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Towards a Theory of the Credit-Risk Balance Sheet (II).

The Evolution of its Structure

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Abstract: This article has an immediate predecessor, upon which it is based and with which readers must necessarily be familiar: *Towards a Theory of the Credit-Risk Balance Sheet* (Vallverdú, Somoza and Moya, 2006). The Balance Sheet is conceptualised on the basis of the duality of a credit-based transaction; it deals with its theoretical foundations, providing evidence of a causal credit-risk duality, that is, a true causal relationship; its characteristics, properties and its static and dynamic characteristics are analyzed.
This article, which provides a logical continuation to the previous one, studies the evolution of the structure of the Credit-Risk Balance Sheet as a consequence of a business's dynamics in the credit area. Given the Credit-Risk Balance Sheet of a company at any given time, it attempts to estimate, by means of sequential analysis, its structural evolution, showing its usefulness in the management and control of credit and risk. To do this, it bases itself, with the necessary adaptations, on the by-now classic works of Palomba and Cutolo. The establishment of the corresponding transformation matrices allows one to move from an initial balance sheet structure to a final, future one, to understand its credit-risk situation trends, as well as to make possible its monitoring and control, basic elements in providing support for risk management.

Key words: bad debts, business risk, commercial credit, credit information, credit management, credit risk, credit-risk balance sheet, insolvency, probabilities matrix, risk, transformation matrix.
JEL classification: M10, M20, M41

Resum: Aquest article té un precedent immediat, en el qual se sustenta, que s'ha de conèixer necessàriament, *Towards a Theory of the Credit-Risk Balance Sheet* (Vallverdú, Somoza i Moya, 2006). Es conceptualitza aquest balanç basant-se en el principi de dualitat d'una transacció de crèdit; se'n planteja la fonamentació teòrica, evidenciant una dualitat causal risc-crèdit, és a dir, una veritable relació causal; se n'analitzen les característiques, les propietats i la seva estàtica i dinàmica.
En el present article, continuació lògica de l'anterior, s'estudia l'evolució de l'estructura del Balanç de Crèdit-Risc com a conseqüència de la dinàmica de l'empresa en l'àmbit del crèdit. Donat el Balanç de Crèdit-Risc d'una empresa en un moment determinat, es tracta

d'estimar-ne, a través de l'anàlisi seqüencial, l'evolució estructural i mostrar-ne la utilitat en la gestió i el control del crèdit i el risc. Així doncs, ens basem en els treballs ja clàssics de Palomba i de Cutolo, amb les adaptacions necessàries. L'establiment de les corresponents matrius de transformació permet passar d'una estructura de balanç inicial a una de final, futura, conèixer la tendència de la seva situació de crèdit-risc, així com possibilitar-ne el seguiment i control, aspectes bàsics per al suport de la gerència de riscos.

Resumen: Este artículo tiene un precedente inmediato, en el que se sustenta, que hay que conocer necesariamente, *Towards a Theory of the Credit-Risk Balance Sheet* (Vallverdú, Somoza y Moya, 2006). Se conceptualiza dicho balance en base al principio de dualidad de una transacción de crédito; se plantea su fundamentación teórica, evidenciando una dualidad causal riesgo-crédito, esto es, una verdadera relación causal; se analizan sus características, sus propiedades y su estática y dinámica.

En el presente artículo, continuación lógica del anterior, se estudia la evolución de la estructura del Balance de Crédito-Riesgo como consecuencia de la dinámica de la empresa en el ámbito del crédito. Dado el Balance de Crédito-Riesgo de una empresa en un momento determinado, se trata de estimar, a través del análisis secuencial, su evolución estructural, mostrando su utilidad en la gestión y el control del crédito y el riesgo. Para ello nos basamos en los ya trabajos clásicos de Palomba y de Cutolo, con las necesarias adaptaciones. El establecimiento de las correspondientes matrices de transformación permite pasar de una estructura de balance inicial a una final, futura, conocer la tendencia de su situación de crédito-riesgo, así como posibilitar su seguimiento y control, aspectos básicos para el soporte de la gerencia de riesgos.

1. THE EVOLUTION OF THE STRUCTURE OF THE CREDIT-RISK BALANCE SHEET AND ITS VARIATIONS

1.1. OPENING SITUATION OF THE CREDIT-RISK BALANCE SHEET AND ITS VARIATIONS

Starting with our credit-risk balance sheet[1] (Vallverdú, Somoza, Moya, 2006):

CREDIT-RISK BALANCE SHEET

CREDIT		**RISK**			
CS_1 Credit w/ certain collection ……..	C_1	**NON-ASSUMED (TRANSFERRED) RISK**			RT
CS_2 Credit w/ high prob. collection.	C_2	Credit insurance ……………		R_I	
CS_3 Credit w/ av. prob. collection.	C_3	Non-compliance (guarantee)	$R_{I.1}$		
CS_4 Credit w/ av./low prob. collection.	C_4	Insolvency (credit)	$R_{I.2}$		
CS_5 Credit w/ low prob. collection.	C_5	Factoring ……………………………………		R_{II}	
… … …	...	Forfeiting ……………………………………		R_{III}	
---		Commission agents ……………………………..		R_{IV}	
---		… … …		...	

---		**ASSUMED (NON-TRANSFERRED) RISK**		R_m	RN
TOTAL CREDIT	C	TOTAL RISK			R

we shall now consider the Credit-Risk Balance Sheet of a company at a specific moment of time and shall try to estimate, by means of sequential analysis, the structural evolution that may arise as a result of its own dynamics, as well as its usefulness in the control of credit and risk. To do this, we shall take as our basis, *mutatis mutandis*, different works by Palomba [1950, 1952, 1967, 1972], Cutolo [1951, 1952, 1955, 1963, 1965] and Palomba and Cutolo [1961][2], although in our adaptation we shall introduce a certain degree of modification. Thus, Owner's Equity, which are not considered by said authors in their approach, for us –in accordance with the isomorphism of the conventional Balance Sheet and our own Credit-Risk Balance Sheet– would have its equivalent in "assumed (non-transferred) risk", which we *will* take into account, as will be explained below.

We shall make the structural representation of said balance sheet on the basis of a transformation into percentages, of the sum total of credit or of risk, of each of the elements of said credit and risk structures. Nonetheless, unlike the model proposed by Palomba and Cutolo, which excludes the owner's equity, we have not excluded "assumed (non-transferred) risk", since said authors seek to study a business's liquidity and do not, therefore, consider non-short-term items. Consequently, it ignores any accounting event that is non-permutative, or the non-permutative part of a mixed event. We, on the other hand, when studying risk, include both that transferred, or not assumed by the company, and that which is not transferred, or assumed, because the movement between the two, in both directions, can be interesting.

With this, the aim is to establish and analyse the configuration of a company's Credit-Risk Balance Sheet over time, as a consequence of its activity, based on a law of transformation.

Schematically, our Credit-Risk Balance Sheet is arranged as follows:

CREDIT (C)	**RISK (R)**
C_1	R_I
C_2	R_{II}
C_3	R_{III}
…	…
$\underline{C_n}$	$\underline{R_m}$
$\sum_{h=1}^{n} C_h$	$\sum_{f=I}^{m} R_f$

Let

$\kappa_1 ,\ \kappa_2 ,\ \kappa_3\ \ldots\ \kappa_n$

the percentage relationships between each of the n elements of the overall credit C and its total on our Credit-Risk Balance Sheet, with

$$\kappa_h = \frac{C_h}{\sum_{h=1}^{n} C_h} \cdot 100\ \%$$

And let

$\rho_I, \rho_{II}, \rho_{III} \ldots \rho_m$

be the percentage relationships between each of the m elements of the overall risk R and its total on our Credit-Risk Balance Sheet, with

$$\rho_f = \frac{R_f}{\sum_{f=I}^{m} R_f} \cdot 100\ \%$$

It should logically follow that

$$\text{[I]} \qquad \sum_{h=1}^{n} \kappa_h = \sum_{f=I}^{m} \rho_f = 100\ \%$$

If we now call [$C^{(0)}$] and [$R^{(0)}$] the matrices that represent, respectively, the structures of credit and risk at the time t_o, obtained from the aforementioned non-cumulative percentages, the following is the result:

$$[\ C^{(0)}\ ] = \begin{bmatrix} \kappa_{1(0)} \\ \kappa_{2(0)} \\ \ldots \\ \ldots \\ \kappa_{n(0)} \end{bmatrix} \qquad \text{y} \qquad [\ R^{(0)}\ ] = \begin{bmatrix} \rho_{I(0)} \\ \rho_{II(0)} \\ \ldots \\ \ldots \\ \rho_{m(0)} \end{bmatrix}$$

Where $[CR^{(0)}]$, or rather $[C^{(0)}, R^{(0)}]$, is the structural configuration of the Credit-Risk Balance Sheet at the time t_o:

$$[CR^{(0)}] = [C^{(0)}, R^{(0)}] = [\ \kappa_1^{(0)}\ \kappa_2^{(0)}\ \dots\ \kappa_n^{(0)}\ \rho_I^{(0)}\ \rho_{II}^{(0)}\ \dots\ \rho_m^{(0)}\ ]$$

Similarly, we would obtain the matrices $[\ C^{(j)}\ ]$ and $[\ R^{(j)}\ ]$, representing the credit and risk structures at any subsequent time t_j.

The structural configurations at each of said times t_0 and t_j can be symbolised by $[\ C^{(0)}, R^{(0)}\ ]$ and $[\ C^{(j)}, R^{(j)}\ ]$, respectively.

In the interval $(t_0 - t_j)$, and as a consequence of the business's activity, variations may have arisen in that initial structural configuration $[C^{(0)}, R^{(0)}\ ]$.

If $[C^{(0)}, R^{(0)}\ ] = [C^{(j)}, R^{(j)}\ ]$, the company has undergone no alterations or has remained stationary*("stazionarietà")*. In other words, the structural configurations remain the same[3].

