Informational Accuracy and the Optimal Monetary Regime

David Demery
Nigel W. Duck

Discussion Paper No. 05/571

April 2005

Department of Economics
University of Bristol
8 Woodland Road
Bristol BS8 1TN

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David Demery
Nigel W. Duck

Department of Economics
University of Bristol
8 Woodland Road
Bristol BS8 1TN
UK

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Abstract

King (1997) develops a framework for assessing four monetary regimes: an optimal state-contingent rule; a non-contingent rule; pure discretion; and a Rogoffian conservative central banker. Using this framework we show (a) that King is wrong to claim that it implies that an optimally-conservative central banker always dominates a fixed-rule monetary regime; (b) that if the private sector has a signal of the shock to which monetary policy responds - the accuracy of which is exogenously fixed -then either the optimal state-contingent rule or the optimally-conservative central bank can dominate; and (c) that if the private sector optimally chooses the accuracy of its signal then *any* regime can dominate.

Classification Code: E3 E52
Key Words: monetary policy, expectations
Rogoffian central banker

[1] We would like to thank Barbara Annicchiarico, Andrew Pickering, and Runchana Pongsaparn for helpful comments on this paper. The authors are responsible for any errors in the paper.

1 Introduction

King (1997) presents a simple, standard model of monetary policy and uses it to assess four monetary regimes: an optimal state-contingent rule; a non-contingent rule; pure discretion; and a Rogoffian conservative central banker. In each case the monetary authority sets the quantity of money in *full* knowledge of the realization of the economy's single (aggregate supply) shock whereas the private sector forms its expectations before it observes this shock.[1] King draws a number of conclusions from this model, notably that 'both a non-contingent rule and the exercise of discretion are dominated by the optimal state-contingent 'rule' ' (King 1997, p.85), and that a conservative central banker dominates 'both a simple rule and the exercise of discretion by a 'representative' central banker.' (King 1997, p. 89).[2]

In this paper we make three main points: first, King's own model, unmodified, does *not* in fact imply that a conservative central banker dominates a simple rule; secondly, a modification to King's model which permits the private sector to have some signal of the aggregate shock, the accuracy of which is exogenously fixed, implies that the exercise of discretion by a Rogoffian conservative central banker can dominate the optimal state-contingent rule; and thirdly, if the private sector is assumed to be able to select the accuracy of its signal then *any* of the regimes may dominate the others. The second and, especially, the third points draw on a growing macroeconomic literature which examines the implications of incomplete information. Typically in this literature, agents balance the costs and benefits of making more accurate expectations and will choose neither to be perfectly ill-informed nor to use *all* available information.[3] In King's framework this approach suggests that the private sector will choose to acquire *some* information about the current realization of the aggregate supply shock rather than none at all, but will stop short of being perfectly well-informed of its value. It also implies, we suggest, that the accuracy of the information the private sector chooses to acquire will depend upon the monetary regime in operation: different monetary regimes imply different costs of being misinformed and therefore alter the private sector's optimal level of information.

[1] This approach can be traced to Barro and Gordon (1983), Barro (1985), and Rogoff (1987) via Kydland and Prescott (1977).

[2] In fact at this point the article actually reads: 'Hence the conservative central banker determinates [*sic*] both a simple rule and the exercise of discretion by a 'representative' central banker'. We have assumed that the word 'determinates' is a misprint for 'dominates'.

[3] This litereraturе is traceable at least to Feige and Pearce (1976) and Buiter (1980), who argue that 'economically rational' agents may select an incomplete information set. A more recent example in the theory of consumption is Pischke's (1995) version of the permanent income model. More appropriately in the context of this paper, Mankiw and Reis (2002), Ball, Mankiw and Reis (2003) and Carroll (2003) analyse the effects on monetary policy and inflation of 'rational inattentiveness'.

The paper is in three sections. In the first we outline King's framework and establish the first point with a simple example. In the second we introduce the idea that the private sector has some signal of the aggregate shock and we show how this can change the ordering of regimes. In this section we take the accuracy of the signal as exogenously given and the same for each regime. In the third section we consider the implications of allowing the private sector to *select* the degree of accuracy of its signal.

