Reference-Day Risk and the Use of Monthly Returns Data: A Warning Note

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Abstract

Investment practitioners and the empirical finance literature make extensive use of monthly stock returns, where a monthly return is based on the change in stock price between one particular day of the calendar month – which we term the reference day - and the corresponding day of the following month. We present evidence that the choice of reference day can seriously affect the estimated statistical properties of monthly returns, properties that include mean and median returns, variances of returns, correlations, CAPM betas and Fama and French, 1993, 3-factor model betas. These effects are present both in individual stocks and in market indices. Our evidence suggests that the results of academic studies that use estimates of these properties as inputs, and portfolio decisions based on similar estimates, should be tested for robustness against different choices of reference day.

Classification Code: G0, G10, G11, G14, G15
Keywords: Estimation risk, reference-day risk, betas, correlations

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REFERENCE-DAY RISK AND THE USE OF MONTHLY RETURNS DATA

1. INTRODUCTION

Investment practitioners and the empirical finance literature make extensive use of monthly stock returns, where a monthly return is the proportionate change in a stock's price between one particular day of the month - which we term the reference day - and the corresponding day of the following month, adjusted for dividends when appropriate. Such series are commonly used to provide point estimates of, for example, means and variance-covariance matrices of stock returns, and of company betas. These estimates are used in a variety of ways. They can be used to make broad inferences about financial markets and stocks - for example whether international stock markets are becoming more or less correlated, or whether a particular stock is a 'defensive' or 'aggressive' investment. In academic studies they are used to test asset pricing models, to investigate stock market efficiency and as inputs in event studies. Investment practitioners might use them to construct portfolios or to appraise a portfolio's performance.

Point estimates based on independent, finite samples drawn from a common population will, of course, be subject to conventional sampling variation. The uncertainty that this sampling variation implies for the true value of the properties of any monthly return series is an example of what is termed 'estimation risk' in the finance literature. This problem is well known and widespread,[1] but is normally thought of as applying to independent samples drawn from a common population. In this paper we focus on a different form of sampling variation and associated estimation risk, which is implicitly regarded by academics and practitioners as negligible and which has consequently been overlooked. To illustrate it, consider an estimate of a stock's beta using 60 months' returns over a particular five year period. Given daily data, one

[1] For example, much work has been done on reducing the impact of sampling variation on beta estimation, by, say, Bayesian-type adjustments or the use of portfolio-level data; parameter uncertainty is generally acknowledged to cause standard optimising methods of portfolio selection to be highly unreliable (see, for example, Jobson and

can choose the reference day from which to calculate these monthly returns, and thereby generate a number of monthly returns series that cover more or less the same period and can therefore be presumed to be drawn from the same population. However, they are not independent samples from this population, since there will be some daily returns common to the construction of each monthly series. Despite these shared observations, series constructed from different reference days will not be identical and will therefore produce different estimates of the population's characteristics and, in the specific example we are considering, will produce different estimates of the stock's beta. We term this type of variation 'reference-day variation' and describe the associated estimation risk as 'reference-day risk'. We emphasise at this point that it is quite distinct from the turn-of-the-month effect noted by some authors, such as Ziemba and Hensel, 1995: higher returns on certain days of the month should affect each set of monthly returns equally, since whichever reference day is used as the base, each month incorporates returns from a full set of trading days.

As we have said, in any two series of monthly returns that cover the same overall period but are calculated from different reference days, pairs of corresponding monthly observations will contain at least one common daily return - and many more than one if the reference days are close. It is natural therefore to assume that reference-day risk will be small compared with estimation risk in general, and might possibly be negligible. This is presumably the main reason why, to our knowledge, neither academic researchers nor investment practitioners generally check that their estimates of company betas (or any other property of their monthly series) are robust to different choices of reference day, even though it is quite possible to do so where daily data are available.[2]

Korkie, 1980 and Frankfurter *et al.*, 1971); and, more recently, the problem has attracted the attention of the VaR literature (for example, see Figlewski, 2003 and Jondeau and Rockinger, 2003).

[2] Many data sources are available only in monthly form and the fact that there is no choice as to the reference day on which these returns are based is a further indication that reference-day risk is not considered to be a problem.

We report evidence below that estimates of company betas are, in fact, highly sensitive to the choice of reference day. For example, we report that over the five-year period ending December 1995 the beta of one stock in our sample is estimated to be +2 using one reference day, and –2 using another; that between the two five-year periods ending December 1995 and December 2000 another stock's beta is estimated to have **fallen** by 0.931 using one reference day, and **risen** by 3.454 using another. Furthermore, we find that estimates of many of the basic characteristics of a stock's monthly returns, such as their mean, median, variance, and correlation with other stocks' returns, are also generally highly sensitive to the choice of reference day. For example, the mean monthly (non-annualised) return on one stock over the five-year period ending in December 2005 is estimated to be –0.239% using one reference day and +0.934% using another; the median (non-annualised) monthly return on another stock over the 5-year period ending December 1995 is estimated to be +6.07% using one reference day, but -1.49% using a reference day approximately a week later; and the variance of one stock's monthly return over the 5-year period ending December 2005 is estimated to be 0.06 using one reference day and 0.21 using another. We also report evidence that reference-day sensitivity is not just a feature of the returns on individual stocks: it is also a feature of equity indices. For example, we report that between the two 5-year periods 1980/84 and 1985/89 the correlations between the US and six other countries' stock market indices could be said either to have fallen or to have risen, depending on the reference day used; and we report that the choice of reference day can determine whether the correlations between international stock markets appear to increase or decrease during bull and bear markets.

Our overall conclusion is that the choice of reference day can have serious effects on estimates of population characteristics made from monthly returns data and that the type of inferences that are typically drawn from such estimates should be regarded as highly tentative if they are based on a monthly series calculated from a single reference day.

The emphasis in this article is on establishing the existence of reference-day risk and on assessing its effect on point estimates of population parameters. Hence it consists mainly of demonstrations of the sensitivity of estimates of commonly used financial characteristics to the choice of reference day. However, we also make a preliminary exploration of two issues that are related to this main theme. First, we consider whether standard methods of dealing with estimation risk reduce or even eliminate reference-day risk. We do this in the context of estimates of company betas. We consider the Blume, 1971, regression method, the Vasicek, 1973, Bayesian approach, and the Dimson, 1979, adjustment for thin trading; and we investigate the technique of working with portfolio-level returns, used extensively in the asset-pricing literature (such as in Fama and French, 1993). We find that the Blume and Vasicek methods reduce only the most severe reference-day variability, while the Dimson adjustment has little or no effect. The use of portfolio-level returns is noticeably more successful at reducing reference-day risk, though it by no means eliminates it.

The second issue that we explore is possible sources of reference-day risk. Reasons for the variability that have been suggested to us include extreme and rare market movements, such as 9/11 or the October 1987 crash; thin trading of smaller companies' stocks; window-dressing by market participants who are appraised on month-end returns; and institutional features such as the expiry of derivatives at month-, or quarter-ends. We can eliminate the first two suggestions since we have repeated our tests on numerous different periods, some of which contain rare events and some of which do not, but all of which exhibit similar degrees of reference-day variability; and our work on the Dimson betas shows that adjustment for thin trading does not reduce the variability. That leaves window-dressing and institutional features, and our own view, which is that the variability that we find is a form of sampling variation that is surprising only because it has not previously attracted the attention of the literature. To address the window-dressing/institutional features possibility we test for the systematic influence of particular reference days on the characteristics of monthly returns. Our general finding is, first, that whilst there are sub-periods when certain reference days seem to exert a significant influence on estimates of financial characteristics, these effects are highly unstable from one period to the next, and much less obvious over longer periods. Furthermore there is no evidence that it is month-ends that are consistently different from other reference days. Since these results are consistent with our view that the variability that we find might simply be due to sampling variation, we simulate the scale of reference-day risk we would expect to find in a sample of stocks with similar characteristics to our own sample, but which has Normally-distributed daily stock returns. We find that the degree of reference-day variation in our simulated data is similar to that observed in our actual data, a result that is consistent with the view that reference-day risk is indeed a form of sampling variation, one that applies to non-independent samples.

The evidence of these tests, the ineffectiveness of the thin-trading Dimson betas at reducing the variability, and the fact that reference-day variation is evident in market indices as well as individual stock returns, all suggest that reference-day variation is little more than random variation across non-independent samples.

The paper is organised as follows. Our analysis of reference day effects in sections 2 to 4 moves from individual stocks up to international equity markets. We start in section 2 by examining characteristics of individual stocks' monthly returns – their mean, median and variance; in section 3 we incorporate interaction between stocks by looking at correlations, and between stocks and indices by looking at CAPM and Fama-French three-factor model betas. In section 4, we examine correlations between monthly returns of pairs of country equity markets, and correlations between these markets during extreme markets. The remainder of the paper investigates more deeply the reference-day variation that we find in sections 2 to 4. In section 5 we explore the Blume, Vasicek, and Dimson beta adjustments and betas constructed at portfolio level. In section 6 we carry out our preliminary investigations into the source of reference-day risk. We end with a brief summary and conclusions, and make suggestions for further research.

2. REFERENCE-DAY RISK AND INDIVIDUAL STOCKS

2.1 The data

Our sample of stocks is drawn from the S&P500 index at 1 March, 2006, and all our data are downloaded from Datastream.[3, 4] For all stocks and the S&P500 index we calculate monthly returns in log form,[5] as $\ln\left[RI_{J,t+1}/RI_{J,t}\right]$, where $RI_{J,t}$ is the Datastream 'Return Index' at the close of reference day J (J= 1, 2, …, 28) in month t, and $RI_{J,t+1}$ is the Return Index at the close of the same reference day in the following month.[6] The Return Index is an index that reflects the stock price and, when applicable, the dividend paid on the stock. On non-trading days the Return Index is held at the same value as the previous day, so the daily return is recorded as zero. The monthly reference-day J return is therefore the sum of the daily close-to-close log returns from day $J+1$ in month t to day J in month $t+1$.

We use data for the 15-year period ended 31 December, 2005, which gives us three non-overlapping five-year sub-periods on which to base our parameter estimates, sub-periods that end in December 1995, December 2000, and December 2005. To ensure that these estimates are calculated for sufficiently mature companies, we omit those that have fewer than eight years of continuous price data. This leaves us with 459 companies: 374 in all three sub-periods, a further 68 in the second and third sub-periods only, and 17 in the last sub-period only.

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[3] This is the main data source for studies carried out by UK academics. We thank our anonymous referees for confirming our general results using CRSP data.
[4] We have repeated all the tests described below using companies in the UK FTSE All-Share Index and obtained similar results.
[5] We repeated all the tests using buy-and-hold returns, with similar results.
[6] To avoid the distraction of month-ends we restricted our sample to reference days 1 to 28. Otherwise our results would be muddied by the fact that, for example, February returns based on reference days 29, 30 and 31 would all end on 28 February. We did repeat the tests using conventional calendar month returns, with similar results.

2.2 Means, medians and variances of stocks' monthly returns

Tables 1 to 3 present evidence on the sensitivity to the choice of reference day of the means, medians and variances of the stocks' monthly returns for the three sub-periods. For each period and each company we make 28 estimates of each parameter - one for each reference day - and calculate for that company the mean estimate across reference days, the highest estimate, the lowest estimate and the difference between these two, the range. To give readers an idea of reference-day variation at its most and least severe, we present in the first few rows of each table the results for the four companies with the highest ranges and the four with the lowest ranges. In the last rows of the table we present the results for our market proxy, the S&P500 index and a summary over all the companies used in that particular period. For the individual companies and the market we also show the reference days that generate the maximum/minimum value of each parameter. To save space, we do not give individual companies' names, but we can report that few companies appear in more than one table. That is, the companies that are most/least subject to reference-day variation in the context of one parameter are not the same as those that are most/least subject to it in the context of another.

