Tacit Collusion under Fairness and Reciprocity*

Doruk Iris and Luís Santos-Pinto†

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Abstract

This paper explores the implications of fairness and reciprocity in dynamic market games. A reciprocal player responds to kind behavior of rivals with unkind actions (destructive reciprocity), while at the same time, it responds to kind behavior of rivals with kind actions (constructive reciprocity). The paper shows that for general perceptions of fairness, reciprocity facilitates collusion in dynamic market games. The paper also shows that this is a robust result. It holds when players' choices are strategic complements and strategic substitutes. It also holds under grim trigger punishments and optimal punishments.

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†Doruk Iris is a graduate student at Universidade Nova de Lisboa. Luís Santos-Pinto is a professor at University of Lausanne. Corresponding author: Luís Santos-Pinto, University of Lausanne, Faculty of Business and Economics, Extranef 211, CH-1015, Lausanne, Switzerland. Ph: 41-216923658. Fax: 41-. E-mail address: LuisPedro.SantosPinto@unil.ch.

1 Introduction

The assumption that individuals behave as if maximizing their material payoffs, despite its central role in economic analysis, is at odds with a large body of evidence from psychology and from experimental economics. Economic agents often pursue objectives other than actual payoff maximization. Many observed departures from material payoff maximizing behavior arise through actions that favor fairness or reciprocity.

Rabin (1993) argues that the parties of a transaction care about fairness in the sense that they "like to help those who are helping them, and hurt those who are hurting them" (pp. 1281). Fairness and reciprocity have been shown to explain behavior in bargaining games and in trust games. For example, in ultimatum games offers are usually much more generous than predicted by equilibrium and low offers are often rejected. These offers are consistent with an equilibrium in which players make offers knowing that other players may reject allocations that appear unfair.[1]

Motivated by this evidence, we address the research question: "can fairness and reciprocity facilitate cooperation?" Since this is a very broad question we focus on infinitely repeated market games. This important class of games tells us how firms can sustain collusive outcomes when they interact repeatedly. This literature also tell us what are the factors that help or hinder collusion. For example, it is now well know that concentration, barriers to entry, cross-ownership, symmetry and multi-market contracts facilitate collusion.

The main finding of our paper is that fairness and reciprocity is yet another factor that might facilitate collusion in infinitely repeated market games. This result is consistent with findings in Rabin (1993) and (1997) which show that in a fairness equilibrium it is possible to sustain cooperation in the one shot Prisoners' Dilemma and in every period of the finitely repeated Prisoners' Dilemma.

To model fairness and reciprocity we follow Segal and Sobel (2007). We assume that a player's utility is additively separable in monetary and fairness payoffs. Monetary payoffs are revenues minus costs and fairness payoffs are a weighted average of the rivals' monetary payoffs where the weights depend on how the rivals' actions are expected to differ from the fair ones. If a player expects a rival to play a fair action then he places zero weight on that rivals' monetary payoff. However, if a player expects a rival to play a kind (mean) action, then he places a positive (negative) weight on that rival's monetary payoff. We also assume that monetary payoffs are large by comparison with fairness payoffs.[2]

The paper starts by studying the impact of fairness and reciprocity on incen-

[1] Sobel (2005) argues that models of interdependent preferences such as reciprocity can provide clearer and more intuitive explanations of interesting economic phenomena.

[2] In Rabin (1993) and (1997) utility is additively separable in monetary and fairness payoffs but the weight a player places on rivals' monetary payoffs depends on his perception of the rivals' intentions, which are evaluated using (i) beliefs about the rivals' strategy choices, and (ii) beliefs about the rivals' beliefs about his strategy.

tives for collusion in dynamic price-setting games where players use grim trigger punishments and where prices are strategic complements. A natural benchmark in a collusive framework is to assume that reciprocal players believe that a rival is fair when the rival charges exactly its agreed upon collusive price. Our first result shows that if this is the case, then it is easier to sustain collusion in a dynamic price game with reciprocal players than with self-interested ones.

The intuition behind this result is straightforward. If reciprocal players think that the fair price of each of their rivals is the rival's collusive price, then the prices set under grim trigger punishments will be lower than those that self-interested players would set. This happens because when players are reciprocators and expect their rivals to set unkind prices in the punishment phase, they have an incentive to lower their own price since they derive pleasure from hurting the rivals for their nasty behavior. If the grim trigger punishments prices of reciprocators are lower than those of self-interested players, then Nash punishments are harsher with reciprocal players than with self-interested ones. Additionally, since the collusive prices are considered to be fair, there is no impact of fairness and reciprocity on the collusive payoffs nor on the single period deviation payoffs. Thus, the critical discount factor needed to sustain a collusive outcome must be lower with reciprocal players than with self-interested ones.

What if players have more general perceptions of what the fair prices of their rivals should be? Our second result shows that if players think that fair prices of the rivals are between the largest Nash prices of the stage game with self-interested players and the collusive prices, and marginal costs are constant, then it is easier to sustain collusion when players are reciprocal than when they are self-interested.

The intuition for this result is as follows. If reciprocal players think that the fair prices of the rivals are greater than the largest Nash prices of the stage game with self-interested players, then Nash punishments are harsher with reciprocal players than with self-interested ones. This is the same effect that drives our first result. Additionally, if reciprocal players think that the fair prices are smaller than the collusive prices, then players have an incentive to increase their own collusive prices since they expect their rivals to set kind prices in the collusive phase and they derive pleasure from rewarding the rivals for this kind behavior. Clearly, these two effects make collusion *more* attractive to reciprocal players than to self-interested ones. However, the unilateral single period deviation payoff is higher with reciprocal players than with self-interested ones. This happens because the unilateral single period deviation payoff of a reciprocal player also includes the benefit that player derives from being treated kindly by the rivals (the rivals are playing their collusive prices). This effect of fairness and reciprocity makes collusion *less* attractive to reciprocal players than to self-interested ones. The assumption that monetary payoffs are large by comparison with fairness payoffs implies that the increase in collusive payoff is of first-order whereas the increase in the unilateral single period deviation payoff is of second-order.

Next we argue that our main finding is robust. To do that we show that fairness and reciprocity also facilitate collusion in dynamic quantity-setting games where players use grim trigger punishments and where quantities are strategic substitutes. Thus, fairness and reciprocity can facilitate collusion not only when players actions are strategic complements but also when they are strategic substitutes.

We also show that fairness and reciprocity can facilitate collusion in infinitely repeated market games where players behave according to Abreu's (1988) theory of optimal punishments. To do that we compare the benefit of deviating today (the unilateral single period deviation payoff minus the collusive payoff) with the benefit of not deviating in the long run (the collusive payoff minus the payoff of entering a punishment stage).

First, the benefit of deviating today when players use optimal punishments is the same as when they use grim trigger punishments. As we have seen, the assumption that monetary payoffs are large by comparison with fairness payoffs implies that the increase in the collusive payoff due to fairness considerations is of first-order whereas the increase in the unilateral single period deviation payoff is of second-order. Thus, the benefit of deviating today is smaller for reciprocators than for self-interested players no matter if players use optimal punishments or grim trigger punishments.

Second, if reciprocal players think that the fair prices of the rivals are smaller than the collusive prices, then the prices set on the initial path are perceived as kind behavior and lead to positive fairness payoffs. Therefore, when the prices of the initial path are set, the payoffs for reciprocal players are higher than those for self-interested players . Third, it is well known that punishments during the punishment phase are more severe when players use optimal punishments than when they use Nash reversion strategies. If reciprocal players think that the fair prices of the rivals are greater than the largest Nash prices of the stage game with self-interested players, then seeing the rivals setting punishment prices lower than Nash prices will be perceived as nastier behavior than seeing the rivals setting Nash prices. Therefore, reciprocal players will set lower prices than self-interested players during the punishment phase under optimal punishments. The second and the third effects imply that the benefit from not deviating in the long run is larger for reciprocal players than for self-interested players no matter if players use optimal punishments or grim trigger punishments.

Our paper is related to papers that show that fairness and reciprocity can change market outcomes in imperfectly competitive settings. Rabin (1993) introduces fairness considerations into game theory using psychological games and shows, among many other things, that a monopolist ought to set price lower than "the monopoly price" if consumers have concerns about fairness. Rotemberg (2006) develops a model where consumers expect firms to be somewhat altruistic towards them and they react with anger if firms prove to be insufficiently altruistic. He shows that many of the implications of his model are consistent with several pricing practices of firms and consumer behavior in the lab and in actual markets.

