Disaster Insurance or a Disastrous Insurance - Natural Disaster Insurance in France*

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Abstract

We model natural disaster insurance in France. We explicitly take into account the main institutional features of the system, such as the uniform premium rate in both high and low risk regions and the existence of a state reinsurance company. Our model indicates that the institutional set-up is fundamentally flawed. We find that the market is likely to lead to "specialist" equilibria, where insurers specialize in serving either high or low risk regions. As a result the reinsurance company, which offers cover to all insurers at the same price, is likely to suffer from a portfolio with mainly "bad" risks. We show that increasing the premium rate customers have to pay, a policy undertaken by the French authorities, will not necessarily solve these problems and comes at a high cost to the final consumer (and taxpayer).

Key words: Property insurance, reinsurance, risk selection.

JEL: L11, G22, D78.

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1 Introduction

The recent evolution of climatic conditions in many parts of the world has brought insurance against natural disasters back on the policy agenda. Both the costs of floods and the violence of storms (e.g. the storm Lothar at the end of 1999 has caused damage of several billion Euro in France, Germany and Switzerland) seem to have increased in the last decade. Natural disasters have specific characteristics that lead to market failure in the absence of government intervention. In particular the "objective" distribution of damage is very hard to specify (or varies substantially across time) and the distribution of claims is very uneven across space. In Britain for example, damage due to subsidence (unheard of prior to the 1970s) has caused claims of the order of £3.5 billion since 1975. Only areas with clay soils are affected, since this type of soil strongly contracts when it dries out.

Natural disasters are often classified as "uninsurable". Jaffee and Russell (1996) indicate three characteristics that are often mentioned in this context: i) the market displays problems of adverse selection and moral hazard, ii) the potential loss is very large and iii) it is difficult (impossible) to obtain precise estimates of the probability of the event.

In many countries, such particularities have led to some form of government intervention in the market for natural disaster insurance. Von Ungern-Sternberg (2004) examines the property insurance market in five countries (Britain, Spain, France, Germany and Switzerland). He shows that there is a wide variety of regulatory systems in these countries, ranging form integrated state insurance in some parts of Switzerland to laissez-faire in Britain.

In the present paper, we study the French system of natural disaster insurance. Its main specificity is the fact that it is a mixed system in which both the state and the private insurance sector play an important role. Let us start off by briefly outlining its basic structure[1]. Natural disaster insurance was introduced in France in 1982 as a reaction to a severe flood a few months earlier. The term "natural disaster" ("catastrophe naturelle") is not defined in the law creating the system. A commission, formed of representatives of the Ministries of Interior, Finance and Environment, has to decide whether a given occurrence is deemed a natural disaster and hence makes claimants eligible for reimbursement. The conditions of the insurance (e.g. premium rates and excesses) are fixed by decree and uniform across the country. Insurance is compulsory, presumably to reduce problems of adverse selection among property owners. Similarly, all insurance companies offering (other types of) property insurance in a specific area are obliged to include protection against natural disasters. Premium rates are defined as a percentage of other property insurance premiums (in particular fire), while excesses (for non-commercial buildings) are fixed amounts per contract and event.

An important institutional feature for our analysis is the existence of a publicly owned reinsurance company, the Caisse Centrale de Réassurance (CCR). Reinsurance is not compulsory, and insurers are free to contract with other, private, reinsurance companies. Reinsurance with the state reinsurance company is, however, particularly attractive, both because of the reinsurance premiums it charges and because it can offer unlimited cover (it is covered by a state guarantee). Insurance companies that decide to reinsure with the public reinsurer are offered two types of contracts; proportional contracts (for a given percentage of premium income the reinsurance company covers a given percentage of claims) and stop-loss contracts (the reinsurance company covers all claims that exceed a given multiple of annual premium income). The combination of these two types of reinsurance necessarily implies that the CCR (and ultimately the tax-payer) will bear most of the cost when a large-scale disaster occurs.

In the first 20 years of its existence, the CCR never managed to accumulate any substantial level of reserves. In 1999, it was on the verge of bankruptcy, and had to be recapitalized with FF 1 billion. All this occurred in spite of the fact that the claims/premium ratio for the entire period was only of the order of 55% (c.f. von Ungern (2004), p. 91). Parallel

[1] The description below is drawn from von Ungern (2004). The author also provides a more in depth description of the natural disaster insurance in France.

to the recapitalization, premiums and excesses for the property owners were raised by 30%, and the amount of coverage the CCR was willing to supply on a proportional basis was limited to 50%[2].

The purpose of this paper is to build a simple model which allows us to understand why the reinsurance company was unable to participate in the profits of the system to any significant extent. In our model, two forces drive this result. First, there is a problem of regulatory capture. The CCR paid the French insurance companies a large proportion of premium to cover, mainly fictitious, "administrative costs".[3] Second, the CCR is subject to important problems of risk selection. Given the structure of the system, there is a natural tendency for certain insurers to concentrate only on the "good" regions and buy little reinsurance, while others insure the "bad" regions and then reinsure with the CCR.

Our model shows that increasing the premium rate might not be an adequate solution to this problem. Indeed, the higher the premium rate, the greater the incentives to "separate" the good from the bad risks. If one takes into account this adjustment effect, it is not clear that an increase in the customer premium rate will in fact solve the CCR's problems.

In a more general discussion we address the question of the full "social" cost of the system. This cost is defined as the sum of final costumer premium payments and expected losses of the reinsurance company. The latter presumably covered by the state. The model illustrates that increasing the premium rate to improve the financial situation of the reinsurance comes at a high cost to the taxpayer.

Within the simple model, we work with risk neutral insurers. This necessarily implies that the choice to reinsure is discrete (either full or no reinsurance). If expected damage is lower than the reinsurance premium the insurer will not buy reinsurance cover. If the inverse holds, it will buy complete cover, instead. This in turn implies that the reinsurance company cannot be financially viable (reinsurance premium income will be lower than expected cost). This property does not necessarily hold if one were to work with risk averse insurers. For this reason we introduce a more general (and more complex) variant of the model, where insurers are risk averse. This change results in "interior" solutions for reinsurance and potentially positive (expected) profits for the reinsurer. This more general model is no longer analytically tractable, but we conjecture that the outcome would be very similar to our basic model, especially the fact that an increase in the premium rate is likely to lead to a higher degree of risk selection.

As mentioned above, von Ungern (2004) provides a critical appraisal of the French natural disaster insurance. He describes empirically most of the problems studied in this paper. However, he does not proceed to a formal analysis. A more favourable assessment of the system is given in Michel-Kerjan (2001), concluding that "the partnership between public and private sectors has passed the test of twenty years of operation". This author does not address the problem of risk selection, and he assigns the depletion of reserves of the public reinsurance company (which lead to the collapse of the system in 1999) to an exceptional succession of high damage years in the 1990s. This assessment seems unconvincing, since, as shown in von Ungern (2004) using data from the CCR, the accumulated excess premium income (premium income minus damage payments) over the period 1990-1998 was over 10 billion French Francs. This amount alone would have been enough to pay the entire damage payments for the year 1999 of 6.6 billion French Francs.

The paper is organised as follows: Section 2 presents the model and characterizes potential equilibrium outcomes. Section 3 discusses which of the potential equilibria are Nash equilibria. Section 4 presents the effect of an increase in the premium rate. Section 5 addresses the situation of the reinsurance company, demonstrating that the effect from an increase in the premium on its expected profits is dampened with a higher degree of risk selection. Section 6 presents a discussion of our results, while Section 7 concludes.

[2] Stop loss cover continues to be available.

[3] In Spain, working with an identical system of premium collection, commission for administrative costs is 5%, while it was, until recently, 24% in France.

2 The model

2.1 Setup

Consider a country with two regions $i \in [H; L]$. A region has a stock of $\mathcal{H}$ houses, each of (normalized) value 1. We assume that a house can be affected by damage, which destroys the house completely (damage value = 1). Damage occurrence for each house in region i is assumed to follow a binomial distribution with the probability of damage equal to p_i. We assume that damage occurrence is independent both among houses and among regions. Hence, for each region the expected damage is $\mathcal{H}p_i$. Regions differ only in the respective probability of an event occurring, with the H-region having a higher probability of damage than the L-region ($p_H > p_L$).

Housing insurance is compulsory, and its price is fixed at rate θ independent of p_i. The service is provided by identical insurers in a competitive market. Therefore, the market outcome in the model will be determined by a symmetric, free-entry equilibrium. Whenever an insurer j decides to offer service in a particular region i, its market share will be given by $1/N_i^*$, where N_i^* is the number of insurers active in the region.[4]

There exists a single reinsurance company which offers proportional contracts to the market in all regions.[5] Reinsurance is voluntary, and each insurer can decide on the fraction of the portfolio that it wants to reinsure. We denote by $r_j \in [0, 1]$ the rate of retention for insurer j.[6] Note that an insurer can reinsure only its entire portfolio and therefore can only choose one level of r_j, even when active in both regions.

The premium income from the fraction of the portfolio that is reinsured is divided as follows. The insurer keeps a percentage γ to cover administrative costs and passes $(1 - \gamma)$ to the reinsurance company. For the remaining portfolio, the insurer keeps the entire premium income. The reinsurer covers the fraction $(1 - r_j)$ of claims payments. In terms of a particular house in region i, revenue for insurer j from signing a natural disaster insurance contract can be expressed by the following random variable:

$$Rev_j\,(i) = \left\{ \begin{array}{c} \theta\gamma + r_j\theta(1-\gamma) \\ \theta\gamma + r_j\left[\theta(1-\gamma) - 1\right] \end{array} \right. \; with\,prob. \; \begin{array}{c} (1-p_i) \\ p_i \end{array}$$

The insurer is certain to obtain $\theta\gamma$ for each house it has under contract. Further, with probability $(1 - p_i)$ the house does not suffer a damage. The insurer then receives the remaining premium income for the fraction of retained risks (r_j). With probability p_i the house is destroyed (damage value equals 1) and "net revenue" to the insurer is the fraction of remaining premium income minus the fraction of damage that the insurance company bears.

All costs other than claims payments are assumed to be fixed. However, we consider two types of fixed costs. A region specific fixed cost (f) and a country specific fixed cost (F). F represents the fact that some costs, such as setting up a countrywide representation, must be incurred independently of the number of regions covered. Other fixed costs, such as local branch activities, are specific to a particular region. For simplicity we assume that regional fixed costs (f) are the same in both regions.[7] The country specific fixed cost (F) plays a crucial role in this model. Without it, there is no incentive for an insurer to serve both regions. In fact, in the absence of F and under a competitive insurance market characterized by free entry, each individual insurer will seek a homogeneous risk portfolio in order to choose an optimal rate of retention. Only through the introduction of country fixed costs might an insurer actually want to provide service in both regions. We will show below

[4] We work with free-entry equilibria to make the model simple. A high N in the model represents the substantial profits earned by the French insurers.

[5] This is an exact model of the proportional reinsurance cover the CCR offered prior to 2001.

[6] Hence $(1 - r_j)$ is the fraction of the portfolio that is reinsured.

[7] One could argue that administrative costs in the H-regions are higher since there will be, on average, more cases to evaluate and forms to fill out. We abstract from this.

that the French institutional setup leads to a tendency for risk selection despite country fixed costs.

There can be two types of insurers in our model. A *specialist (insurer)* serves only one specific region. Specialist j's expected profit if serving region i is

$$E[\pi_j] = \frac{\mathcal{H}}{N_i^*}\left[\theta\gamma + r_j\left(\theta\left(1-\gamma\right) - p_i\right)\right] - f - F, \tag{1}$$

where N_i^* is the total number of insurers (of both types) operating in region i. Similarly, a *generalist (insurer)* serves both regions, with expected profits given by:

$$E[\pi_j] = \frac{\mathcal{H}}{N_L^*}\left[\theta\gamma + r_j\left(\theta\left(1-\gamma\right) - p_L\right)\right] + \frac{\mathcal{H}}{N_H^*}\left[\theta\gamma + r_j\left(\theta\left(1-\gamma\right) - p_H\right)\right] - 2f - F. \tag{2}$$

Each insurer is assumed to be risk neutral, and hence will choose r_j to maximize expected profits and decide to enter as long as expected profits are positive. This implies that there can be three candidates of market equilibria.

Definition 1 *i) in a* specialist equilibrium *(SE), all insurers serve only one region. Such an equilibrium is characterized by $SE(r_L; r_H)$, where r_i is the rate of retention for specialists operating in region i.*

ii) in a generalist equilibrium *(GE), all insurers serve both regions. Such an equilibrium is characterized by $GE(r_G)$.*

iii) in a hybrid equilibrium *(HE), specialist and generalist insurers coexist. Such an equilibrium is characterized by $HEL(r_S; r_G)$ if the specialists operate in the L-region and by $HEH(r_S; r_G)$ if the specialists operate in the H-region. r_S is the rate of retention of specialists and r_G the one for generalists.*

Let us start off by looking at the optimal choice for r_j for the insurance companies. The expression for expected profits is linear in r_j, for both the specialist (1) and the generalist (2) characterization. Hence, the optimal choice of retention will be either zero or one. Specifically, from (1) we have for a specialist insurer:

$$\begin{array}{ll} r_j = 1 & if \ \ \theta\left(1-\gamma\right) - p_i \geq 0 \\ r_j = 0 & otherwise \end{array}. \tag{3}$$

Note that the choice of retention is independent of the number of firms in the region. Similarly, using (2) we see that for a generalist insurer we have:

$$\begin{array}{ll} r_j = 1 & if \ \ \frac{\mathcal{H}}{N_L^*}\left(\theta\left(1-\gamma\right) - p_L\right) + \frac{\mathcal{H}}{N_H^*}\left(\theta\left(1-\gamma\right) - p_H\right) \geq 0 \\ r_j = 0 & otherwise \end{array}. \tag{4}$$

In this case the optimal rate of retention depends on the number of insurers, since it is potentially different across regions. We next turn to each of the equilibrium candidates.

2.2 Equilibrium candidates

Equilibrium candidates are characterized by two factors: the optimal choice of the rate of retention for each insurer and the number of each type of insurers, which is obtained by setting the corresponding profit, (1) or (2), equal to zero. Table 1 presents the eight equilibrium candidates and Appendix A provides more detail on how these candidates are obtained.

We observe that there exist three candidates for specialist equilibria (SE), one where specialists in both regions fully reinsure, one where only the insurers in the H-region reinsure and one with no reinsurance in both regions. The number of insurers in each market depends on the equilibrium rate of retention. Similarly, there are two equilibrium candidates for generalist equilibria, no reinsurance or full reinsurance. Again the number of insurer varies according to the rate of retention. Finally, there are three candidates of hybrid equilibria:[8] one where generalists fully reinsure and specialists, without reinsuring, operate in the L-region; one with specialists in the L-region but no reinsurance for either type of firms; and one with specialists, fully reinsuring, in the H-region and generalists not reinsuring. In addition to the number of generalists, Table 1 displays the total number of insurers in each of the hybrid equilibrium candidates. The number of specialists can be obtained by difference.

Appendix A presents comparative statics results of the number of insurance companies in each equilibrium candidate with respect to the parameters of the model. Generally, the number of houses ($\mathcal{H}$), the premium rate (θ) and the commission rate (γ) increase the number of insurers, while the damage probabilities (p_i) and the fixed costs (f, F) decrease the number of them. There are some exceptions to this pattern (as shown in Appendix B), mainly for the hybrid equilibrium candidates, where a change in a parameter also implies a redistribution between the types of insurers, and thus the comparative static effects can be ambiguous.

