Fiscal harmonization and portfolio choice*

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March 2002

Abstract

In this paper, we consider a two-country model based on Svensson (1989) in order to analyze how fiscal harmonization impacts on economic growth and welfare through its effects on agents portfolio decisions in an uncertain world. We derive the conditions under which fiscal harmonization proves to be welfare enhancing and analyse how the set of initial tax rates leading to a welfare improving harmonization is affected by uncertainty and assets returns correlation. In particular, the results obtained suggest that the probability for tax harmonization to be welfare improving is first increasing and then decreasing with uncertainty while it monotonically decreases with the correlation between the assets returns shocks.

Key words : fiscal harmonization, growth, uncertainty
JEL classification : E62, F21, F41, H22, O16

*The authors thank Anne Epaulard, William Smith, Santanu Chatterjee and participants at 2001 Conference of the Southern Economic Association (Tampa, Florida, November 2001), for theircomments. All remaining errors are the authors'.

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1 Introduction

The issue of fiscal harmonization is a classic in the economic literature and is frequently inscribed both on research and economic policy agendas. This is particularly true for Europe where the integration process taking place under the auspice of the European Union explicitly contains objectives of reducing national discrepancies in fiscal domains. The debate seems particularly controversial as far as financial assets are concerned. While fiscal harmonization was the option favored during the 70's-80's, today's approach is more oriented towards countries keeping some sovereignty and developing different forms of cooperation. To a large extent, the cooperation/harmonization issue has mostly been analyzed by focusing on fiscal competition aspects. Cooperation indeed allows to solve a coordination failure between the authorities collecting taxes and is therefore shown to generate gains in efficiency and welfare. Nevertheless, it may conflict with the principle of subsidiarity which pushes public decisions to be taken at the most local level. This generates an interesting trade-off concerning the appropriate organization of fiscal matters[1].

However, the piece of literature ignores the stochastic nature of the environment surrounding economic decisions. In fact, taxing uncertain flows may not be similar to taxing flows that are known for sure. Indeed, considering taxes collected on financial assets, Smith (1991) shows how uncertainty affects the relationship between taxes and economic decisions in a non trivial way. The idea is that when taxing the stochastic return of an asset, both the average return and its associated variance are reduced. Thus, in the presence of risk averse agents, taxes may have an a priori ambiguous effect on welfare even if they are not used to finance something like a public good. Since such effects of taxes are related to the stochastic nature of the environment, it seems interesting to reconsider the problem of the relevance of a fiscal harmonization under uncertainty. Abstracting from any fiscal competition consideration, we emphasize the conditions under which fiscal harmonization is desirable and show how its relevance crucially depends on uncertainty. This implies that taking decisions about tax harmonization without considering the stochastic nature of the economy may clearly be misleading.

In this paper, we therefore propose a two-country model based on Svensson (1989) or Obstfeld (1994) in a multi-country setting, in order to analyze how fiscal harmonization impacts on welfare through its effects on agents portfolio decisions[2]. Note that these assets may be viewed as being the production of the country. We then derive the conditions under which fiscal harmonization proves to be welfare enhancing by considering the way taxes affect the assets characteristics and hence portfolio choices. The results obtained suggest that

[1] See the literature on tax competition and fiscal harmonization, for instance, Keen (1989), Sinn (1990 and 1994), Kanbur and Keen (1993), Cremer and Gahvari (2000).

[2] Therefore, our setting may be viewed as a particular case of a more general problem : taxation choice of different competing assets without paying attention to the number and identity of fiscal authorities.

the probability for tax harmonization to be welfare improving is dependent on the size of uncertainty as well as on the correlation between the assets returns shocks. Finally, the paper provides one simple example based on calibrations using German and French data.

Section 2 presents the model and the relevant resolution method. The way taxes affect portfolio decisions is analyzed in section 3. Section 4 describes the characteristics of an optimal taxation scheme. Section 5 then examines the issue of fiscal harmonization and section 6 concludes.

2 The Model

We consider a two-country continuous-time model allowing for an analytical resolution. In this section, we first recall the methodological interest of the recursive utility function; we then specify the assets portfolio characteristics. Finally we introduce taxes.

2.1 The recursive utility function

Following Kreps and Porteus (1978), Epstein and Zin (1991) and Weil (1990), the representative agent in each country maximizes the same recursive utility function which disentangles between risk aversion and intertemporal elasticity of substitution. Indeed, there is no reason for tastes so different in nature as the one for intertemporal substitution and the one for risk to be captured by the same parameter. In this utility function, the intertemporal utility at time t depends on consumption at this date and on the certainty equivalent of future utility at time $t + dt$ which is written $\hat{U}(t + dt)$; the intertemporal elasticity of substitution between current consumption and the certainty equivalent of future utility is supposed to be constant:

$$U(t) = \left[C(t)^{\frac{\varepsilon-1}{\varepsilon}} + e^{-\delta dt}\left(\hat{U}(t+dt)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

where ε is the intertemporal elasticity of substitution and δ the time preference rate. Note that $1/\varepsilon$ may also be understood as the aversion to fluctuations.
The certainty equivalent at time t of the intertemporal utility at time $t + dt$ depends on the agent's attitude with respect to risk which is taken into account through the usual Constant Relative Risk Aversion (*CRRA)* functional form:

$$F(U(t+dt)) = [U(t+dt)]^{1-\gamma} \quad \Rightarrow \qquad U(t+dt) = \left[E\left([U(t+dt)]^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}} \tag{2}$$

with $U(t + dt)$ being the future intertemporal stochastic utility. One finally obtains[3] a *recursive utility function*:

$$U(t)=\left[\frac{\varepsilon}{\varepsilon-1}C(t)^{\frac{\varepsilon-1}{\varepsilon}}+e^{-\delta dt}\left[E\left(U(t+dt)^{\frac{\varepsilon(1-\gamma)}{\varepsilon-1}}\right)\right]^{\frac{\varepsilon-1}{\varepsilon}\frac{1}{1-\gamma}}\right] \tag{3}$$

Note that in the special case in which the risk aversion is the inverse of the intertemporal elasticity of substitution ($\gamma = 1/\varepsilon$), this utility function adopts the rather usual following form:

$$U(t)=E_t\int_t^{+\infty}\frac{C(s)^{1-\gamma}}{1-\gamma}e^{-\delta(s-t)}ds \tag{4}$$

2.2 Portfolio assets characteristics and wealth

There exists one risky asset in each country. Both asset returns (q_i, $i = 1,2$) are assumed to follow geometric Brownian motions[4]:

$$\frac{dq_i(t)}{q_i(t)}=\alpha dt+\sigma dz_i(t),\quad i=1,2$$

where z_i is the vector of the increment of a Wiener process *i.e.* $dz_i(t) = \epsilon_i(t)\sqrt{dt}$; $\epsilon_i(t) \sim iid\ N(0,1)$. The returns consist of two components: α is the deterministic component and $\sigma dz_i(t)$ is the stochastic one where σ is the standard error. Finally, the Wiener process increments are correlated: $dz_1(t)dz_2(t) = \rho_{1,2}dt$. Note that such a specification implies the deterministic part of the assets returns as well as the size of the risk on each return to be identical across countries. Nevertheless, returns are subject to different shocks that may be imperfectly correlated.

In each country, the representative agent's wealth only comes from the return on her portfolio. This portfolio consists of shares of the two countries assets. Since agents have the same utility function and the same access to both assets irrespective of the country where they live, they choose the same assets shares and we therefore may consider only one representative agent for both countries. Her wealth is written:

$$\frac{dW(t)}{W(t)}=\alpha dt+\omega\sigma dz_1(t)+(1-\omega)\sigma dz_2(t)-\frac{C(t)}{W(t)}$$

where ω is the share of country 1 asset in the representative agent portfolio.

2.3 Taxes

Each country's government levies a tax which is proportional to the asset return, $\tau_i \in (0,1)$ denoting the tax rate on asset i return. This tax affects the deterministic part of the return as well as its stochastic component. The after-tax

[3] Using a transformation of the type $(U(t)/a)^a$ as proposed by Duffie and Epstein (1992), with $a = \varepsilon/(\varepsilon-1)$.

