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Col·lecció d'Economia

Term Structure of Interest Rates.

European Financial Integration

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Abstract: In this paper we estimate, analyze and compare the term structures of interest rates in six different countries over the period 1992-2004. We apply the Nelson-Siegel model to obtain the term structures of interest rates at weekly intervals. A total of 4,038 curves are estimated and analyzed.

Four European Monetary Union countries—Spain, France, Germany and Italy—are included. The UK is also included as a European non-member of the Monetary Union. Finally the US completes the analysis.

The goal is to determine the differences in the shapes of the term structure of interest rates among these countries. Likewise, we can determine the most usual term structure shapes that appear for each country.

Key words: term structure of interest rate, parsimonious models, level parameter, slope parameter, European interest rate.

JEL classification: C14, C51, C82, E43, G15

Resum: L'objectiu d'aquest article és estimar, analitzar i comparar les estructures de tipus d'interès de sis països diferents durant el període 1992-2004. Apliquem el model de Nelson-Siegel per obtenir les corbes de tipus d'interès setmanals. S'han estimat i analitzat un total de 4.038 corbes.

L'estudi inclou quatres països europeus, Espanya, França, Alemanya i Itàlia. El Regne Unit s'inclou com a país europeu, però que no pertany a la Unió Monetària. Finalment els Estats Units s'han incorporat a l'estudi per la seva importància dins dels mercats financers mundials.

L'objectiu del treball és determinar las diferencies en les formes de les corbes de tipus entre països i respecte el temps. Per altra banda també hem pogut establir quín és el tipus de forma més comú en cada país, degut a les particulars característiques del model utilitzat.

1. Introduction

Increasing capital mobility and decreasing market regulation are the main features of the current international economic system. This new situation directly affects monetary policy decisions. Financial markets provide information about the economic situation, thereby allowing some outcomes of possible economic decisions to be anticipated. Monetary authorities use variables such as monetary aggregates, interest rates and exchange rates to set monetary policy, paying special attention to the information contained in the yield curve. Among such financial indicators, the term structure of interest rates provides a valuable source of information for policymakers.

We define the term structure of interest rates as a function of the interest rate over a specific term. The best source for obtaining the required information is public debt.

Since we cannot determine the interest rate directly from the market for a wide range of maturities, we use various methodologies to obtain it. Professionals choose a model depending on the aim of their study; models based on *splines* have been widely accepted among them. These kinds of models fit the particularities all along the curve well. However, they usually have explosive tails, that is, long-term interest rates are not as asymptotic as desired.

Most central banks prefer to apply parsimonious functional forms (Anderson et al., 1996), specifically the model proposed by Nelson and Siegel (1987) and the extended version of Svensson (1994). Both are widely used in monetary policy analysis (Bank for International Settlements, 2005). In general, these models smooth the curve but respect its asymptotic properties.

Table 1 relates the models applied by many central banks. Most of the countries apply Nelson-Siegel (NS) or the extended version of Svensson (SV); however, Japan, the United Kingdom and the United States do not. Nevertheless, the United Kingdom used the Svensson model from January 1982 to April 1998.

The purpose of this paper is to obtain the term structures of interest rates with the NS model to analyze these term structures over the last decade. We are interested in finding the parameters (level, slope and curvature) of the model and compare the evolution of these curves in the context of the Monetary Union. The study covers a period of thirteen years, from 1992 to 2004, and we analyze the evolution of these term structures in six different countries: Spain, France, Germany, Italy, the United Kingdom and the United States. The first four countries are members of the European Monetary Union (EMU). Germany, France and Italy initially participated in the creation of the European Monetary System (EMS). Spain didn't join the EMS until 1986 but it is interesting to note that, like Italy, starting from an economic situation very different from France and Germany, it was able to fulfill the conditions set down at Maastricht. Moreover, both Italy and Spain were able to join the Economic and Monetary Union on 1st January 1999. Therefore, the evolution of Germany, France, Italy and Spain allows us to analyze the process of convergence across the single-currency countries in contrast to the United Kingdom and the United States, employed as outside references. Furthermore, the UK and the USA allow us to determine if the convergence is limited to single-currency countries or exists in other financial markets as well.

We estimate the term structures of interest rates applying the NS model, because of its widespread application and international relevance in the monetary policy context.

Section 2 details the construction of the database. In section 3 we define the model. The process used to estimate the term structures is specified in section 4. Section 5 reports the results from comparing the different parameter vectors among countries. Finally, section 6 sets out conclusions.

Table 1. Methodologies applied by central banks.

Source: Bank for International Settlements 2005, Zero-Coupon Yield Curves: Technical Documentation, Monetary and Economic Department.

Central Bank	**Estimation method**[1]	**Estimates available since**	**Frequency**	**Minimised error**	**Adjustments for tax distortions**	**Relevant maturity spectrum**
Belgium	SV NS	1 Sept 1997	daily	Weighted prices	No	Couple of days to 16 years
Canada[2]	SV	23 Jun 1998	daily	Weighted prices	Effectively by excluding bonds	1 to 30 years
Finland	NS	3 Nov 1998	weekly; daily from 4 Jan 199 9	Weighted prices	No	1 to 12 years
France	SV NS	3 Jan 1992	weekly	Weighted prices	No	Up to 10 years
Germany	SV	7 Aug 1997 Jan 1973	daily monthly	Yields	No	1 to 10 years
Italy	NS	1 Jan 1996	daily	Weighted prices	No	Up to 30 yrs. Up to 10 yrs. (bef. Feb.02)
Japan	SS	29 Jul 1998 to 19 Apr 2000	weekly	Prices	Effectively by price adjustments for bills	1 to 10 years
Norway	SV	21 Jan 1998	± monthly	Yields	No	Up to 10 years
Spain	SV NS	Jan 1995 Jan 1991	daily monthly	Weighted prices	No	Up to 10 yrs. Up to 10 yrs.
Sweden	SV	9 Dec 1992	at least once a week	Yields	No	Up to 10 years
Switzerland	SV SV	4 Jan 1998 Jan 1998	daily monthly	Yields	No	1 to 30 years
United Kingdom[3]	SV	4 Jan 1982 to 30 Ap.1998	daily monthly	Weighted prices	No	1 week to 30 years
	VRP VRP	4 Jan 1982 15 Jan 1985	daily daily	Weighted prices	No	1 week to 30 years
United States	SS	14 Jun 1961	Daily	Bills: Weighted p. Bonds p.	No	Up to 1 year

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[1] NS = Nelson-Siegel, SV = Svensson, SS = smoothing splines, VRP = variable roughness penalty.
[2] Canada is reviewing its current estimation method.
[3] The UK used the Svensson model from January 1982 to April 1998

2. Data base

The most common way to obtain the term structure of interest rates is from national debt, since it carries practically null insolvency risk. According to the Bank for International Settlement (2005), most central banks use national debt to obtain their curves.

The required information has mainly been provided by Bloomberg and some central banks. Specifically, we have requested the average prices of daily traded national debt and the characteristics of these bonds for every country and all years, namely the ISIN code, the bond type, the nominal interest rate, the issue date, the maturity date and the coupon frequency, as well as the accrual date and the first coupon payment date.