The results in tables 1 to 3, especially those in tables 2 and 3, suggest that the choice of reference day can have considerable effects on estimates of the population characteristics of an individual stock's monthly returns. In the more extreme cases the choice of reference day can double the estimate of a stock's mean monthly return, and can alter the estimate from positive to negative. The effects on the estimated median are more dramatic: the ranges are larger and the estimated median frequently changes sign. For the market the maximum is twice as high as the minimum in the 1995 sub-period and many times higher in the last. The results reported in table 3 also indicate a strong reference day effect on estimates of the variance of a stock's monthly return. At its most extreme, changing the reference day can more than double the estimate of the variance, not only for an individual company, but also for the market index.

In all these results, the highest and lowest estimates are often a large number of reference days apart, as one might expect, but this is not always the case. For example, the highest-ranked range of mean returns in the 2000 sub-period (columns 6 and 7, row 1, table 1) has its minimum based on reference day 7, with the maximum based on day 3; the market's minimum median return in the 1995 sub-period (columns 2 and 3, table 2) is based on reference day 7, while its maximum is based on reference day 4. It also seems that certain reference days might be more likely than others to be associated with particularly high or particularly low estimates. For example, in table 3 reference day 22 generates 3 of the four minima for the low-range companies in sub-period 2000. This type of pattern is also evident in some of the results we present below, and we investigate this issue further in section 6.

All in all these results suggest a surprisingly high degree of reference-day variability, given that the parameters are derived from almost exactly the same time period – even reference days 1 and 28 have only two 'unshared' months out of 60.

3 REFERENCE-DAY RISK AND RELATIONSHIPS AMONG STOCKS, AND BETWEEN STOCKS AND THE MARKET

3.1 Relationships among stocks

This section examines the effect of reference-day risk on parameters that are the result of interactions between series of different returns. In table 4 we present results on the reference-day sensitivity of correlations between different stocks' returns using the dataset described above and the same three 5-year sub-periods. Due to the large number of correlations that can be calculated, we restrict our sample by subdividing the 459 stocks into nine groups, each containing 51 stocks. For each group of 51 stocks and each sub-period we calculate correlations between monthly returns of all stocks within the group. These are summarised in table 4 for all the groups.

We adopt the same format for presenting the results and show, for each period, correlations for the four company pairs with the highest range, the four pairs with the lowest range, and a summary for all pairs in our groups. Here too the effects of reference day selection can be striking. In all three periods the selection of a different reference day can, in the more extreme cases, cause a change in the sign of the correlation, and can cause estimated correlations to change by up to 80 percentage points. In fact the mean (and median) absolute change in correlation between **consecutive** reference days is between 4 and 5 percentage points, depending on the sub-period, with a maximum absolute change between consecutive reference days of 38, 46 and 64 percentage points in 1991/95, 1996/2000 and 2001/5 respectively. These are very large variations when compared with a mean overall correlation of between 17% and 28%, depending on the sub-period.

3.2 Relationships between stocks and the market

We now turn to interactions between individual stock returns and index returns, calculating both CAPM betas and Fama and French, 1993 (FF1993) 3-factor model betas. The FF1993 model is given by:

$$XR_{i,t}^{J} = \alpha_i + \beta_{i,1}^{J} XRM_t^{J} + \beta_{i,2}^{J} SMB_t^{J} + \beta_{i,3}^{J} HML_t^{J} + \varepsilon_{i,t}^{J} \qquad J = 1, 2, \ldots, 28 \qquad (1)$$

where

$XR_{i,t}^{J}$ is the excess return of stock *i* over the risk-free rate in month *t* based on reference day *J*;

XRM_t^{J} is the excess return of the market index in month *t* based on reference day *J*;

SMB_t^J is the additional return to holding small-cap, rather than large-cap stocks, in month t based on reference day J; and

HML_t^J is the additional return to holding high, rather than low, book-to-market stocks in month t based on reference day J.

The empirical CAPM is effectively a restricted form of this model, with $\beta_{i,2}^J$ and $\beta_{i,3}^J$ both set to zero.

Our data for these estimates include the same stocks as before, using the S&P500 index to proxy the market. We use the US 3-month Treasury Bill Rate as the risk-free rate. This is available on Datastream on a daily basis in annualised form, so for each reference day we convert the rate into a monthly one using the formula $rf_m^J = \left(1 + rf_y^J\right)^{\frac{1}{12}} - 1$, where rf_m^J is the monthly rate for the month starting on reference day J and rf_y^J is the annualised rate reported by Datastream on that day. Daily measures of market returns (based on value-weighted returns on all NYSE, AMEX and NASDAQ stocks, from CRSP) and the risk-free rate (the one-month Treasury bill rate from Ibbotson) are also available on French's website. We repeated the tests using these data to construct our monthly market return and risk-free rate series, with qualitatively similar results. French's website also provides daily (simple) returns on the *SMB* and *HML* indices, which we converted into 28 sets of logged monthly returns (as mentioned earlier, all tests were repeated using buy-and-hold returns, with similar results).

The CAPM betas are summarised in table 5. The first four rows of the table show that estimates of betas can change by as much as 4 (from approximately –2 to +2) as a result of a change of reference day. Even for the four companies with the smallest degree of sensitivity a change of reference day can influence whether the company's beta is classified as aggressive (above 1) or defensive (below 1). In total, 53% of companies have betas that vary from below 1 to above 1 as the reference day changes; and 8% have betas that change from negative to positive on different reference days. As with earlier tables, although the minimum and maximum betas generally lie several reference days apart, this is not always true. For example, the second-ranked company in the 2005 sub-period (columns 10 and 11, row 2, table 5) has a beta of 1.4 when reference day 20 is used, and 4.7 using reference day 25.

We do not present *t*-statistics in any of the tables, partly to save space and partly because they are not our main interest. Betas are generally used as inputs into other tests and processes, such as the calculation of abnormal returns, without making adjustments for their significance so *t*-statistics are not of primary relevance in this article (although see our description of Vasicek betas below). Nevertheless, for interest, we report that where the range between the minimum and maximum beta is large, the minimum beta is often, though not always, not significantly different from zero, while the maximum beta is usually highly significant. Where the range is low, the minimum and the maximum might both be significantly different from zero, neither could be, or one of them could be (the maximum usually has a higher *t*-statistic than the minimum, but this is not always the case).

Table 6 uses the estimated CAPM betas to examine the effect of reference-day variation on the assessment of the change of a company's beta over time. It summarises, in our standard format, the statistics $\left(\hat{\beta}^{J}_{i,2000} - \hat{\beta}^{J}_{i,1995}\right)$ and $\left(\hat{\beta}^{J}_{i,2005} - \hat{\beta}^{J}_{i,2000}\right)$, where the subscript indicates the end of the 5-year period for which company *i*'s beta is estimated, and *J* indicates the reference day on which the returns are based. The results clearly demonstrate that estimates of these changes can be highly sensitive to the selection of a reference day. As the first four rows indicate, estimates of changes, especially between the 1995 and 2000 sub-periods, can be positive or negative depending upon the choice of reference day, and can vary up to a range of about 4. Overall, approximately 75% of all the observations could be classified as positive or negative by an appropriate selection of reference day.

The results presented in Panels A to C of table 7 show that the betas estimated using the FF1993 three-factor model are just as sensitive to the selection of reference day. The highest market beta ranges, and the ranges for companies overall (shown in the first four and last two rows of each panel respectively) are similar to those shown in table 5. The *SMB* and *HML* betas (factor loadings) are just as variable as the market betas, exhibiting patterns similar to the market betas.

Overall these results strongly suggest that estimates of key statistics on the relationship between a stock's return and the return of an index, and the inferences drawn from them, can be highly sensitive to the selection of reference day.

4 REFERENCE-DAY RISK AND INTERNATIONAL EQUITY MARKETS

The results presented in the previous two sections related primarily to the returns on individual stocks and might conceivably be due to idiosyncratic effects on individual stocks. To test the generality of our findings, we consider the scale of reference-day risk in an international context and use a dataset consisting of monthly returns on country equity indices. This allows us to abstract entirely from idiosyncratic effects on individual stocks, and also provides long-run data that are not subject to survivorship bias.

4.1 The data

With this dataset we concentrate on the correlation between equity market returns since these are of interest to those researchers investigating the benefits of international diversification, and the effects of globalisation and market integration. We consider the sensitivity to reference day selection of three specific characteristics of those correlations: first, their level; second, their change over time; and finally their change in extreme (i.e. bull and bear) markets. To check on the robustness of our earlier results to a different data period we select a different 15-year period (1975 - 1989) to investigate the first two of these issues. To investigate the third issue we use a data period of more than 30 years, since we need a large sample when dealing with extremes of distributions.

We construct the series used for this section as before, using the log of the Datastream datatype RI, this time converted to US dollars and applied to the total equity market index of each of the G7 countries (US, UK, Canada, France, Germany, Italy and Japan). Daily data are available for these indices from January 1973 and we use the whole period from January 1973 to December 2005 for our tests of extreme markets.

4.2 International equity market correlations

In table 8 we present statistics on the degree to which the monthly returns on the US market are correlated with those of the other six countries, summarised across our 28 reference-day returns distributions. We also show, in table 9, the estimated change in correlations between the first and second five-year periods, and between the second and third.

The results in table 8 show that reference-day risk is as important for the investigation of the behaviour of international stock markets as it is for the investigation of the behaviour of individual stocks within a single stock market. The point estimate of the correlation with the US market can vary by as much as 46 percentage points when the choice of reference day changes; for Italy and Japan the highest correlation is at least twice as high as the lowest, and, in the final 5-year period, over five times higher.

4.3 Changes in international equity market correlations between time periods

Not surprisingly in view of the previous results, table 9 shows that estimates of the change in correlations between one period and the next are highly dependent upon the choice of reference day. For all the countries considered, the choice of reference day determines whether the correlation appears to have increased or decreased between the second and the third 5-year periods; and for all countries except Canada the same is true of the change in correlation between the first and second five-year periods. Nor are the differences between the highest and the lowest estimates of the change generally negligible. In the case of the UK, for example, on one reference day the correlation between its market and that of the US is estimated to have increased by over 32 percentage points between 1980/84 and 1985/89, whilst on another it is estimated to have decreased by almost 26 percentage points. The ranges for other countries are somewhat smaller than this but are typically in the region of 20 to 30 percentage points.

4.4 International equity market correlations during extreme market conditions

For our final investigation of the influence of reference-day risk on international correlations we consider an issue that has been the focus of much recent research, the question of whether these correlations rise or fall when markets exhibit extreme behaviour (see for example Longin and Solnik, 2001). We do so by examining the correlations of market returns in the tails of the distributions.

We estimate correlations in the standard way, with a different set of correlations for each reference day, but applying them to return ‘exceedances’, the returns that lie above or below a pre-defined threshold. The thresholds are defined around the means of the relevant countries' returns and are in steps of +/-2.5 percentage points. So, for example, a threshold of ±5% indicates that the correlations are based on only those observations where both countries' returns are more than 5 percentage points away from their respective means. A negative (positive) threshold denotes correlations based on the lower (upper) tails of the distributions.

Table 10 shows the summary data for correlations between monthly return exceedances of the US paired with the largest of the six markets considered earlier, the UK, France, Germany and Japan. It also summarises the numbers of co-exceedances, that is the numbers of observations that lie within the tails, a statistic that might be used instead of correlations (see, for example, Bae *et al*., 2003).

Again, we have considerable variation across reference days, both for the correlations and the numbers of co-exceedances. As we would expect, the variation tends to be more severe the more extreme the tail and the fewer the observations in the tail. It is worth pointing out that the standard deviations of the monthly returns of the five countries over the whole sample period vary between about 4.5% and 6.5%, depending on the country and reference day chosen, so the thresholds are set at a maximum of about 1 standard deviation away from the mean, which is by no means extreme. Furthermore, the thresholds of ±0 generate tails with observations roughly double the size of the samples we have used elsewhere, and the range of estimates of both correlations and numbers of co-exceedances is startlingly large.