[4] Such a modelling allows to take into account that the smaller the forecast interval, the sharper the evaluation of an asset value.

return of each asset may thus be expressed as follows:

$$\frac{dq_i(t)}{q_i(t)} = (1-\tau_i)\alpha dt + (1-\tau_i)\sigma dz_i(t), \quad i = 1,2$$

We assume that the government uses this tax to finance the provision of a public good which enters neither the utility function nor wealth[5]. Importantly, it is this assumption that to eliminate fiscal competition aspects from the analysis. The evolution of the representative agent's wealth is then:

$$\frac{dW(t)}{W(t)} = \omega(1-\tau_1)(\alpha dt + \sigma dz_1(t)) + (1-\omega)(1-\tau_2)(\alpha dt + \sigma dz_2(t)) - \frac{C(t)}{W(t)} \quad (5)$$

3 Exogenous tax and diversification

Considering an exogenous taxation, the representative agent program is simply:

$$\left\{ \begin{array}{l} \max_{\omega(t),C(t)} U(t) = \left[C(t)^{\frac{\varepsilon-1}{\varepsilon}} + \beta \left[E\left(U(t+dt)^{1-\gamma} \right) \right]^{\frac{\varepsilon-1}{\varepsilon}\frac{1}{1-\gamma}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ \text{sc.} \left| \begin{array}{l} \frac{dW(t)}{W(t)} = \omega(1-\tau_1)(\alpha dt + \sigma dz_1(t)) + (1-\omega)(1-\tau_2)(\alpha dt + \sigma dz_2(t)) - \frac{C(t)}{W(t)} \\ W(0) \text{ given} \end{array} \right. \end{array} \right.$$

This is a standard consumption/saving arbitrage and portfolio choice problem under uncertainty similar to the one which has initially been considered by Merton (1969, 1971). It has already been solved by Svensson (1989) and leads to the following first order conditions (see appendix 1):

$$C^*(t) = [\varepsilon\delta - (\varepsilon - 1)\chi^*(t)] W(t) \quad (6)$$

$$\omega(t)^* = \frac{\alpha(\tau_2 - \tau_1) - \gamma\sigma^2(1-\tau_2)\left[\rho_{1,2}(1-\tau_1) - (1-\tau_2)\right]}{\gamma\sigma^2\left[(1-\tau_1)^2 - 2\rho_{1,2}(1-\tau_1)(1-\tau_2) + (1-\tau_2)^2\right]} \quad (7)$$

where $\chi^*(t)$ is the certainty equivalent of the portfolio rate of return when the asset shares are optimally chosen:

$$\begin{aligned} \chi^*(t) = {} & \alpha(1-\tau_1)\omega(t)^* + \alpha(1-\tau_2)(1-\omega(t)^*) - \frac{1}{2}\gamma\sigma^2 \left[(1-\tau_1)^2\omega(t)^{*2} \right. \\ & \left. + 2\rho_{1,2}(1-\tau_1)(1-\tau_2)\omega(t)^*(1-\omega(t)^*) + (1-\tau_2)^2(1-\omega(t)^*)^2 \right] \end{aligned} \quad (8)$$

One may note (see appendix 1) that maximizing the expected value of the intertemporal utility with respect to the assets shares reduces to the maximization of the certainty equivalent of the portfolio rate of return $\chi(t)$ with respect

[5] Note that the government revenue is : $dT(t) = \omega\tau_1 (adt + \sigma dz_1(t)) + (1-\omega)\tau_2 (adt + \sigma dz_2(t))$. So, it may be the case for negative shocks, that this revenue becomes negative, that is, the government has to provide subsidies. The model of Smith (1996) exhibits the same feature. This introduces a partial equilibrium flavour in this type of models since the intertemporal budget constraint of the government is not explicitely modelled. The government may finance its subsidies by creating an external debt or by levying taxes on other sources of revenues.

to these same shares. Portfolio allocation and consumption/saving trade-off decisions are indeed separable: first, the representative agent determines her portfolio structure, whose certainty equivalent rate of return depends on the risk aversion and second, she decides the amount to consume depending on her intertemporal elasticity of substitution. This is rather intuitive: in the absence of reallocation cost inside the portfolio, there is no reason for the intertemporal elasticity of substitution to affect its composition. Moreover, once the certainty equivalent of the portfolio rate of return has been determined, it seems obvious that the risk aversion no longer affects the consumption decision.

Note also that it is the use of a recursive utility function which allows to show that the optimal portfolio composition is independent from the intertemporal elasticity of substitution while this parameter enters the expression of the optimal consumption.

With regard to the effect of uncertainty, we have that, for given assets shares, the volatility negatively affects the certainty equivalent of the optimal portfolio rate of return. Of course, the higher the risk aversion *i.e.* the higher the agent's sensitivity to uncertainty, the larger the effect of uncertainty on $\chi^*(t)$. The partial derivative of $\chi^*(t)$ with respect to σ^2 indeed writes[6]:

$$\begin{aligned}
\frac{\partial \chi^*(t)}{\partial \sigma^2} &= -\gamma \left[(1-\tau_1)^2\omega(t)^{*2} + 2\rho_{1,2}(1-\tau_1)(1-\tau_2)\omega(t)^*(1-\omega(t)^*) + (1-\tau_2)^2(1-\omega(t)^*)^2\right] \\
&\quad + \underbrace{\frac{\partial \chi^*(t)}{\partial \omega^*(t)}\frac{\partial \omega^*(t)}{\partial \sigma^2}}_{0} < 0 \\
\frac{\partial C^*(t)}{\partial \sigma^2} &= (1-\varepsilon)W(t)\frac{\partial \chi^*(t)}{\partial \sigma^2}, \ > 0 \text{ if } \varepsilon > 1 \text{ and } < 0 \text{ if } \varepsilon < 1
\end{aligned}$$

For a given current wealth, the optimal consumption depends on the certainty equivalent of the portfolio rate of return and on the intertemporal elasticity of substitution. Moreover, the direction of the effect of uncertainty on the optimal consumption level is determined by the degree of intertemporal substitution: if the intertemporal elasticity of substitution is greater than unity[7], an increase in uncertainty, which induces a smaller certainty equivalent for the portfolio rate of return, eventually leads to a higher consumption today. The intuition runs as follows: in a standard way, a decrease of the certainty equivalent of the portfolio rate of return generates a substitution effect, which urges the representative agent to consume more today, together with an income effect which has the opposite effect. As soon as the intertemporal elasticity of substitution is sufficiently high (greater than unity) the substitution effect prevails.

[6] Note that we only consider the effect of uncertainty on the certainty equivalent of the portfolio rate of return for given assets shares. The effect *via* $\omega(t)^*$ indeed needs not to be taken into account : since we only consider marginal variations of uncertainty, such an effect is only of second order magnitude by the envelope theorem.

[7] Note that an intertemporal elasticity of substitution greater than unity is implausible in a *CRRA* utility function since it is largely admitted that the relative risk aversion coefficient is greater than unity.

The value function, that is the indirect utility function which gives the welfare of the representative agent, is then:

$$\begin{aligned} V(t) &= \frac{\varepsilon}{\varepsilon-1}\left[\varepsilon\delta-(\varepsilon-1)\left(\chi^*(t)\right)\right]^{-1/\varepsilon} W(t)^{\frac{\varepsilon-1}{\varepsilon}} \\ \frac{\partial V(t)}{\partial\chi^*(t)} &= \left[\varepsilon\delta-(\varepsilon-1)\left(\chi^*(t)\right)\right]^{\frac{\varepsilon-1}{\varepsilon}} W(t)^{\frac{\varepsilon-1}{\varepsilon}} > 0 \end{aligned} \tag{9}$$

Whatever the value of the intertemporal elasticity of substitution, a larger uncertainty reduces the certainty equivalent of the portfolio rate of return which unambiguously reduces welfare at each date.

3.1 Portfolio choice and taxes

Intuition suggests that the optimal share of a particular asset should be a decreasing function of the tax on its return. Nevertheless, as pointed out by Smith (1996), the tax affects both the deterministic part of the return and its stochastic component. As we show below, this generates opposite effects leading to a more complex relationship between the optimal share and the tax.