3 Nash equilibria

3.1 Equilibrium conditions

Having described equilibrium candidates we now determine under which conditions these are Nash equilibria.[9]

For this we explore whether, and under which conditions, a single insurer has an incentive to deviate from its candidate equilibrium strategy, under the assumption that all the other insurers play the equilibrium strategies. Note that the set of possible deviations is not very large. Retiring from the market is never a profitable deviation, since equilibria are characterized by zero expected profits. Deviation strategies are defined by entering or retiring from a particular region choosing the optimal rate of retention.[10] In particular, we have the following deviation strategies: i) in a specialist equilibrium the deviating insurer will want to serve both regions, ii) in a generalist equilibrium the deviating insurer will specialize in the L-region and iii) in a HEL a generalist will deviate by specializing into the H-region, while in a HEH a generalist will deviate by specializing into the L-region. Appendix B provides a detailed description of these equilibrium conditions. The results are displayed in Table 2.

The second column in Table 2 indicates the rate of retention (r_D) chosen by the deviating insurer. Note that in most equilibrium candidates this choice is uniquely determined by the conditions which must hold for the equilibrium candidate. Only in the $SE(r_L = 1; r_H = 0)$ and $GE\,(r_G = 0)$ is it possible that the rate of retention r_D can be both zero or one, depending on parameter values. The last two columns indicate the conditions under which deviation is not profitable for each of the regions. Thus, for the specialist equilibrium candidates there is a condition that must hold in each region. In the generalist equilibrium, since deviation is into the L-region, the condition must only hold in the L-region. Similarly, for the HEL candidates deviation is to specialize into the H-region, thus the condition must hold in this region and inversely for the HEH candidate.

For the sake of completeness, Table 2 also includes conditions that must hold such that at least one insurer is active in the market.

[8] Appendix A explains why other potential characterizations such as $HEL\,(r_S = 0; r_G = 0)$ are in fact not equilibrium candidates.

[9] Any equilibria we describe below are Nash.

[10] Note that the set of relevant deviation strategies is finite due to risk neutrality, even though the strategy space is not.

To sum up, three conditions must hold for a candidate to be an equilibrium. First, the choice of the rate of retention of the candidate must indeed be optimal according to (3) and (4). Second, the corresponding equilibrium condition displayed in Table 2 must hold. Third, the condition that at least one insurer is active in the market must also hold.

3.2 Areas of equilibrium

We can describe these equilibrium conditions graphically. This tool will also help us to illustrate the analysis of an increase in the premium rate. For this we introduce the concepts of "switch" and "jump" lines.

Definition 2 Switch lines *are defined by situations where either (3) or (4) hold with equality. They are represented by ri in the graphs, where i corresponds to the type of firm.*

Jump-lines *are defined by a situation such that a particular equilibrium condition holds with equality. These lines are represented by* $J(r_H; r_L; r_D)$ *for specialist equilibria,* $J(r_G; r_D)$ *for generalist equilibria and* $J(r_S; r_G; r_D)$ *for hybrid equilibria.*

Intuitively, switch lines indicate where an individual insurer will "switch" regime from zero to full retention. Similarly, jump lines indicate when a single insurer will "jump" from the actual equilibrium configuration. In a broader sense, by crossing these lines we change from one equilibrium characterization to another.

These graphical devices allow us to illustrate the area of equilibria for particular parameter values, defined by the area where a single insurer does neither want to switch retention regime nor wants to jump to another equilibrium configuration. For the graphical analysis, and without loss of generality, we define p_H and p_L in terms of a mean preserving spread such that:

$$\overline{p} = \frac{p_H + p_L}{2}$$

Figure 1 illustrates the area of different types of equilibria under a set of parameter values in the $(\theta,\, p_H)$-space.[11] Each graph includes the relevant jump- and switch-lines for a given equilibrium candidate. These lines, together with the minimum-number-of-firms condition determine the (shaded) area of equilibrium. A more detailed description of the graphical analysis can be found in Appendix B.

We observe that a $GE\,(r_G = 0)$ equilibrium can only exist for low premium rates and rather small differences in the damage probability.[12] Similarly, a $GE\,(r_G = 1)$ is sustained with high premium rates and small differences in the damage probabilities. Thus, as mentioned above, the introduction of country specific fixed costs is not sufficient to generate an outcome of insurers offering in both regions. In fact, the cost of not being able to adjust the rate of retention optimally when offering in both regions outweighs the benefit of "diluting" the country fixed cost, at least for intermediate and high differences in damage probabilities across the regions.

Quite intuitively, the opposite occurs for the $SE(r_L = 1; r_H = 0)$. This equilibrium will be sustained in cases where there are important differences in the damage probabilities and for intermediate values of the premium rate. In such cases, insurers prefer to have a homogeneous risk portfolio, even though they have to support the full country fixed cost.

The $HEL\,(r_S = 1; r_G = 0)$ is an equilibrium for intermediate values of both the premium rate and the difference in damage probabilities, while the $HEL\,(r_S = 1; r_G = 1)$ can be sustained both with low and high differences in the damage probabilities for sufficiently high premium rates. Finally, a $HEH\,(r_S = 0; r_G = 1)$ requires intermediate premium rates and rather important differences in the damage probabilities.

[11] The types of equilibria not displayed in Figure 1 do not exist under the chosen set of parameter values.

[12] Note that, due to the specification of a mean preserving spread of p_L, p_H around $\overline{p}$, a high p_H in the graph implies a low p_L.

Appendix B also presents a sensitivity analysis of the equilibrium areas with respect to the relative weight of the two fixed costs. Quite intuitively, we observe that relatively high country fixed costs expand the equilibrium areas of the generalist and hybrid equilibria, while relatively high regional fixed costs expand the area of the $SE(r_L = 1; r_H = 0)$ and (for the parameter values chosen) make a $SE(r_L = 0; r_H = 0)$ possible.

Figure 2 presents the different equilibrium areas jointly. To reduce the number of areas, the area for a $GE\,(r_G = 1)$ has been omitted, as well as the condition on the minimum number of firms.

We see that for each point in the graph there exists at least one equilibrium. In fact, our model satisfies the conditions of Proposition 8.D.3 of Mas-Colell, Whinston and Green (1995) and hence we have existence of a pure strategy equilibrium. Further we observe that some equilibrium areas overlap. Particularly the $HEL\,(r_S = 1; r_G = 0)$ is embedded in the $SE(r_L = 1; r_H = 0)$ area and the $GE\,(r_G = 0)$ area. The intuition behind this overlap is that the number of insurers in the market can be such that one or the other equilibrium occurs. For example, in the area of overlap of the $HEL\,(r_S = 1; r_G = 0)$ and the $GE\,(r_G = 0)$ we have that the number of insurers is higher in the generalist equilibrium. Loosely speaking, we will end up with a generalist equilibrium if "many" insurers decide to enter the market.

3.3 Situation under redistributive premium rates

We next briefly explore the outcome of our model if the premium rate is chosen with a goal of redistribution between high and low probability regions. The feature of a unique premium rate within the country, without distinguishing different, regional risk exposition is a clear indication that the French legislator willingly imposed a certain redistribution among high and low risk areas. It is therefore informative to analyze what the outcomes are in our model when the low risk areas subsidize damage payments in more exposed areas. Formally, we assume $p_L < \theta < p_H$ to represent a redistributive premium rate. Further, we assume that the premium rate is chosen such that

$$\theta\,(1-\gamma) - p_L + \theta\,(1-\gamma) - p_H \geq 0.$$

This additional assumption ensures that if all houses where reinsured the reinsurance company would at least break even.

In such a situation only few of the candidates are actually potential equilibria. First of all, using (3) and the assumptions on the redistributive premium rate, we can observe that for specialist insurers: $r_L = 1$ and $r_H = 0$. This directly eliminates the $SE(r_L = 0; r_H = 0)$ and $SE(r_L = 1; r_H = 1)$ as equilibrium candidates. Further, due to $\theta < p_H$ we have that the equilibrium conditions for $SE(r_L = 1; r_H = 0)$ Case 2 are always satisfied. Finally, conditions in $SE(r_L = 1; r_H = 0)$ Case 1 remain unchanged.

Regarding the generalist equilibrium candidates we observe that our assumptions on the redistributive premium rate implies that only $GE(r_G = 1)$ is a valid candidate. However, in that case the equilibrium condition is never satisfied, such that no generalist equilibrium exists.

Lastly, the only hybrid equilibrium characterization that is a potential equilibrium is $HEL(r_S = 1; r_G = 0)$. The other characterizations are ruled out by redistributive premium rates.

Remark 3 *Under the realistic assumption of redistributive premium rates across the country, we observe that only a few of our equilibrium candidates are actually potential equilibria. Note especially that no generalist equilibrium is possible under redistributive premium rates. Thus, the model clearly illustrates the tendency towards risk selection of the institutional setup.*

4 Effect of an increase in the premium rate

The effect of an increase in the premium rate should not be summarized in a simple comparative statics exercise. In general, (marginally) increasing the premium rate would not alter the situation in the market. As shown in Appendix A, increasing the premium rate would in our model lead to an increased number of firms under each specific type of equilibrium. Much more interestingly, discrete increases in θ might provoke dramatic changes in the market structure in the sense that the type of insurers serving each region might change. From the graphical analysis above we have seen that changes in the premium rate can easily lead to a situation where the situation after the change is a different type of equilibrium. Analyzing discrete changes are in line with the decision of the French authorities when increasing the premium rate by 30%.

Our graphical tools are useful for this analysis. We describe sequences of the equilibrium outcome for different levels of the premium rate. Naturally, one can think of several different initial situations. We chose to examine sequences starting from a generalist equilibrium with full reinsurance and varying degrees of differences in damage probabilities across regions. In such a situation, three potential sequences can be found.[13] These sequences are illustrated in Figure 3.[14]

Proposition 4 *Starting from a generalist equilibrium with full reinsurance, the effect of an increase in the premium rate leads to the following sequences:*

i) small difference in damage probabilities $\Longrightarrow GE\,(r_G = 0) \rightarrow GE\,(r_G = 1)$.

ii) intermediate difference in damage probabilities $\Longrightarrow GE\,(r_G = 0) \rightarrow HEL\,(r_S = 1; r_G = 0) \rightarrow SE(r_L = 1; r_H = 0) \rightarrow HEL\,(r_S = 1; r_G = 1)$.

iii) large difference in damage probabilities $\Longrightarrow GE\,(r_G = 0) \rightarrow SE(r_L = 1; r_H = 0) \rightarrow HEL\,(r_S = 1; r_G = 1)$.

Starting from a GE with small differences in the damage probabilities across the high and low regions, an increase in the premium rate corresponds to moving up along the vertical line starting from point A. We observe that a sufficient increase in the premium rate implies a switch in the retention regime (point B).

In the case of intermediate differences in damage probabilities, the initial starting equilibrium can be illustrated by point C. A sufficient increase implies that we jump to a $HEL\,(r_S = 1; r_G = 0)$, illustrated by point D. Choosing an even higher premium rate implies a specialist equilibrium of the $SE(r_L = 1; r_H = 0)$ type (point E). Finally, with even further increases we end up in a $HEL\,(r_S = 1; r_G = 1)$ (point F).

In the case of large differences in damage probabilities, the sequence is similar to that described above. The difference is that the passage via the $HEL\,(r_S = 1; r_G = 0)$ area is bypassed. In this case the market configuration changes directly from a generalist to a specialist equilibrium (passage from point G to H). Further increases of the premium rate imply again a $HEL\,(r_S = 1; r_G = 1)$ (point I).

Remark 5 *Starting from a generalist equilibrium configuration with a low premium rate, we observe that only one of the three sequences following increases in the premium rate involves no risk selection. In fact, only with small differences of damage probabilities can generalist equilibria be sustained for all choices of the premium rate. As soon as the differences in probabilities exceed a certain threshold, sufficient increases in the premium rate imply partial or full risk selection. Furthermore, we observe that neither sequence ii) nor iii) ever return to a generalist equilibrium.*

[13] Theoretically there could be a sequence involving a jump into a $HEH\,(r_S = 0; r_G = 1)$. However, as shown above, deviation from a GE implies specialization into the L-region, hence such a sequence could not be triggered by the deviation of a *single* insurer.

[14] The areas of equilibrium indicated in the graph do not necessarily correspond to the full area of each type of equilibrium.

Obviously, this result has important consequences for the financial situation of the reinsurance company. An increase in the premium rate does not necessarily leave the market structure unchanged, and thus improve the situation of the reinsurer. The implied changes in market structure might actually make the situation worse.

5 Profits of the reinsurance company

In our model, the position of the reinsurer is obviously very weak. It is a direct result of risk neutral insurers that expected profits of the reinsurance company are negative in equilibrium. This (unrealistic) property would obviously change if one were to work with risk averse insurers.[15] Bearing this in mind, one can nevertheless gain useful insights from our model regarding the relative change in expected profits with changes in the premium rate.

Table 3 indicates the expressions for expected profits in decreasing order of expected profits for the reinsurance company in the different potential equilibria.

Three equilibria ($SE(r_L = 1; r_H = 1)$, $GE\,(r_G = 1)$, $HEL\,(r_S = 1;\ r_G = 1)$) have no role for the reinsurer. Next, under the $GE\,(r_G = 0)$ and $SE(r_L = 0; r_H = 0)$ we have full reinsurance, and hence the reinsurer obtains both the "good" and the "bad" risks. Under a $HEL\,(r_S = 1; r_G = 0)$ the reinsurer also gets both types of risks. However, in this case only a fraction of the houses in the low probability region are reinsured. Finally, in the $HEH\,(r_S = 0;\ r_G = 1)$ and $SE(r_L = 1; r_H = 0)$ the reinsurer gets only "bad" risks. While it is only a fraction of bad risks in the $HEH\,(r_S = 0; r_G = 1)$ it is the full portfolio of high damage houses in the $SE(r_L = 1; r_H = 0)$.

Table 4 presents the comparative statics with respect to the premium rate.

We note that, not considering eventual changes in the equilibrium characterization, a (marginal) increase in the premium rate decreases expected losses for the reinsurance company.[16] There is in fact an ordering in the magnitude of the derivative. We can show that

$$\left.\frac{\partial \pi_R}{\partial \theta}\right|_{GE(r_G=0)} > \left.\frac{\partial \pi_R}{\partial \theta}\right|_{HEL(r_S=1;r_G=0)} > \left.\frac{\partial \pi_R}{\partial \theta}\right|_{SE(r_L=1;r_H=0)}.^{17}$$

This ordering is intuitive. When the reinsurer has the full portfolio, the effect of an increase in the premium rate is strongest. For equilibrium outcomes with more and more risk selection, the reinsurer "loses" part or all of the good risks, and thus the effect of an increase in the premium rate becomes less effective. In other words, *if increasing the premium rate aims at improving the situation of the reinsurer, then this policy is less effective in situations with a high degree of risk selection.*

As mentioned above, a comparative statics exercise does not give us the full picture of the effect of an increase in the premium rate. One must also include the effect of the changes in equilibrium structure, which were outlined with the sequences of equilibrium. This effect is twofold: first, a higher degree of risk selection implies that the slope of the reinsurer's expected profits is reduced, and, second, the change in the portfolio of the reinsurer can imply drops in expected profits.

This is illustrated in Figure 4 displaying the expected profits of the reinsurer as a function of the premium rate for the three sequences of equilibria. It should be noted that the only difference between the sequences displayed is the difference between the damage probability

[15] See the next section for a discussion.

