MODELING FINANCIAL FRAGILITY IN TRANSITION ECONOMIES

Jeffrey H. Nilsen

Studienzentrum Gerzensee and *Copenhagen Business School*

Tel. +41.31.780.3108; Fax +41.31.780.3100

e-mail: jhn@szgerzensee.ch

Riccardo Rovelli

University of Bologna

Tel. (+39) 051 .6402 .601; Fax (+39) 051 .6402 .664

e-mail: rovelli@spbo.unibo.it

This version: July 27, 1999

Abstract

Capital inflows have an enormous importance in the financing of investment in emerging and transition economies. However short-term inflows, intermediated by the banking sector of the emerging economy, may be subject to early withdrawals. We model a situation where such withdrawals are motivated by a change in either the domestic or the foreign fundamentals. We show that, for a given change in fundamentals, a reversal in the capital flows (and hence a currency crisis) is more likely the more risk averse are the foreign investors into the emerging economy. We also show that a policy to tax early withdrawals may discourage capital inflows which are more likely to give rise to fundamental runs, by helping to select relatively less risk averse investors. However, such a policy would have to be fine tuned in order not to discourage all capital inflows.

Keywords: Short-term capital inflows, Currency crisis, Financial Fragility, Chilean tax.

JEL codes: G21, G28, O16, 023.

This research was undertaken with support from the European Union's Phare-ACE Programme 1996, Project No. P96-6114-R.

Both authors thank the Studienzentrum Gerzensee for providing logistic support during this research. We also thank seminar participants at the 1998 ASSET Conference at the University of Bologna for useful comments.

1. Introduction

In the last six years, emerging market economies have been affected by a long series of financial crises, starting with Mexico in 1994-95, followed by the crisis in Asia in 1997, in Russia in 1998 and in Brazil in 1999. Each of these crises has its own history and determinants. However, they also share some common factors: they all demonstrated the "potential for sharp changes in investor sentiment", triggered in most cases by some combination of unsustainable external imbalance, overvalued exchange rate, financial fragility, and unsustainable fiscal policy. The elements to be considered in the etiology, prevention and intervention in the case of a financial crisis are numerous and interrelated. In this paper we concentrate on one crucial step in the development of many such crises: the interplay between financial fragility and deteriorating economic fundamentals. More precisely, we want to concentrate on the analysis of a single factor of crisis, and on a single possible cure. The factor we want to focus on is the financing of capital accumulation through short-term capital inflows; the candidate therapy is a tax on short term inflows (a so-called *Chilean tax*).

We propose a simple unified theoretical framework to evaluate these issues. While extremely simple, our model is useful to analyze one aspect of how a Chilean tax works: not simply by deterring short term capital inflows, but more precisely by *helping to select less risk averse lenders*. In fact we shall show that, ceteris paribus, a financial crisis is more likely to occur the more risk averse are the investors entering into an emerging market economy: by selecting out such investors, the tax renders a crisis less probable, for any given configuration of the "fundamentals" which characterize the emerging economy.

The paper is organized as follows. In section 2 we review the background motivation for our analysis. In section 3 we set up the basic model which we shall use to examine the interaction between foreign investors and the financial and industrial sector of an emerging economy. In section 4 we examine under which conditions foreign investment will take place, and in section 5 we characterize those conditions under which a run will take place. In section 6 we evaluate alternative policies to avoid such runs. Section 7 concludes.

2. The role of short-term capital inflows

Although most remarks in this paper would apply to emerging economies in general, we focus more precisely on the analysis of economies in transition in Central and Eastern Europe (EIT). Lack of domestic savings is a constraining factor in the growth of developing economies, and of transition economies in particular. Inflows of foreign capital may help sustain a phase of intensive capital accumulation. Several factors may however deter foreign investments:

- *Lack of an appropriate financial environment*. For instance, sometimes direct foreign ownership in EIT is discouraged. Also, monitoring portfolio equity investments in local firms may be difficult for a foreigner, possibly due to insufficient transparency of local stock markets. Issuing and purchasing bonds in a domestic bond market may also be difficult, as these markets are often at an initial stage of development. From the point of view of EIT, even if “footloose” funds may be less desirable than direct investments, they are still better than nothing.

- *Demand for liquidity*. While acknowledging the need for long term investments, investors may want to keep their options open, in case of either a sudden worsening of the political perspectives in the EIT or a sudden change of the financial outlook in their home country or in the rest of the world. Thus, demand for liquidity is often an integral part of foreign investors' requirements.

- *Risk of devaluation.* Once an investment has been made, will the foreign investor be able to repatriate her investment into her own currency? Will a sudden devaluation reduce the value of the investment in terms of her consumption basket? Or might the repayment be rescheduled or partially written-off due to balance of payments problems?

Often, the first two issues direct foreign investors to local banks, for they can provide solutions to both the monitoring and liquidity problems. As for the foreign exchange risks, although one can hedge the currency exposure in forward markets, it is much more difficult to insure oneself against rescheduling and write-offs.

Building upon these considerations, our analysis models foreign investments as channeled through bank deposits in the EIT. We obviously do not neglect the importance of other financial instruments, however we aim at identifying within the simplest framework what we perceive to be the single most critical aspect of foreign investments into EIT: that of funds which are committed for less than the required period of investment, and that need to be repaid into a foreign currency, which may ultimately be acquired only as a result of the success of that investment.

That this is a relevant issue is now widely acknowledged. As Radelet and Sachs (1998) observe: "Much of the economic activity supported by the capital inflows was highly productive, and the loss of economic activity resulting from the sudden and enormous reversal in capital flows has been enormous". And on a related note, a recent OECD report states that: “ensuring that banks are reasonably sound may not be sufficient to prevent self-fulfilling crises: it is also important to ensure that banks are not exposed to liquidity crises. ... Short-term maturity debt is risky because it increases the potential magnitude of capital outflows. ... It is the ratio of short-term debt to international reserves that matters ... . In some instances a case can be made for limiting short-term capital inflows through taxes on capital imports, foreign deposit reserve requirements, or similar measures, but global financial integration is driven by forces that cannot easily be reined in and that generally carry many benefits” (OECD, 1999, pp.83-87).

There is a sense in which the normative implications addressed in the second quotation above are rather obvious: disincentives to invest into short-term versus longer-term debt would obviously mitigate liquidity problems (provided they do not deter investment altogether). In the model to be developed below, however, we want to point out a rather less obvious, but nevertheless desirable, feature of these disincentives: that of reducing the degree of risk aversion of investors.

3. The model

As the analysis will soon become inherently complicated, we design the model in the simplest possible way. Thus we assume only three agents in the world economy: the productive sector in the EIT ("the firm"); the consolidated central bank and commercial banking sector in the EIT ("the bank") and foreign investors. The central bank holds foreign reserves as a counterpart to the monetary base (bank reserves) but does not exercise any independent monetary policy action, as the structure of domestic rates is a function of technology and of investors preferences. The size and composition of the current and capital account depend on the capital accumulation decisions of domestic firms. There is no fiscal policy and no foreign debt of the government. While these omissions obviously reduce the overall “realism “ of the model, the main point of the analysis will stand up more clearly in this simplified setting.

The overall setup and discussion of the model takes places through the following sequence:

a) The firm demands a 2-period loan from the bank, to finance its demand for investment
b) In order to finance the loan, the bank collects deposit from foreign investors
c) The deposit contract is of the Diamond-Dybvig type. It is set for 2 periods, with an option to withdraw after 1 period
d) Foreign investors also have the outside option to invest in the foreign market
e) New information on fundamentals of the domestic economy accrues in period 1. Under the new circumstances, some foreign investors who would not otherwise have done so decide to seek liquidity, hence to withdraw in period 1
f) A “run” is equivalent to the decision to withdraw in period 1 by depositors who would otherwise have stayed in for 2 periods
g) A run causes a currency crises, as the bank is subsequently unable to face its obligation towards foreign investors

Having examined these issues, we conclude by addressing one specific policy issue: for a given change in fundamentals, as in point (e) above, can a “Chilean tax” reduce the likelihood that it will result in a run on the domestic currency?

We examine points (a-c) in this section, point (d) in section 4, points (e-g) in section 5, and we address the analysis of the "Chilean tax" in section 6. [1]

The firm. Domestic production starts off with no initial resources. Thus the firm requires initial finance (time = 0) to purchase imported inputs. The production process is long (2 periods) and totally illiquid in the intermediate (time = 1) period. Each unit of investment at time 0 yields a total return R > 1 at time 2. Thus the periodic rate of return is the square root of (R-1).

Output produced at time 2 is exported. Proceeds from exports are used to repay the initial foreign investment.

[1] Two recent papers also use a framework derived from Diamond and Dybvigs's to address the issues of financial fragility in emerging markets. Chang and Velasco (1997), differently from us, characterize the domestic residents of the emerging economy as Diamond-Dybvig risk-averse investors. Goldfajn and Valdes (1997) model runs in a way similar to ours, and emphasize the role of liquidity-creating intermediaries in facilitating runs, but they do not analyze the role of investors' risk aversion in this context, which is the main focus of our analysis.

Foreign inflows are the only possible source of funds for financing domestic production. While this is an extreme assumption, it helps us to concentrate on what we perceive to be the main ingredient in our story. No additional insight would be gained by having a parallel domestic source of funds. Moreover, it is quite realistic to assume that EIT need a large amount of investments to rebuild their capital stock, and this largely exceeds the amount of domestic savings.

Foreign investors. Foreign investment takes the form of deposits with the domestic bank. Deposits inflow at time 0, and may be left for one or two periods, with no pre-commitment. A proportion $0 < \varepsilon < 1$ of all depositors will seek reimbursement at time 1, receiving the pre-agreed sum of r_1 in domestic currency per unit of deposit. The remaining depositors are expected to be reimbursed at time 2, and to receive $(r_2)^2$ per unit of initial deposit (see below for more on the characteristics of depositors).