For given values of the parameters characterizing the after tax assets return (α, σ τ_1, τ_2 and ρ_{12}), we have two conditions ensuring that the optimal share ω of asset 1 (and thus of asset 2 as well) is between zero and unity. Imposing that $\omega^* \in [0,1]$ rules out degenerate solutions for which the representative agent optimally chooses to sell short one of the asset to acquire a value of the other asset which is larger than her wealth. These conditions are:

$$\begin{aligned} \alpha_0 &< \alpha < \alpha_1 \quad \text{iff } \tau_1 < \tau_2 \\ \alpha_0 &> \alpha > \alpha_1 \quad \text{iff } \tau_2 < \tau_1 \end{aligned}$$

with

$$\begin{aligned} \alpha_0 &= \frac{\gamma\sigma^2(1-\tau_2)\left[(1-\tau_2)-\rho_{1,2}(1-\tau_1)\right]}{\tau_1-\tau_2} \\ \alpha_1 &= \frac{\gamma\sigma^2(1-\tau_1)\left[\rho_{1,2}(1-\tau_2)-(1-\tau_1)\right]}{\tau_1-\tau_2} \end{aligned}$$

Moreover, it can be shown that the optimal share ω^* of asset 1 is such that:

- $\omega^*(\tau_1)=0$ has one root
- $\frac{\partial\omega^*(\tau_1)}{\partial\tau_1}=0$ has to two roots
- $\lim_{\tau_1\to-\infty}\omega^*(\tau_1)=\lim_{\tau_1\to+\infty}\omega^*(\tau_1)=0$

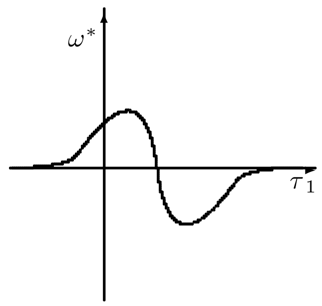

Figure 1: $\omega^*(\tau_1)$

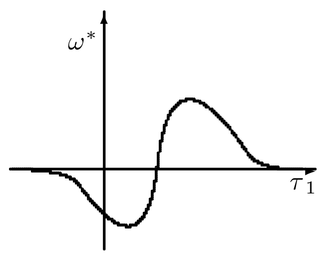

Figure 2: $\omega^*(\tau_1)$

which implies (see figures 1 and 2) that, when only considering the range of τ_1 for which asset shares are between 0 and unity, we have:

$$\frac{\partial \omega^*(\tau_1)}{\partial \tau_1} > 0 \text{ for } \tau_1 < \widehat{\tau}_1 \text{ and } \frac{\partial \omega^*(\tau_1)}{\partial \tau_1} < 0 \text{ for } \tau_1 > \widehat{\tau}_1$$

That is, this optimal share ω^* is first an increasing function and then a decreasing function of τ_1. Of course, nothing guarantees that $\widehat{\tau}_1$ is in the range of τ_1 that we consider (*i.e.* the range of τ_1 between zero and unity and leading to asset shares between zero and unity): if $\widehat{\tau}_1$ is on the left (*resp.* right) side of the lower (*resp.* higher) bound of the range, only the decreasing (*resp.* the increasing) part appears for the values of τ_1 we consider, while for $\widehat{\tau}_1$ belonging to the range, ω^* is maximum for $\tau_1 = \widehat{\tau}_1$.

The expression of $\frac{\partial \omega^*}{\partial \tau_1}$ is tedious and difficult to interpret. In order to give the intuition concerning the shape of the relationship between ω^* and τ_1, we can nevertheless use the properties of $\chi^*(t)$. Appendix 2 shows that the sign of $\frac{\partial \omega^*}{\partial \tau_1}$ is that of $\left(\frac{\partial^2 \chi}{\partial \tau_1 \partial \omega}\right)_{[\omega=\omega^*(\tau_1)]}$ whose expression is relatively simple:

$$\left(\frac{\partial^2 \chi}{\partial \tau_1 \partial \omega}\right)_{[\omega=\omega^*(\tau_1)]} = -\alpha + 2\gamma\sigma^2(1-\tau_1)\omega^*(\tau_1) - \gamma\rho_{1,2}\sigma^2(1-\tau_2)\left[\omega^*(\tau_1) - (1-\omega^*(\tau_1))\right]$$

The effect of τ_1 on ω^*, can then be decomposed as follows:

- First, the deterministic effect of the tax ($-\alpha$) is negative. The increase in the tax rate affecting asset 1 indeed lowers the after tax deterministic part of the asset return which reduces the incentive to hold this asset.
- Second, the stochastic effect of the tax encompasses
  - the volatility component ($2\gamma\sigma^2(1-\tau_1)\omega^*$) which is positive and whose magnitude depends on ω^* and τ_1. An increase in τ_1 reduces the after tax volatility of asset 1 which in turns increases the certainty equivalent of this asset rate of return, urging for a larger ω^*.
  - the correlation component ($-\gamma\rho_{1,2}\sigma^2(1-\tau_2)[\omega^* - (1-\omega^*)]$) whose sign depends on $\rho_{1,2}$ and ω^*. For instance, for $\rho_{1,2} > 0$ an increase in τ_1 reduces the negative effect due to the correlation on the certainty equivalent of the portfolio rate of return. Note that this effect is maximum for asset shares equal to 1/2 (see equation(8)). Thus a lower after tax correlation generates an incentive to hold shares that are closer to the (1/2,1/2) combination. Finally, ω^* should increase *iff* its initial value is less than 1/2.

One now clearly checks that the impact of the tax on the assets shares results from a complex mix of positive and negative effects. The global effect is such that for small values of the tax rate, the optimal asset share is an increasing function of the tax while it is decreasing with the tax for larger values. Of course, according to parameters values, only the increasing (resp. decreasing) part of the relationship may appear in the relevant range for the tax. We therefore derive the following general result :

Proposition 1 *The optimal share of one particular asset is not monotonically decreasing with the tax on its return.*

This result is due to a complex combination of three different effects of the tax on the corresponding optimal asset share:

(i) the first effect reduces the deterministic part of its return

(ii) the second effect reduces the stochastic part of its return

(iii) the third effect reduces the absolute value of the correlation between its return and that of the other asset

The same reasoning applies for the optimal share of asset 2 when considering variations in the tax rate of the second country. Since the two shares must add up to one, we can also deduce that there may exist a negative relationship between the tax rate of country 2 and the optimal share of asset 1.

3.2 Optimal consumption, welfare and taxes

As for the optimal share of one particular asset, the tax affects the certainty equivalent of the optimal portfolio as well as welfare and consumption in a rather complex way through its deterministic and stochastic effects. In particular, these variables are shown not to be always decreasing functions of the tax.

Let us first consider the effect of taxes on the certainty equivalent of the optimal portfolio rate of return. By the envelope theorem:

$$\begin{aligned}\left(\frac{d\chi(\omega^*(\tau_1),\tau_1)}{d\tau_1}\right)_{[\omega=\omega^*(\tau_1)]} &= \left(\frac{\partial\chi(\omega,\tau_1)}{\partial\tau_1}\right)_{[\omega=\omega^*(\tau_1)]} + \underbrace{\left(\frac{\partial\chi(\omega,\tau_1)}{\partial\omega}\right)_{[\omega=\omega^*(\tau_1)]} \frac{\partial\omega^*(\tau_1)}{\partial\tau_1}}_{0} \\ &= -\alpha\omega(t)^* + \gamma\sigma^2(1-\tau_1)\omega(t)^{*2} + \gamma\sigma^2\rho_{1,2}(1-\tau_2)\omega(t)^*(1-\omega(t)^*)\end{aligned}$$

Thus, the tax generates effects of the same nature as those stressed in the previous section (*i.e.* one through the deterministic part of the return, one through its volatility and one through the correlation with the other asset return) and finally, an increase in one of an increase in the tax rate may increase or reduce the certainty equivalent of the optimal portfolio depending on the relative magnitude of the three effects. We then have the following proposition :

Proposition 2 *On the relevant range for the tax levied on one of the assets, the certainty equivalent of the optimal portfolio is not necessarily monotonically decreasing with the tax.*

More precisely, it is shown in appendix 3 that it is first increasing and then decreasing, or possibly only increasing or decreasing depending on the relevant tax range, with the tax.

The effect of the tax on the optimal consumption applies through the certainty equivalent of the optimal portfolio rate of return (see equation (6)).

Corollary 3 *The direction of the effect of taxes on welfare is the same as the one of their effect on $\chi^*(t)$ (see equation (9)).*

Note that, as for uncertainty, the direction of the effect of the tax on asset 1 rate of return on the optimal consumption depends first on the effect of this tax on the certainty equivalent of the portfolio rate of return and second on the value of the intertemporal elasticity of substitution:

$$\frac{\partial C^*(t)}{\partial \tau_1} = (1-\varepsilon)W(t)\frac{\partial \chi^*(t)}{\partial \tau_1}$$

If an increase in the tax reduces the certainty equivalent of the portfolio rate of return, this in turn reduces (*resp.* increases) the optimal current consumption if the intertemporal elasticity of substitution is less (*resp.* greater) than one (the argument is exactly the same as the one developed when studying the effect of uncertainty on the optimal consumption).

Corollary 4 *If the intertemporal elasticity of substitution is less (resp. greater) than unity, the optimal consumption is first increasing (resp. decreasing) and then decreasing (resp. increasing) with the tax rates.*

Again, depending on the relevant range for the tax, the consumption may be only increasing or only decreasing with the tax rate.

4 Endogenous tax

In this section, we derive an optimal taxation scheme for the two risky assets. This might seem at first glance irrelevant because taxes are usually seen as distortionnary and are moreover not used here to increase welfare through the provision of public goods. Nevertheless, due to the various effects of the tax described in the preceding section, an optimal taxation scheme may exist.