As a pictorial demonstration of the effect of the choice of reference day, figure 1 shows for each reference day the correlations between return exceedances for the US with the UK. It is clear from the diagrams that, while each reference day exhibits a pattern similar to the reference days around it, if one considers reference days that are say, a week apart (by looking down the columns, rather than across the rows), the data present highly contrasting pictures. A holding period starting on day 5, say, suggests that correlations rise with increasing truncation in bear markets and fall in bull markets, in accordance with most work in this area. However, a holding period starting on day 12 suggests that they fall in bull markets only, and one starting on day 19, that they fall in bear markets and rise in bull markets.

This evidence reinforces the general finding reported in previous sections, that the choice of reference day can have dramatic effects on the estimates of population characteristics made from monthly return data and that the type of inferences that are typically drawn from such estimates should be regarded as highly tentative if they are based on monthly series calculated from a single reference day.

5 ADJUSTING FOR ESTIMATION RISK

As we mentioned in the introduction to this paper, the existence of conventional sampling variation and the estimation risk to which it leads has long been recognised, and a number of attempts have been made to address it, particularly in the context of beta estimation. In this section we consider whether some of these methods also reduce reference-day risk. We investigate three methods of adjusting individual stock betas: the Blume method; the Vasicek Bayesian approach; and the Dimson adjustment for thin trading. We then investigate the technique of using portfolio-level returns as in Fama and French, 1993, amongst many others.

5.1 Adjusting individual stock betas

The Blume, 1971, method is based on Blume's finding that betas show a rate of regression towards the grand mean of betas that is fairly stable over time. It uses betas estimated in the standard way from two consecutive, non-overlapping periods to estimate 'predicted' betas for a third non-overlapping period. The period 2 betas are regressed on those for period 1, generating an estimated intercept and slope. These estimated coefficients are then applied to the period 2 betas to obtain a prediction of period 3 betas, which are used in place of the standard period 3 betas. We apply this technique to the 374 companies in our sample that had a beta for the five years ended December 1995, first estimating 28 regressions:

$$\hat{\beta}_{i,2000}^{J} = \gamma_1^J + \gamma_2^J \hat{\beta}_{i,1995}^{J} + \varepsilon_i^J \qquad J = 1, 2, \ldots, 28 \qquad (2)$$

where the subscript indicates the end of the 5-year period for which company *i*'s beta is estimated and *J* indicates the reference day on which the returns are based.

Then we calculate the 2005 betas as $\hat{\beta}_{i,2005}^{J} = \hat{\gamma}_1^J + \hat{\gamma}_2^J \hat{\beta}_{i,2000}^{J}$.

Table 11 compares the estimated betas with the unadjusted ones.[7] In general the effect of Blume's procedure is to sharply reduce the range of beta estimates for companies for which the unadjusted beta is highly sensitive to the choice of reference day. This is illustrated in the first four rows of table 11 where the highest four ranges of Blume-beta estimates are between a third and a quarter of the equivalent figure for the unadjusted betas. However, at the lower end, the range of the Blume betas is mostly slightly higher – in fact for 131 out of the 374 companies the range on the Blume beta was larger than the range on the unadjusted beta. So the overall average range for the Blume betas is, as the last two rows of the table show, reduced by much less than two-thirds. It is also worth noting that the overall mean Blume beta is 0.774, whereas the mean of the unadjusted betas is 0.965, so the reduction in the range is partly the result of all beta estimates being contracted to below their theoretical level of 1. The result is that the range for Blume betas is on average about 60% of the mean beta across reference days, while it is about 70% for the unadjusted betas.

[7] The summary of unadjusted betas in this table is different from the 2005 sub-period betas shown in table 5 because it relates to the sub-sample of stocks that have observations for all three sub-periods.

The Vasicek, 1973, Bayesian beta combines information from individual betas and from the overall distribution of betas in the sample:

$$\hat{\beta}_{V,i} = \frac{\bar{\beta}\Big/\sigma_{\beta}^{2} + \hat{\beta}_i\Big/SE_{\beta_i}^{2}}{1\Big/\sigma_{\beta}^{2} + 1\Big/SE_{\beta_i}^{2}} \tag{3}$$

where $\bar{\beta}$ and σ_{β}^{2} are the mean and variance respectively of the cross-sectional distribution of betas in the sample; and $\hat{\beta}_i$ and SE_{β_i} are the traditional beta and its standard error (so here the significance of the beta does play a role in its estimation).

To maintain comparability with the Blume method, table 12 shows the results of the Vasicek method applied to the same sample of stocks used to produce table 11. In general the effect of Vasicek procedure is similar to the effect of Blume's though less marked. It sharply reduces the range of beta estimates for companies for which the unadjusted beta is highly sensitive to the choice of reference day and slightly reduces the average range over all companies. The choice of reference day can still cause estimates of Vasicek betas to be twice those obtained from other reference days, and can cause them to be reclassified as aggressive or defensive.

To demonstrate the effect of the Blume and Vasicek methods graphically, figure 2 shows the ranges for the adjusted betas of the companies summarised in tables 11 and 12 plotted against the ranges of the unadjusted betas. Each set of ranges is plotted in order of size, rather than by company. The unbroken line in the figure is the 45^{o} line. The figure shows that the prevalence of ranges of up to about 0.5 is about the same for unadjusted and Blume betas, this figure rising to about 0.8 for Vasicek betas. Above these levels, ranges are attenuated, more so by the Blume method than the Vasicek method (although the maximum ranges are still 0.934 for Blume and 1.851 for Vasicek). So the most severe effects of reference-day variation are reduced, but this still leaves a large degree of variability, together with, in the Blume case, possible downward bias of the underlying beta itself.

The Dimson, 1979, adjustment takes account of thin trading. We showed above that the reference day effect occurs in total market indices as well as individual stocks, so it is unlikely that it is caused by thin trading. However it is possible that it is amplified by thin trading, so we estimate a third set of adjusted betas using the same sample of stocks as for the Blume and Vasicek adjustments. The Dimson beta is the sum of the slopes on CAPM-style regressions that include lag and lead measures of the excess market return as independent variables. Here we include one lag only. We tried including different combinations of leads and lags, up to 2 of each, on the data of the 2000 sub-period, with results similar to those reported below: the reference-day variation using the Dimson method was no better, and in some cases, worse, than with the unadjusted betas.

Table 13 shows the results of the Dimson method applied to the same sample of stocks used to produce tables 11 and 12. In general the Dimson method appears to reduce slightly the highest range of beta estimates and very slightly reduces the mean and median ranges over all companies. Typical estimates of Dimson betas can still straddle one or zero, and can be almost doubled as a result of a different choice of reference day. It does not appear that correcting beta estimates for thin trading makes any appreciable difference to reference-day variation.

Overall, these methods for dealing with conventional estimation risk do reduce the importance of reference-day risk on the estimation of company betas, but they by no means eliminate it. Even after employing these methods estimates of company betas can still be more than doubled as a result of choosing a different reference day and can change from being classified as aggressive to defensive.

5.2 Working with portfolio-level returns

Sample variation in betas and other characteristics can be addressed by forming portfolios of stocks and basing estimates on portfolio, rather than individual stock, returns. This method is used extensively in tests of asset-pricing models and investigations of stock market 'anomalies'. It is useful for tests where a large number of stocks can be grouped together under some broad common characteristic.

FF1993 use 25 stock portfolios formed by starting with five portfolios constructed according to book-to-market quintiles, and subdividing them into five portfolios based on market size quintiles. The total number of firms in their sample is, on average, 3,174 per annum, with an average annual number of firms in each portfolio of between 512.7 and 23.6 depending on the portfolio (the mean (median) over all portfolios is 127 (78) - see FF1993 table 1, page 11). As above, we use the daily data that are provided on French's website, this time to construct 28 sets of monthly returns for each of their 25 portfolios, as well as the three index returns and the risk-free rate of interest. We then use the portfolio returns to calculate CAPM and 3-factor model betas for our three sub-periods.[8]

Table 14 summarises the CAPM betas and table 15 summarises the 3-factor model betas.[9] The format is the same as the previous tables, although it is worth bearing in mind that only 25 portfolios are summarised here, as opposed to the 400-odd companies represented in the earlier tables. Table 14 shows that both the range of betas across portfolios, and the range between maximum and minimum across reference day are considerably reduced when compared with table 5: the averaging both eliminates extreme betas and reduces variation between reference days. However, at the upper end, the FF betas can still vary between, say, 1.26 and 1.93

[8] One way of reducing sampling variation is to use a longer time series – FF1993 use 342 months, for example. We address this point in section 6, but it is worth noting that the longer the period sampled, the less representative it is likely to be of the current relationship between the stock and the market.

(columns 2 and 3, row 1, table 14), and can straddle 1 (in the first and third sub-periods, around 50% of the betas straddle 1, although only 16% do so in the second sub-period). Not shown in this, or the subsequent table of 3-factor model betas, are the *t*-statistics. Unlike the unadjusted betas, on the whole both CAPM and 3-factor model betas were highly significantly different from zero, with *t*-statistics as high as 30.

Figure 3 shows the ranges of the CAPM betas of the FF 1993 portfolios for each of the three sub-periods, plotted in order of size. It is clear from the figure that the highest ranges are concentrated in about 5 portfolios, while ranges of the other 20 are relatively low. However, five portfolios is one fifth of the total, and despite the large numbers of companies contained within the portfolios and the care which has been taken to eliminate estimation risk, it is still possible obtain a beta estimate that, even at the low end, varies from, for example, 0.38 to 0.52 (columns 6 and 7, table 14, company with second lowest range).

Turning to Panels A, B and C of table 15, the market betas are even more stable and the low ranges on the *SMB* and *HML* betas suggest, at first sight, that they are relatively unaffected by reference-day variation, although there are some striking variations, such as between 0.02 and 0.32 for *SMB* betas in the 1995 sub-period (columns 6 and 7, row 1, Panel A). This example emphasises that the betas on these two indices are lower than the market betas, so although the ranges are small in absolute terms, they are relatively large when compared to the underlying betas. In all sub-periods the median range on the market betas is about 13% of the mean beta on average, while the equivalent percentage for *SMB* betas lies between around 25% and 50% depending on the sub-period, and between around 35% and 55% for *HML* betas.

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[9] For this section, the betas calculated using buy-and-hold returns show noticeably more variation across reference days than those based on log returns, though the differences are insufficiently large to merit reporting.

To conclude, it is clear, and perhaps unsurprising, that the most successful method of dealing with reference-day risk (and, presumably, conventional estimation risk) is to work with portfolio-level returns which involve a considerable degree of averaging. However, considering the large number of firms contained within the portfolios, a surprising amount of variability still remains in CAPM betas and in the 3-factor model *SMB* and *HML* betas. Furthermore, the technique is not suitable in many cases, since any averaging procedure loses information about the constituents of that average; for example, the market betas of all the portfolios are very closely clustered around 1, since all extremes have been averaged away.

6. POSSIBLE CAUSES OF REFERENCE-DAY VARIATION

As explained in the introduction, we start this section by carrying out a preliminary investigation into whether the reference day effect might be caused by any systematic stock market behaviour; for example, window-dressing by market participants who are appraised on month-end performance might cause returns based on month ends to exhibit characteristics that are systematically different from those of returns based on the middle of months; a similar effect might be caused by the expiry of derivatives at month ends. We then carry out simulations to establish the degree of variability we would expect to find in returns that are not subject to such influences.