[16] Note that $\frac{\partial\left(\frac{N_S}{N_S+N_G}\right)}{\partial \theta}$ and $\mathcal{H}\left(\theta\left(1-\gamma\right)-p_H\right) < 0$ and hence the positive derivative for the $HEH\,(r_S = 0; r_G = 1)$.

[17] We have $\frac{\partial\left(\frac{N_G}{N_S+N_G}\right)}{\partial \theta} < 0$ and thus $\left.\frac{\partial \pi_R}{\partial \theta}\right|_{GE(r_G=0)} > \left.\frac{\partial \pi_R}{\partial \theta}\right|_{HEL(r_S=1;r_G=0)}$. Further, manipulating the expression for the $HEL\,(r_S = 1; r_G = 0)$ we can show that $\left.\frac{\partial \pi_R}{\partial \theta}\right|_{HEL(r_S=1;r_G=0)} > \mathcal{H}\left(1-\gamma\right)$.

in the H and L-region. In particular, overall expected damages ($\mathcal{H}\overline{p}$) are identical for all sequences. In sequence *i)* (the change of retention regime within a generalist equilibrium) the transition from one type of equilibrium to another is smooth and occurs at zero expected profits for the reinsurer.

For sequence *ii),* the passage from the $GE\,(r_G = 0)$ to the $HEL\,(r_S = 1;\ r_G = 0)$ implies a change of slope (point I). Further, at point II, the passage from the $HEL\,(r_S = 1;\ r_G = 0)$ to the $SE\,(r_L = 1; r_H = 0)$, expected profits drop first and then increase at an even lower rate. Finally, for sequence *iii)* there exists an early drop and change in slope of expected profits, when changing from the $GE\,(r_G = 0)$ to a $SE\,(r_L = 1; r_H = 0)$ (point III).

The graph illustrates that the tendency to risk selection can have important effects for the situation of the reinsurance company. For example, given our parameter values, for a premium rate of 0.06, corresponding to a situation where the reinsurer almost breaks even, expected losses are more than four times larger with partial risk selection (sequence *ii)*), and eight times larger with complete risk selection (sequence *iii)*).

6 Discussion

Within the more general discussion we want to address the following issues: 1) the evolution of the rate of reinsurance and profits in France; 2) the "social" cost of increasing the premium rate; 3) regions of different size; 4) risk-averse insurance companies; 5) introduction of a risk-pool transfer scheme as observed for some health insurance systems.

6.1 The evolution of the rate of reinsurance and profits in France

Our model, based on the institutional setup in France, predicts that an increase in the premium rate is likely to lead to an increased degree of risk selection, which in turn, generally, reduces the average rate of reinsurance. Table 5 illustrates this result.[18] It displays the evolution of the rate of reinsurance from the beginning of the system in 1982/83 to 1997.[19] This period includes an initial raise in the premium rate occurring after 1984. In the years after this first premium increase, the rate of reinsurance decreased from close to 100% to 40%. Obviously, this could simply be an indication that the premium rate is on average "comfortably" high, such that the insurance companies need not reinsure fully. Unfortunately, data on the amount of risk selection is not available. Von Ungern provides some factual evidence stating that "(i)n 1995, the share of premiums ceded to the CCR was 45 percent. In the same year, the CCR had to pay 98 per cent of the damage caused by the three hurricanes in the DOM. With a premium volume of F24m (from the DOM), the CCR had to pay claims in the order of F650m..." (von Ungern (2004), p.91).

The model also predicts that the reinsurance company cannot benefit from the profits generated in the system. This is also illustrated in Table 5 representing an estimation of the aggregate profits of the system as well as the evolution of revenues of the CCR. In the period 1982-1997 the system (and mainly the private insurers) accumulated an estimated 33,5 billion French Francs. At the same time, premium income of the CCR remained fairly stable over time, leading to the already mentioned need of refinancing of the reinsurer.

6.2 The "social" cost of increasing the premium rate

Our model allows us to calculate the "social" cost of the system. This cost is the sum of the expected losses of the reinsurer and the premium payments of the final consumers.[20] Note that, given the unique premium rate, the direct cost of insurance to final customers is simply $2\mathcal{H}\theta$, i.e. the number of houses in the country times the premium rate.

[18] Data in this Table are drawn from von Ungern (2004).
[19] Recall that starting in 1999, the CCR only allowed a maximum of 50% reinsurance.
[20] Recall that profits of the insurance companies are zero by the determination of equilibrium. Further, this is not a measure of "net welfare" as the (exogenous) cost of damage is not considered.

Figure 5 illustrates the social cost as a function of the premium rate for the three equilibrium sequences defined. First of all, the figure illustrates that improving the situation of the reinsurance company by increasing the premium rate comes at an important cost to the taxpayer (and final consumer). This effect is accentuated (higher slope of the social cost curve) with more risk selection. Finally, we observe that bigger differences in the damage probabilities, sequences *ii)* and *iii)*, might imply considerably higher social costs.

One should not overemphasize this simple welfare calculation. For one thing we do not include an analysis of why this institutional framework was chosen. Presumably, there is some benefit from the fact that a public reinsurance company exists, even if it generates losses. Our positive framework does not allow us to address the issue of the optimal institutional setup. For this, one would have to build a more complex model, which takes into account the preferences of the consumers.

However, we can analyze a situation where the private insurance companies are obliged to fully reinsure, and hence act as pure intermediaries. Figure 5 includes the social cost of such a system (the dashed-dotted line). Clearly, this simple change in the setup implies an important reduction of the total cost of the system. We can distinguish two effects that differentiate a policy of full reinsurance from the current institutional setup. First, when risk selection occurs, the social cost of the system increases much more strongly than under full reinsurance. Second, at a certain point the reinsurer actually makes positive profits and thus dampens the effect of higher premium rates.

6.3 Different sized regions

Next we want to include different sized regions. It is easy to introduce different sizes of regions (see Appendix C), where it is realistic to assume that the L-region is bigger than the H-region.[21]

One might think that a bigger L-region would attenuate the problems outlined in our model. Thus, since an *a priori* overlook of the distribution of low and high damage probability areas in France would lead to the impression that the L-region is much bigger, one would be tempted to dismiss our results as hinging crucially on the identical size of the regions.

This is not the case. In fact, a bigger L-region might make things worse for the reinsurance company. Especially under a constant average damage probability, the consequences from risk selection are even more devastating, since now the reinsurance company must support a high fraction of total damages while only reinsuring a small part of the country housing stock.

Appendix C shows that the tendencies to risk selection persist virtually unchanged under different size regions. In a similar vein, Figure 6 below represents the total social cost of the system under a bigger L-region. For the example, we assume that the L-region is 10 times the size of the H-region and take slightly different values for p_H. The other parameters, in particular $\overline{p}$ remain unchanged. Sequences a) and b) in the figure are similar to sequences ii) and iii) above. See Appendix C for details.

The illustration shows that the problems of risk selection and their consequences on the cost of the system are present also under different size regions. In order to illustrate the additional costs causes by the risk selection we again included the cost of the system under the assumption that reinsurance is compulsory.

6.4 Risk averse insurance companies

In this section we sketch a modification of the model to introduce risk-averse insurers. This implies interior solutions for the choice of the rate of retention. However, even though

[21] For clarity there is a slight change in notation in Appendix $A3$. The housing stock in region i is denoted by M_i (for maison).

we introduce risk aversion in the simplest possible way, the model becomes intractable analytically, particularly for hybrid equilibrium situations.

We return to the initial model with equally sized regions. The setup is identical to the one outlined before, with the modification that insurers maximize expected utility

$$E\left[u\left(\pi_j\right)\right] = E\left[\pi_j\right] - \frac{\lambda}{2} Var\left[\pi_j\right]$$

where λ is the risk-aversion parameter, $E\left[\pi_j\right]$ is given by (1) or (2) and

$$Var\left[\pi_j\right] = r_j^2 \frac{\mathcal{H}}{N_i} p_i \left(1 - p_i\right)$$

for specialists and

$$Var\left[\pi_j\right] = r_j^2 \left[\frac{\mathcal{H}}{N_L^*} p_L \left(1 - p_L\right) + \frac{\mathcal{H}}{N_H^*} p_H \left(1 - p_H\right)\right]$$

for generalists, choosing their rate of retention r_j. Before describing the equilibrium candidates, it is useful to introduce some further notation. We denote by ω_{ij} the "risk-adjusted unitary profit" for insurer j in region i

$$\omega_{ij} = \left[\theta\gamma + r_j\left[\theta\left(1-\gamma\right) - p_i\right] - \frac{\lambda}{2}\left(r_j\right)^2 p_i\left(1 - p_i\right)\right].$$

Equilibrium candidates are displayed in Table 6.[22]

As for the main model, the next step is to derive conditions under which a given candidate is indeed an equilibrium. Since the candidate characterization for the hybrid equilibrium is not tractable, it is not possible to obtain analytical conditions in this case. One would have to proceed to a simulation exercise. For this reason we restrict the analysis to specialist and generalist equilibria. The equilibrium conditions are displayed in Table 7, recall the deviation strategy (D) for a specialist candidate is to offer in both regions, while it is to retreat to the L-region for the generalist candidate.

As for the model with linear utility we observe that some conditions must be satisfied for a specific candidate to be an equilibrium. Thus, as before, a discrete increase in the premium rate might imply a change in the equilibrium configuration and lead to an increase in risk-selection, with the already explained consequences for the financial situation of the reinsurer. Thus the conclusions of this more general model would be essentially the same as for the model with risk-neutral insurers. The main difference in the results for the reinsurer is that its problems are somewhat attenuated, since for sufficient risk-aversion expected profit of the reinsurer might actually be positive.

6.5 Risk-pool transfer scheme

So far we have considered one simple "mechanism" to improve the financial situation of the reinsurance company via the introduction of compulsory reinsurance. We have shown that such a scheme would significantly reduce the social cost of the system. Below we want to sketch an alternative; the introduction of a risk pool similar to the ones often encountered in health insurance. Generally, a risk pool is a mechanism to redistribute premium income from insurers with a good risk portfolio to insurers with a bad one.

We model the risk-pool as a payment (rp) for each good risk (i.e. insurance contract in the L-region). The payment rp is transferred to the reinsurance company, and is set such that the latter brakes even.

Thus, expected profits in the L-region are given by

[22] The number of insurers in the HEL specification is not a tractable expression, and thus is omitted in the table. For brevity we did not consider other types of hybrid equilibria.

$$E[\pi_j] = \frac{\mathcal{H}}{N_L^*}[\theta\gamma - rp + r_j(\theta(1-\gamma) - p_i)] - f - F,$$

for a specialist, and

$$E[\pi_j] = \frac{\mathcal{H}}{N_L^*}[\theta\gamma - rp + r_j(\theta(1-\gamma) - p_L)] + \frac{\mathcal{H}}{N_H^*}[\theta\gamma + r_j(\theta(1-\gamma) - p_H)] - 2f - F,$$

for a generalist. Note that expected profit for a specialist in the H-region are still given by (1), and the choice of the rate of retention for any insurer by (3) and (4). Similarly, expected profits of the reinsurer for the different equilibrium candidates are still given by the expressions in Table 3 adding $\mathcal{H}rp$ to each expression.

Equilibrium candidates are now characterized by a system of equations considering the zero-profit conditions for each type of insurer in the candidate and, additionally, by the zero expected profit condition for the reinsurer to determine rp.

Descriptions of the equilibrium candidates are displayed in Table 8.[23]

Equilibrium candidates where there is no reinsurance are not displayed, since in this case $rp = 0$ and the expressions are as in Table III-1. Further, the hybrid equilibrium expressions are messy and are thus not displayed either. We observe similar expressions for the number of insurers as in our basic model. The main difference is that, in general, the number of insurers operating in the L-region is lower, due to the additional financing of the reinsurer.

From here the analysis is essentially the same as for the basic model, i.e. one could again define areas of equilibrium with the consequence that an discrete increase in the premium rate may lead to a change in the equilibrium configuration with an increase in risk-selection. All this is not needed to consider the social cost of such a system, since by construction expected profits of both the insurers and the reinsurer are zero. Hence, social cost is now determined uniquely by the cost to final customers. However, it should be mentioned that the risk-pool scheme is only feasible for premium rates that satisfy

$$(\theta - p_L) + (\theta - p_H) > 0,$$

since, without this condition the number of insurers would become negative.

Figure 7 takes up Figure 5 and includes the social cost of the risk-pool scheme. Quite intuitively, the slope of the risk-pool scheme is the highest and coincides with the segments of the above mentioned sequences for situations where there is no reinsurance. In this sense, the risk pool has (weakly) lower costs than the current situation of free choice of reinsurance, whenever the risk-pool is feasible. Further, we can also detect a range of premium rate (in our case between 0.05 and slightly more than 0.06), where the risk-pool scheme is also better than the alternative of full reinsurance.

7 Conclusions

Natural disaster insurance has recently come back into the policy agenda due to increased occurrence and severity of floods and other catastrophes. The situation in France merits particular attention because of some stylized facts one can observe. Although the system is financially viable (the claims/premium ratio since introduction of the insurance is around 55%), the existing, publicly owned, reinsurance company (Caisse Centrale de Réassurance - CCR) was never able to build up vital reserves and had to be recapitalized in 1999. This injection of public funds was accompanied by an increase in premium rates and excesses of 30%.

Our claim is that this situation is the outcome of flaws in the institutional setup. The private insurance companies are free to contract reinsurance at any degree with the CCR.

[23] Since the risk-pool does not affect the H-region, the expressions for the number of insurers in this region are identical to the ones in Table 1.

This leads to a situation of risk selection by the insurance companies. Further, the increase in the premium rates to improve the situation for the financially distressed CCR, comes at a high cost for the tax payer and final consumer of the insurance.

In order to sustain this claim we build a model that represents the situation of the French property insurance market. Although the model is kept as simple as possible, we include the main features of the institutional setup, such as the uniform premium rate across the country and the existence of a (publicly owned) reinsurer that offers (proportional) reinsurance cover. The equilibrium outcomes of our model confirms the inherent tendency of risk selection. In particular, assuming redistributive premium rates, where low damage probability regions subsidize high damage probability regions, no equilibrium where all insurance companies serve the whole country ("generalist equilibrium") can exist. Further, we find that a higher premium rate is likely to increase the tendency for risk selection. Note that the increase in risk selection is not gradual. Rather, a sufficient increase in the premium rate eventually leads to a change in the equilibrium market configuration, one with a higher degree of risk selection.

These changes in equilibrium configuration have their effect on the reinsurer. While a marginal increase in the premium rate improves the expected profits of the reinsurer, this effect is dampened with a higher degree of risk selection. When calculating the "social" cost of the system, which is defined as the sum of (final) consumer premium payments and expected profits (losses) of the reinsurer, we show that the current institutional setup leads to high social cost. In particular, social costs of the current setup are (in general) significantly higher than in a situation where all insurers are obliged to fully reinsure and act as pure intermediaries.

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A Determination of equilibrium candidates

Below we describe in some more detail the determination of the equilibrium candidates for each of the equilibrium defined in the text.

