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Breaking the panels. An application to the GDP per capita

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Abstract

This paper proposes a test statistic for the null hypothesis of panel stationarity that allows for the presence of multiple structural breaks. Two different specifications are considered depending on the structural breaks affecting the individual effects and/or the time trend. The model is flexible enough to allow the number of breaks and their position to differ across individuals. The test is shown to have an exact limit distribution with a good finite sample performance. Its application to a typical panel data set of real per capita GDP gives support to the trend stationarity of these series.

Keywords: multiple structural changes, panel data, stationarity test, GDP per capita

JEL codes: C12, C22

Resum

Aquest article proposa un estadístic de prova per contrastar la hipòtesi nul·la d'estacionarietat en panell permetent la presència de múltiples canvis estructurals. Es consideren dues especificacions diferents en funció de si els canvis estructurals afecten els efectes individuals i/o la tendència temporal. El model és el suficientment flexible com per permetre que tant el nombre de canvis com la seva posició puguin diferir entre els individus. El treball mostra que la distribució asimptòtica de l'estadístic és exacta. Experiments de simulació indiquen que el comportament del contrast en mides mostrals finites és bo. La seva aplicació a un panell típic de PIB per capita real proporciona evidència a favor de l'estacionarietat de les sèries.

Paraules clau: Múltiples canvis estructurals, dades de panell, contrast d'estacionarietat, PIB per capita

Classificació JEL: C12, C22

1. Introduction

The econometric literature on nonstationary time series has seen the emergence of a wide set of new developments centred on panel data models. The attractiveness of the panel approach lies in the assumption that each time series is a realization of a common underlying data generating process so that better power is expected by exploiting the cross-section dimension of the panel when performing unit root tests. Thus, the combination of the time and cross-section information mitigates the lack of power that the time series based unit root and cointegration tests show when they are applied to the current available samples. The seminal proposals in the panel data framework are those by Levin, Lin and Chu (2002), Breitung and Meyer (1994), Quah (1994) and Phillips and Moon (1999). Banerjee (1999), Baltagi and Kao (2000) and Baltagi (2001) provide comprehensive surveys of the subject. While several tests have already been proposed in this area, less attention has been paid to the presence of structural changes in each of the time series in the panel. Now it is well known that the erroneous omission of structural breaks in the series can lead to deceptive conclusions when performing the univariate integration order analysis - see Perron (1989). Two exceptions that address this concern in the panel data field are the papers by Im and Lee (2001) and Carrion, Del Barrio and López-Bazo (2001b). The first of these papers extends the univariate LM unit root tests proposed by Schmidt and Phillips (1992) and Amsler and Lee (1995) to the panel data framework. Their specifications, which consider individual effects and a time trend, allow for one structural break that shifts the mean of the individual time series. The authors show that the limiting distribution of the new test does not depend on any nuisance parameter. More precisely, the asymptotic distribution does not depend on the location of the break point provided that the limiting distribution of the individual tests is invariant to this nuisance parameter. However, they note that this result of invariance does not hold in finite samples. For their part, Carrion et al. (2001b) generalize the model that specifies individual effects in Harris and Tzavalis (1999) to take into account a structural change that shifts the mean of each of the individual time series at the same date. This panel data unit root test considers the time dimension T as fixed; this is particularly attractive for practitioners, as a variety of macroeconomic panel data sets are characterized by a limited number of temporal observations. The application of the infimum functional makes the limiting distribution of the test free of the break fraction parameter.

In the spirit of the contributions cited above, in this paper we design a test for the null hypothesis of stationarity that takes multiple structural breaks into account. The procedure is based on the panel data version of the KPSS univariate test developed in Hadri (2000) and generalizes existing proposals in this field. The null hypothesis of stationarity can be considered to be more natural than the null hypothesis of a unit root for many economic problems - see Bai and Ng (2001). This implies that there has to be strong evidence against trend stationarity to conclude in favor of the nonstationarity of the panel. Some authors have proposed using both types of test statistics, that is to say, unit root and stationarity tests, to carry out a sort of confirmatory analysis - see Maddala and Kim (1998) for a summary.

Besides, our approach is general enough to allow for the structural changes to shift the mean and/or the trend of the individual time series. Additionally, each individual in the panel can have a different number of breaks located at different dates. The limit distribution of the test statistic is obtained, as the first stage, using the sequential limits. However, following Phillips and Moon (1999), it is shown that the same limiting distribution result is reached if we apply joint limit asymptotics with the additional assumption $N/T \to 0$. These findings are confirmed in the Monte Carlo analysis since, in general, the test shows good finite sample performance when T is large compared to N. The increasing availability of macroeconomic panel data sets, spanning longer time periods and larger numbers of economies, gives rise to many situations in which our proposal can be applied. This is supported by the fact that the probability of a break occurrence increases as the time dimension expands. As an illustration, we test the null hypothesis of panel stationarity in real GDP per capita of fifteen developed countries from 1870 to 1994. These time series have been extensively analysed in applied economics - see Ben-David and Papell (1995) and Ben-David, Lumsdaine and Papell (1996), among others.

The plan of the paper is as follows. In Section 2 we describe the models and the test, and present its limiting distribution. Section 3 deals with the estimation of the number of structural breaks and the determination of the break points. Section 4 analyses the finite sample performance of the test through a Monte Carlo experiment. Our proposal is used to assess the stochastic properties of one typical macroeconomic panel data set in Section 5. Section 6 concludes. All the proofs are compiled in the Appendix.