If $[C^{(0)}\ , R^{(0)}\ ] \neq [C^{(j)}\ , R^{(j)}\ ]$, or in the preceding case of a stationary situation, the variations which must have arisen as a consequence of the business's consubstantial dynamic will affect credit, risk or credit and risk simultaneously. All the possible variations are displayed, systematised, in the following table:

MUTATIONS OR MIGRATIONS [4]

Overall categories	CREDIT	RISK
CREDIT	CC [1]	CR [2]
RISK	RC [3]	RR [4]

1. **INTERNAL MUTATIONS**, ENDOGENOUS TRANSFORMATIONS or INTRA-SYSTEM TRANSACTIONS (that take place within the credit or risk structures).

1.1. In Credit Area [1]: Credit ↔ Credit mutation (CC).

An example of the real-life situation this represents: a) *An arrears situation which requires a credit to given a lower classification, due to its poorer quality, or there are reasonable indications of a reduction of the debtor's solvency;* b) *the opposite case, of the improvement in quality of a credit with the provision of new guarantees or an improvement in the intrinsic solvency status of the debtor, reclassifying the credit.*

1.2. In Risk Area [4]: Risk ↔ Risk mutation (RR).

An example of the real-life situation this represents: A change arises in the source of the assumption of the risk, such as when an initially-assumed risk is subsequently outsourced, or vice versa. The former case would involve, subsequent to formally entering into a sale or credit, the commission agent, credit assurance company, etc., assuming the risk. In the second case, it is the other way around: a risk initially assumed by a credit assurance company which, due to some contractual problem (we shall go into this in further detail later on), stops assuming it and transfers it to the vendor or another figure.

2. **EXTERNAL MUTATIONS**, EXOGENOUS TRANSFORMATIONS OR INTER-SYSTEM TRANSACTIONS (take place affecting both credit and risk structures simultaneously).

2.1. In Credit-Risk Area [2]: Credit → Risk mutation (CR).

Example of the real-life situation this represents: The settlement of a credit making unnecessary the assumption of risk which is, consequently, also cancelled.

2.2. In Risk-Credit Area [3]: Risk → Credit mutation (RC) (See Vallverdú, Somoza and Moya, 2006, p. 12).

Example of the real-life situation this represents: The new assumption of risk allowing for the granting of a new credit. In such a case, thanks to the risk assumed by a credit assurance company, the insured party decides to make a credit sale to a customer, which it could not otherwise have done: in other words, it would not have been prepared to assume this risk and the sale would not have taken place (on a credit basis, at least).

With regard to Category 1.2, we believe there is a need to make some clarifications which should permit one to actually calibrate the possibility of internal Risk ↔ Risk mutation. The change from transferred to non-transferred risk will probably be the most frequent case, but we cannot rule out the opposite (non-transferred to transferred) nor mutations within transferred risk itself. The fact is that, with credit insurance, for example, initial cover (the assumption of risk by the insurer) may be revoked at a given time due to certain circumstances, a matter we shall look at in more detail later on.

1.2. THE PROBABILITIES OF VARIATION IN THE ELEMENTS OF THE CREDIT-RISK BALANCE SHEET

We can now establish the probability of the occurrence of said mutations. In other words, we can study the relevant *law of transformation.* To do this, with a_{rs} we symbolise the probability that one unit of the heading r moves to the heading s.

Given that, as noted above, we can talk of four different types of mutations, the matrix that we shall establish, whose elements are the probability a_{rs}, shall also consist of four parts or areas too:

AREAS OF THE MUTATION PROBABILITIES MATRIX

1. PROBABILITY OF INTERNAL MUTATIONS
   1.1. Probability of mutation between credit structure elements [1].
   1.2. Probability of mutation between risk structure elements [4].

2. PROBABILITY OF EXTERNAL MUTATIONS:
   2.1. Probability of mutation from credit structure elements to risk structure elements [2].
   2.2. Probability of mutation from risk structure elements to credit structure

elements [3].

Thus, each of the elements C_h $(\forall h = 1, 2, 3 \ldots n)$, as well as any R_f $(\forall f = I, II \ldots m)$, may move to any other C_h or an R_f element:

$$C_1 \begin{cases} C_1 \\ C_2 \\ \ldots \\ C_n \\ R_I \\ R_{II} \\ \ldots \\ R_m \end{cases} \quad C_2 \begin{cases} C_1 \\ C_2 \\ \ldots \\ C_n \\ R_I \\ R_{II} \\ \ldots \\ R_m \end{cases} \quad \ldots \quad \ldots \quad \ldots \quad C_n \begin{cases} C_1 \\ C_2 \\ \ldots \\ C_n \\ R_I \\ R_{II} \\ \ldots \\ R_m \end{cases}$$

$$R_I \begin{cases} C_1 \\ C_2 \\ \ldots \\ C_n \\ R_I \\ R_{II} \\ \ldots \\ R_m \end{cases} \quad R_{II} \begin{cases} C_1 \\ C_2 \\ \ldots \\ C_n \\ R_I \\ R_{II} \\ \ldots \\ R_m \end{cases} \quad \ldots \quad \ldots \quad \ldots \quad R_m \begin{cases} C_1 \\ C_2 \\ \ldots \\ C_n \\ R_I \\ R_{II} \\ \ldots \\ R_m \end{cases}$$

With the following probabilities p_{rs}:

$$C_I \begin{cases} C_1 \rightarrow C_1 : p_{11} \\ C_1 \rightarrow C_2 : p_{12} \\ \ldots \\ C_1 \rightarrow C_n : p_{1n} \\ C_1 \rightarrow R_I : p_{1I} \\ C_1 \rightarrow R_{II} : p_{1II} \\ \ldots \\ C_1 \rightarrow R_m : p_{1m} \end{cases} \quad C_2 \begin{cases} C_2 \rightarrow C_1 : p_{21} \\ C_2 \rightarrow C_2 : p_{22} \\ \ldots \\ C_2 \rightarrow C_n : p_{2n} \\ C_2 \rightarrow R_I : p_{2I} \\ C_2 \rightarrow R_{II} : p_{2II} \\ \ldots \\ C_2 \rightarrow R_m : p_{2m} \end{cases} \quad \ldots \quad \ldots \quad \ldots \quad C_n \begin{cases} C_n \rightarrow C_1 : p_{n1} \\ C_n \rightarrow C_2 : p_{n2} \\ \ldots \\ C_n \rightarrow C_n : p_{nn} \\ C_n \rightarrow R_I : p_{nI} \\ C_n \rightarrow R_{II} : p_{nII} \\ \ldots \\ C_n \rightarrow R_m : p_{nm} \end{cases}$$

$$R_I \begin{cases} R_I \rightarrow C_1 : p_{I1} \\ R_I \rightarrow C_2 : p_{I2} \\ \ldots \\ R_I \rightarrow C_n : p_{In} \\ R_I \rightarrow R_I : p_{I.I} \\ R_I \rightarrow R_{II} : p_{I.II} \\ \ldots \\ R_I \rightarrow R_m : p_{I.m} \end{cases} \quad R_{II} \begin{cases} R_{II} \rightarrow C_1 : p_{II1} \\ R_{II} \rightarrow C_2 : p_{II2} \\ \ldots \\ R_{II} \rightarrow C_n : p_{IIn} \\ R_{II} \rightarrow R_I : p_{II.I} \\ R_{II} \rightarrow R_{II} : p_{II.II} \\ \ldots \\ R_{II} \rightarrow R_m : p_{II.m} \end{cases} \quad \ldots \quad \ldots \quad \ldots \quad R_m \begin{cases} R_m \rightarrow C_1 : p_{m1} \\ R_m \rightarrow C_2 : p_{m2} \\ \ldots \\ R_m \rightarrow C_n : p_{mn} \\ R_m \rightarrow R_I : p_{mI} \\ R_m \rightarrow R_{II} : p_{mII} \\ \ldots \\ R_m \rightarrow R_m : p_{mm} \end{cases}$$

To better visualise the credit-risk flows, we can summarise all the above in the following table:

PROBABILITIES OF THE CREDIT-RISK INTERRELATIONS

TO: ***FROM:***	C_1	C_2	...	C_n	R_I	R_{II}	...	R_m
C_1	p_{11}	p_{12}	…	p_{1n}	p_{1I}	p_{1II}	…	p_{1m}
C_2	p_{21}	p_{22}	…	p_{2n}	p_{2I}	p_{2II}	…	p_{2m}
...	…	…	…	…	…	…	…	…
C_n	p_{n1}	p_{n2}	…	p_{nn}	p_{nI}	p_{nII}	…	p_{nm}
R_I	p_{I1}	p_{I2}	…	p_{In}	$p_{I\,I}$	$p_{I\,II}$	…	p_{Im}
R_{II}	p_{II1}	p_{II2}	…	p_{IIn}	$p_{II\,I}$	$p_{II\,II}$	…	$p_{II\,m}$
...	…	…	…	…	…	…	…	…
R_m	p_{m1}	p_{m2}	…	p_{mn}	p_{mI}	p_{mII}	…	p_{mm}

So, we can establish the following *probabilities matrix* or *stochastic matrix*, which we shall call [a_{rs}], and in which we shall delimit four quadrants or sectors, viz.:

$$
\begin{array}{cc}
 & \begin{array}{cc} [1] \qquad\qquad\qquad & [2] \end{array} \\
[\, a_{rs} \,] = & \left[\begin{array}{cccccc}
a_{11} & a_{12} \; \ldots\ldots \; a_{1n} & & a_{1I} & a_{1II} \; \ldots\ldots \; a_{1m} \\
a_{21} & a_{22} \; \ldots\ldots \; a_{2n} & & a_{2I} & a_{2II} \; \ldots\ldots \; a_{2m} \\
\ldots & \ldots\;\ldots\;\ldots\;\ldots & & \ldots & \ldots\;\ldots\;\ldots\;\ldots \\
a_{n1} & a_{n2} \; \ldots\ldots \; a_{nn} & & a_{nI} & a_{nII} \; \ldots\ldots \; a_{nm} \\
 & & & & \\
a_{I1} & a_{I2} \; \ldots\ldots \; a_{In} & & a_{I\,I} & a_{I\,II} \; \ldots\ldots \; a_{I.m} \\
a_{II1} & a_{II2} \; \ldots\ldots \; a_{IIn} & & a_{II\,I} & a_{II\,II} \; \ldots\ldots \; a_{II.m} \\
\ldots & \ldots\;\ldots\;\ldots\;\ldots & & \ldots & \ldots\;\ldots\;\ldots\;\ldots \\
a_{m1} & a_{m2} \; \ldots\ldots \; a_{mn} & & a_{mI} & a_{mII} \; \ldots\ldots \; a_{mm}
\end{array} \right] \\
 & \begin{array}{cc} [3] \qquad\qquad\qquad & [4] \end{array}
\end{array}
$$

Or, schematically,

$$a_{rs} = \begin{bmatrix} a_{hh} & a_{hf} \\ a_{fh} & a_{ff} \end{bmatrix} \quad h = 1, 2, ..n; f = I, II, ...m$$

and

$$[\, a_{rs} \,] = \begin{bmatrix} CC & CR \\ RC & RR \end{bmatrix}$$

The elements a_{rs} constitute the probability that a monetary unit included in the Credit-Risk Balance Sheet in the position r at the time t_0 is included in the position s at the subsequent time t_j. With Arabic-numbered sub-indices we refer elements of the credit structure and with Roman-numbered sub-indices to elements of the risk structure, and where n and m, as can be seen, symbolise generic Arabic and Roman numbers, respectively.