Specialist equilibrium candidates

Due to the discrete optimal rate of retention we can distinguish three characterizations of a potential specialist equilibrium:

Characterization	$\theta(1-\gamma)-p_L$	$\theta(1-\gamma)-p_H$
$SE(r_L=0;r_H=0)$	<0	<0
$SE(r_L=1;r_H=0)$	>0	<0
$SE(r_L=1;r_H=1)$	>0	>0

In each case the equilibrium number of firms will be determined by setting (1) equal to zero and solving for the equilibrium number of firms. Denoting by N_i^S the number of firms in region i in the specialist equilibrium characterization we obtain:

Eqm. Candidate	N_L^S	N_H^S
$SE(r_L=0;r_H=0)$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$
$SE(r_L=1;r_H=0)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$
$SE(r_L=1;r_H=1)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$	$\frac{\mathcal{H}(\theta-p_H)}{(f+F)}$

Generalist equilibrium candidates

For the generalist equilibrium candidates there are only two characterizations according to (4). The equilibrium number of firms, denoted by N_G, is determined by setting (2) equal to zero:

Eqm. Candidate	N_G
$GE(r_G=0)$	$\frac{2\mathcal{H}\theta\gamma}{(2f+F)}$
$GE(r_G=1)$	$\frac{\mathcal{H}(\theta-p_L+\theta-p_H)}{(2f+F)}$

Hybrid equilibrium characterization

In a so called hybrid equilibrium candidate (HE) the choice of the rate of retention will again be determined by (3) and (4), respectively.[24]

The equilibrium number of insurers is obtained by a zero-profit condition for each type of firm, using (1) and (2). Note that we must distinguish between HEL and HEH candidates. For example, in a HEH characterization the conditions to determine the number of each type of insurer are:

[24] One could imagine a hybrid equilibrium with three type of firms: specialist in either region and generalists. However, for the characterization of such an equilibrium a condition, which is independent of the number of firms of any type, must hold. We do not consider this particular case.

$$E[\pi_S] = \frac{\mathcal{H}}{N_H^*}[\theta\gamma + r_S(\theta(1-\gamma) - p_H)] - f - F = 0$$
$$E[\pi_G] = \frac{\mathcal{H}}{N_G}[\theta\gamma + r_G(\theta(1-\gamma) - p_L)] + \frac{\mathcal{H}}{N_H^*}[\theta\gamma + r_G(\theta(1-\gamma) - p_H)] - 2f - F = 0$$

where π_S and π_G are profits of the specialists and generalists, respectively, and $N_H^* = N_S + N_G$ is the total number of insurers operating in the H-region (and also the total number of firms in the country). Again, we must consider the four cases of possible rates of retention.

Below we present the equilibrium number of insurers for the hybrid equilibrium candidates when the specialists operate in the L-region and the H-region. We indicate the total as well as the generalist number of insurers. The number of specialists can easily be obtained by difference. Note that the table only excludes situations, denoted by (-), where the number of one type of firms is negative under all parameter constellations.[25]

Eqm. Candidate	N_L^*	N_G^{HEL}
$HEL\,(r_S=0; r_G=0)$	$-$	$-$
$HEL\,(r_S=0; r_G=1)$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$	$\frac{\mathcal{H}(\theta-p_H)}{f\left(2-\frac{\theta-p_L}{\theta\gamma}\right)+F\left(1-\frac{\theta-p_L}{\theta\gamma}\right)}$
$HEL\,(r_S=1; r_G=0)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$	$\frac{\mathcal{H}\theta\gamma}{f\left(2-\frac{\theta\gamma}{\theta-p_L}\right)+F\left(1-\frac{\theta\gamma}{\theta-p_L}\right)}$
$HEL\,(r_S=1; r_G=1)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$	$\frac{\mathcal{H}(\theta-p_H)}{f}$

Eqm. Candidate	N_H^*	N_G^{HEH}
$HEH\,(r_S=0; r_G=0)$	$-$	$-$
$HEH\,(r_S=0; r_G=1)$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$	$\frac{\mathcal{H}(\theta-p_L)}{f\left(2-\frac{\theta-p_H}{\theta\gamma}\right)+F\left(1-\frac{\theta-p_H}{\theta\gamma}\right)}$
$HEH\,(r_S=1; r_G=0)$	$\frac{\mathcal{H}(\theta-p_H)}{(f+F)}$	$\frac{\mathcal{H}\theta\gamma}{f\left(2-\frac{\theta\gamma}{\theta-p_H}\right)+F\left(1-\frac{\theta\gamma}{\theta-p_H}\right)}$
$HEH\,(r_S=1; r_G=1)$	$-$	$-$

We observe that a HE candidate with a rate of retention of zero is not possible. In fact, under such a situation the number of specialists would be negative. This result is quite intuitive, under zero retention an insurer which operates in both regions has an advantage over an insurer specializing in one region, since it can "dilute" the country fixed costs. Note further, that within the $HEH\,(r_S = 1; r_G = 1)$ candidate there is additionally no possibility of a situation with full retention. Again in this case the number of specialist firms would be negative.

Next we must check whether the potential equilibrium characterizations are feasible in the sense that there exist situations where the rate of retention of the equilibrium characterization indeed is an optimal choice for the reinsurer. If this is not the case, we can eliminate further candidates.

For example, within the $HEL\,(r_S = 0; r_G = 1)$ characterization we have that $r_S = 0$. This implies for an optimal choice, from (3), that $\theta(1-\gamma) - p_L < 0$. Hence, under $p_H > p_L$ we also have $\theta(1-\gamma) - p_H < 0$, which implies using (4), that $r_G = 1$, is never an optimal choice. We are thus able to exclude the $HEL\,(r_S = 1; r_G = 1)$ candidate. For the other two cases there are no restrictions that would render them a priori infeasible.

An inverse reasoning shows that the $HEH\,(r_S = 1; r_G = 0)$ candidate is not feasible. Indeed, $r_S = 1$ implies $\theta(1-\gamma) - p_H > 0$, which implies that the optimal choice for a generalist is $r_G = 1$. Thus we can also eliminate this characterization.

Summarizing, the potential hybrid equilibrium candidates are:

[25] We therefore, so far, do not rule out potential negative numbers of firms under certain parameter values. We will introduce this restriction later on in the analysis.

Eqm. Candidate	N_i^*	N_G^{HEi}
$HEL\,(r_S=1;r_G=0)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$	$\frac{\mathcal{H}\theta\gamma}{f\left(2-\frac{\theta\gamma}{\theta-p_L}\right)+F\left(1-\frac{\theta\gamma}{\theta-p_L}\right)}$
$HEL\,(r_S=1;r_G=1)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$	$\frac{\mathcal{H}(\theta-p_H)}{f}$
$HEH\,(r_S=0;r_G=1)$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$	$\frac{\mathcal{H}(\theta-p_L)}{f\left(2-\frac{\theta-p_H}{\theta\gamma}\right)+F\left(1-\frac{\theta-p_H}{\theta\gamma}\right)}$

Comparative statics of the equilibrium characterizations

Table A-1 below summarizes the comparative statics effects of the parameters in the model on the number of insurers in each equilibrium candidate[26]. Results are in general straightforward and intuitive. The only somewhat different effects occur in the hybrid equilibrium candidates. There we observe that some derivatives cannot be signed, or have the opposite sign as the ones observed for the specialist and generalist candidates. This is due to the fact that one type of insurer can "benefit" from an increase in one of the parameters at the cost of the other type in the equilibrium. Generally, the pattern is the following: the number of houses ($\mathcal{H}$), the premium rate (θ) as well as the commission rate (γ) have a positive effect on the number of insurers, while the damage probabilities (p_i) and fixed costs have a negative effect. As exceptions to these effects we can note that the damage probability in the L-region affects positively the number of generalist firms in a $HEL\,(r_S = 1, r_G = 0)$ situation. Whereas for the same candidate the rate of commissions (γ) affects negatively the number of specialists. The effect of an increase in the premium rate for this candidate is ambiguous. Further, note that the damage probabilities have a positive effect on the number of specialists in the $HEH\,(r_S = 0; r_G = 1)$ candidate.

B Equilibrium conditions

We consider the deviation strategies for the three types of equilibrium: specialist, generalist and hybrid.

Specialist equilibrium

Under the specialist equilibrium candidates we have separation of markets. We must consider deviation strategies for the H and the L regions. However, the possible deviations are of the same kind in both cases. Does an insurer have an incentive to offer in both regions, choosing its optimal rate of retention given by (4)? It seems intuitive that an insurer operating in the H-region more likely has an incentive to deviate. For this reason we concentrate on the situation for a specialist firm operating in the high probability region. Below we present the corresponding conditions for a specialist in the L-region. Note that the deviation implies that the number of market participants does not change in the region where the insurer was already active, while it increases by one in the other region. For deviation from the H-region this implies:

$$\begin{aligned} N_H &= N_H^S \\ N_L &= N_L^S + 1 \end{aligned}$$

Further we can distinguish four possible cases that potentially occur, characterized in the table below. We denote by r_D the optimal choice of the rate of retention for a deviation strategy.

[26] Note that the effect of an increase in the premium rate (θ) is further analyzed in a subsequent section.

Char.	Case	r_D
$SE(r_L = 0; r_H = 0)$		0
$SE(r_L = 1; r_H = 0)$	1	0
$SE(r_L = 1; r_H = 0)$	2	1
$SE(r_L = 1; r_H = 1)$		1

To obtain the condition under which a deviation is not profitable we compute the expected profits of a deviating insurer, taking into account the number of firms in the market. This expression must be negative, implying the conditions we are looking for.

SE($\mathbf{r}_L$= 0; $\mathbf{r}_H$= 0)

Deviating expected profits are given by:

$$E[\pi_D] = \frac{\mathcal{H}}{N_L}\theta\gamma + \frac{\mathcal{H}}{N_H}\theta\gamma - 2f - F < 0$$

Using the expression for the number of insurers we obtain:

$$\begin{aligned} E[\pi_D] &= \frac{\mathcal{H}\theta\gamma}{\frac{\mathcal{H}\theta\gamma+f+F}{f+F}} + \frac{\mathcal{H}\theta\gamma}{\frac{\mathcal{H}\theta\gamma}{f+F}} - 2f - F < 0 & (B-1) \\ &= \frac{\mathcal{H}\theta\gamma}{\mathcal{H}\theta\gamma + f + F}(f+F) - f < 0 & (5) \end{aligned}$$

If this condition holds, then a typical insurer of a $SE(r_L = 0; r_H = 0)$ operating in the H-region, will not have an incentive to deviate from the equilibrium characterization[27].

SE($\mathbf{r}_L$= 1; $\mathbf{r}_H$= 0) Case 1

The same procedure leads to the following condition:

$$E[\pi_D] = \frac{\mathcal{H}\theta\gamma}{\mathcal{H}(\theta - p_L) + f + F}(f+F) - f < 0 \qquad (B-2)$$

The condition states that deviation is not profitable if generated income from entering the L-region does not cover the additional regional fixed costs.

SE($\mathbf{r}_L$= 1; $\mathbf{r}_H$= 0) Case 2

The condition for this case is somewhat more complicated. Again, determining the expected profits from deviation, using the equilibrium number of firms and then manipulating the expression somewhat, we obtain:

$$E[\pi_D] = \begin{array}{c} f\left[\frac{\mathcal{H}(\theta-p_L)}{\mathcal{H}(\theta-p_L)+f+F} + \frac{(\theta-p_H)}{\theta\gamma} - 2\right] \\ +F\left[\frac{\mathcal{H}(\theta-p_L)}{\mathcal{H}(\theta-p_L)+f+F} + \frac{(\theta-p_H)}{\theta\gamma} - 1\right] \end{array} < 0 \qquad (B-3)$$

Note that the term in f is strictly negative. This comes from the fact that here (3) in the H-region is negative and thus: $\theta - p_H < \theta\gamma$. On the other hand, the term in F is potentially positive.

SE($\mathbf{r}_L$= 1; $\mathbf{r}_H$= 1)

[27] In this case, as is shown in the appendix, the condition for a L-insurer is the same, since both have the same initial rate of retention.

Finally the condition a in $SE(r_L = 1; r_H = 1)$ is given by:

$$E[\pi_D] = \frac{\mathcal{H}(\theta - p_L)}{\mathcal{H}(\theta - p_L) + f + F}(f + F) - f < 0 \qquad (B-4)$$

The corresponding conditions for a specialist in the L-region are the following:

Candidate	Condition
$SE(r_L = 0; r_H = 0)$	$\frac{\mathcal{H}\theta\gamma}{\mathcal{H}\theta\gamma+f+F}(f+F) - f < 0$
$SE(r_L = 1; r_H = 0)$ Case 1	$f\left[\frac{\theta\gamma}{(\theta-p_L)} + \frac{\mathcal{H}\theta\gamma}{\mathcal{H}\theta\gamma+f+F} - 2\right] + F\left[\frac{\theta\gamma}{(\theta-p_L)} + \frac{\mathcal{H}\theta\gamma}{\mathcal{H}\theta\gamma+f+F} - 1\right] < 0$
$SE(r_L = 1; r_H = 0)$ Case 2	$\frac{\mathcal{H}(\theta-p_H)}{\mathcal{H}\theta\gamma+f+F}(f+F) - f < 0$
$SE(r_L = 1; r_H = 1)$	$\frac{\mathcal{H}(\theta-p_H)}{\mathcal{H}(\theta-p_H)+f+F}(f+F) - f < 0$

Generalist equilibrium

To determine the conditions under which deviation from a generalist equilibrium candidate is not profitable we proceed in the exact same way as for the specialist equilibrium characterization. Here the potential deviation strategy is to restrict service into one of the regions. However, we can show that deviation into the high probability region is never profitable.

Thus, the conditions we must check corresponds to deviation into the L-region, choosing the rate of retention according to (3). Note that restricting service into one region does not change the number of insurers active in that particular region. Here we can distinguish three cases.

Candidate	Case	r_D
$GE(r_G = 0)$	1	0
$GE(r_G = 0)$	2	1
$GE(r_G = 1)$		1

$\mathbf{GE(r_G = 0)}$ **Case 1**

Expected profits from deviation are given by:

$$\begin{aligned} E[\pi_D] &= \frac{\mathcal{H}\theta\gamma}{N_G} - f - F \\ &= \frac{\mathcal{H}\theta\gamma}{\frac{2H\theta\gamma}{(2f+F)}} - f - F \\ &= f + \frac{F}{2} - f - F < 0 \end{aligned}$$

Thus, deviation from a $GE(r_G = 0)$ is never profitable. A situation which is quite intuitive. Here insurers fully reinsure and the zero profit condition is met with firms "diluting" the country fixed cost over two regions. Retiring from one region in such a situation, only reducing the regional fixed costs, is not profitable.

GE $(\mathbf{r_G = 0})$ Case 2

Calculating expected profits from deviating, using the equilibrium number of firms and then manipulating the expression we obtain the following condition:

$$E[\pi_D] = f\left[\frac{(\theta - p_L)}{\theta\gamma} - 1\right] + F\left[\frac{(\theta - p_L)}{2\theta\gamma} - 1\right] < 0 \qquad (B-5)$$

Note that here the term in f is strictly positive, since $r_D = 1$ implies $(\theta - p_L) > \theta\gamma$.

GE $(\mathbf{r_G = 1})$

Similarly, the condition for $GE(r_G = 1)$ is given by:

$$E[\pi_D] = f\left[\frac{2(\theta - p_L)}{(\theta - p_L + \theta - p_H)} - 1\right] + F\left[\frac{(\theta - p_L)}{(\theta - p_L + \theta - p_H)} - 1\right] < 0$$

Note again that in this case the term in f is strictly positive. Further it is interesting to note that in the case of $p_H > \theta$, the condition is never satisfied, i.e. in a situation where there is redistribution between the low and the high probability region, a $GE(r_G = 1)$ candidate is never an equilibrium. We will come back to this issue later on.