2 The King framework

The framework, using King's notation, consists of four equations

$$y = y^* + b[\pi - \widehat{\pi}] + \varepsilon \tag{1}$$

$$m = \pi + y \tag{2}$$

$$m = \lambda_1 + \lambda_2 \varepsilon \tag{3}$$

$$L = aE\pi^2 + E(y - ky^*)^2 \tag{4}$$

where equation (1) is an aggregate supply relationship with standard notation: y is (the log of) real aggregate output; y^* is the natural level of output rate; π is the inflation rate; $\widehat{\pi}$ is the private sector's expected inflation rate; ε is an aggregate supply shock; and b is a positive parameter; equation (2) is the quantity theory with constant velocity; equation (3) represents the policy regime; and equation (4) is the loss function of the 'representative agent' in which a is the weight attached to inflation, $k \geq 1$ and E is the expectations operator. Equations (1)-(3) imply

$$\pi = \frac{\lambda_1 - y^* + (\lambda_2 - 1)\varepsilon + b\widehat{\pi}}{1 + b} \tag{5}$$

$$y - ky^* = \left[\frac{1 - k(1 + b)}{1 + b}\right] y^* + \frac{b\lambda_1}{1 + b} + \frac{1 + b\lambda_2}{1 + b}\varepsilon - \frac{b\widehat{\pi}}{1 + b} \tag{6}$$

King's first regime - the optimal state-contingent rule - is defined by the values of λ_1 and λ_2 that minimise the (unconditional) loss function on the assumption that expectations are rational and that the rule can be enforced. Formally, King uses equation (5) to derive an expression for $\widehat{\pi}$ (by running the $\hat{}$ operator through it and solving for $\widehat{\pi}$), and then minimising L with respect to λ_1 and λ_2. The second regime - the non-contingent rule - is defined by setting λ_2 to zero and repeating the previous exercise to find the optimal value of λ_1.

The third and fourth regimes - pure discretion and the Rogoffian conservative central banker - are derived by minimising L with respect to λ_1 and λ_2 taking $\widehat{\pi}$ as given; in the Rogoffian case χ replaces a , where $\chi = a/\rho$ and, if the central banker is 'conservative', $0 < \rho < 1$.[4] The value of $\widehat{\pi}$ is

[4] The regime that King calls the 'inflation nutter' regime corresponds to $\rho = 0$; the 'employment nutter' regime corresponds to $\rho \to \infty$. Pure discretion corresponds to $\rho = 1$.

then derived from the implied optimising inflation rate and the assumption of rational expectations.

The value of the loss function under each regime can be written:

$$L_O = z^2 + \frac{a}{a+b^2}\sigma_\varepsilon^2 \tag{7}$$

$$L_R = z^2 + \frac{1+a}{(1+b)^2}\sigma_\varepsilon^2 \tag{8}$$

$$L_D = z^2(1+\frac{b^2}{a}) + \frac{a}{a+b^2}\sigma_\varepsilon^2 \tag{9}$$

$$L_C = z^2(1+\frac{ab^2}{\chi^2}) + \frac{ab^2+\chi^2}{(\chi+b^2)^2}\sigma_\varepsilon^2 \tag{10}$$

where $z = (k-1)y^*$; σ_ε^2 is the variance of ε; and the O, R, D and C subscripts refer, respectively, to the optimal state-contingent rule, the non-contingent rule, pure discretion, and the conservative central banker.

King claims (1997, p. 89) that it 'is straightforward to show that the optimal value of ρ satisfies $0 < \rho < 1$. Hence it is optimal to delegate control of a central bank which exercises discretion to a 'conservative' central bank governor ($L_C < L_D$). However, at the optimal ρ, $L_C < L_R$.' On the basis of these results King claims that the conservative central banker dominates 'both a simple rule and the exercise of discretion by a 'representative' central banker.'

King's assertion that at the optimal value of ρ $L_C < L_R$, can easily be shown to be wrong. Assume values of 1 for the key parameters a, σ_ε^2, and z; and assume that $b = 2$. Under these conditions the optimal value of ρ is 0.1721,[5] the value of L_R is 1.2222, and the value of L_C (with $\rho =$ 0.1721) is 1.5108.[6] Clearly under these conditions the conservative central banker is dominated by a non-contingent rule. These conditions are not in this respect unique: there are many parameter values which imply that the non-contingent rule dominates the Rogoffian optimally-conservative central banker; and many in which the reverse is true; neither regime dominates the other in all circumstances.

[5] This optimal value was found numerically.

[6] Under these same conditions the value of L_O is 1.2; and the value of L_D is 5.2.