In other words, the row a_{1s} reflects the probability that the "credit with certain collection" (the first credit heading on the Credit-Risk Balance Sheet) transforms into itself (a_{11}), into the remaining credit elements (a_{12} to a_{1n}) and into risk elements (a_{1I} to a_{1m}). The a_{2s} row reflects the probability that the "credit with a high probability of collection" (the second credit heading on the Credit-Risk Balance Sheet) transforms into itself (a_{22}), into the remaining credit elements (a_{21} to a_{2n}, with the exception of a_{22}) and into risk elements (a_{2I} to a_{2m}). The same is the case, *mutatis mutandis*, with all the remaining a_h rows $(\forall h = 1, 2 \ldots n)$. The a_{Is} row reflects the probability that "transferred risk R_I (credit insurance)", the first risk heading, transforms into itself ($a_{I\,I}$), into the remaining credit elements (a_{I1} to a_{In}) and into the risk elements ($a_{I\,II}$ to a_{Im}). The same is the case, *mutatis mutandis*, with all the remaining a_f rows $(\forall f = I, II \ldots m)$.

This is a squared matrix, with $(n+m)^2$ elements, in which four areas or quadrants are delimited - [1], [2], [3], [4] – whose meaning is included in the following table:

MEANING OF THE FOUR AREAS OF THE PROBABILITIES MATRIX

1. PROBABILITIES OF INTERNAL MUTATIONS:

Area [1]: the squared matrix of n^2 elements, symbolised with two Arabic-numbered sub-indices, which identifies mutations within credit elements. It shows the probability that each credit element transforms into another credit element[5].

Area [4]: the squared matrix of m^2 elements, symbolised with two Roman-numbered sub-indices, which identifies mutations within risk elements. It shows the probability that each risk element transforms into another risk element[6].

2. PROBABILITIES OF EXTERNAL MUTATIONS:

Area [2]: matrix of nm elements, symbolised with two sub-indices, in which an Arabic-numbered one preceding a Roman-numbered one. It identifies mutations between credit and risk elements and shows the probability that each credit element transforms into a risk element[7].

Area [3]: matrix of mn elements, symbolised with two sub-indices, in which a Roman-numbered one preceding an Arabic-numbered one. It identifies mutations between risk and credit elements and shows the probability that each risk element transforms into a credit element[8].

The a_{rs} elements of the probability matrix [a_{rs}] are statistical frequencies of the period $(t_0 - t_j)$ which are inferred from the accounting records, on the basis of the accounts open to each credit and risk element. It is an *a posteriori* calculation, but we could be in the presence of an *a priori* calculation and it would be an "objective probability". It is a matter of calculating the probability that a monetary unit included in the Credit-Risk Balance Sheet, in the position r at the time t_0, is included in the position s at the subsequent time t_j. For this [cf. Cutolo, 1965: 422], it is established that an element a_{rs} of said matrix may be quantified as follows:

[II] $$a_{rs} = \frac{I_{rs}}{T_r + S_r} \qquad (\text{with } r \neq s)$$

where:

I_{rs} : The sum, in monetary units, of the mutations of the element r towards the element s, of the Credit-Risk Balance Sheet. This information shall be obtained from the general ledger of the *ad hoc* credit-risk accountancy that should be available, where, through analysis of the movements of each account, and, through the rule of the connection or coordination thereof, the relevant mutations can be obtained.

E.g. (in accordance with the symbology used for the Credit-Risk Balance Sheet).

In credit class C_3 which, at the start of a period has a balance of 500 m.u, the following movements are recorded:

[i] A new credit *(C_3)* of 100 m.u. whose risk is assumed by commission agents *(R_{IV})*.

[ii] Another, of 200 m.u., whose risk is assumed only partially by commission agents (R_{IV}: 70 m.u.); the other part is assumed by credit insurance (R_I: 130 m.u.).

[iii] A final credit, of 150 m.u., whose risk is assumed by the company itself *(R_m)*.

[iv] A credit, of 80 m.u., is reclassified as C_5.

[v], [vi], [vii]. Additionally, the following credits were settled, whose risk was assumed by credit insurance *(R_I)*: two from category C_3, of 10 m.u. [v] and 20 m.u. [vi]; [vii] one of C_2, of 30 m.u.

[viii] A credit in category C_3 of 40 m.u. was settled, whose risk had not been transferred to third parties *(R_m)*.

The mutations that have arising are schematised as follows:

MOVEMENT		**AREA OR QUADRANT OF MUTATION**
No.	**Schematic**	
i	$R_{IV} \rightarrow C_3$: 100 m.u.	[3]
ii	$R_{IV} \rightarrow C_3$: 70 m.u. $R_I \rightarrow C_3$: 130 m.u.	[3] [3]
iii	$R_m \rightarrow C_3$: 150 m.u.	[3]
iv	$C_3 \rightarrow C_5$: 80 m.u.	[1]
v	$C_3 \rightarrow R_I$: 10 m.u.	[2]
vi	$C_3 \rightarrow R_I$: 20 m.u.	[2]
vii	$C_2 \rightarrow R_I$: 30 m.u.	[2]
viii	$C_3 \rightarrow R_m$: 40 m.u.	[2]

C_3			
Opening balance:	**500**	80 : C_5	
R_{IV}	100	10: R_I	
R_{IV}	70	20: R_I	
R_I	130	40: R_m	
R_m	150	**800**:	**Closing balance**
	950	950	

I_{rs}	Value	
I_{31}	0	
I_{32}	0	
I_{34}	0	
I_{35}	80	
...	0	
I_{3I}	30	(10 + 20)
I_{3II}	0	
I_{3III}	0	
I_{3IV}	0	
...	0	
I_{3m}	40	
TOTAL	150	

T_r: The sum, in monetary units, of the entries recorded in the account of element r, relating to *headings that move* from element r to other headings.

In the example under review, $T_r = T_3 = 80 + 10 + 20 + 40 = 150$.

S_r: Closing balance of account r. That is, *headings that do not move* and which therefore remain in the same account.

In the example under review, $S_r = S_3 = 800$.

And, as a particular case,

[III] $$a_{rr} = \frac{S_r}{T_r + S_r}$$

in which the relationship of the account balance with regard to the sum total of the entries therein can be interpreted as the probability that the element r is self-supplying.

If no (T_r) movements are recorded, $a_{rr} = 1$, that is, everything remains in the account itself, it must be, logically, that

$$0 \leq a_{rs} \leq 1 \quad \text{and} \quad 0 \leq a_{rr} \leq 1$$

Thus, the probabilities that (C_3) commercial credits transform into themselves, into the remaining classes of credit and into different classes of risk are:

$a_{rs} = a_{3s}$	$\frac{I_{rs}}{T_r+S_r}$	$\frac{I_{rs}}{T_r+S_r}$ ***values***	***TOTAL***
a_{31}	$\frac{I_{31}}{T_3+S_3}$	$\frac{0}{150+800}$	0
a_{32}	$\frac{I_{32}}{T_3+S_3}$	$\frac{0}{150+800}$	0
a_{33}	$\frac{S_3}{T_3+S_3}$	$\frac{800}{150+800}$	0.8421
a_{34}	$\frac{I_{34}}{T_3+S_3}$	$\frac{0}{150+800}$	0
a_{35}	$\frac{I_{35}}{T_3+S_3}$	$\frac{80}{150+800}$	0.0842
...	…	…	...
a_{3I}	$\frac{I_{3I}}{T_3+S_3}$	$\frac{30}{150+800}$	0.0316
a_{3II}	$\frac{I_{3II}}{T_3+S_3}$	$\frac{0}{150+800}$	0

a_{3III}	$\frac{I_{3III}}{T_3+S_3}$	$\frac{0}{150+800}$	0
a_{3IV}	$\frac{I_{3IV}}{T_3+S_3}$	$\frac{0}{150+800}$	0
…	…	…	…
a_{3m}	$\frac{I_{3m}}{T_3+S_3}$	$\frac{40}{150+800}$	0.0421
$\sum_{s=1;s=I}^{5;m} a_{3s}$		$\frac{950}{150+800}$	1

$\sum_{s=1;s=I}^{5;m} a_{3s} = 1$, since the sum of each horizontal line in the probabilities matrix is equal to 1, given that each of them covers the probability that each element mutates into itself and into the other elements.

To conclude this section, we would like to highlight the isomorphism that exists between the above *Mutations* table, *Credit-Risk Interrelations* and *Probabilities Matrix* tables and the *Esquema de las interrelaciones económicas* (economic interrelations schematic) presented by Calafell [1967: 94 bis], in his "*Fundamentos de la teoría lineal de la contabilidad*" (Fundaments of the linear theory of accounting)[9].