2. The model and test statistic

In this Section we describe the models defined to test the null hypothesis of stationarity allowing for two different types of multiple structural break effect. Let $\{y_{i,t}\}$ be the set of stochastic processes given by:

$$y_{i,t} = \alpha_{i,t} + \beta_i \, t + \varepsilon_{i,t}, \tag{1}$$

$$\alpha_{i,t} = \sum_{k=1}^{m_i} \theta_{i,k} D\left(T_{b,k}^i\right)_t + \sum_{k=1}^{m_i} \gamma_{i,k} DU_{i,k,t} + \alpha_{i,t-1} + \upsilon_{i,t}, \tag{2}$$

where $\upsilon_{i,t} \sim iid\left(0, \sigma_{\upsilon,i}^2\right)$ and $\alpha_{i,0} = \alpha_i$, a constant, with $i = 1, \ldots, N$ individuals and $t = 1, \ldots, T$ time periods. The dummy variables $D\left(T_{b,k}^i\right)_t$ and $DU_{i,k,t}$ are defined as $D\left(T_{b,k}^i\right)_t = 1$ for $t = T_{b,k}^i + 1$ and 0 elsewhere, and $DU_{i,k,t} = 1$ for $t > T_{b,k}^i$ and 0 elsewhere, with $T_{b,k}^i$ denoting the k-th date of the break for the i-th individual, $k = 1, \ldots, m_i$, $m_i \geq 1$. The data generating process (DGP) given by (1) and (2) decomposes $\{y_{i,t}\}$ as the sum of a random walk, $\{\alpha_{i,t}\}$, and a stochastic process, $\{\varepsilon_{i,t}\}$, which is assumed to be a sequence of mixingales - this includes stochastic processes satisfying the strong mixing regularity conditions defined in Phillips and Perron (1988). Moreover, we assume that $\{\varepsilon_{i,t}\}$ and $\{v_{i,t}\}$ are mutually independent across the two dimensions of the panel data set. Hence, the null hypothesis of a stationary panel is equivalent to set $\sigma^2_{v,i} = 0$, $\forall i = 1, \ldots, N$, under which the model given by (1) and (2) becomes:

$$y_{i,t} = \alpha_i + \sum_{k=1}^{m_i} \theta_{i,k} DU_{i,k,t} + \beta_i\, t + \sum_{k=1}^{m_i} \gamma_{i,k} DT^*_{i,k,t} + \varepsilon_{i,t}, \tag{3}$$

with the dummy variable $DT^*_{i,k,t} = t - T^i_{b,k}$ for $t > T^i_{b,k}$ and 0 elsewhere, $k = 1, \ldots, m_i$, $m_i \geq 1$. The model in (3) includes individuals effects, *individual structural break* effects - that is, shifts in the mean caused by the structural breaks -, temporal effects - if $\beta_i \neq 0$ - and *temporal structural break* effects - if $\gamma_{i,k} \neq 0$, that is, when there are shifts in the individual time trend. This specification is the panel data counterpart of models with breaks proposed in the univariate framework. Thus, when $\beta_i = \gamma_{i,k} = 0$ the model in (3) is the counterpart of the one analysed by Perron and Vogelsang (1992) - hereafter denoted as model 1 - whereas when $\beta_i \neq \gamma_{i,k} \neq 0$ we have the specification given by Perron (1989)'s model C, to which we will refer as model 2. Although other specifications might be adopted - *e.g.* the panel data counterparts of models A and B in Perron (1989) - the asymptotic distribution of the test proposed below for those cases cannot be distinguished from the one in model 2. So, these models can be rewritten in a way that their representation becomes equivalent, and they thus share the limit distribution. This feature is deduced from the derivations in the Appendix.

The specification given by (3) is general enough to allow the following characteristics: (i) the structural breaks may have different effects on each individual time series - the effects are measured by $\theta_{i,k}$ and $\gamma_{i,k}$; (ii) they may be located at different dates since we do not restrict the dates of the breaks to satisfy $T_{b,k}^{i} = T_{b,k}$, $\forall i = 1, \ldots, N$ and, (iii) individuals may have different numbers of structural breaks $m_i \neq m_j$, $\forall i \neq j$, $i, j = 1, \ldots, N$. The test of the null hypothesis of a stationary panel follows the proposal of Hadri (2000), who designed a test statistic that is simply the average of the univariate stationarity test in Kwiatkowski, Phillips, Schmidt and Shin (1992). The general expression for the test statistic is:

$$LM(\lambda) = N^{-1} \sum_{i=1}^{N} \left(\hat{\omega}_i^{-2} T^{-2} \sum_{t=1}^{T} S_{i,t}^2 \right), \tag{4}$$