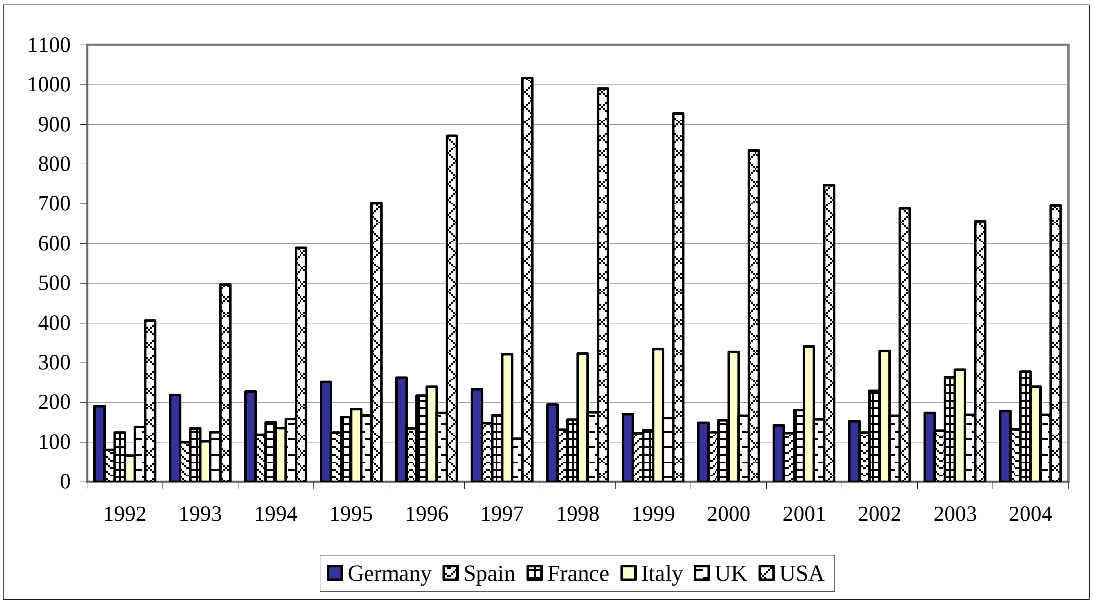
The two fundamental types of public debt are treasury bills and bonds. Generally, these two instruments represent a high percentage of the market debt issued by a government. Treasury bills, which do not pay periodic interest, are issued at a discounted rate and mature at different terms according to the country. Government bonds pay a fixed interest rate and have a fixed maturity date. All those bonds that have additional features or indexed variable coupons are excluded from the data set. Likewise, benchmark or reference bonds are used to cover those necessary terms in which no information is provided. Table 2 summarizes the government debt issued by each central bank included.

The resulting data set contains a total of 1,116,397 references, involving 64,998,548 observations. Figure 1 shows weekly averages for bonds over a year given for each country. We can see that the liquidity of the public debt market varies from country to country since its issuance is related to the financial necessities of each government. Figure 1 provides a summary of the available data for different maturities in the six countries.

Table 2. Public debt issued by every government.

	Instrument	**Maturity date**	**Issued**
Germany (*Deutschen Finanzagentur*)	Bubills	6 months	Discount
	Bobls	5 years	Annual coupon
	Bunds	10-30 years	Annual coupon
Spain (Departamento del Tesoro)	Letras Tesoro	6, 12 and 18 months	Discount
	Bonos del Estado	3 or 5 years	Annual coupon
	Obligaciones del Estado	15 or 30 years	Annual coupon
France (*Agence France Trésor*)	BTF	3, 6 and 12 months	Discount
	BTAN	From 2 to 5 years	Annual coupon
	OAT	From 7 to 30 years	Annual coupon
Italy (*Dipartamento del Tesoro*)	BOT	3, 6 and 12 months	Discount
	CTZ	18 or 24 months	Discount
	BPT	3, 5, 10 and 30 years	Semi-annual coupon
United Kingdom (*Debt Management Office*)	T-bills	1, 3, 6 and 12 months	Discount
	Conventional gilts	5, 10 and 30 years	Semi-annual coupon
United States of America (*Bureau of the Public Debt*)	T-bills	4, 13 and 26 weeks	Discount
	T-notes	2, 3, 5 and 10 years	Semi-annual coupon
	T-bonds	between 10 and 30 years	Semi-annual coupon

Figure 1. Weekly averages for bonds over a year.



3. The model

The NS model defines the term structure of instantaneous forward rate. It assumes that the instantaneous forward rate is the solution to a second-order differential equation with two equal roots. Hence, the NS model establishes that the functional form for the instantaneous forward rate at time t is the following:

$$f_m(\beta) = \beta_0 + \beta_1 \exp\left(\frac{-m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(\frac{-m}{\tau_1}\right), \qquad (1)$$

where $\beta = (\beta_0, \beta_1, \beta_2, \tau_1)$ denotes the vector of the parameters to estimate and m is the maturity.

Consequently, the spot rate $z_{t,m}$ with time to maturity m is calculated by integrating the forward rate. The discount factor δ_m for this period is equal to the exponential term $\exp(-z_{t,m} \cdot m)$. So, the discount factor is obtained by applying the previous mathematical relationship:

$$\delta_m(\beta) = \exp\left[-\beta_0 m - (\beta_1 + \beta_2)\tau_1\left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) + \beta_2 m \exp\left(-\frac{m}{\tau_1}\right)\right]. \qquad (2)$$

The spot rate, the discount factor and the forward rate have convenient properties. The limit:

$$\lim_{m\to\infty} f_m = \beta_0$$

shows that the forward rate curve converges asymptotically towards the parameter β_0, which can be interpreted as a long-term interest rate. If the maturity approaches zero, $\lim_{m\to 0} f_m = \beta_0 + \beta_1$, the forward rate equals the parameter combination $\beta_0 + \beta_1$, which can be interpreted accordingly as a very short rate (instantaneous interest rate). Thus, the forward rates approach a constant for long maturities and settlements. The parameter β_1 shows the *spread* between the short and long interest rates. This is an interesting parameter, since variations between the starting and the ending point of the curve generate changes in the slope.

The parameters β_2 and τ_1 do not appear in the very short or long term and do not have a comparable direct interpretation. Since the curvature is shown in the medium term, they influence the shape of the curve between these limits.

The *forward* function can present a stationary point in $\frac{m}{\tau_1} = 1 - \frac{\beta_1}{\beta_2}$, under the condition $|\beta_1| < |\beta_2|$. Then, the parameter β_2 determines the magnitude and shape of the curvature. If β_2 is positive, the curve will have an interior maximum and if β_2 is negative, it will have an interior minimum. When β_2 equals zero, monotocity will occur in the term structure of *forward* rate.

The stationary point will be nearer or farther from the value of the parameter β_0, depending on if β_1 and β_2 are positive or negative. If the shape of the curve is U-inverted, that is to say

$\beta_2 > 0$, and the condition $\beta_1 < 0$ is given, then the function starts from a point under β_0 and crosses the horizontal asymptote. Otherwise, when $\beta_1 > 0$, the function starts from a point over β_0 and it never crosses the asymptote.

Analogously, if the function is U-shaped, that is $\beta_2 < 0$, and the condition $\beta_1 < 0$ is given, then the function starts from a point under β_0 and remains under the horizontal asymptote. Otherwise, if $\beta_1 > 0$, the function falls from a point over β_0 and crosses the horizontal asymptote.