1.3. PROPERTIES OF THE MATRIX OF PROBABILITIES OF VARIATION IN THE ELEMENTS OF THE CREDIT-RISK BALANCE SHEET

1. The values of each of the $(n+m)^2$ elements are non-negative $(a_{rs} \geq 0)$.

2. The sum of the values of each row is always and invariably one. Each of them represents the probability that the element in question emigrates to other Credit-Risk Balance Sheet elements, including transforming into itself. This makes sense, since the overall combination of elements must, at any given time, be in some position.

[IV] $$\sum_{s=1;\,s=I}^{n;\,m} a_{rs} = 1 \qquad (s = 1, 2 \dots n, I, II \dots m)$$

3. From the above property, it is deduced that the sum of all the elements in the matrix [a_{rs}] is $n+m$, which is the number of rows it contains.

4. The elements of the main diagonal a_{rr} express the probabilities that each elements transforms into itself.

5. As a consequence of the above property, if the matrix [a_{rs}] is reduced to a unit matrix, that is, in which every $a_{rr} = 1$, consequently, and any other element $a_{rs} = 0$, by virtue of property number 2, the company will not have operated in the credit-risk field during the period under review, since there are no mutations between the elements of said Credit-Risk Balance Sheet [III].

   In such a case, we would be dealing with the static situation ("*statizità*"), which should be distinguished from a stationary situation ("*stazionarietà*"), which occurs when the structural configuration is retained.

1.4. FINAL SITUATION OF THE CREDIT-RISK BALANCE SHEET

We shall now establish the structural configuration of the Credit-Risk Balance Sheet at time t_j , [$C^{(j)}$, $R^{(j)}$], based on the configuration at time t_0 , [$C^{(0)}$, $R^{(0)}$]. To do this, we shall consider all the possible mutations. Thus, each κ_h at time t_0, that is, $\kappa_h^{(0)}$, can change into $\kappa_h^{(j)}$, that is, κ_h at time t_j.

To do this, we must consider all the possible changes that may be experienced by each of the elements of the Credit-Risk Balance Sheet, transformed into percentage relationships $\kappa_h^{(0)}$ and $\rho_f^{(0)}$, at the time t_0:

$$
\left.\begin{array}{l}\kappa_{1(0)}\\ \kappa_{2(0)}\\ \dots\\ \kappa_{n(0)}\\ \rho_{I(0)}\\ \rho_{II(0)}\\ \dots\\ \rho_{m(0)}\end{array}\right\}\kappa_1^{(j)}
\qquad
\left.\begin{array}{l}\kappa_{1(0)}\\ \kappa_{2(0)}\\ \dots\\ \kappa_{n(0)}\\ \rho_{I(0)}\\ \rho_{II(0)}\\ \dots\\ \rho_{m(0)}\end{array}\right\}\kappa_2^{(j)}
\qquad \dots\ \dots\ \dots \qquad
\left.\begin{array}{l}\kappa_{1(0)}\\ \kappa_{2(0)}\\ \dots\\ \kappa_{n(0)}\\ \rho_{I(0)}\\ \rho_{II(0)}\\ \dots\\ \rho_{m(0)}\end{array}\right\}\kappa_n^{(j)}
$$

$$
\left.\begin{array}{l}\kappa_{1(0)}\\ \kappa_{2(0)}\\ \dots\\ \kappa_{n(0)}\\ \rho_{I(0)}\\ \rho_{II(0)}\\ \dots\\ \rho_{m(0)}\end{array}\right\}\rho_I^{(j)}
\qquad
\left.\begin{array}{l}\kappa_{1(0)}\\ \kappa_{2(0)}\\ \dots\\ \kappa_{n(0)}\\ \rho_{I(0)}\\ \rho_{II(0)}\\ \dots\\ \rho_{m(0)}\end{array}\right\}\rho_{II}^{(j)}
\qquad \dots\ \dots\ \dots \qquad
\left.\begin{array}{l}\kappa_{1(0)}\\ \kappa_{2(0)}\\ \dots\\ \kappa_{n(0)}\\ \rho_{I(0)}\\ \rho_{II(0)}\\ \dots\\ \rho_{m(0)}\end{array}\right\}\rho_m^{(j)}
$$

Thus we have all the possible outcomes of each $\kappa_h^{(0)}$ and of each $\rho_f^{(0)}$ at the time t_j, that is, $\kappa_h^{(j)}$ y $\rho_f^{(j)}$.

Now, applying the transformation probabilities of each initial element to each of the other elements supplied by the probability matrix [a_{rs}], which is configured, as we can demonstrate, as a *transition matrix*, or transformation matrix, we can move from an initial structure *(0)* to a final one *(j)*:

[V]

$$
\left|\begin{array}{l}
\kappa_1^{(j)} = \kappa_1^{(0)} a_{11} + \kappa_2^{(0)} a_{21} + \ldots + \kappa_n^{(0)} a_{n1} + \rho_I^{(0)} a_{I1} + \rho_{II}^{(0)} a_{II\,1} + \ldots + \rho_m^{(0)} a_{m1} \\
\kappa_2^{(j)} = \kappa_1^{(0)} a_{12} + \kappa_2^{(0)} a_{22} + \ldots + \kappa_n^{(0)} a_{n2} + \rho_I^{(0)} a_{I2} + \rho_{II}^{(0)} a_{II\,2} + \ldots + \rho_m^{(0)} a_{m2} \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
\kappa_n^{(j)} = \kappa_1^{(0)} a_{1n} + \kappa_2^{(0)} a_{2n} + \ldots + \kappa_n^{(0)} a_{nn} + \rho_I^{(0)} a_{In} + \rho_{II}^{(0)} a_{II\,n} + \ldots + \rho_m^{(0)} a_{mn} \\
\rho_I^{(j)} = \kappa_1^{(0)} a_{1I} + \kappa_2^{(0)} a_{2I} + \ldots + \kappa_n^{(0)} a_{nI} + \rho_I^{(0)} a_{I\,I} + \rho_{II}^{(0)} a_{II\,I} + \ldots + \rho_m^{(0)} a_{mI} \\
\rho_{II}^{(j)} = \kappa_1^{(0)} a_{1II} + \kappa_2^{(0)} a_{2\,II} + \ldots + \kappa_n^{(0)} a_{nII} + \rho_I^{(0)} a_{I\,II} + \rho_{II}^{(0)} a_{II\,II} + \ldots + \rho_m^{(0)} a_{mII} \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
\rho_m^{(j)} = \kappa_1^{(0)} a_{1m} + \kappa_2^{(0)} a_{2m} + \ldots + \kappa_n^{(0)} a_{nm} + \rho_I^{(0)} a_{Im} + \rho_{II}^{(0)} a_{IIm} + \ldots + \rho_m^{(0)} a_{mm}
\end{array}\right.
$$

Where, adding term to term gives:

$$
\sum_{h=1}^{n} \kappa_h^{(j)} + \sum_{f=I}^{m} \rho_f^{(j)} = \kappa_1^{(0)} \sum_{s=1;s=I}^{n;m} a_{1s} + \kappa_2^{(0)} \sum_{s=1;s=I}^{n;m} a_{2s} + \ldots + \kappa_n^{(0)} \sum_{s=1;s=I}^{n;m} a_{ns} +
$$

$$
+ \rho_I^{(0)} \sum_{s=1;s=I}^{n;m} a_{Is} + \rho_{II}^{(0)} \sum_{s=1;s=I}^{n;m} a_{IIs} + \ldots + \rho_m^{(0)} \sum_{s=1;s=I}^{n;m} a_{ms}
$$

Given that the first term is worth 200 [I] and that all the sums of the second term are equal to one, since they relate to the sum of all the terms in both rows of the probabilities matrix [IV], the result is:

$$
200 = \kappa_1^{(0)} + \kappa_2^{(0)} + \ldots + \kappa_n^{(0)} + \rho_I^{(0)} + \rho_{II}^{(0)} + \ldots + \rho_m^{(0)}
$$

Let us now see that if the structural configuration of the initial Credit-Risk Balance Sheet (time t_0)

$$
[\, CR^{(0)} \,] = [\, \kappa_1^{(0)} \;\; \kappa_2^{(0)} \;\; \ldots \;\; \kappa_n^{(0)} \;\; \rho_I^{(0)} \;\; \rho_{II}^{(0)} \;\; \ldots \;\; \rho_m^{(0)} \,]
$$

multiplied by the transformation matrix [a_{rs}], gives [$CR^{(j)}$], with

[$CR^{(j)}$] = [$\kappa_I^{(j)}$ $\kappa_2^{(j)}$ ... $\kappa_n^{(j)}$ $\rho_I^{(j)}$ $\rho_{II}^{(j)}$ ... $\rho_m^{(j)}$],

in other words, the structural configuration of the Credit-Risk Balance Sheet at the final time (t_j).