Our paper is also related to Malueg (1992). This paper studies the impact of cross ownerships between firms on incentives for collusion. In this paper a firm's payoff is a weighted average of its own profit and the rivals' profits. The greater the share of the rivals' profits in the payoff of a firm, the greater the level of cross ownership. Malueg shows that increasing the level of cross ownership may decrease incentives for collusion for some demand functions, especially when the level of cross ownership is high. Finally, our paper is related to literature in industrial organization that analyzes how firms will choose prices and product characteristics when managers have behavioral biases. An example is Al-Najjar et al. (2006) work on the pricing decision of firms whose managers confound fixed, sunk and variable costs.

The rest of the paper proceeds as follows. Section 2 sets-up the model. Section 3 discusses the impact that fairness and reciprocity have on incentives for collusion when players use grim trigger strategies. Section 4 studies the impact of fairness and reciprocity on collusion when choice variables are strategic complements. Section 5 considers the case when choice variable are strategic substitutes. Section 6 shows that the results also extend to optimal punishments. Section 7 discusses the possibility that fairness and reciprocity also facilitate collusion in other models used to study to collusive behavior. Section 8 concludes the paper. The Appendix contains the proofs of all results.

2 Set-up

Segal and Sobel (2007) provide an axiomatic foundation for interdependent preferences that can reflect reciprocity, inequity aversion, altruism as well as spitefulness. They assume that in addition to conventional preferences over outcomes, players in a strategic environment also have preferences over strategy profiles. We use their approach to study the impact of fairness and reciprocity on collusion in dynamic market games.[3]

Consider a dynamic game, where players $i = 1, 2, \ldots, n$, play the same stage game over an infinite horizon $t = 0, 1, 2, \ldots$. In each period player i chooses an action $x_i \in X_i$, and his payoff in that period is given by

$$u_i(x_i, x_{-i}, x^f_{-i}) = \pi_i(x_i, x_{-i}) + \alpha \sum_{j \neq i} w_{ij}(x_j, x^f_{ij})\pi_j(x_i, x_{-i}), \qquad (1)$$

where $\pi_i(x_i, x_{-i})$ represents player i's monetary payoff, a function of player i's action and the actions of the rivals, x_{-i}. The term $\alpha \sum_{j \neq i} w_{ij}(x_j, x^f_{ij})\pi_j(x_i, x_{-i})$ represents player i's fairness payoff. The function $w_{ij}(x_j, x^f_{ij})$ is the weight that player i places on player j's monetary payoff, $\pi_j(x_i, x_{-i})$, which depends on player j's action, x_j, and player i's perception of the fair action of player j, x^f_{ij}. The parameter $\alpha > 0$ is a normalization.

[3] Preferences for fairness and reciprocity were first modeled in the economics literature by Rabin (1993) in the context of static games using the theory of psychological game by Geanakoplos et al. (1989). In Rabin's model the weight a player places on a rival's monetary payoffs depends on the interpretation of that rival's intentions which are evaluated using beliefs (and beliefs about beliefs) over strategy choices.

The central behavioral feature of these preferences is the assumption that players care about the intentions of the rivals. If player i expects player j to treat him kindly, then $w_{ij}(\cdot)$ will be positive, and player i will wish to treat player j kindly. If player i expects player j to treat him badly, then $w_{ij}(\cdot)$ will be negative, and player i will wish to treat player j badly. If player i expects player j to play the fair action, then $w_{ij}(\cdot)$ will be zero, and there is issue of reciprocity.

We assume throughout that players' preferences as well as their exogenous perceptions of the fair actions of the rivals, $x_{-i}^f = (x_{i1}^f, ..., x_{ii-1}^f, x_{ii+1}^f, ..., x_{in}^f)$ for $i = 1, ..., n$, are common knowledge. Players discount the future at rate $\delta \in (0,1)$. The repeated game payoff of player i of choosing action x_i when rivals play x_{-i} is given by

$$U_i = \sum_{t=0}^{\infty} u_i(x_i, x_{-i}, x_{-i}^f)\delta^t.$$

Denote the dynamic game with reciprocal players by $\Gamma_\infty^r(u, x)$, where $u = (u_1, ..., u_n)$ and $x = (x_1, ..., x_n)$ and the dynamic game with self-interested players by $\Gamma_\infty^s(\pi, x)$, where $\pi = (\pi_1, ..., \pi_n)$.

Players are able to sustain a collusive outcome when the payoff from collusion is no less than the payoff from deviation. To understand how fairness and reciprocity influence collusion we will compare the incentive compatibility condition of self-interested players in $\Gamma_\infty^s(\pi, x)$ to that of reciprocal players in $\Gamma_\infty^r(u, x)$ assuming that these two games are identical in all respects with the exception of players' social preferences.

To perform this analysis we focus on infinitely repeated market games. More specifically, we consider the cases where players' actions are strategic complements (e.g., price competition with products that are imperfect substitutes) and strategic substitutes (e.g., quantity competition with products that are perfect substitutes). We also consider two alternative modes of punishments after deviations: grim trigger and optimal punishments.

3 Grim Trigger Punishments

When players use grim trigger strategies any deviation from collusion leads to a switch to a Nash equilibrium of the stage game forever after. Thus, when self-interested player use grim trigger punishments in $\Gamma_\infty^s(\pi, x)$, each player i will prefer to play his collusive action x_i^c if the payoff from collusion, $\pi_i(x_i^c, x_{-i}^c)/(1-\delta)$, is no less than the payoff from defection which consists of the one period gain from deviating $\pi_i(BR_i^s(x_{-i}^c), x_{-i}^c)$ plus the discounted payoff of inducing Nash reversion forever $\delta\pi_i(x_i^{ns}, x_{-i}^{ns})/(1-\delta)$, that is,

$$\pi_i(BR_i^s(x_{-i}^c), x_{-i}^c) + \frac{\delta}{1-\delta}\pi_i(x^{ns}) \leq \frac{1}{1-\delta}\pi_i(x^c).$$

Solving for δ we obtain

$$\delta^s_{x^c} = \frac{\pi_i(BR^s_i(x^c_{-i}), x^c_{-i}) - \pi_i(x^c)}{\pi_i(BR^s_i(x^c_{-i}), x^c_{-i}) - \pi_i(x^{ns})} \leq \delta. \quad (2)$$

The collusion action profile x^c can be sustained by self-interested players who are patient enough such that $\delta^s_{x^c} \leq \delta$ where $\delta^s_{x^c}$ is the critical discount factor above which x^c can be sustained by self-interested players.

The same reasoning applies when players are reciprocal. Thus, for a reciprocal player i to play the collusive action x^c_i in $\Gamma^r_\infty(u, x)$ using a grim trigger strategy the following condition must be satisfied

$$u_i(BR^r_i(x^c_{-i}), x^c_{-i}, x^f_{-i}) + \frac{\delta}{1-\delta} u_i(x^{nr}, x^f_{-i}) \leq \frac{1}{1-\delta} u_i(x^c, x^f_{-i}).$$

Solving for δ we obtain

$$\delta^r_{x^c} = \frac{u_i(BR^r_i(x^c_{-i}), x^c_{-i}, x^f_{-i}) - u_i(x^c, x^f_{-i})}{u_i(BR^r_i(x^c_{-i}), x^c_{-i}, x^f_{-i}) - u_i(x^{nr}, x^f_{-i})} \leq \delta. \quad (3)$$

When players are reciprocal it follows that the collusive action profile x^c can be sustained if players are patient enough such that $\delta^r_{x^c} \leq \delta$ where $\delta^r_{x^c}$ is the critical discount factor above which x^c can be sustained by reciprocal players.

We will use (2) and (3) to characterize the impact that fairness and reciprocity have on collusion when players use grim trigger strategies. To perform this analysis we compare the critical discount factor above which the collusive action profile can be sustained when players are self-interested to the critical discount factor when players are reciprocal. We assume that the two games are identical in all respects (demand, costs, and number of players) with the exception of players' preferences. We say that fairness and reciprocity facilitate collusion when the collusive action profile can be sustained at a lower critical discount factor when players are reciprocal than when they are self-interested. If the opposite happens we say that fairness and reciprocity make collusion harder.

3.1 Strategic Complements

We now specialize the model by assuming that choice variables are strategic complements. This assumption means that a player's incremental returns from increasing his own action are increasing in the rivals' actions. The canonical market game where players' actions are strategic complements is price competition with products that are imperfect substitutes. We use this game to study the impact of fairness and reciprocity on collusion when players' choices are strategic complements.