Hybrid equilibrium

HEL

For these candidates the only possible deviating strategy is to restrict service to the H-region. In fact, if a specialist would want to become a generalist, then the number of generalist insurers would increase by one, leading to negative expected profits due to the zero-profit condition imposed on the characterization. Similarly, if a generalist restricts service to the L-region the number of insurers in that region does not change. Hence, its situation becomes identical to the already active specialists in that region, which have zero expected profits. Finally, note that we can restrict the analysis to generalists. If one of them deviates, the number of insurers in the H-region does not change, whereas if a specialist deviates the number of firms in the market increases making the situation worse.

As we have seen before, there remain two possible candidates for this hybrid equilibrium: $HEL(r_S = 1; r_G = 0)$ and $HEL(r_S = 1; r_G = 1)$.

In the first case, we can show that $r_G = 0$ implies that $\theta < p_H$. Thus, the optimal rate of retention for the deviating insurer is $r_D = 0$. Expected profits become:

$$E[\pi_D] = \frac{\mathcal{H}\theta\gamma}{N_G} - f - F < 0$$

using the expression for N_G in this case we obtain:

$$E[\pi_D] = f - \frac{\theta\gamma}{(\theta - p_L)}(f + F) < 0$$

It should be recalled that $r_S = 1$ implies $(\theta - p_L) > \theta\gamma$, and thus the condition is not necessarily negative.

For the $HEL(r_S = 1; r_G = 1)$ we can show that deviation is never profitable.

HEH

Within the hybrid equilibrium candidate where specialists are active in the high probability region, only one potential case is actually possible. The condition for an equilibrium in this case can be found in the same way as before. The deviation strategy for a generalist is to restrict service to the L-region. The condition on the rate of retention for the generalist implies that $(\theta - p_L) > \theta\gamma$ must hold. Thus the optimal rate of retention for deviation is $r_D = 1$. Expected profits from deviation can then be written as:

$$E[\pi_D] = \frac{\mathcal{H}(\theta - p_L)}{N_G} - f - F$$

which becomes, using the expression for the number of insurers:

$$E[\pi_D] = f - \frac{(\theta - p_H)}{\theta\gamma}(f + F) < 0$$

We can note here that deviation is always profitable in situations where $\theta < p_H$.

Minimum number of insurers

Finally, before moving to the next step, we want to restrict the set of equilibria such that at least one firm is active in the market. We do this by specifying conditions ensuring that within the most unfavorable situation there is still at least one firm in the market. For the specialist equilibrium this is given by:

$$\begin{aligned} N_H^{SE} &= \frac{\mathcal{H}\theta\gamma}{(f+F)} \geq 1 \\ &\Longrightarrow \mathcal{H}\theta\gamma \geq (f+F) \end{aligned}$$

and for the generalist equilibrium:

$$\begin{aligned} N_G &= \frac{2\mathcal{H}\theta\gamma}{(2f+F)} \geq 1 \\ &\Longrightarrow \mathcal{H}\theta\gamma \geq \frac{(2f+F)}{2} \end{aligned}$$

for the hybrid equilibrium the conditions that at least one firm of each type is active are not very handy. Thus we impose the restriction that the total number of insurers is at least two:

$$\begin{aligned} N_i^* &= \frac{\mathcal{H}\theta\gamma}{(f+F)} \geq 2 \\ &\Longrightarrow \mathcal{H}\theta\gamma \geq 2(f+F) \end{aligned}$$

Note that, quite intuitively, the least restrictive condition is the one on the generalist equilibrium.

Description of the graphical analysis

Specialist equilibrium

We start with the specialist equilibrium and present only the lines corresponding to insurers operating in the H-region.[28]

[28] We assume that these conditions are more stringent than the ones for insurers in the L-region. Thus we assume that, at the margin, if an H-insurer wants to proceed to a change an L-firm will not want to do so, yet.

Figure B-1 below presents the switch-lines for the $SE(r_L = 1; r_H = 0)$ in the premium rate (θ) - high risk area probability (p_H) space. The upper increasing line (rh) represents situations where an H-insurer wants to change retention regime. Above rh the premium rate is favorable enough that the insurer retains all risks ($r_H = 1$), whereas below the line the insurer fully reinsures ($r_H = 0$). Similarly, the lower increasing line (rd) represents the switch-line for the deviating insurer.[29] Again, above rd the deviating insurer chooses full retention. Finally, the decreasing line (rl) corresponds to the switch-line for an L-insurer, with full retention above the line. Thus the area for a $SE(r_L = 1; r_H = 0)$ Case 1 is delimited above by rd and below by rl. Correspondingly, $SE(r_L = 1; r_H = 0)$ Case 2 is delimited above by rh and below by rd.

The relevant jump-lines for the $SE(r_L = 1; r_H = 0)$ are presented in Figure B-2. The increasing line corresponds to the jump-line for $SE(r_L = 1; r_H = 0)$ Case 1, i.e. situations where $(B - 2)$ holds with equality. Below the line the equilibrium condition is satisfied. Similarly, the decreasing line represents situations where $(B - 3)$ holds with equality. In other words, this is the jump-line for $SE(r_L = 1; r_H = 0)$ Case 2. Here the condition holds everywhere above the line.

Figure B-3 takes the two graphs together. It allows to determine the area of equilibria for the particular candidate under study ($SE(r_L = 1; r_H = 0)$).[30] In our example this area is principally determined by the relevant jump-lines. However, the switch-lines are also important because they delimit the range where a specific jump line is actually relevant. In order to illustrate this interplay we have included in Figure B-3 the switch-line for the deviating firm (rd). In the end, the area of equilibria is determined by the area which is both below the dotted ($J(r_L = 1,\ r_H = 0,\ r_D = 1)$) and above the solid line ($J(r_L = 1,\ r_H = 0,\ r_D = 0)$). To get to this conclusion we must consider also the switch-line for the deviating insurer (rd). Take for example the solid jump-line ($J(r_L = 1,\ r_H = 0,\ r_D = 0)$). This line, by definition is only valid for situation where the deviating insurer chooses a zero rate of retention, hence situation which are delimited from above by rd. Anywhere else this is not the relevant jump-line. Similar argumentation implies that another part of the area of equilibria is delimited by $J(r_L = 1,\ r_H = 0,\ r_D = 1)$ from above and rd from below. Taking these parts together, we obtain the full (shaded) area of equilibria.

Finally, Figure B-4 includes the last restriction, the fact that at least one insurer should be active in every region, line denoted by NF.

Similar steps can be applied to obtain the equilibrium area for all the SE-configurations, which are shown in the first column of Figure B-5. Note that under our parameter specifications there are no equilibria of type $SE(r_L = 0; r_H = 0)$ and $SE(r_L = 1; r_H = 1)$, due to the fact that some conditions are mutually exclusive. For the $SE(r_L = 0; r_H = 0)$ it is the fact that the jump-line and the restriction for the minimum number of firms coincide, while for the $SE(r_L = 1; r_H = 1)$ the switch and the jump-line delimit exclusive areas.

In the same figure we have a look at how the "shape" of the equilibrium area changes under different parameter constellations. Of particular interest are changes in the structure of the fixed costs. For this we specify two alternative scenarios, one where country fixed costs (F) are high and one where regional fixed costs (f) are high. In both cases we leave the sum of fixed costs ($f + F$) identical to the base scenario. Column two and three present the resulting area of equilibria.

For the $SE(r_L = 0; r_H = 0)$ we have that equilibrium is delimited from above the by the jump-line $J\left(r_L = 0,\ r_H = 0,\ r_D = 0\right)$. As mentioned above, there is no equilibrium of this characterization under the base parameter values. Similarly, a high country fixed cost (F) does not give an equilibrium, whereas high regional fixed costs (f) imply existence of $SE(r_L = 0; r_H = 0)$-type equilibria. This result is quite intuitive, only in situations where regional fixed costs are high relative to country fixed costs does it make sense to specialize into one type of market in a situation of full reinsurance. In the other situations it is more

[29] Recall that a deviating insurer from a specialist equilibrium characterization will necessarily choose to be a generalist. Further, note that by considering H-region conditions, deviation implies that $N_L = N_L^{SE} + 1$.

[30] For clarity the lines irrelevant for equilibrium determination are not drawn.

appropriate to offer in both regions, in order to dilute the country fixed cost.

Regarding the $SE(r_L = 1; r_H = 0)$ (in row two) we can observe that a change in the relative composition of fixed costs moves the relevant jump-lines around. The result is that a $SE(r_L = 1; r_H = 0)$ equilibrium can be sustained only in case of extreme differences in damage probabilities in the case of high F, whereas the area of equilibrium is much bigger under high f. Again, this result is intuitive in that high country fixed cost provide an incentive for specialization only in the case of important differences across the regions.

Finally, under the considered parameter constellations there is no $SE(r_L = 1; r_H = 1)$ equilibrium. Again we observe that changes in the fixed cost composition move around the jump-line.

Generalist Equilibrium

Having introduced the different concepts we use for the graphical analysis, we can proceed much quicker for the other two equilibrium characterizations. As before, within the Generalist Equilibrium candidates, equilibria are determined through the interplay between the relevant switch- and jump-lines. Figure B-6 presents these results for the two candidates of GE ($GE\,(r_G = 0)$, $GE\,(r_G = 1)$). The figure also includes the effects of a change in the relative composition of fixed costs.

Above we have shown that a $GE\,(r_G = 0)$ with $r_D = 0$ is always an equilibrium. Since, in our case $r_D = r_L$, we have that all points below the rl line are Generalist Equilibria, except for the restriction on the minimum number of insurers. Further, points below the jump-line for $GE\,(r_G = 0)$ are also part of the area of equilibria. Finally, note that here an increase in the relative magnitude of F implies an increase in the area of equilibrium.

For the $GE\,(r_G = 1)$ we have that situations with high premium rates and little differences in damage probabilities constitute an equilibrium. Again the area of equilibria expands when F is relatively high and contracts for high f.

Hybrid Equilibrium

Figure B-7 presents the area of equilibrium for the three HE candidates ($HEL\,(r_S = 1; r_G = 0)$, $HEL\,(r_S = 1; r_G = 1)$, $HEH\,(r_S = 0; r_G = 1)$) including the effects of a change in the relative composition of fixed costs.

We observe that a $HEL\,(r_S = 1; r_G = 0)$ equilibrium is delimited from below by rl and from above by rg and the relevant jump-line. Note that, as in the GE case, a high F increases the area of equilibria.

As for the $HEL\,(r_S = 1; r_G = 1)$, we have shown above that it is always an equilibrium. Thus, all points above rg represent an equilibrium. Note that changes in the composition of fixed costs only slightly alter rg, and hence the area of equilibria.

Finally, within our parameter values we observe that the area of a $HEH\,(r_S = 0; r_G = 1)$ is rather small, again with its size increasing with relatively high country fixed costs.

C Different size regions

In this appendix we describe the outcome of our model when introducing different size regions. For this we introduce a slight change of notation. We denote by M_i $(i \in L, H)$[31] the stock of houses in each region assuming that $M_L > M_H$. The steps to analyze the model are the same as in the text. Hence, we first present the equilibrium candidates, then the conditions that must hold for (Nash) equilibrium. Finally, we present the situation of the reinsurance company and the resulting social cost analysis.

Table C-1 summarizes the equilibrium candidates. The only notable change is the fact that a $HEL\,(r_S = 0; r_G = 0)$ is now *ex ante* possible.

[31] M for "maison".

Equilibria are obtained by determining the conditions under which a single insurer does not have an incentive to deviate. Table C-2 presents these conditions.

These conditions are very similar to the ones for equally sized regions. Thus one can again determine the area of equilibrium for each candidate, and identify sequences of equilibrium configurations for increases in the premium rate. Similarly, the expressions for the expected profits of the reinsurer are almost identical to the ones in Table 3. Note particularly, that these profits are again non-positive in all cases.

In order to illustrate the social cost of the system under different size regions we specified two sequences a) and b). We assume that M_L is ten times bigger ($1,000,000$ houses) than M_H ($100,000$ houses), while the other basic parameters are identical to the scenarios before: $\overline{p} = 0.05$, $f = F = 200$, $\gamma = 0.2$.

Sequence a) corresponds to sequence ii) in the illustration before, i.e. a small difference in damage probability across regions. We assumed $p_L = 0.049$ and $p_H = 0.06$. Similarly, sequence b) represents sequence iii) from above with $p_L = 0.04$ and $p_H = 0.15$.

Figure 7 in the text presents the social cost for these sequences.

Table 1
Equilibrium candidates

Equilibrium	Number of ins. in L-region	Number of ins. in H-region	Number of generalists
$SE(r_L = 0; r_H = 0)$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$	
$SE(r_L = 1; r_H = 0)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$	$\frac{\mathcal{H}\theta\gamma}{(f+F)}$	
$SE(r_L = 1; r_H = 1)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$	$\frac{\mathcal{H}(\theta-p_H)}{(f+F)}$	
$GE\,(r_G = 0)$			$\frac{2\mathcal{H}\theta\gamma}{(2f+F)}$
$GE\,(r_G = 1)$			$\frac{\mathcal{H}(\theta-p_L+\theta-p_H)}{(2f+F)}$
$HEL\,(r_S = 1; r_G = 0)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$		$\frac{\mathcal{H}\theta\gamma}{f\left(2-\frac{\theta\gamma}{\theta-p_L}\right)+F\left(1-\frac{\theta\gamma}{\theta-p_L}\right)}$
$HEL\,(r_S = 1; r_G = 1)$	$\frac{\mathcal{H}(\theta-p_L)}{(f+F)}$		$\frac{\mathcal{H}(\theta-p_H)}{f}$
$HEH\,(r_S = 0; r_G = 1)$		$\frac{\mathcal{H}\theta\gamma}{(f+F)}$	$\frac{\mathcal{H}(\theta-p_L)}{f\left(2-\frac{\theta-p_H}{\theta\gamma}\right)+F\left(1-\frac{\theta-p_H}{\theta\gamma}\right)}$