That is, [$CR^{(0)}$] · [a_{rs}] = [$CR^{(j)}$].

$$[CR^{(0)}] \cdot [a_{rs}] =$$

$$= [\ \kappa_I^{(0)}\ \ \kappa_2^{(0)} \dots \kappa_n^{(0)}\ \ \rho_I^{(0)}\ \ \rho_{II}^{(0)} \dots \rho_m^{(0)}\ ] \cdot \begin{bmatrix} a_{11}\ a_{12} \dots\dots a_{1n}\ a_{1I}\ a_{1II} \dots\dots a_{1m} \\ a_{21}\ a_{22} \dots\dots a_{2n}\ a_{2I}\ a_{2II} \dots\dots a_{2m} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}\ a_{n2} \dots\dots a_{nn}\ a_{nI}\ a_{nII} \dots\dots a_{nm} \\ a_{I1}\ a_{I2} \dots\dots a_{In}\ a_{II}\ a_{III} \dots\dots a_{I.m} \\ a_{II1}\ a_{II2} \dots\dots a_{IIn}\ a_{III}\ a_{IIII} \dots a_{II.m} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}\ a_{m2} \dots\dots a_{mn}\ a_{mI}\ a_{mII} \dots a_{mm} \end{bmatrix} =$$

$$= [\ \kappa_I^{(j)}\ \ \kappa_2^{(j)}\ \dots\ \kappa_n^{(j)}\ \ \rho_I^{(j)}\ \ \rho_{II}^{(j)}\ \dots\ \rho_m^{(j)}\ ] \qquad \text{[VI]}$$

And that the product [$CR^{(0)}$] · [a_{rs}], in accordance with the rule of the multiplication of the dimension vector $(n+m)$ by the dimension matrix $(n+m)\cdot(n+m)$ is:

$$[\ CR^{(0)}\ ] \cdot [\ a_{rs}\ ] = [\ \sum_{r=1}^{n} \kappa_r^{(0)} a_{r1} + \sum_{r=I}^{m} \rho_r^{(0)} a_{r1} \quad \sum_{r=1}^{n} \kappa_r^{(0)} a_{r2} + \sum_{r=I}^{m} \rho_r^{(0)} a_{r2} \quad \dots$$
$$\sum_{r=1}^{n} \kappa_r^{(0)} a_{rn} + \sum_{r=I}^{m} \rho_r^{(0)} a_{rn} \quad \sum_{r=1}^{n} \kappa_r^{(0)} a_{rI} + \sum_{r=I}^{m} \rho_r^{(0)} a_{rI} \quad \sum_{r=1}^{n} \kappa_r^{(0)} a_{rII} +$$
$$\sum_{r=I}^{m} \rho_r^{(0)} a_{rII} \quad \dots \quad \sum_{r=1}^{n} \kappa_r^{(0)} a_{rm} + \sum_{r=I}^{m} \rho_r^{(0)} a_{rm}\ ]$$

And the developments and the values of each of the pairs of sums are the following:

$$\sum_{r=1}^{n} \kappa_r^{(0)} a_{r1} + \sum_{r=I}^{m} \rho_r^{(0)} a_{r1} = \kappa_1^{(0)} a_{11} + \kappa_2^{(0)} a_{21} + \ldots + \kappa_n^{(0)} a_{n1} + \rho_I^{(0)} a_{I1} + \rho_{II}^{(0)} a_{II\,1} + \ldots + \rho_m^{(0)} a_{m1} = \kappa_1^{(j)}$$

The a_{r1} $(\forall r = 1, 2 \ldots n, I, II \ldots m)$ of this expression, in accordance with the second sub-index $(= 1)$, indicate, in both cases, probabilities that each $\kappa_h^{(0)}$ $(\forall h = 1, 2 \ldots n)$ and each $\rho_f^{(0)}$ $(\forall f = I, II \ldots m)$ may transform into $\kappa_1^{(j)}$.

$$\sum_{r=1}^{n} \kappa_r^{(0)} a_{r2} + \sum_{r=I}^{m} \rho_r^{(0)} a_{r2} = \kappa_1^{(0)} a_{12} + \kappa_2^{(0)} a_{22} + \ldots + \kappa_n^{(0)} a_{n2} + \rho_I^{(0)} a_{I2} + \rho_{II}^{(0)} a_{II\,2} + \ldots + \rho_m^{(0)} a_{m2} = \kappa_2^{(j)}$$

The a_{r2} $(\forall r = 1, 2 \ldots n, I, II \ldots m)$ of this expression, in accordance with the second sub-index $(= 2)$, indicate, in both cases, probabilities that each $\kappa_h^{(0)}$ $(\forall h = 1, 2 \ldots n)$ and each $\rho_f^{(0)}$ $(\forall f = I, II \ldots m)$ may transform into $\kappa_2^{(j)}$.

And so on, successively, in each of the other expressions, in which the transformations into... $\kappa_n^{(j)}$, $\rho_I^{(j)}$, $\rho_{II}^{(j)}$ ... $\rho_m^{(j)}$ will arise:

... ...

$$\sum_{r=1}^{n} \kappa_r^{(0)} a_{rn} + \sum_{r=I}^{m} \rho_r^{(0)} a_{rn} = \kappa_1^{(0)} a_{1n} + \kappa_2^{(0)} a_{2n} + \ldots + \kappa_n^{(0)} a_{nn} + \rho_I^{(0)} a_{In} + \rho_{II}^{(0)} a_{II\,n} + \ldots + \rho_m^{(0)} a_{mn} = \kappa_n^{(j)}$$

$$\sum_{r=1}^{n} \kappa_r^{(0)} a_{rI} + \sum_{r=I}^{m} \rho_r^{(0)} a_{rI} = \kappa_1^{(0)} a_{1I} + \kappa_2^{(0)} a_{2I} + \ldots + \kappa_n^{(0)} a_{nI} + \rho_I^{(0)} a_{I\,I} + \rho_{II}^{(0)} a_{II\,I} + \ldots + \rho_m^{(0)} a_{mI} = \rho_I^{(j)}$$

$$\sum_{r=1}^{n} \kappa_r^{(0)} a_{rII} + \sum_{r=I}^{m} \rho_r^{(0)} a_{rII} = \kappa_1^{(0)} a_{1II} + \kappa_2^{(0)} a_{2\,II} + \ \dots\ \kappa_n^{(0)} a_{nII} + \rho_I^{(0)} a_{I\,II} +$$

$$\rho_{II}^{(0)} a_{II\,II} + \dots + \rho_m^{(0)} a_{mII} = \rho_{II}^{(j)}$$

... ...

$$\sum_{r=1}^{n} \kappa_r^{(0)} a_{rm} + \sum_{r=I}^{m} \rho_r^{(0)} a_{rm} = \kappa_1^{(0)} a_{1m} + \kappa_2^{(0)} a_{2m} + \dots + \kappa_n^{(0)} a_{nm} + \rho_I^{(0)} a_{Im}$$

$$+ \rho_{II}^{(0)} a_{IIm} + \dots + \rho_m^{(0)} a_{mm} = \rho_m^{(j)}$$

which is what we had reflected in [VI] for the product $[CR^{(0)}] \cdot [a_{rs}]$ and what we had obtained in [V]. Thus it is that the matrix $[\ a_{rs}\ ]$ transforms the initial structural configuration $[CR^{(0)}]$ into the final structural configuration $[\ CR^{(j)}\ ]$.

1.5. THE LIMITING STRUCTURE OF THE CREDIT-RISK BALANCE SHEET

We shall assume a certain stationary nature in the business's internal policies and make a projection of the evolution of the Credit-Risk Balance Sheet.

The Credit-Risk Balance Sheet of a business may be considered as an evolutive system to which the procedures of the Markov chain or the Markovian process can be applied[10]. "It is shown that for a regular-type Markov chain, there is a *state of equilibrium* which is unique and independent of the initial configuration, which means that the *reiterated application of the transition matrix (...) to any initial configuration produces, after a certain number of steps, always the same, well-defined configuration, which is one of equilibrium*. To obtain the configuration of equilibrium there is, in general, a need to carry out a rather large number of steps, that is, to repeatedly apply the stochastic matrix a large number of time to the initial vector" (the italics in the quotes are ours) [Cutolo, 1963: 1202].

Starting from the already-noted vector representing the initial structure of the Credit-Risk Balance Sheet:

$$[\ CR^{(0)}\ ] = [\ \kappa_1^{(0)}\ \ \kappa_2^{(0)}\ \dots\ \kappa_n^{(0)}\ \ \rho_I^{(0)}\ \ \rho_{II}^{(0)}\ \dots\ \rho_m^{(0)}\ ]$$

and from the transition matrix [a_{rs}], we have for following stages:

$$[\ CR^{(1)}\ ] = [\ CR^{(0)}] \cdot [\ a_{rs}\ ]$$
$$[\ CR^{(2)}\ ] = [\ CR^{(1)}\ ] \cdot [\ a_{rs}\ ] = [\ CR^{(0)}\ ] \cdot [\ a_{rs}\ ]^2$$

... ...

$$[\ CR^{(j)}\ ] = [\ CR^{(j-1)}] \cdot [\ a_{rs}\ ] = [\ CR^{(0)}\ ] \cdot [\ a_{rs}\ ]^j$$

That is, the transition (or transformation) matrix has been successively transforming the initial structure of the Credit-Risk Balance Sheet.

It is shown[11] that, for a sufficiently-large j $(j \to \infty)$, [a_{rs}]$^\infty$ is a matrix in which all the rows are the same

$$[\ a_{rs}\ ]^\infty = \begin{bmatrix} a_1 & a_2 & \dots\dots & a_n & a_I & a_{II} & \dots\dots & a_m \\ a_1 & a_2 & \dots\dots & a_n & a_I & a_{II} & \dots\dots & a_m \\ \dots & \dots & \dots\dots & \dots & \dots & \dots & \dots\dots & \dots \\ a_1 & a_2 & \dots\dots & a_n & a_I & a_{II} & \dots\dots & a_m \\ a_1 & a_2 & \dots\dots & a_n & a_I & a_{II} & \dots\dots & a_m \\ a_1 & a_2 & \dots\dots & a_n & a_I & a_{II} & \dots\dots & a_m \\ \dots & \dots & \dots\dots & \dots & \dots & \dots & \dots\dots & \dots \\ a_1 & a_2 & \dots\dots & a_n & a_I & a_{II} & \dots\dots & a_m \end{bmatrix}$$

and which we shall write, in abbreviated form, as

$$a_i = [\ a_1\ \ a_2\ \ \dots\ \ a_n\ \ a_I\ \ a_{II}\ \ \dots\ \ a_m\ ]$$

Then,

$$[\ CR^{(0)}\ ] \cdot [\ a_{rs}\ ]^j = [\ CR^{(j)}\ ] = [\ \kappa_1^{(j)}\ \ \kappa_2^{(j)}\ \dots\ \kappa_n^{(j)}\ \ \rho_I^{(j)}\ \ \rho_{II}^{(j)}\ \dots\ \rho_m^{(j)}\ ]$$

represents the *structural stability distribution of the system*, since if $j \to \infty$,

$$[\ CR^{(0)}\ ] \cdot [\ a_{rs}\ ]^{\infty} = [\ \kappa_1^{(0)}\ \ \kappa_2^{(0)}\ \dots\ \kappa_n^{(0)}\ \ \rho_I^{(0)}\ \ \rho_{II}^{(0)}\ \dots\ \rho_m^{(0)}\ ] \cdot [\ a_1\ \ a_2\ \dots\ a_n\ \ a_I\ \ a_{II}\ \dots\ a_m] = [\ CR^{(j)}\ ]$$