where $S_{i,t} = \sum_{j=1}^{t} \hat{\varepsilon}_{i,j}$ denotes the partial sum process that is obtained using the estimated OLS residuals of (3), with $\hat{\omega}_i^2$ being a consistent estimate of the long-run variance of $\varepsilon_{i,t}$, $\omega_i^2 = \lim_{T \to \infty} T^{-1} S_{i,T}^2$, $i = 1, \ldots, N$. This allows the disturbances to be heteroscedastic across the cross sectional dimension, that is to say, there is some sort of heterogeneity across individuals - see McCoskey and Kao (1998) and Hadri (2000). The non-parametric method described by Newey and West (1994) and the parametric method in Shin and Snell (2000) can be applied to obtain consistent estimates of ω_i^2. However, care should be taken when applying the non-parametric methods jointly with the use of optimal lag selection for the bandwidth. As Lee (1996b) and Kurozumi (2002) have shown, the procedure of lag selection in Andrews and Monahan (1992) should not be applied to compute the long-run variance for the KPSS test as it makes the test inconsistent. Note that the test in (4) can be also computed assuming homogeneity of the long-run variance across individuals. Instead of computing the test as in (4) we can formulate it as $LM(\lambda) = N^{-1} \sum_{i=1}^{N} \left(\hat{\omega}^{-2} T^{-2} \sum_{t=1}^{T} S_{i,t}^2 \right)$ with $\hat{\omega}^2 = N^{-1} \sum_{i=1}^{N} \hat{\omega}_i^2$. Finally, λ is used in (4) to denote the dependence of the test on the dates of the break. For each individual i it is defined as the vector $\lambda_i = (\lambda_{i,1}, ..., \lambda_{i,m_i})' = \left(T_{b,1}^{i}/T, \ldots, T_{b,m_i}^{i}/T \right)'$ which indicates the relative positions of the dates of the breaks on the entire time period, T.

The derivation of the asymptotic distribution of (4) only requires knowing the expectation and the variance of the limiting distribution of $\eta_i(\lambda_i) = \hat{\omega}_i^{-2} T^{-2} \sum_{t=1}^{T} S_{i,t}^2$ in order to apply the Central Limit Theorem (CLT). It can be shown that under the null hypothesis of stationarity the univariate KPSS test with multiple shifts, $\eta_i(\lambda_i)$, has the following limit distribution - the proof of this statement is sketched in the Appendix for completeness:

$$\eta_i(\lambda_i) \Rightarrow \sum_{k=1}^{m_i+1} \left[(\lambda_{i,k} - \lambda_{i,k-1})^2 \int_0^1 V_i(b_k)^2 \, db_k \right] = H_i(\lambda_i), \tag{5}$$

with $\lambda_{i,0} = 0$ and $\lambda_{i,m_i+1} = 1$, where $\Rightarrow$ denotes weak convergence of the associated measure of probability. This limiting distribution encompasses the one in Lee (1996a), Lee and Strazicich (2001), Busetti and Havey (2001) and Kurozumi (2002) for the model that considers one structural break that shifts the level of non-trending variables and the model that takes account for one break that shifts the level and slope of trending variables. Moreover, it also encompasses the limiting distributiond in Carrion (1999) for the KPSS with two structural breaks. For notational convenience, we have followed Lee (1996a) and Lee and Strazicich (2001) when expressing the limiting distributions, although it is straightforward to show that the results of the papers mentioned above are equivalent.[1] Carrion, Sansó and Artís (2001a) and Bartley, Lee and Strazicich (2001) obtain similar limiting distributions when testing the null hypothesis of cointegration with one structural break using the KPSS.

For the two specifications considered in the paper the first two moments of (5) are given in the following Proposition.

[1] Notice that departing from Busetti and Havey (2001) we just need to rescale the Brownian motions of each subsample to obtain the limiting distributions in Lee (1996a) and Lee and Strazicich (2001). Kurozumi (2002) prefers to express the limiting distributions in terms of standard Brownian motions instead of detrended Brownian motions.

Proposition 1 *Let $\{y_{i,t}\}$ be the stochastic process given by (3) with $\{\varepsilon_{i,t}\}$ a sequence of mixingales, $i = 1, \ldots, N$, $t = 1, \ldots, T$. Thus, the expectation (ξ_i) and variance $\left(\varsigma_i^2\right)$ of $H_i(\lambda_i)$ are given by:*

$$\xi_i = A \sum_{k=1}^{m_i+1} (\lambda_{i,k} - \lambda_{i,k-1})^2; \quad \varsigma_i^2 = B \sum_{k=1}^{m_i+1} (\lambda_{i,k} - \lambda_{i,k-1})^4,$$

$\lambda_{i,0} = 0$ and $\lambda_{i,m_i+1} = 1$, being $A = \frac{1}{6}$ and $B = \frac{1}{45}$ for model 1 $\left(\beta_i = \gamma_{i,k} = 0\right)$, and $A = \frac{1}{15}$ and $B = \frac{11}{6300}$ for model 2 $\left(\beta_i \neq \gamma_{i,k} \neq 0\right)$.

The proof of Proposition 1 is outlined in the Appendix. Some remarks are in order. First, when either $\lambda_i = (0, 0, ..., 0)'$ or $\lambda_i = (1, 1, ..., 1)'$, $\forall i = 1, \ldots, N$, - that is, when there are no structural breaks affecting the time series - the mean and the variance of $H_i(\lambda_i)$ in Proposition 1 equal the values of the moments in Hadri (2000), $\xi_i = 1/6$ $(1/15)$ and $\varsigma_i^2 = 1/45$ $(11/6300)$ for model 1 (2). Second, under the presence of structural breaks, the asymptotic distribution of $\eta_i(\lambda_i)$ depends on λ_i. In the rest of this section we are assuming λ_i known for all i. The case in which these break fraction parameters should be estimated will be addressed in Section 3.