Assume that in each period player i chooses price, p_i, and his payoff in that period is given by

$$u_i(p_i, p_{-i}, p^f_{-i}) = \pi_i(p_i, p_{-i}) + \alpha \sum_{j \neq i} w_{ij}(p_{ij}, p^f_{ij}) \pi_j(p_i, p_{-i}). \quad (4)$$

As usual player i's monetary payoff, $\pi_i(p_i, p_{-i})$, is given by the difference between revenue and cost, that is,

$$\begin{aligned}\pi_i(p_i, p_{-i}) &= R_i\,(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i})) \\ &= p_i D_i(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i})),\end{aligned}$$

where $R_i(p_i, p_{-i})$ is revenue, $C_i(D_i(\cdot))$ is the cost of production, and $D_i(p_i, p_{-i})$ is the demand faced by player i. We assume that $D_i(\cdot)$ is decreasing with p_i, increasing with p_{-i}, and $C_i(\cdot)$ is increasing with $D_i(\cdot)$. Furthermore, we assume that

$$w_{ij}(p_j, p_{ij}^f)\begin{cases} > 0 \text{ if } p_j > p_{ij}^f \\ = 0 \text{ if } p_j = p_{ij}^f \\ < 0 \text{ otherwise} \end{cases}. \tag{5}$$

The assumptions on $w_{ij}(\cdot)$ capture the fact that a reciprocal player cares about the intentions of the rivals. The first condition expresses constructive or positive reciprocity. If player i expects player j's price to be greater than the fair price, p_{ij}^f, then player i places a positive weight on j's monetary payoffs. In this case player i is willing to sacrifice some of its monetary payoff to increase player j's monetary payoff. The second condition says that if player i expects player j to choose the fair price, then player i places no weight on j's monetary payoffs. The third condition expresses destructive or negative reciprocity. If player i expects player j to set a price lower than the fair price, then player places a negative weight on j's monetary payoffs. In this case player i is willing to sacrifice some of its monetary payoff to reduce player j's monetary payoff.

Let

$$A_i(p_i, p_{-i}, p_{-i}^f) = \underset{p_i \in P_i}{\arg\max}\ \pi_i(p_i, p_{-i}) + \alpha \sum_{j \neq i} w_{ij}(p_j, p_{ij}^f)\pi_j(p_i, p_{-i}),$$

denote the set of maximizers of player i's stage game problem as a function of p_i, p_{-i} and p_{-i}^f. For finite quantities, the players will never choose an infinite price. Hence, the players' price choice set is compact set in $\mathcal{R}$. We assume that u_i is order upper semi-continuous in p_i. The choice set being compact with this assumption guarantees that the set of maximizers $A_i(p_i, p_{-i}, p_{-i}^f)$ is nonempty.

We also assume that u_i has increasing differences in (p_i, p_{-i}), that is, for any fixed p_{-i}^f, $u_i(p_i, p'_{-i}, p_{-i}^f) - u_i(p_i, p''_{-i}, p_{-i}^f)$ is increasing in p_i for all $p'_{-i} \geq p''_{-i}$. This assumption guarantees that prices are strategic complements.[4] Together,

[4] If the payoff function is differentiable, then u_i having increasing differences in (p_i, p_{-i}), is equivalent to the assumption that the cross partial derivatives of u_i with respect to p_i and p_j for any player j, is non-negative, that is,

$$\frac{\partial^2 u_i}{\partial^2 p_i p_j} = \underbrace{\frac{\partial^2 \pi_i}{\partial^2 p_i p_j}}_{\geq 0} + \alpha w_{ij}(p_j, p_{ij}^f)\underbrace{\frac{\partial^2 \pi_j}{\partial^2 p_i p_j}}_{\geq 0} + \alpha \frac{\partial w_{ij}(p_j, p_{ij}^f)}{\partial p_j}\frac{\partial \pi_j}{\partial p_i} \geq 0.$$

Note that if a player cares only about monetary payoffs and if the payoff function is differentiable, then the increasing differences assumption boils down to $\frac{\partial^2 \pi_i}{\partial^2 p_i p_j} > 0$. In the game these assumptions imply that $\Gamma^r(u,p)$ is a supermodular game. This result is stated formally in Lemma 1.

Lemma 1: *If (i) P_i is a compact interval in $\mathcal{R}$, (ii) $u_i(p_i, p_{-i}, p^f_{-i})$ is order upper semi-continuous in p_i for fixed p_{-i} and order continuous in p_{-i} for a fixed p_i, and $u_i(p_i, p_{-i}, p^f_{-i})$ has a finite upper bound, (iii) u_i is supermodular in p_i for fixed p_{-i}, and (iv) $u_i(p_i, p_{-i}, p^f_{-i})$ has increasing differences in (p_i, p_{-i}), then $\Gamma^r(u,p)$ is a supermodular game.*

By Milgrom and Roberts (1990) we know that if $\Gamma^r(u,p)$ is a supermodular game, then there exist largest and smallest serially undominated strategies for each player, $\overline{p}_i$ and $\underline{p_i}$. Moreover, the strategy profiles $\underline{p}$ and $\overline{p}$ are pure-strategy Nash equilibrium profiles. Thus, the existence of a Nash equilibrium of the stage game is guaranteed. Our next result shows how players' perceptions of the fair prices of the rivals influence the extremal equilibrium prices of this game**.**

Proposition 1: *If $\Gamma^r(u,p)$ is a supermodular game and u_i has decreasing differences in (p_i, p^f_{-i}), then the smallest and the largest pure-strategy Nash equilibria of $\Gamma^r(u,p)$, i.e., $\underline{p}^{nr}$ and $\overline{p}^{nr}$, are nonincreasing functions of $p^f = (p^f_{-1}, ..., p^f_{-n})$.*

Proposition 1 is a comparative statics result that characterizes the impact that players' perceptions of the fair prices of their rivals have on the Nash equilibrium prices of the stage game. This result says that the higher are players' perceptions of what the fair prices of the rivals should be, the lower will the equilibrium prices be. The critical condition that drives this result is the assumption that the payoff function u_i has decreasing differences in (p_i, p^f_{-i}).[5] This assumption says that the marginal returns from increasing prices are decreasing with a player's perception of the fair prices of the rivals. This implies that an increase in p^f_{-i} shifts the best reply of a reciprocal player i towards origin. In other words, the higher player i perceives the fair price for the other players to be, the more it would like to set a smaller price for any price of the other players.

Recall that the main goal of the paper is to study how fairness and reciprocity change the nature of dynamic price competition. An intermediate step in the analysis is to understand how these preferences change the nature of static price competition. To do that we will compare the set of Nash equilibria of the stage game $\Gamma^s(\pi,p)$ to that of $\Gamma^r(u,p)$. The findings are summarized in Corollary 1.

Corollary 1: *If $\Gamma^r(u,p)$ is a supermodular game, u_i has decreasing differences in (p_i, p^f_{-i}), and $\overline{p}^{ns}_{-i} \leq p^f_{-i}$, then (i) $\overline{p}^{nr} \leq \overline{p}^{ns}$ and $u_i(\overline{p}^{nr}, p^f_{-i}) \leq \pi_i(\overline{p}^{ns})$, for all i, and (ii) $\underline{p}^{nr} \leq \underline{p}^{ns}$ and $u_i(\underline{p}^{nr}, p^f_{-i}) \leq \pi_i(\underline{p}^{ns})$, for all i.*

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with reciprocal players and differentiable payoff functions, the assumption will be satisfied if $\frac{\partial^2 \pi_i}{\partial^2 p_i p_j} > 0$ and α is sufficiently small.

[5] If u_i is differentiable this assumption is equivalent to $\frac{\partial w_{ij}(p_j, p^f_{ij})}{\partial p^f_{ij}} \frac{\partial \pi_j}{\partial p_i} < 0$ for all j.