The bank. The bank at time = 0 receives foreign currency from foreign depositors. This sum is in part converted into local currency and loaned to the firm, which however uses it to purchase foreign currency in order to pay for imported inputs. The part of the initial deposits that are not loaned to the firm is kept as (foreign currency) reserves in the consolidated banking system. At time = 1, the bank uses reserves to pay back depositors wishing to withdraw early [2, 3].

The consolidated balance sheet of the bank is defined by the following three equations (with inflows on the left, outflows on the right):

(1.1) $t_0: \quad D_0 = F_0 + L_0$

(1.2) $t_1: \quad r_1^* F_0 = \varepsilon\, r_1\, D_0$

(1.3) $t_2: \quad R\, L_0 = (r_2)^2 (1-\varepsilon)\, D_0$

where subscripts refer to the time period and :

D = foreign deposits,

---

[2] We could alternatively assume that the bank may also finance these intermediate-deposit withdrawals borrowing (on the world financial market, at the going riskless rate, r_2^*) on account of the proceeds from final-period exports. This strategy would not improve on the one described in the text.

[3] The role of the central bank, as described here, is similar to that of a currency board. In fact, we could assume that the central bank creates additional high-powered money as counterpart to, say, loans to the banking sector, in order to finance domestic credit creation. However, no additional insights would be gained by adding such realistic complication to our model.

F = foreign currency reserves;

L = loan to domestic firms (also equal to imports at time 0)

r = one-period total return to domestic deposits (1 + interest rate)

r*= one-period total return to foreign deposits

R = two-period total return to investments

Eqs. (1.1)-(1.3) are defined in domestic currency. They can be easily written in foreign currency terms by deflating both sides with the current nominal exchange rate (e_i ; $i = 0, 1, 2$, where e is the price of one euro in EIT currency). Also note that, since all and only foreign transactions are intermediated through the bank, the same equations also define the balance of payments of the economy across the three periods.[4]

Determination of interest rates on domestic deposits. In order to evaluate the resource constraint in the final period, substitute (1.1) and (1.2) in (1.3), to obtain:

$$R\left(1-\frac{\varepsilon\, r_1}{r_1^*}\right)=(r_2)^2(1-\varepsilon)$$

that is:

(2) $$(r_2)^2=\frac{R}{1-\varepsilon}\left(1-\frac{\varepsilon\, r_1}{r_1^*}\right)$$

Also, we assume now but shall justify later (see section 4) the following relationship between domestic and foreign returns:

(3) $R \geq r_1^* r_2^*$, with both $r_1^*,\ r_2^* \geq 1$.

In order to find explicit solutions for r_1 and r_2 (for parametric r_1^*, R, and ε) we need to model foreign investors' characteristics and preferences. We assume, following Diamond and Dybvig (1983), that foreign investors can be either "patient" (i.e., they want to withdraw and consume at time 1) or "impatient" (they prefer to withdraw and consume at time 2). Their type becomes privately known before the beginning of time 1, that is, after time 0 decisions have been made

[4] From the balance of payments perspective, D_0 is the capital inflow, F_0 is the capital outflow and L_0 are imports, all at time 0. At time 1, $r_1^*F_0$ is the capital and interest inflow and $\varepsilon\, r_1\, D_0$ is the capital

but before making time 1 (withdrawal/reinvestment) decisions. This is however private information unobservable to others. There is no aggregate uncertainty concerning the proportion of impatient vs. patient investors (ε vs. $1-\varepsilon$). Apart from their type (which is unknown at time 0) investors are homogeneous: in particular they all have the same endowment and the same degree of aversion to risk. Later we shall assume that investors differ also in their aversion to risk..

If investors were *risk neutral*, they would maximize the expected return to their portfolios, thus choosing $r_1 = \varepsilon\, r_1 = 0$. Hence impatient types would not obtain a return. In this case, there would be no need to allow for "liquidity" contracts, i.e. for intermediate withdrawals. Financial fragility would never become an issue (within this model).

Let us then turn to the more interesting case of *risk averse investors*. Assume the consolidated banking system to include a central bank and a competitive banking system. Banks would then act as if they were maximizing investors' expected utility [5]. Thus:

$$\text{(3)} \qquad \text{Max}\left[\varepsilon\, U(\frac{r_1}{e_1}) + (1-\varepsilon)\, U[\frac{(r_2)^2}{e_2}]\right]$$

Substitute eq.(2) in (3). We have:

$$\text{(4)} \qquad \text{Max}\left[\varepsilon\, U(r_1) + (1-\varepsilon)\, U\left(\frac{R}{1-\varepsilon}\left\{1-\varepsilon\ {r_1}\big/{r_1^*}\right\}\right)\right]$$

and the associated FOC:

$$\text{(5)} \qquad U'(r_1) - U'[(r_2)^2]\cdot\frac{R}{r_1^*} = 0$$

To obtain an explicit solution, assume U(.) to have CRRA. Thus:

---

and interest outflow. Finally, at time 2, $R\, L_0$ are exports and $(1-\varepsilon)\,(r_2)^2\, D_0$ is the capital and interest outflow.

[5] This can be justified, given that competitors face the same budget constraints, either because domestic bank competition forces them to offer contractual conditions which cannot be improved (and to zero profits) or because the investor chooses among investments of many EIT, thus forcing locally monopolistic banking sectors to behave competitively vis-à-vis banks of other EIT.

(6) $$U(r) = \frac{r^{\gamma}}{\gamma} \quad ; \quad \gamma < 1 \qquad \leftrightarrow \qquad RRA = 1 - \gamma$$

In this case, (5) becomes:

(7) $$(r_1)^{\gamma - 1} + \left(\frac{R}{1-\varepsilon} \left\{ 1 - \varepsilon \; r_1 \big/ r_1^* \right\} \right)^{\gamma - 1} \frac{R}{r_1^*} = 0$$

Solving for the first period interest rate we obtain:

(8) $$r_1 = \frac{r_1^*}{\varepsilon + (1-\varepsilon) \left(R \big/ r_1^* \right)^{\gamma/(1-\gamma)}}$$

which we can then use in (2) to solve for the second-period interest rate.

Properties of deposit contract. The competitive (utility maximizing) contract is obtained by sequentially solving eqs. (8) and (2).[6] Similarly to Diamond and Dybvig (1983) we notice that, for RRA>1 (greater than log utility)

$$r_1^* < r_1 < r_2 < R$$

We also notice by inspection of eq. (8) that the first period interest rate (r_1) is:

- decreasing in ε
- increasing (but less than one-for-one) in both R and r_1^*
- increasing as risk aversion increases (γ decreases).

On the other hand, from eq. (2), upon substitution of eq.(8), we notice that r_2 decreases with greater risk aversion. Hence there exists a degree of risk aversion (1 - γ°) in correspondence to which the two interest rates are equal to each other (See von Thadden, 1997). For γ < γ° the term structure of deposit rates slopes downwards

The analytical expression for γ° is rather complicated and by itself uninteresting (see the Appendix, A.1). Numerical values of γ° are quite reasonable, however. For example, using equation (A.1), we find that:

---

[6] Note that, in assessing the degree of risk aversion, a benchmark case is that of the log utility function. In this case, relative risk aversion is equal to 1, and this is equivalent to assuming γ = 0 in equation (6).

- with $\varepsilon = .3$, $R = 1.3$, and $r_1^* = 1.0$, then $\gamma° = -1.463$ and $r_1 = r_2 = 1.112$
- with $\varepsilon = .2$, $R = 1.2$, and $r_1^* = 1.05$, then $\gamma° = -.546$ and $r_1 = r_2 = 1.09$

We comment below on the implications that an inverted term structure would have in our setup.

4. To invest or not to invest?

At time 0 all foreign investors face the following opportunities, among which they choose so as to maximize their expected utility:

1. stay out of country: obtain r_1^* (and, if of patient type, then also r_2^* later)
2. invest (deposit) in the transition country for one period and:
   - 2.1. <u>either</u> withdraw and disappear (if impatient), after getting r_1/e_1 (in foreign currency)
   - 2.2. <u>or</u> (if patient) reinvest for a second period according to one of the following three alternatives :
     - 2.2.1. get r_1/e_1 and reinvest in same country, getting (for both periods) $r_1\, r_1/e_2$
     - 2.2.2. get r_1/e_1 and reinvest in foreign market, getting (for both periods) $(r_1/e_1)\, r_1^*$
     - 2.2.3. stay invested in deposit contract, getting for the whole period $(r_2)^2/e_2$.

Since at time = 0 all investors have identical risk aversion and endowments, they will make the same decision. (They differ by type, but do not discover this until time = 1). Assume that a deposit contract of the type analyzed in the previous paragraph is being offered, with interest rates determined on the basis of eqs. (8) and (2) respectively. We call such contract a Diamond-Dybvig (D-D) contract. In particular, this allows for the possibility of <u>either</u> an intermediate withdrawal (time = 1) with total return r_1 <u>or</u> a final withdrawal with total return $(r_2)^2$, at the demand of the depositor.

We are now interested, first, to identify the set of parameter for which such a contract shall be proposed and accepted. Second, by appropriately modifying the model in equation (4), and assuming that the contract has been accepted on the basis of a given expected distribution of the future variables (R, r_2^*) we want to see whether observing the realized values of these variables might induce depositors of the patient type to reconsider their choice and withdraw in the intermediate period.

In this section we examine the first issue, while the second issue is discussed in section 5. Here we ask under which circumstances will the following decisions be made:

- *to __invest__ in the EIT,* and
- (conditional on having invested and on being of patient type) *to expect to __continue__ the investment throughout the second period ?*

The following *constraints*, to be evaluated at time 0, ensure a positive answer to both questions:

- **C.1**: do not stay out: $EU(r_1, r_2) > EU(r_1^*, r_2^*)$
- **C.2**: do not go short: $EU(r_1, r_2) > EU(r_1 , r_1)$ => $r_2 \geq r_1$
- **C.3**: expect to stay in: $EU(r_1, r_2) > EU(r_1 , r_2^*)$ => $(r_2)^2 > r_1 r_2^*$

If these constraints are satisfied, the foreign deposit (and the corresponding capital inflow) will take place and domestic production will be financed. This process will be interrupted only if a run (unexpected at time 0) will occur at time 1. Before we discuss this case, however, we must find under which conditions the constraints C.1-C.3 will be satisfied. This is done below.