4.1 The central planner program

Considering an endogenous taxation problem, the representative agent program remains the same. The central planner (who may be any of the two countries governments since nothing allows to distinguish between agents of each country) maximizes the value function of the representative agent with respect to the taxes on each asset.

$$\begin{cases} \max_{\tau_1,\tau_2} V(t) = \frac{\varepsilon}{\varepsilon-1}\left[\varepsilon\delta - (\varepsilon-1)\left(\chi^*(t)\right)\right]^{-1/\varepsilon} W(t)^{\frac{\varepsilon-1}{\varepsilon}} \\ \text{sc.} \left| \begin{array}{c} \chi^*(t) = \alpha(1-\tau_1)\omega(t)^* - \alpha(1-\tau_2)(1-\omega(t)^*) - \frac{1}{2}\gamma\sigma^2\left[(1-\tau_1)^2\omega(t)^{*2}\right. \\ \left. +2\rho_{1,2}(1-\tau_1)(1-\tau_2)\omega(t)^*(1-\omega(t)^*) + (1-\tau_2)^2(1-\omega(t)^*)^2\right] \\ \omega(t)^* = \frac{\alpha(\tau_2-\tau_1)-\gamma\sigma^2(1-\tau_2)\left[\rho_{1,2}(1-\tau_1)-(1-\tau_2)\right]}{\gamma\sigma^2\left[(1-\tau_1)^2-2\rho_{1,2}(1-\tau_1)(1-\tau_2)+(1-\tau_2)^2\right]} \end{array}\right. \end{cases}$$

The use of a recursive utility function highlights that this program only involves the portfolio optimally chosen by the agents which is not affected by the intertemporal elasticity of substitution; thus, the optimal tax rates on the risky assets returns are independent from the intertemporal elasticity of substitution.

The first order conditions leads to:

$$\begin{aligned}
\tau_1^*(\tau_2) &= \frac{\alpha(1-\tau_2)}{\alpha - \gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)} + 1 \\
\tau_2^*(\tau_1) &= \frac{\alpha(1-\tau_1)}{\alpha - \gamma\sigma^2(1+\rho_{1,2})(1-\tau_1)} + 1
\end{aligned} \tag{10}$$

In fact two solutions for each tax rate are obtained. Since there are no asymptotes in the functions $\chi(\tau_1)$ and $\chi(\tau_2)$, one of the solutions corresponds to a minimum and can therefore be eliminated[8]. If $\tau_1^* \in [0,1]$, it is a global maximum on this range since it can be shown that $\chi(\tau_1^*) > \chi(1)$ and $\chi(\tau_1^*) > \chi(0)$.

The optimal tax structure (τ_1^*, τ_2^*) is given by the system (10). In fact, the two relationships are identical so that the solution is obtained through only one equation. Therefore, there is no unique optimal tax structure but instead a infinite-sized set of optimal couples (τ_1^*, τ_2^*) given by one of the equations of (10). Importantly, even though assets share the same characteristics and only differ according to shocks on their return, the optimal taxation scheme is not necessarily one equalizing tax rates.

If taxes are optimal, we then obtain:

$$\omega(t)^* = 1 - \frac{\alpha}{\gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)}$$

Assets optimal shares are then between 0 and 1 as soon as

$$0 < \alpha < \gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)$$

Moreover, the condition ensuring that τ_1^* is between 0 and 1 is simply[9]:

$$0 < \alpha < \frac{\gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)}{2-\tau_2}$$

Note that considering public expenditures in the representative agent wealth would generate a wedge between the ways the two countries are affected by a particular tax, which would lead to a unique tax structure but would also introduce fiscal competition.

[8] This minimum corresponds to the case for which the representative agent does not want to hold both assets *i.e.* one of the share is zero (see appendix 2).

[9] Symetric conditions apply for τ_2^*.

4.2 Comparative statics

$$
\begin{aligned}
\frac{\partial \tau_1^*(\tau_2)}{\partial \sigma^2} &= \frac{\partial \tau_1^*(\tau_2)}{\partial \gamma} = \frac{\alpha\gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)^2}{\left[\alpha+\gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)\right]^2} > 0 \\
\frac{\partial \tau_1^*(\tau_2)}{\partial \alpha} &= \frac{-\gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)^2}{\left[\alpha+\gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)\right]^2} < 0 \\
\frac{\partial \tau_1^*(\tau_2)}{\partial \rho_{1,2}} &= \frac{\alpha\gamma\sigma^2(1-\tau_2)^2}{\left[\alpha+\gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)\right]^2} > 0
\end{aligned}
$$

The larger the uncertainty (or the risk aversion which enhances the effect of uncertainty), the larger the optimal tax in one country given that of the other country. This is explained by the fact that a larger uncertainty generates a stronger need for the insurance role of the tax. On the contrary, an increase in the deterministic part of the return makes this insurance less necessary and therefore implies a lower level of the optimal tax rate.

Finally, it may be shown that there exists a negative relationship between τ_1^* and τ_2 (or between τ_2^* and τ_1):

$$
\frac{\partial \tau_1^*(\tau_2)}{\partial \tau_2} = \frac{-\alpha^2}{\left[\alpha+\gamma\sigma^2(1+\rho_{1,2})(1-\tau_2)\right]^2} < 0
$$

5 Fiscal harmonization

Using the framework developed above, we now address the issue of fiscal harmonization. As an example, we proceed by considering that the agreement between the two countries implies that a common a symmetric tax structure is chosen as the average of the pre-harmonization one[10]. Formally, denoting τ^a the harmonized tax rate, we have $\tau_1^a = \tau_2^a = \tau^a$ with $\tau^a = \frac{\tau_1+\tau_2}{2}$ and τ_i corresponding to the pre-harmonization tax rate of country i.

5.1 Analytical results

Of course, starting from an optimal taxation scheme, fiscal harmonization can never be welfare enhancing. This is trivial since, in this case, harmonization implies to depart from a tax structure which has precisely been chosen to maximize welfare. This leads to the result that fiscal harmonization may not always be welfare enhancing even if assets exhibit the same characteristics (only shocks differ). Therefore in some cases a less constraining cooperation scheme such as

[10] Of course, many other forms of harmonization could be considered without altering the qualitative results.

the one currently favored by the EU is a choice with a clear economic justification. Nevertheless, there may be cases for which a fiscal harmonization allows to get closer to the tax structure that maximises welfare ; in such cases, a fiscal harmonization is welfare improving.

Starting from any pre-harmonization exogenous tax structure, we can define a set $S(\tau_1, \tau_2)$ such that harmonization increases welfare. Fiscal harmonization can therefore be considered as welfare improving for any initial tax structure belonging to this set.

Let us first derive the set of taxes starting from which a fiscal harmonization leaves welfare unchanged. Solving this problem yields two relationships linking τ_1 and τ_2:

$$\begin{aligned}
\tau_{2l}(\tau_1) &= \frac{-(A+B)}{\gamma\sigma^2(1+\rho_{1,2})} \quad \text{and} \quad \tau_{2u}(\tau_1) = \frac{-(A-B)}{\gamma\sigma^2(1+\rho_{1,2})} \\
\text{with } A &= 4\alpha - \gamma\sigma^2(1+\rho_{1,2})\left[3-\rho_{1,2}(1-\tau_1)-2\tau_1\right] \\
\text{and } B &= \sqrt{(1-\rho_{1,2})\left[\left(2\alpha-\gamma\sigma^2(1+\rho_{1,2})(1-\tau_1)\right)^2+\gamma^2\sigma^4(1+\rho_{1,2})^2(1-\tau_1)^2(1-\rho_{1,2})/2\right]}
\end{aligned}$$

One may show that, in the space (τ_1, τ_2), these frontiers are both decreasing with respect to τ_1, and that $\tau_{2l}(\tau_1)$ is concave while $\tau_{2u}(\tau_1)$ is convex. Moreover, at their intersection with the 45° line, both curves feature a slope of (-1).

Second, we show in appendix 4 that for any τ_1, the two frontiers define an interval $[\tau_{2l}, \tau_{2u}]$ of τ_2 *inside which* fiscal harmonization proves to be welfare improving. Consequently, $S(\tau_1, \tau_2)$ corresponds to the area defined by couples (τ_1, τ_2) simultaneously between 0 and 1 and the two frontiers.