6.1 Regression tests

First of all we carry out a set of regression tests using reference day dummies. The first series of tests consists of regressions of each stock's monthly returns on 27 reference day zero/one dummies, with reference day 28 representing the 'baseline'. The regressions are carried out for each sub-period individually and for the whole 15-year period ended December 2005. None of the resulting *t*-statistics suggested that any reference day dummy was significantly different from zero for any individual stock at any conventional significance level (to save space, none of the regression results in this section are tabulated, but details are available from the authors).

In a second series of tests, applied both to the overall period and to each of the three sub-periods, we stack the estimated mean returns (or medians or variances or CAPM betas) - 28 for each company - and estimate fixed-effects regressions of the resulting series on 27 reference-day dummies. That is, for each variable covered in tables 1, 2, 3 and 5 we estimate

$$Y_{i,J} = \alpha + \sum_{J=1}^{27} \beta_J RDAY_J + \upsilon_i + \varepsilon_{i,J} \qquad (4)$$

where $Y_{i,J}$ is the value of the mean return (or median return, etc.) on company *i* estimated using reference day *J*;

$RDAY_J$ is a dummy variable which takes a value of 1 if the estimate is based on reference day *J* and zero otherwise;

υ_i is a company-specific residual, which may be interpreted as a dummy variable for each company, with value 1 for company *i* and zero otherwise; and

$\varepsilon_{i,J}$ is the error term, with all the standard properties.

Our estimates of equation (4) for the overall 15-year period suggested little evidence of any *systematic* reference day influence on the means, medians or variances: in the regressions involving means and medians none of the *t*-statistics on the reference day dummy variables had absolute values above 1.4, and most were much lower; and in the regressions involving variances, although five dummies had associated *t*-statistics above 1.4 none was above 1.9 and most were much lower. There was slightly stronger evidence of a systematic effect in the regressions involving betas, in that the day 7 to day 18 dummies had *t*-statistics varying between 2.33 and 3.95. In each of the three sub-periods we found evidence of some strong systematic reference day effects for all four variables tested, a substantial number of dummies in each regression having highly significant coefficients. However, we could find no consistency in the sign or significance of the dummies across the three sub-periods: reference day dummies that were highly significant and negative in one period were frequently found to be insignificant, or significant and positive, in another. All in all these results appear to indicate that, whilst within sub-periods certain reference days might exert an influence on estimated population characteristics, any such effects are unstable or are specific to that particular historical period.

6.2 Simulations

We explore this further by carrying out a simulation exercise designed to investigate the reference-day variation that we would expect to find in a sample of stocks that has the same overall characteristics as our own sample, but imposing a Normal distribution on the daily returns.[10] Our simulated, 'well-behaved' data will not suffer from any distortions caused by market participants or from institutional frictions, so if we find the same degree of reference-day variation as in our empirical data this will support the view that reference-day variation is a form of sampling variation. We therefore re-create the 28 sets of parameters summarised in tables 1, 2, 3 and 5, using daily returns that are simulated with our best estimate of the underlying parameters of the empirical distributions, using information from all reference days, as follows.

For each sub-period we first simulate a 5-year series of log-Normal daily returns with the same monthly mean and standard deviation as the actual S&P500 returns over the period. We then, for each stock, simulate a similar daily series with the monthly characteristics (mean, standard deviation and covariance with the market) of that stock's monthly returns over the period. For both the S&P500 and the individual stocks the monthly means, standard deviations and covariances are the average empirical values over the 28 reference days. From these daily returns we construct 28 series of monthly returns - one for each reference day - and finally estimate 28 sets of mean and median monthly returns, variances and betas. We generate 5,000 such simulations for each stock and calculate the mean parameters across all simulations. We

[10] It is well known that daily returns are not Normally distributed, but we are using this assumption as a plausible benchmark.

use these to construct the equivalents of tables 1, 2, 3 and 5, but for the simulated data with known parameters and a known distribution. [11]

To limit the number of tables we present in table 16 only the summaries over all companies and the S&P500 index results. The table shows that the simulated data have reference-day variation of order of magnitude similar to the actual data.[12] The ranges across reference days of the simulated mean and median returns are, if anything, slightly higher than those of the actual data. There are larger differences between simulated and actual data for variances and betas: here the range of the simulated data is consistently smaller than that of the actual data, but, again, of the same order of magnitude, except perhaps the S&P variance in the 2005 sub-period. These results suggest that the reference-day variability that we find is, to a large extent, something that we would expect to occur in a standard, well-behaved sample of sums of a Normally-distributed variable. The simulated variances and betas suggest that we have not captured all of the variability in our simulations, so our assumption of Normally distributed daily returns does not mimic the true distributions sufficiently closely (the excess kurtosis that has been found in empirical returns is most likely to affect simulated variances, and parameters based on these variances). Nevertheless, the simulations present persuasive evidence that reference-day risk is largely due to sampling variation, albeit of a type not normally considered, since it relates to non-independent samples.

[11] We also carried out three series of similar simulations using a 15-year period of simulated daily returns with the characteristics of the three sub-periods respectively. The use of this longer period to calculate estimates of means, medians and so on markedly reduced the range of estimates of each characteristic reported in table 16: in most cases the range (Max-Min) fell to about half of the level reported in table 16.

[12] The fact that the simulated means of the mean returns, variance of returns and betas are very close to the true values is of no significance, since the simulations were generated using these as inputs.

7. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

Estimation risk is a well known problem. Much less well known is reference-day risk – the possibility that monthly returns that cover essentially the same five-year period, but that are estimated from different reference days, might produce radically different estimates of the population's characteristics. This phenomenon has not hitherto been studied, presumably because it is natural to assume that variation between non-independent samples is negligible when compared with conventional sampling variation.

The evidence we have presented in this paper strongly suggests that this assumption is unfounded: the choice of reference day from which to calculate monthly returns can seriously influence the estimated properties of those monthly returns. We have shown that it can lead to widely differing estimates of the means, medians, variances and correlations of individual assets' returns; it can reverse inferences about the nature of companies' betas and about their change over time; it can produce marked differences in estimates of the degree of correlation between different countries' stock markets; and it can determine whether such correlations are reported as increasing or decreasing over time, and whether they increase or decrease in extreme markets.

This degree of reference day risk suggests that results of academic studies that use as inputs estimated properties of monthly returns based on a single reference day may be insecure, and should be tested for robustness against different choices of reference day. Portfolio decisions based on estimates of these properties from a single reference day are also vulnerable to reference day risk and are very unlikely to be optimal.

Further research is required into two main issues: first, the extent to which results of existing studies are indeed sensitive to reference day risk; and secondly, whether the possibility of estimating monthly returns series from different reference days offers an unexploited opportunity to improve the robustness of hypothesis testing and portfolio selection.

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TABLES

Table 1 5-year company monthly returns (%). Means 1991-1995, 1996-2000, 2001-2005

	5 years ended December 1995				5 years ended December 2000				5 years ended December 2005			
Summary of mean returns across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range												
	2.069	1.403	2.500	1.098	1.212	0.718	1.843	1.125	0.298	-0.239	0.934	1.173
		3	*24*			*7*	*3*			*5*	*22*	
	3.426	2.978	3.875	0.897	7.499	6.898	7.986	1.088	-4.206	-4.785	-3.641	1.144
		28	*4*			*21*	*8*			*8*	*21*	
	2.417	2.014	2.838	0.824	4.095	3.637	4.662	1.026	-1.680	-2.150	-1.092	1.057
		2	*28*			*19*	*5*			*8*	*21*	
	3.290	2.913	3.719	0.806	5.372	4.771	5.785	1.014	-5.660	-6.147	-5.119	1.027
		28	*2*			*26*	*18*			*12*	*21*	
Companies with smallest range												
	1.071	1.040	1.098	0.058	0.229	0.192	0.288	0.096	0.996	0.929	1.047	0.118
		12	*4*			*17*	*19*			*28*	*18*	
	1.148	1.119	1.176	0.057	0.943	0.901	0.986	0.084	0.392	0.332	0.445	0.113
		4	*19*			*7*	*28*			*28*	*20*	
	1.268	1.245	1.299	0.055	1.641	1.602	1.685	0.083	0.821	0.782	0.872	0.090
		5	*19*			*27*	*19*			*18*	*7*	
	0.790	0.772	0.823	0.051	1.146	1.108	1.188	0.080	1.404	1.360	1.446	0.086
		12	*17*			*17*	*20*			*27*	*13*	
S&P500 index	1.285	1.253	1.308	0.055	1.419	1.359	1.469	0.110	0.055	-0.015	0.131	0.146
		20	*11*			*20*	*5*			*11*	*20*	
Summary over all companies												
Mean over all companies	1.743	1.616	1.876	0.260	1.326	1.158	1.505	0.347	0.520	0.341	0.683	0.342
Median over all companies	1.556	1.466	1.658	0.215	1.192	1.041	1.336	0.305	0.672	0.512	0.801	0.292

Notes:

1. Returns were calculated using the log of Datastream datatype RI, which incorporates stock price and dividends. The companies were those in the S&P500 at 1 March, 2006 which had continuous price data for at least 8 years.
2. Mean, Min and Max are the mean, minimum and maximum of the 28 estimates of the stock's mean return over the period from the 28 series of actual monthly returns on the company's stock or the S&P500 index.
3. Reference days on which the maximum/minimum mean returns are based are shown in italics directly below the relevant return.

Table 2 5-year company monthly returns (%). Medians 1991-1995, 1996-2000, 2001-2005

	5 years ended December 1995				5 years ended December 2000				5 years ended December 2005			
Summary of median returns across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range												
	3.630	-1.490	6.070	7.560	-1.167	-6.282	2.219	8.501	-3.298	-7.614	0.141	7.755
		9	*1*			*1*	*16*			*8*	*22*	
	3.918	0.762	7.627	6.865	2.750	-1.636	6.679	8.315	-1.913	-6.246	0.795	7.042
		25	*4*			*1*	*12*			*1*	*20*	
	0.926	-2.172	4.362	6.533	4.895	0.809	8.928	8.119	0.800	-2.551	4.127	6.678
		8	*21*			*2*	*15*			*23*	*9*	
	7.461	4.736	10.749	6.013	3.340	-0.870	6.949	7.819	2.617	-1.287	5.285	6.572
		26	*16*			*5*	*18*			*9*	*25*	
Companies with smallest range												
	1.025	0.493	1.439	0.946	0.682	0.158	1.330	1.172	0.592	0.034	1.085	1.051
		3	*14*			*26*	*3*			*26*	*10*	
	1.421	1.042	1.946	0.904	0.992	0.444	1.600	1.156	1.291	0.790	1.787	0.997
		24	*21*			*12*	*6*			*4*	*11*	
	0.972	0.499	1.393	0.894	1.286	0.681	1.728	1.047	1.580	1.145	2.140	0.995
		28	*26*			*2*	*16*			*12*	*14*	
	0.738	0.234	1.122	0.888	1.267	0.720	1.730	1.010	1.462	0.894	1.882	0.988
		8	*23*			*25*	*4*			*24*	*13*	
S&P500 index	1.226	0.792	1.715	0.923	1.482	0.829	2.320	1.491	0.622	0.026	1.199	1.173
		7	*4*			*14*	*10*			*4*	*28*	
Summary over all companies												
Mean over all companies	1.772	0.533	3.000	2.467	1.707	0.158	3.223	3.065	1.081	-0.216	2.362	2.577
Median over all companies	1.553	0.521	2.615	2.206	1.516	0.158	2.900	2.827	1.150	0.014	2.284	2.329

Notes:

1. Returns were calculated using the log of Datastream datatype RI, which incorporates stock price and dividends. The companies were those in the S&P500 at 1 March, 2006 which had continuous price data for at least 8 years.
2. Mean, Min and Max are the mean, minimum and maximum of the 28 estimates of the stock's median return over the period from the 28 series of actual monthly returns on the company's stock or the S&P500 index.
3. Reference days on which the maximum/minimum median returns are based are shown in italics directly below the relevant return.