Assumptions:

- **A.0** $r_1^*, r_2^* \geq 1$
- **A.1** $1 > \gamma \geq 0$ and $R > r_1^* r_2^* > 1$
- **A.2** $0 > \gamma \geq \gamma°$ and $R > r_1^* (r_2^*)^{1-\gamma} > 1$

Proposition 1.

If assumptions A.0 and __either__ A.1 __or__ A.2 are satisfied, then constraints C1-C.3 are also satisfied.

Proof (sketch):

- A.0 is required in order to assure that investing in the foreign market is preferable than holding currency throughout
- Rearrange C.1 and notice that it is satisfied iff $R > r_1^* r_2^*$.
- Observe that C.2 is always satisfied for $1 > \gamma > 0$. Also, as we show in eq. (A.1) in the Appendix, it is always possible to find a value $\gamma° < 0$ such that, for all $\gamma \geq \gamma°$, C.2 is satisfied, and for all $\gamma < \gamma°$ C.2 is not satisfied. Hence $\gamma \geq \gamma°$ is required to satisfy C.2.

- Rearrange C.3 and observe that it is satisfied as long as $R > r_1{*}(r_2{*})^{1-\gamma}$. This is more stringent than the requirement for C.1 only if $\gamma < 0$.
- Hence the joint satisfaction of all three constraint requires assumption A.0 and either A.1 or A.2 to be satisfied.

QED

5. To run or not to run?

In order to discuss whether the good equilibrium of the model, as characterized so far, will in fact be realized, or whether instead a run on the currency will occur, it is first useful to summarize how events and decisions occur along the time line. See Table 1.

Table 1. The time line

Time 0	Firm makes initial investment decision and demands loan to the bank. If {C.1, C.2, C.3} satisfied, then foreign investors make deposit at the bank. Loan is given to firm, and production starts.
Time 1- -	Type of investors becomes privately known
Time 1-	*Patient* depositors observe realized values of uncertain exogenous variables (R, $r_2{*}$)
Time 1	Patient depositors either confirm decision to invest until period 2 or decide to withdraw. If also (some) *patient-types withdraw*, then *run* occurs and, since not enough foreign currency is available at time 1, currency devalues. Production continues in all cases.
Time 2	Production realized and exported Exports are used to pay depositors (at the same exchange rate which had been expected at time 0, provided a run has not occurred)

In particular, we are interested to know for which parameter values a deposit which was made at time 0 by an investor who turned out (at time 1--) to be of the "patient" type will be maintained in the second period or will instead be withdrawn on the basis of the information observed at time 1-. Moreover, for all cases where a run would happen, we want to enquire whether it could have been avoided if alternative policies had been adopted. However, we leave this last matter to section 6. Here instead we concentrate on analyzing the "illness": how the accrual of information at time 1- might change investors perspectives, and induce a "run" out of the EIT which was not expected at time 0.

At time 1- new information may be observed relatively to two different set of events. New information may arrive, relative to:

- profitability of the real investment in the EIT : R
- state of the world in period 2: r_2*

If either of these variables is uncertain, then the Max problem analyzed so far must be modified, and the corresponding solution for r_1 will in general be different (specifically, equation (8) will no longer be valid). Equation (2) will continue to be valid in expectation. Constraints C1-C.3 will also have to be evaluated to take into account that EU is now a function of both the variability of investors' types and of *either* future profitability *or* future state of the world (or both).

For tractability, we assume a binomial distribution (for either R or r_2*), taking one source of uncertainty at a time. Below, we define these two cases separately (sections 5.1 and 5.2 respectively). Then, in section 5.3, we state the main proposition on the likelihood that a run will occur after a change in fundamentals is observed.

5.1 Uncertainty in final outcome

In order to accommodate uncertainty in the realized value R in our model, we must reformulate the Max problem, substituting equation (4) with the following:

(4')

$$\mathrm{Max}\left[\varepsilon\,U(r_1)+(1-\varepsilon)\left\langle q\,U\left(\frac{R^u}{1-\varepsilon}\left\{1-\varepsilon\ r_1\!\Big/\!r_1^*\right\}\right)+(1-q)\,U\left(\frac{R^d}{1-\varepsilon}\left\{1-\varepsilon\ r_1\!\Big/\!r_1^*\right\}\right)\right\rangle\right]$$

where R^u , R^d are the possible values which the final return may assume, with probabilities q and (1-q) respectively.

Note that, as a result of this uncertainty, r_2 will *de facto* be indexed to the realized value of R. This might seem counterfactual, given the usual specification of deposit contracts. However it need not be so if one interprets the value of $r_2(R^d)$ as either the result of a partial write-off of debt or as being isomorphic to a devaluation of the currency which would occur if too little output had been produced for foreign debt to be repaid on the basis of a constant (from time 0 to time 2) exchange rate.

As a result of the uncertainty on r_2, the decision to "stay in" the transition economy will have to be reassessed at time 1, on the basis of the realized value of r_2. If this is "low", then it may become more convenient for "patient" types to withdraw and reinvest for the second period in the world market. They will instead confirm their investment in the EIT if the following constraint is satisfied:

- **C.4.1:** continue to stay in: $(r_2^d)^2 > r_1 r_2^*$

5.2 Uncertainty in the world interest rate

The role of external factors in precipitating banking crises in emerging markets has been acknowledged by several authors. For instance, Eichengreen and Rose (1998) find the Northern interest rates are strongly associated with the onset of banking crises in developing countries. In this section, we extend our model to take account of this occurrence. In particular, we introduce some volatility in the second period value of the foreign interest rate, r_2^*. To do this, we retain equation (4) but we must modify the specification of the constraints which depend on the value of r_2^*. Thus, for constraint C.1, we specify the EU of staying out as:

$$\text{(9)} \qquad EU\,(r_1^*, r_2^*) = \varepsilon\, U(r_1^*) + (1-\varepsilon) \left\langle p\, U\left(r_2^{*u}\right) + (1-p)\, U\left(r_2^{*d}\right) \right\rangle,$$

where we have assumed, similarly to the previous section, a binomial distribution for the foreign interest rate in the second period.

Assuming constraints C.1-C.3 (the former specified so as to take (9) into account) to be satisfied, the question is: as time 1- occurs and the realized value of r_2^* is observed, will the decision to stay invested in the EIT be confirmed by individuals whose type is "patient"? Of course, the question is relevant only in case of a "high" realized value, i.e. r_2^{*u}. If patient investors "stay in", they will obtain - with certainty - $(r_2)^2$. If instead they withdraw, they will obtain the total compound return $r_1 r_2{*}^u$. Thus the stay in constraint at time 1 is:

- **C.4.2:** continue to stay in: $(r_2)^2 > r_1 r_2{*}^u$

5.3 Likelihood of a run upon observing new information

We first define a fundamental run as follows:

Definition: Fundamental Run

A fundamental run occurs when {C.1, C.2, C.3} are satisfied and either C.4.1 or C.4.2 are not.

Proposition 2.1

A fundamental run in case of uncertain profitability of the real investment is more likely:

- *the more variable is R*
- *the more risk averse are the investors.*

Proof*:* See Appendix, A.2.

Proposition 2.2

A fundamental run in case of uncertain value of world interest rates is more likely:

- *the more variable are world interest rates*
- *the more risk averse are the investors.*

Proof*:* See Appendix, A.3.

In order to get an intuition about what is happening in this case, consider the examples presented in Table 2. We examine three cases, all characterized by uncertainty in r_2*: this can take the values 1.2 or 1.0 with equal probability (equivalent to an interest rate of 20% and 0 respectively). The three cases differ only for the proportion of "impatient" investors: respectively 0.1, 0.5, 0.9. Risk aversion and all the other parameters are instead constant. We see that in the first too cases the capital inflow will not take place: constraint C.2 is binding in both cases. Instead, in the third case, (ε = 0.9) the inflow will take place, but it will be reversed (a run will occur) if the "bad" scenario (or high world interest rates, r_2* = 20%) materializes.

Table 2. Example: Uncertain world interest rates, with or without a run

Data:				***rstar2***		
rstar1	*1.1*	***pr(rstar2=high)***	*0.5*	***-high***	*1.2*	
R	*1.5*			***-low***	*1*	
gamma	*-1.5*					
epsilon	*0.1*					
Domestic rates:		***Expected Utility of:***		***Constraints:***		
r1	1.29843	***(r1,r2)***	-0.38173	**C.1**	0.13391	
r2	1.21241	***(r1*, r2H*, r2*L)***	-0.51564	**C.2**	-0.08602	X
E(r2*)	1.10000			**C.3**	0.04166	
RRA	2.50			**C.4**	-0.08818	

Data:				***rstar2***		
rstar1	*1.1*	***pr(rstar2=high)***	*0.5*	***-high***	*1.2*	
R	*1.5*			***-low***	*1*	
gamma	*-1.5*					
epsilon	*0.5*					
Domestic rates:		***Expected Utility of:***		***Constraints:***		
r1	1.20206	***(r1,r2)***	-0.46290	**C.1**	0.08039	
r2	1.16655	***(r1*, r2H*, r2*L)***	-0.54329	**C.2**	-0.03551	X
E(r2*)	1.10000			**C.3**	0.03857	
RRA	2.50			**C.4**	-0.08164	

Data:				***rstar2***		
rstar1	*1.1*	***pr(rstar2=high)***	*0.5*	***-high***	*1.2*	
R	*1.5*			***-low***	*1*	
gamma	*-1.5*					
epsilon	*0.9*					
Domestic rates:		***Expected Utility of:***		***Constraints:***		
r1	1.11900	***(r1,r2)***	-0.55364	**C.1**	0.01731	
r2	1.12552	***(r1*, r2H*, r2*L)***	-0.57094	**C.2**	0.00652	
E(r2*)	1.10000			**C.3**	0.03590	
RRA	2.50			**C.4**	-0.07600	R

6. Policies to prevent currency runs

In section 5, we have identified how, for certain parameter values, a "fundamental" run may take place. Is it possible to prevent, at least up to reduce the likelihood of such events, without getting into some kind of radical therapy (à la Mahatir) - i.e. without discouraging capital inflows *tout court*? In general, several policies might be considered:

- tax (on an ex-ante basis) short-run inflows, or impose a reserve requirement on such deposits
- increase reserves ex ante, i.e. reduce lending to firms
- borrow in intermediate period to finance withdrawals, and repay in final period by taxing realized output (which also gives to the taxing authority access to the foreign exchange obtained from exports).