3 The King framework with an exogenously informed private sector

In deriving the loss associated with each regime, King assumes that the private sector's expectation of ε is zero. This has been a common-enough assumption in such models, but, in the light of the recent literature noted above in which agents *choose* their information, it is now somewhat jarring. Why should the private sector choose to be totally ignorant of something that is important to it and about which the government is perfectly well-informed? If the private sector has *any* control over its ability to predict the value of ε then the normal economic calculus - in this case the balancing of the costs and benefits of predicting ε more accurately - will lead it to choose to be completely ill-informed only under very special circumstances. We consider what these costs and benefits might be more fully in the next section but for the moment we shall assume that agent j in the private sector has access in period t to a signal of this error, η_j, where

$$\eta_j = \varepsilon + \omega_j$$

ω_j is the noise in the signal which we assume is Gaussian white noise; its variance, σ_ω^2, is a measure of the (in)accuracy of the agent's information.[7] We assume that each agent can, by incurring the costs of obtaining more, or higher-quality, information, choose the variance of ω_j.[8] For simplicity we also assume that the terms in ω_j sum to zero over all individuals. We therefore write the private sector's expectation of ε as

$$\widehat{\varepsilon} = \gamma\varepsilon$$

where $\gamma = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_\omega^2}$ and hence $0 \leq \gamma \leq 1$. A value of zero for γ corresponds to the standard King case; a value of 1 corresponds to the private sector being as well-informed as the monetary authority.

In this section we shall treat the value of γ as exogenous and the same in each regime. Using a procedure similar to the one described in the previous section - and explained more fully in Appendix A - but replacing, where appropriate, $\widehat{\varepsilon} = 0$ with $\widehat{\varepsilon} = \gamma\varepsilon$, we can derive the following loss functions for each regime.

- Optimal state-contingent rule

$$L_O = \phi_O \sigma_\varepsilon^2 + [(1-k)\,y^*]^2 \tag{11}$$

where

$$\phi_O = a\left[\frac{b(1-\gamma)(1+b\gamma)}{a(1+b\gamma)^2 + b^2(1-\gamma)^2}\right]^2 + \left[\frac{a(1+b\gamma)^2}{a(1+b\gamma)^2 + b^2(1-\gamma)^2}\right]^2$$

[7] To keep the notation as uncluttered as possible we ignore the time subscript.
[8] Agents are assumed to know the variance of ε.

- A non-contingent rule

$$L_R = \phi_R \sigma_\varepsilon^2 + [(1-k)\, y^*]^2 \tag{12}$$

where

$$\phi_R = [1+a]\left[\frac{1+b\gamma}{1+b}\right]^2$$

- Pure discretion

$$L_D = \phi_D \sigma_\varepsilon^2 + \left[1+\frac{b^2}{a}\right][(1-k)\, y^*]^2 \tag{13}$$

where

$$\begin{aligned}\phi_D &= a\left[\left(\frac{b}{a}\right)\left[\frac{a+b^2-(1-\gamma)b^2}{a+b^2}\right]\right]^2 \\ &\quad + \left[1-\left(\frac{b^2}{a}\right)\left[\frac{a+b^2-(1-\gamma)b^2}{a+b^2}\right][1-\gamma]\right]^2\end{aligned}$$

- Rogoffian conservative central banker[9]

$$L_C = \phi_C \sigma_\varepsilon^2 + \left[1+\frac{ab^2}{\chi^2}\right][(1-k)\, y^*]^2 \tag{14}$$

where

$$\begin{aligned}\phi_C &= a\left[\left(\frac{b}{\chi}\right)\left[\frac{\chi+b^2-(1-\gamma)b^2}{\chi+b^2}\right]\right]^2 \\ &\quad + \left[1-\left(\frac{b^2}{\chi}\right)\left[\frac{\chi+b^2-(1-\gamma)b^2}{\chi+b^2}\right][1-\gamma]\right]^2\end{aligned}$$

Table 1 shows the losses that would occur in each regime for a small number of selected parameter values, and illustrates three main features of our initial extension to the King framework:[10] first as γ rises, the losses for each regime tend to rise, though this is not always true in the case of pure discretion; second a change in γ can alter the ordering of the regimes' losses; and third,when $\gamma = 0$ or $\gamma = 1$ L_O is always as low or lower than L_R, L_D, and L_C.

[9] Note that this function contains both a and χ: the former because the loss function is for 'society' not for the central banker; the latter because the central banker's weighting of inflation against output determines the actual policy response.

[10] Another feature illustrated by the table is that for all values of γ, $L_O \leq L_R$, and $L_D \leq L_C$. This is because because the non-contingent rule is a restricted case of the optimal state-contingent rule, and the Rogoffian conservative central banker is the optimal case of pure discretion.