The speed at which the instantaneous forward rate approaches its asymptotic level β_0 depends on τ_1. An increase in τ_1 shifts the curvature towards the right, so that the bigger τ_1 is, the more slowly the forward interest rate will approach β_0. The parameter τ_1 can only take positive values in order to guarantee the long-term convergence of β_0. Given β_1 and β_2, τ_1 is determined by *m* according to the expression $\frac{m}{\tau_1} = 1 - \frac{\beta_1}{\beta_2}$ (see Meier 1999 and Schich 1996).

Under the condition $|\beta_1| \geq |\beta_2|$, the term structure does not present a stationary point. Then, if the parameter β_1 is negative, the term structure is monotonically rising and if β_1 is positive, the function will be monotonically falling. In this case, if $\beta_2 > 0$ and $\beta_1 < 0$, the second derivative is negative and the function will be concave. On the contrary, if $\beta_2 < 0$ and $\beta_1 > 0$, the second derivative is positive and the function is convex. Otherwise, the concavity or convexity of the function cannot be determined since it depends on the value and magnitude of these parameters.

Table 3 summarizes the different shapes of the term structure of interest rates according to the relationships and signs of the different parameters that define the NS model.

Table 3. Term structures shapes and relation between parameters.

Shape	β_0	β_1	β_2	τ_1	**Condition**
Increasing, concave	+	−	+	+	$\|\beta_1\| \geq \|\beta_2\|$
Increasing	+	−	−	+	$\|\beta_1\| \geq \|\beta_2\|$
Decreasing, convex	+	+	−	+	$\|\beta_1\| \geq \|\beta_2\|$
Decreasing	+	+	+	+	$\|\beta_1\| \geq \|\beta_2\|$
Hump, above β_0	+	+	+	+	$\|\beta_1\| < \|\beta_2\|$
Hump, crosses β_0	+	−	+	+	$\|\beta_1\| < \|\beta_2\|$
Through, below β_0	+	−	−	+	$\|\beta_1\| < \|\beta_2\|$
Through, crosses β_0	+	+	−	+	$\|\beta_1\| < \|\beta_2\|$

Thus, we are able to carry out a descriptive analysis of the term structures of different countries by applying these relationships between parameters.

4. Estimating the term structures

Prior to obtaining the term structures, it is necessary to decide the frequency of estimation. The frequency of estimation must reflect all the shifts of the curve. Likewise, we need to ensure that enough data is provided in every period in order to estimate the optimal parameters. From the analysis of the daily and weekly distribution data in each of the countries, we consider that the optimal frequency for estimating the term structures is weekly, as this frequency ensures representation of the evolution of the term structure in each country for the complete period in question. As a result, the total number of curves to estimate rises to 4,038, one for each week from 1992 to 2004, for each of the six countries.

Before proceeding to the estimation of the parameters, we need to carry out a process of depuration on the data set. The yield-to-maturity curves are a useful instrument before obtaining the parameters of the term structure of interest rates. The yield-to-maturity $r_{t,i}$ of a bond i at a point in time t can be calculated from the equation of the price of a bond, using an iterative process:

$$P_{t,i} = \sum_{m_i=1}^{M_i} C_i \cdot \left(1 + r_{t,i}\right)^{-m_i} + N_i \cdot \left(1 + r_{t,i}\right)^{-M_i} . \tag{3}$$

The yield $r_{t,i}$ for the maturity M_i represents the average rate of return from holding a bond for M_i years, assuming that all coupon payments are reinvested at exactly the same interest rate $r_{t,i}$ during the maturity of the bond. The depuration process is carried out in order to eliminate short-term outliers. These outliers hinder the estimation of the parameters. Likewise, we eliminate all those observations with negative yield-to-maturity or those that remain very far from the yield curve.

We estimate the parameter vector $\beta_t = \left(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t}\right)$ separately for each week. The non-linear model to estimate is:

$$\begin{aligned} P_{t,i} = C_i \cdot \sum_{m_i=1}^{M_i} \left(\exp\left[-\beta_{0,t} m_i - \left(\beta_{1,t} + \beta_{2,t}\right)\tau_{1,t} \left(1 - \exp\left(-\frac{m_i}{\tau_{1,t}}\right)\right) + \beta_{2,t} m_i \exp\left(-\frac{m_i}{\tau_{1,t}}\right)\right]\right) + \\ + N_i \cdot \left(\exp\left[-\beta_{0,t} M_i - \left(\beta_{1,t} + \beta_{2,t}\right)\tau_{1,t} \left(1 - \exp\left(-\frac{M_i}{\tau_{1,t}}\right)\right) + \beta_2 M_i \exp\left(-\frac{M_i}{\tau_{1,t}}\right)\right]\right) + \varepsilon_{t,i}, \end{aligned} \tag{4}$$

where $P_{t,i}$ corresponds to the price of a bond i at a point in time t, C_i is the coupon payment, N_i represents the redemption payment, m_i corresponds to the moments at which the different coupon payments and redemption payment take place and $\varepsilon_{t,i}$ $\forall t,i$ are random errors, identically and independently distributed (iid), for which we assume normal distribution with average 0 and variance σ_ε^2.

We minimize the sum-squared error (SSE) between the observed price $\left(P_{i,t}\right)$ and the fitted price by the model $\left(\hat{P}_{i,t}\right)$ to estimate the parameter vector $\beta_t = \left(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t}\right)$. The difference between the two prices is weighted by an inverse factor proportional to the maturity. The optimization criterion of the models with parsimonious forms can be defined according to the price or, if not, according to the yield to maturity. When the minimization of the error in price is applied, a factor inversely proportional to the duration is often included. The inclusion of this factor is due to the difficulty in fitting the short term of the term structure. In general, the fitting in the long term is quite good, since the function is asymptotic by definition. However, the estimated term structure sometimes does not fit very well at the short and medium-terms, where the slope and curvature are given. In order to improve the fitting, it is alternatively proposed to use an error on prices weighted by a factor inversely proportional to the duration (Ricart and Sicsic, 1995; Bolder and Stréliski, 1999).

The sum squares weighted error (SSWE) that we apply to obtain the estimated parameter vector corresponds to:

$$SECP_t\left(\hat{\beta}_t\right) = \sum_{i=1}^{n_i} \alpha_{t,i} \left(e_{t,i}\left(\hat{\beta}_t\right)\right)^2 = \sum_{i=1}^{n_i} \left(\frac{1 / d_{t,i}}{\sum_{s=1}^{n_t} 1 / d_{t,s}} \right) \left(P_{t,i} - \hat{P}_{t,i}\right)^2, \tag{5}$$

where $d_{i,t}$ is the duration of a bond i in a week t and n_t is the number of bonds in a week t. In this case, the duration definition corresponds to the Macauley modified duration.

A Newton iterative algorithm is used to minimize the objective function. The software used is SAS 9.1. In order to start the iterative algorithm, initial values for the parameter vector β_t must be provided. These initial values significantly influence the final estimated parameter vector. If the initial parameter vector is very different from the optimal one, the algorithm does not converge. In general, the parameter vector from the previous week is suitable to obtain the optimal parameter vector for the following week. However, in periods of instability, the parameters vary between weeks. In such cases it is necessary to find other alternative values to initialize the iterative process.