Note that in each one of these three cases the resource constraint (eq.2) will be changed accordingly. In particular in the latter case the constraint would become more “lax”, as the policy of borrowing to finance withdrawals allows to reduce, or in the limit to eliminate initial reserves. This in turns yields an increased amount of output in period 2.

The most interesting policy suggestion is certainly the first one mentioned above. Thus we propose to examine it analytically within the framework of our model.

6.1 Chilean tax

We define as a "Chilean tax", in our context, a tax imposed on the return paid to foreign investors who withdraw their deposit after the first period (either because they turn out to be "impatient", or because they are patient but have observed unfavorable information at time 1-. In our context, such a tax may be structured in either of two ways:

- The tax reduces the amount to be paid to first period withdrawals, hence no reserves are held against this tax, hence the proceeds from the tax are kept invested for two periods in the firm.
- Reserves must be equal to amount of gross first period interest, of which net interest is paid to depositors while the tax is paid to government (consolidated with the central bank), which invests it at the foreign rate r_1*.

In the first case the tax is arbitraged away, since the contract is endogenous to it. Hence we consider only the second case. Proposition 3 establishes our results in evaluating the effects of the tax on the structure of interest rates and checks that the tax does not discourage investments (by making the constraints C1-C.3 binding).

Proposition 3.

A Chilean tax has the following effects:

- *lowers r_1 (net of tax)*
- *for any given other parameter values, makes the constraint C.1 more binding*
- *hence, may discourage foreign investments that would otherwise have occurred had the tax been nil.*

***Proof**:* See Appendix, A.4.

The result in Proposition 3 might seem rather obvious (despite the lengthy proof). However, it should really be seen as a preliminary result towards a more subtle, or more thought-provoking statement (which however requires a change in the scenario of our model, towards a more realistic set of assumptions). Suppose that we relax the assumption of homogenous investors (apart from their type). Assume instead that foreign investors do not have the same degree of aversion to risk; for simplicity, let us have two types of investors, characterized by high (H) or low (L) risk aversion.

Now conduct the following experiment. Take a situation in which both H and L investors are willing to invest in the EIT. Introduce a Chilean tax, and gradually increase the tax rate. The tax will affect the constraints of both groups of investors. But whose constraints will start biting first, those of the H or of the L investors? And is the answer to this question related to the results stated in Propositions 2.1 and 2.2? The answer, as shown in the statement of Proposition 4, turns out to be rather favorable.

Proposition 4. *A Chilean tax helps to select the less risk averse investors*

Proof. See Appendix, A.5.

Discussion. Figure 1 shows the highest tax for which H accepts the contract (it remains acceptable to L) for given $r_1^* = r_2^* = 1, \ R = 1.5$ (the graph does not change appreciably with other values of r_1^*, r_2^*, or R). With higher ε, a lower tax rate separates the H and L investors. A higher ε places greater weight on first period, taxed, consumption in the utility of

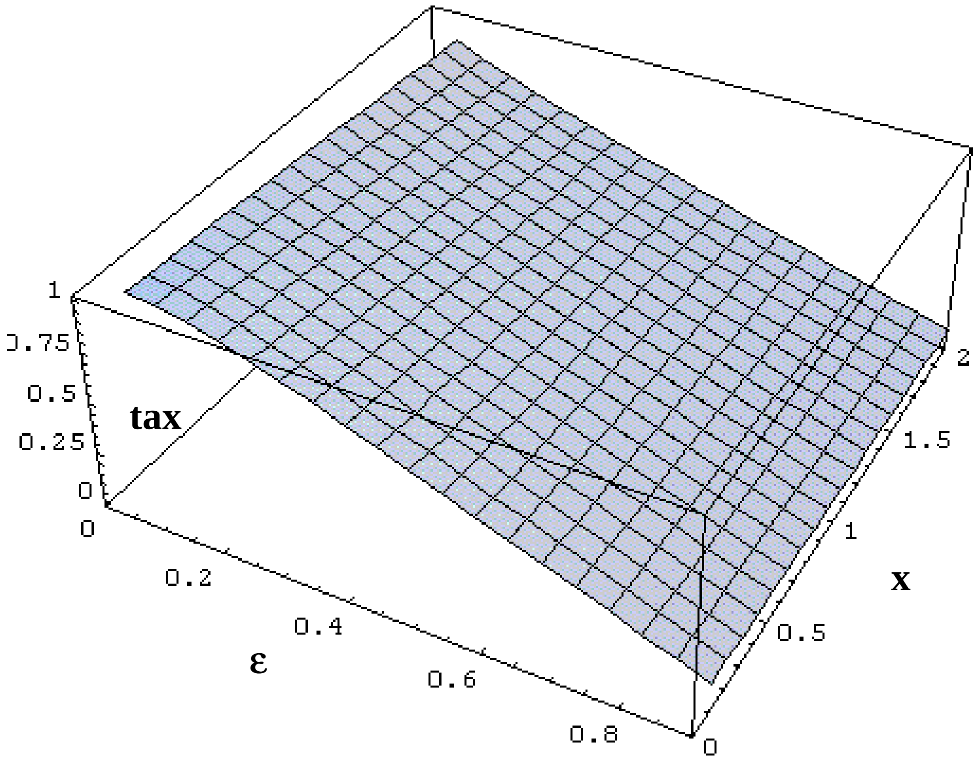

Figure 1

investors. Thus a rise in the tax will decrease utility more strongly than when second period consumption is more heavily weighted.

The larger is x, the difference in risk aversion between the H and L investors, the lower the tax. Since we offer investors the combination of rates optimal for those investors whose CRRA approaches 0, x reflects the H investor's risk aversion coefficient. The larger x thus reflects that it is easier to prevent an investor with a higher degree of risk aversion from accepting the rate package. Note that with x < .2, the investors' risk-aversion coefficients are insufficiently different for the tax to separate the investors. The tax is also impotent below ε of approximately .05.

In Table 3, we develop some numerical examples to calculate the lowest tax which is required to induce the H or L investor's indifference. For H, use equation (A.29) or equation (A.23). To calculate the lowest tax inducing investor L indifference, use the limit of equation (A.24) as $\gamma_L \to 0$, i.e.: $\ln\left(\frac{R}{r_1^*}\right) - \varepsilon \ln\left(\frac{R - r_1^* r_2^* t}{r_1^*(1-t)}\right) - (1-\varepsilon)\ln\left(r_2^*\right)$. In this case, the column γ_H entries are the same as -x (see the text of the proof, in the Appendix, A.5). Also recall that the analysis has been carried out using a tax rate applied to gross interest rates. To find the net tax rate that yields an equal after-tax return, solve the following equation:

$(1+r)(1-t_G) = 1 + r(1-t_N)$, and find: $t_N = \frac{(1+r)}{r} t_G$. This is, however, deceptive since the pre-tax rate r_1 rises with a higher tax. Thus, we include the third and sixth columns in the following table which show the lowest r_1 rate acceptable to the H and L investors, respectively.

Table 3. Choice of tax rates to select out investors with different aversion to risk

***Parameter values*:**					***Tax s.t. H rejects***	***Pre-tax r_1 when H rejects***	***H's lowest $r_1(1-t)$***	***Tax s.t. L rejects***	***Pre-tax r_1 when L rejects***	***L's lowest $r_1(1-t)$***
R	r_{1*}	r_2*	ε	γ_H						
1.5	1.0	1.0	.3	-1.5	.65	1.77	.62	.83	2.22	.39
1.5	1.05	1.05	.3	-1.1	.68	2.11	.67	.80	2.54	.51
1.3	1.0	1.0	.7	-3.6	.22	1.21	.94	.34	1.35	.89
1.3	1.05	1.05	.7	-1.5	.27	1.37	.99	.33	1.45	.98
1.5	1.0	1.0	.7	-3.7	.19	1.15	.93	.36	1.32	.84
1.5	1.0	1.1	.7	-3.7	.16	1.13	.95	.35	1.34	.88
1.5	1.1	1.0	.7	-1.2	.29	1.40	.99	.35	1.48	.96

The γ_H entry in each line corresponds to the risk aversion of the most risk averse investor accepting a contract optimally designed for herself according to constraint C.2. That is, the value in the table is identical to γ^0 cited in proposition 1, calculated by setting $r_1 = r_2$ given that $t = 0$. Note that γ^0 is given as a benchmark only since the contract offers rates optimal for the investor with risk aversion coefficient 0. Note that in this case, the second period rate is always R.

To understand numerically the implications of Proposition 4, let us examine for instance the fourth line of the Table. With world interest rates at 5% and a 2 period compound return in the EIT of 30%, and a fairly high proportion of impatient investors (70%), we see that imposing a tax rate on first-period withdrawals equal to, say, 30%, is sufficient to discourage the more risk averse investors, while leaving the less risk averse still wanting to invest in the EIT. The wedge between the two tax rates (the one that discourage the H investors and the one that discourages the L investors) increases with the difference in attitudes towards risk. From the examples it is also apparent that, when the proportion of impatient investors is low (30%), then only implausibly high tax rates would do the job (see the first 2 rows of the table).