TABLE 1 LOSSES UNDER EACH REGIME

γ	L_O	L_R	L_D	L_C	ρ^*
$\sigma_\varepsilon^2 = 1$; $a = 0.1$; $b = 0.1$; $k = 1.1$; $y^* = 1.$					
0.00	0.919	0.919	0.920	0.920	0.987
0.25	0.959	0.965	0.963	0.957	0.741
0.50	0.988	1.012	1.009	0.986	0.494
0.75	1.005	1.061	1.058	1.004	0.247
1.00	1.010	1.110	1.111	1.010	0.000
$\sigma_\varepsilon^2 = 1$; $a = 0.1$; $b = 2.0$; $k = 1.1$; $y^* = 1.0$					
0.000	0.034	0.132	0.434	0.057	0.171
0.250	0.101	0.285	52.975	0.055	0.072
0.500	0.296	0.499	96.353	0.103	0.072
0.750	0.724	0.774	99.324	0.299	0.088
1.000	1.010	1.110	41.410	1.010	0.000
$\sigma_\varepsilon^2 = 1$; $a = 2.0$; $b = 0.1$; $k = 1.1$; $y^* = 1.0$					
0.000	1.005	2.489	1.005	1.005	0.990
0.250	1.007	2.615	1.008	1.007	0.742
0.500	1.009	2.743	1.010	1.009	0.495
0.750	1.010	2.875	1.013	1.010	0.247
1.000	1.010	3.010	1.015	1.010	0.000

The underlying cause of all these features is that, in this class of model, if $\gamma < 1$ a supply shock will tend to create *unexpected* inflation and therefore cause output to deviate from its natural level, $y^* + \varepsilon$. For example, if there were a negative aggregate supply shock then, in the case of the non-contingent rule, where there is a fixed quantity of money and fixed velocity - and therefore a fixed level of nominal aggregate demand - the reduction in output will inevitably raise prices. If $\gamma = 0$ then this rise in prices will be entirely unexpected and will tend to stimulate output, thereby, to some extent, offsetting its initial fall. The resulting combination of higher inflation and reduced output will deliver the loss caused by this shock under this regime.

The other regimes exercise some choice over the extent to which they allow the supply shock to generate unexpected inflation and hence the extent to which they allow output to deviate from its natural rate. The various regimes differ in the amount of unexpected inflation they allow the supply shock to generate, and thereby the extent to which they allow it to cause deviations of output and inflation from their respective desired levels, but the underlying mechanism is the same. For those regimes that can exercise discretion, the fact that they have some power to select the level of output has a second effect: it leads the private sector to anticipate the exercise of that power and hence leads it to expect a higher level of inflation. This is the source of the well-known inflationary bias in such regimes.

A rise in γ generally increases the losses under any regime because it reduces the extent to which the supply shock creates unexpected inflation, and hence reduces the scope each regime has to move output to its desired level (ky^*). The reason it can change the ranking of the regimes is that the rise in γ affects them differentially. One reason for this is that, for those regimes which allow discretion, the rise in γ will alter the private sector's assessment of the extent to which the monetary authority will exploit its ability to cause output to deviate from its natural rate. It will therefore alter the degree of inflationary bias. For such regimes, this second, and potentially beneficial, effect of a rise in γ can offset the effect of the first and lead to lower losses. Because a rise in γ has differential effects on the four regimes - especially as between those regimes that allow discretion and those that do not - it can change the ordering of the losses associated with each of them.

The optimal state-contingent rule dominates when $\gamma = 0$ because, under this condition, a monetary authority that can credibly pre-commit to a state-contingent rule can select from outcomes where, whatever the value of the shock and the selected value of λ_2, expected inflation is zero. Under pure discretion, expectations of inflation are also independent of the shock when $\gamma = 0$, but, because of its associated inflationary bias, pure discretion can, in contrast, only select outcomes involving expected inflation of $\frac{b}{a}(k-1)y^*$. For each possible value of ε, the optimal state-contingent regime can always select the *same* output as would pure discretion but, given expected inflation of zero, the level of inflation associated with each outcome would be *lower*. Hence the optimal state-contingent regime will always dominate pure discretion. For the same reason it will dominate the Rogoffian conservative central banker, although the dominance will be less the more conservative the central banker is. And, of course, it dominates the non-contingent rule since that rule is a special case of the optimal state-contingent rule.

When $0 < \gamma \leq 1$ it is no longer true that, under an optimal state-contingent regime, expected inflation will be zero whatever the value of the shock and the selected value of λ_2. For example, a negative shock will to some extent now be anticipated and this will tend to produce an expectation of positive inflation. This can be reduced by an appropriate selection of λ_2 but this in turn will, provided $\gamma < 1$, have implications for *unexpected* inflation and output volatility. The set of outcomes from which an optimal state-contingent regime can choose is therefore restricted by the rise in γ. So too are the sets of outcomes from which the other regimes can choose, but in the case of the Rogoffian conservative central banker this restriction can, to some degree, be offset by changing the degree of conservativeness. As a result the Rogoffian conservative central banker can, as Table 1 illustrates, dominate the optimal state-contingent rule when $0 < \gamma \leq 1$. This is a further reason why a rise in γ can change the rankings of the regimes' losses.