Carrying out the operation indicated in the second term gives the result[12]:

$$[\ (\kappa_1^{(0)} \cdot a_1 + \kappa_2^{(0)} \cdot a_1 + \dots \kappa_n^{(0)} \cdot a_1 + \rho_I^{(0)} \cdot a_1 + \rho_{II}^{(0)} \cdot a_1 + \dots + \rho_m^{(0)} \cdot a_1)\ (\kappa_1^{(0)} \cdot a_2 + \kappa_2^{(0)} \cdot a_2 + \dots \kappa_n^{(0)} \cdot a_2 + \rho_I^{(0)} \cdot a_2 + \rho_{II}^{(0)} \cdot a_2 + \dots + \rho_m^{(0)} \cdot a_2) \dots (\kappa_1^{(0)} \cdot a_m + \kappa_2^{(0)} \cdot a_m + \dots \kappa_n^{(0)} \cdot a_m + \rho_I^{(0)} \cdot a_m + \rho_{II}^{(0)} \cdot a_m + \dots + \rho_m^{(0)} \cdot a_m)\ ] = [\ a_1 (\sum_{h=1}^{n} \kappa_h^{(0)} + \sum_{f=I}^{m} \rho_f^{(0)})\ \ a_2 (\sum_{h=1}^{n} \kappa_h^{(0)} + \sum_{f=I}^{m} \rho_f^{(0)})\ \dots\ a_m (\sum_{h=1}^{n} \kappa_h^{(0)} + \sum_{f=I}^{m} \rho_f^{(0)})\ ]$$

Given that, as we have seen in [I], every parenthesis has a value of 200, the following is the value of the preceding expression:

$$200\ [\ a_1\ \ a_2\ \dots\ a_n\ \ a_I\ \ a_{II}\ \dots\ a_m\ ]$$

that is,

$$200\ a_i$$

Thus it is that:

$$[\ CR^{(0)}\ ] \cdot [\ a_{rs}\ ]^{\infty} = 200\ a_i = [\ CR^{(j)}\ ]$$

which means that the final configuration, which is that of equilibrium, shall depend, solely and exclusively, on a_i (the stabilised transformation matrix), with complete independence from the initial configuration, vector [$CR^{(0)}$]. So the tendency of the initial structural evolution towards stability depends only upon the policy followed in the period under review.

For Cutolo [1963: 1211], “the fate of the business is linked solely to the structure of the transition matrix. That is, to the internal dynamics of the business, obviously to management, and is independent of the conditions in which the business finds itself at the time of implementation of a development project”.

The Markovian process requires that, during the entire evolutionary process of the balance sheet, the transition coefficients remain constant. Cutolo [1965: 1058-1073], in a specific real-life case, which is also alluded to by Palomba [1972: 565], works with the balance sheets for the years 1953 to 1962 of the Turin company Fiat, where he obtains the transition matrix (constant) as the arithmetical average of the nine transition matrices calculated based on the balance sheets for the ten years under review. He obtains the transition matrix for each year indirectly [Cutolo, 1965: 1053], since he does not have he required information on the endogenous dynamics of the system and does not know the company’s balance sheet mutation flows. From said average transition matrix, he arrives at the final structural distribution of stability, applying the procedure we have seen above.

2. A READING OF THE TRANSITION MATRIX

2.1. INTRODUCTION

If a business at time t_j retains the same Credit-Risk Balance Sheet structure it had at a previous time t_0, it is in a stationary situation *(stazionarietà)*, which does not mean that there have been no movements:

$$\forall \kappa_h^{(0)} \ \exists \kappa_h^{(j)} \ \wedge \ \forall \rho_h^{(0)} \ \exists \rho_h^{(j)} \ / \ (\kappa^{(0)} \cong \kappa^{(j)}) \ \wedge \ (\rho^{(0)} \cong \rho^{(j)})$$

The above situation should be distinguished from a static one *(statizità)* which assumes that [a_{rs}] = 1, as has been seen in previous sections and implies that all the elements of [a_{rs}], except those of the main diagonal, are equal to 0, that is:

$$a_{11} = a_{22} = \ldots = a_{nn} = a_{I\,I} = a_{II\,II} = \ldots = a_{mm} = 1$$

And $a_{rs} = 0$, with $r \neq s$

However, on the other hand, the business at time t_j may not have the same Credit-Risk Balance Sheet structure as it had at a previous time t_0 :

$$\forall \kappa_h^{(0)} \ \exists \kappa_h^{(j)} \ \wedge \ \forall \rho_h^{(0)} \ \exists \rho_h^{(j)} \ / \ (\kappa^{(0)} \neq \kappa^{(j)}) \ \wedge \ (\rho^{(0)} \neq \rho^{(j)})$$

To properly judge the credit-risk situation of the business, it will not be enough to evaluate the structure it presents at any given time(s): it would also be wise to analyse the *flow* of values arising in the transformation. Next, we shall study, using the transition matrix as a basis, trends that can be observed in credit, risk, and credit and risk taken together, based on dividing said matrix into the four areas we have been considering.

2.2. ANALYSIS BY AREA AND CONSIDERING THE ROWS

Analysis by area-row will allows us to understand the evolution of credit and risk.

2.2.1. AREA 1 (CREDIT ↔ CREDIT)

Given that this area of the transition matrix [a_{rs}] contains the probabilities of mutation between the different classes of credit, we can consider an evolution of the credit portfolio to be unfavourable or favourable, with the former being reflected in a tendency to always move to lower-quality classes of credit, something that will happen if the rows make up growing sequences:

$$
\begin{array}{ccccccc}
a_{11} & < & a_{12} & < & \ldots & < & a_{1n} \\
a_{21} & < & a_{22} & < & \ldots & < & a_{2n} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \\
a_{n1} & < & a_{n2} & < & \ldots & < & a_{nn}
\end{array}
$$

which provide evidence of arrears in payment, at the very least, or even signs of insolvency.

If, on the other hand, the sequences of the rows are decreasing:

$$
\begin{array}{ccccccc}
a_{11} & > & a_{12} & > & \ldots & > & a_{1n} \\
a_{21} & > & a_{22} & > & \ldots & > & a_{2n} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \\
a_{n1} & > & a_{n2} & > & \ldots & > & a_{nn}
\end{array}
$$

we are witnessing an evolutionary situation of improvement in the quality of the credit portfolio.

As such variations in quality are detected, it will be necessary to reclassify credit into the appropriate level (not only in our credit-risk system, but also, in some cases, in the asset-based system) indicating the new quality associated with it.

If these situations are analysed together with those shown by the Area 4-rows (Risk ↔ Risk) of the transition matrix, a global view of the business's credit-risk situation can be provided.

Analysis of said Area 4-rows will provide us with conclusions on the assumption (own and external) of risk which, together with knowledge of the trends in credit quality and its flows, should give an understanding of the outlook for bad debts affecting the business.

Thus, a situation of worsening/improvement in credit (Area 1) may exist with a polarisation of risk towards transferred/non-transferred.

The gravity of a situation, by classes of credit (by Area 1 rows), may be assessed on the basis of how close the a_{rn} values $(\forall r = 1, 2 \dots n)$ come to 1. In a_{rn} we consider n to be constant, given that the credit category n corresponds to that of the worst quality, which is the category towards which it converges, from others, when there are arrears in payment or indications of potential losses due to bad debts.

Taking into account, as we saw in [IV], that

$$\sum_{s=1;s=I}^{n;m} \boldsymbol{a_{rs}} = \boldsymbol{1}$$

and that a situation in which the values of the elements in Area 2 of the transition matrix (which are, respectively, the continuation of each of the above rows of Area 1, which we are considering, in said matrix)

$$\begin{matrix} a_{1I} & a_{1II} & \dots & a_{1m} \\ a_{2I} & a_{2II} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{nI} & a_{nII} & \dots & a_{nm} \end{matrix}$$

tend towards 0 indicates the non-settlement of the credits in the period under review, that is, the low probability of settlement of the credits, then, the significant values of Area 1 of the transition matrix would tend to concentrate in a_{rn} $(\forall r = 1, 2 \dots n)$.

2.2.2. AREA 2 (CREDIT → RISK)

$$\begin{matrix} a_{1I} & a_{1II} & \dots & a_{1m} \\ a_{2I} & a_{2II} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots & \dots \end{matrix}$$

$$a_{nI} \quad a_{nII} \quad \ldots \quad a_{nm}$$

This area of the transition matrix shows the settlement of credit and permits the assessment by rows of how this has developed in each of the classes of credit *(1, 2 ... n)*, based on the different entities that have assumed the risk *(I, II ... m)*.

Values of the terms of said area tending towards *0* may indicate non-settlement (late payment), in other words that there has been no mutation of credit to risk, as has already been indicated above, with regard to Area 1, *in fine*. But it may also indicate that a particular source of risk did not "finance" a particular class of credit and, consequently, not give rise to any settlement.

The values of each row, irrespective of whether or not they make up a trend (increasing/decreasing), could provide evidence on whether the settlement of different classes of credits *(1, 2 ... n)* is influenced by the source of risk *(I, II ... m)*, that is, if the credits are settled homogenously irrespective of the source of the risk; in short, if there is a correlation between the entity that assumes the risk and settlement of credit. All this, however, without losing sight, as noted above that, a particular source may not have "financed", in risk terms, a particular class of credit and, consequently, in this case $a_{rs} = 0$.

An ordering of the terms in the above rows does not signify evolutive stages. Nevertheless, it could be interesting to consider:

$$\sum_{s=I}^{m-1} a_{1s} \geq \text{ó} \leq a_{1m}$$

$$\sum_{s=I}^{m-1} a_{2s} \geq \text{ó} \leq a_{2m}$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\sum_{s=I}^{m-1} a_{ns} \geq ó \leq a_{nm}$$

where the issue raised is the settlement of each of the classes of credit by external sources of risk *(I, II ... m-1)*, against the credits made possible by the assumption of risk by the vendor company *(m)*.