We can now state, without proving, a corollary of Proposition 4:

Corollary.

A Chilean tax may help to reduce the likelihood of a fundamental run.

This follows from Proposition 4 together with Proposition 2.1 and 2.2.

Discussion

The main objection to a Chilean tax is, of course, that it may throw away the baby with the hot water, i.e. discourage capital inflows *tout court*. Thus the main problem in devising such a tax is to fine tune it so that it will discourage only the most run-prone (i.e., the more risk-averse) investors. However, as we have shown in Table 3, there is room for such tuning, especially if the differences in the attitude to risk of different investor groups are wide enough.

One aspect which we have not discussed so far is the possibility that investors have different probabilities of facing a liquidity need in period 1. In this case, it may be desirable from the point of view of the EIT to attract only one type of investors, or to offer different contracts. A model of this sort has been examined by Smith (1984) in the context of a closed economy. This would be a worthwhile extension of our model, and we leave this issue to future research.

7. Conclusions

Although capital inflows have a crucial importance in the financing of investment in emerging and transition economies, they also play a crucial role in the developing of financial crises. In this paper we have focused on the role of short-term inflows, intermediated by the banking sector of the emerging economy, in the financing of domestic production in emerging and transition economies. It is in the nature of such inflows to be subject to the possibility of early withdrawals. We have modeled a situation where such withdrawals are motivated by a change in fundamentals: this can be of a domestic (a change in the productivity of invested capital) or foreign (a change in world interest rates) origin. In either case, such a change in fundamentals may induce the withdrawal of foreign deposits that would otherwise have been invested for a longer period, thus activating a banking and currency crisis (a "run").

Our main result is that, for a given change in fundamentals, a reversal in the capital flows (and hence a currency crisis) is more likely the *more risk averse* are the foreign investors into the emerging economy. Essentially, more risk averse investors demand higher first-period returns to deposits (at the cost of relatively lower second period returns). This makes them more likely to reconsider their decision to continue to invest in a country during a second period, after the release of unfavorable information.

We have also shown that a policy to tax early withdrawals (a "Chilean tax") may discourage capital inflows which are more likely to give rise to fundamental runs, by helping to select relatively less risk averse investors. However, such a policy would have to be fine tuned in order not to discourage all capital inflows. In particular we have shown, under reasonable hypotheses, that the range of values for which a tax on early withdrawals may help to discriminate between less and more risk averse investors may well be in the range of 20-30%, that is within admissible values.

REFERENCES

Chang, R. and A. Velasco, 1997, *Financial fragility and the exchange rate regime*, FRB of Atlanta W.P. 97-16, November.

Diamond, D.W. and P.H. Dybvig, 1983, *Bank runs, deposit insurance, and liquidity*, Journal of Political Economy, 91, 401-419.

Eichengreen, B. and A.K. Rose, 1998, *Staying afloat when the wind shifts: external factors and emerging-market banking crises*, CEPR D.P. 1828, April.

Goldfajn, I. and R.O. Valdés, 1997, *Capital flows and the twin crises: the role of liquidity,* IMF WP/97/87, July

OECD, 1999, *World Economic Outlook*, May.

Radelet, S. and J. Sachs, 1998, *The onset of the East Asian Financial Crisis,* mimeo, March.

Smith, B., 1984, *Private information, deposit interest rates, and the 'stability' of the banking system*, Journal of Monetary Economics, 14, 293-317.

von Thadden, E.-L., 1997, *The term-structure of investment and the banks' insurance function*, European Economic Review, 7, 1355-1374.

APPENDIX

A.1 Value of γ^o in section 2

To set $r_1 = r_2$, use equations (2) and (8):

$$(r_2)^2 = \frac{R}{1-\varepsilon}\left(1 - \frac{\varepsilon\, r_1}{r_1^*}\right) = \frac{r_1^{*2}}{\left[(1-\varepsilon)\left(\frac{R^D}{r_1^*}\right)^{\frac{\gamma_0}{(1-\gamma_0)}} + \varepsilon\right]^2}$$

Rearrange this condition into a form easily to translate into the quadratic formula.

$$\left[(1-\varepsilon)\left(\frac{R^D}{r_1^*}\right)^{\frac{\gamma_0}{(1-\gamma_0)}} + \varepsilon\right]^2 - \varepsilon\left[(1-\varepsilon)\left(\frac{R^D}{r_1^*}\right)^{\frac{\gamma_0}{(1-\gamma_0)}} + \varepsilon\right] - \frac{r_1^{*2}(1-\varepsilon)}{R} = 0$$

This yields the following value for γ^o (the other, negative, root is imaginary):

(A.0) $$\gamma^o = \frac{\ln\left(\frac{\left(\varepsilon^2 + \frac{4r_1^{*2}(1-\varepsilon)}{R}\right)^{\frac{1}{2}} - \varepsilon}{2(1-\varepsilon)}\right)}{\ln\left[\frac{R}{r_1^*}\right] + \ln\left(\frac{\left(\varepsilon^2 + \frac{4r_1^{*2}(1-\varepsilon)}{R}\right)^{\frac{1}{2}} - \varepsilon}{2(1-\varepsilon)}\right)}$$

A.2 Proof of Proposition 2.1 (Uncertain profitability of real investment)

First, we transform constraint C.4.1 into terms of parameters only. We then use the equation as basis to prove a more binding constraint in the case of a) higher variance and b) more risk averse investors.

Maximizing equation (4') with respect to r_1 and solving yields the following expression:

(A.1) $$r_1 = \frac{r_1^*}{(1-\varepsilon)\left[q\left(\frac{R^U}{r_1^*}\right)^{\gamma} + (1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\right]^{\frac{1}{(1-\gamma)}} + \varepsilon}$$

Constraint C.4.1 requires satisfying the following expression:

(A.2) $$\frac{R^D}{(1-\varepsilon)}\left(\frac{r_1^* - \varepsilon r_1}{r_1^*}\right) > r_1 r_2^*$$

Isolating the r_1 term in (A.2) and substituting expression equation (A.1) for r_1 yields:

(A.3)

$$R^D r_1^* > \frac{r_1^*}{(1-\varepsilon)\left[q\left(\frac{R^U}{r_1^*}\right)^{\gamma} + (1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\right]^{\frac{1}{(1-\gamma)}} + \varepsilon}\left[r_1^* r_2^*(1-\varepsilon) + R^D\varepsilon\right]$$

Divide both sides by r_1*, multiply both sides by the denominator to obtain:

(A.4)

$$R^D(1-\varepsilon)\left[q\left(\frac{R^U}{r_1^*}\right)^{\gamma} + (1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\right]^{\frac{1}{(1-\gamma)}} + R^D\varepsilon > r_1^* r_2^*(1-\varepsilon) + R^D\varepsilon$$

Subtract $R^D\varepsilon$ from both sides and divide by (1-ε) to get:

(A.5) $$R^D\left[q\left(\frac{R^U}{r_1^*}\right)^{g} + (1-q)\left(\frac{R^D}{r_1^*}\right)^{g}\right]^{\frac{1}{(1-g)}} > r_1^* r_2^*$$

Now take logs and rearrange:

(A.6) $$\mathrm{Ln}\left[q\left(\frac{R^U}{r_1^*}\right)^{\gamma} + (1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\right] - (1-\gamma)\left[\mathrm{Ln}(r_1^* r_2^*) - \mathrm{Ln}(R^D)\right] > 0$$

We can now prove both parts of the Proposition.

Part a). We show that constraint C.4.1 is more likely to bind with an increase in variance.

The variance of the binomial return distribution assumed in equation (4'), is easily shown to be: $Var(R) = q(1-q)(R^U - R^D)^2$. To show that constraint C.4.1 becomes more binding (i.e. the left hand side is reduced) we impose a mean preserving spread on R, thus increasing its variance. That is, we increase R^U and decrease R^D to maintain the original mean of the distribution, $qR^U + (1-q)R^D$. For this to occur, $dR^U = -\frac{(1-q)}{q}dR^D$.

Take the derivative of (A.6) w.r.t. R^D and R^U and substitute out dR^U according to the restriction that the mean remains the same:

(A.7)
$$\frac{\left[\frac{(1-q)\gamma}{r_1^*}\left[\left(\frac{R^D}{r_1^*}\right)^{\gamma-1} - \left(\frac{R^U}{r_1^*}\right)^{\gamma-1}\right]\right]dR^D}{q\left(\frac{R^U}{r_1^*}\right)^{\gamma} + (1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}} + \frac{(1-\gamma)}{R^D}dR^D > 0$$

Now, combine in the numerator:

$$\frac{\left[\left(\frac{R^D}{r_1^*}\right)^{\gamma-1}\left(\frac{(1-\gamma)(1-q)}{R^D}\left(\frac{R^D}{r_1^*}\right) + \frac{(1-q)\gamma}{r_1^*}\right) + \left(\frac{R^U}{r_1^*}\right)^{\gamma-1}\left(\left(\frac{R^U}{r_1^*}\right)\frac{(1-\gamma)q}{R^D} - \frac{(1-q)\gamma}{r_1^*}\right)\right]dR^D}{q\left(\frac{R^U}{r_1^*}\right)^{\gamma} + (1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}} > 0$$

Factoring:

(A.8)
$$\frac{\left[\left(\frac{R^D}{r_1^*}\right)^{\gamma-1}(1-q) + \left(\frac{R^U}{r_1^*}\right)^{\gamma-1}\left(\left(\frac{R^U}{R^D}\right)(1-\gamma)q - (1-q)\gamma\right)\right]dR^D}{r_1^*\left(q\left(\frac{R^U}{r_1^*}\right)^{\gamma} + (1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\right)} > 0$$

To understand why (A.8) is positive, note that the denominator is always > 0. For $\gamma \le 0$, the numerator > 0. For the case $1 > \gamma > 0$, let $R^U = k\,R^D$, where $k \ge 1$. Rearrange (A.8):

(A.9)
$$\frac{\left[\left(\frac{R}{r_1^*}\right)^{\gamma-1}(1-q)+\left(\frac{kR}{r_1^*}\right)^{\gamma-1}\left(kq+\gamma(q(1-k)-1)\right)\right]dR^D}{r_1^*\left(q\left(\frac{R}{r_1^*}\right)^{\gamma}+(1-q)\left(\frac{kR}{r_1^*}\right)^{\gamma}\right)}>0$$

It is now easy to see that while the right hand term in the numerator may be negative, the term is largest in absolute value when gamma = 1, and this is when the numerator is equal to 0.