In order to appraise how uncertainty, the correlation between the shocks on the assets returns and other parameters of the model (the assets rate of return, the risk aversion) affect the set $S(\tau_1, \tau_2)$, we approximate the curve $\tau_{2l}(\tau_1)$ and $\tau_{2u}(\tau_1)$ using a first order Taylor expansion around their intersection with the 45° line. Let us denote these approximations by $\widetilde{\tau}_{2l}(\tau_1)$ and $\widetilde{\tau}_{2u}(\tau_1)$. We have:

$$\widetilde{\tau}_{2l}(\tau_1) = (2-\tau_1) - \frac{4\alpha}{(1+\rho_{1,2})\gamma\sigma^2} \tag{11}$$

$$\widetilde{\tau}_{2u}(\tau_1) = (2-\tau_1) - \frac{4\alpha}{(3-\rho_{1,2})\gamma\sigma^2} \tag{12}$$

The horizontal distance between the two approximations is given by:

$$D = \widetilde{\tau}_{2u}(\tau_1) - \widetilde{\tau}_{2l}(\tau_1) = \frac{8\alpha(1-\rho_{1,2})}{(3-\rho_{1,2})(1+\rho_{1,2})\gamma\sigma^2}$$

Moreover we use $\widetilde{\tau}_{2u}(\tau_1)$ and $\widetilde{\tau}_{2l}(\tau_1)$ to approximate $S(\tau_1, \tau_2)$ by $\widetilde{S}(\tau_1, \tau_2)$[11]. Remember that $S(\tau_1, \tau_2)$ corresponds to the area between the two frontiers for tax rates between 0 and 1. It is then not true that when curves are getting closer, $S(\tau_1, \tau_2)$ decreases. This implies that in order to analyze the sensitivity of $\widetilde{S}(\tau_1, \tau_2)$ with respect to $\alpha, \sigma, \rho_{1,2}$ and γ, we must distinguish the following cases (expressions for these areas as well as illustrative figures are given in appendix 5):

- $\widetilde{S}(\tau_1, \tau_2) = \widetilde{S}_1(\tau_1, \tau_2)$ when only $\widetilde{\tau}_{2u}(\tau_1)$ contains couples of taxes with both elements between 0 and 1
- $\widetilde{S}(\tau_1, \tau_2) = \widetilde{S}_2(\tau_1, \tau_2)$ when both $\widetilde{\tau}_{2l}(\tau_1)$ and $\widetilde{\tau}_{2u}(\tau_1)$ contains couples of taxes with both elements between 0 and 1/2
- $\widetilde{S}(\tau_1, \tau_2) = \widetilde{S}_3(\tau_1, \tau_2)$ when $\widetilde{\tau}_{2l}(\tau_1)$ contains couples of taxes with both elements between 0 and 1/2 and $\widetilde{\tau}_{2u}(\tau_1)$ contains couples of taxes with both elements between 1/2 and 1.
- $\widetilde{S}(\tau_1, \tau_2) = \widetilde{S}_4(\tau_1, \tau_2)$ when both $\widetilde{\tau}_{2l}(\tau_1)$ and $\widetilde{\tau}_{2u}(\tau_1)$ contains couples of taxes with both elements between 1/2 and 1
- $\widetilde{S}(\tau_1, \tau_2) = \widetilde{S}_5(\tau_1, \tau_2)$ when only $\widetilde{\tau}_{2l}(\tau_1)$ contains couples of taxes with both elements between 0 and 1.

The following figure illustrate one of these cases (namely the one for which $\widetilde{S}(\tau_1, \tau_2) = \widetilde{S}_3(\tau_1, \tau_2)$) with $\widetilde{S}(\tau_1, \tau_2)$ being represented by the shaded area.

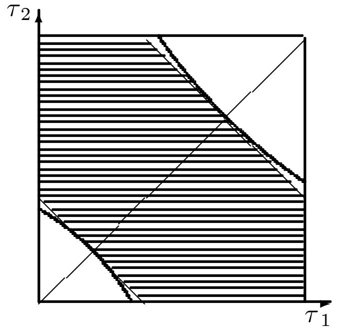

Figure 3: $S(\tau_1, \tau_2)$ and $\tilde{S}(\tau_1, \tau_2)$

The first derivative of these areas ($\widetilde{S}_1 ... \widetilde{S}_5$) with respect to the model parameters are given in table 1.

[11] Appendix 6 contains numerical simulations suggesting that both $S(\tau_1, \tau_2)$ and $\widetilde{S}(\tau_1, \tau_2)$ behave exactly in the same way with the model's parameters.

Table 1: Sensitivity of the welfare improving area with respect to the parameters

Sign of the first derivative of → with respect to ↓	$\widetilde{\tau}_{2l}(\tau_1)$	$\widetilde{\tau}_{2u}(\tau_1)$	D	$\widetilde{S}_1$	$\widetilde{S}_2$	$\widetilde{S}_3$	$\widetilde{S}_4$	$\widetilde{S}_5$
σ	+	+	-	+	+	+ for $\sigma^2 < \sigma_S^2$ - for $\sigma^2 > \sigma_S^2$	-	-
γ	+	+	-	+	+	+ for $\gamma < \gamma_S$ - for $\gamma > \gamma_S$	-	-
α	-	-	+	-	-	+ for $\alpha < \alpha_S$ - for $\alpha > \alpha_S$	+	+
$\rho_{1,2}$	+	-	-	-	-	-	-	-

with[12] $\alpha_S = \frac{\gamma(3-\rho_{1,2})^2(1+\rho_{1,2})\sigma^2}{4(5-2\rho_{1,2}+\rho_{1,2}^2)}$, $\sigma_S^2 = \frac{4\alpha(5-2\rho_{1,2}+\rho_{1,2}^2)}{\gamma(3-\rho_{1,2})^2(1+\rho_{1,2})}$, and
$\gamma_S = \frac{4\alpha(5-2\rho_{1,2}+\rho_{1,2}^2)}{(3-\rho_{1,2})^2(1+\rho_{1,2})\sigma^2}$

Considering the effect of an increase in uncertainty, we first note that the distance between the approximated frontiers becomes smaller while both frontiers move upwards. Moreover, an increase in the deterministic part of the return has exactly the opposite effect.

Proposition 5 *An increase in uncertainty or in risk aversion leads the set of welfare improving pre-harmonization taxes to contain higher tax rates.*

An increase in the deterministic rate of return leads the set of welfare improving pre-harmonization taxes to contain lower tax rates.

In other words, if uncertainty is strong, harmonization is likely to be worth on condition that intial tax rates are relatively high. This is quite intuitive since the larger the uncertainty the higher the optimal tax rate, while the larger the determinist part of the return, the smaller the optimal tax rate. Following an increase in uncertainty, the optimal frontier goes up, and the deviation from the optimal tax structure will thus be reduced by the fiscal harmonization for more "too high" tax rates and less "too small" tax rates. In the same way, following an increase in the determinist part of the return, the optimal frontier goes down and the deviation from it will get smaller thanks to the fiscal harmonization for more "too low" tax rates and less "too high" tax rates.

However this does not tell us much on how uncertainty affects the size of $S(\tau_1, \tau_2)$ or, in other words, the likelihood to observe a welfare improving fiscal harmonization. In fact is possible to rank $(\widetilde{S}_1...\widetilde{S}_5)$ depending on the values of

[12] Of course, these three expressions define the same relationship between the model parameters.

the model's parameters. In particular, $\widetilde{S}_1$ corresponds to small values of σ (or of γ), and when uncertainty (or the risk aversion) increases, $\widetilde{S}_2$, $\widetilde{S}_3$, $\widetilde{S}_4$, and finally $\widetilde{S}_5$ are reached in this precise order. For α, the exact opposite ranking is obtained. Using these properties and the results presented in table 1, we have the following proposition:

Proposition 6 *The probability for harmonization to be welfare improving for any couple of initial taxes between zero and unity is :*

- *first increasing and then decreasing with uncertainty, risk aversion and with the deterministic part of the assets return.*
- *monotonically decreasing with the correlation between the assets returns shocks.*

  Finally, this process is independent of the intertemporal elasticity of substitution.

Turning to welfare, consumption and growth, we should note that since the relationship between welfare and consumption depends on the value of the intertemporal elasticity of substitution relative to unity (*cf.* equation (6)), a welfare improving tax harmonization may not be consumption improving. Moreover, the expected value of the economy rate of growth may be written:

$$g = \frac{E_t(dW(t)/W(t))}{dt} = \underbrace{[\omega^*(1-\tau_1) + (1-\omega^*)(1-\tau_2)]\,\alpha}_{A} \underbrace{-\,[\varepsilon\delta - (\varepsilon - 1)\chi^*]}_{B}$$

On the one hand, it may easily be shown that a fiscal harmonization always reduces A. On the other hand, since both welfare and χ^* reacts in the same way to tax variations, a welfare improving fiscal harmonization may increase or reduce B depending on the value of the intertemporal elasticity of substitution with respect to unity. Thus, if the intertemporal elasticity of substitution is less than unity, a welfare improving fiscal harmonization is growth reducing.