The assumption of risk by the vendor business may take place by default, that is, when it does not have the possibility of transferring it, for example as a consequence of the credit-elasticity of demand in which a more aggressive sales policy leads it to sell (on credit) to customers with a lower probability of collection, that is, with a lower credit score, and independently of any consideration of the assumption of risk. This situation would be made clear in inequalities of the type

$$\sum a_{rs} < a_{rm} \quad (\forall s = I, II \ \dots \ m\text{-}1)$$

2.2.3. AREA 3 (RISK → CREDIT)

$$\begin{array}{cccc} a_{I1} & a_{I2} & \dots & a_{In} \\ a_{II1} & a_{II2} & \dots & a_{IIn} \\ \dots & \dots & \dots & \dots \quad \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{array}$$

This area of the transition matrix shows the assumptions of risk (own and external) which have made the credits possible, and allows for evaluation by rows of how each source of risk has been applied to each of the classes of credit.

If the sequencing is decreasing

$$a_{I1} > a_{I2} > \dots > a_{In}$$
$$a_{II1} > a_{II2} > \dots > a_{IIn}$$

$$
\begin{array}{l}
\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\
a_{m1} > a_{m2} > \ldots > a_{mn}
\end{array}
$$

it shows a trend towards higher quality in the credit portfolio.

If the sequencing is increasing

$$
\begin{array}{l}
a_{I1} < a_{I2} < \ldots < a_{In} \\
a_{II1} < a_{II2} < \ldots < a_{IIn} \\
\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\
a_{m1} < a_{m2} < \ldots < a_{mn}
\end{array}
$$

it shows a trend towards lower quality in the credit portfolio.

Which will have to be analysed in the context of the overall general economic situation and/or of the credit-elasticity of demand, of a possible relaxation of the business's credit policy, etc.

We would like to highlight the importance of the last row, given that it reflects the flows arising from the risk assumed by the business.

2.2.4. AREA 4 (RISK ↔ RISK)

$$
\begin{array}{ccccc}
a_{I\,I} & a_{I\,II} & \ldots & a_{I\,m-1} & a_{Im} \\
a_{II\,I} & a_{II\,II} & \ldots & a_{II\,m-1} & a_{IIm} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
a_{m-1I} & a_{m-1II} & \ldots & a_{m-1\,m-1} & a_{m-1\,m} \\
a_{mI} & a_{mII} & \ldots & a_{m\,m-1} & a_{mm}
\end{array}
$$

This area of the transition matrix shows, by rows, how a risk initially assumed by a particular entity ends up being assumed by another.

Now, given that this does not signify evolutive stages –which does not mean that it cannot change from one situation to another– it would be important to consider:

$$\sum_{s=I}^{m-1} a_{Is} \geq ó \leq a_{Im}$$

$$\sum_{s=I}^{m-1} a_{IIs} \geq ó \leq a_{IIm}$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots$$

$$\sum_{s=I}^{m-1} a_{m\text{-}1\ s} \geq ó \leq a_{m\text{-}1\ m}$$

$$\sum_{s=I}^{m-1} a_{ms} \geq ó \leq a_{mm}$$

in whose second terms of the *m-1* first inequalities we have mutations from transferred risk (situations *I, II ... m-1*) to non-transferred risk (situation *m*).

If the inequalities are of the type $\sum a_{rs} < a_{rm}$, they show a tendency of polarisation towards non-transferred risk. In other words, particular risks that had initially been transferred end up being assumed.

Let us remember that, in the Credit-Risk Balance Sheet, risk is arranged into two large basic categories, transferred risk R_f *(*$\forall f$ *= I, II ... m-1)*, which can be subdivided into the different sources, and non-transferred risk R_m.

In fact, although any mutation can theoretically arise between one source of risk and another, in practice those that may be of greatest interest are those which take place between transferred risk (situations *I, II ... m-1*) and non-transferred risk (situation *m*). In principle, however, once the risk has been assumed by someone there ought not to be any mutation until the settlement of the credit.

Let us analyse one particular interesting case which may give rise to the mutations

$$a_{I\,I} \quad a_{I\,II} \quad \dots \quad a_{I\,m\text{-}1} \quad a_{Im}$$

The first risk category on the Credit-Risk Balance Sheet is “credit insurance” (R_I), and a_{Is} $(\forall s = I, II \dots m)$ expresses the probability that said source with be replaced by another or others.

Should there be circumstances in the contractual regime of the credit insurance which lead to an inhibition of responsibility on the part of the insurer, as the consequence of an incident of the kind noted below, a mutation to a_{Im} would arise. In other words, the vendor company would assume the risk originally assumed by the insurance company.

This would take place, in the specific case Category I source of risk (credit insurance), in the case of improper administration by the insured vendor business of the insurance policy, such as: *a)* the application of the proportional insurance rule in the case of failing to declare for cover purposes all declarable credit trading transactions (hiding of credit sales for insurance purposes); *b)* having inadequate supporting documentation for the sale, or it having formal defects, which makes it impossible to make any claim against the customer; *c)* failure to inform the insurer in due time and form of a non-payment situation; *d)* failure to inform the insurer of the worsening situation of the insured credits (failure to make payments, extended payment deadlines or other circumstances); *e)* the existence of an unpaid credit arising from a “commercial disagreement” in accordance with the terms established in Spain’s Commercial Code (Arts. 336 and 342)[13] and that does not arise due to liquidity problems or lack of a desire to make payment; *f)* breach of the timetable for notifying the insurer of non-payment, whose contractual consequence is a reduction in or cancellation of the guarantee percentage; *g)* failure to pay the insurance premiums; etc.

All these cases result in movement from risk initially transferred (to the insurer) to risk finally not transferred, or assumed by the business.

Thus, credits not covered by the insurer are subject to reclassification in the Credit-Risk Balance Sheet, which shall be equivalent, in the majority of cases, to it being the vendor business that assumes the risk. The mutation shall, in such a case, be that represented by a_{Im}, so that $a_{Im} > 0$ would provide evidence of ineffective management of the risk transferred to the insurer.

If we have, until this point, spoken of *exclusions* with regard to risk initially assumed by an entity (risk source *I* in our case), in the opposite sense, *inclusions* with regard to risk not initially assumed may arise (continuing with risk source *I*). Hence, the mutation of a_r $(\forall r = II, III \ldots m)$ to a_{rI}. Initial cover provided by the company insuring credit operations taking place for the first time with a particular customer maybe *a priori*, automatically, a pre-determined percentage, subsequently increased when the insurer approves the credit classification initially requested by the insured party.

The reasoning in the case of other external sources of risk would, *mutatis mutandis*, be analogous.

With regard to the rows *II* to *m-1*

$$\begin{matrix} a_{II\,I} & a_{II\,II} & \ldots & a_{II\,m-1} & a_{IIm} \\ a_{III\,I} & a_{III\,II} & \ldots & a_{IIIm-1} & a_{IIIm} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{m-1\,I} & a_{m-1\,II} & \ldots & a_{m-1\,m-1} & a_{m-1\,m} \end{matrix}$$

In principle, as already noted, once the risk has been assumed by someone, there should be no mutation at all until the credit is settled, so values other than 0 for the above elements would, with the exception of a_{rr} provide evidence of singular cases worthy of individual analysis.

And, finally, the row *m*

$$a_{m\,I} \quad a_{m\,II} \quad \ldots \quad a_{m\,m-1} \quad a_{mm}$$

shows situations of the shedding of risk by the vendor business as it is assumed by other entities.

2.3. ANALYSIS BY AREA AND CONSIDERING THE COLUMNS

2.3.1. AREA 1

If, in this area, we can establish an increasing sequence of type

$$a_{11} < a_{21} < \ldots < a_{n1}$$

and in the remaining a_{rs} terms the highest values relate to the probabilities of moving towards higher-quality classes of credit, with

$$a_{11} > a_{22} > \ldots > a_{nn}$$

such a structure would related to the profile of a business that sees an increase in its credit portfolio *per se* or because additional guarantees have been supplied.

As noted above, it is supposed that the business periodically monitors its credits, recalculating their credit score and, consequently, reclassifying them.

In the case of a decreasing sequence, the reasoning would be the other way round.

2.3.2. AREA 2

In this area, one can assess by rows, for each of the sources of risk *(I, II ... m)*, the different quality of the credits that have been settled. Values tending towards *0* may be indicative of non-settlement and, hence, of arrears.

A decreasing sequence could provide evidence of greater settlement of the credits of greater quality.

2.3.3. AREA 3

In this area, one can assess by rows the different sources of risk that come together in a particular class of credit.

It would be important in this case to consider:

$$\sum_{r=I}^{m-1} a_{rI} \geq \text{ó} \leq a_{mI}$$

$$\sum_{r=I}^{m-1} a_{r2} \geq \text{ó} \leq a_{m2}$$

$$\dots \quad \dots \quad \dots \quad \dots$$

$$\sum_{r=I}^{m-1} a_{rn} \geq \text{ó} \leq a_{mn}$$

Inequalities of the type $\Sigma a_{rs} < a_{ms}$ would be compatible with the profile of a business with an aggressive sales policy, in which the risks it assumes, irrespective of the quality of the credit, exceed any other risks assumed by third parties in any classes of credit.

Inequalities of the type $\Sigma a_{rs} > a_{ms}$ would be compatible with the profile of a business with a more conservative policy. They could provide evidence that it is the transferred risks, basically, that will make the credits possible.

This would be a situation compatible with a business policy of entrusting the study of credits to third parties, as well as the assumption of the relevant risk. A business situation of controlled expansion would be in line with this profile: in other words, growth made possible by the outside assumption of risk. The growth of many companies is, in fact, slowed by the desire not to assume such risks.