Table 2
Summary of Nash equilibrium and minimum number of insurer conditions

Equilibrium Candidates	r_D	Nash-equilibrium condition in L-region or cond. on minimum number of insurers	Nash-equilibrium condition in H-region
$SE(r_L = 0; r_H = 0)$	0	$\frac{\mathcal{H}\theta\gamma}{\mathcal{H}\theta\gamma+f+F}(f+F) - f \leq 0$	$\frac{\mathcal{H}\theta\gamma}{\mathcal{H}\theta\gamma+f+F}(f+F) - f \leq 0$
$SE(r_L = 1; r_H = 0)$ Case 1	0	$f\left[\frac{\theta\gamma}{(\theta-p_L)} + \frac{\mathcal{H}\theta\gamma}{\mathcal{H}\theta\gamma+f+F} - 2\right] + F\left[\frac{\theta\gamma}{(\theta-p_L)} + \frac{\mathcal{H}\theta\gamma}{\mathcal{H}\theta\gamma+f+F} - 1\right] \leq 0$	$\frac{\mathcal{H}\theta\gamma}{\mathcal{H}(\theta-p_L)+f+F}(f+F) - f \leq 0$
$SE(r_L = 1; r_H = 0)$ Case 2	1	$\frac{\mathcal{H}(\theta-p_H)}{\mathcal{H}\theta\gamma+f+F}(f+F) - f \leq 0$	$f\left[\frac{\mathcal{H}(\theta-p_L)}{\mathcal{H}(\theta-p_L)+f+F} + \frac{(\theta-p_H)}{\theta\gamma} - 2\right] + F\left[\frac{\mathcal{H}(\theta-p_L)}{\mathcal{H}(\theta-p_L)+f+F} + \frac{(\theta-p_H)}{\theta\gamma} - 1\right] \leq 0$
$SE(r_L = 1; r_H = 1)$	1	$\frac{\mathcal{H}(\theta-p_H)}{\mathcal{H}(\theta-p_H)+f+F}(f+F) - f \leq 0$	$\frac{\mathcal{H}(\theta-p_L)}{\mathcal{H}(\theta-p_L)+f+F}(f+F) - f \leq 0$
$GE\,(r_G = 0)$ Case 1	0	always Nash	
$GE\,(r_G = 0)$ Case 2	1	$f\left[\frac{(\theta-p_L)}{\theta\gamma} - 1\right] + F\left[\frac{(\theta-p_L)}{2\theta\gamma} - 1\right] \leq 0$	
$GE\,(r_G = 1)$	1	$f\left[\frac{2(\theta-p_L)}{(\theta-p_L+\theta-p_H)} - 1\right] + F\left[\frac{(\theta-p_L)}{(\theta-p_L+\theta-p_H)} - 1\right] \leq 0$	
$HEL\,(r_S = 1; r_G = 0)$	0		$f - \frac{\theta\gamma}{(\theta-p_L)}(f+F) \leq 0$
$HEL\,(r_S = 1; r_G = 1)$	1		always Nash
$HEH\,(r_S = 0; r_G = 1)$	1	$f - \frac{(\theta-p_H)}{\theta\gamma}(f+F) \leq 0$	
Min. number in SE		$\mathcal{H}\theta\gamma \geq (f+F)$	
Min. number in GE		$\mathcal{H}\theta\gamma \geq \frac{(2f+F)}{2}$	
Min. number in HE		$\mathcal{H}\theta\gamma \geq 2(f+F)$	

Table 3
Expected profits of the reinsurance company

Equilibria	Expected profit (π_R)
$SE(r_L = 1; r_H = 1)$, $GE\,(r_G = 1)$, $HEL\,(r_S = 1; r_G = 1)$	0
$SE(r_L = 0; r_H = 0)$, $GE\,(r_G = 0)$	$\mathcal{H}\left[(\theta\,(1-\gamma) - p_L) + (\theta\,(1-\gamma) - p_H)\right] < 0$
$HEL\,(r_S = 1; r_G = 0)$	$\frac{N_G}{N_S+N_G}\mathcal{H}\,(\theta\,(1-\gamma) - p_L)$ $+\mathcal{H}\,(\theta\,(1-\gamma) - p_H) < 0$
$HEH\,(r_S = 0; r_G = 1)$	$\frac{N_S}{N_S+N_G}\mathcal{H}\,(\theta\,(1-\gamma) - p_H) < 0$
$SE(r_L = 1; r_H = 0)$	$\mathcal{H}\,(\theta\,(1-\gamma) - p_H) < 0$

Table 4
Comparative statics with respect to the premium rate

Equilibria	$\frac{\partial \pi_R}{\partial \theta}$
$SE(r_L = 1; r_H = 1)$, $GE\,(r_G = 1)$, $HEL\,(r_S = 1; r_G = 1)$	0
$SE(r_L = 0; r_H = 0)$, $GE\,(r_G = 0)$	$2\mathcal{H}\,(1-\gamma) > 0$
$HEL\,(r_S = 1; r_G = 0)$	$\frac{\partial\left(\frac{N_G}{N_S+N_G}\right)}{\partial\theta}\mathcal{H}\,(\theta\,(1-\gamma) - p_L)$ $+\frac{N_G}{N_S+N_G}\mathcal{H}\,(1-\gamma) + \mathcal{H}\,(1-\gamma) > 0$
$HEH\,(r_S = 0; r_G = 1)$	$\frac{\partial\left(\frac{N_S}{N_S+N_G}\right)}{\partial\theta}\mathcal{H}\,(\theta\,(1-\gamma) - p_H)$ $+\frac{N_S}{N_S+N_G}\mathcal{H}\,(1-\gamma) > 0$
$SE(r_L = 1; r_H = 0)$	$\mathcal{H}\,(1-\gamma) > 0$

Table 5
Evolution of the rate of reinsurance, profits of the system and premium income of the CCR

Year	Rate of reinsurance %	Accumulated system's profit billion FF	Premium income of the CCR billion FF
82/83	83	-1.4	1.6
84	75	0.5	1.9
85	75	3.2	2.2
86	73	5.9	2.2
87	52	8.2	1.8
88	41	10.1	1.5
89	51	13.3	1.5
90	43	14.0	1.5
91	40	17.4	1.5
92	38	19.7	1.5
93	41	20.8	1.7
94	41	24.4	1.9
95	45	26.5	2.1
96	39	29.0	2.0
97	40	33.5	2.1

Source: von Ungern (2004).

Table 6
Risk-averse insurers: Equilibrium candidates

Eqm. candidate	Type of insurer	Rate of retention	Number of insurers
SE		$r_j^* = \frac{\theta(1-\gamma)-p_i}{\lambda p_i(1-p_i)}$	$N_i = \frac{\mathcal{H}\omega_i}{f+F}$
GE		$r_G^* = \frac{\theta(1-\gamma)-p_L+\theta(1-\gamma)-p_H}{\lambda[p_L(1-p_L)+p_H(1-p_H)]}$	$N_G = \frac{\mathcal{H}\omega_G}{2f+F}$
HEL	Specialist	$r_S^* = \frac{\theta(1-\gamma)-p_L}{\lambda p_L(1-p_L)}$	n.a
HEL	Generalist	$r_G^* = \frac{N_G[\theta(1-\gamma)-p_L]+N_L^*(\theta(1-\gamma)-p_H)}{\lambda\left[N_G p_L(1-p_L)+N_L^* p_H(1-p_H)\right]}$	n.a

Note: The displayed expression for the number of insurers corresponds to an interior solution for the rate of retention.

Table 7
Risk-averse insurers: Equilibrium conditions

Equilibrium	Eqm. condition
SE	$f\left[\frac{\mathcal{H}\omega_{LD}}{\mathcal{H}\omega_L+f+F}+\frac{\omega_{HD}}{\omega_H}-2\right]+ F\left[\frac{\mathcal{H}\omega_{LD}}{\mathcal{H}\omega_L+f+F}+\frac{\omega_{HD}}{\omega_H}-1\right]<0$
GE	$\frac{\omega_L}{\omega_G}(2f+F)-f-F<0$

Table 8
Equilibrium candidates with risk pool

Equilibrium	Number of ins. in L-region	Number of generalists
$SE(r_L=0; r_H=0)$	$\frac{\mathcal{H}[\theta-p_L+\theta(1-\gamma)-p_H]}{(f+F)}$	
$SE(r_L=1; r_H=0)$	$\frac{\mathcal{H}[\theta-p_L+\theta(1-\gamma)-p_L]}{(f+F)}$	
$GE\,(r_G=0)$		$\frac{\mathcal{H}[\theta-p_L+\theta-p_H]}{(2f+F)}$

Table B-1
Comparative Statics of equilibrium candidates

Eqm. candidate	Type of firm	$\mathcal{H}$	θ	p_L	p_H	γ	f, (F)
$SE(r_L = 0; r_H = 0)$	N_L	$>$	$>$	0	0	$>$	$<$
	N_H	$>$	$>$	0	0	$>$	$<$
$SE(r_L = 1; r_H = 0)$	N_L	$>$	$>$	$<$	0	0	$<$
	N_H	$>$	$>$	0	0	$>$	$<$
$SE(r_L = 1; r_H = 1)$	N_L	$>$	$>$	$<$	0	0	$<$
	N_H	$>$	$>$	0	$<$	0	$<$
$GE\,(r_G = 0)$	N_G	$>$	$>$	0	0	$>$	$<$
$GE\,(r_G = 1)$	N_G	$>$	$>$	$<$	$<$	0	$<$
$HEL\,(r_S = 1; r_G = 0)$	N_G	$>$	$\lesseqgtr$	$>$	0	$>$	$<$
	N_S	$>$	$\lesseqgtr$	$<$	0	$<$	$\lesseqgtr$
$HEL\,(r_S = 1; r_G = 1)$	N_G	$>$	$>$	0	$<$	0	$<$ (0)
	N_S	$\lesseqgtr$	$<$	$<$	$>$	0	$\lesseqgtr$ $(<)$
$HEH\,(r_S = 0; r_G = 1)$	N_G	$>$	$\lesseqgtr$	$<$	$<$	$\lesseqgtr$	$<$
	N_S	$\lesseqgtr$	$\lesseqgtr$	$>$	$>$	$\lesseqgtr$	$\lesseqgtr$

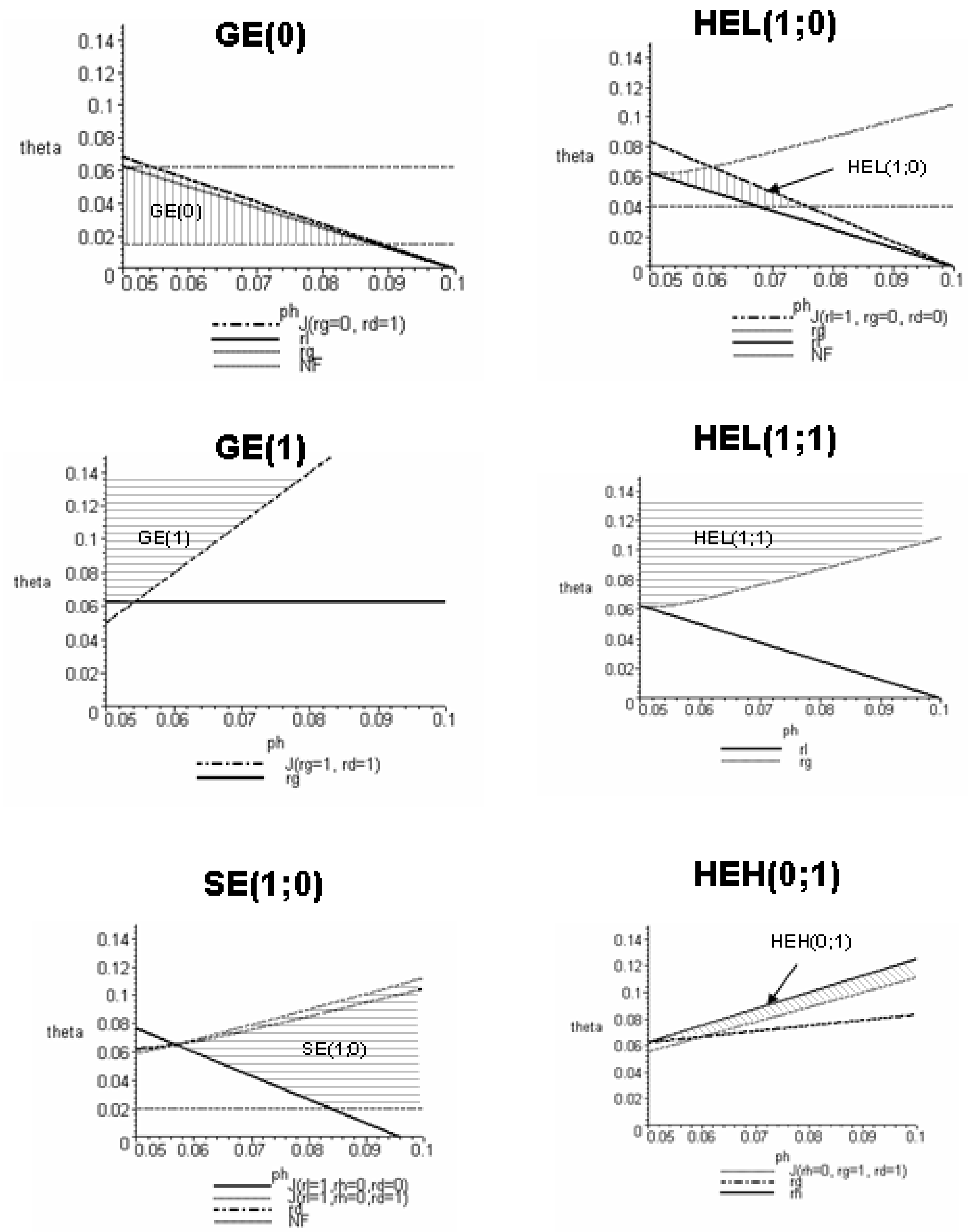
Table C-1
Equilibrium candidates with different size regions

Equilibrium	Number of ins. in L-region	Number of ins. in H-region	Number of generalists
$SE(r_L = 0; r_H = 0)$	$\frac{M_L \theta\gamma}{(f+F)}$	$\frac{M_H \theta\gamma}{(f+F)}$	
$SE(r_L = 1; r_H = 0)$	$\frac{M_L(\theta - p_L)}{(f+F)}$	$\frac{M_H \theta\gamma}{(f+F)}$	
$SE(r_L = 1; r_H = 1)$	$\frac{M_L(\theta - p_L)}{(f+F)}$	$\frac{M_H(\theta - p_L)}{(f+F)}$	
$GE\,(r_G = 0)$			$\frac{(M_L+M_H)\theta\gamma}{(2f+F)}$
$GE\,(r_G = 1)$			$\frac{M_L(\theta-p_L)+M_H(\theta-p_H)}{(2f+F)}$
$HEL\,(r_S = 0; r_G = 0)$	$\frac{M_L \theta\gamma}{(f+F)}$		$\frac{M_H \theta\gamma}{f}$
$HEL\,(r_S = 1; r_G = 0)$	$\frac{M_L(\theta - p_L)}{(f+F)}$		$\frac{M_H \theta\gamma}{f\left(2-\frac{\theta\gamma}{\theta-p_L}\right)+F\left(1-\frac{\theta\gamma}{\theta-p_L}\right)}$
$HEL\,(r_S = 1; r_G = 1)$	$\frac{M_L(\theta - p_L)}{(f+F)}$		$\frac{M_H(\theta-p_H)}{f}$
$HEH\,(r_S = 0; r_G = 1)$		$\frac{M_H \theta\gamma}{(f+F)}$	$\frac{M_L(\theta-p_L)}{f\left(2-\frac{\theta-p_H}{\theta\gamma}\right)+F\left(1-\frac{\theta-p_H}{\theta\gamma}\right)}$

Table C-2
Summary of Nash equilibrium and minimum number of insurer conditions with different size regions