This result tells us if reciprocal players believe that the fair prices of the rivals satisfy the inequality $p^f_{-i} \geq \overline{p}^{ns}_{-i}$, then the set of Nash equilibria of $\Gamma^r(u,p)$ will be lower than the set of Nash equilibria of $\Gamma^s(\pi,p)$. In other words, if reciprocal players believe that the fair prices of the rivals must be greater than or equal to the largest pure strategy Nash equilibrium prices of the rivals in $\Gamma^s(\pi,p)$, then prices set by reciprocators will be lower than those set by self-interested players.[6]

The result also states that if reciprocal players believe that $p^f_{-i} \geq \overline{p}^{ns}_{-i}$, then the set of Nash equilibrium payoffs of $\Gamma^r(u,p)$ will be lower than the set of Nash equilibrium payoffs of $\Gamma^s(\pi,p)$. This happens because for these perceptions of fair prices, the smallest and the largest Nash equilibria of the game with reciprocal players are destructive reciprocity states. The belief that $p^f_{-i} \geq \overline{p}^{ns}_{-i}$ implies that reciprocal players expect their rivals to charge unfair prices. This implies that reciprocal players wish to punish their rivals. They do it by setting lower prices than self-interested players. The lower equilibrium prices reduce players' monetary payoffs and in addition lead to payoff losses given that players feel that their rivals are being unfair.

We now turn our attention to the infinitely repeated game. For the dynamic game with self-interested players, $\Gamma^s_\infty(\pi,p)$, we know from Friedman (1971) that for a sufficiently high discount factor, there is a subgame-perfect Nash equilibrium of $\Gamma^s_\infty(\pi,p)$ at p^c with payoff $\pi(p^c)$ where $\pi(p^c)$ is any payoff which gives every player strictly more than the payoff of the largest Nash equilibrium of $\Gamma^s(\pi,p)$, that is, $\pi_i(p^c) > \pi_i(\overline{p}^{ns})$, for all i. Lemma 2 applies this result to the dynamic game with reciprocal players, $\Gamma^r_\infty(u,p)$.

Lemma 2: *Let $\Gamma^r(u,p)$ be a supermodular game where (i) u_i has decreasing differences in (p_i, p^f_{-i}), for all i, (ii) $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$. Let p^c satisfy $\pi_i(p^c) > \pi_i(\overline{p}^{ns})$ for all i. Under these conditions there is a sufficiently high discount factor such that there exists a subgame-perfect Nash equilibrium of $\Gamma^r_\infty(u,p)$ at p^c.*

This result states that given the fair prices profile, p^f, for any p^c such that the players' payoffs at the collusive prices are higher than their payoffs at the largest Nash equilibrium of the stage game, collusion can be sustained by reciprocal players at the strategy profile p^c. We are now ready to state our first result about the impact of fairness and reciprocity on collusion.

Proposition 2: *Let $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$ be supermodular games where (i) u_i has decreasing differences in (p_i, p^f_{-i}), all i, and (ii) $p^f_{ij} = p^c_j$ for all i and $j \neq i$. Let Nash punishments in $\Gamma^r_\infty(u,p)$ and in $\Gamma^s_\infty(\pi,p)$ be either at the smallest or largest pure strategy Nash equilibria of $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$, respectively.*

[6] The opposite result would hold if reciprocal players think that the fair prices of the rivals are less than or equal to the lowest pure-strategy Nash equilibrium prices of the rivals in $\Gamma^s(\pi,p)$. However, the assumption that fair prices are greater than the largest Nash prices in $\Gamma^s(\pi,p)$ is more compelling than the assumption that they are less than the smallest Nash prices in $\Gamma^s(\pi,p)$. In fact, a natural benchmark in a collusive framework is to assume that players believe that a rival is fair when the rival charges exactly its agreed upon collusive price.

Let p^c satisfy $\pi_i(p^c) > \pi_i(\overline{p}^{ns})$ for all i. Under these assumptions, the critical (minimum) discount factor needed to sustain collusion at p^c is lower in $\Gamma^r_\infty(u,p)$ than in $\Gamma^s_\infty(\pi,p)$, that is $\delta^r_{p^c} < \delta^s_{p^c}$.

A natural benchmark in a collusive framework is to assume that reciprocal players believe that a rival is fair when the rival charges exactly its agreed upon collusive price. Proposition 2 says that if reciprocal players think that the fair price of each of their rivals is the rival's collusive price, then it is easier to sustain collusion when players are reciprocal than when they are self-interested.[7]

If reciprocal players feel that rivals who charge exactly their agreed upon collusive price are being fair, then a unilateral deviation from the collusive price is punished not only because this allows collusion to be sustained in equilibrium but also because players like to punish what they consider to be an mean behavior by a rival. It follows immediately that if all players think that the fair price of each of the rivals is the rival's collusive price, then they will set lower prices than the self-interested Nash prices if they wish to punish the rivals for their unfair behavior. In other words, Nash reversion punishments are harsher in the game with reciprocal players than in the game with self-interested players since playing Nash reversion in the game with reciprocal players leads to a destructive reciprocity state.

Additionally, if reciprocal players feel that rivals who charge exactly their agreed upon collusive price are being fair, then playing the collusive price implies neither gains nor losses from fairness and reciprocity. In this case the collusive payoff is exactly the same in the game with reciprocal players and in the game with self-interested players. This follows from the assumption that a reciprocal player places no weight on the rivals' payoffs when the rivals charge exactly the prices that a reciprocal player perceives to be fair. This also implies that the one period deviation payoff in the game with reciprocal players is identical to the one period deviation payoff in game with self-interested players.

Does the result obtained in Proposition 2 extend to more general perceptions of fair prices? Proposition 3 provides conditions under which the answer to this question is positive.

Proposition 3: *Let $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$ be supermodular games where (i) u_i has decreasing differences in (p_i, p^f_{-i}), for all i, (ii) $\pi_i(p_i,p_{-i}) = (p_i - c_i)D_i(p_i,p_{-i})$, and (iii) $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$. Let Nash punishments in $\Gamma^r_\infty(u,p)$ and in $\Gamma^s_\infty(\pi,p)$ be either at the smallest or largest pure strategy Nash equilibria of $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$, respectively. Let p^c satisfy $\pi_i(p^c) > \pi_i(\overline{p}^{ns})$ for all i. Under these assumptions, the critical (minimum) discount factor needed to sustain collusion at p^c is lower in $\Gamma^r_\infty(u,p)$ than in $\Gamma^s_\infty(\pi,p)$, that is $\delta^r_{p^c} < \delta^s_{p^c}$.*

[7] The assumption that Nash punishments in both games are either at the smallest or largest pure strategy Nash equilibria of these games is essentially a technical condition. This condition is necessary when the stage game has multiple equilibria since in a supermodular game we can state unambiguous comparative static results for the largest and the smallest Nash equilibria but not for other Nash equilibria.

This result shows that if players think that the fair prices of the rivals are greater than or equal to the largest Nash equilibrium prices of the stage game with self-interest players but less than or equal to the collusive prices, and marginal costs are constant, then it is easier to sustain collusion when players are reciprocal than when they are self-interested.

The intuition behind this result as follows. If players think that the fair prices of the rivals are less than the collusive prices, then collusion becomes a constructive reciprocity state. In this case players' monetary payoffs from collusion are the same as the ones obtained in the game with self-interested players but in addition there are fairness payoff gains since players think that their rivals are being kind. This effect makes collusion more attractive when players are reciprocal than when they are self-interested.

Additionally, if players think that the fair prices of the rivals are greater than the largest Nash equilibrium prices of the stage game with self-interested players, then Nash reversion becomes a destructive reciprocity state. This implies that the punishment imposed after cheating occurs is more severe when players are reciprocal than when they are self-interested. This happens because monetary payoffs are lower than the payoffs of self-interested players and in addition there are fairness payoff loses since players think that the rivals are being unkind. This effect also makes collusion more attractive when players are reciprocal than when they are self-interested.

However, the single period deviation payoff is higher in the game with reciprocal players than in the game with self-interested players. This effect makes collusion *less* attractive when players are reciprocal than when they are self-interested. This happens because the unilateral single period deviation payoff of a reciprocal player also includes the benefit that player derives from being treated kindly by the rivals (the rivals are playing their collusive price levels). However, the assumption that monetary payoffs are large by comparison with fairness payoffs implies that this effect is of second-order.[8]

Proposition 3 shows that this result holds provided certain conditions are met. Clearly, the most important condition is the one about players' perceptions of fair prices of the rivals. As we mentioned before, we think that it is natural to assume that players would think that if the rivals play their collusive prices the rivals are being fair. However, it could be that players also place some weight on consumer surplus. If this were the case, then players could think that the fair prices of the rivals should be less than the collusive prices. We are not able to show that our method of proof also works when we relax the assumption that marginal costs are constant. However, we are convinced that it does since we found that to be the case for several types of increasing marginal costs.