Table 3 5-year company monthly returns. Variances 1991-1995, 1996-2000, 2001-2005 (all multiplied by 100)

Summary of variances across reference days:	5 years ended December 1995 Mean	Min	Max	Range (Max-Min)	5 years ended December 2000 Mean	Min	Max	Range (Max-Min)	5 years ended December 2005 Mean	Min	Max	Range (Max-Min)
Companies with largest range												
	4.554	2.942	6.450	3.507	5.890	4.783	7.769	2.986	10.340	6.335	21.054	14.720
		10	*22*			*23*	*3*			*1*	*25*	
	4.086	3.094	5.447	2.353	4.681	2.896	5.830	2.933	6.776	3.663	15.472	11.809
		9	*24*			*5*	*22*			*3*	*25*	
	3.517	2.818	4.631	1.813	4.848	3.710	6.389	2.678	7.485	5.456	10.789	5.333
		3	*8*			*3*	*10*			*12*	*27*	
	4.550	3.802	5.157	1.356	4.162	3.369	5.959	2.590	5.104	2.943	7.689	4.746
		22	*1*			*5*	*9*			*13*	*4*	
Companies with smallest range												
	0.159	0.138	0.187	0.048	0.307	0.241	0.353	0.112	0.277	0.223	0.323	0.100
		18	*25*			*22*	*13*			*15*	*23*	
	0.175	0.156	0.204	0.048	0.475	0.419	0.520	0.101	0.249	0.196	0.295	0.100
		9	*16*			*22*	*8*			*13*	*9*	
	0.117	0.097	0.132	0.035	0.254	0.204	0.295	0.091	0.268	0.240	0.322	0.082
		25	*21*			*22*	*10*			*26*	*20*	
	0.116	0.101	0.130	0.030	0.369	0.326	0.414	0.088	0.302	0.272	0.348	0.076
		28	*20*			*5*	*1*			*17*	*21*	
S&P500 index	0.076	0.052	0.107	0.056	0.201	0.161	0.246	0.085	0.247	0.175	0.398	0.223
		21	*9*			*22*	*10*			*15*	*23*	
Summary over all companies												
Mean over all companies	0.738	0.584	0.914	0.330	1.329	1.055	1.655	0.601	1.139	0.839	1.536	0.697
Median over all companies	0.505	0.389	0.615	0.231	0.991	0.759	1.245	0.481	0.760	0.556	1.001	0.416

Notes:

1. Returns were calculated using the log of Datastream datatype RI, which incorporates stock price and dividends. The companies were those in the S&P500 at 1 March, 2006 which had continuous price data for at least 8 years.
2. Mean, Min and Max are the mean, minimum and maximum of the 28 estimates of the variance of the stock's return over the period from the 28 series of actual monthly returns on the company's stock or the S&P500 index.
3. Reference days on which the maximum/minimum variances are based are shown in italics directly below the relevant variance.

Table 4 5-year company monthly returns. Inter-company correlations 1991-1995, 1996-2000, 2001-2005

Summary of correlations across reference days:	5 years ended December 1995 Mean	Min	Max	Range (Max-Min)	5 years ended December 2000 Mean	Min	Max	Range (Max-Min)	5 years ended December 2005 Mean	Min	Max	Range (Max-Min)
Company pairs with largest range												
	-0.0548	-0.4305	0.2446	0.6750	0.0502	-0.2175	0.5837	0.8011	0.1969	-0.2416	0.5703	0.8119
		4	*23*			*12*	*5*			*3*	*19*	
	0.1049	-0.1938	0.4540	0.6478	0.3105	-0.0856	0.6409	0.7265	0.1274	-0.2435	0.5560	0.7995
		22	*8*			*20*	*10*			*4*	*23*	
	0.0718	-0.2622	0.3788	0.6410	0.2847	-0.0741	0.6458	0.7199	0.2157	-0.1879	0.5966	0.7845
		23	*12*			*25*	*7*			*2*	*23*	
	-0.0006	-0.3593	0.2792	0.6384	0.0271	-0.2993	0.4184	0.7176	0.1240	-0.2455	0.5377	0.7833
		23	*7*			*28*	*11*			*8*	*23*	
Company pairs with smallest range												
	0.2829	0.2218	0.3376	0.1158	0.8767	0.8286	0.9292	0.1006	0.8018	0.7430	0.8461	0.1030
		14	*7*			*28*	*16*			*8*	*19*	
	0.1707	0.1273	0.2398	0.1125	0.7634	0.6984	0.7974	0.0990	0.8943	0.8462	0.9289	0.0828
		23	*17*			*4*	*15*			*11*	*7*	
	0.1471	0.0896	0.1967	0.1071	0.5131	0.4708	0.5665	0.0957	0.8600	0.8242	0.9057	0.0815
		25	*15*			*5*	*24*			*14*	*3*	
	0.3761	0.3327	0.4389	0.1062	-0.1209	-0.1721	-0.0865	0.0856	0.8139	0.7823	0.8461	0.0638
		23	*15*			*26*	*19*			*16*	*28*	
Summary over all company pairs												
Mean over all pairs	0.1664	0.0125	0.3123	0.2998	0.1868	0.0331	0.3355	0.3024	0.2790	0.1058	0.4470	0.3412
Median over all pairs	0.1656	0.0097	0.3137	0.2926	0.1872	0.0307	0.3384	0.2936	0.2755	0.0100	0.4508	0.3307

Notes:

1. Returns were calculated using the log of Datastream datatype RI, which incorporates stock price and dividends. The companies were those in the S&P500 at 1 March, 2006 which had continuous price data for at least 8 years.
2. Mean, Min and Max are the mean, minimum and maximum of the 28 estimates of correlations between returns on pairs of stocks divided into 9 groups, as described in the text, over the period shown.
3. Reference days on which the maximum/minimum correlations are based are shown in italics directly below the relevant correlation.

Table 5 5-year company betas: 1991-1995, 1996-2000, 2001-2005

	5 years ended December 1995				5 years ended December 2000				5 years ended December 2005			
Summary of betas across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range												
	0.176	-2.002	2.043	4.045	0.748	-0.468	1.499	1.967	2.895	1.652	5.272	3.620
		1	*19*			*24*	*8*			*17*	*28*	
	1.803	0.426	3.480	3.054	1.277	0.503	2.189	1.686	2.860	1.415	4.666	3.251
		6	*21*			*5*	*21*			*20*	*25*	
	1.884	0.378	3.350	2.972	1.134	0.226	1.857	1.631	2.397	0.993	3.736	2.744
		6	*25*			*7*	*28*			*21*	*28*	
	1.813	0.506	3.468	2.963	2.028	1.095	2.693	1.598	3.691	2.408	4.960	2.552
		7	*21*			*28*	*20*			*19*	*1*	
Companies with smallest range												
	0.947	0.813	1.139	0.326	0.082	-0.029	0.207	0.236	0.868	0.741	0.967	0.226
		2	*14*			*22*	*1*			*4*	*20*	
	1.108	0.956	1.279	0.323	0.510	0.400	0.632	0.232	0.847	0.757	0.979	0.221
		4	*18*			*14*	*21*			*26*	*22*	
	0.821	0.660	0.956	0.297	0.711	0.589	0.816	0.227	0.826	0.702	0.905	0.203
		21	*13*			*20*	*17*			*25*	*4*	
	0.570	0.434	0.719	0.285	0.004	-0.098	0.113	0.211	0.920	0.835	1.035	0.201
		10	*28*			*3*	*6*			*21*	*3*	
Summary over all companies												
Mean over all companies	1.112	0.656	1.576	0.920	0.910	0.544	1.275	0.731	1.026	0.680	1.380	0.700
Median over all companies	1.069	0.657	1.438	0.782	0.874	0.536	1.250	0.672	0.864	0.602	1.144	0.563

Notes:

1. Returns were calculated using the log of Datastream datatype RI, which incorporates stock price and dividends. The companies were those in the S&P500 at 1 March, 2006 which had continuous price data for at least 8 years. The S&P500 index proxies the market.
2. Betas were calculated using the CAPM applied to monthly returns for the five calendar years shown.
3. Mean, Min and Max are the mean, minimum and maximum of the 28 betas estimated from the 28 series of actual monthly returns on the company's stock and the market over the 5-year period to which the table refers.
4. Reference days on which the maximum/minimum betas are based are shown in italics directly below the relevant beta.

Table 6 Change in betas between December 1995 and December 2000; and between December 2005 and December 2000

	Betas for 5 years ended Dec 2000 minus betas for 5 years ended Dec 1995				Betas for 5 years ended Dec 2005 minus betas for 5 years ended Dec 2000			
Summary of change in betas across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range								
	1.095	-0.931	3.454	4.386	2.769	1.411	4.971	3.560
		19	*2*			*17*	*28*	
	-0.661	-2.811	1.222	4.034	2.210	0.441	3.966	3.525
		25	*6*			*20*	*28*	
	-0.930	-2.549	0.738	3.287	1.845	0.179	3.562	3.384
		20	*5*			*23*	*8*	
	0.508	-1.078	1.896	2.974	1.748	-0.048	3.331	3.379
		20	*14*			*22*	*3*	
Companies with smallest range								
	-0.229	-0.463	-0.048	0.414	0.028	-0.111	0.281	0.392
		26	*15*			*7*	*22*	
	-0.142	-0.307	0.091	0.399	0.125	-0.070	0.311	0.381
		7	*25*			*15*	*3*	
	-0.196	-0.353	0.015	0.368	0.196	0.057	0.410	0.353
		22	*5*			*6*	*25*	
	-0.163	-0.344	-0.005	0.339	0.156	0.014	0.255	0.241
		11	*7*			*8*	*4*	
Summary over all companies								
Mean over all companies	-0.226	-0.801	0.325	1.126	0.082	-0.428	0.599	1.026
Median over all companies	-0.222	-0.706	0.299	0.985	0.037	-0.439	0.496	0.910

Notes:

1. The table summarises across reference days the increase (+ive) or decrease (-ive) in CAPM betas between the two pairs of five-year periods described in the title.
2. Returns were calculated using the log of the Datastream datatype RI.
3. The reference days on which the maximum/minimum changes are based are shown in italics directly below the relevant change.

Table 7 Fama and French 3-factor model beta coefficients

Panel A 5 years ended December 1995

	Market betas				SMB betas				HML betas			
Summary of betas across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range												
	-0.344	-2.458	1.122	3.579	2.666	1.246	3.981	2.735	1.320	0.189	2.510	2.321
		27	*19*			*28*	*13*			*28*	*13*	
	1.947	0.656	3.839	3.184	2.152	0.951	3.488	2.536	0.025	-1.315	0.824	2.139
		18	*26*			*15*	*4*			*25*	*7*	
	1.246	-0.017	2.683	2.700	4.349	3.052	5.414	2.362	-0.579	-1.765	0.258	2.022
		6	*20*			*9*	*25*			*6*	*22*	
	1.309	0.207	2.686	2.479	1.267	0.137	2.453	2.316	0.412	-0.561	1.421	1.983
		7	*20*			*2*	*10*			*27*	*19*	
Companies with smallest range												
	0.677	0.503	0.828	0.325	-0.555	-0.693	-0.379	0.314	0.867	0.758	1.000	0.242
		8	*28*			*23*	*18*			*10*	*18*	
	0.358	0.182	0.496	0.314	0.524	0.412	0.716	0.304	0.068	-0.077	0.159	0.236
		7	*12*			*3*	*21*			*7*	*19*	
	1.039	0.919	1.227	0.308	0.056	-0.117	0.176	0.293	0.301	0.181	0.387	0.206
		20	*18*			*21*	*2*			*2*	*19*	
	0.587	0.438	0.733	0.295	-0.402	-0.515	-0.229	0.286	0.148	0.044	0.234	0.190
		1	*15*			*10*	*28*			*9*	*24*	
Summary over all companies												
Mean over all companies	1.054	0.593	1.504	0.911	0.490	0.048	0.934	0.886	0.046	-0.307	0.394	0.701
Median over all companies	0.989	0.560	1.411	0.815	0.380	-0.036	0.794	0.805	0.156	-0.137	0.443	0.609

Notes:

1. Stock and market returns were calculated using the log of Datastream datatype RI, which incorporates stock price and dividends. The companies were those in the S&P500 at 1 March, 2006 which had continuous price data for at least 8 years. The *SMB* and *HML* returns were constructed using daily data obtained from French's website.
2. The coefficients shown are the result of regressing excess stock returns on the excess market return and the *SMB* and *HML* returns, for the five calendar years shown.
3. Mean, Min and Max are the mean, minimum and maximum of the 28 betas estimated from the 28 series of monthly returns.
4. Reference days on which the maximum/minimum betas are based are shown in italics directly below the relevant beta.