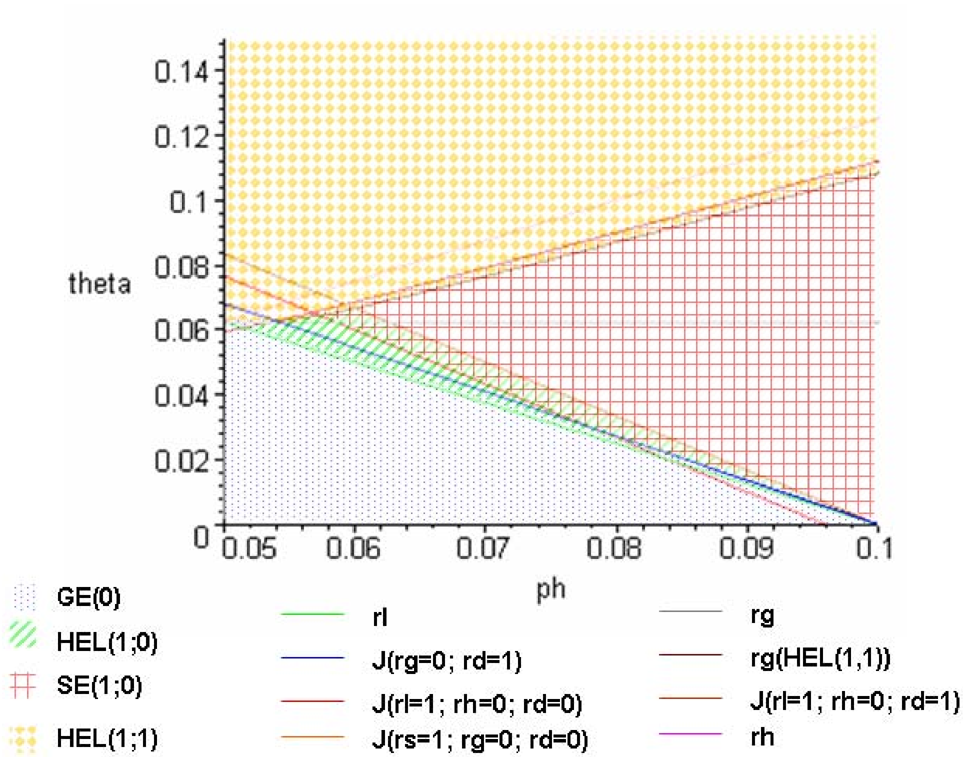
Equilibrium Candidates	r_D	Nash-equilibrium condition
$SE(r_L = 0; r_H = 0)$	0	$\frac{M_L\theta\gamma}{M_L\theta\gamma+f+F}(f+F) - f \leq 0$
$SE(r_L = 1; r_H = 0)$ Case 1	0	$\frac{M_L\theta\gamma}{M_L(\theta-p_L)+f+F}(f+F) - f \leq 0$
$SE(r_L = 1; r_H = 0)$ Case 2	1	$f\left[\frac{M_L(\theta-p_L)}{M_L(\theta-p_L)+f+F} + \frac{(\theta-p_H)}{\theta\gamma} - 2\right] + F\left[\frac{M_L(\theta-p_L)}{M_L(\theta-p_L)+f+F} + \frac{(\theta-p_H)}{\theta\gamma} - 1\right] \leq 0$
$SE(r_L = 1; r_H = 1)$	1	$\frac{M_L(\theta-p_L)}{M_L(\theta-p_L)+f+F}(f+F) - f \leq 0$
$GE\,(r_G = 0)$ Case 1	0	$f\left[\frac{2M_L}{M_L+M_H} - 1\right] + F\left[\frac{M_L}{M_L+M_H} - 1\right] < 0$
$GE\,(r_G = 0)$ Case 2	1	$f\left[\frac{2M_L}{M_L+M_H}\frac{(\theta-p_L)}{\theta\gamma} - 1\right] + F\left[\frac{M_L}{M_L+M_H}\frac{(\theta-p_L)}{2\theta\gamma} - 1\right] \leq 0$
$GE\,(r_G = 1)$	1	$f\left[\frac{2M_L(\theta-p_L)}{M_L(\theta-p_L)+M_H(\theta-p_H)} - 1\right] + F\left[\frac{M_L(\theta-p_L)}{M_L(\theta-p_L)+M_H(\theta-p_H)} - 1\right] \leq 0$
$HEL\,(r_S = 0; r_G = 0)$	0	always Nash
$HEL\,(r_S = 1; r_G = 0)$	0	$f - \frac{\theta\gamma}{\theta-p_L}(f+F) \leq 0$
$HEL\,(r_S = 1; r_G = 1)$	1	always Nash
$HEH\,(r_S = 0; r_G = 1)$	1	$f - \frac{(\theta-p_H)}{\theta\gamma}(f+F) \leq 0$

Figure 1
Areas of equilibrium



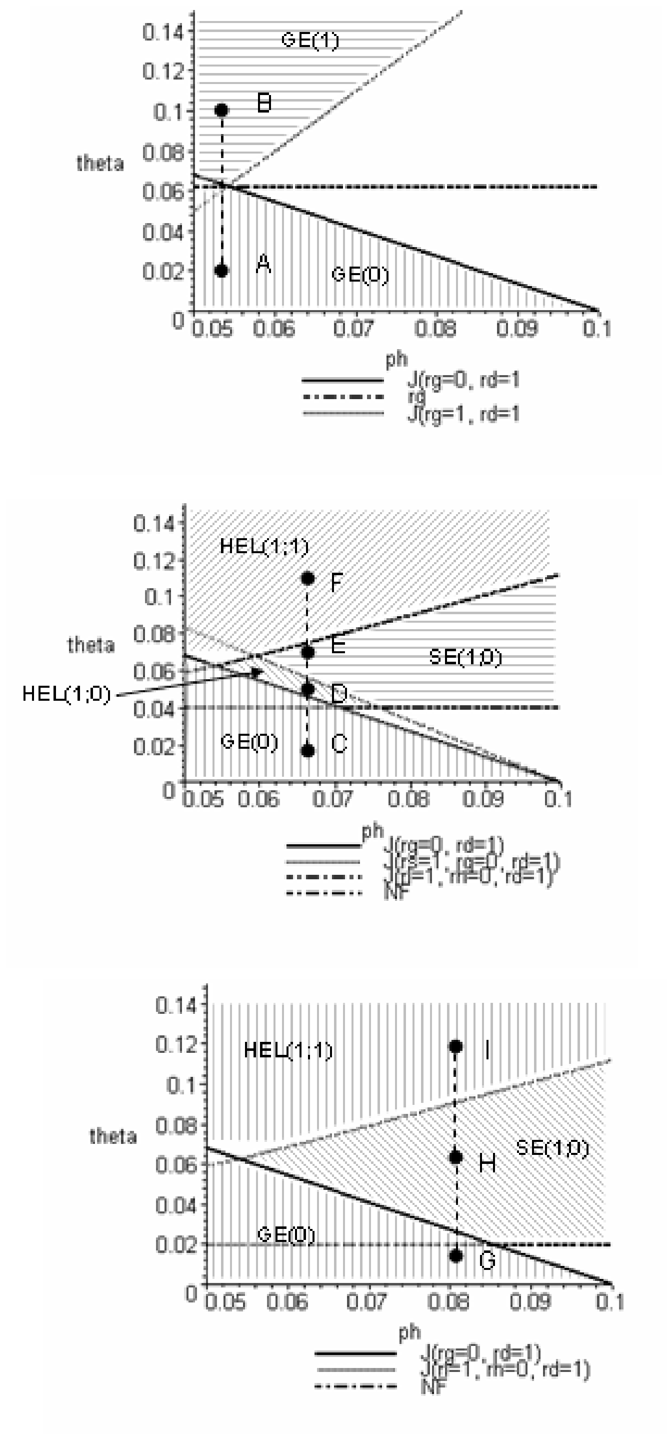
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$.

Figure 2
Joint areas of equilibrium



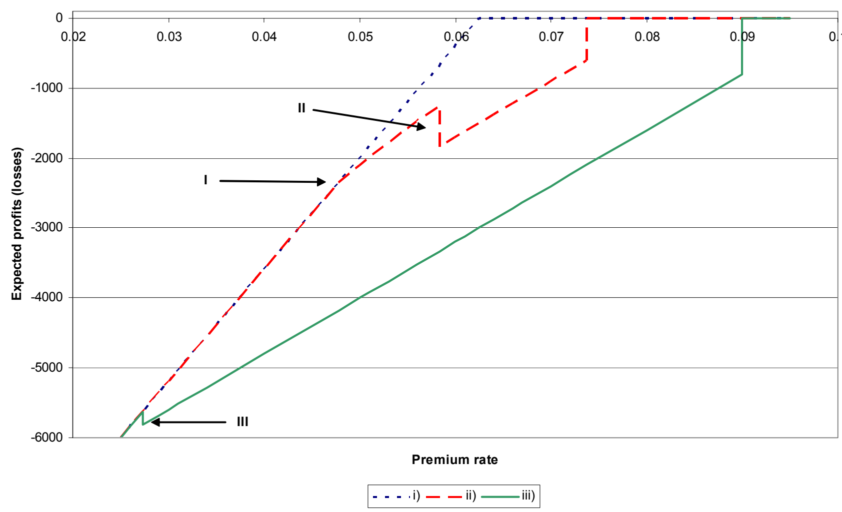
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$.

Figure 3
Equilibrium Sequences with increase in premium rate



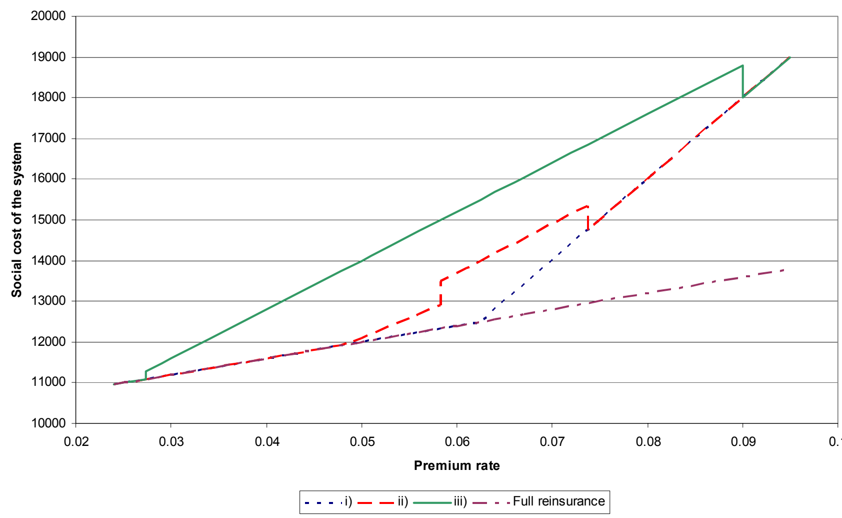
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$.

Figure 4
Evolution of expected profits of reinsurer



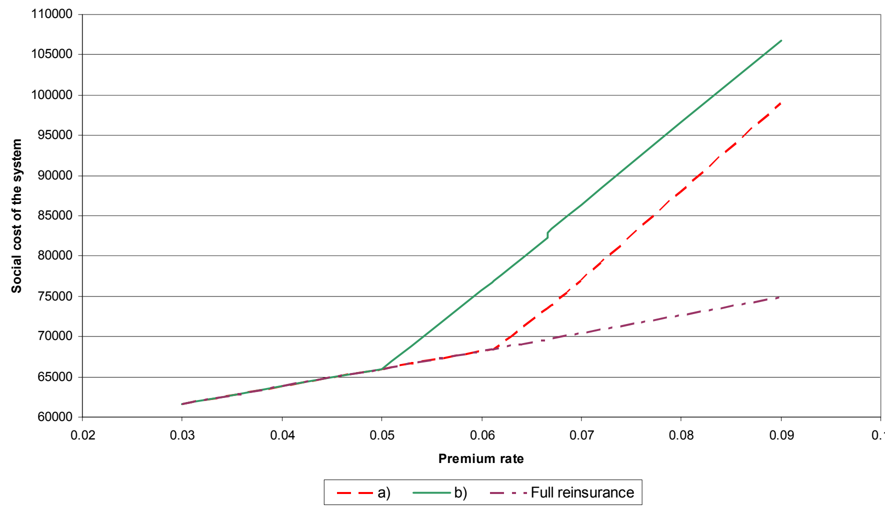
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$. $p_H = 0.0525$ for sequence i), $p_H = 0.065$ for sequence ii) and $p_H = 0.08$ for sequence iii).

Figure 5
Social cost of the system



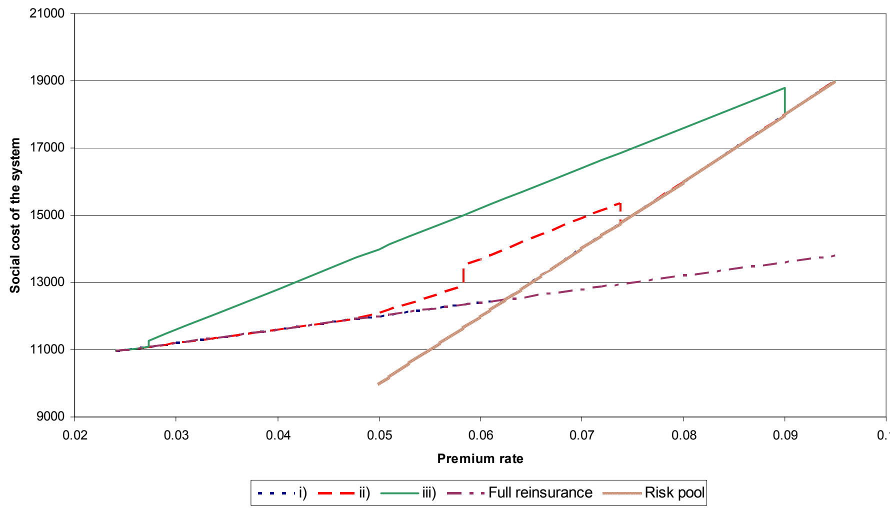
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$. $p_H = 0.0525$ for sequence i), $p_H = 0.065$ for sequence ii) and $p_H = 0.08$ for sequence iii).

Figure 6
Social cost of the system - different size regions



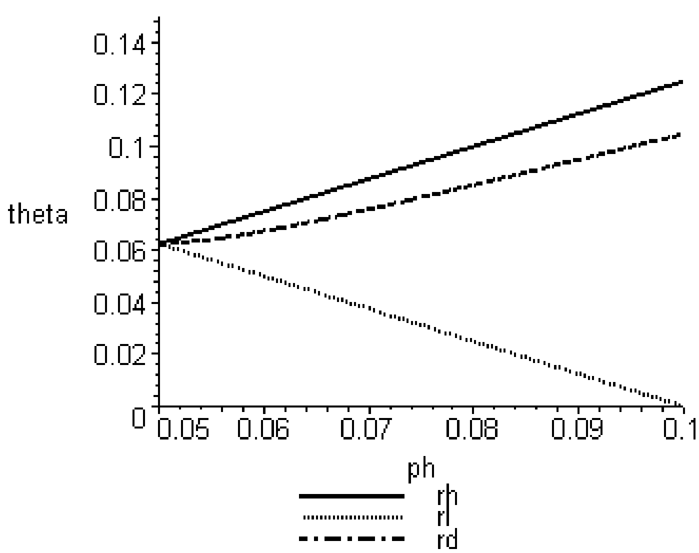
Parameter values: $M_L = 1,000,000$; $M_H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$.
$p_H = 0.06$ for sequence a), $p_H = 0.15$ for sequence b).

Figure 7
Social cost - risk pool



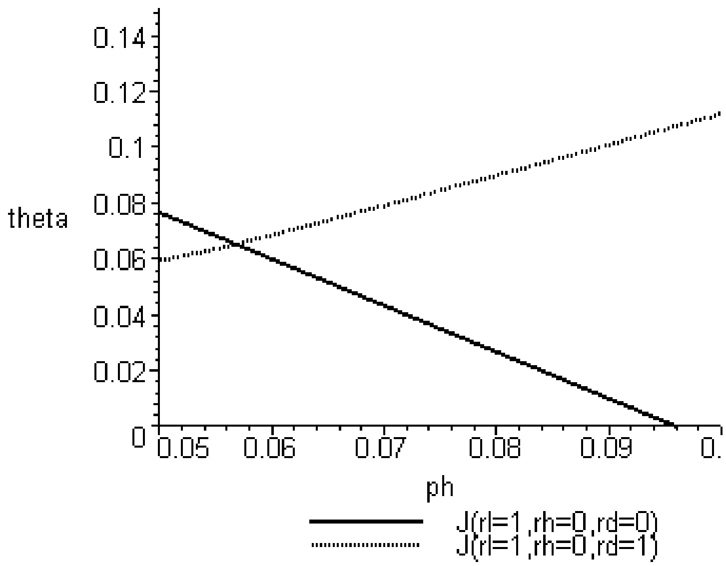
Parameter values same as in Figure 5.