3.2 Strategic Substitutes

When choice variables are strategic substitutes a player's incremental returns from increasing his own action are decreasing in the rivals' actions. The canon-

[8] The assumption that u_i has increasing differences in (p_i, p_{-i}) implies that monetary payoffs are large by comparison with fairness payoffs.

ical market game where players' actions are strategic substitutes is quantity competition with products that are perfect substitutes. We use this game to study the impact of fairness and reciprocity on collusion when players' choices are strategic substitutes.

Assume that in each period player i chooses quantity, q_i, and his payoff in that period is given by

$$u_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + \alpha w_i(Q_{-i}, Q_{-i}^f) \sum_{j \neq i} \pi_j(q_i, Q_{-i}),$$

where $\pi_i(q_i, Q_{-i})$ is the monetary payoff and $\alpha w_i(Q_{-i}, Q_{-i}^f) \sum_{j \neq i} \pi_j(q_i, Q_{-i})$ is the fairness payoff, with $\alpha > 0$. Player i's monetary payoff, $\pi_i(q_i, q_{-i})$, is the difference between revenue and cost, that is,

$$\begin{aligned} \pi_i(q_i, Q_{-i}) &= R_i(q_i, Q_{-i}) - C_i(q_i) \\ &= P(Q)q_i - C_i(q_i), \end{aligned}$$

where $R_i(q_i, Q_{-i})$ is revenue, $C_i(q_i)$ is the cost of production, and $P(Q)$ is the inverse market demand with $Q = \sum q_i$. We assume that $P(Q)$ is strictly positive on some bounded interval $(0, \bar{Q})$ with $P(Q) = 0$ for $Q \geq \bar{Q}$. We also assume that $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ (in the interval for which $P(Q) > 0$). Players costs of production are assumed to be twice continuously differentiable with $C_i'(q_i) \geq 0$. It is also assumed that the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)\, q_i \leq 0$, and $P'(Q) - C_i''(q_i) < 0$. Furthermore, we assume that the weight that player i places on the rivals' aggregate monetary payoffs depends on player i's perception of the fair aggregate output of the rivals, Q_{-i}^f, and on the actual aggregate output of the rivals such that

$$w_i(Q_{-i}, Q_{-i}^f) \begin{cases} > 0 \text{ if } Q_{-i} < Q_{-i}^f \\ = 0 \text{ if } Q_{-i} = Q_{-i}^f \\ < 0 \text{ otherwise} \end{cases},$$

where $w_i(Q_{-i}, Q_{-i}^F)$ is assumed to be differentiable in both arguments with $\partial w_i / \partial Q_{-i} < 0$ and $\partial w_i / \partial Q_{-i}^F > 0$.

These conditions capture the fact that a player with reciprocal preferences cares about the intentions of the rivals. The first condition in (5) expresses constructive reciprocity. If player i expects the aggregate output of the rivals to fall short of its own perception of the fair aggregate output of the rivals, then player i is willing to sacrifice some of his payoffs to increase the rivals' monetary payoffs. The third condition in (5) expresses destructive reciprocity. When player i expects the rivals to produce more than player i's perception of the fair aggregate output of the rivals, then player i is willing to sacrifice some of its profit to reduce the rivals' profit.[9]

[9] Santos-Pinto (2006) shows that if monetary payoffs are not large enough by comparison with fairness payoffs, then best replies of reciprocal players in a static Cournot oligopoly may no longer have a negative slope accross all quantities.

The next result shows that reciprocity facilitates collusion if players think that the fair output of their rivals is greater than or equal to the rivals' joint self-interested collusive output but smaller than or equal to the rivals' joint self-interested Nash output.

Proposition 4: *If $\Gamma^r(u,p)$ and $\Gamma^s(\pi,p)$ satisfy the conditions stated and $Q_{-i}^f \in [Q_{-i}^c, Q_{-i}^{ns}]$ for all i, then the critical (minimum) discount factor needed to sustain collusion at q^c is lower in $\Gamma_\infty^r(u,q)$ than in $\Gamma_\infty^s(\pi,q)$, that is, $\delta_{q^c}^r \leq \delta_{q^c}^s$.*

Proposition 4 shows that fairness and reciprocity also facilitate collusion when players' choices are strategic substitutes. It says that if players think that the fair aggregate output of the rivals is greater than or equal to the collusive output but less than or equal to the aggregate output of the rivals in the Nash equilibrium of the stage game with self-interested players, then it is easier to sustain collusion when players are reciprocal than when they are self-interested.

The intuition is similar to that of Proposition 3. If reciprocal players think that the fair output of their rivals is greater than the joint self-interested collusive output of the rivals, then playing the collusive output is more attractive in the dynamic quantity-setting game with reciprocal players than in the game with self-interested players. This happens because the collusive monetary payoffs are the same as the ones obtained in the game with self-interested players but in addition there are payoff gains from constructive reciprocity since reciprocal players think that their rivals are being kind.

Additionally, if reciprocal players perceive that the fair output of their rivals is smaller than the aggregate output of the rivals in the Nash equilibrium of the stage game with self-interested players, then the punishment imposed after cheating occurs becomes more severe in the dynamic game with reciprocal players than in the dynamic game with self-interested players. This happens because, the Nash equilibrium of the stage game with reciprocal players becomes a destructive reciprocity state. This is bad for players since it reduces monetary payoffs (by comparison with the monetary payoffs of self-interested players) and leads to payoff loses from destructive reciprocity since reciprocal players think that the rivals are being mean.

In contrast, the single period deviation payoff in the game with reciprocal players is larger than the single period deviation payoff in the game with self-interested players. This happens because the unilateral single period deviation payoff of a reciprocal player also includes the benefit that player derives from being treated kindly by the rivals (the rivals are playing their collusive outputs). However, this effect is of second-order since monetary payoffs are larger by comparison with fairness payoffs.

4 Optimal Punishments

So far the paper has indicated that fairness and reciprocity facilitate collusion when players use Nash reversion to punish deviations. However, Abreu's (1988) theory of optimal punishments can be an alternative framework of analysis.

This section shows that our finding also extends to the optimal punishments framework.[10]

Abreu (1988) introduces a rule which consists of an initial path (that is an infinite stream of one period action profiles) and punishments (that are also infinite streams for any deviation from the initial path or from a prescribed punishment). He introduces the notion of *simple* strategy profile in which a specific punishments take place after any deviation for each particular player. Thus, the simple strategy profiles have a description of (n+1) paths for an n-player game, on the other hand an arbitrary strategy profile may consist of infinite amount of punishments and depends on complex history-dependent formulas.

We begin by introducing additional notations and definitions, after we show an optimal simple penal code exists. Finally, we state conditions under which it is easier to sustain collusion with reciprocal players than with self-interested ones under optimal punishments.

A pure strategy of player i is denoted σ_i. The function for all periods t determines player i's action at t as a function of the actions of all players in previous periods. Formally, at $t = 1, \sigma_i(1) \in P_i$ and for $t = 2, 3, ...,$ $\sigma_i(t)$: $P^{t-1} \to P_i$.[11] Player i's strategy set is denoted Σ_i, and the set of strategy profiles is denoted $\Sigma \equiv \Sigma_1 \times \Sigma_2 \times ... \times \Sigma_n$.

A path (or punishment), $\widetilde{P}$, is a stream of action profiles $\{p(t)\}_{t=1}^{\infty}$ and let $\Omega \equiv P^{\infty}$ be the set of punishments. Any strategy profile $\sigma \in \Sigma$ generates a path denoted $\widetilde{P}(\sigma) = \{p(\sigma)(t)\}_{t=1}^{\infty}$, and it is defined as follows: $p(\sigma)(1) = \sigma(1)$, and $p(\sigma)(t) = \sigma(t)(p(\sigma)(1), ..., (p(\sigma)(t))$.

Player i's payoff from path $\widetilde{P} \in \Omega$ is given by $v_i^x : \Omega \to \mathcal{R}$ for $x = \{r, s\}$ such that

$$v_i^x(\widetilde{P}) = \begin{cases} \sum_{t=1}^{\infty} \delta^t u_i(p(t)) \text{ if } x = r \\ \sum_{t=1}^{\infty} \delta^t \pi_i(p(t)) \text{if } x = s \end{cases}$$

where u_i is given by (1) and (5). Player i's payoff function is given by $\widetilde{v}_i^x : \Sigma \to \mathcal{R}$ such that $\widetilde{v}_i^x(\sigma) = v_i(\widetilde{P}(\sigma))$.