5.2 Example based on France and Germany

To obtain results that may be directly interpreted as a way to appraise the desirability of a fiscal harmonization between European countries, one should extend the model to n countries. This would only be possible at the cost of giving up analytical resolutions and turning to numerical simulations, which is out of the scope of the paper since it aims at explaining the effect of taxes. In this section we only intend to give an illustration of the analytical results we derived. Let us consider two countries, Germany and France. Data for the returns of the risky asset in each country is the annual return index of Datastream (1973-2001). From these data, we derive an average deterministic rate of return: $\alpha = 12.35\%$, an average standard deviation $\sigma = 26.185\%$ and a correlation coefficient $\rho_{1,2} = 0.685$. Moreover, taxes on risky returns in France and in Germany are respectively (see Carey and Tchilinguirian (2000)): $\tau_F = 26.8\%$ and $\tau_G = 25.1\%$. This allows us to compute the optimal tax rates combinations as well as the frontiers $\tau_{2l}(\tau_1)$ and $\tau_{2u}(\tau_1)$ defining the welfare improving tax harmonization area and to compare their position with respect to the observed tax couple.

5.2.1 Optimal taxes

The location of the observed tax couple relative to the optimal tax structure $\tau_2^*(\tau_1)$ (or $\tau_1^*(\tau_2)$) is highly dependent on the value of γ. Using analytical results above, we have that the smaller (*resp.* larger) γ, the greater the chance for the observed taxes to be too high (*resp.* too low) compared to what would be optimal. The is confirmed by the analysis of the special case of Germany and France (see figure 1). If $\gamma = 2.5$ taxes need to be reduced to reach an optimal couple. The opposite applies for $\gamma = 3.5$. It may be computed that in the particular case of $\gamma = 2.95$, the observed tax structure coincides with an optimal one.

Insert figure 4

5.2.2 Welfare improving harmonization

It is rather intuitive that as for the optimal tax structure, the location of the observed tax couple relative to the frontiers $\tau_{2l}(\tau_1)$ and $\tau_{2u}(\tau_1)$ depends on the value of γ. For given values of the average deterministic return and volatility, we have seen that the frontiers move up (while the gap between them is decreasing) with γ. This is confirmed by figure 2 which represents these frontiers for $\gamma = 2.5$ and $\gamma = 3$. In fact, for values of γ in the range $[2.125, 2.925]$, it may be shown that a tax harmonization starting from the observed tax couple for France and Germany would be welfare improving.

Insert figure 5

6 Conclusion

In this paper, we developed a two-country model in order to analyze how fiscal harmonization impacts on economic growth and welfare through its effects on agents portfolio decisions. In such a framework, taxes affect growth because, as underlined by Smith (1996), they change the riskiness of disposable income. The interest to examine the impact of fiscal harmonization using this kind of approach is that it eliminates standard fiscal competition aspects to isolate alternative channels through which tax harmonization affects economic performance. We therefore derived the conditions under which a fiscal harmonization between the two countries considered proved to be growth and/or welfare enhancing. These conditions are related to the stochastic nature of the environment and in particular to the relevant characteristics of the assets contained in the agents' portfolio (after tax risk, after tax correlation between assets return) as well as the preferences of agents towards risk. The main results of the paper suggest (i) that an increase in uncertainty or in risk aversion (resp. in the deterministic rate of return) leads the set of welfare improving pre-harmonization taxes to contain higher (resp. lower) tax rates and (ii) that the probability for tax harmonization to be welfare improving is first increasing and then decreasing with uncertainty while it monotonically decreases with the correlation between the assets returns shocks. A direct application of these results is to highlight a policy choice such as the one faced by EU authorities between harmonization and a less demanding cooperation scheme : a less constraining cooperation scheme such as the one currently favored by the EU may be a choice with a clear economic justification.

7 References

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7.1 Appendix

Appendix 1: Model resolution

The program is:

$$V(t) = \max_{\omega(t),C(t)} \left[\frac{\varepsilon}{\varepsilon-1}C(t)^{\frac{\varepsilon-1}{\varepsilon}} + e^{-\delta dt}\left[E\left(V(t+dt)^{\frac{\varepsilon(1-\gamma)}{\varepsilon-1}}\right)\right]^{\frac{\varepsilon-1}{\varepsilon}\frac{1}{1-\gamma}}\right] \tag{13}$$

subject to the constraint of wealth evolution (5) and where $V((t))$ is the value function.

By analogy with the solution of this program when using a standard utility function, we solve this problem by postulating a value function of the form:

$$V(W(t)) = A\frac{\varepsilon}{\varepsilon-1}W(t)^{\frac{\varepsilon-1}{\varepsilon}}, \quad A>0 \tag{14}$$

Replacing in the Bellman equation (13), we obtain:

$$A\frac{\varepsilon}{\varepsilon-1}W(t)^{\frac{\varepsilon-1}{\varepsilon}} = \max_{\omega(t),C(t)} \left[\frac{\varepsilon}{\varepsilon-1}C(t)^{\frac{\varepsilon-1}{\varepsilon}} + e^{-\delta dt}A\frac{\varepsilon}{\varepsilon-1}\left[E\left(W(t+dt)^{(1-\gamma)}\right)\right]^{\frac{\varepsilon-1}{\varepsilon}\frac{1}{1-\gamma}}\right] \tag{15}$$

Using Itô’s lemma leads to:

$$\begin{aligned} E_t(W(t+dt)^{1-\gamma} &= \Big[(1-\gamma)\Big[[\omega(1-\tau_1)+(1-\omega)(1-\tau_2)]\,\alpha - \frac{C(t)}{W(t)} - \frac{1}{2}(1-\tau_1)^2\gamma\sigma^2\omega^2 \\ &\quad -\frac{1}{2}(1-\tau_2)^2\gamma\sigma^2(1-\omega)^2 - \rho_{1,2}\gamma\sigma^2(1-\tau_1)(1-\tau_2)\omega(1-\omega)\Big]\,dt+1\Big]\,W(t)^{1-\gamma} \end{aligned}$$

Replacing in the Bellman equation (15) and using a first order Taylor expansion, we obtain:

$$\begin{aligned} AW(t)^{\frac{\varepsilon-1}{\varepsilon}} &= \max_{\omega(t),C(t)} C(t)^{\frac{\varepsilon-1}{\varepsilon}} dt + AW(t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\frac{\varepsilon-1}{\varepsilon} \left[[\omega(1-\tau_1)+(1-\omega)(1-\tau_2)]\,\alpha - \frac{C(t)}{W(t)} - \frac{1}{2}(1-\tau_1)^2\gamma\sigma^2\omega^2 \right.\right. \\ & \left.\left. -\frac{1}{2}(1-\tau_2)^2\gamma\sigma^2(1-\omega)^2 - \rho_{1,2}\gamma\sigma^2(1-\tau_1)(1-\tau_2)\omega(1-\omega) - \frac{\varepsilon\delta}{\varepsilon-1} \right] dt + 1 \right] \qquad (16) \end{aligned}$$

The first order conditions give equation (7) of the text and $C(t)^*$:

$$C(t)^* = A^{-\varepsilon}W(t)$$

Note that maximizing the expected value of the intertemporal utility with respect to $\omega(t)$ reduces to the maximization of the certainty equivalent of the portfolio rate of return which is:

$$\begin{aligned} \chi(t) &= [\omega(1-\tau_1)+(1-\omega)(1-\tau_2)]\,\alpha - \frac{1}{2}(1-\tau_1)^2\gamma\sigma^2\omega^2 \\ & -\frac{1}{2}(1-\tau_2)^2\gamma\sigma^2(1-\omega)^2 - \rho_{1,2}\gamma\sigma^2(1-\tau_1)(1-\tau_2)\omega(1-\omega) \end{aligned}$$

Then, replacing $\omega(t)^*$ and $C(t)^*$ by their expressions in the Bellman equation (16) allows to identify A :

$$A = [\varepsilon\delta - (\varepsilon-1)\chi^*(t)]^{-1/\varepsilon}$$

with

$$\begin{aligned} \chi^*(t) &= \alpha(1-\tau_1)\omega(t)^* + \alpha(1-\tau_2)(1-\omega(t)^*) \\ & -\frac{1}{2}\gamma\sigma^2\left[(1-\tau_1)^2\omega(t)^{*2} + 2\rho_{1,2}(1-\tau_1)(1-\tau_2)\omega(t)^*(1-\omega(t)^*) + (1-\tau_2)^2(1-\omega(t)^*)^2\right] \end{aligned}$$

which leads to equation (6) in the text.

Appendix 2 :Relationship between ω^* and τ_1.