Figure B-1
Switch-lines for $SE(r_L = 1; r_H = 0)$ characterization



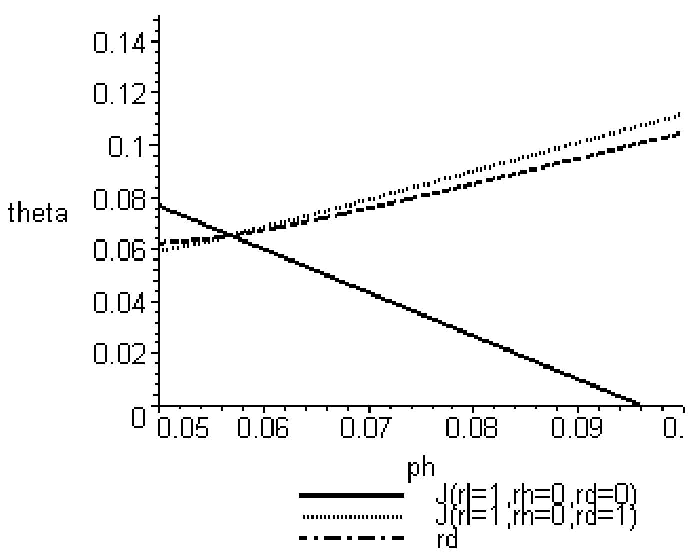
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$.

Figure B-2
Jump-lines for the $SE(r_L = 1; r_H = 0)$



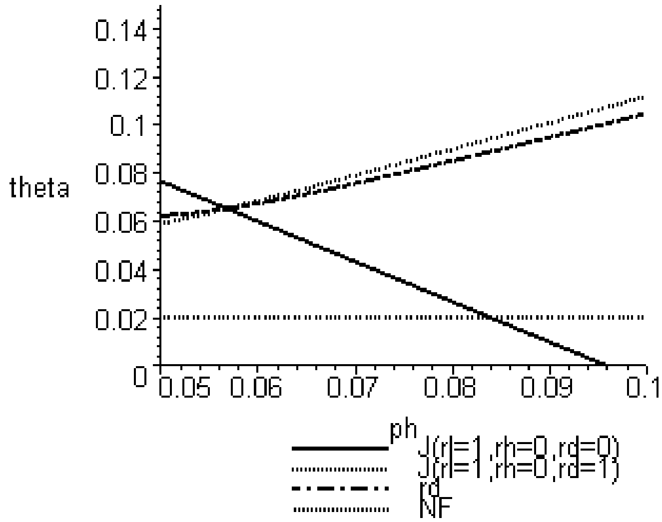
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$.

Figure B-3
Equilibrium Area for the $SE(r_L = 1; r_H = 0)$



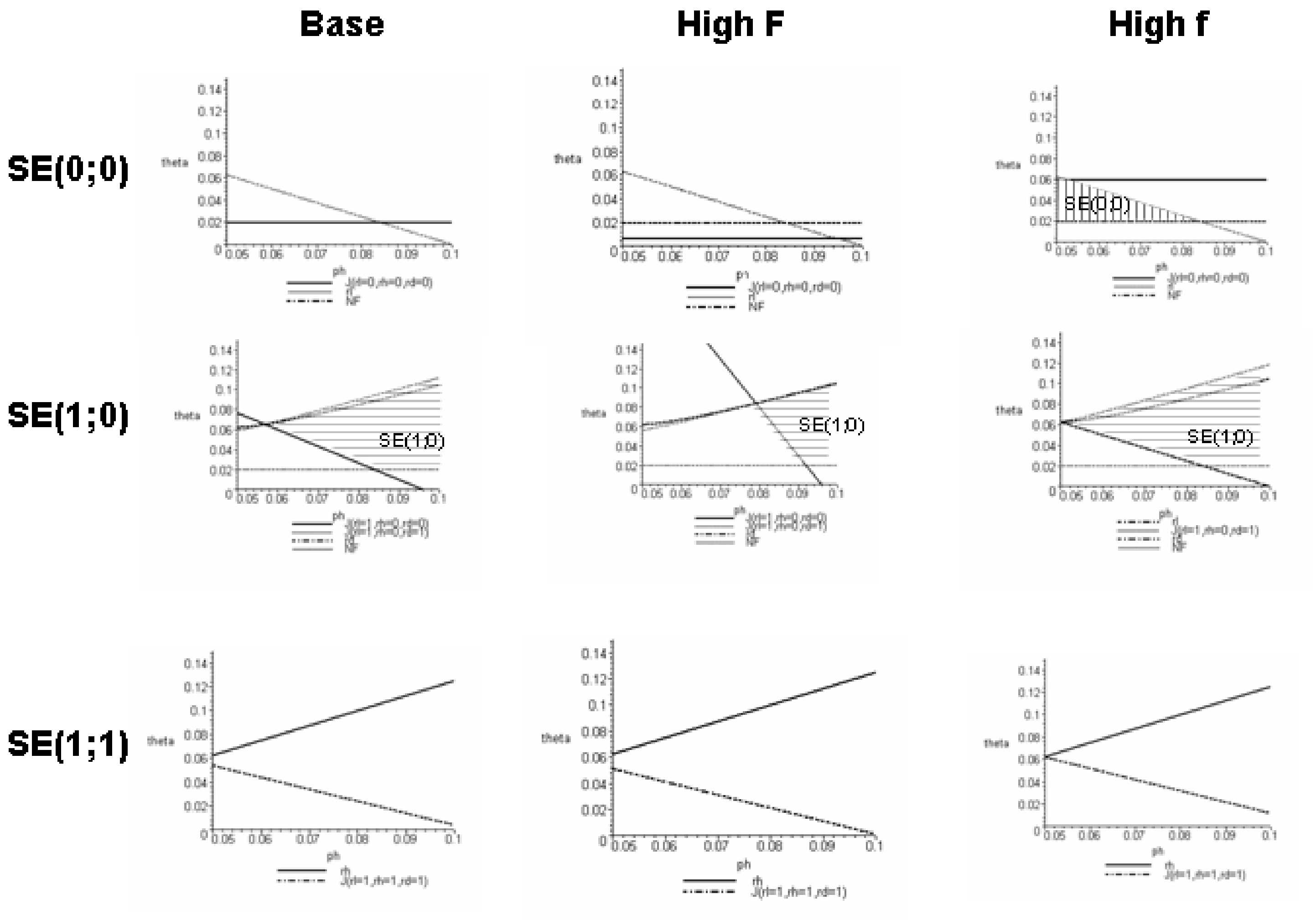
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$.

Figure B-4
Equilibrium Area for the $SE(r_L = 1; r_H = 0)$
Including minimum number of insurers



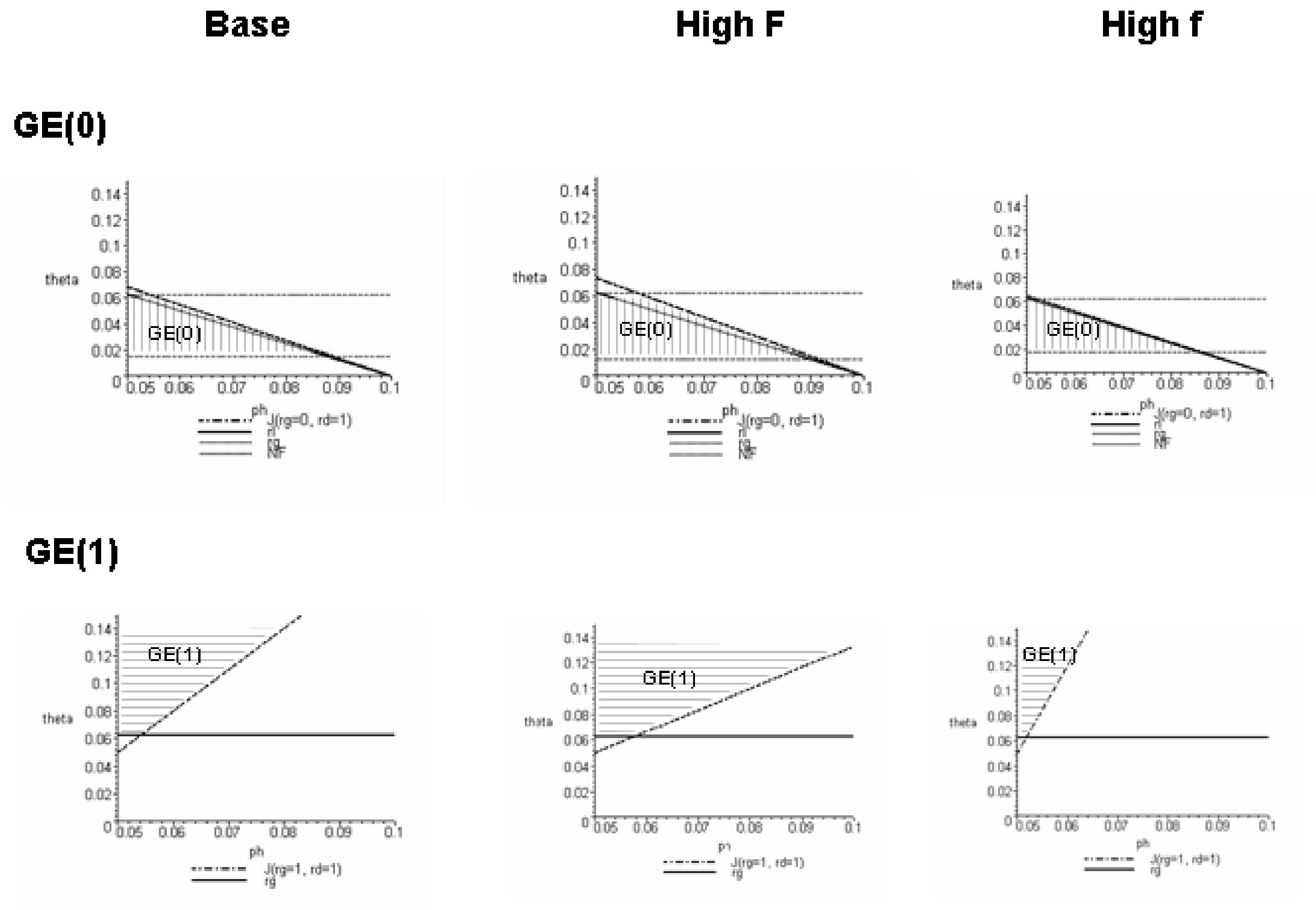
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$.

Figure B-5
Specialist equilibrium



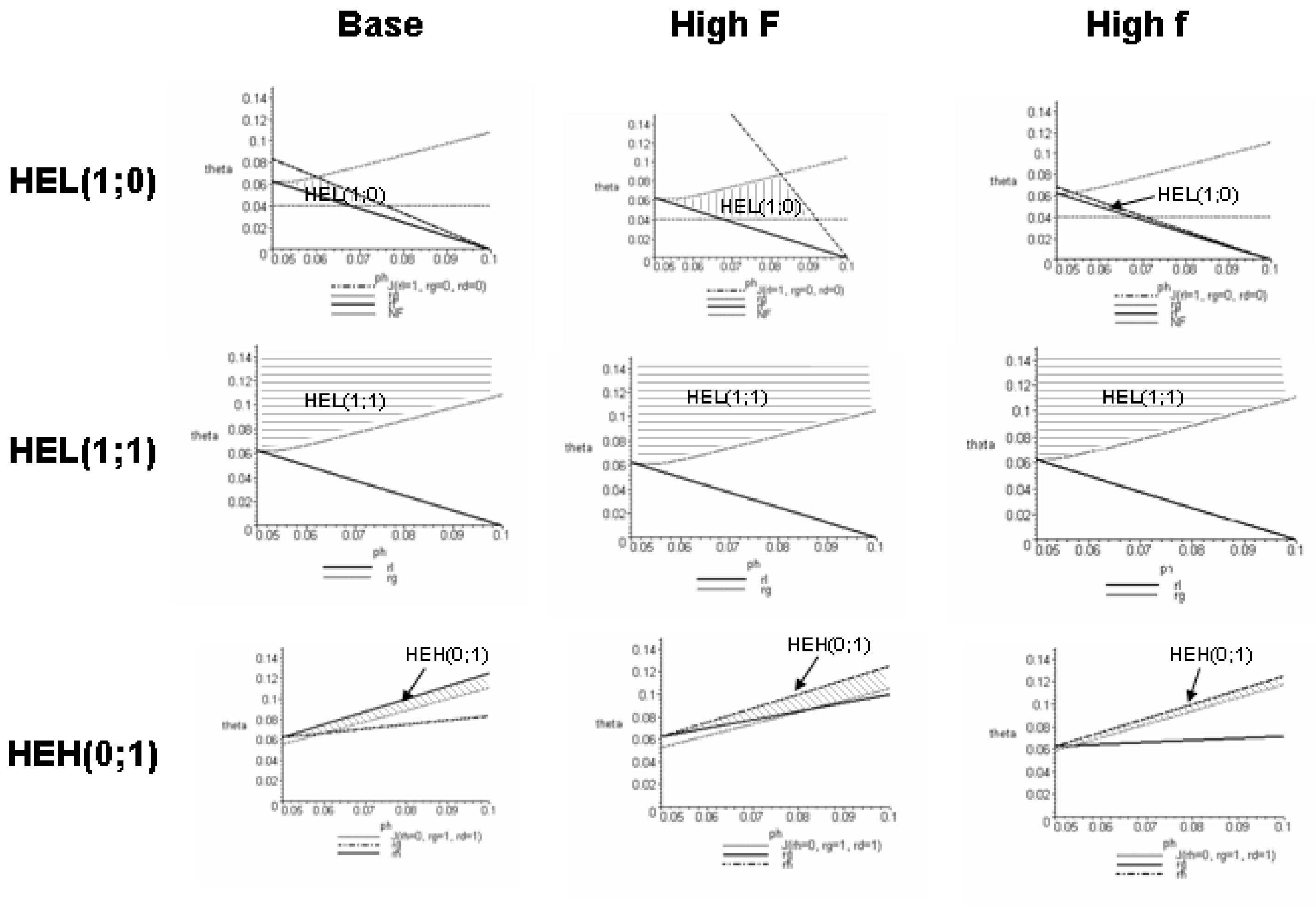
Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$ for base scenario.
$f = 100$; $F = 300$ for "High F" and $f = 300$; $F = 100$ for "High f".

Figure B-6
Generalist equilibrium



Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$ for base scenario.
$f = 100$; $F = 300$ for "High F" and $f = 300$; $F = 100$ for "High f".

Figure B-7
Hybrid equilibrium



Parameter values: $H = 100,000$; $\delta = 0.2$; $f = 200$; $F = 200$; $\overline{p} = 0.05$ for base scenario.
$f = 100$; $F = 300$ for "High F" and $f = 300$; $F = 100$ for "High f".