When $\gamma = 1$ the set of possible inflation-output combinations available under each regime is restricted to those where output equals $y^* + \varepsilon$. A state-contingent rule exists which ensures that at each such output level inflation is zero; in the other regimes (unless the Rogoffian conservative central banker is an 'inflation nutter') these output levels generally involve non-zero inflation rates. Hence the state-contingent rule is again inevitably dominant.

4 The King framework with an optimally informed private sector

The previous section assumes that γ is exogenous and the same in each regime. But different regimes clearly create different economic environments within which private agents have to operate. It is odd to assume that agents' behaviour is unresponsive to these differences. For example, each of the regimes has different implications for the variance of inflation around its expected value. Being wrong about inflation will impose some costs on individuals in the private sector: it inevitably means that they will make important decisions which they later regret.[11] To the extent that they can control the degree of inaccuracy of their inflationary expectations we would expect private agents to select different γs for different regimes: regimes which typically produce little unexpected inflation will allow agents to economise on information gathering and operate with a lower γ.

To capture this idea as simply as possible we assume that the total expected benefits and the total costs of being well-informed are functions solely of $E(\pi - \widehat{\pi})^2$. Again for simplicity, we specify the functions as

$$TB = A - e^{c_1 x} \tag{15}$$

$$TC = e^{-c_2 x} \tag{16}$$

where TB represents the expected benefits, and TC the expected costs, of being informed; $x \equiv E(\pi - \widehat{\pi})^2$; A, c_1, and c_2 are arbitrary constants. Equation (15) is consistent with agents' total benefits declining at an increasing rate as the inaccuracy of their expectations increases. Equation (16) is consistent with their total costs declining at a decreasing rate as the inaccuracy of their expectations increases. These functional forms also imply an 'optimal' value of $E(\pi - \widehat{\pi})^2$ which is constant across regimes (see Appendix B) but an optimal value of γ which differs across regimes.

[11] The model itself implies that mistakes about inflation induce suppliers to supply a different amount of output from the one they would have supplied if they had known the true value of the shock.

Table 2 regime ordering and parameter values

	$\sigma_\varepsilon^2 = 1; k = 1.1; y^* = 1; A = 1; c_1 = 1.25; c_2 = 3$			
Regime Ordering	$L_O < L_C < L_R < L_D$	$L_R < L_C < L_O < L_D$	$L_D < L_C < L_O < L_R$	$L_C < L_O < L_R < L_D$
	Parameters			
a	1.000	0.100	0.800	0.100
b	1.500	0.300	1.900	0.200
	Optimal γs and ρ			
γ_O^*	0.065	0.553	0.111	0.479
γ_R^*	0.167	0.485	0.237	0.505
γ_D^*	0.569	0.836	0.016	0.756
γ_C^*	0.147	0.578	0.231	0.497
ρ^*	0.630	0.404	0.412	0.492

These optimal values of γ are highly complex functions of the underlying parameters of the model and may not lie in the economically meaningful region between 0 and 1. We therefore use numerical methods to derive the losses associated with each regime under the assumption that the private sector selects the optimal values of γ in the region between zero and one, and, in the case of the conservative central banker, that the central banker has the optimal degree of 'conservativeness'.[12] These are sufficient to demonstrate the central point of this section: that, depending upon the economy's parameter values, any of the four monetary regimes can dominate the others. In Table 2 we illustrate this point with examples which are not intended to be realistic but which demonstrate that different values for the model's underlying parameters can lead to a re-ordering of the regimes and to *any* of the regimes being dominant.

At the head of each column we show the ordering of the regimes' losses for the assumed parameter values, and in the rows below we show the associated optimal values of γ and ρ. So, taking the second column as an example: if the values of a and b are 0.1 and 0.3 respectively, then a non-contingent rule dominates the three other regimes; the optimal value of γ ranges from 0.485 in the non-contingent rule regime, to 0.836 in the case of pure discretion; and the optimal degree of conservativeness in the case of the Rogoffian conservative central banker is 0.404.