For example, in the simplest case set $R^U = R^D$ prior to applying the mean preserving spread, and find that equation (A.8) simplifies to:

$$\frac{(1-\gamma)dR^D}{R}>0$$

QED

Part b) We show that an increase in risk aversion, i.e. a decline in γ , decreases the left hand side of equation (A.6), hence it makes it more difficult to satisfy constraint C.4.1.

Take the derivative of (A.6) with respect to γ.

(A.10)
$$\frac{\left[q\left(\frac{R^U}{r_1^*}\right)^{\gamma}\ln\left(\frac{R^U}{r_1^*}\right)+(1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\ln\left(\frac{R^D}{r_1^*}\right)\right]d\gamma}{\left(q\left(\frac{R^U}{r_1^*}\right)^{\gamma}+(1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\right)}+\left[\ln(r_1^*r_2^*)-\ln(R^D)\right]d\gamma>0$$

Bring terms under the common denominator and factor out the terms to the γ power.

(A.11)
$$\frac{\left[q\left(\frac{R^U}{r_1^*}\right)^{\gamma}\left[\ln\left(\frac{R^U}{r_1^*}\right)+\ln(r_1^*r_2^*)-\ln(R^D)\right]+(1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\left[\ln\left(\frac{R^D}{r_1^*}\right)+\ln(r_1^*r_2^*)-\ln(R^D)\right]\right]d\gamma}{\left(q\left(\frac{R^U}{r_1^*}\right)^{\gamma}+(1-q)\left(\frac{R^D}{r_1^*}\right)^{\gamma}\right)}>0$$

Simplifying:

(A.12)

$$\frac{\left[q\left(\frac{R^{U}}{r_1^{*}}\right)^{\gamma}\left[\ln\left(\frac{R^{U}}{R^{D}}\right)+\ln(r_2^{*})\right]+(1-q)\left(\frac{R^{D}}{r_1^{*}}\right)^{\gamma}\left[\ln(r_2^{*})\right]\right]d\gamma}{\left(q\left(\frac{R^{U}}{r_1^{*}}\right)^{\gamma}+(1-q)\left(\frac{R^{D}}{r_1^{*}}\right)^{\gamma}\right)} > 0$$

QED

A.3 Proof of Proposition 2.2. (Uncertain value of world interest rates in period 2)

Note first that in the case that the central bank holds foreign exchange reserves, a stochastic r_2^* will not alter the utility maximizing choice of r_1 and r_2 selected by agents relative to the non-stochastic case.

Constraint (C.1) will be altered at time 0, but the assumption $R > r_1^* r_2^*$ ensures the constraint does not bind to the extent that the expected value in the stochastic case is the same as the value of r_2^* in the deterministic case.

In *part a),* we show that constraint (C.4.2) is more likely to bind with a higher variance of r_2. In *part b)* we show it is more likely to bind when investors are more risk-averse.

Constraint C.4.2 requires satisfying the following expression:

(A.13) $$\frac{R}{(1-\varepsilon)}\left(\frac{r_1^{*}-\varepsilon r_1}{r_1^{*}}\right) > r_1 r_2^{U*}$$

Isolating the r_1 term in (A.13) and substituting expression equation (A.1) for r_1 yields:

(A.14) $$Rr_1^{*} > \frac{r_1^{*}}{(1-\varepsilon)\left(\frac{R}{r_1^{*}}\right)^{\frac{\gamma}{(1-\gamma)}}+\varepsilon}\left[r_1^{*}r_2^{U*}(1-\varepsilon)+R\varepsilon\right]$$

Divide both sides by r_1^*, multiply both sides by the denominator to obtain:

(A.15) $$R(1-\varepsilon)\left(\frac{R}{r_1^{*}}\right)^{\frac{\gamma}{(1-\gamma)}}+R\varepsilon > \left[r_1^{*}r_2^{U*}(1-\varepsilon)+R\varepsilon\right]$$

Subtract $R\varepsilon$ from both sides and divide by $(1-\varepsilon)$ to get:

(A.16) $$R\left(\frac{R}{r_1^{*}}\right)^{\frac{\gamma}{(1-\gamma)}} > \left[r_1^{*}r_2^{U*}\right]$$

Divide both sides by r_1^*:

(A.17) $$\left(\frac{R}{r_1^*}\right)^{\frac{1}{(1-\gamma)}} > r_2^{U*}$$

Part a). We show below that constraint C.4.2 is more likely to bind with an increase in variance.

The variance of the binomial return distribution implicit in equation (8), is easily shown to be: $Var(r_2*) = p(1-p)(r_2^U - r_2^D)^2$. By observation, (A.17) becomes more binding with a higher r_2^U that would be implicit in a mean preserving spread which increased the variance of r_2^*.

QED

Part b) We show that an increase in risk aversion, i.e. a decline in γ , decreases the left hand side of equation (A.17), i.e. makes it more difficult to satisfy constraint C.4.2.

Take the derivative of (A.17) with respect to γ.

(A.18) $$\frac{\left(\frac{R}{r_1*}\right)^{\frac{1}{(1-\gamma)}} \ln\left(\frac{R}{r_1*}\right)}{(1-\gamma)^2} > 0$$

QED

A.4 Proof of Proposition 3 (Imposition of a Chilean-type tax)

Part i. Derivation of the equilibrium contract in presence of taxes.

Imposing a Chilean type tax alters the balance of payments and balance sheet constraints. First, note that flows in period 0 are unaffected, thus (1.1) is unchanged. In period 1, the gross foreign exchange earnings from investing reserves abroad in period 0 are divided between the (net of tax) payment to impatient investors and the tax payment to the government, expressed in the equation[7] $r_1^* F_0 = er_1 D_0 t + er_1 D_0 (1-t)$

Finally, in period 2, the project yields export goods. The proceeds of this sale reimburses (as in the base case) patient investors. In this case, however, tax receipts augment patient investors' returns. This implies that equation (1.3) changes:

(1.3') $$RL_0 = (r_2^2)(1-\varepsilon)D_0 - \varepsilon r_2^* t r_1 D_0$$

[7] Note that this is identical to equation (1.2) in the text; i.e. the balance sheet and balance of payments restrictions remain identical. Note also that since the government invests the tax revenue abroad at period 1 it will earn foreign riskless rate r_2* affecting period 2 accounts.

where the second term on the right hand side is the return from investing abroad the tax revenue from impatient investors.

Combining equations (1.1), (1.2), and (1.3') and solving for r_2^2

(2') $$r_2^2 = \frac{R}{(1-\varepsilon)} - \frac{\varepsilon r_1}{(1-\varepsilon) r_1^*}\left[R - r_1^* r_2^* t\right]$$

Note this resource constraint reflects that a higher r_2^* increases the return on the government tax revenue and hence the return to patient investors.

Maximization of expected utility now requires to solve the following:

(4') $$\text{Max}\left[\varepsilon U\left(r_1(1-t)\right) + (1-\varepsilon)U\left(\frac{R}{(1-\varepsilon)} - \frac{\varepsilon r_1}{(1-\varepsilon) r_1^*}\left[R - r_1^* r_2^* t\right]\right)\right]$$

In the case of CRRA preferences, the solution to this problem is obtained by satisfying the first order condition:

(7') $$\left(r_1(1-t)\right)^{\gamma-1}(1-t) + \left(\frac{R}{(1-\varepsilon)} - \frac{\varepsilon r_1}{(1-\varepsilon) r_1^*}\left[R - r_1^* r_2^* t\right]\right)^{\gamma-1}\left[r_1^* t - \frac{R}{r_1^*}\right] = 0$$

Solving this equation for r_1 yields:

(8') $$r_1 = \frac{R r_1^*}{(R - r_1^* r_2^* t)\left\langle \varepsilon + (1-\varepsilon)\left[\frac{R - r_1^* r_2^* t}{(1-t) r_1^*}\right]^{\frac{\gamma}{1-\gamma}} \right\rangle}$$

Part ii. Net-of-tax r_1 declines with a higher tax

First, take the partial derivative of r_1 (1 - t) w.r.t. t to show that net of tax r1 decreases:

$$\frac{\partial r_1(1-t)}{\partial t} = \frac{R r_1^*(1-t)\left[r_1^* r_2^* \varepsilon + (1-\varepsilon)\left[\frac{(R - r_1^* r_2^* t)}{(1-t) r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\left\langle r_1^* r_2^* - \frac{\gamma}{1-\gamma}\frac{(R - r_1^* r_2^*)}{(1-t)}\right\rangle\right]}{\left[(R - r_1^* r_2^* t)\left\langle \varepsilon + (1-\varepsilon)\left[\frac{(R - r_1^* r_2^* t)}{(1-t) r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\right\rangle\right]^2} - \frac{R r_1^*}{(R - r_1^* r_2^* t)\left\langle \varepsilon + (1-\varepsilon)\left[\frac{(R - r_1^* r_2^* t)}{(1-t) r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\right\rangle}$$