As the test in (4) is in essence the average of the N individual statistics, its limiting distribution can be obtained as the average of $H_i(\lambda_i)$. Therefore, by defining $\bar{\xi} = N^{-1} \sum_{i=1}^{N} \xi_i$ and $\bar{\varsigma}^2 = N^{-1} \sum_{i=1}^{N} \varsigma_i^2$, the test statistic for the null hypothesis of a stationary panel with multiple shifts is:

$$Z(\lambda) = \frac{\sqrt{N}\left(LM(\lambda) - \bar{\xi}\right)}{\bar{\varsigma}}. \tag{6}$$

The following Theorem establishes the sequential limit distribution of $Z(\lambda)$.

Theorem 1 *Let $\{y_{i,t}\}$ be the stochastic process given by (3) with $\{\varepsilon_{i,t}\}$ a sequence of mixingales, $i = 1, \ldots, N$, $t = 1, \ldots, T$. Thus, as $T \to \infty$ followed by $N \to \infty$:*

$$Z(\lambda) = \frac{\sqrt{N}\left(LM(\lambda) - \bar{\xi}\right)}{\bar{\varsigma}} \xrightarrow{d} N(0, 1),$$

where $\xrightarrow{d}$ denotes weak convergence in distribution.

The proof of Theorem 1 relies on the application of the Lindberg-Lévy Central Limit Theorem (CLT) to the average of independent random variables. As in the case of the univariate KPSS test statistic, the null hypothesis of stationarity in the panel is rejected for large values of $Z(\lambda)$. It should be stressed that the limit distribution of the $Z(\lambda)$ test is standard normal and, hence, no new set of critical values needs to be computed. Note that the limiting distribution of the test has been obtained through the application of sequential limits. However, Phillips and Moon (1999) recommend the application of joint asymptotic limits in order to obtain the limit distribution of panel data based unit root and stationarity tests. This suggestion is addressed in Shin and Snell (2000) for the KPSS panel data-based stationarity test; they show that the joint asymptotic distribution of the test proposed by Hadri (2000) equals the sequential limiting distribution if the additional condition of $N/T \to 0$ is imposed. This result can be straightforwardly extended for the test that has been presented in this paper. Hence, following the developments in Shin and Snell (2000), under the null hypothesis, as $T \to \infty$, $T_{b,k}^{i} \to \infty \; \forall k = 1, \ldots, m_i$ - in such a way that $\lambda_{i,k}$ remains constant - and $N \to \infty$ with $N/T \to 0$, the $Z\left(\hat{\lambda}\right)$ test statistic (*jointly*) converges to the standard normal distribution. This result indicates that the test statistic derived here is suitable for panels with larger T compared to N, so that $N/T \to 0$. The Monte Carlo results in Section 4 support this statement.

3. Estimating and testing the breaks

The break fraction vector is usually unknown and must therefore be estimated. Hence, in order to compute the test statistic we need to detect the breaks in each one of the individual time series as a first step. As mentioned above, the test statistics here proposed aim at allowing each time series to have different numbers of breaks located at different dates. We suggest applying the proposal in Bai and Perron (1998). In brief, it consists in, specifying a maximum number of break points ($m^{\max}$), estimating their position for each $m_i \leq m^{\max}$, $i = 1, \ldots, N$, testing for the significance of the breaks and, then, obtaining their optimum number and position for each series.

Different methods based on the application of the infimum functional have been used in the literature to estimate the dates of the breaks. On this matter, Carrion et al. (2001a) showed that, for a cointegration test based on the multivariate KPSS test that allows for one structural break, the best finite sample results were achieved when using the procedure of Bai and Perron (1998) that computes the global minimization of the sum of squared residuals (SSR). Here we use this procedure and choose as the estimate of the dates of the breaks the argument that minimizes the sequence of individual $SSR\left(T_{b,1}^i, \ldots, T_{b,m_i}^i\right)$ computed from (3):

$$\left(\hat{T}_{b,1}^i, \ldots, \hat{T}_{b,m_i}^i\right) = \arg\min_{T_{b,1}^i, \ldots, T_{b,m_i}^i} SSR\left(T_{b,1}^i, \ldots, T_{b,m_i}^i\right).$$

Notice that it is necessary to do some trimming when computing estimates of the break points. Though the amount of trimming is somewhat arbitrary some practitioners have specified $T_b^i \in [0.15T, 0.85T]$ - see among others Zivot and Andrews (1992). Bai (1994, 1997), for $m_i = 1$, shows that if either θ_i is assumed to be fixed or $\theta_i \to 0$ as $T \to \infty$ - shrinking structural break - $\hat{T}_b^i = T_b^i + O_p\left(\|\theta_i\|^{-2}\right)$ and, hence, the estimate of the break date is consistent. This result is extended for $m_i > 1$ by Bai and Perron (1998) for the case of trending and non trending regressors. They also show the consistency of the vector of break fractions $\hat{\lambda}_i$ for each individual.