Since there is a relationship between the term structure of interest rates and the yield curve, we use this yield curve as an approach to the starting vector of parameters for the first week in 1992. In addition, it is useful in all those cases where the previous week's parameter vector is not suitable.

The interpretation of β_0 and β_1 provides starting values for both of these parameters. β_0 is the long term interest rate implied by the model. Therefore, the smoothed yield with the longest maturity is used as the starting value for β_0. The difference between the smoothed yield with the longest and shortest maturity is used as a starting value for the spread parameter β_1. For β_2 and τ_1 there is no specific economic interpretation. Notwithstanding, when a maximum or minimum exists, and if there is enough data, we can approach the parameter β_2 with coordinates (*r, m*) as:

$$r = \beta_0 + \beta_2 \cdot \exp\left(-1 + \frac{\beta_1}{\beta_2}\right).$$

$$\frac{m}{\tau_1} = 1 - \frac{\beta_1}{\beta_2}$$

Otherwise, when the yield curve does not present a stationary point or the starting values are not convenient, the estimation is carried out by specifying several reasonable starting values. The estimated parameters that provide the best fit are then used.

Moreover, constraints are imposed on the parameters and on the spot-rate curve in order to exclude implausible and unrealistic estimation results. Thus we reject negative values in $\beta_{t,0}$

and $\tau_{t,1}$. Likewise, we must guarantee that a short-term interest rate, equivalent to the sum of parameters $\beta_0 + \beta_1$, is always positive.

Over most of the thirteen-year sample period and for all countries considered, the model has produced reliable and reasonable estimation results. Once the 679 parameter vectors for each country have been estimated, we can obtain the forward function, the discount factor and the spot function for any time to maturity m. From all possible parameter constellations, the term structure shapes are found for every country and week from 1992 to 2004. The analysis of these parameters and the different shapes are one of the main results of this paper.

The descriptive analysis of the weekly estimated parameters for every year is presented in detail in appendix 1. In order to consult any weekly term structure of interest rates from January 1992 to December 2004, see http://guillen.eco.ub.es/~eruizd.

5. Results

The method of estimation, shown in the previous section, has generated the weekly spot and forward curves for the six countries from 1992 to 2004 inclusive.

According to the values and the relationship between the parameters detailed in table 3, the term structure defined by the NS model can take different shapes. Thus we are able to apply these shapes to the obtained parameter vectors for each country. The results are detailed in table 4.

It is noteworthy that the most frequent forms in the EMU countries are the increasing and the trough below β_0 shapes. These two shapes are given in roughly 70% of the total 679 analyzed weeks for each country. In the United Kingdom, in addition to these two shapes, the humped curve above β_0 can be seen. The occurrence of this last shape is limited practically to the

United Kingdom. In the United States, alongside the increasing and trough below β_0 shapes, we can find humped curves crossing β_0.

Table 4. Frequency of the term structure shapes.

	Germany	Spain	France	Italy	United Kingdom	United States
Increasing, concave	8%	13%	13%	15%	5%	15%
Increasing	40%	28%	40%	37%	17%	34%
Decreasing, convex	0%	2%	2%	2%	2%	0%
Decreasing	4%	7%	3%	2%	7%	1%
Hump, above β_0	1%	1%	0%	3%	26%	1%
Hump, crosses β_0	0%	4%	0%	10%	8%	21%
Through, below β_0	38%	39%	37%	32%	30%	32%
Through, crosses β_0	8%	6%	5%	0%	4%	2%

Since we are interested in comparing the time serial of the term structure of interest rate, the first step is to observe the evolution of each parameter in the NS model. In figures 2 to 4 the parameters β_0, β_1 and β_2 are represented. From these figures we realize that the values of the parameters are very similar among EMU countries from 1999, mainly for the β_0 and β_1 parameters.

Consequently, we would like to test whether their term structures of interest rates are actually the same or whether the EMU countries maintain some differences between shape curves.

Figure 2. Weekly level parameter (β_0) values from January 1992 to December 2004.