Raising the second term to a common denominator, we find after simplifying:

$$\frac{\partial r_1(1-t)}{\partial t}=\frac{Rr_1^*\left\langle\left[r_1^*r_2^*\varepsilon+(1-t)(1-\varepsilon)\left[\frac{\left(R-r_1^*r_2^*t\right)}{(1-t)r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\left\langle\frac{\gamma}{1-\gamma}\frac{\left(R-r_1^*r_2^*\right)}{(1-t)}\right\rangle\right]-\left[R\varepsilon+R(1-\varepsilon)\left[\frac{\left(R-r_1^*r_2^*t\right)}{(1-t)r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\right]\right\rangle}{\left[\left(R-r_1^*r_2^*t\right)\left\langle\varepsilon+(1-\varepsilon)\left[\frac{\left(R-r_1^*r_2^*t\right)}{(1-t)r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\right\rangle\right]^2}$$

Factoring the term in exponents and simplifying:

$$\frac{\partial r_1(1-t)}{\partial t}=\frac{Rr_1^*\left\langle(1-\varepsilon)\left[\frac{\left(R-r_1^*r_2^*t\right)}{(1-t)r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\left(r_1^*r_2^*-\frac{\gamma}{1-\gamma}\left(R-r_1^*r_2^*\right)-R\right)+\varepsilon\left[r_1^*r_2^*-R\right]\right\rangle}{\left[\left(R-r_1^*r_2^*t\right)\left\langle\varepsilon+(1-\varepsilon)\left[\frac{\left(R-r_1^*r_2^*t\right)}{(1-t)r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\right\rangle\right]^2}$$

Further simplifying:

(A.19)
$$\frac{\partial r_1(1-t)}{\partial t}=\frac{-Rr_1^*\left(R-r_1^*r_2^*\right)\left\langle\frac{(1-\varepsilon)}{(1-\gamma)}\left[\frac{\left(R-r_1^*r_2^*t\right)}{(1-t)r_1^*}\right]^{\frac{\gamma}{1-\gamma}}+\varepsilon\right\rangle}{\left[\left(R-r_1^*r_2^*t\right)\left\langle\varepsilon+(1-\varepsilon)\left[\frac{\left(R-r_1^*r_2^*t\right)}{(1-t)r_1^*}\right]^{\frac{\gamma}{1-\gamma}}\right\rangle\right]^2}$$

This expression is clearly negative.

Part iii. Partial derivative of r_2^2 with a change in tax rates.

Substitute eq. (8') into (2') for the equilibrium r_2^2 in a world with taxes. Simplifying:

$$\text{(A.20)} \qquad r_2{}^2 = \frac{R}{\varepsilon\left(\frac{\left(R - r_1^* r_2^* t\right)}{r_1^*(1-t)}\right)^{\frac{\gamma}{(\gamma-1)}} + (1-\varepsilon)}$$

And taking the partial derivative for use in *part iv* below:

$$\text{(A.21)} \qquad \frac{\partial r_2{}^2}{\partial t} = \frac{\varepsilon\left(\frac{\left(R - r_1^* r_2^* t\right)}{r_1^*(1-t)}\right)^{\frac{1}{(\gamma-1)}} R\gamma\left(R - r_1^* r_2^*\right)}{(1-\gamma) r_1^*(1-t)^2\left[\varepsilon\left(\frac{\left(R - r_1^* r_2^* t\right)}{r_1^*(1-t)}\right)^{\frac{\gamma}{(\gamma-1)}} + (1-\varepsilon)\right]^2}$$

where we observe that, for the case where $\gamma < 0$, this derivative is also < 0.

Part iv. Investors may be discouraged if a tax is applied.

An investor may be discouraged if for any parameter values, the imposition of a tax brings any constraint to become more binding. We show that if a tax is imposed for any given parameter values, C.1 becomes more binding. That is, $\frac{\partial(C1)}{\partial t} < 0$, where:

$$C1 = \frac{\varepsilon}{\gamma}\left(r_1(1-t)\right)^{\gamma} + \frac{(1-\varepsilon)}{\gamma}\left(r_2^2\right)^{\gamma} - \frac{r_1^{*\gamma}}{\gamma}\left(\varepsilon + (1-\varepsilon) r_2^{*\gamma}\right)$$

This will be true if

$$\text{(A.22)} \qquad \frac{\partial C1}{\partial t} = e\left(r_1(1-t)\right)^{g-1}\frac{\partial r_1(1-t)}{\partial t} + (1-e)\left(r_2^2\right)^{g-1}\frac{\partial r_2^{2g}}{\partial t} < 0.$$

Since both (A.19) and (A.21) are negative, (A.22) is also negative.

QED

A.5 Proof of Proposition 4 (A Chilean tax selects less-risk averse investors)

The proof is in 3 parts:

i. For given interest rates, r_1*, r_2*, R, a given proportion of impatient investors, ε, and two investors with given levels of risk aversion, $\gamma_H < \gamma_L < 0$, we seek a level of tax, t, that satisfies constraint C.1 for investor L, but does NOT satisfy C.1 for investor H.

ii. For these parameters, we show that in the absence of a tax, the H investor would accept the contract, i.e. when t = 0, constraint C.1 is satisfied for investor H.

iii. We then show that no investor will be influenced by constraints C.2 or C.3, so we do not analyze these constraints.

Part i. ***.*** *There exists a tax rate t simultaneously solving C.1 for investor L, but not for investor H.*

We consider a contract offering rates r_1 and r_2 that would be the optimal choice of the L investor as if she were the only investor. These rates are taken as a *given* by the H person in considering whether to accept the contract or not. That is, constraint C.1 is weighted by the risk-aversion coefficient of investor H.

A tax rate, t, to keep H out must then satisfy (insert in C.1 equations (2') and (8'), equilibrium r_1 and r_2 that optimally satisfy the L investor):

$$\frac{\varepsilon}{\gamma_H}\left(\frac{(1-t)Rr_1^*}{(R-r_1^*r_2^*t)\left\{\varepsilon+(1-\varepsilon)\left[\frac{(R-r_1^*r_2^*t)}{r_1^*(1-t)}\right]^{\frac{\gamma_L}{1-\gamma_L}}\right\}}\right)^{\gamma_H}$$

$$+\frac{1-\varepsilon}{\gamma_H}\left[\frac{R}{1-\varepsilon}-\frac{(R-r_1^*r_2^*t)\varepsilon}{r_1^*(1-\varepsilon)}\left(\frac{Rr_1^*}{(R-r_1^*r_2^*t)\left\{\varepsilon+(1-\varepsilon)\left[\frac{(R-r_1^*r_2^*t)}{r_1^*(1-t)}\right]^{\frac{\gamma_L}{1-\gamma_L}}\right\}}\right)\right]^{\gamma_H}$$

$$< \frac{r_1^{*\gamma_H}\left[\varepsilon+(1-\varepsilon)r_2^{*\gamma_H}\right]}{\gamma_H}$$

Simplify the second term and create new variables A and B, where:

$$A = \frac{(R - r_1^* r_2^* t)}{(1-t)r_1^*} \qquad \text{and} \qquad B = \frac{\gamma_L}{(1-\gamma_L)}$$

to obtain the following equation:

$$\frac{\varepsilon}{\gamma_H}\left(\frac{(1-t)Rr_1^*}{(R - r_1^* r_2^* t)\left\{\varepsilon + (1-\varepsilon)A^B\right\}}\right)^{\gamma_H} + \frac{1-\varepsilon}{\gamma_H}\left(\frac{R}{1-\varepsilon}\right)^{\gamma_H}\left[\frac{(1-\varepsilon)A^B}{\left\{\varepsilon + (1-\varepsilon)A^B\right\}}\right]^{\gamma_H}$$

$$< \frac{r_1^{*\gamma_H}\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_H}\right]}{\gamma_H}$$

Divide both sides by γ_H, factor the term from the left hand side to get:

$$\left(\frac{R}{\left\{\varepsilon + (1-\varepsilon)A^B\right\}}\right)^{\gamma_H}\left[\varepsilon\left(\frac{(1-t)r_1^*}{(R - r_1^* r_2^* t)}\right)^{\gamma_H} + (1-\varepsilon)\left[\frac{(1-\varepsilon)A^B}{(1-\varepsilon)}\right]^{\gamma_H}\right]$$

$$> r_1^{*\gamma_H}\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_H}\right]$$

Simplifying, we establish the condition for "H out":

$$\text{(A.23)} \qquad \frac{\varepsilon A^{-\gamma_H} + (1-\varepsilon)A^{B\gamma_H}}{\left[\varepsilon+(1-\varepsilon)A^B\right]^{\gamma_H}} > \left(\frac{R}{r_1^*}\right)^{-\gamma_H}\left[\varepsilon+(1-\varepsilon)r_2^{*\gamma_H}\right]$$

We use similar steps to obtain the condition required for L to accept the contract ("L in"):

$$\text{(A.24)} \qquad \frac{\varepsilon A^{-\gamma_L} + (1-\varepsilon)A^{B\gamma_L}}{\gamma_L\left[\varepsilon + (1-\varepsilon)A^B\right]^{\gamma_L}} \geq \left(\frac{R}{r_1^*}\right)^{-\gamma_L}\frac{\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_L}\right]}{\gamma_L}$$

Notice that variable A is increasing in the tax rate. Notice also that if R = r_1*r_2* (the lowest real return by assumption A.1), $A = \frac{R}{r_1^*}$, independent of the tax. That is, a tax can have no effect in this circumstance since both investors have C.1 constraints that are at the point of binding.