Once the dates for all possible $m_i \leq m^{\max}$, $i = 1, \ldots, N$, have been estimated, the point is to select the suitable number of structural breaks, if any, for each i, that is, to obtain the optimal m_i. Bai and Perron (1998) address this concern using two different procedures. Briefly speaking, the first procedure relie on the use of information criteria - the Bayesian information criterion (BIC) and the modified Schwarz information criterion (LWZ) of Liu, Wu and Zidek (1997). The second procedure is based on the sequential computation - and detection - of structural breaks with the application of *pseudo* F-type test statistics, though the asymptotic distribution of these test statistics is only derived for the case of non trending regressors. Bai and Perron (2001) compare the procedures and conclude that the second one presents better performance. Following their recommendations, when the model under the null hypothesis of panel stationarity does not include trending regressors our suggestion is to estimate the number of structural breaks using the sequential procedure. For trending regressors the number of structural breaks should be estimated using the information criteria; they conclude that the LWZ criterion performs better than the BIC.

As a result $\hat{\lambda}_i$, $i = 1, \ldots, N$, is obtained and, hence, the test statistic is defined as:

$$Z\left(\hat{\lambda}\right) = \frac{\sqrt{N}\left(LM\left(\hat{\lambda}\right) - \bar{\xi}\right)}{\bar{\varsigma}},$$

where $\bar{\xi} = N^{-1}\sum_{i=1}^{N}\xi_i$ and $\bar{\varsigma}^2 = N^{-1}\sum_{i=1}^{N}\varsigma_i^2$ with ξ_i and ς_i^2 defined as in Proposition 1 using $\hat{\lambda}_i$. The consistency of the estimation of λ and the statement given in Theorem 1 shows that $Z\left(\hat{\lambda}\right)$ has a standard normal distribution.

4. Finite sample performance

The behaviour of the test statistics derived above in finite samples is assessed by computing their empirical size, considering up to two structural breaks. For simplicity, we assume the date of breaks to be known. The DGP is given by (3) with $\alpha_i \sim U[0,1]$, $\theta_{i,k} \sim U[-5,5]$, $\beta_i \sim U[0.3, 0.8]$, $\gamma_{i,k} \sim U[-1,1]$, $m_i = \{1, 2\}$ $\forall i$, where U denotes the uniform distribution. The disturbance term has been specified as $\varepsilon_{i,t} \sim iid\ N(0,1)$. Note that this specification assumes homogeneous long-run variance across i. In fact, note also that there might be some individuals for which there are no structural breaks as 0 belongs to the range of values for $\theta_{i,k}$ and $\gamma_{i,k}$. When $m_i = 1$ the break fraction is randomly generated as $\lambda_i \sim U[0.15, 0.85]$ whereas, for computational convenience, $\lambda_i = (0.25, 0.75)'$ when $m_i = 2$. We have also conducted the Monte Carlo with fixed $\lambda_i \in \{0.25, 0.5, 0.75\}$ for $m_i = 1$, obtaining similar results to the ones reported in Table 1. The Monte Carlo is carried out for $T \in \{50, 100, 200\}$ and $N \in \{10, 25, 50, 100\}$ using $n = 5,000$ replications. The test is performed on the upper tail of the asymptotic distribution.

Table 1 reports the empirical size for the test statistic that assumes heterogeneity in the computation of the long-run variance, although similar results were obtained when homogeneity was imposed. This indicates that the estimation of the long-run variance is not affected when it is assumed to be heterogeneous when in fact homogeneity across i should be considered. In general, the empirical size of the tests is quite close to the 5% nominal size for those situations in which $N/T \to 0$, that is to say, situations in which the time dimension is much larger than the cross section dimension. Thus, the Monte Carlo analysis supports the (joint) asymptotic derivations in the sense that for the test to have good performance it is required that $N/T \to 0$. Finally, it is also observed that the empirical size decreases in the case of $m_i = 2$, particularly for model 2. In this case a large T is required for the empirical size to equal the nominal size.

The analysis of the empirical power for different values of the ratio $\pi_i = \sigma^2_{v,i}/\sigma^2_{\varepsilon,i}$ is presented in Table 2 for model 1 with $m_i = 1$. Similar results were obtained for the specification given by model 2 so that they are not reported for reasons of space. As expected, the power increases with T and N. Interestingly, power improves with N for fixed T. That is, the best inference on the stochastic properties of the time series can be achieved when exploiting the cross-section information in the panel. Besides, notice that the power increases as the ratio π_i grows for small T and N.

Table 1: Empirical size of the test for models 1 and 2

Panel A: $m_i = 1, \forall i$

	Model 1			Model 2		
$N \backslash T$	50	100	200	50	100	200
10	0.046	0.054	0.067	0.052	0.063	0.061
25	0.052	0.058	0.056	0.042	0.045	0.054
50	0.048	0.057	0.062	0.037	0.047	0.052
100	0.043	0.050	0.053	0.031	0.050	0.049

Panel B: $m_i = 2, \forall i$

	Model 1			Model 2		
$N \backslash T$	50	100	200	50	100	200
10	0.052	0.062	0.062	0.037	0.050	0.054
25	0.049	0.046	0.057	0.025	0.048	0.053
50	0.041	0.051	0.050	0.019	0.043	0.050
100	0.036	0.050	0.057	0.017	0.037	0.050

DGP: $y_{i,t} = \alpha_i + \sum_{k=1}^{m_i} \theta_{i,k} DU_{i,k,t} + \beta_i\ t + \sum_{k=1}^{m_i} \gamma_{i,k} DT^*_{i,k,t} + \varepsilon_{i,t}$, with $\alpha_i \sim U[0,1]$, $\theta_{i,k} \sim U[-5,5]$, $\beta_i \sim U[0.3, 0.8]$, $\gamma_{i,k} \sim U[-1,1]$, $m_i = \{1,2\}$, and $\varepsilon_{i,t} \sim iid\ N(0,1)$. The critical value was 1.645 and $n = 5,000$ replications were carried out.

