965	5,431	5,400	5,245
Likelihood Ratio (χ^2)[c]	4,807.11	4,073.30	3,160.69	5,111.88	4,375.59	3,417.10
P-value	0.000	0.000	0.000	0.000	0.000	0.000
Pseudo R^2	0.674	0.574	0.467	0.679	0.796	0.480
BRMR test of Heteroskedasticity (χ^2)[d]	1.48	0.60	0.68	1.33	1.83	0.74
P-value	0.993	0.999	0.995	0.995	0.969	0.994
Proportion of entrant won patents in full sample[e]	0.651	0.670	0.738	0.651	0.670	0.738
Proportion of entrant won patents in estimation sample	0.492	0.509	0.579	0.493	0.513	0.584

Panel B: estimates of marginal effects. All estimates are computed at the sample median of all variables, unless noted.

Variables	[1]Lagged two years: Winner is an entrant if youngest own citations is older than: 20 years	10 years	5 years	[1]Lagged three years: 20 years	10 years	5 years
Incumbent's cash / assets[1]	0.100e-2	0.101e-2	0.079e-2	0.648e-3	0.904e-3	0.561e-3
$\Delta P_{(1\to 9)}$	0.5894	0.6005	0.5156	0.4048	0.5385	0.4025
Entrants' cash / assets[1]	-0.120e-2	-0.118e-2	-0.099e-2	-0.085e-2	-0.120e-2	-0.086e-2
$\Delta P_{(1\to 9)}$	0.6211	0.6195	0.5994	0.458	0.6145	0.5629

Notes: same as Table 4.

$\Delta P_{(1\to 9)}$ is the predicted average difference between the probability that the incumbent wins when the incumbent's or the entrant's cash/assets correspond to the 9th and the 1st decile of their distributions.