Abreu (1988) introduces the simple strategy profile, which is defined by $(n+1)$-vector of paths $(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ and a rule. The initial path is $\widetilde{P}^0$, and for each player $i \in \{1, ..., n\}$, $\widetilde{P}^i$ is the punishment for player i. For any alone deviation of player i from the ongoing path is responded by imposing $\widetilde{P}^i$. If more than one player deviate, the ongoing path continues to be followed and deviators will not be punished. Formally:

Let $\widetilde{P}^i \in \Omega$, $i = 0, 1, ..., n$. The *simple strategy profile* $\sigma(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ specifies: (i) play $\widetilde{P}^0$ until some player deviates singly from $\widetilde{P}^0$; (ii) for any $j \in \{1, ..., n\}$, play $\widetilde{P}^j$ if the jth player deviates singly from $\widetilde{P}^i$, $i = 0, 1, ..., n$, where $\widetilde{P}^i$ is an ongoing previously specified path; continue with $\widetilde{P}^i$ if no deviations occur or if two or more players deviate simultaneously.

[10] We only discuss the dynamic price-setting market game since the quantity-setting case is very similar.

[11] In optimal punishment section, the starting period is 1 for convenience.

We interest in the simple strategy profile being perfect. The following proposition indicates that assuming the set of payoffs of the stage game is bounded (i.e.: $\{u(p)|p \in P\}$ is bounded), the simple strategy $\sigma(\widetilde{P}^0, \widetilde{P}^1, ..., \widetilde{P}^n)$ profile is perfect if and only if no one-shot deviation by any player $j \in \{1, ..., n\}$ from $\widetilde{P}^i$,$i = 0, 1, ..., n$, yields player j a higher payoff, when all players conform with $\widetilde{P}^j$ after the deviation.

Let Σ^p denote the set of perfect equilibrium strategy profiles of $\Gamma_\infty(\delta)$. The perfect equilibrium paths $\Omega^p = \{\widetilde{P}(\sigma)|\sigma \in \Sigma^p\}$, and payoffs $V = \{v(\widetilde{P})|\widetilde{P} \in \Omega^p\}$.

We introduce three more definitions from Abreu (1988) before the existence result. An *optimal penal code* is an n-vector of the strategy profiles $\{\underline{\sigma}^1, ..., \underline{\sigma}^n\}$ such that for all i,

$$\underline{\sigma}^i \in \Sigma^p \text{ and } \widetilde{v}_i(\underline{\sigma}^i) = \min\{\widetilde{v}_i(\sigma)|\sigma \in \Sigma^p\}.$$

Let $\sigma^i(\widetilde{P}^1, ..., \widetilde{P}^n) = \sigma(\widetilde{P}^i, \widetilde{P}^1, ..., \widetilde{P}^n)$. The *simple penal code* $(\widetilde{P}^1, ..., \widetilde{P}^n)$ is the n-vector of the strategy profiles

$$\sigma^1(\widetilde{P}^1, ..., \widetilde{P}^n), ..., \sigma^n(\widetilde{P}^1, ..., \widetilde{P}^n).$$

Finally, a simple penal code $(\widetilde{P}^1, ..., \widetilde{P}^n)$ is an *optimal simple penal code* if it is an optimal penal code.

Lemma 3: *If Σ^p is non-empty, P is a compact topological space and given p^f, $u : P \times p^f \to R^n$ is continuous, then an optimal simple penal code exists.*

Similarly, an optimal simple penal code exists for a continuous payoff function $\pi : P \to \mathcal{R}^n$. The following result indicates the use of optimal penal code to characterize the set of perfect equilibrium paths. Let

$$v_i^x(\widetilde{P}; t+1) = \begin{cases} \sum_{k=1}^{\infty} \delta^k u_i(p(t+k)) \text{ if } x = r \\ \sum_{k=1}^{\infty} \delta^k \pi_i(p(t+k)) \text{ if } x = s \end{cases},$$

denote player i's present discounted payoffs from the period $t+1$ to ∞ along the path $\widetilde{P}$ and $\underline{v}_i^x = \widetilde{v}_i^x(\underline{\sigma}^i)$, the payoff, player i will get under her optimal penal code, where r stands for reciprocal and s for self-interested.

By Abreu (1998) we know that if an optimal penal code exists, then $\widetilde{P}^0 \in \Omega^p$ if and only if

$$u_i(p_i^{dr}, p_{-i}^0(t)) - u_i(p^0) \leq v_i^r(\widetilde{P}^0; t+1) - \underline{v}_i^r \quad (6)$$

$$\pi_i(p_i^{ds}, p_{-i}^0(t)) - \pi_i(p^0) \leq v_i^s(\widetilde{P}^0; t+1) - \underline{v}_i^s \quad (7)$$

The left-hand-side of inequalities (6) and (7) are the benefit of deviating today for reciprocators and self interested players, respectively. The right-hand-side is the benefit of not deviating in the long-run. Note that the prices in each period of the initial path can be simply considered as the collusive prices, but we allow them to be more general than the collusive prices.

Since the existence of optimal simple penal code is guaranteed under the given assumptions, our final result shows that fairness and reciprocity facilitate collusion when players use optimal simple penal codes.

Proposition 5: *Assume (i) u_i has decreasing differences in (p_i, p^f_{-i}), for all i, (ii) $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$, and (iii) $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$. Let p^0 satisfy $\pi_i(p^o) > \pi_i(\overline{p}^{ns})$ for all i. If an optimal simple penal code exist, then the critical (minimum) discount level to sustain collusion at $\widetilde{P}^0$ is lower in the game with reciprocal players $\Gamma^r_\infty(n, u, p, P^f)$ than in the game with self-interested players $\Gamma^s_\infty(n, \pi, p)$, that is $\delta^r_{p^0} < \delta^s_{p^0}$.*

The intuition of this result is as follows. First, the benefit of deviating today when players use optimal punishments is the same as when they use grim trigger punishments. Proposition 3 shows that if monetary payoffs are large by comparison with fairness payoffs, then the increase in the collusive payoff due to fairness considerations is of first-order whereas the increase in the unilateral single period deviation payoff is of second-order. Thus, the benefit of deviating today is smaller for reciprocators than for self-interested players no matter if players use optimal punishments or grim trigger punishments.

Second, if reciprocal players think that the fair prices are smaller than the collusive prices, then the prices set on the initial path are perceived as kind behavior by the other players and lead to positive fairness payoffs. Therefore, the payoffs for reciprocal players are higher than for self-interested players when the prices of the initial path are set. Finally, it is well known that punishments are more severe when players use optimal punishments than the punishment phase of Nash reversion strategies. Setting such low punishment prices is perceived as extremely unkind behavior if reciprocal players think that the fair prices of the rivals are greater than the largest Nash prices of the stage game with self-interested players. Therefore, reciprocal players set smaller prices than self-interested players during the punishment phase no matter if players use optimal punishments or grim trigger punishments. These three effects imply that fairness and reciprocity also facilitate collusion when players use optimal punishments.

5 Conclusion

This paper shows that fairness and reciprocity can facilitate collusion in infinitely repeated market games. This result is valid not only when players' choices are strategic complements but also when they are strategic substitutes. The result also holds no matter if players use grim trigger punishments or optimal punishments.

Our analysis was focused on models of collusion without uncertainty or private information. Studying the impact of fairness and reciprocity in more realistic models of collusion is a promising avenue for research.

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6 Appendix

Proof of Lemma 1: According to Milgrom and Roberts (1990), a game $\Gamma(u, x)$ is supermodular if (i) the choice set is a compact interval in $\mathcal{R}$, (ii) u_i is order upper semi-continuous in x_i for x_{-i} and order continuous in x_{-i} for a fixed x_i, and it has a finite upper bound, (iii) u_i is supermodular in x_i for fixed x_{-i}, and (iv) u_i has increasing differences in (x_i, x_{-i}).
The price stage game with reciprocal players $\Gamma^r(u, p)$ satisfies condition (i) since it is never optimal for players to choose infinite price as their action for any finite quantity. We have assumed that u_i also satisfies all the requirements of condition (ii). Condition (iii) is satisfied since the choice variables of players are scalars. Condition (iv) is satisfied if for any two aggregate actions of the others p'_{-i}, p''_{-i} with $p'_{-i} \geq p''_{-i}$ (product order) the difference $u_i(p_i, p'_{-i}, P_i^f) - u_i(p_i, p''_{-i}, P_i^f)$ is increasing (or non-decreasing) in p_i, which is assumed as well. Therefore $\Gamma^r(u, p)$ is supermodular game. *Q.E.D.*