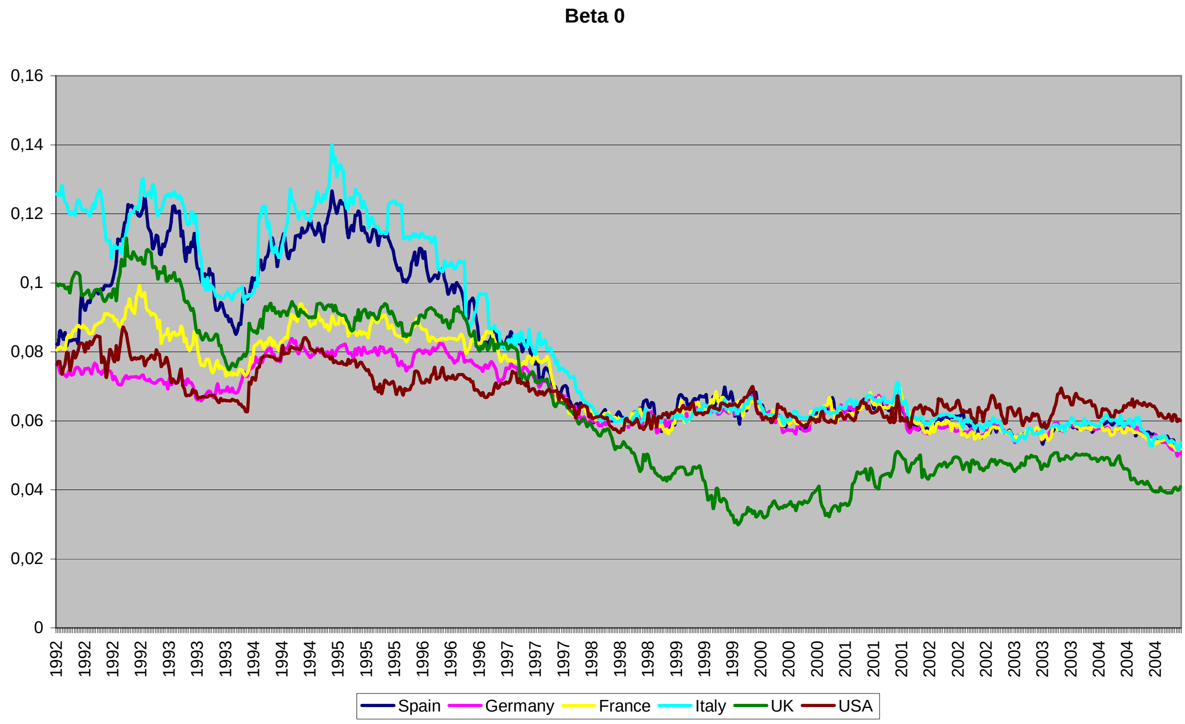


Figure 3. Weekly slope parameter (β_1) values from January 1992 to December 2004

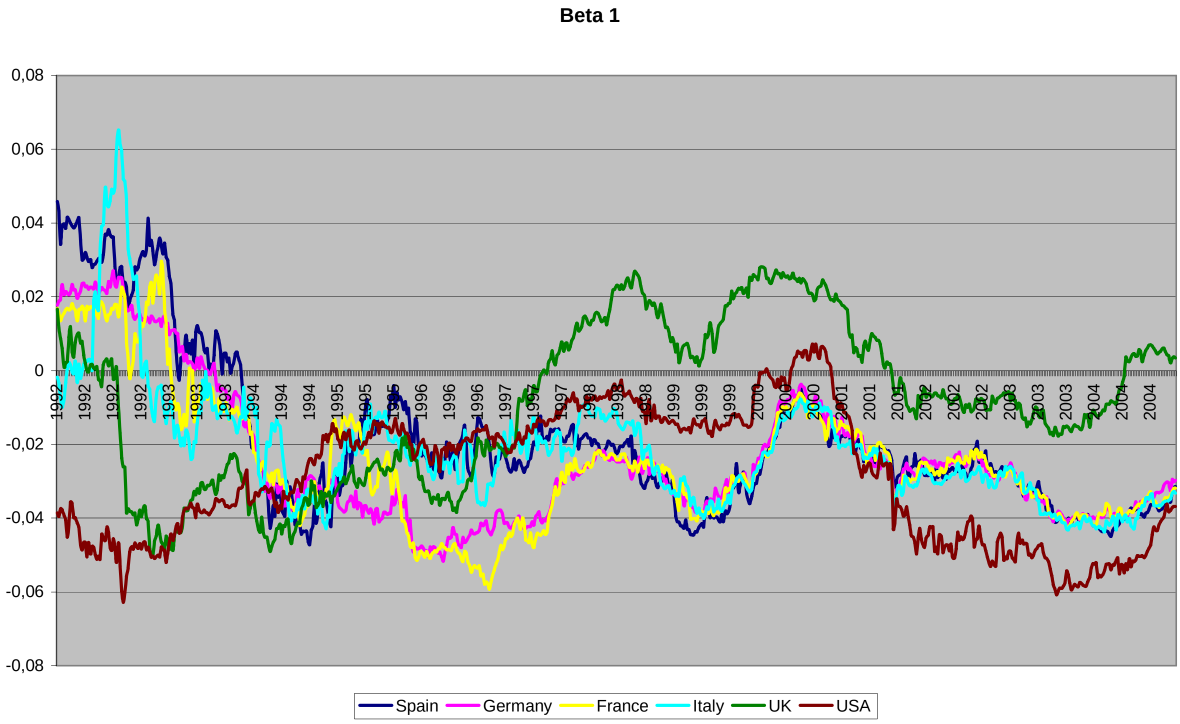
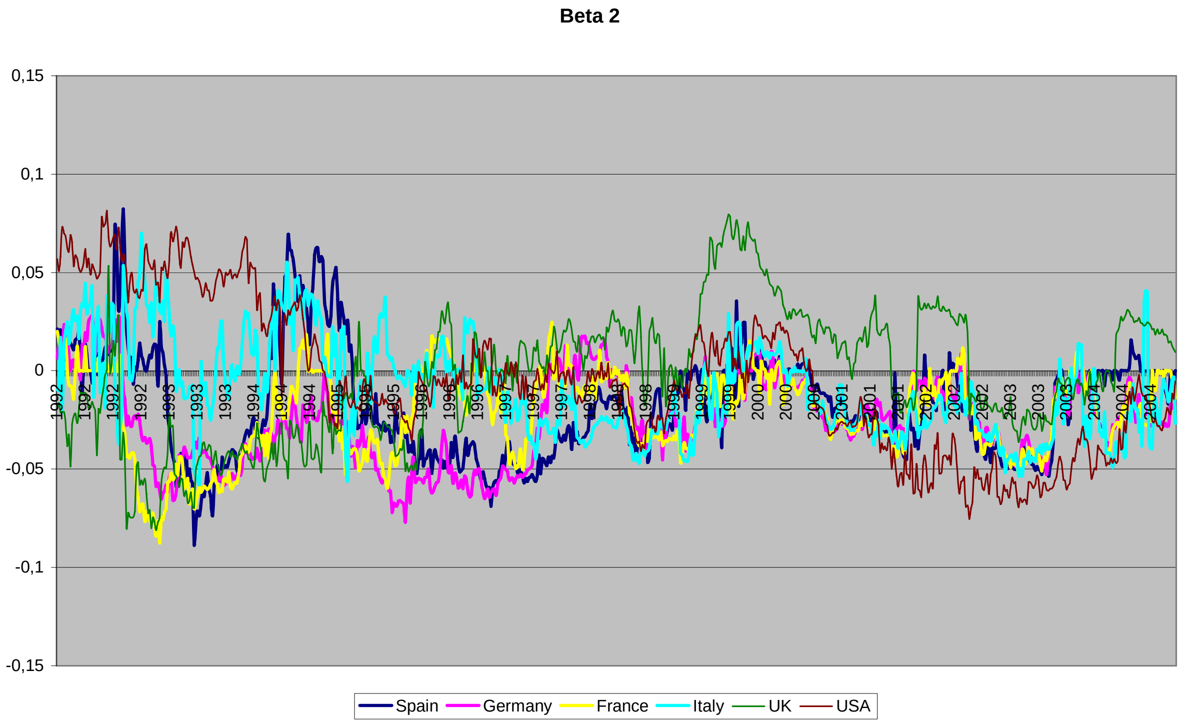


Figure 4. Weekly parameter β_2 values from January 1992 to December 2004



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In order to test whether the term structures of interest rates are equal among these four countries, we apply univariate and multivariate inference. The univariate inference is used for each of the four parameters ($\beta_0, \beta_1, \beta_2, \tau_1$) defined by the NS model for the EMU countries. The null hypothesis in the test is defined as equal parameters between countries. That is to say:

$$\begin{aligned}
H_0 &: \beta_0^{Sp} = \beta_0^{It} = \beta_0^{Fr} = \beta_0^{G} \\
H_0 &: \beta_1^{Sp} = \beta_1^{It} = \beta_1^{Fr} = \beta_1^{G} \\
H_0 &: \beta_2^{Sp} = \beta_2^{It} = \beta_2^{Fr} = \beta_2^{G} \\
H_0 &: \tau_1^{Sp} = \tau_1^{It} = \tau_1^{Fr} = \tau_1^{G}
\end{aligned},$$

where ‘Sp’ is Spain, ‘It’ Italy, ‘Fr’ France and ‘G’ Germany.

This inference is drawn for each year from 1999 to 2004. We calculate the F-Fisher statistic in the variance analysis for each year in order to test the null hypothesis. It allows us to compare the differences of the parameters between countries with the differences of the parameters inside countries for each year.

In general, let Y_t^i be the variable associated with a country *i* at time *t* (where in our case *t* indicates a week in a year), $\overline{Y}^i$ is the average in the country *i* and $\overline{Y}$ is the average for all times and all countries. The sum of squares between countries is:

$$E = \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\overline{Y}^i - \overline{Y}\right)^2 = T\sum_{i=1}^{N}\left(\overline{Y}^i - \overline{Y}\right)^2$$

and the sum of squares within countries is:

$$D = \sum_{t=1}^{T}\sum_{i=1}^{N}\left(Y_t^i - \overline{Y}^i\right)^2 ,$$

where *N* is the number of countries and *T* is the number of times.