Table 7 continued

Panel B   5 years ended December 2000

	Market betas				SMB betas				HML betas			
Summary of betas across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range												
	2.097	0.635	3.153	2.518	1.696	0.397	2.774	2.377	0.337	-1.047	1.486	2.533
		24	*7*			*2*	*17*			*7*	*19*	
	2.661	1.876	3.786	1.910	2.367	1.368	3.446	2.078	-0.559	-1.821	0.565	2.386
		20	*11*			*16*	*28*			*3*	*18*	
	1.112	0.042	1.952	1.910	1.766	0.876	2.823	1.947	-1.479	-2.762	-0.491	2.272
		4	*18*			*16*	*7*			*24*	*13*	
	1.357	0.240	2.112	1.872	1.168	0.300	2.219	1.919	-0.278	-1.254	1.006	2.260
		4	*18*			*24*	*13*			*5*	*21*	
Companies with smallest range												
	0.370	0.265	0.530	0.265	-0.549	-0.680	-0.390	0.290	0.757	0.590	0.908	0.318
		22	*16*			*13*	*1*			*20*	*25*	
	0.417	0.290	0.540	0.250	-0.442	-0.598	-0.310	0.288	0.891	0.708	1.025	0.317
		21	*15*			*22*	*18*			*19*	*1*	
	0.715	0.583	0.815	0.233	0.119	-0.006	0.276	0.283	1.026	0.911	1.178	0.267
		20	*16*			*2*	*20*			*17*	*1*	
	0.994	0.899	1.080	0.182	0.351	0.229	0.471	0.242	0.857	0.785	0.926	0.141
		20	*12*			*23*	*1*			*21*	*17*	
Summary over all companies												
Mean over all companies	1.157	0.764	1.545	0.781	0.286	-0.092	0.667	0.759	0.551	0.099	1.000	0.901
Median over all companies	1.177	0.780	1.535	0.717	0.192	-0.128	0.556	0.681	0.688	0.319	1.063	0.826

Table 7 continued

Panel C 5 years ended December 2005

Summary of betas across reference days:	Market betas				SMB betas				HML betas			
	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range												
	3.739	2.143	7.111	4.968	1.570	-0.568	3.844	4.412	3.564	1.631	6.021	4.390
		22	*28*			*8*	*15*			*22*	*25*	
	3.503	1.881	6.302	4.421	0.253	-2.515	1.831	4.346	2.814	1.529	5.222	3.693
		20	*28*			*25*	*15*			*21*	*28*	
	2.231	0.813	3.764	2.951	2.160	0.309	4.009	3.699	-0.044	-1.688	1.886	3.573
		21	*8*			*7*	*19*			*23*	*9*	
	2.899	1.715	4.441	2.726	0.429	-1.219	2.449	3.668	1.704	-0.316	3.068	3.384
		21	*28*			*27*	*20*			*6*	*23*	
Companies with smallest range												
	0.501	0.399	0.648	0.249	0.120	-0.083	0.318	0.402	1.003	0.852	1.213	0.361
		1	*10*			*11*	*16*			*12*	*19*	
	0.660	0.557	0.794	0.237	-0.124	-0.315	0.055	0.370	0.210	0.030	0.336	0.306
		15	*6*			*3*	*13*			*22*	*7*	
	0.914	0.817	1.043	0.227	-0.065	-0.289	0.068	0.357	0.578	0.443	0.723	0.279
		12	*9*			*3*	*17*			*3*	*28*	
	0.571	0.483	0.692	0.209	0.250	0.108	0.406	0.298	0.157	0.024	0.288	0.264
		13	*23*			*17*	*2*			*12*	*25*	
Summary over all companies												
Mean over all companies	1.076	0.705	1.464	0.760	0.305	-0.260	0.866	1.125	0.332	-0.185	0.877	1.062
Median over all companies	0.960	0.648	1.253	0.624	0.217	-0.269	0.673	0.969	0.396	0.008	0.803	0.910

Table 8 5-year monthly returns on market indices of G7 countries: Correlations 1975-1979, 1980-1984, 1985-1989 (all in %)

	5 years ended December 1979				5 years ended December 1984				5 years ended December 1989			
Summary of correlations across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
US/UK	50.22	36.94	65.46	28.51	48.10	36.95	57.93	20.97	54.25	23.68	69.55	45.87
		24	*8*			*5*	*24*			*19*	*28*	
US/Canada	61.46	53.90	66.13	12.23	79.03	70.96	87.07	16.11	80.41	66.56	87.46	20.90
		12	*27*			*2*	*21*			*19*	*6*	
US/France	40.05	30.41	46.57	16.16	27.41	18.27	37.29	19.02	47.92	33.11	61.75	28.64
		12	*28*			*11*	*1*			*18*	*28*	
US/Germany	33.43	25.52	43.89	18.37	35.22	22.81	50.24	27.43	41.39	23.86	52.47	28.61
		27	*15*			*5*	*24*			*19*	*10*	
US/Italy	29.13	18.95	36.65	17.71	20.16	12.61	26.58	13.98	25.26	6.73	36.31	29.58
		28	*15*			*4*	*15*			*19*	*9*	
US/Japan	32.83	20.50	45.36	24.86	36.13	19.90	49.47	29.57	19.65	5.34	39.22	33.88
		10	*17*			*4*	*17*			*5*	*20*	

Notes:

1. Returns were calculated using the log of Datastream datatype RI.
2. Reference days on which the maximum/minimum correlations are based are shown in italics directly below the relevant correlation.

Table 9 5-year monthly returns on market indices of G7 countries: Changes in correlations between 1975/79 and 1980/84; and between 1980/84 and 1985/89

	1980/84 correlation minus 1975/79 correlation				1985/89 correlation minus 1980/84 correlation			
	Mean %	Min %	Max %	Range%	Mean %	Min %	Max %	Range%
US/UK	-2.13	-27.75	20.98	48.73	6.16	-25.95	32.44	58.39
		8	*24*			*19*	*5*	
US/Canada	17.58	7.63	24.96	17.33	1.38	-17.15	13.82	30.97
		2	*12*			*19*	*6*	
US/France	-12.65	-22.26	-3.84	18.42	20.51	-3.58	39.56	43.14
		8	*13*			*16*	*4*	
US/Germany	1.79	-11.85	20.55	32.39	6.17	-14.08	25.53	39.62
		6	*27*			*25*	*10*	
US/Italy	-8.97	-22.39	3.18	25.57	5.09	-14.82	23.70	38.52
		9	*26*			*19*	*9*	
US/Japan	3.30	-16.26	21.56	37.82	-16.49	-30.11	3.12	33.23
		2	*12*			*17*	*20*	

Notes:
1 The table summarises across reference days the increase (+ive) or decrease (-ive) in international monthly return correlations between the two pairs of five-year periods described in the title.
2 Returns were calculated using the log of Datastream datatype RI.
3 The reference days on which the maximum/minimum changes are based are shown in italics directly below the relevant change.

Table 10 5-year monthly returns on market indices of G7 countries: Correlations between monthly return exceedances; numbers of co-exceedances

US with:	Thr'hld %	Correlations (%)				Number of co-exceedances (%)			
		Mean	Min	Max	Range	Mean	Min	Max	Range
UK	-5.0	55.87	0.62	78.85	78.23	23.54	17	29	12
	-2.5	61.59	34.42	74.87	40.45	54.93	46	68	22
	-0	58.48	43.48	66.70	23.21	125.43	120	132	12
	+0	35.09	28.09	42.19	14.11	139.39	129	147	18
	2.5	34.06	13.04	54.45	41.40	56.11	48	63	15
	5.0	23.53	-10.45	68.27	78.72	19.96	16	25	9
France	-5.0	56.29	21.66	83.78	62.12	23.86	19	28	9
	-2.5	58.61	44.66	78.54	33.88	53.93	43	62	19
	-0	53.04	44.52	60.18	15.66	118.18	104	129	25
	+0	21.82	15.38	30.92	15.54	141.18	134	148	14
	2.5	22.35	-7.97	48.00	55.97	57.46	50	67	17
	5.0	9.26	-47.38	59.32	106.70	19.46	16	23	7
Germany	-5.0	58.69	18.14	76.16	58.02	22.50	20	27	7
	-2.5	58.46	32.67	69.74	37.07	53.04	43	61	18
	-0	59.75	42.01	67.48	25.47	116.36	110	126	16
	+0	20.16	5.83	30.41	24.58	134.64	126	148	22
	2.5	16.60	-7.38	51.33	58.71	52.54	44	59	15
	5.0	20.01	-61.06	64.56	125.61	13.39	10	17	7
Japan	-5.0	17.95	-48.90	58.09	106.99	20.00	15	25	10
	-2.5	24.53	-13.20	48.52	61.72	49.07	39	63	24
	-0	27.99	15.75	43.72	27.97	115.21	106	123	17
	+0	18.32	6.54	29.46	22.92	118.93	111	127	16
	2.5	21.63	-3.76	45.78	49.54	50.25	43	57	14
	5.0	21.77	-25.25	63.07	88.32	15.39	10	21	11

Notes:

1. The table presents summary statistics across the different reference days of the relationships between monthly returns in the tails of the bivariate distributions of each pair of markets.
2. The first block shows the correlations between monthly returns in the tails above (for positive thresholds) or below (negative thresholds) the threshold shown. The second block shows the number of observations used to estimate the correlations (the numbers of co-exceedances).
3. Thresholds are defined around the mean of the relevant country and are expressed in terms of percentage points.

Table 11 Blume betas: estimates of betas for five years ended December 2005

	Blume betas				Unadjusted betas			
Summary of betas across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range								
	0.927	0.398	1.332	0.934	2.895	1.652	5.272	3.620
		23	*8*			*17*	*28*	
	1.179	0.841	1.701	0.860	2.860	1.415	4.666	3.251
		22	*8*			*20*	*25*	
	0.594	0.193	1.050	0.858	2.617	1.238	3.747	2.509
		22	*8*			*19*	*1*	
	0.997	0.561	1.407	0.845	1.661	0.956	2.998	2.042
		28	*5*			*21*	*11*	
Companies with smallest range								
	0.893	0.801	1.037	0.236	0.868	0.741	0.967	0.226
		15	*7*			*4*	*20*	
	0.662	0.533	0.761	0.228	0.847	0.757	0.979	0.221
		24	*18*			*26*	*22*	
	1.062	0.936	1.153	0.217	0.826	0.702	0.905	0.203
		24	*3*			*25*	*4*	
	0.785	0.691	0.869	0.178	0.920	0.835	1.035	0.201
		8	*2*			*21*	*3*	
Summary over all companies								
Mean over all companies	0.774	0.533	0.995	0.462	0.965	0.643	1.298	0.655
Median over all companies	0.761	0.529	0.980	0.452	0.845	0.597	1.110	0.543

Notes:

1. The Blume betas are calculated by applying the equation $\hat{\beta}^J_{i,2005} = \hat{\gamma}^J_1 + \hat{\gamma}^J_2 \beta^J_{i,2000}$, where the subscript indicates the end of the 5-year period for which company *i*'s beta is estimated and *J* indicates the reference day on which the returns are based.
2. The unadjusted betas are the $\hat{\beta}^J_{i,2005}$ for those companies that also had betas for the five years ended December 1995.
3. Reference days on which the maximum/minimum betas are based are shown in italics directly below the relevant beta.