Table 2: Empirical power of the test for model 1

		$\pi_i = \sigma^2_{\upsilon,i}/\sigma^2_{\varepsilon,i}$		
N	T	0.001	0.01	0.1
10	50	0.108	0.668	1
	100	0.328	0.994	1
	200	0.879	1	1
25	50	0.124	0.908	1
	100	0.505	1	1
	200	0.990	1	1
50	50	0.154	0.993	1
	100	0.732	1	1
	200	1.000	1	1
100	50	0.202	1	1
	100	0.905	1	1
	200	1	1	1

DGP: $y_{i,t} = \alpha_i + \sum_{k=1}^{m_i} \theta_{i,k} DU_{i,k,t} + \sum_{j=1}^{t} \upsilon_{i,j} + \varepsilon_{i,t}$, with $\alpha_i \sim U[0,1]$, $\theta_{i,k} \sim U[-5,5]$, $m_i = 1$, $\varepsilon_{i,t} \sim iid\ N(0,1)$ and $\upsilon_{i,t} \sim iid\ N(0,1)$. The critical value was 1.645 and $n = 5,000$ replications were carried out.

5. Empirical application

To illustrate the ease of application of the test proposed here we will consider the panel data set made up of annual (logarithms of) real per capita GDP for fifteen OECD countries from 1870 to 1994 (125 observations). These are the developed countries in Maddison (1997) for which data is available for the full period: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Netherlands, New Zealand, Norway, Sweden, United Kingdom and United States of America. This panel is particularly attractive as a flurry of papers have discussed trend versus difference stationarity of output series (aggregate and per capita). The puzzle concerning the determination of the stochastic properties of the GDP has lead to considerable debate in the econometric literature. Indeed, the distinction between neoclassical and endogenous economic growth models can be settled in terms of the stochastic properties of the output - see Ben-David and Papell (1995) for a discussion. There are many empirical applications in which evidence supporting the unit root hypothesis in aggregate as well as in per capita real GDP is found - see Kormendi and Meguire (1990) and Ben-David and Papell (1995). Nowadays, it is well known that integration analysis critically relies on the specification assumed for the deterministic trend. Thus, the evidence in favour of non stationarity is weakened when the occurrence of structural breaks is allowed - see Perron (1989, 1994), Banerjee, Lumsdaine and Stock (1992), Zivot and Andrews (1992), Ben-David and Papell (1995), Ben-David et al. (1996) and Ben-David and Papell (1998), among others. The analysis in McCoskey and Selden (1998), McCoskey and Kao (1999) and Gerdtham and Löthgren (2000) focus on testing the unit root hypothesis on the GDP (either the aggregate, or measured per capita or per worker) for different panels of countries and conclude in favour of nonstationarity. In this regard, Phillips and Moon (2000) indicate that per capita GDP growth from the Penn World Tables, extensively used in applied cross-country analysis, exhibits strong nonstationarity. Unlike the case of the univariate analysis mentioned above, little attention has been paid to the effect of structural breaks on panel data-based unit root tests. Thus, given the inconsistency that might be caused by a misspecification error in the deterministic component of the panel data-based tests and the evidence drawn from the univariate analysis, it seems desirable to carry out the study of the panel stationarity properties allowing for the presence of structural changes. As real per capita GDP is a trending variable, throughout this section the deterministic component is assumed to include a trend.

As a first exploratory analysis, results from the individual KPSS, not shown here to save space, indicate that the null hypothesis of stationarity is rejected at the 5% level of significance for all time series. These results agree with those concluding in favor of the strong nonstationarity of the per capita GDP and are confirmed when the analysis is performed using the panel data test of Hadri (2000). The value of the test statistic is 28.386 with the corresponding p-value of 0.000, which indicates that the null hypothesis of panel stationarity is strongly rejected - see Panel B in Table 3.

The long time period covered by the variables, on the one hand, and the information shown in the graphs, on the other, indicate that there might be some structural breaks affecting the time series. Let us now allow for the presence of structural breaks through the specification given by model 2 with up to $m^{\max} = 5$ structural breaks and using the LWZ information criteria to determine the number of structural breaks. The long-run variance estimate is obtained using the quadratic spectral window with the optimal bandwidth determined as described in Kurozumi (2002). The allowance of structural breaks changes the previous results since now the null hypothesis of panel stationarity cannot be rejected at the 5% level of significance. Therefore, this result extends the support for trend stationarity in GDP per capita series in Ben-David and Papell (1995) and Ben-

David et al. (1996). Applying the univariate ADF unit root test to the data set considered here with one and two structural breaks respectively, they were unable to reject the unit root hypothesis for some of the countries. Not surprisingly, our specification is more flexible than the one considered by these authors - more structural breaks are allowed - and the inference uses two sources of information -the time and cross-section dimensions.

In general, the use of the general panel data stationarity test proposed in this paper may challenge previous conclusions on the non-stationarity of some typical panels.