The F-Fisher statistic for the equality averages contrast is:

$$F = \frac{T\sum_{i=1}^{N}\left(\overline{Y}^i - \overline{Y}\right)^2}{\sum_{t=1}^{T}\sum_{i=1}^{N}\left(Y_t^i - \overline{Y}^i\right)^2}\frac{T-N}{N-1} ,$$

and it is distributed as a F-Fisher with (*N*-1) degrees of freedom in the numerator and (*T*-*N*) degrees of freedom in the denominator.

In order to test the null hypothesis for each defined parameter, we consider Y_t^i as an estimated parameter for a country *i* in a week *t*.

We calculate the F-statistic for each year in the period in question. The Fisher contrast is based on the independence hypothesis of the data. We select the first week of each four, we can consider this as a representative random sample of the data. We have calculated the results for the second, third and fourth weeks and they were very similar.

The results obtained for each year between 1999 and 2004 are shown in table 5. We can see that, for the years 1999 and 2003, we cannot reject that the term structures of interest rates for Germany, France, Italy and Spain are the same. Thus, the term structures of interest rates are

Table 5. The F-Fisher statistics results.

	β_0	β_1	β_2	τ_1
1999	1.30	2.44	2.31	1.70
2000	0.83	0.36	1.77	3.21*
2001	1.35	0.67	0.53	10.94*
2002	8.33*	8.62*	0.24	1.65
2003	0.61	0.09	0.16	2.05
2004	1.64	2.61	7.88*	5.44*
All	**2.1**	**0.82**	**1.90**	**5.41***

***Indicates statistical significance at 5%.**

statistically equal for the EMU countries in these two years. Moreover, for the years 2000 and 2001 the difference is in τ_1. With respect to 2002, there arise significant differences among countries in β_0 and β_1, although β_2 and τ_1 are statistically equal. Finally, the opposite situation is given in 2004. In general, the β_0 and β_1 parameter values are not statistically different, so the term structures in the EMU countries present the same level and slope from 1999 onwards, except for 2002. Thus, we accept that short and long-term interest rates are equal but we reject that medium-term interest rates are the same in these four countries, since the curvature is manifested in the medium term.

In order to complete the univariate inference, we have calculated the multivariate inference.

Now, the null hypothesis is:

$$H_0 : \beta^{Sp} = \beta^{It} = \beta^{Fr} = \beta^{G} ,$$

where $\beta^i = \left(\beta_0^i, \beta_1^i, \beta_2^i, \tau_1^i\right)$ is the parameter vector for a country *i*. We calculate the Wilk's Λ statistic for the multivariate contrast in each year of the period. This statistic is calculated as the following:

$$\Lambda = \frac{|E|}{|T|}$$

where |·| denotes the determinant. In this case, *E* is a matrix of the sum of squares between countries and *T*=*E*+*D*, being *D* the matrix of the sum of squares within countries, that is:

$$E = \begin{pmatrix} T\sum_{i=1}^{N}\left(\overline{\beta}_0^i - \overline{\beta}_0\right)^2 & T\sum_{i=1}^{N}\left(\overline{\beta}_0^i - \overline{\beta}_0\right)\left(\overline{\beta}_1^i - \overline{\beta}_1\right) & T\sum_{i=1}^{N}\left(\overline{\beta}_0^i - \overline{\beta}_0\right)\left(\overline{\beta}_2^i - \overline{\beta}_2\right) & T\sum_{i=1}^{N}\left(\overline{\beta}_0^i - \overline{\beta}_0\right)\left(\overline{\tau}_1^i - \overline{\tau}_1\right) \\ T\sum_{i=1}^{N}\left(\overline{\beta}_1^i - \overline{\beta}_1\right)\left(\overline{\beta}_0^i - \overline{\beta}_0\right) & T\sum_{i=1}^{N}\left(\overline{\beta}_1^i - \overline{\beta}_1\right)^2 & T\sum_{i=1}^{N}\left(\overline{\beta}_1^i - \overline{\beta}_1\right)\left(\overline{\beta}_2^i - \overline{\beta}_2\right) & T\sum_{i=1}^{N}\left(\overline{\beta}_1^i - \overline{\beta}_1\right)\left(\overline{\tau}_1^i - \overline{\tau}_1\right) \\ T\sum_{i=1}^{N}\left(\overline{\beta}_2^i - \overline{\beta}_2\right)\left(\overline{\beta}_0^i - \overline{\beta}_0\right) & T\sum_{i=1}^{N}\left(\overline{\beta}_2^i - \overline{\beta}_2\right)\left(\overline{\beta}_1^i - \overline{\beta}_1\right) & T\sum_{i=1}^{N}\left(\overline{\beta}_2^i - \overline{\beta}_2\right)^2 & T\sum_{i=1}^{N}\left(\overline{\beta}_2^i - \overline{\beta}_2\right)\left(\overline{\tau}_1^i - \overline{\tau}_1\right) \\ T\sum_{i=1}^{N}\left(\overline{\tau}_1^i - \overline{\tau}_1\right)\left(\overline{\beta}_0^i - \overline{\beta}_0\right) & T\sum_{i=1}^{N}\left(\overline{\tau}_1^i - \overline{\tau}_1\right)\left(\overline{\beta}_1^i - \overline{\beta}_1\right) & T\sum_{i=1}^{N}\left(\overline{\tau}_1^i - \overline{\tau}_1\right)\left(\overline{\beta}_2^i - \overline{\beta}_2\right) & T\sum_{i=1}^{N}\left(\overline{\tau}_1^i - \overline{\tau}_1\right)^2 \end{pmatrix}$$

and

$$D = \begin{pmatrix} \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{0t}^i - \overline{\beta}_0^i\right)^2 & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{0t}^i - \overline{\beta}_0^i\right)\left(\beta_{1t}^i - \overline{\beta}_1^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{0t}^i - \overline{\beta}_0^i\right)\left(\beta_{2t}^i - \overline{\beta}_2^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{0t}^i - \overline{\beta}_0^i\right)\left(\tau_{1t}^i - \overline{\tau}_1^i\right) \\ \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{1t}^i - \overline{\beta}_1^i\right)\left(\beta_{0t}^i - \overline{\beta}_0^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{0t}^i - \overline{\beta}_0^i\right)^2 & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{1t}^i - \overline{\beta}_1^i\right)\left(\beta_{2t}^i - \overline{\beta}_2^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{1t}^i - \overline{\beta}_1^i\right)\left(\tau_{1t}^i - \overline{\tau}_1^i\right) \\ \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{2t}^i - \overline{\beta}_2^i\right)\left(\beta_{0t}^i - \overline{\beta}_0^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{2t}^i - \overline{\beta}_2^i\right)\left(\beta_{1t}^i - \overline{\beta}_1^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{0t}^i - \overline{\beta}_0^i\right)^2 & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{2t}^i - \overline{\beta}_2^i\right)\left(\tau_{1t}^i - \overline{\tau}_1^i\right) \\ \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\tau_{1t}^i - \overline{\tau}_1^i\right)\left(\beta_{0t}^i - \overline{\beta}_0^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\tau_{1t}^i - \overline{\tau}_1^i\right)\left(\beta_{1t}^i - \overline{\beta}_1^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\tau_{1t}^i - \overline{\tau}_1^i\right)\left(\beta_{2t}^i - \overline{\beta}_2^i\right) & \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\beta_{0t}^i - \overline{\beta}_0^i\right)^2 \end{pmatrix}$$

There exists a relation between the Wilk's Λ statistic and the F-Fisher statistic (Rao, 1951).