Let us define $f(\omega,\tau_1) = \frac{\partial\chi}{\partial\omega}$. Since $f(\omega,\tau_1)_{[\omega=\omega^*(\tau_1)]} = 0$, the implicit functions theorem implies:

$$df(\omega,\tau_1)_{[\omega=\omega^*(\tau_1)]} = \left(\frac{\partial f(\omega,\tau_1)}{\partial\tau_1}\right)_{[\omega=\omega^*(\tau_1)]} d\tau_1 + \left(\frac{\partial f(\omega,\tau_1)}{\partial\omega}\right)_{[\omega=\omega^*(\tau_1)]} \frac{\partial\omega^*(\tau_1)}{\partial\tau_1} d\tau_1 = 0$$

$$\Rightarrow \frac{\partial\omega^*(\tau_1)}{\partial\tau_1} = -\left(\frac{\partial f(\omega,\tau_1)}{\partial\tau_1}\right)_{[\omega=\omega^*(\tau_1)]} / \left(\frac{\partial f(\omega,\tau_1)}{\partial\omega}\right)_{[\omega=\omega^*(\tau_1)]}$$

Moreover, $\left(\frac{\partial f(\omega,\tau_1)}{\partial\omega}\right)_{[\omega=\omega^*(\tau_1)]} = \left(\frac{\partial^2\chi(\omega)}{\partial\omega^2}\right)_{[\omega=\omega^*(\tau_1)]} < 0$ since $\omega^*$is a maximum

$$\text{and finally : } \left(\frac{\partial f(\omega,\tau_1)}{\partial \tau_1}\right)_{[\omega=\omega^*(\tau_1)]} = \left(\frac{\partial^2 \chi}{\partial \tau_1 \partial \omega}\right)_{[\omega=\omega^*(\tau_1)]}$$

Appendix 3: Proof of proposition 2

It may easily be shown that $\chi(\omega^*,\tau_1)$ has no asymptotes and that $\lim_{\tau_1\to-\infty}\chi(\omega^*,\tau_1) = \lim_{\tau_1\to+\infty}\chi(\omega^*,\tau_1)$ and is finite. Moreover $\left(\frac{\partial \chi(\omega,\tau_1)}{\partial \tau_1}\right)_{[\omega=\omega^*(\tau_1)]} = 0$ exhibits two roots, with one such that $\omega^* = 0$. Moreover,

$$\left(\frac{d^2\chi(\omega^*(\tau_1),\tau_1)}{d\tau_1^2}\right)_{[\omega=\omega^*(\tau_1)]} = \underbrace{\left(\frac{\partial^2\chi(\omega,\tau_1)}{\partial\tau_1^2}\right)_{[\omega=\omega^*(\tau_1)]}}_{A} + \underbrace{\left(\frac{\partial\chi(\omega,\tau_1)}{\partial\tau_1\partial\omega}\right)_{[\omega=\omega^*(\tau_1)]}\frac{\partial\omega^*(\tau_1)}{\partial\tau_1}}_{B}$$

Since $\left(\frac{\partial \chi(\omega,\tau_1)}{\partial \tau_1\partial\omega}\right)_{[\omega=\omega^*(\tau_1)]}$ and $\frac{\partial\omega^*(\tau_1)}{\partial\tau_1}$ (see appendix 2) always have the same sign, we deduce that $B \geq 0$. We also have $A = -\gamma\sigma^2\omega^*$.

Thus, for $\omega^* = 0$, $\left(\frac{d^2\chi(\omega^*(\tau_1),\tau_1)}{d\tau_1^2}\right)_{[\omega=\omega^*(\tau_1)]} \geq 0$ which implies that this root for $\left(\frac{\partial \chi(\omega,\tau_1)}{\partial \tau_1}\right)_{[\omega=\omega^*(\tau_1)]} = 0$ corresponds to a minimum of $\chi(\omega^*,\tau_1)$ while the other root corresponds to a maximum.

Since $\omega^* = 0$ has only one root (see section 3.1), we may deduce that we only keep one side of $\chi(\omega^*,\tau_1)$ with respect to its minimum to eliminate cases in which $\omega^* < 0$. Thus we obtain that it is not possible to have $\chi(\omega^*,\tau_1)$ decreasing and then decreasing with τ_1 in the relevant range for the tax.

Appendix 4: Defining $S(\tau_1,\tau_2)$

The objective is to determine whether $S(\tau_1,\tau_2)$ corresponds to the area between or outside the two frontiers $\tau_{2l}(\tau_1)$ and $\tau_{2u}(\tau_1)$. One easily shows that both $\tau_2^*(\tau_1)$ and $\tau_{2l}(\tau_1)$ cross the 45° line at the same point with a (-1) slope. Moreover, at this point $\frac{\partial^2\tau_2^*(\tau_1)}{\partial\tau_1^2} < \frac{\partial^2\tau_{2l}(\tau_1)}{\partial\tau_1^2}$ which imply that at least one point of $\tau_2^*(\tau_1)$ is below $\tau_{2l}(\tau_1)$. Since by definition, all couples (τ_1,τ_2) belonging to the optimal tax locus $\tau_2^*(\tau_1)$ are outside $S(\tau_1,\tau_2)$, it follows that it is the area *between* the two frontiers which is welfare improving.

Appendix 5: Expressions and illustrations for $\widetilde{\mathbf{S}}_1(\tau_1,\tau_2), \widetilde{\mathbf{S}}_2(\tau_1,\tau_2), \widetilde{\mathbf{S}}_3(\tau_1,\tau_2), \widetilde{\mathbf{S}}_4(\tau_1,\tau_2)$ and $\widetilde{\mathbf{S}}_5(\tau_1,\tau_2)$

Using (11) and (12) we obtain that:

- If $\widetilde{\tau}_{2u}(\tau_1)$ contains couples of taxes with both elements between 0 and 1/2: $\widetilde{\mathbf{S}}_1(\tau_1,\tau_2) = 2\left[\frac{2\alpha+(\rho_{1,2}-3)\gamma\sigma^2}{(\rho_{1,2}-3)\gamma\sigma^2}\right]^2$

If $\widetilde{\tau}_{2u}(\tau_1)$ contains couples of taxes with both elements between 1/2 and 1: $\widetilde{\mathbf{S}}_1(\tau_1,\tau_2) = 1-2\left[\frac{2\alpha}{(3-\rho_{1,2})\gamma\sigma^2}\right]^2$

- $\widetilde{\mathbf{S}}_2(\tau_1,\tau_2) = 2\left[\left(\frac{2\alpha+(\rho_{1,2}-3)\gamma\sigma^2}{(\rho_{1,2}-3)\gamma\sigma^2}\right)^2 - \left(\frac{-2\alpha+(\rho_{1,2}+1)\gamma\sigma^2}{(\rho_{1,2}+1)\gamma\sigma^2}\right)^2\right]$
- $\widetilde{\mathbf{S}}_3(\tau_1,\tau_2) = 1-2\left[\frac{4\alpha+(\rho_{1,2}-3)\gamma\sigma^2}{(3-\rho_{1,2})\gamma\sigma^2}\right]$
- $\widetilde{\mathbf{S}}_4(\tau_1,\tau_2) = 2\left[\left(\frac{2\alpha}{(1+\rho_{1,2})\gamma\sigma^2}\right)^2 - \left(\frac{2\alpha}{(3-\rho_{1,2})\gamma\sigma^2}\right)^2\right]$
- If $\widetilde{\tau}_{2l}(\tau_1)$ contains couples of taxes with both elements between 0 and 1/2: $\widetilde{\mathbf{S}}_5(\tau_1,\tau_2) = 1-2\left(\frac{-2\alpha+(\rho_{1,2}+1)\gamma\sigma^2}{(\rho_{1,2}+1)\gamma\sigma^2}\right)^2$

  If $\widetilde{\tau}_{2l}(\tau_1)$ does not contains couples of taxes with both elements between 0 and 1/2: $\widetilde{\mathbf{S}}_5(\tau_1,\tau_2) = 2\left(\frac{2\alpha}{(1+\rho_{1,2})\gamma\sigma^2}\right)^2$

Figures 6 to 12 illustrate these cases.

Appendix 6: Appraising the error due to the approximation

The analytical results obtained in section 5 have been derived using a linear approximation. In order to appraise whether ignoring the curvature of both curves affects these results we now turn to numerical simulations.

We consider the evolution of the welfare improving initial taxes area successively, depending on one of the different model parameters, the other parameters being fixed to the values we used in the example based on France and Germany. The result appearing on figures 13 to 16 is that both the real area and the approximated one behave exactly in the same way when the model parameters vary.