[12] We used a grid search in steps of 0.0001 over the range zero to one to find the optimal values of γ for regimes O, R, and D, defined as the value of γ which maximises $TB - TC$. For regime C the process is more complicated because there are two optimising processes: for any initial value of ρ the private sector selects the optimal value of γ, i.e. the value of γ which maximises $TB - TC$; for this value of γ, there is an optimal value of ρ, i.e. the value which minimises L_C; but this optimal value of ρ may not equal the initial value. The value of γ_C^* shown in the table is the value which minimised the squared difference between the initial and optimal values of ρ; the value of ρ^* shown is the optimal value of ρ for this value of γ. The grid search took place over the range of 0 to 1 for γ in steps of 0.0001, and over the range 0.0001 to 1 for ρ in steps of 0.0001.

5 Conclusions

In this paper we have shown first that, contrary to King's assertion and with no modification to his framework, a Rogoffian conservative central banker with an optimal degree of 'conservativeness' will *not* always dominate a simple non-contingent rule. More importantly and more generally, we have used King's framework to explore the implications of allowing the private sector to have some signal of the shock to which monetary policy responds. We have first assumed that the accuracy of this signal is exogenously given and the same under each monetary regime. We have then, in as simple a way as possible, assumed that the private sector chooses the degree of this accuracy. The main implication of our first modification is that dominance of the optimal state-contingent rule is undermined: which regime dominates the others is to some extent dependent upon the accuracy of the signal, together with the particular values of the economy's other parameters. The second modification takes this argument one stage further: any regime can in fact dominate the others and there is therefore no clear preference for an optimal state-contingent rule or a Rogoffian conservative central banker, or any other conventional monetary regime.

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Appendix A Derivation of each regime's loss function

- The optimal state-contingent rule

From equation (5) of the text derive

$$\widehat{\pi} = \lambda_1 + (\lambda_2 - 1)\gamma\varepsilon - y^*$$

and hence

$$\pi = \lambda_1 - y^* + \frac{(\lambda_2 - 1)(1 + b\gamma)\varepsilon}{1 + b}$$

$$y = y^* + \phi_1\varepsilon$$

where

$$\phi_1 = \frac{1 + b + b(1 - \gamma)(\lambda_2 - 1)}{1 + b}$$

The unconditional expectation of π^2 and $(y - ky^*)^2$ can be written

$$E\pi^2 = (\lambda_1 - y^*)^2 + \left(\frac{(\lambda_2 - 1)(1 + b\gamma)}{1 + b}\right)^2 \sigma_\varepsilon^2$$

$$E(y - ky^*)^2 = (y^*(1 - k))^2 + \phi_1^2\sigma_\varepsilon^2$$

So the loss under this regime is

$$\begin{aligned} L_O &= a\left[(\lambda_1 - y^*)^2 + \left(\frac{(\lambda_2 - 1)(1 + b\gamma)}{1 + b}\right)^2 \sigma_\varepsilon^2\right] \\ &\quad +(y^*(1 - k))^2 + \phi_1^2\sigma_\varepsilon^2 \end{aligned}$$

Minimising L_O with respect to λ_1 and λ_2 gives

$$\lambda_1 = y^*$$

$$\lambda_2 = \frac{a(1 + b\gamma)^2 - b(1 - \gamma)\,(1 + b\gamma)}{a(1 + b\gamma)^2 + b(1 - \gamma)b(1 - \gamma)}$$

and hence

$$\pi_O = -\frac{b(1 - \gamma)(1 + b\gamma)}{a(1 + b\gamma)^2 + b^2(1 - \gamma)^2}\varepsilon$$

$$y_O = y^* + \frac{a(1 + b\gamma)^2}{a(1 + b\gamma)^2 + b^2(1 - \gamma)^2}\varepsilon$$

$$L_O = \phi_O\sigma_\varepsilon^2 + [(1 - k)\,y^*]^2$$

where

$$\phi_O = a\left[\frac{b(1 - \gamma)(1 + b\gamma)}{a(1 + b\gamma)^2 + b^2(1 - \gamma)^2}\right]^2 + \left[\frac{a(1 + b\gamma)^2}{a(1 + b\gamma)^2 + b^2(1 - \gamma)^2}\right]^2$$

- The non-contingent rule

Under this regime $\lambda_2 = 0$ but the value of λ_1 remains at y^*. These values generate the following expressions for π and y

$$\pi_R = -\frac{1+b\gamma}{1+b}\varepsilon$$

$$y_R = y^* + \left(\frac{1+b\gamma}{1+b}\right)\varepsilon$$

and hence

$$L_R = (y^*(1-k))^2 + [1+a]\left[\frac{1+b\gamma}{1+b}\right]^2 \sigma_\varepsilon^2$$