Proof of Proposition 1: It is an application of Theorem 6 in Milgrom and Roberts (1990) with a slight difference. In their setting, the smallest and largest pure strategy of the game depends on a scalar, but in our model it depends on a vector. Nevertheless, the proof is immediate since we propose the smallest and largest equilibria is nonincreasing with the fair price perception for any player j, which is a scalar. As a result, if the vector increases in every component, then the smallest and largest equilibria do not increase. *Q.E.D.*

Proof of Corollary 1: The stage game $\Gamma^s(\pi, p)$ is obtained from the stage game $\Gamma^r(u, p)$ by setting $\alpha = 0$. Thus, if $\Gamma^r(u, p)$ is a supermodular game so is $\Gamma^s(\pi, p)$. This means that $\Gamma^s(\pi, p)$ also has a smallest and a largest Nash equilibria in pure-strategies. Denote these two equilibria by $\underline{p}^{ns}$ and $\overline{p}^{ns}$, respectively. (i) If $p^f_{-i} = \overline{p}^{ns}_{-i}$ then $\overline{p}^{nr} = \overline{p}^{ns} = \overline{p}^n$ and $u_i(\overline{p}^n, p^f_{-i}) = \pi_i(\overline{p}^n)$ since $w_{ij}(\overline{p}^n_j, p^f_{ij}) = 0$ for all j. If $\overline{p}^{ns}_{-i} < p^f_{-i}$, then $\overline{p}^{nr} < \overline{p}^{ns}$ by Proposition 1. These two inequalities imply $\overline{p}^{nr}_{-i} < p^f_{-i}$ which together with (5) imply $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) < 0$ for all j. But then it follows that $u_i(\overline{p}^{nr}, p^f_{-i}) < \pi_i(\overline{p}^{ns})$ since $\overline{p}^{nr} < \overline{p}^{ns}$ implies that $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) < 0$ for all j and $\pi_i(\overline{p}^{nr}) < \pi_i(\overline{p}^{ns})$ for all i. The proof of (ii) is similar. *Q.E.D.*

Proof of Lemma 2: If $p^f_{ij} \in [\overline{p}^{ns}_j, p^c_j]$ for all i and $j \neq i$, then $w_{ij}(p^c_j, p^f_{ij}) \geq 0$ and $w_{ij}(\overline{p}^{nr}_j, p^f_{ij}) \leq 0$, for all i and $j \neq i$. This in turn implies that

$$u_i(p^c, p^f_{-i}) \geq \pi_i(p^c). \tag{8}$$

We also know that

$$\pi_i(p^c) > \pi_i(\overline{p}^{ns}) > \pi_i(\underline{p}^{ns}). \tag{9}$$

If $p^f_{ij} \geq \overline{p}^{ns}_j$ for all i and $j \neq i$, then we know from Corollary 1 that

$$u_i(\overline{p}^{nr}, p^f_{-i}) \leq \pi_i(\overline{p}^{ns}), \text{ and } u_i(\underline{p}^{nr}, p^f_{-i}) \leq \pi_i(\underline{p}^{ns}) \tag{10}$$

for all i. From (8), (9) and (10) we obtain

$$u_i(p^c, p^f_{-i}) > u_i(\overline{p}^{nr}, p^f_{-i}) \text{ and } u_i(p^c, p^f_{-i}) > u_i(\underline{p}^{nr}, p^f_{-i})$$

for all i, which by Friedman (1971) implies that there exists a discount factor such that p^c is a subgame-perfect Nash equilibrium of $\Gamma^r(u,p)$. *Q.E.D.*

Proof of Proposition 2: By Friedman (1971) and Lemma 2, the assumptions made imply that p^c is a subgame-perfect Nash equilibrium of $\Gamma^s(\pi,p)$ and of $\Gamma^r(u,p)$. Next we show that the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^r_\infty(u,p)$ is lower than the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^s_\infty(\pi,p)$, that is, $\delta^r_{p^c} < \delta^s_{p^c}$. From (2) and (3) sufficient conditions are that

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^c, p^f_{-i}) \le \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c), \qquad (11)$$

and

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^{nr}, p^f_{-i}) \ge \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^{ns}), \qquad (12)$$

where $p^{nr} = \underline{p}^{nr}$ and $p^{ns} = \underline{p}^{ns}$ or $p^{nr} = \overline{p}^{nr}$ and $p^{ns} = \overline{p}^{ns}$. If $p^f_{ij} = p^c_j$ for all i and $j \ne i$, then $w_{ij}(p^c_j, p^f_{ij}) = 0$ for all i and $j \ne i$ which implies that

$$u_i(p^c, p^f_{-i}) = \pi_i(p^c), \qquad (13)$$

for all i, and

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) = \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}), \qquad (14)$$

for all i. It follows from (13) and (14) that (11) is satisfied as an equality. If $p^f_{ij} = p^c_j > \overline{p}^{ns}_j$ for all i and $j \ne i$, then we know from Corollary 1 that

$$u_i(\overline{p}^{nr}, p^f_{-i}) \le \pi_i(\overline{p}^{ns}), \text{ and } u_i(\underline{p}^{nr}, p^f_{-i}) \le \pi_i(\underline{p}^{ns}) \qquad (15)$$

for all i. It follows from (15) and (14) that (12) is satisfied. Thus $\delta^r_{p^c} \le \delta^s_{p^c}$. *Q.E.D.*

Proof of Proposition 3: By Friedman (1971) and Lemma 2, the assumptions made imply that p^c is a subgame-perfect Nash equilibrium of $\Gamma^s(\pi,p)$ and of $\Gamma^r(u,p)$. We want to show that the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^r_\infty(u,p)$ is lower than the critical discount factor at which p^c can be sustained using grim trigger punishments in $\Gamma^s_\infty(\pi,p)$, that is, $\delta^r_{p^c} < \delta^s_{p^c}$. From (2) and (3) sufficient conditions are that

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^c, p^f_{-i}) \le \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^c), \qquad (16)$$

and

$$u_i(BR^r_i(p^c_{-i}), p^c_{-i}, p^f_{-i}) - u_i(p^{nr}, p^f_{-i}) \ge \pi_i(BR^s_i(p^c_{-i}), p^c_{-i}) - \pi_i(p^{ns}). \qquad (17)$$

We start by showing that $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$ and $p_{ij}^f \leq p_j^c$ for all $j \neq i$ imply that (16) is satisfied as a strict inequality. We have that

$$\begin{aligned} u_i(BR_i^r(p_{-i}^c), p_{-i}^c, p_{-i}^f) - u_i(p^c, p_{-i}^f) &= \pi_i(BR_i^r(p_{-i}^c), p_{-i}^c) - \pi_i(p^c) \\ &+ \alpha \sum_{j \neq i} w_{ij}(p_j^c, p_{ij}^f)(p_j^c - c_j)[D_j(BR_i^r(p_{-i}^c), p_{-i}^c) - D_j(p^c)] \\ &\leq \pi_i(BR_i^r(p_{-i}^c), p_{-i}^c) - \pi_i(p^c) < \pi_i(BR_i^s(p_{-i}^c), p_{-i}^c) - \pi_i(p^c) \end{aligned}$$

The equality is obtained from (4) and from the assumption $\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})$. The weak inequality comes from the assumption that $p_{ij}^f \leq p_j^c$ which implies $w_{ij}(p_j^c, p_{ij}^f) \geq 0$, and the assumption that D_j is increasing with p_i which together with $p_i^{dr} < p_i^c$ imply $D_j(BR_i^r(p_{-i}^c), p_{-i}^c) - D_j(p^c) < 0$. The strict inequality comes from the fact that $BR_i^s(p_{-i}^c)$ is the best-reply to p_{-i}^c by a self-interested player.
We now show that if $\overline{p}_j^{ns} \leq p_{ij}^f$ for all $j \neq i$ and α is sufficiently small, then (17) is satisfied. Rewrite (17) as

$$[u_i(BR_i^r(p_{-i}^c), p_{-i}^c, p_{-i}^f) - \pi_i(BR_i^s(p_{-i}^c), p_{-i}^c)] + [\pi_i(p^{ns}) - u_i(p^{nr}, p_{-i}^f)] \geq 0.$$