2.3.4. AREA 4

In this area, the ordering of the terms does not signify evolutive stages and what would be significant would be to consider the moves between transferred and non-transferred risks, that is:

$$\sum_{r=I}^{m-1} a_{rI} \geq ó \leq a_{mI}$$

$$\sum_{r=I}^{m-1} a_{rII}, \geq ó \leq a_{mII}$$

$$\dots \quad \dots \quad \dots \quad \dots$$

$$\sum_{r=I}^{m-1} a_{rm} \geq ó \leq a_{mm}$$

A structure such that

$$a_{ms}\ (\forall s = I, II \dots m\text{-}1) \neq 0$$

would correspond to the profile of a business which, subsequent to the sale, is managing to transfer own risk to third parties, which would require analysis of the sales procedures and circumstances of the business. Above, we have already specifically considered the case $a_{mI} \neq 0$.

3. CONCLUSIONS

1. Both credit and risk can evolve over time: credit quality can change, as can risk, which can move from transferred to non-transferred and vice-versa. The flow from transferred to non-transferred will, probably, be more frequent, but neither the reverse (non-transferred to transferred), nor movements within transferred risk itself should be ruled out.

2. The structural evolution of the Credit-Risk Balance Sheet allows for the identification of four types of mutations or migrations: *a)* internal mutations (i.e. endogenous transformations or intra-system transactions), which are those that take place within the credit or risk structure: Credit $\leftrightarrow$ Credit; Risk $\leftrightarrow$ Risk. *b)* External mutations (i.e. exogenous transformations or inter-system transactions), which are those that take place affecting credit and risk structures simultaneously: Credit $\rightarrow$ Risk; Risk $\rightarrow$ Credit.

3. Consideration of all the credit-risk interrelations or flows permits the establishment of a matrix of the probabilities of mutations – internal and external – of the elements of the Credit-Risk Balance Sheet, in which four areas can be delimited: *(i)* Credit-Credit, *(ii)* Credit-Risk, *(iii)* Risk-Credit y *(iv)* Risk-Risk.

4. The probabilities matrix, which is configured as a transition or transformation matrix, has the following properties: *a)* the values of all of the elements are non-negative or zero; *b)* the sum of the values of each row is always one; *c)* the sum of all the element of the matrix is the number of rows of the latter; *d)* the elements of the main diagonal express the probabilities that each element transforms into itself; *e)* if the matrix is reduced to a unit matrix (all the elements of the main diagonal are equal to one and the other elements are equal to zero), the business would not have operated in the credit-risk field in the period under review, since there are no mutations between the elements of said Credit-Risk Balance Sheet. In such a case, we would be in the presence of a *static situation*,

which should be distinguished from a *stationary situation*, which occurs when the structural configuration is retained.

5. Starting with the structural configuration of the Credit-Risk Balance Sheet at a given time, one can move to the structural configuration at a subsequent time by multiplying the vector representing that initial structure by the transformation matrix.

6. To properly judge the credit-risk situation of the business, it is not enough to evaluate the structure arising at given time, but one also needs to analyse the flows of values arising in the transformation.

7. Using the transition matrix, one can study trends that can be observed in credit, risk and in credit and risk taken together, on the basis of dividing said matrix into the four areas: Credit-Credit, Credit-Risk, Risk-Credit and Risk-Risk, either considering the rows or columns in each of said areas.

8. The validity of the evolutive analysis is subject to the permanence of the conditions reflected in the transformation coefficients employed.

NOTES

[1] In our Credit-Risk Balance Sheet, *Credit* is classified on the basis of its quality, whilst *Risk* is on the basis of its source, i.e. who assumes it or makes the credit possible. "Credit" also includes guarantees pledged, which are reflected in "non-transferred risk". One way of achieving the aforementioned ranking of credit is to employ some form of credit scoring. Each customer will receive a specific discriminant function for the company, thus giving a credit score or cs_i score for the *i-st* customer ($i = 1, 2, \ldots$), classifying them in accordance with their status and circumstances and which will take into account their economic-financial situation, their behaviour or problems with them over the length of their commercial relationship (failure to pay, payment arrears, disagreements over the merchandise sold, sales returns due to disagreements with the order, etc.); state of the economic subsector to which it belongs, holding of which it forms part, as the case may be, etc.

It can be concluded that:

i. A credit transaction, from the vendor's viewpoint, permits the identification of the origin of the credit facility (the existence of someone prepared to assume a credit risk) and the destination of the credit facility (a credit granted the customer). Consequently, it can be said that a causal credit-risk relationship is established and that there is, therefore, a duality of a causal, and not merely classificatory, nature.

ii. The Credit-Risk Balance Sheet drawn up, which is the result of an approach compatible with the causal duality, classifies credit on the basis of its quality (credit score) and risk on the basis of its source, that is, the establishment of who assumes said risk or makes the credit possible. The following dichotomy can be established in the overall risk grouping: non-assumed risk (that transferred to third parties) and assumed risk (not transferred). Its generic meaning is the representation of credit and the risk of default and its variations.

iii. The Credit-Risk Balance Sheet has a certain similarity with the conventional balance sheet. In our credit-risk sub-system, the nine laws of the patrimonial system are complied with, with it being possible to find a semantic or specific empirical meaning in all of them, with some slight qualification. It is also possible, following the analogy of the patrimonial system, to typify "credit-risk accounting events" as modificatory (involving increases and decreases) and permutative and in turn, from a double viewpoint depending upon whether their impact is upon "non-transferred risk" or "credit and risk structures".

[2] Professor Rivero Torre [1987], in his work *Análisis de balances y estados complementarios* (Analysis of balance sheets and complementary statements) dedicates Chapter 15 to "Sequential analysis", where he brings together the approaches of Palomba and Cutolo.

[3] The possible mutations will not have altered the percentages. This stationary situation *("stazionarietà")* must be distinguished from a static one *("statizità")*, which arises, as noted below, when no credit-risk movement occurs in the period under review.

[4] In the assessment of credit risk which, with a view to standardisation, has been drawn up by J.P. Morgan, the term "migration" is employed.

[5] Cf. meanings in the *Mutations* table above.

[6] *Ibid.*

[7] *Ibid.*

[8] *Ibid.*

[9] The currents or flows are configured into four areas:

INTRA-SYSTEM PERIOD TRANSACTIONS

Interrelations between accounts for:

A. Investment-Investment.
B. Financing-Financing.

INTER-SYSTEM PERIOD TRANSACTIONS

Interrelations between accounts for:

C. Financing-Investment.
D. Investment-Financing.

The structure of this is as follows:

[A] [D]

$$\begin{bmatrix}
x_{11} & x_{12} & \ldots\ldots & x_{19} & x_{1.10} & x_{1.11} & \ldots\ldots & x_{1.18} \\
x_{21} & x_{22} & \ldots\ldots & x_{29} & x_{2.10} & x_{2.11} & \ldots\ldots & x_{2.18} \\
\ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\
x_{91} & x_{92} & \ldots\ldots & x_{99} & x_{9.10} & x_{9.11} & \ldots\ldots & x_{9.18} \\
 & & & & & & & \\
x_{10.1} & x_{10.2} & \ldots\ldots & x_{10.9} & x_{10.10} & x_{10.11} & \ldots\ldots & x_{10.18} \\
x_{11.1} & x_{11.2} & \ldots\ldots & x_{11.9} & x_{11.10} & x_{11.11} & \ldots\ldots & x_{11.18} \\
\ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\
x_{18.1} & x_{18.2} & \ldots\ldots & x_{18.9} & x_{18.10} & x_{18.11} & \ldots\ldots & x_{18.18}
\end{bmatrix}$$

[C] [B]

The rows and columns relate to:

INVESTMENT:

1. Available sums in liquid money.
2. Certain collection, convertible into cash in the period.
3. Certain collection, convertible into cash in other periods.
4. Collectable, dependent upon production (production factors).
5. Collectable, dependent upon sales (semi-finished and finished).
6. Collectable, not connected with operating activities.
7. Net fixed assets, part of business's economic structure.
8. Fictitious net fixed assets, from business's economic restructuring.
9. Financial investments in other businesses and in own current assets for operating activities.

FINANCING:

10. Operating income from the sale of goods or services.
11. Non-operating income unrelated to the sale of goods or services (also includes extraordinary income).
12. Calculation of profit/loss for period (operating, non-operating and extraordinary).
13. External financing through supplier credits.
14. External financing through financing credits.
15. Own financing through capital contributions.
16. Own financing through reserves.
17. Own financing through results.
18. Own financing through depreciation/amortization.

We have not included in the above the sectors for own funds [E], [F], [G] and [H].

[10] Cf. amongst others, Kemeny, Schleifer, Snell and Thompson [1964]; Eppen and Gould [1987]; Gordon [1967].
As Gordon [1967: X-XI] notes in his preface, "What is a *Markov chain*? It is, *grosso modo*, a story: that of a that of a *system* which undergoes, over time, *changes of state* or random transitions and which, without entirely lacking in memory, only retains the *most recent* memory of its past".
Cyert, Davidson and Thompson [1962] and Díaz [1985] provide applications of Markov chains to accounts receivable.

[11] An elementary demonstration can be seen in Kemeny, Mirkil, Snell and Thompson [1959, Ch. 6, par. 3]. And in Cutolo [1963, par. 5 and 6] an example of application can be found. Also, in Cutolo [1965, par. 11 to 14].

[12] Remember that we multiply the originating line vector by a squared matrix of the order $n+m$ all of whose rows are the same, which we had symbolised by a_i.

[13] Art. 336 of Spain's Commercial Code: Those purchasers who, when receiving the goods, examine their content, shall have no cause for action for recovery against the vendor alleging faults or defects in the quantity or quality of the goods.- Purchasers shall have the right to action for recovery for *defect in the quantity or quality* of goods received in bundled or packaged form, provided that the action is carried out *within four days of their receipt* and that the fault does not arise from Act of God, defect inherent in the thing or fraud.- In these cases, the purchaser shall be entitled to opt for the rescission of the agreement or for compliance therewith in accordance with what has been agreed, but always with compensation for any harm caused by the defects or missing items.- *The vendor may avoid this action by requiring at the time of delivery that an inspection of the quantity and quality is carried out to the purchaser's satisfaction.*

Art. 342, of Spain's Commercial Code: Purchasers who have not made any complaint based on the *internal faults* of the thing sold, *within thirty days of its delivery,* lose all entitlements and the right to action for recovery for this cause against the vendor (the italics are ours).

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