Table 6 illustrates the results that we have obtained with the multivariate inference.

This results show that we reject the null hypothesis that the parameter vectors are the same in the EMU countries in all years except for 1999. That year reflects the effort made for the four countries to fulfill the conditions set down at Maastricht and consequently the term structure of interest rates throughout that year takes the same shape.

Table 6. The Wilks' Λ statistics results.

	Wilks' Λ	*F*	*Num DF*	*Den DF*
1999	0.72	1.43	12	119.35
2000	0.34	5.01*	12	119.35
2001	0.34	5.66*	12	119.35
2002	0.49	3.08*	12	119.35
2003	0.55	2.54*	12	119.35
2004	0.51	2.92*	12	119.35
All	**0.88**	**3.19***	**12**	**807.25**

*** Indicates statistical significance at 5%.**

6. Conclusions

We estimate, analyze and compare the weekly term structure of interest rates for six countries. Four EMU countries -Spain, France, Germany and Italy- are included. Moreover, the United Kingdom is included as a European non-member of the EMU and the United States as a reference in international financial markets. The time period covers 13 years, from 1992 to 2004 inclusive.

Firstly, the parameters of the NS model are estimated minimizing the sum-squared error in price at weekly intervals. A total of 4,074 curves are obtained.

We obtain two levels of results. First, we analyse the vector parameter $\beta_t = \left(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t}\right)$ for each week and country, and using the relationships between parameters summarized in table 3, we establish the different shapes of the term structure of interest rates. The most frequent shapes in the EMU countries are the increasing and the trough below β_0 shapes, involving roughly 70% of the total. Only in the United Kingdom is the humped curve above

β_0 also observed. In the United States, alongside the increasing and trough below β_0 shapes, we can also find humped curves crossing β_0.

The second result is related to the difference in the parameters between countries from 1999 to 2004. Over this period, the level and the slope parameters show similar values (figures 2 and 3). The univariate test reveals that we can accept similar curves for Germany, France, Italy and Spain in the years 1999 and 2003. However, there is no significant difference in level, slope and curvature parameters in 2000 and 2001. β_0 and β_1 parameter values are different in 2002, although β_2 and τ_1 are statistically equal. Finally, the opposite situation is given in 2004. In general, the β_0 and β_1 parameter values are coincident statistically, so that the term structures in the EMU countries present the same level and slope from 1999 to 2004, except for 2002. Thus, we accept that the short and long-term interest rates are equal but we reject that the medium-term interest rates are the same in these four countries. The Wilk's Λ statistic for the multivariate test in each year of the period shows the parameter vectors are not the same in the EMU countries, except for 1999.

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Appendix 1

Average and deviation of the parameters

		Germany	France	Spain	Italy	United Kingdom	United States
1992	β_0	0.07371086 *0.00142222*	0.08838092 *0.00434239*	0.10015208 *0.013906*	0.1196958 *0.00568269*	0.10065034 *0.00462974*	0.07976331 *0.00319536*
	β_1	0.09449418 *0.00367855*	0.10289887 *0.00442534*	0.1321648 *0.00926355*	0.14023015 *0.01847026*	0.09316272 *0.01461283*	0.03306322 *0.00384309*
	β_2	0.00484996 *0.01867941*	-0.011219 *0.02461469*	0.01495299 *0.0195769*	0.01847716 *0.02393766*	-0.0259119 *0.02745378*	0.05675558 *0.01132018*
	τ_1	0.97159862 *0.22127982*	1.078838 *0.66197065*	4.26874898 *2.1561845*	2.2162232 *7.289596*	0.76422489 *0.48323933*	5.67696487 *1.60566231*
1993	β_0	0.06990486 *0.00196337*	0.0824119 *0.00604893*	0.10761204 *0.00974947*	0.11397649 *0.01225823*	0.09355741 *0.01021421*	0.07103213 *0.00494754*
	β_1	0.07501203 *0.00837738*	0.08246705 *0.01813551*	0.12197596 *0.01954236*	0.10268545 *0.01246528*	0.05561954 *0.00430038*	0.02924474 *0.00142393*
	β_2	-0.0489967 *0.00940503*	-0.0622998 *0.01018897*	-0.0379996 *0.0298678*	0.00422122 *0.02676646*	-0.0531599 *0.01412124*	0.05210225 *0.00989326*
	τ_1	1.24139298 *0.31862643*	1.72940174 *0.34051903*	1.32137731 *0.53018412*	2.21687055 *0.94087192*	1.14697764 *0.29823645*	7.5872027 *0.99875159*
1994	β_0	0.07752021 *0.00427188*	0.08280443 *0.00616698*	0.10542625 *0.00947*	0.1109108 *0.0106627*	0.08786309 *0.00576409*	0.07584461 *0.00651932*
	β_1	0.05162105 *0.0060849*	0.0564736 *0.00400902*	0.07795709 *0.00660356*	0.08752314 *0.00424947*	0.0500257 *0.00366921*	0.0435105 *0.00823518*
	β_2	-0.0345397 *0.01161094*	-0.0235886 *0.02100524*	0.00429538 *0.03842263*	0.02047093 *0.01929772*	-0.0405104 *0.00783472*	0.03377371 *0.01713895*
	τ_1	1.19589304 *0.35118596*	1.47190282 *0.53026878*	1.55454074 *0.3297322*	3.85419862 *3.031062*	0.50010591 *0.2703182*	3.79815113 *1.80821227*
1995	β_0	0.07991234 *0.00128798*	0.08748219 *0.00205055*	0.11536718 *0.00535141*	0.12385152 *0.00611115*	0.09103426 *0.0020645*	0.0746745 *0.00386964*
	β_1	0.04408074 *0.00278009*	0.06366144 *0.00804701*	0.09426087 *0.00721348*	0.10146719 *0.0070543*	0.06170307 *0.00305199*	0.05692955 *0.00259231*
	β_2	-0.0388052 *0.01739502*	-0.0326837 *0.01935809*	0.00074388 *0.03278327*	0.00183558 *0.01985253*	-0.0234214 *0.01496932*	-0.0130138 *0.00926378*
	τ_1	1.26809548 *0.22954746*	1.43737477 *0.28718239*	1.11548011 *0.56962115*	2.08766085 *2.05580913*	0.88125312 *0.46439149*	2.43068784 *1.45379101*
1996	β_0	0.07831796 *0.00223476*	0.08426352 *0.00192597*	0.09819263 *0.00766136*	0.1049909 *0.00846549*	0.08787254 *0.00371227*	0.07143687 *0.00227932*
	β_1	0.03276608 *0.00217398*	0.03460449 *0.00401943*	0.07780811 *0.00820318*	0.07948699 *0.0109219*	0.05868467 *0.00400658*	0.05152724 *0.00153928*
	β_2	-0.0553614 *0.00828567*	-0.0019622 *0.01405251*	-0.043282 *0.00761734*	0.00204155 *0.01105892*	-0.0090896 *0.02587461*	-0.0075477 *0.01216173*
	τ_1	1.36620582 *0.1087861*	3.48500571 *0.77837546*	1.37476982 *0.38419047*	4.9334002 *1.57698254*	2.10595946 *1.12506409*	2.02687205 *1.25073874*