Table 3: GDP per capita panel data set

Panel A: Estimation of the number of structural breaks

	Indiv. test	Break dates
Australia	0.033	1891;1928
Austria	0.019	1913;44;62
Belgium	0.016	1903;21;41;71
Canada	0.027	1904;39
Denmark	0.016	1889;1914;39;73
Finland	0.032	1916;39;71
France	0.023	1940;69
Germany	0.017	1914;45;63
Italy	0.018	1896;1918;43;67
Netherlands	0.024	1925;45;74
New Zealand	0.018	1893;1911;35;76
Norway	0.025	1903;41;76
Sweden	0.024	1894;1916;69
U. Kingdom	0.028	1919;45
U. States	0.021	1930;48

Panel B: Stationarity panel data tests

	Test	p-value
No breaks	29.386	0.000
Breaks	1.372	0.085

The second and third columns in panel A offer the individual KPSS test value and the estimated break dates respectively. The number of break points has been estimated using the LWZ information criteria allowing for up to $m^{max} = 5$ structural breaks. Panel B presents the corresponding panel data stationarity test. The long-run variance is estimated using the quadratic kernel with automatic spectral window bandwidth selection.

6. Conclusions

In this paper we have extended the panel data stationarity test proposed in Hadri (2000) to allow for multiple breaks under the null hypothesis of stationarity. The specification is flexible enough to account for a large amount of heterogeneity. It considers (i) multiple structural breaks, (ii) multiple structural breaks positioned at different unknown dates, and (iii) a different number of breaks for each individual. In addition, the test is derived for panels including individual fixed effects and/or an individual-specific time trend.

The limit distribution is proved to be standard normal. This result is obtained using sequential as well as joint limits. Monte Carlo results confirm the good performance of the test in finite samples, particularly when $N/T \rightarrow 0$. This makes our proposal particularly attractive considering the increasing availability of panels with number of cross-sections and time periods that meet this criteria. Besides, the use of longer periods increases the probability that structural breaks will affect the series.

The application of the test proposed here may provide further evidence on the stochastic time series properties of widely used economic panel data sets. As an example, we have obtained evidence that points to the trend stationarity of GDP per capita in a set of developed countries, once breaks in the series are considered.

Appendix

The following Lemma presents some useful statements that involve the proof of the limit results of the paper.

Lemma 1 *Let* $\{\varepsilon_{i,t}\}_{t=1}^{T}$ *be a sequence of mixingales and* $S_{i,t} = \sum_{j=1}^{t} \varepsilon_{i,j}$ *the partial sum process,* $i = 1, \ldots, N$*. Thus, as* $T \to \infty$*,* $\omega_i^{-1} T^{-1/2} S_{i,t} \Rightarrow W_i(r)$*,* $t/T \leq r < (t+1)/T$*,* $t = 1, ..., T$*, where* $\Rightarrow$ *denotes weak convergence of the associated probability measures and* $W_i(r)$ *is a standard Wiener processes defined on* $C[0,1]$ *with* $\omega_i^2 = \lim_{T\to\infty} T^{-1} E\left(S_{i,T}^2\right)$*.*

Proof: see Herrndorf (1984).

Proof of Proposition 1

The regression equation given by (3) can be rewritten in terms of a block diagonal regression model as:

$$y_i = \left[z_{i1}\ z_{i2}\ \ldots\ z_{im_i+1}\right] \delta_i + \varepsilon_i = z_i \delta_i + \varepsilon_i, \tag{7}$$

with $z_{i,k,t} = 1$ for $T_{i,k-1}^b < t \leq T_{i,k}^b$ and 0 elsewhere, $k = \{1, \ldots, m_i + 1\}$, $T_{i,0}^b = 0$, $T_{i,m_i+1}^b = T$. The estimated OLS residuals from (7), $\hat{\varepsilon}_{i,t} = \varepsilon_{i,t} - z_{i,t}\left(z_i' z_i\right)^{-1} z_i' \varepsilon_i$, define the (rescaled) partial sum processes $\omega_i^{-1} T^{-1/2} S_{i,t} = \omega_i^{-1} T^{-1/2} \sum_{j=1}^{t} \hat{\varepsilon}_{i,j}$ - hereafter we assume heterogeneity of the long-run variance across i. Note that for $T_{i,k-1}^b < t \leq T_{i,k}^b$ the partial sum processes are $\omega_i^{-1} T^{-1/2} S_{i,t} = \omega_i^{-1} T^{-1/2} \sum_{j=1}^{t} \left(\varepsilon_{i,j} - z_{i,k,j} P \left(P z_{i,k}' z_{i,k} P\right)^{-1} P z_{i,k}' \varepsilon_i\right)$ which for $k = 1$ converges to $\omega_i^{-1} T^{-1/2} S_{i,t} \Rightarrow W_i(r) - r/\lambda_{i,1} W_i(\lambda_{i,1})$, with $P = T^{-1/2}$ a rescaling matrix and $W_i(r)$ being a standard Brownian motion process - see Lemma 1. Let us define $b_1 = r/\lambda_{i,1}$ so that $0 < b_1 < 1$. Thus, using the properties of the Brownian motions the limiting distribution can be expressed in terms of b_1 as $\omega_i^{-1} T^{-1/2} S_{i,t} \Rightarrow \sqrt{\lambda_{i,1}} W_i(b_1) - b_1 \sqrt{\lambda_{i,1}} W_i(1) = \sqrt{\lambda_{i,1}} \left(W_i(b_1) - b_1 W_i(1)\right)$.