- Pure discretion

Under this regime the authorities take $\widehat{\pi}$ as given and select the optimal values of λ_1 and λ_2. This implies

$$\begin{aligned}\pi^2 &= \left[\frac{\lambda_1 - y^*}{1+b}\right]^2 + \left[\frac{b}{1+b}\right]^2 \widehat{\pi}^2 + \left[\frac{\lambda_2 - 1}{1+b}\right]^2 \varepsilon^2 \\ &\quad +2\left[\frac{\lambda_1 - y^*}{1+b}\right]\left[\frac{b\widehat{\pi}}{1+b}\right] + 2\left[\frac{\lambda_1 - y^*}{1+b}\right]\left[\frac{(\lambda_2 - 1)\,\varepsilon}{1+b}\right] \\ &\quad +2b\left[\frac{(\lambda_2 - 1)\,\widehat{\pi}\varepsilon}{(1+b)^2}\right]\end{aligned}$$

$$\begin{aligned}(y-ky^*)^2 &= \left(\theta_1 y^* + \frac{b\lambda_1}{1+b}\right)^2 + \frac{b^2\widehat{\pi}^2}{(1+b)^2} + \left(\frac{1+b\lambda_2}{1+b}\right)^2 \varepsilon^2 \\ &\quad +2\left[\theta_1 y^* + \frac{b\lambda_1}{1+b}\right]\left(\frac{1+b\lambda_2}{1+b}\right)\varepsilon - 2\left[\theta_1 y^* + \frac{b\lambda_1}{1+b}\right]\frac{b\widehat{\pi}}{(1+b)} \\ &\quad -2\left[\frac{1+b\lambda_2}{1+b}\right]\frac{b\widehat{\pi}\varepsilon}{(1+b)}\end{aligned}$$

where

$$\theta_1 = \frac{1}{1+b} - k$$

It follows that

$$\begin{aligned}aE\pi^2 &= a\left[\frac{\lambda_1 - y^*}{1+b}\right]^2 + a\left[\frac{b}{1+b}\right]^2 \widehat{\pi}^2 + a\left[\frac{\lambda_2 - 1}{1+b}\right]^2 \sigma_\varepsilon^2 \\ &\quad +2a\left[\frac{\lambda_1 - y^*}{1+b}\right]\left[\frac{b\widehat{\pi}}{1+b}\right]\end{aligned}$$

$$
\begin{aligned}
E\,(y-ky^*)^2 \quad &= \quad \left(\theta_1 y^* + \frac{b\lambda_1}{1+b}\right)^2 + \frac{b^2\widehat{\pi}^2}{(1+b)^2} + \left(\frac{1+b\lambda_2}{1+b}\right)^2 \sigma_\varepsilon^2 \\
&\quad -2\left[\theta_1 y^* + \frac{b\lambda_1}{1+b}\right]\frac{b\widehat{\pi}}{(1+b)}
\end{aligned}
$$

Optimising with respect to λ_1 and λ_2 implies

$$\lambda_1 = \frac{(a-b\theta_1(1+b))y^* + b\,(b-a)\,\widehat{\pi}}{a+b^2}$$

$$\lambda_2 = \frac{a-b}{a+b^2}$$

So in this case we have

$$\pi = \theta_2 y^* - \frac{b(1+b)\varepsilon}{(a+b^2)\,(1+b)} + \theta_3\widehat{\pi}$$

where

$$\theta_2 = -\frac{b\theta_1(1+b)-b^2}{(1+b)\,(a+b^2)}$$

$$\theta_3 = \frac{b^2}{a+b^2}$$

From this we can derive that

$$\widehat{\pi} = \left[\theta_2 y^* - \frac{b(1+b)\gamma\varepsilon}{(a+b^2)\,(1+b)}\right]\left[\frac{1}{1-\theta_3}\right]$$

$$\pi = \frac{\theta_2}{1-\theta_3}y^* - \left[\frac{b(1+b)}{(a+b^2)\,(1+b)}\right]\left[\frac{1-\theta_3\,(1-\gamma)}{1-\theta_3}\right]\varepsilon$$

Since $1-\theta_3 = \frac{a}{a+b^2}$ it follows that $\frac{\theta_2}{1-\theta_3} = -\frac{b\theta_1(1+b)-b^2}{a(1+b)}$ and so

$$\pi_D = \frac{b}{a}(k-1)y^* - \frac{b}{a}\left[\frac{a+b^2-(1-\gamma)b^2}{a+b^2}\right]\varepsilon$$

$$y_D = y^* + \left[1 - \frac{b^2}{a}\left[\frac{a+b^2-(1-\gamma)b^2}{a+b^2}\right][1-\gamma]\right]\varepsilon$$

and hence

$$
\begin{aligned}
L_D \quad &= \quad a\left[\frac{b}{a}\,(k-1)\,y^*\right]^2 + [(1-k)\,y^*]^2 \\
&\quad +a\left[\left(\frac{b}{a}\right)\left[\frac{a+b^2-(1-\gamma)b^2}{a+b^2}\right]\right]^2\sigma_\varepsilon^2 \\
&\quad +\left[1-\left(\frac{b^2}{a}\right)\left[\frac{a+b^2-(1-\gamma)b^2}{a+b^2}\right][1-\gamma]\right]^2\sigma_\varepsilon^2
\end{aligned}
$$