1997	β_0	0.07156695 *0.003934*	0.07507311 *0.00626527*	0.07720743 *0.00678252*	0.08108679 *0.00427686*	0.07335496 *0.00699072*	0.06900675 *0.00241436*
	β_1	0.03332369 *0.0019011*	0.03282712 *0.00279879*	0.05607594 *0.00390315*	0.06046589 *0.00484622*	0.06562827 *0.00426476*	0.0533323 *0.00200056*
	β_2	-0.0335415 *0.02552283*	-0.0168361 *0.01957171*	-0.0468774 *0.00939762*	-0.0203422 *0.01116403*	0.00704948 *0.00912261*	-0.0031486 *0.007144*
	τ_1	2.25547914 *0.94777718*	3.18718309 *1.34602191*	1.65361676 *0.16590728*	3.26258112 *1.44294432*	1.72115131 *1.41506067*	2.48175508 *2.03779584*
1998	β_0	0.06006525 *0.00131339*	0.06105371 *0.00135634*	0.06184466 *0.00222756*	0.06194938 *0.00266643*	0.05307159 *0.0052102*	0.06016004 *0.00203186*
	β_1	0.03458379 *0.00180107*	0.03629852 *0.00162233*	0.03936029 *0.00490846*	0.0464027 *0.00627057*	0.07101809 *0.00429432*	0.05212811 *0.00234107*
	β_2	-0.0073851 *0.01805302*	-0.0150109 *0.01429201*	-0.0251864 *0.01073025*	-0.0314849 *0.0085891*	0.01537151 *0.00996749*	-0.0147364 *0.01263188*
	τ_1	5.24126749 *1.48760764*	4.21500524 *1.13189881*	2.72415952 *0.72148315*	2.71864077 *0.96267928*	0.80208333 *0.31803517*	2.36029996 *1.16066974*
1999	β_0	0.06247436 *0.00165276*	0.06306985 *0.00294321*	0.06432582 *0.00316815*	0.062759 *0.00146612*	0.03996706 *0.00560065*	0.06367364 *0.00141982*
	β_1	0.02929593 *0.00320159*	0.0292854 *0.00340385*	0.02732404 *0.00348167*	0.02875766 *0.0028537*	0.05116721 *0.0038753*	0.04920792 *0.00191149*
	β_2	-0.0197248 *0.01289792*	-0.0217319 *0.01358005*	-0.0088411 *0.01511659*	-0.0171246 *0.01891058*	0.03443916 *0.03238355*	0.00089957 *0.01399857*
	τ_1	3.18233874 *1.03849443*	3.05659462 *0.64362141*	3.97430812 *1.76766588*	2.71406424 *1.11177352*	2.4909761 *0.83742148*	2.6326028 *0.94078814*
2000	β_0	0.06082846 *0.00292171*	0.0617397 *0.00232076*	0.0619012 *0.00300414*	0.06242637 *0.00183478*	0.03524671 *0.00210112*	0.06151974 *0.00260587*
	β_1	0.04688296 *0.00614783*	0.04595463 *0.00620968*	0.04483653 *0.00640414*	0.04629705 *0.00660243*	0.05941436 *0.00200675*	0.06117269 *0.00369028*
	β_2	-0.0077033 *0.01093717*	-0.0078245 *0.01035339*	-0.0011072 *0.00639814*	-0.0012551 *0.01301461*	0.03717097 *0.01623702*	0.00585217 *0.01702843*
	τ_1	4.15680164 *1.52431432*	3.61378911 *0.94228842*	3.56117422 *1.16059161*	2.66802368 *0.9167527*	3.50914318 *0.78740665*	1.1892144 *0.72749886*
2001	β_0	0.06336994 *0.00315572*	0.06363558 *0.00268308*	0.06309503 *0.00275842*	0.06488708 *0.00236196*	0.04378319 *0.00478593*	0.06208784 *0.00197635*
	β_1	0.04029884 *0.00627688*	0.04112215 *0.00596474*	0.04034163 *0.00496195*	0.04019583 *0.00587885*	0.04813574 *0.00545594*	0.03560195 *0.01153195*
	β_2	-0.0246722 *0.007786*	-0.0286502 *0.00852395*	-0.0266789 *0.00922029*	-0.0263602 *0.00781231*	0.00619244 *0.01616879*	-0.0357196 *0.01241431*
	τ_1	3.47695255 *0.78866985*	2.7814723 *0.38012603*	2.43714693 *0.75167605*	2.16120384 *0.57586854*	3.59641275 *2.55723938*	1.48170415 *0.51928568*
2002	β_0	0.05750121 *0.00119263*	0.05735725 *0.00146467*	0.05875757 *0.00197195*	0.05979687 *0.00140155*	0.04701379 *0.0016198*	0.06322957 *0.00194081*
	β_1	0.03287399 *0.00301039*	0.03186089 *0.00161234*	0.03226529 *0.00187435*	0.03106096 *0.00138083*	0.03896145 *0.0016531*	0.01678697 *0.00209779*
	β_2	-0.018226 *0.01286741*	-0.017765 *0.01559262*	-0.0210933 *0.01667668*	-0.023552 *0.00918516*	0.01096389 *0.02376458*	-0.0536378 *0.01028689*
	τ_1	1.94556499 *0.58599232*	1.79258842 *0.4587025*	1.74461792 *0.514673*	1.34933709 *0.4448862*	1.89134793 *1.17430261*	1.24767323 *0.49247668*

2003	β_0	0.05716799 *0.00140579*	0.05708945 *0.0013913*	0.05700261 *0.00137872*	0.05766184 *0.00172194*	0.04872277 *0.00148184*	0.06344306 *0.00309212*
	β_1	0.02164779 *0.00364276*	0.02158897 *0.00376235*	0.02169311 *0.00407178*	0.02156364 *0.00412994*	0.03574959 *0.00234349*	0.01036652 *0.00221523*
	β_2	-0.0284172 *0.01782092*	-0.02716 *0.02058286*	-0.030786 *0.02090119*	-0.0304344 *0.01864066*	-0.01868 *0.00949116*	-0.0543566 *0.01024459*
	τ_1	2.4451144 *0.59758556*	2.51410344 *0.77269125*	2.32393256 *0.71621549*	2.00612935 *0.80488499*	0.88000235 *0.34546377*	1.87998297 *0.22780485*
2004	β_0	0.05582698 *0.00231794*	0.05570261 *0.00205938*	0.05674936 *0.00229606*	0.05739823 *0.00285811*	0.0438102 *0.0038686*	0.06294541 *0.00170569*
	β_1	0.01951538 *0.0011985*	0.01858605 *0.0008588*	0.01756475 *0.00124147*	0.01865637 *0.00115645*	0.04267371 *0.00369618*	0.01413227 *0.00520992*
	β_2	-0.0224215 *0.00757202*	-0.0165416 *0.0131045*	-0.0006421 *0.00509345*	-0.0217033 *0.02009585*	0.01378239 *0.01345427*	-0.0286783 *0.01479081*
	τ_1	2.46501117 *0.31730944*	2.8435505 *1.01506694*	3.95673001 *0.45545746*	2.58235042 *1.59551113*	3.91497532 *2.60803719*	2.50191505 *0.71668594*

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