Now define, for later convenience, $\gamma_H = (\gamma_L - x)$, where x is a positive constant. Rewriting (A.23):

$$\frac{\varepsilon A^{-\gamma_L} A^{x} + (1-\varepsilon)A^{B\gamma_L} A^{-Bx}}{\left[\varepsilon + (1-\varepsilon)A^{B}\right]^{\gamma_L}\left[\varepsilon + (1-\varepsilon)A^{B}\right]^{-x}} > \left(\frac{R}{r_1^*}\right)^{-\gamma_L}\left(\frac{R}{r_1^*}\right)^{x}\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_L} r_2^{*-x}\right]$$

Rearranging:

(A.25)

$$\frac{\varepsilon A^{-\gamma_L} A^{x} + (1-\varepsilon)A^{B\gamma_L} A^{-Bx}}{\left[\varepsilon + (1-\varepsilon)A^{B}\right]^{-x}\left(\frac{R}{r_1^*}\right)^{x}\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_L} r_2^{*-x}\right]} > \left(\frac{R}{r_1^*}\right)^{-\gamma_L}\left[\varepsilon + (1-\varepsilon)A^{B}\right]^{\gamma_L}$$

Now rearrange (A.24) so the right hand side is identical to that of equation (A.25):

(A.24')

$$\frac{\varepsilon A^{-\gamma_L} + (1-\varepsilon)A^{B\gamma_L}}{\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_L}\right]} \le \left(\frac{R}{r_1^*}\right)^{-\gamma_L}\left[\varepsilon + (1-\varepsilon)A^{B}\right]^{\gamma_L}$$

Thus, to separate out investors having relatively risk-averse preferences, we require a tax rate t satisfying simultaneously (A.23) and (A.24'), i.e. to find a tax rate t_0 solving the inequality:

(A.26)

$$\frac{\varepsilon A^{-\gamma_L} + (1-\varepsilon)A^{B\gamma_L}}{\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_L}\right]} < \frac{\varepsilon A^{-\gamma_L} A^{x} + (1-\varepsilon)A^{B\gamma_L} A^{-Bx}}{\left[\varepsilon + (1-\varepsilon)A^{B}\right]^{-x}\left(\frac{R}{r_1^*}\right)^{x}\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_L} r_2^{*-x}\right]}$$

In order to simplify, assume $\lim g_L \to 0$ (which $\Rightarrow B \to 0$):

$$\frac{\varepsilon + (1-\varepsilon)}{\left[\varepsilon + (1-\varepsilon)\right]} < \frac{\varepsilon A^{x} + (1-\varepsilon)}{\left[\varepsilon + (1-\varepsilon)\right]^{-x}\left(\frac{R}{r_1^*}\right)^{x}\left[\varepsilon + (1-\varepsilon)r_2^{*-x}\right]}$$

or:

(A.27) $$\left(\frac{R}{r_1^*}\right)^x \left[\varepsilon + (1-\varepsilon) r_2^{*-x}\right] < \varepsilon A^x + (1-\varepsilon)$$

Now decompose A and solve for t_0:

(A.28)
$$\frac{R^x\left[\varepsilon r_2^{*x} + (1-\varepsilon)\right] - (1-\varepsilon) r_1^{*x} r_2^{*x}}{r_1^{*x} r_2^{*x}} < \varepsilon\left[\frac{(R - r_1^* r_2^* t_0)}{(1-t_0) r_1^*}\right]^x$$

Now define:

$$C = \frac{\varepsilon r_2^{*x} R^x + \left[(1-\varepsilon)\left\langle R^x - r_1^{*x} r_2^{*x}\right\rangle\right]}{\varepsilon r_1^{*x} r_2^{*x}},$$

and solve (A.28) for t_0:

(A.29) $$\frac{C^{1/x} - \frac{R}{r_1^*}}{C^{1/x} - r_2^*} < t_0$$

Now, obviously $C > 1$, implying $t_0 < 1$ for some parameter values. Thus (A.29) gives the condition for a tax that simultaneously satisfies C.1 for L but not for H.

QED

Note that since we use the C.1 constraint for the H investor as an equality, t_0 gives the lowest tax that makes the contract unacceptable to the H investor. It does not inform about the maximum tax for which investor L accepts the contract. We examine this issue in the text.

We now turn to the next part of the proof.

Part ii. The H investor rejecting the above contract would accept the same contract in the absence of a tax.

Consider again equation (A.23) and set t = 0. Note $A = \frac{R}{r_1^*}$ in this case. It implies the H investor has an incentive to participate if:

$$\text{(A.30)}\qquad \frac{\varepsilon\left(\frac{R}{r_1^*}\right)^{-\gamma_H} + (1-\varepsilon)\left(\frac{R}{r_1^*}\right)^{B\gamma_H}}{\gamma_H\left[\varepsilon + (1-\varepsilon)\left(\frac{R}{r_1^*}\right)^{B}\right]^{\gamma_H}} > \left(\frac{R}{r_1^*}\right)^{-\gamma_H}\frac{\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_H}\right]}{\gamma_H}$$

Since $\gamma_H < 0$ and $\gamma_L \rightarrow 0 \Rightarrow B \rightarrow 0$, rewrite (A.30) as:

$$\frac{\varepsilon\left(\frac{R}{r_1^*}\right)^{-\gamma_H} + (1-\varepsilon)}{\left[\varepsilon + (1-\varepsilon)\right]^{\gamma_H}} < \left(\frac{R}{r_1^*}\right)^{-\gamma_H}\left[\varepsilon + (1-\varepsilon)r_2^{*\gamma_H}\right]$$

Simplifying:

$$\text{(A.31)}\qquad \left(\frac{R}{r_1^*}\right)^{\gamma_H} < r_2^{*\gamma_H}$$

This is true by assumption (A.1).

QED

Part iii. Constraints C.2 and C.3 never bind with the parameter values associated with the contract of part i.

In the case a tax is imposed, constraint C.2 "do not go short" must be > 0.

$$C2 = \frac{\varepsilon}{\gamma}\left(r_1(1-t)\right)^{\gamma} + \frac{(1-\varepsilon)}{\gamma}\left(r_2^2\right)^{\gamma} - \frac{\varepsilon}{\gamma}\left(r_1(1-t)\right)^{\gamma} - \frac{(1-\varepsilon)}{\gamma}\left(r_1(1-t)\right)^{2\gamma}$$

It can be easily shown that the above will be satisfied so long as the spread is positive. The investor may rather invest in short-term deposits if the tax causes the spread to become negative. Simplify and insert equilibrium contract values, equations (A.20) and (8'), where the latter is multiplied by (1-t):

$$r_2^2 - (r_1(1-t))^2 = \left[\frac{R}{\varepsilon A^{\frac{\gamma}{(\gamma-1)}} + (1-\varepsilon)}\right] - \left[\frac{R}{A\left(\varepsilon + (1-\varepsilon)A^{\frac{\gamma}{(1-\gamma)}}\right)}\right]^2$$

where $$A = \left[\frac{R - r_1^* r_2^* t}{(1-t)r_1^*}\right].$$

Using the assumptions from *part i.* above, that the contract offers an interest rate combination set optimally for an L investor with a small negative risk aversion coefficient that approaches 0, the spread above is:

$$r_2^2 - (r_1(1-t))^2 = R - \left[\frac{R}{A}\right]^2$$

Since A = R/r_1* in the case of t = 0 and since A is increasing in the tax, this spread is always positive.

QED

In the case a tax is imposed, constraint C.3, "expect to stay in" must be > 0:

$$C3 = \frac{\varepsilon}{\gamma}(r_1(1-t))^{\gamma} + \frac{(1-\varepsilon)}{\gamma}\left(r_2^2\right)^{\gamma} - \frac{\varepsilon}{\gamma}(r_1(1-t))^{\gamma} - \frac{(1-\varepsilon)}{\gamma}\left(r_1(1-t)r_2^*\right)^{\gamma}$$

Simplify and insert equilibrium contract values, equations (A.20) and (8'), where the latter is multiplied by (1-t):

$$\frac{(1-\varepsilon)}{\gamma}\left[\frac{R}{\varepsilon A^{\frac{\gamma}{(\gamma-1)}} + (1-\varepsilon)}\right]^{\gamma} - \frac{(1-\varepsilon)r_2^{*\gamma}}{\gamma}\left[\frac{R}{A\left(\varepsilon + (1-\varepsilon)A^{\frac{\gamma}{(1-\gamma)}}\right)}\right]^{\gamma}$$

where A is defined as above. Factor R^{γ} and prepare to combine terms:

$$\frac{(1-\varepsilon)R^{\gamma}}{\gamma}\left\{\frac{1}{\left(A^{\frac{\gamma}{(\gamma-1)}}\left[\varepsilon + (1-\varepsilon)A^{\frac{\gamma}{(1-\gamma)}}\right]\right)^{\gamma}} - \frac{r_2^{*\gamma}}{A^{\gamma}\left(\varepsilon + (1-\varepsilon)A^{\frac{\gamma}{(1-\gamma)}}\right)^{\gamma}}\right\}$$

Factor the A term and combine. For constraint C.3 to be satisfied the following must be true:

$$\frac{(1-\varepsilon)R^{\gamma}}{\gamma A^{\frac{\gamma^2}{(\gamma-1)}}}\left\{\frac{1-r_2^{*\gamma}A^{\frac{\gamma}{(\gamma-1)}}}{\left[\left(\varepsilon+(1-\varepsilon)A^{\frac{\gamma}{(1-\gamma)}}\right)^{\gamma}\right]}\right\}>0$$

For the important case when $\gamma < 0$, the numerator must be negative. Decomposing A, that means:

$$\left((1-t)r_1^*\right)^{\frac{\gamma}{(\gamma-1)}} < r_2^{*\gamma}\left(R - r_1^* r_2^* t\right)^{\frac{\gamma}{(\gamma-1)}}$$

Take the root of both sides and solve for r_1^*

$$r_1^*\left((1-t(1-r_2^{*\gamma})\right) < r_2^{*(\gamma-1)}R$$

Dividing both sides by : $r_2^{*(\gamma-1)}\left((1-t(1-r_2^{*\gamma})\right)$, we obtain:

$$r_1^* r_2^{*(1-\gamma)} < \frac{R}{\left((1-t(1-r_2^{*\gamma})\right)}$$

This is true by assumption A.2

QED.