From Corollary 1 we have that

$$\pi_i(p^{ns}) - u_i(p^{nr}, p_{-i}^f) \geq 0.$$

If $\overline{p}_j^{ns} \leq p_{ij}^f$ for all $j \neq i$, then $w_{ij}(p_j, p_{ij}^f) \geq 0$ for all $j \neq i$. Taking a first-order Taylor series expansion of $u_i(BR_i^r(p_{-i}^c), p_{-i}^c, p_{-i}^f)$ around $\alpha = 0$ we obtain

$$\begin{aligned} u_i(BR_i^r(p_{-i}^c), p_{-i}^c, p_{-i}^f) &\approx \pi_i(BR_i^s(p_{-i}^c), p_{-i}^c)) \\ &+ \alpha[\sum_{j \neq i} w_{ij}(p_j, p_{ij}^f)\pi_j(BR_i^s(p_{-i}^c), p_{-i}^c)], \end{aligned}$$

which is equivalent to

$$\begin{aligned} u_i(BR_i^r(p_{-i}^c), p_{-i}^c, p_{-i}^f) - \pi_i(BR_i^s(p_{-i}^c), p_{-i}^c) &\approx \\ \alpha[\sum_{j \neq i} w_{ij}(p_j, p_{ij}^f)\pi_j(BR_i^s(p_{-i}^c), p_{-i}^c)] &\geq 0 \end{aligned}$$

Thus $\delta_{p^c}^r \leq \delta_{p^c}^s$. *Q.E.D.*

Proof of Proposition 4: We need to show that $Q_{-i}^f \in [Q_{-i}^c, Q_{-i}^{ns}]$ for all i, implies $\delta_{q^c}^r \leq \delta_{q^c}^s$, where $\delta_{q^c}^r$ is the critical discount factor above which q^c can be sustained in $\Gamma_\infty^r(u, q)$ and $\delta_{q^c}^s$ is the critical discount factor above which q^c can be sustained in $\Gamma_\infty^s(\pi, q)$. From (2) and (3) sufficient conditions are that

$$u_i(BR_i^r(Q_{-i}^c), Q_{-i}^c) - u_i(q^c) \leq \pi_i(BR_i^s(Q_{-i}^c), Q_{-i}^c) - \pi_i(q^c) \quad (18)$$

and

$$u_i(BR_i^r(Q_{-i}^c), Q_{-i}^{cs}) - u_i(q^{nr}) \geq \pi_i(BR_i^s(Q_{-i}^c), Q_{-i}^{cs}) - \pi_i(q^{ns}). \quad (19)$$

(i) We start by showing that $Q^f_{-i} \in [Q^c_{-i}, Q^{ns}_{-i}]$ implies (18) is satisfied as a strict inequality. We have that

$$\begin{aligned} u_i(BR^r_i(Q^c_{-i}), Q^c_{-i}) - u_i(q^c) &= \pi_i(BR^r_i(Q^c_{-i}), Q^c_{-i}) - \pi_i(q^c) \\ &+ \alpha w_i(Q^c_{-i}, Q^f_{-i}) \left[P(BR^r_i(Q^c_{-i}) + Q^c_{-i}) - P(Q^c) \right] Q^c_{-i} \\ &\leq \pi_i(BR^r_i(Q^c_{-i}), Q^c_{-i}) - \pi_i(q^c) < \pi_i(BR^s_i(Q^c_{-i}), Q^c_{-i}) - \pi_i(q^c) \end{aligned}$$

The strict inequality follows from the fact that $BR^s_i(Q^c_{-i})$ is the best reply to Q^c_{-i} for self-interested players. If $Q^c_{-i} \leq Q^f_{-i}$ then $w_i(Q^c_{-i}, Q^f_{-i}) \geq 0$. Furthermore, $Q^f_{-i} \leq Q^{ns}_{-i}$ implies $BR^r_i(Q^c_{-i}) > q^c_i$ which in turn implies $P(BR^r_i(Q^c_{-i}) + Q^c_{-i}) < P(Q^c)$, since $P'(\cdot) < 0$.
(ii) We now show that $Q^f_{-i} \in [Q^c_{-i}, Q^{ns}_{-i}]$ implies that (19) is satisfied. Rewrite (19) as

$$[u_i(BR^r_i(Q^c_{-i}), Q^c_{-i}) - \pi_i(BR^s_i(Q^c_{-i}), Q^c_{-i})] + [\pi_i(q^{ns}) - u_i(q^{nr})] \geq 0.$$

We have that

$$u_i(q^{nr}) = \pi_i(q^{nr}) + \alpha w_i(Q^{nr}_{-i}, Q^f_{-i}) \sum_{j \neq i} \pi_j(q^{nr}) \leq \pi_i(q^{ns}).$$

The inequality follows from $w_i(Q^{nr}_{-i}, Q^f_{-i}) \leq 0$ and Proposition 3 in Santos-Pinto (2006) which shows that $Q^f_{-i} \leq Q^{ns}_{-i}$ implies $q^{ns}_i \leq q^{nr}_i$ and $\pi_i(q^{nr}) \leq \pi_i(q^{ns})$, for all i. Taking a first-order Taylor series expansion of $u_i(BR^r_i(Q^c_{-i}), Q^{cs}_{-i})$ around $\alpha = 0$ we have that

$$\begin{aligned} u_i(BR^r_i(Q^c_{-i}), Q^c_{-i}) &\approx \pi_i(BR^s_i(Q^c_{-i}), Q^c_{-i}) \\ &\quad + \alpha[w_i(Q^c_{-i}, Q^f_{-i}) \sum_{j \neq i} \pi_j(BR^s_i(Q^c_{-i}), Q^c_{-i})]. \end{aligned}$$

which is equivalent to

$$\begin{aligned} & u_i(BR^r_i(Q^c_{-i}), Q^c_{-i}) - \pi_i(BR^s_i(Q^c_{-i}), Q^c_{-i}) \approx \\ & \qquad \alpha[w_i(Q^c_{-i}, Q^f_{-i}) \sum_{j \neq i} \pi_j(BR^s_i(Q^c_{-i}), Q^c_{-i})] \geq 0 \end{aligned}$$

since $Q^c_{-i} \leq Q^f_{-i}$ implies that $w_i(Q^c_{-i}, Q^f_{-i}) \geq 0$. Thus, $Q^f_{-i} \in [Q^c_{-i}, Q^{ns}_{-i}]$ for all i, implies $\delta^r_{q^c} \leq \delta^s_{q^c}$. *Q.E.D.*

Proof of Lemma 3: Since the assumptions are satisfied for $u(.)$, this is an application of Abreu (1988). *Q.E.D.*

Proof of Proposition 5: The minimum critical discount factor will be obtained if the inequality (6) and (7) hold with equality respectively for reciprocators and self-interested players, otherwise the discount factor can be decreased by a small amount without violating the inequality.

In Proposition 3, we proved the LHS of the equations being smaller for reciprocators, hence a smaller discount level is possible for the reciprocators. In addition, the following condition is immediate

$$v_i^r(\widetilde{P}^0; t+1) \geq v_i^s(\widetilde{P}^0; t+1)$$

considering the initial path where each player i sets at least the collusive price p_i^c for each stage, until one deviates. Hence for any fair price perception $p_{ij}^f \in [\overline{p}_j^{ns}, p_j^c]$ for all i and $j \neq i$, the prices set at the initial path will be perceived as kind behavior, thus the condition holds. Note that, if the prices set at the initial path are equal to collusive prices p_i^c for each player i and $p_{ij}^f = p_j^c$ for all i and $i \neq i$, then the condition holds with equality. Finally, to complete the proof we need to compare the payoff of any player i in the optimal penal code $\underline{v}_i^x$. In the optimal penal code, the players punish the deviated player i via playing a pure strategy profile $\underline{\sigma}^i \in \Sigma^p$, which gives the lowest possible payoff to player i. Let ${}^{nx}\underline{\sigma}$ denote the strategy profile where in each stage players set Nash prices. Since ${}^{nx}\underline{\sigma} \in \Sigma^p$, in each stage the optimal penal code for player i, $\underline{v}_i^x$, is at least as severe as ${}^{nx}\underline{\sigma}$, which means each player j sets $\underline{p}_j \leq p_j^{ns}$. Note that, if the set prices in the penal code is such that $\underline{p}_j = p_j^{ns} = p_{ji}^f$ for all j and $i \neq j$, then the payoffs from the penal code are equal for self interested and reciprocal player i, $\underline{v}_i^r = \underline{v}_i^s$. Otherwise, the reciprocal players perceive the unkind behavior and destructive reciprocity state implies the payoff under optimal penal code is harsher for reciprocal players than self-interested players, that is $\underline{v}_i^r < \underline{v}_i^s$. Since all these conditions leads the reciprocators to have a smaller critical discount factor than self-interested players, that is $\delta_{p^0}^r < \delta_{p^0}^s$, we are done. *Q.E.D.*