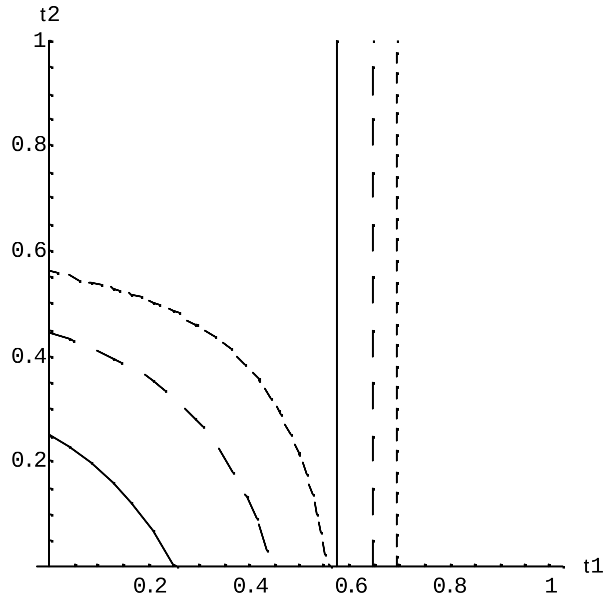

Figure 4: solid line : $\gamma = 2.5$; big-dashed line : $\gamma = 3$; small-dashed line $\gamma = 3.5$

Appendix 6: Approximations $S(\tau_1,\tau_2)$

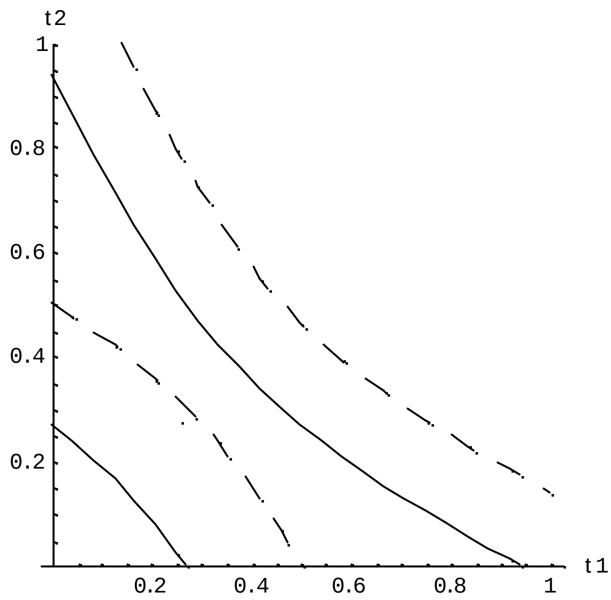

Figure 5: Solid line : $\gamma = 2.5$; dashed-line $\gamma = 3$

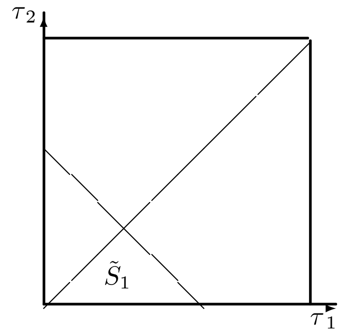

Figure 6: $\tilde{S}_1(\tau_1, \tau_2)$

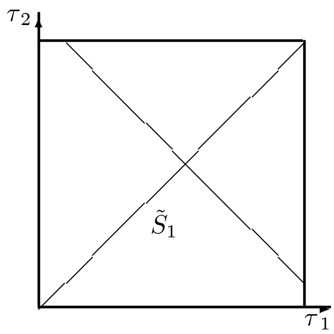

Figure 7: $\tilde{S}_1(\tau_1, \tau_2)$

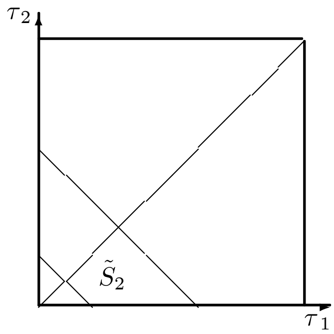

Figure 8: $\tilde{S}_2(\tau_1, \tau_2)$

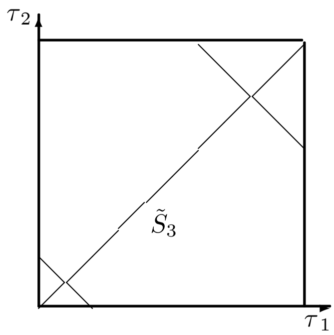

Figure 9: $\tilde{S}_3(\tau_1, \tau_2)$

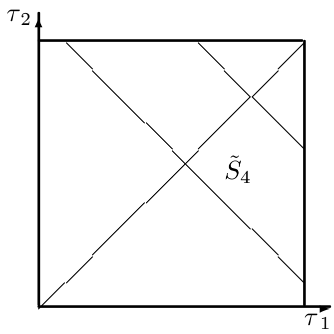

Figure 10: $\tilde{S}_4(\tau_1, \tau_2)$

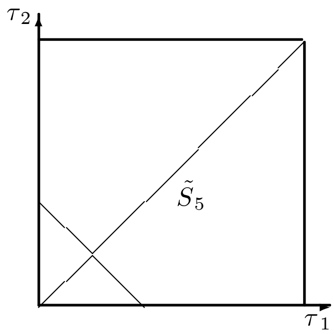

Figure 11: $\tilde{S}_5(\tau_1, \tau_2)$

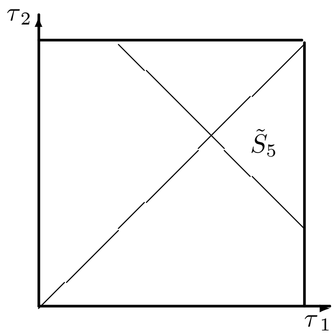

Figure 12: $\tilde{S}_5(\tau_1, \tau_2)$

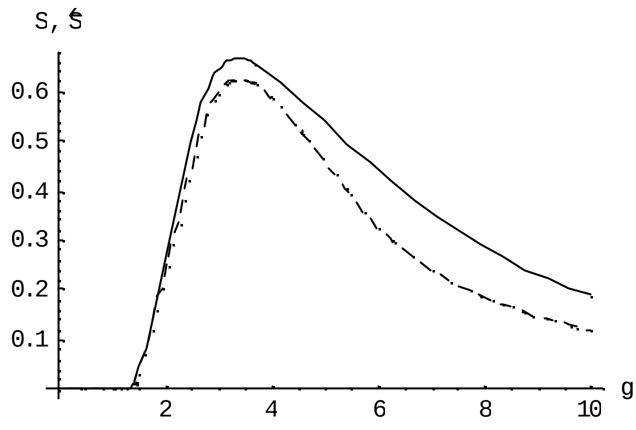

Figure 13:

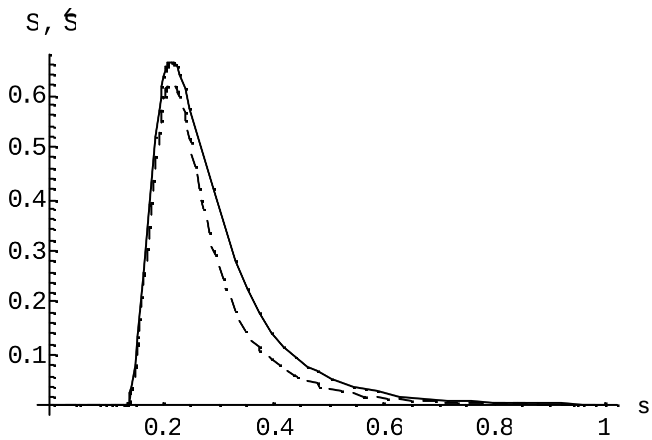

Figure 14:

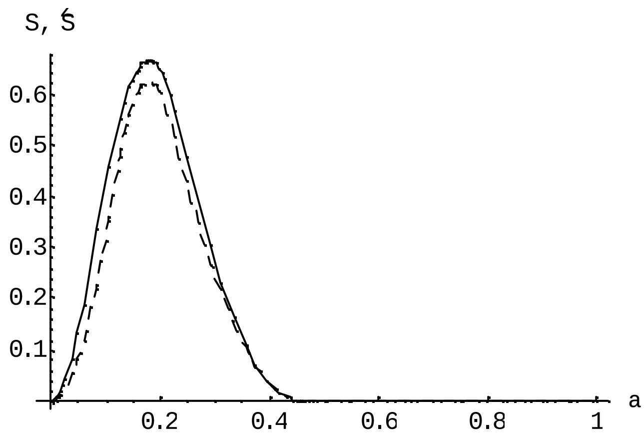

Figure 15:

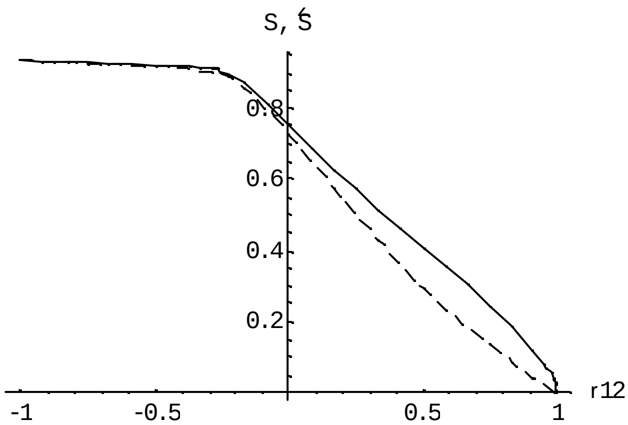

Figure 16: