The Role of the Agent's Outside Options in Principal-Agent Relationships

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THE ROLE OF THE AGENT'S OUTSIDE OPTIONS IN PRINCIPAL-AGENT RELATIONSHIPS*

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Abstract

We consider a principal-agent model of adverse selection where, in order to trade with the principal, the agent must undertake a relationship-specific investment which affects his outside option to trade, i.e. the payoff that he can obtain by trading with an alternative principal. This creates a distinction between the agent's *ex ante* (before investment) and *ex post* (after investment) outside options to trade. We investigate the consequences of this distinction, and show that whenever an agent's *ex ante* and *ex post* outside options differ, this equips the principal with an additional tool for screening among different agent types, by randomizing over the probability with which trade occurs once the agent has undertaken the investment. In turn, this may enhance the efficiency of the optimal second-best contract.

JEL Classification: D21, L14.

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1 Introduction

In many forms of bilateral exchange, one party often has to undertake relationship-specific investments before trade can occur with their partner. An important consequence of such specific investments is that they typically change the investing party's outside option to trade, namely the payoff that he would obtain by trading with an alternative partner. For example, a firm that tailors its machinery in order to produce a specific widget required by a certain buyer, will change its production possibilities when trading with alternative buyers whose requirements need not be the same.[1]

A key distinction therefore exists between the firm's *ex ante* outside option, before the relationship-specific investment is undertaken, and their *ex post* outside option, after the investment has occurred. This paper investigates the consequences of this distinction in principal-agent models of adverse selection, where the agent's type is his private information, and both parties are risk neutral. We show that whenever an agent's *ex ante* and *ex post* outside options differ, this may equip the principal with an additional tool for screening among different agent types, by randomizing over the probability with which trade occurs once the agent has undertaken the specific investment. In turn, this may enhance the efficiency of the optimal second-best contracts.

This paper contributes to the literature on mechanism design when agents have type-dependent outside options (Lewis and Sappington 1989, Maggi and Rodriguez-Clare 1995, Jullien 2000). The earlier literature on adverse selection identifies several cases in which the optimal mechanism can involve randomization, such as when agents have different levels of risk aversion (Stiglitz 1982, Arnott and Stiglitz 1988, Brito *et al* 1995), when the agent's type-space is multi-dimensional (Baron and Myerson 1982, Rochet 1984 and Thanassoulis 2004), or when randomization might allow non-monotonic allocation schedules to become incentive compatible (Strausz 2006). We add to this literature by considering situations where relationship-specific investments affect the agent's future prospects, so that his type-dependent *ex ante* and *ex post* outside options differ. This provides a novel rationale of why randomization may be optimal in principal-agent settings.

The remainder of the paper is organized as follows. In Section 2 we develop the principal-agent model. Section 3 solves for the optimal second best contracts. Section 4 discusses the efficiency consequences of having both types of outside option. All proofs are in the Appendix.

2 Model

Preliminaries We consider a principal-agent model with a principal P and an agent A, who contract over the production of output, q. Production is assumed to be observable and verifiable. The

[1] This phenomenon is not confined to bilateral exchange between firms. Consider a traveller who wants to travel from A to B at 8pm on a given day. The traveller can choose whether to travel by train or bus. The specific investment undertaken by the traveller in order to access a certain type of travel takes the form of him being physically present at a particular location – the bus or train station – at a particular time. While from an *ex ante* perspective the traveller's outside option to catching the 8pm bus would be to take the 8pm train, once he has made the specific investment of arriving at the bus station prior to 8pm, his *ex post* outside option to catching the 8pm bus will be quite different. While he may for example catch the 9pm train, the 8pm train has been ruled infeasible by his earlier specific investment.

agent's marginal cost of production, θ, which defines his type, is not observed by the principal, and we assume $\theta \in \{\theta_H, \theta_L\}$, where $\theta_H > \theta_L > 0$, and $prob(\theta = \theta_H) = \lambda$. In order to trade with the principal, the agent must undertake a relationship-specific investment, with cost normalized to zero. The agent's decision to undertake the investment is observable and verifiable. A contract between the principal and the agent is denoted $\{\phi, \pi, q, T\}$, where $\phi \in \{0, 1\}$ specifies whether the agent must undertake the investment[2], $\pi \in [0, 1]$ denotes the probability with which trade occurs between the parties, $q \in [0, \overline{q}]$ denotes the output that the agent must produce in case of trade, and $T \in \mathbb{R}^+$ indicates the payment from the principal to the agent (independent of whether trade actually occurs or not). We assume trade can only occur if the agent has made the relationship-specific investment so that if $\phi_i = 0$, $\pi_i = 0$.[3]

The principal's problem consists of designing the optimal menu of contracts from which the agent makes his preferred choice. The revelation principle states this search can be confined to the set of direct revelation mechanisms, whereby the agent is requested to report his type and is offered a contract that is contingent upon this report. The timing of actions is then as follows.

t=0 P offers A a menu of contracts $M = \{M_H, M_L\}$, where $M_i = \{\phi_i, \pi_i, q_i, T_i\}$ is the contract offered to the agent when his reported type is θ_i, $i = H, L$.

t=0.5 If A accepts M_i and M_i specifies $\phi_i = 1$, A undertakes the relationship-specific investment.

t=1 Conditional on $\phi_i = 1$, trade occurs with probability π_i, in which case A produces q_i. With probability $1 - \pi_i$ trade between A and P does not occur. If $\phi_i = 0$, trade between A and P does not occur with certainty.

t=1.5 Provided that he has respected the terms of the contract, A receives T_i.

Without loss of generality we restrict attention to contracts that always induce truthtelling and participation by the agent.

Agent's *Ex ante* and *Ex post* Outside Options If the agent does not accept the principal's contract, or if his contract prescribes $\phi = 0$, then the agent does not undertake any relationship-specific investment, and obtains a payoff $B_i \geq 0$ from alternative trade, where $i = H, L$. This defines the agent's *ex ante* outside option. Importantly, we allow for the possibility that *ex ante* outside options differ across types, so that $B_H \neq B_L$. If the agent undertakes the relationship-specific investment, but trade between the parties does not occur, then the agent obtains a payoff $C_i < B_i$ from alternative trade. C_i captures the agent's *ex post* outside option, namely the value of him trading prospects with alternative principals, after having undertaken the relationship-specific investment with the previous principal. *Ex post* outside options may also be type-dependent, so that $C_H \neq C_L$. The expression $B_i - C_i > 0$ reflects the loss in terms of the agent's alternative trading prospects from undertaking the relationship-specific investment, which tailors his production to the principal's needs. We refer to this as the opportunity cost of randomization, since this cost is only incurred when $\pi < 1$.

[2] Allowing the contract to specify ϕ enables us to restrict attention to contracts that are always accepted by the agent. We thank an anonymous referee for providing this suggestion.

[3] By restricting attention to $\phi \in \{0,1\}$ we rule out the possibility of the principal randomizing over ϕ. This is done to shorten the exposition of our results. The reader may readily verify that randomization over ϕ is never optimal for the principal.

Payoffs Both parties are assumed to be risk neutral with respect to monetary transfers and production. If a type θ_i agent accepts a contract $\{\phi, \pi, q, T\}$, his net expected utility is,

$$u(\theta_i) = T + \phi\{-\theta_i \pi q + (1-\pi)C_i - B_i\}. \tag{1}$$

The principal's expected payoff is $U_P = \pi v q - T$, where $v > \theta_H$. u_i denotes the utility obtained by a type θ_i agent when he truthfully declares his type. From (1), the value of T is determined for any given values of u_i, ϕ, π and q. In what follows we will therefore characterize a contract as $M_i = \{\phi_i, \pi_i, q_i, u_i\}$. Finally, we denote $\theta_H - \theta_L$ as $\Delta\theta$, $C_H - C_L$ as ΔC, $B_H - B_L$ as ΔB and $u_H - u_L$ as Δu.

3 Results

The participation constraint for a type θ_i agent is $u_i = T_i + \phi_i[-\theta_i \pi_i q_i + (1-\pi_i)C_i - B_i] \geq 0$. The incentive compatibility constraints which ensure agents find it optimal to declare their true type are,

$$IC_H : u_H \geq u_L + \phi_L[-\pi_L q_L \Delta\theta + (1-\pi_L)\Delta C - \Delta B].$$
$$IC_L : u_L \geq u_H + \phi_H[\pi_H q_H \Delta\theta - (1-\pi_H)\Delta C + \Delta B].$$

Suppose full information contracts are offered so that $\phi_i = \pi_i = 1$, $q_i = \overline{q}$, and $u_i = 0$ for $i = H, L$. Constraint IC_H becomes, $0 \geq -\overline{q}\Delta\theta - \Delta B$, and IC_L becomes, $0 \geq \overline{q}\Delta\theta + \Delta B$. We focus on the more intuitive case in which $\overline{q}\Delta\theta + \Delta B > 0$ so θ_L types have incentives to overstate their costs and mimic θ_H types. This is embodied in assumption A1 below.[4] To ensure that under full information the optimal contract prescribes $\phi_i = \pi_i = 1$, $q_i = \overline{q}$ for both types, assumption A2 below is required. Assumptions A3 and A4 ensure that if $\pi_H = 0$ and/or $q_H = 0$, the principal cannot gain from asking type θ_H to undertake the relationship-specific investment. To summarize, the assumptions on the exogenous parameters are,

A1: $\overline{q}\Delta\theta + \Delta B > 0$
A2: $\overline{q}(v - \theta_i) \geq B_i$, $i = H, L$
A3: $\lambda(C_H - B_H) - (1-\lambda)(\Delta B - \Delta C) < 0$
A4: $\lambda B_H + (1-\lambda)\Delta B > 0$

Our first result provides a partial characterization of type θ_H's optimal contract whenever θ_H agents are required to undertake the relationship-specific investment.

Lemma 1: *It is never optimal for the principal to offer $\phi_H = 1$ in conjunction with π_H and q_H satisfying,*

$$\pi_H q_H \Delta\theta - (1-\pi_H)\Delta C + \Delta B < 0. \tag{2}$$

[4] For completeness, in the Appendix, we state the main results for the case in which the parameter values are such that high types have incentives to understate their type and mimic low cost types. These two cases arise because of the existence of the type-dependent *ex ante* outside options, B_i, as has been analyzed in detail by Maggi and Rodriguez-Clare (1995). Note that in the knife-edge case where $\overline{q}\Delta\theta + \Delta B = 0$ the principal can offer the full information contract to both types without inducing either to mimic the other, so this is clearly her favored course of action.

Under A1 the full information contracts would violate IC_L. By offering type θ_H agents a contract such that $\pi_H q_H \Delta\theta - (1-\pi_H)\Delta C + \Delta B = 0$, the principal ensures both that IC_L is satisfied and that no rents are offered to θ_L agents. Offering θ_H agents a contract such that (2) holds would only increase the distortions of π_H and/or q_H from their full information values (1 and $\overline{q}$ respectively) without generating any gain for the principal. This is essentially the rationale for Lemma 1.

An implication of Lemma 1 is that the participation constraint of type θ_L will not bind at the optimum because given type θ_H's participation, IC_L implies $u_L > u_H \geq 0$. In what follows, we therefore allow IC_L to hold with equality, let $u_H = 0$, and ignore constraint IC_H. We then later verify that the solution of the relaxed problem indeed satisfies IC_H. The principal's problem then is,

$$\max_{\substack{q_i \in [0,\overline{q}],\ \pi_i \in [0,1],\\ \phi_i \in \{0,1\},\ i=H,L}} U_P = \begin{array}{l} \lambda\phi_H\left[\pi_H q_H(v-\theta_H) + (1-\pi_H)C_H - B_H\right] + \\ (1-\lambda)\phi_L\left[\pi_L q_L(v-\theta_L) + (1-\pi_L)C_L - B_L\right] - \\ (1-\lambda)\phi_H\left[\pi_H q_H \Delta\theta - (1-\pi_H)\Delta C + \Delta B\right] \end{array} \quad (\mathbf{P})$$

$$\text{subject to} \quad \phi_H\left[\pi_H q_H \Delta\theta - (1-\pi_H)\Delta C + \Delta B\right] \geq 0. \quad (\mathbf{C1})$$

where (**C1**) derives from Lemma 1. We first solve (**P**) ignoring (**C1**). If this solution satisfies (**C1**), it is the solution to the overall problem. Otherwise (**C1**) binds.

The principal faces a standard trade-off between efficiency and informational rents. If she offers θ_H types the efficient (full-information) contract where $\phi_H = \pi_H = 1$, $q_H = \overline{q}$, then she must also offer positive rents to θ_L types to prevent mimicking. In this case (**C1**) is slack. If the principal wishes to eliminate θ_L's rents, then she must distort type θ_H's contract away from the efficient contract.[5] In this case (**C1**) binds so, conditional on $\phi_H = 1$, we have,

$$q_H = \frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H \Delta\theta}. \quad (3)$$

As $q_H \in [0,\overline{q}]$, condition (3) may restrict the range of values of π_H the principal can offer. As a result, the optimal contract may prescribe randomization so $\pi_H \in (0,1)$. Our main result fully describes the optimal second best contracts.[6]

Proposition 1: *For type θ_L, the optimal contract always prescribes $\phi_L = \pi_L = 1$, $q_L = \overline{q}$. If*

$$\lambda \geq \max\left\{\frac{\Delta C + \overline{q}\Delta\theta}{\overline{q}\,(v-\theta_L) - C_L}, \frac{\Delta\theta}{v-\theta_L}, \frac{\Delta B + \overline{q}\Delta\theta}{\overline{q}\,(v-\theta_L) - B_L}\right\}, \quad (4)$$

*then (**C1**) is slack, and the optimal contract for type θ_H has $\phi_H = \pi_H = 1$, $q_H = \overline{q}$. If (4) does not hold, then (**C1**) binds, and the optimal contract for type θ_H is,*

(i) if $C_H > \frac{C_L(v-\theta_H)}{v-\theta_L}$ and $\Delta C - \Delta B > \frac{(B_H - C_H)(\overline{q}\Delta\theta + \Delta C)}{\overline{q}(v-\theta_H) - C_H} > 0$: $\phi_H = 1$, $\pi_H = \frac{\Delta C - \Delta B}{\overline{q}\Delta\theta + \Delta C}$ and $q_H = \overline{q}$.

(ii) if $C_H < \frac{C_L(v-\theta_H)}{v-\theta_L}$ and $\Delta B < -\frac{B_H \Delta\theta}{v-\theta_H} < 0$: $\phi_H = \pi_H = 1$ and $q_H = -\frac{\Delta B}{\Delta\theta}$.

(iii) in all the other cases: $\phi_H = 0$.

[5] Given the linearity of her payoff, the principal would never select contracts between these extremes.

[6] We adopt the convention that if P is indifferent between setting $\phi_i = 1$ or $\phi_i = 0$ for $i = H, L$, then she selects $\phi_i = 0$.

If *prob*($\theta = \theta_H$) $= \lambda$ is sufficiently high, then the principal finds it optimal to offer θ_H types the efficient contract, so as to maximize her profit when trading with θ_H types, even if this implies that positive rents are relinquished to agents of type θ_L. Conversely, if λ is sufficiently low, then the principal prefers to allow (**C1**) to bind and so eliminate any rents to θ_L types.

Proposition 1 makes precise the optimal contract for θ_H types will prescribe randomization if two conditions hold – (a) $C_H > \frac{C_L(v-\theta_H)}{v-\theta_L}$; (b) $\Delta C - \Delta B > 0$. The intuition why these conditions lead the optimal second best contract to involve randomization is as follows.

Condition (a) requires that C_H should not be too low. If C_H is low, then the transfer needed by θ_H types to accept a contract that involves randomization is high and the principal prefers to set $\pi_H = 1$ even if this implies a lower prescribed q_H.

Condition (b) requires the opportunity cost of randomization to be higher for θ_L types than for θ_H. Hence, by offering θ_H types a contract involving randomization, the principal can lower the incentives of θ_L types to overstate their costs and mimic θ_H types. In contrast, if $\Delta C - \Delta B \leq 0$, then θ_H types stand to lose more from randomization than θ_L types, and so randomization would not help deter θ_H from mimicking θ_L. Condition (b) also requires $\Delta C - \Delta B$ to be sufficiently large, which ensures the principal can obtain a positive expected profit when trading with type θ_L.

A Numerical Example Suppose $\theta_H = 0.75$, $\theta_L = 0.25$, $\overline{q} = v = 2$, and agent's *ex ante* and *ex post* outside options are $B_H = 1.85$, $B_L = 2.35$, $C_H = 1.75$, and $C_L = 1.95$. For (4) to hold we require $\lambda \geq 2/7$. If $\lambda < 2/7$, then (**C1**) must bind in the optimal contract. From (3), this implies $q_H = \frac{2}{5} + \frac{3}{5\pi_H}$, and to ensure $q_H \leq \overline{q} = 2$, we require $\pi_H \geq 0.375$. Conditional on $\phi_H = 1$, the principal then selects $\pi_H \in [0.375, 1]$ to maximize her expected payoff when dealing with a type θ_H agent, $U_P = \pi_H \left[\left(2/5 + \frac{3}{5\pi_H}\right) 1.25 - 1.75\right] - 0.1$. This is decreasing in π_H – a lower π_H decreases the probability of trade, but it also increases q_H, and hence the value of trade. In this numerical example, the latter effect is stronger than the former, so the principal selects the lowest π_H compatible with (**C1**). The optimal contract for θ_H then is, $\phi_H = 1, \pi_H = 0.375, q_H = \overline{q} = 2$, and when dealing with type θ_H agents, the principal's expected payoff is 0.18.

4 Discussion

Efficiency Proposition 1 highlights the impact of having two type-dependent outside options on the optimal second best contracts. Suppose that, on the contrary, $C_i = B_i$ for both $i = H, L$, so $\Delta C = \Delta B$. From (3), the only for (**C1**) to then bind is to set $q_H = -\frac{\Delta B}{\Delta \theta}$. If (4) does not hold and $\Delta B \geq -\frac{B_H \Delta \theta}{(v-\theta_H)}$, then the optimal contract prescribes $\phi_H = 0$, i.e. no trade between the principal and agents of type θ_H, since with $q_H = -\frac{\Delta B}{\Delta \theta}$ the principal would never obtain a non-negative profit when dealing with type θ_H. In contrast when $B_i \neq C_i$, trade between the principal and agents of type θ_H may occur with positive probability even if $\Delta B \geq -\frac{B_H \Delta \theta}{(v-\theta_H)}$.[7]

[7] This is the case for example in the numerical example, where $\Delta B = -0.5 > -\frac{B_H \Delta \theta}{(v-\theta_H)} = -0.74$.

Hence, in a complete contracting environment, the need for agents to undertake relationship-specific investments *ex ante* that decrease the agent's outside option, can result in greater *ex post* efficiency, that is, at the production stage. This is because such investments enable the principal to utilize randomization as a tool to screen between agent types. To our knowledge, the earlier literature has not noted this potentially useful role for *ex ante* relationship-specific investments to improve on *ex post* efficiency. The literature has emphasized rather, that in the presence of contractual incompleteness, investment specificity results in *ex ante* inefficiencies, i.e. inefficiencies at the investment stage (Grout 1984, Grossman and Hart 1986, Hart and Moore 1990).

Relaxing the Linearity Assumption The restriction to linear payoff functions allows us to abstract from risk-aversion considerations, and to differentiate our results from the existing literature on randomization in mechanism design (Stiglitz 1982, Arnott and Stiglitz 1988, Brito *et al* 1995). However, our results extend also to non-linear settings. To see one particular example of this, suppose agents have quadratic production costs, so the net utility of an type θ_i agent when accepting a contract $\{\phi, \pi, q, T\}$ is,

$$T + \phi\left[-0.5\pi\theta_i q^2 + (1-\pi)C_i - B_i\right]. \tag{5}$$

The full-information contracts prescribe $\phi_i = \pi_i = 1$, $q_i = \frac{v}{\theta_i}$ and $u_i = 0$ for $i = H, L$. Suppose $\frac{0.5v^2\Delta\theta}{\theta_H^2} + \Delta B > 0$ so that if offered the full-information contract, a type θ_L agent would overstate his cost and mimic type θ_H, as was the case throughout Section 3. Condition (**C1**) then is,

$$\phi_H\left[0.5\pi_H q_H^2 \Delta\theta - (1-\pi_H)\Delta C + \Delta B\right] \geq 0. \tag{$\mathbf{C1'}$}$$

Following the same argument as in Proposition 1, for λ sufficiently low, the optimal contract for type θ_H agents is such that $\mathbf{C1'}$ binds. Then, conditional on $\phi_H = 1$, we have,

$$q_H = \sqrt{\frac{2\left[(1-\pi_H)\Delta C - \Delta B\right]}{\pi_H \Delta\theta}}. \tag{6}$$

As in the linear case, whether randomization is optimal or not depends on the precise parameter values. To see this we continue the numerical example discussed above but where the restriction that q may not exceed $\overline{q}$ is relaxed – as we no longer have linear payoffs it is not necessary to impose an upper bound on q.

Expression (6) then becomes $q_H = \sqrt{0.8 + \frac{1.2}{\pi_H}}$. Conditional on $\phi_H = 1$, the principal's expected payoff when dealing with a type θ_H agent is $U_P = \pi_H\left[2\sqrt{0.8 + \frac{1.2}{\pi_H}} - 0.75\left(0.4 + \frac{0.6}{\pi_H}\right) - 1.75\right] - 0.1$, which is concave in π_H. The optimal contract for θ_H is $\phi_H = 1$, $\pi_H = 0.78$, and $q_H = 1.53$, and when dealing with type θ_H, the principal's expected profit is 0.23. Hence in this numerical example, for λ sufficiently low the optimal contract for θ_H may again prescribe randomization, although in contrast with the linear case, the optimal q_H is below its first-best value.

5 Appendix

5.1 Proofs

Proof of Lemma 1: We show that any menu of contracts in which $\phi_H = 1$ and (2) holds is necessarily dominated, as P could offer a menu that, whilst violating (2), satisfies both IC_H and IC_L and yields him a strictly higher expected payoff. Consider a menu $M = \{M_H, M_L\} = \{(\phi_H, \pi_H, q_H, u_H), (\phi_L, \pi_L, q_L, u_L)\}$ such that $\phi_H = 1$ and (2) holds. P's expected payoff from M is,

$$\lambda \{\pi_H [q_H(v - \theta_H) - C_H] + C_H - B_H - u_H\} + (1-\lambda)\phi_L \{\pi_L [q_L(v - \theta_L) - C_L] + C_L - B_L - u_L\}. \quad (7)$$

Now consider an alternative menu $\widehat{M} = \left\{\widehat{M}_H, \widehat{M}_L\right\}$, where $\widehat{M}_H = (1, \widehat{\pi}_H, \widehat{q}_H, 0)$ and $\widehat{M}_L = (1, 1, \overline{q}, 0)$. Under A1, $\widehat{M}$ satisfies IC_H. It also satisfies IC_L provided,

$$\widehat{\pi}_H \widehat{q}_H \Delta\theta - (1 - \widehat{\pi}_H)\Delta C + \Delta B \leq 0 \quad (8)$$

Since we are interested in a menu $\widehat{M}$ that violates condition (2), we restrict attention to $\widehat{\pi}_H$ and $\widehat{q}_H$ that satisfy (8) with equality. We now show that there exist values of $\widehat{\pi}_H$ and $\widehat{q}_H$ which satisfy (8) with equality (i.e., violate (2)) and which are such that $\widehat{M}$ yields P a greater expected payoff than M. P's expected payoff from $\widehat{M}$ is,

$$\lambda \{\widehat{\pi}_H [\widehat{q}_H(v - \theta_H) - C_H] + C_H - B_H\} + (1 - \lambda) [\overline{q}(v - \theta_L) - B_L]. \quad (9)$$

A sufficient condition for (9) to exceed (7) is,

$$\widehat{\pi}_H [\widehat{q}_H(v - \theta_H) - C_H] - \pi_H [q_H(v - \theta_H) - C_H] > 0. \quad (10)$$

Condition (10) ensures that P prefers $\widehat{M}$ to M. We distinguish between two cases. First, suppose that $\frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H \Delta\theta} \leq \overline{q}$. Hence setting $\widehat{\pi}_H = \pi_H$ and $\widehat{q}_H = \frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H \Delta\theta}$ ensures (8) holds with equality. Contract $\widehat{M}_H = \left(1, \pi_H, \frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H \Delta\theta}, 0\right)$ is feasible because, if (2) holds, then $q_H < \frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H \Delta\theta}$ which implies $(1 - \pi_H)\Delta C - \Delta B > 0$. With $\widehat{\pi}_H = \pi_H$ the LHS of (10) is $\pi_H (\widehat{q}_H - q_H)(v - \theta_H)$, which is strictly positive. Hence, $\widehat{M}_H$ dominates M_H and so $\widehat{M}$ dominates M.

Second, suppose $\frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H \Delta\theta} > \overline{q}$. Note that since $\overline{q}\Delta\theta + \Delta B > 0$ under A1, -$\frac{\Delta B}{\Delta\theta} < \overline{q} < \frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H \Delta\theta}$, so $\Delta C - \Delta B > 0$. There are then two possibilities to consider.

In the first case, $q_H \Delta\theta + \Delta C > 0$. Inequality (2) can be rewritten as $\pi_H < \frac{\Delta C - \Delta B}{q_H \Delta\theta + \Delta C}$. By setting $\widehat{\pi}_H = \frac{\Delta C - \Delta B}{q_H \Delta\theta + \Delta C}$, $\widehat{q}_H = q_H$ we ensure (8) holds with equality. The LHS of (10) becomes $(\widehat{\pi}_H - \pi_H)[q_H(v - \theta_H) - C_H]$, which is strictly positive. Hence, $\widehat{M}_H = \left(1, \frac{\Delta C - \Delta B}{q_H \Delta\theta + \Delta C}, q_H, 0\right)$ dominates M_H and so $\widehat{M}$ dominates M.

In the second case, $q_H \Delta\theta + \Delta C \leq 0$. For this to hold, we require $\Delta C < 0$. As $\Delta C - \Delta B > 0$, this implies $\Delta B < 0$. By setting $\widehat{\pi}_H = 1$, $\widehat{q}_H = -\frac{\Delta B}{\Delta\theta}$ we ensure (8) holds with equality. The LHS of (10) becomes $\left[-\frac{\Delta B}{\Delta\theta}(v - \theta_H) - C_H\right] - \pi_H [q_H(v - \theta_H) - C_H]$. Under (2), a sufficient condition for this to be positive is that,

$$C_H(v-\theta_L) - C_L(v-\theta_H) < 0. \tag{11}$$

Note however that as $q_H\Delta\theta + \Delta C \leq 0$ in this second case, if (11) does not hold then contract M_H is dominated by a contract that sets $\phi_H = 0$. To see this, note that, by setting $\phi_H = 1$, the extra profit obtained by the principal is non-negative only if $q_H \geq \frac{u_H + B_H - C_H(1-\pi_H)}{(v-\theta_H)\pi_H}$. For this to be consistent with $q_H\Delta\theta + \Delta C \leq 0$ it is necessarily required that $\frac{B_H - C_H(1-\pi_H)}{(v-\theta_H)\pi_H} \leq -\frac{\Delta C}{\Delta\theta}$. In turn, this requires $C_H(v-\theta_L) - C_L(v-\theta_H) < 0$. We therefore conclude that contract M is surely dominated.■

Proof of Proposition 1: Consider first the solution of (**P**) ignoring (**C1**). It is straightforward to see the optimal M_L prescribes $\phi_L = \pi_L = 1$, $q_L = \overline{q}$ and this satisfies IC_H. The FOCs for M_H are,

$$\frac{\partial U_P}{\partial \pi_H} = \phi_H q_H \left[\lambda\left(v-\theta_H\right) - \left(1-\lambda\right)\Delta\theta\right] - \phi_H\left[\lambda C_H + \left(1-\lambda\right)\Delta C\right] \tag{12}$$

$$\frac{\partial U_P}{\partial q_H} = \phi_H \pi_H \left[\lambda\left(v-\theta_H\right) - \left(1-\lambda\right)\Delta\theta\right] \tag{13}$$

$$\frac{\partial U_P}{\partial \phi_H} = \lambda\left[\pi_H q_H (v-\theta_H) + (1-\pi_H)C_H - B_H\right] - \left(1-\lambda\right)\left[\pi_H q_H \Delta\theta - (1-\pi_H)\Delta C + \Delta B\right] \tag{14}$$

Note that **IC1** holds only if $\phi_H = 1$. Hence, the solution to the unconstrained problem satisfies **IC1** only if $\frac{\partial U_P}{\partial \phi_H} \geq 0$. If $\frac{\partial U_P}{\partial \pi_H} \leq 0$, to then have $\frac{\partial U_P}{\partial \phi_H} \geq 0$ requires $\lambda\left(C_H - B_H\right) - \left(1-\lambda\right)\left(\Delta B - \Delta C\right) \geq 0$, which violates A3. Similarly, if $\frac{\partial U_P}{\partial q_H} \leq 0$, to then have $\frac{\partial U_P}{\partial \phi_H} \geq 0$ requires $\lambda\left[(1-\pi_H)C_H - B_H\right] - \left(1-\lambda\right)\left[\Delta B - \left(1-\pi_H\right)\Delta C\right] \geq 0$, which is never true under A3 and A4.[8] We therefore conclude that if the solution to the unconstrained problem satisfies **IC1**, then we must have $\pi_H = \phi_H = 1$, $q_H = \overline{q}$, and all the first order conditions above strictly positive so that,

$$\lambda \geq \max\left\{\frac{\Delta C + \overline{q}\Delta\theta}{\overline{q}\left(v-\theta_L\right) - C_L}, \frac{\Delta\theta}{v-\theta_L}, \frac{\Delta B + \overline{q}\Delta\theta}{\overline{q}\left(v-\theta_L\right) - B_L}\right\}.$$

This establishes the first part of the proposition.
Consider now the second part. When **IC1** binds, $q_H = \frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H \Delta\theta}$, and P's expected payoff is,

$$U_P = \lambda\phi_H\left[\pi_H\left(\frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H\Delta\theta}(v-\theta_H) - C_H\right) + C_H - B_H\right] + (1-\lambda)\phi_L\left[\pi_L q_L(v-\theta_L) + (1-\pi_L)C_L - \right. \tag{15}$$

It is straightforward to see the optimal M_L in this case also prescribes $\phi_L = \pi_L = 1$, $q_L = \overline{q}$ and satisfies IC_H. The optimal M_H maximizes (15) subject to $q_H \in [0, \overline{q}]$. The FOCs are,

[8] To see this, note that $\lambda\left[(1-\pi_H)C_H - B_H\right] - \left(1-\lambda\right)\left[\Delta B - \left(1-\pi_H\right)\Delta C\right] \geq 0$ implies $\frac{\lambda(C_H - B_H) - (1-\lambda)(\Delta B - \Delta C)}{\pi_H} \geq \lambda C_H + \left(1-\lambda\right)\Delta C > \frac{\lambda B_H + (1-\lambda)\Delta B}{1-\pi_H}$. Under A3 and A4, $\lambda B_H + (1-\lambda)\Delta B > \lambda\left(C_H - B_H\right) - \left(1-\lambda\right)\left(\Delta B - \Delta C\right)$ so the previous inequality cannot hold.

$$\frac{\partial U_P}{\partial \pi_H} = \lambda\phi_H\left(\frac{-\Delta C(v-\theta_H)}{\Delta\theta} - C_H\right) = \lambda\phi_H\left[C_L(v-\theta_H) - C_H(v-\theta_L)\right] \quad (16)$$

$$\frac{\partial U_P}{\partial \phi_H} = \lambda\left[\pi_H\left(\frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H\Delta\theta}(v-\theta_H) - C_H\right) + C_H - B_H\right] \quad (17)$$

Two cases can arise. In the first $C_L(v-\theta_H) - C_H(v-\theta_L) < 0$, so conditional on $\phi_H = 1$, $\frac{\partial U_P}{\partial \pi_H} < 0$ and P sets π_H as low as possible. If $\Delta C > \Delta B$ then $\frac{\partial q_H}{\partial \pi_H} < 0$ and the lowest feasible π_H solves $\overline{q} = \frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H\Delta\theta}$, so $\pi_H = \frac{\Delta C - \Delta B}{\overline{q}\Delta\theta + \Delta C}$. Provided $\frac{\Delta C - \Delta B}{\overline{q}\Delta\theta + \Delta C}\left(\overline{q}(v-\theta_H) - C_H\right) + C_H - B_H > 0$, the optimal ϕ_H is 1. If $\Delta C < \Delta B$ then $\frac{\partial q_H}{\partial \pi_H} > 0$ and the lowest feasible π_H is $q_H = 0$. However, from A3 and A4, $\phi_H = 0$ is preferred by P in this case.

In the second case, $C_L(v-\theta_H) - C_H(v-\theta_L) > 0$, so conditional on $\phi_H = 1$, $\frac{\partial U_P}{\partial \pi_H} > 0$ and P sets π_H as high as possible. If $\Delta B < 0$, then $\pi_H = 1$ and $q_H = -\frac{\Delta B}{\Delta\theta}$. Provided $-\frac{\Delta B}{\Delta\theta}(v-\theta_H) - B_H > 0$, it is then optimal to set $\phi_H = 1$. If $\Delta B > 0$, it is then optimal to set $\phi_H = 0$ as this is the only way to ensure (**C1**) binds. To see this, note that we can only be in the case $C_L(v-\theta_H) - C_H(v-\theta_L) > 0$ if $\Delta C < 0$ so that, if $\Delta B > 0$, then $\Delta C < \Delta B$. This implies $\frac{(1-\pi_H)\Delta C - \Delta B}{\pi_H\Delta\theta} < 0$ for all π_H, and therefore (**C1**) never binds unless $\phi_H = 0$.■

5.2 Assumption A1 Does Not Hold

For completeness, we consider the case in which $0 \geq \overline{q}\Delta\theta + \Delta B$ and so θ_H types have incentives to understate their costs and mimic θ_L types. The remaining assumptions A2 to A4 are assumed to still hold. The counterparts for the main results are as follows,

Lemma 1B: *It is never optimal for the principal to offer $\phi_L = 1$ in conjunction with π_L and q_L satisfying,*

$$-\pi_L q_L\Delta\theta + (1-\pi_L)\Delta C - \Delta B < 0. \quad (18)$$

An implication is that the participation constraint of type θ_H will not bind at the optimum. The optimal contracts are now found by letting IC_H hold with equality, setting let $u_L = 0$, and ignoring IC_L. The counterpart to (**C1**) is,

$$\phi_L\left[-\pi_L q_L\Delta\theta + (1-\pi_L)\Delta C - \Delta B\right] \geq 0. \quad \text{(C1B)}$$

Proposition 2B: *For type θ_H, the optimal contract always prescribes $\phi_H = \pi_H = 1$, $q_H = \overline{q}$. If*

$$\lambda \leq \min\left\{\frac{\overline{q}(v-\theta_L) - C_L}{\overline{q}(v-\theta_H) - C_H}, \frac{v-\theta_L}{v-\theta_H}, \frac{\overline{q}(v-\theta_L) - B_L}{\overline{q}(v-\theta_H) - B_H}\right\} \quad (19)$$

*then (**C1B**) is slack, and the optimal contract for type θ_L has $\phi_L = \pi_L = 1$, $q_L = \overline{q}$. If* (19) *doesn't hold, then (**C1B**) binds, and the optimal contract for type θ_L is,*

(i) if $C_H > \frac{C_L(v-\theta_H)}{v-\theta_L}$ and $\Delta C - \Delta B < \frac{(B_L - C_L)(\overline{q}\Delta\theta + \Delta C)}{\overline{q}(v-\theta_L) - C_L} < 0$: $\phi_L = 1$, $\pi_L = \frac{\Delta C - \Delta B}{\overline{q}\Delta\theta + \Delta C}$ and $q_L = \overline{q}$.

(ii) if $C_H < \frac{C_L(v-\theta_H)}{v-\theta_L}$ and $\Delta B < -\frac{B_H\Delta\theta}{v-\theta_H} < 0$: $\phi_L = \pi_L = 1$ and $q_L = -\frac{\Delta B}{\Delta\theta}$.

(iii) in all the other cases: $\phi_L = 0$.

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