In general, for $k = 1, \ldots, m_{i+1}$ we have $T_{i,k-1}^b < t \leq T_{i,k}^b$ and the partial sum processes converge to $\omega_i^{-1} T^{-1/2} S_{i,t} \Rightarrow W_i(r) - (r - \lambda_{i,k-1}) / (\lambda_{i,k} - \lambda_{i,k-1})$ $(W_i(\lambda_{i,k}) - W_i(\lambda_{i,k-1}))$, with $\lambda_{i,0} = 0$ and $\lambda_{i,m_i+1} = 1$. Let us now define

$b_k = (r - \lambda_{i,k-1}) / (\lambda_{i,k} - \lambda_{i,k-1})$ so that $0 < b_k < 1$. As before, the limiting distribution of the partial sum processes is given by $\omega_i^{-1} T^{-1/2} S_{i,t} \Rightarrow \sqrt{\lambda_{i,k} - \lambda_{i,k-1}} (W_i (b_k) - b_k W_i (1))$.

The KPSS test statistic with one structural break affecting the mean can be computed as $\eta_i (\lambda_i) = T^{-2} \hat{\omega}_i^{-2} \sum_{t=1}^{T} S_{i,t}^2 = T^{-2} \hat{\omega}_i^{-2} \left[\sum_{t=1}^{T_{b,1}^i} \left(\sum_{j=1}^{t} \hat{\varepsilon}_j \right)^2 + \cdots + \sum_{t=T_{b,k-1}^i+1}^{T_{b,k}^i} \left(\sum_{j=1}^{t} \hat{\varepsilon}_j \right)^2 + \cdots + \sum_{t=T_{b,m_i}^i+1}^{T} \left(\sum_{j=1}^{t} \hat{\varepsilon}_j \right)^2 \right]$ with limiting distribution given by:

$$\eta_i (\lambda_i) \Rightarrow \lambda_{i,1}^2 \int_0^1 V_i (b_1)^2 \, db_1 + \cdots + (\lambda_{i,k} - \lambda_{i,k-1})^2 \int_0^1 V_i (b_k)^2 \, db_k + \cdots + (1 - \lambda_{i,m_i})^2 \int_0^1 V_i (b_{m_i+1})^2 \, db_{m_i+1}, \quad (8)$$

where $V_i (\cdot)$ is the residual projection onto the space spanned by $z_{i,k}$. The limiting distribution of $\eta_i (\lambda_i)$ is the weighted sum of $(m_i + 1)$ independent Cramér-von Mises distributions -see Harvey (2001). The expectations of these Cramér-von Mises distributions are $E \left[\int_0^1 V_i (b_k)^2 \, db_k \right] = 1/6$ where the variance are $V \left[\int_0^1 V_i (b_k)^2 \, db_1 \right] = 1/45$, $\forall k = 1, \ldots, m_i + 1$. Therefore, $E [\eta_i (\lambda_i)] = (1/6) \sum_{k=1}^{m_i+1} (\lambda_{i,k} - \lambda_{i,k-1})^2$ and $V [\eta_i (\lambda_i)] = (1/45) \sum_{k=1}^{m_i+1} (\lambda_{i,k} - \lambda_{i,k-1})^4$. For instance, for the case of only one structural break $E [\eta_i (\lambda_i)] = \lambda_{i,1}^2 (1/6) + (1 - \lambda_{i,1})^2 (1/6) = (1/6) \left(\lambda_{i,1}^2 + (1 - \lambda_{i,1})^2 \right)$ and $V [\eta_i (\lambda_i)] = \lambda_{i,1}^4 (1/45) + (1 - \lambda_{i,1})^4 (1/45) = (1/45) \left(\lambda_{i,1}^4 + (1 - \lambda_{i,1})^4 \right)$.

Derivations for the model that includes the time trend follow the steps described above but now with $z_{i,k}$ in (7) defined by the row vector $z_{i,k,t} = [1 \; t]$ for $T_{i,k-1}^b < t \leq T_{i,k}^b$ and 0 elsewhere. The limiting distribution of the partial sum processes is established using the rescaling matrix $P = diag \left(T^{-1/2}, T^{-1} \right)$. Similar developments as the ones carried out above show that the limiting distribution of $\eta_i (\lambda_i)$ is given by (8) with $V_i (\cdot)$ being the residual projections onto the space spanned by the new set of regressors. The expectation and variance of this second set of Cramér-von Mises distributions are

$E\left[\int_0^1 V_i\left(b_k\right)^2 db_k\right] = 1/15$ and $V\left[\int_0^1 V_i\left(b_k\right)^2 db_k\right] = 11/6300$, respectively. Therefore, $E\left[\eta_i\left(\lambda_i\right)\right] = (1/15)\sum_{k=1}^{m_i+1}\left(\lambda_{i,k} - \lambda_{i,k-1}\right)^2$ and $V\left[\eta_i\left(\lambda_i\right)\right] = (11/6300)\sum_{k=1}^{m_i+1}\left(\lambda_{i,k} - \lambda_{i,k-1}\right)^4$.

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