Table 12 Vasicek betas: estimates of betas for five years ended December 2005

	Vasicek betas				Unadjusted betas			
Summary of betas across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Companies with largest range								
	1.930	1.025	2.876	1.851	2.895	1.652	5.272	3.620
		19	*5*			*17*	*28*	
	2.197	1.410	3.150	1.740	2.860	1.415	4.666	3.251
		19	*9*			*20*	*25*	
	2.084	1.315	2.873	1.558	2.617	1.238	3.747	2.509
		22	*9*			*19*	*1*	
	1.825	1.092	2.543	1.451	1.661	0.956	2.998	2.042
		23	*8*			*21*	*11*	
Companies with smallest range								
	1.186	1.076	1.293	0.217	0.868	0.741	0.967	0.226
		3	*20*			*4*	*20*	
	0.854	0.770	0.975	0.205	0.847	0.757	0.979	0.221
		26	*22*			*26*	*22*	
	0.925	0.842	1.029	0.186	0.826	0.702	0.905	0.203
						25	*4*	
	0.838	0.724	0.907	0.183	0.920	0.835	1.035	0.201
						21	*3*	
Summary over all companies								
Mean over all companies	0.935	0.655	1.223	0.567	0.965	0.643	1.298	0.655
Median over all companies	0.859	0.630	1.092	0.498	0.845	0.597	1.110	0.543

Notes:

1. The betas are based on log returns for the five years ended December 2005.
2. The Vasicek betas are calculated by applying the equation $\hat{\beta}_{V,i} = \dfrac{\overline{\beta} / \sigma_{\beta}^2 + \hat{\beta}_i / SE_{\beta_i}^2}{1/\sigma_{\beta}^2 + 1/SE_{\beta_i}^2}$ where $\overline{\beta}$ and σ_{β}^2 are the mean and variance respectively of the cross-sectional distribution of betas; and $\hat{\beta}_i$ and SE_{β_i} are the traditional beta and its standard error.
3. The unadjusted betas are the $\hat{\beta}_{i,2005}^{J}$ for those companies that also had betas for the five years ended December 1995.
4. Reference days on which the maximum/minimum betas are based are shown in italics directly below the relevant beta.

Table 13 Dimson betas: estimates of betas for five years ended December 2005

Summary of betas across reference days:	Dimson betas: Mean	Dimson betas: Min	Dimson betas: Max	Dimson betas: Range (Max-Min)	Unadjusted betas: Mean	Unadjusted betas: Min	Unadjusted betas: Max	Unadjusted betas: Range (Max-Min)
Companies with largest range								
	3.753	2.095	5.299	3.204	2.895	1.652	5.272	3.620
		21	*25*			*17*	*28*	
	3.461	2.541	5.374	2.834	2.860	1.415	4.666	3.251
		7	*25*			*20*	*25*	
	2.685	1.838	3.895	2.056	2.617	1.238	3.747	2.509
		5	*10*			*19*	*1*	
	2.726	1.715	3.735	2.019	1.661	0.956	2.998	2.042
		23	*8*			*21*	*11*	
Companies with smallest range								
	0.200	0.094	0.310	0.216	0.868	0.741	0.967	0.226
		1	*5*			*4*	*20*	
	0.442	0.338	0.546	0.208	0.847	0.757	0.979	0.221
		3	*23*			*26*	*22*	
	1.136	1.040	1.245	0.205	0.826	0.702	0.905	0.203
		3	*10*			*25*	*4*	
	0.374	0.296	0.490	0.194	0.920	0.835	1.035	0.201
		17	*22*			*21*	*3*	
Summary over all companies								
Mean over all companies	1.072	0.757	1.392	0.635	0.965	0.643	1.298	0.655
Median over all companies	0.925	0.681	1.196	0.532	0.845	0.597	1.110	0.543

Notes:
1. The betas are based on log returns for the five years ended December 2005.
2. The Dimson beta is the sum of the slopes on CAPM-style regressions that include both the contemporaneous and lagged excess market return as independent variables.
3. The unadjusted betas are the $\hat{\beta}_{i,2005}^{J}$ for those companies that also had betas for the five years ended December 1995.
4. Reference days on which the maximum/minimum betas are based are shown in italics directly below the relevant beta.

Table 14 5-year betas for Fama and French 1993 portfolios 1991-1995, 1996-2000, 2001-2005

	5 years ended December 1995				5 years ended December 2000				5 years ended December 2005			
Summary of betas across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Portfolios with largest range												
	1.565	1.264	1.925	0.661	1.612	1.430	1.857	0.428	1.473	1.190	1.724	0.534
		1	*21*			*7*	*17*			*22*	*13*	
	1.279	1.032	1.589	0.557	1.480	1.279	1.650	0.371	0.994	0.812	1.154	0.342
		1	*21*			*25*	*14*			*3*	*13*	
	1.099	0.883	1.402	0.519	1.200	1.053	1.402	0.349	1.108	0.930	1.264	0.334
		1	*21*			*7*	*17*			*22*	*13*	
	1.157	0.980	1.462	0.482	1.4487	1.357	1.659	0.301	1.427	1.268	1.571	0.302
		1	*21*			*25*	*14*			*22*	*1*	
Portfolios with smallest range												
	1.269	1.196	1.380	0.183	0.569	0.502	0.668	0.166	0.867	0.823	0.939	0.116
		16	*26*			*23*	*18*			*21*	*13*	
	0.876	0.806	0.958	0.152	0.757	0.682	0.826	0.144	1.011	0.953	1.064	0.111
		26	*13*			*26*	*7*			*5*	*13*	
	1.080	0.990	1.141	0.152	0.460	0.377	0.516	0.139	0.967	0.925	1.022	0.097
		28	*18*			*22*	*7*			*13*	*21*	
	1.006	0.968	1.055	0.087	1.005	0.958	1.048	0.090	0.931	0.893	0.974	0.081
		14	*6*			*16*	*7*			*21*	*9*	
Summary over all portfolios												
Mean over all portfolios	1.107	0.966	1.270	0.304	0.889	0.775	1.013	0.238	0.992	0.886	1.104	0.218
Median over all portfolios	1.046	0.938	1.200	0.287	0.821	0.710	0.938	0.242	0.955	0.846	1.045	0.212

Notes:
1. Monthly portfolio returns, market returns and the risk-free rate of interest were constructed from daily data obtained from French's website.
2. Mean, Min and Max are the mean, minimum and maximum of the 28 sets of coefficients estimated from regressions of 28 series of monthly excess portfolio returns on the excess market return, for the five calendar years shown.
3. Reference days on which the maximum/minimum betas are based are shown in italics directly below the relevant beta.

Table 15 5-year 3-factor model beta coefficients for Fama and French 1993 portfolios

Panel A   5 years ended December 1995

	Market betas				SMB betas				HML betas			
Summary of betas across reference days:	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)	Mean	Min	Max	Range (Max-Min)
Portfolios with largest range												
	1.112	0.958	1.198	0.240	0.161	0.018	0.316	0.298	0.616	0.500	0.734	0.234
		27	*14*			*26*	*20*			*2*	*7*	
	1.151	1.040	1.268	0.228	0.157	0.031	0.280	0.249	-0.638	-0.695	-0.522	0.173
		18	*27*			*12*	*17*			*28*	*19*	
	1.208	1.113	1.327	0.214	0.642	0.494	0.740	0.246	-0.254	-0.334	-0.163	0.171
		26	*10*			*15*	*22*			*4*	*1*	
	1.200	1.001	1.176	0.174	0.444	0.317	0.534	0.216	-0.471	-0.560	-0.389	0.171
		10	*17*			*28*	*13*			*1*	*21*	
Portfolios with smallest range												
	1.088	1.032	1.133	0.101	1.376	1.316	1.435	0.119	0.176	0.133	0.219	0.086
		14	*17*			*6*	*17*			*27*	*9*	
	0.944	0.908	0.996	0.088	-0.210	-0.258	-0.150	0.108	0.305	0.261	0.346	0.085
		17	*25*			*28*	*14*			*4*	*28*	
	0.898	0.867	0.938	0.071	1.066	1.023	1.126	0.102	0.146	0.108	0.193	0.084
		25	*17*			*20*	*27*			*4*	*24*	
	0.841	0.811	0.865	0.055	0.703	0.645	0.738	0.093	0.678	0.644	0.724	0.079
		21	*10*			*18*	*23*			*16*	*24*	
Summary over all portfolios												
Mean over all portfolios	1.015	0.944	1.083	0.139	0.540	0.458	0.619	0.161	0.201	0.142	0.264	0.122
Median over all portfolios	0.997	0.918	1.053	0.138	0.545	0.483	0.604	0.144	0.305	0.261	0.346	0.111

Notes:
1. Monthly portfolio, market, and *SMB* and *HML* returns, and the risk-free rate of interest were constructed from daily data obtained from French's website.
2. Mean, Min and Max are the mean, minimum and maximum of the 28 sets of coefficients estimated from regressions of 28 series of monthly excess portfolio returns on the excess market return and the *SMB* and *HML* returns, for the five calendar years shown.
3. Reference days on which the maximum/minimum betas are based are shown in italics directly below the relevant beta.

Table 15 continued

Panel B 5 years ended December 2000

Summary of betas across reference days:	Market betas Mean	Min	Max	Range (Max-Min)	SMB betas Mean	Min	Max	Range (Max-Min)	HML betas Mean	Min	Max	Range (Max-Min)
Portfolios with largest range												
	1.054	0.916	1.163	0.246	0.529	0.373	0.665	0.292	-0.439	-0.619	-0.319	0.300
		27	*15*			*2*	*13*			*28*	*21*	
	0.931	0.784	1.024	0.240	1.427	1.279	1.569	0.290	0.756	0.641	0.877	0.235
		28	*16*			*12*	*21*			*6*	*21*	
	1.007	0.877	1.116	0.239	1.388	1.250	1.504	0.254	0.140	0.003	0.220	0.217
		28	*17*			*10*	*21*			*28*	*21*	
	1.000	0.891	1.130	0.239	-0.190	-0.310	-0.094	0.216	0.383	0.298	0.504	0.206
		25	*16*			*14*	*21*			*11*	*3*	
Portfolios with smallest range												
	1.007	0.963	1.055	0.092	0.707	0.663	0.758	0.094	0.696	0.641	0.765	0.124
		24	*28*			*7*	*19*			*15*	*28*	
	0.940	0.899	0.989	0.090	0.037	-0.012	0.081	0.093	0.599	0.544	0.662	0.119
		10	*27*			*26*	*18*			*8*	*20*	
	0.932	0.893	0.980	0.088	-0.397	-0.439	-0.354	0.084	0.699	0.645	0.763	0.118
		8	*20*			*26*	*20*			*23*	*27*	
	0.903	0.871	0.952	0.080	0.656	0.618	0.692	0.074	-0.358	-0.385	-0.328	0.056
		22	*11*			*16*	*6*			*1*	*12*	
Summary over all portfolios												
Mean over all portfolios	1.035	0.957	1.113	0.156	0.504	0.425	0.581	0.156	0.485	0.396	0.567	0.171
Median over all portfolios	1.022	0.941	1.116	0.154	0.493	0.443	0.563	0.146	0.671	0.583	0.763	0.174