- Rogoffian conservative central banker

Under this regime we replace a with χ in the expressions derived above for π_D and y_D and derive

$$\pi_C = \frac{b}{\chi}(k-1)y^* - \frac{b}{\chi}\left[\frac{\chi + b^2 - (1-\gamma)b^2}{\chi + b^2}\right]\varepsilon$$

$$y_C = y^* + \left[1 - \frac{b^2}{\chi}\left[\frac{\chi + b^2 - (1-\gamma)b^2}{\chi + b^2}\right][1-\gamma]\right]\varepsilon$$

$$\begin{aligned} L_C &= a\left[\frac{b}{\chi}(k-1)\,y^*\right]^2 + \left[(1-k)\,y^*\right]^2 \\ &\quad + a\left[\left(\frac{b}{\chi}\right)\left[\frac{\chi + b^2 - (1-\gamma)b^2}{\chi + b^2}\right]\right]^2 \sigma_\varepsilon^2 \\ &\quad + \left[1 - \left(\frac{b^2}{\chi}\right)\left[\frac{\chi + b^2 - (1-\gamma)b^2}{\chi + b^2}\right][1-\gamma]\right]^2 \sigma_\varepsilon^2 \end{aligned}$$

APPENDIX B THE OPTIMAL DEGREE OF INACCURACY

Assume that the total benefits of accuracy are the following declining function of $x = (E(\pi - \widehat{\pi})^2)$

$$TB = A - e^{f_1(x)}$$

where A is a constant and

$$\frac{\partial f_1(x)}{\partial x} = f_1^{'}(x) > 0; \ \frac{\partial \frac{\partial f_1(x)}{\partial x}}{\partial x} \geq 0$$

So the total benefits decline as x increases and at a rate that itself increases with x.

Assume further that the total costs of being well-informed are also a declining function of x but that in this case they decline at a rate that falls with x. Specifically, assume

$$TC = e^{-f_2(x)}$$

where

$$\frac{\partial f_2(x)}{\partial x} = f_2^{'}(x) > 0; \ \frac{\partial \frac{\partial f_2(x)}{\partial x}}{\partial x} \leq 0$$

The optimal value of x, x^*, can in principle be found by first differentiating $TB - TC$ with respect to x and setting the result to zero to get,

$$-e^{f_1(x)}.f_1^{'}(x) + e^{-f_2(x)}.f_2^{'}(x) = 0$$

It follows from this that

$$Ln[f_1^{'}(x^*)] + f_1(x^*) = Ln[f_2^{'}(x^*)] - f_2(x^*)$$

and x^* is found by solving this equation. If we make the simplifying assumptions used in the text,

$$\begin{aligned} f_1(x) &= c_1 x \\ f_2(x) &= c_2 x \end{aligned}$$

where c_1 and c_2 are positive constants, and $c_2 > c_1$, then we have,

$$x^* = \frac{Ln(c_2) - Ln(c_1)}{c_1 + c_2}$$

The optimal degree of inaccuracy is therefore independent of the regime.

It is straightforward to derive the following expressions for $x = E\left(\pi - \widehat{\pi}\right)^2$ for the different regimes:

$$
\begin{aligned}
x_O &= \left[\frac{-b(1-\gamma)^2(1+b\gamma)}{a(1+b\gamma)^2+b^2(1-\gamma)^2}\right]^2 \sigma_\varepsilon^2 \\
x_R &= \left[\frac{(1-\gamma)\,(1+b\gamma)}{1+b}\right]^2 \sigma_\varepsilon^2 \\
x_D &= \left[\frac{b(1-\gamma)}{a}\left[\frac{a+b^2-(1-\gamma)b^2}{a+b^2}\right]\right]^2 \sigma_\varepsilon^2 \\
x_C &= \left[\frac{b(1-\gamma)}{\chi}\left[\frac{\chi+b^2-(1-\gamma)b^2}{\chi+b^2}\right]\right]^2 \sigma_\varepsilon^2
\end{aligned}
$$

By setting each of these equal to $x^*$and solving for γ we can derive the value of γ that optimising agents will select under each regime.