Table 15 continued

Panel C 5 years ended December 2005

Summary of betas across reference days:	Market betas Mean	Market betas Min	Market betas Max	Market betas Range (Max-Min)	SMB betas Mean	SMB betas Min	SMB betas Max	SMB betas Range (Max-Min)	HML betas Mean	HML betas Min	HML betas Max	HML betas Range (Max-Min)
Portfolios with largest range												
	1.242	1.079	1.411	0.332	0.626	0.440	0.925	0.485	1.053	0.830	1.214	0.384
		22	*13*			*19*	*8*			*25*	*9*	
	1.272	1.118	1.412	0.294	1.345	1.092	1.568	0.476	0.513	0.376	0.693	0.317
		23	*13*			*2*	*19*			*25*	*9*	
	1.210	1.071	1.334	0.263	-0.417	-0.617	-0.162	0.455	-0.209	-0.344	-0.031	0.313
		3	*26*			*4*	*23*			*1*	*18*	
	1.218	1.105	1.367	0.262	0.827	0.691	1.046	0.355	-0.504	-0.638	-0.336	0.302
		25	*10*			*19*	*1*			*8*	*20*	
Portfolios with smallest range												
	0.841	0.793	0.874	0.081	0.389	0.279	0.454	0.174	0.990	0.931	1.049	0.118
		4	*25*			*13*	*28*			*23*	*13*	
	0.701	0.662	0.739	0.078	0.905	0.819	0.983	0.164	0.479	0.430	0.545	0.115
		4	*25*			*7*	*26*			*4*	*18*	
	1.027	0.997	1.066	0.069	1.023	0.950	1.071	0.121	0.357	0.309	0.411	0.102
		15	*22*			*1*	*24*			*26*	*9*	
	0.916	0.887	0.954	0.067	-0.215	-0.251	-0.178	0.073	-0.339	-0.373	-0.285	0.088
		3	*21*			*1*	*24*			*10*	*22*	
Summary over all portfolios												
Mean over all portfolios	1.017	0.943	1.090	0.147	0.531	0.398	0.662	0.264	0.410	0.301	0.517	0.216
Median over all portfolios	0.997	0.922	1.036	0.118	0.510	0.414	0.606	0.256	0.499	0.384	0.633	0.225

Table 16 Results of simulations

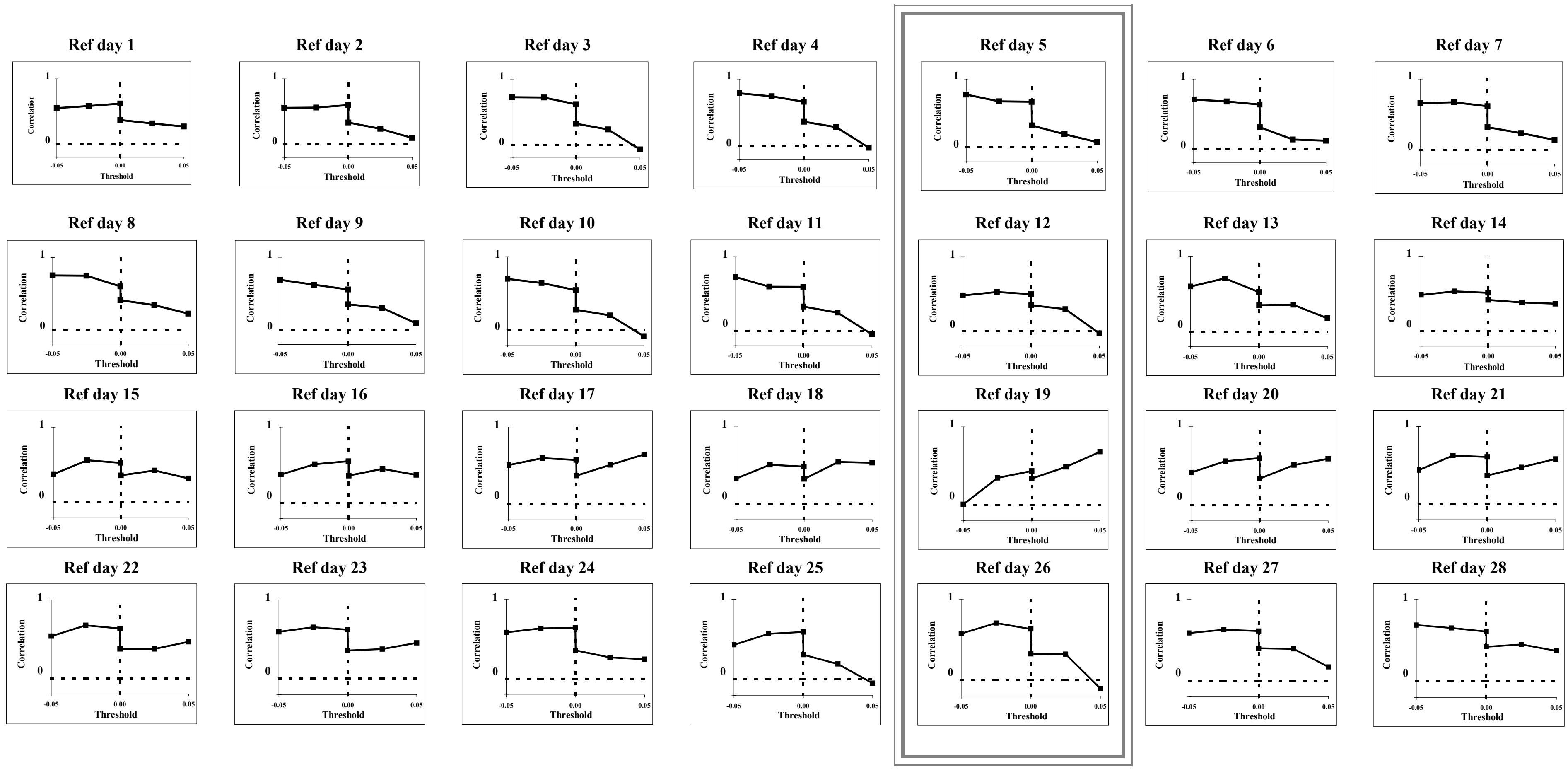
Summary of parameters across reference days:	5 years ended December 1995 Mean	Min	Max	Range (Max-Min)	5 years ended December 2000 Mean	Min	Max	Range (Max-Min)	5 years ended December 2005 Mean	Min	Max	Range (Max-Min)
Mean return (%)												
Mean over all companies												
Simulated	1.744	1.616	1.872	0.256	1.333	1.158	1.507	0.349	0.515	0.359	0.671	0.313
Actual	1.743	1.616	1.876	0.260	1.326	1.158	1.505	0.347	0.520	0.341	0.683	0.342
S&P500 index												
Simulated	1.277	1.233	1.322	0.089	1.404	1.333	1.476	0.143	0.054	-0.025	0.134	0.160
Actual	1.285	1.253	1.308	0.055	1.419	1.359	1.469	0.110	0.055	-0.015	0.131	0.146
Median return (%)												
Mean over all companies												
Simulated	1.744	0.463	3.026	2.563	1.334	-0.415	3.082	3.497	0.515	-1.055	2.085	3.140
Actual	1.772	0.533	3.000	2.467	1.707	0.158	3.223	3.065	1.081	-0.216	2.362	2.577
S&P500 index												
Simulated	1.279	0.835	1.725	0.891	1.399	0.679	2.120	1.441	0.058	-0.743	0.853	1.596
Actual	1.226	0.792	1.715	0.923	1.482	0.829	2.320	1.491	0.622	0.026	1.199	1.173
Variance of returns ($\times$ 100)												
Mean over all companies												
Simulated	0.738	0.605	0.887	0.283	1.328	1.0880	1.597	0.509	1.140	0.934	1.370	0.436
Actual	0.738	0.584	0.914	0.330	1.329	1.055	1.655	0.601	1.139	0.839	1.536	0.697
S&P500 index												
Simulated	0.076	0.062	0.092	0.029	0.201	0.165	0.242	0.077	0.246	0.202	0.297	0.094
Actual	0.076	0.052	0.107	0.056	0.201	0.161	0.246	0.085	0.247	0.175	0.398	0.223
Beta												
Mean over all companies												
Simulated	1.112	0.746	1.478	0.732	0.911	0.601	1.2200	0.619	1.026	0.798	1.254	0.456
Actual	1.112	0.656	1.576	0.920	0.910	0.544	1.275	0.731	1.026	0.680	1.380	0.700

Notes:

1 The table shows the descriptive statistics of the parameters covered in tables 1, 2, 3 and 5, for simulated and actual data (the actual data repeat the company summaries and the S&P500 details that appear in those tables).

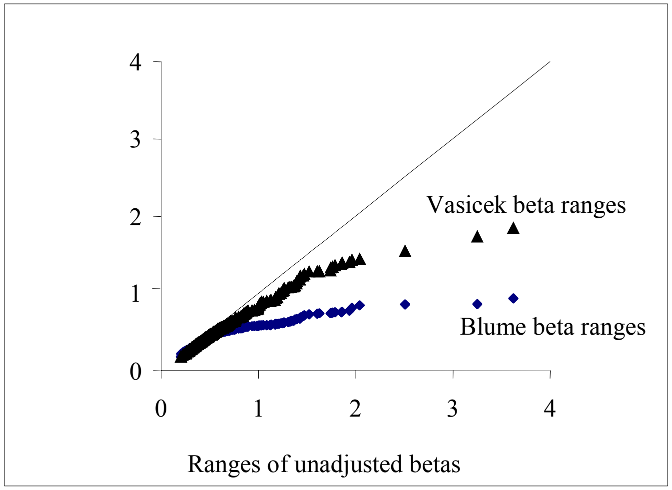
FIGURES

<u>Figure 1 Correlations between return exceedances; US with UK</u>



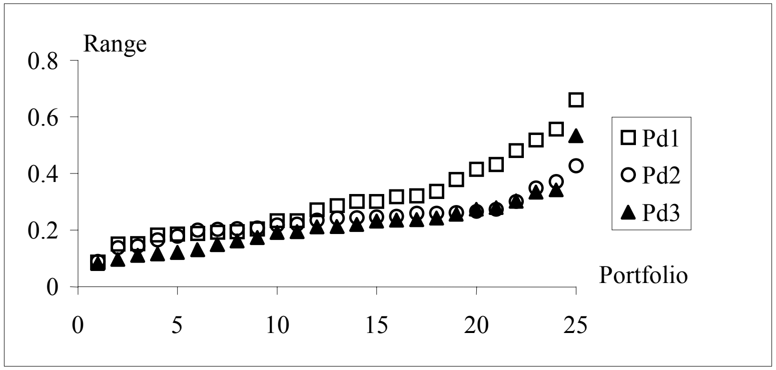
Note:
The figure shows correlations between monthly return exceedances for the UK and the US over the period February 1973 to December 2005, for thresholds -5, -2.5, -0, +0, +2.5 and +5 percentage points around their respective means. Each panel is based on monthly returns calculated using the reference day shown. The double outline emphasises the change in pattern of correlations that can be caused by moving the reference day by one week.

Figure 2 Ranges of unadjusted, Blume and Vasicek betas



Note:
The figure shows the ranges (maximum across reference days – minimum across reference days) of the adjusted betas of the companies summarised in tables 11 and 12 plotted against the ranges of the unadjusted betas. Each set of ranges is arranged in rank order of size and the resultant ranges plotted against one another. The unbroken line in the figure is the 45^{o} line.

Figure 3 Ranges of CAPM betas of Fama and French portfolios



Note:
The figure shows the ranges (maximum across reference days – minimum across reference days) of the betas of the FF 1993 portfolios for each of the three 5-year sub-periods ending in December 1995 (pd1), December 2000 (pd2), and December 2005 (pd3). Each set of ranges is plotted in order of size.