Total Factor Productivity: An Unobserved Components Approach

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Abstract

This work examines the presence of unobserved components in the time series of Total Factor Productivity, which is an idea central to modern Macroeconomics. The main approaches in both the study of economic growth and the study of business cycles rely on certain properties of the different components of the time series of Total Factor Productivity. In the study of economic growth, the Neoclassical growth model explains growth in terms of technical progress as measured by the secular component of Total Factor Productivity. While in the study of business cycles, the Real Business Cycle approach explains short-run fluctuations in the economy as determined by temporary movements in the production function, which are reflected by the cyclical component of the time series of the same variable. The econometric methodology employed in the estimation of these different components is the structural time series approach developed by Harvey (1989), Harvey and Shephard (1993), and others. An application to the time series of Total Factor Productivity for the 1948-2002 U.S. private non-farm business sector is presented. The pattern described by technical progress in this economy is characterised by important growth for the period immediately after War World II, which reaches its peak at the beginning of the 1960s to decline until the earliest 1980s where it shows a modest rebound. On the other hand, the cyclical component of the series seems to be better described by two cycles with periodicity of six and twelve years, respectively.

Keywords: Productivity, Business Cycles, Structural Time Series Models, Unobserved Components.

JEL classification: E23, E32, C22

1. Introduction

The seminal work of Solow's (1957), which derives a methodology to measure technological progress, has been of major importance in Macroeconomics. First, in the growth literature it has become the basis for an extensive theoretical body on growth accounting that tries to quantify the sources of economic growth. Second, the main approach in the study of business cycles, the Real Business Cycle approach, assumes technological innovations (measured by Solow's procedure) as the main driving force of short-run fluctuations in the economy, and employs it in the simulations of quantitative models. And third, as it is believed that technological progress is an important source of economic growth many researchers have attempted to explain it as the endogenous outcome of economic decisions, which has served as the basis of a new body of literature on endogenous economic growth.

Although the main approach in both the study of economic growth and business cycles relies on the time series behaviour of the same variable, technological progress, their interest is focused on different components of the series. Hence, in the study of economic growth the attention is centred on the pattern described by the non-stationary part of the series (which can keep steady, speed up or slow down), while in the study of business cycles, the interest is on the stationary part of this series. This distinction is commonly ignored in the empirical estimation of technical progress, which sometimes could have important effects on our conclusions about the pattern displayed by the secular component of the variable over time.

In this work the presence and characterisation of unobserved components in the time series of Total Factor Productivity is examined. The structure given to the paper is the following: in Section 2 a brief description of the methodology derived by Solow (1957) is presented, and some changes to the specification of the production function are introduced in order to give an explicit account of the different components of the series in accordance with the main approaches in the study of economic growth and the business cycle. In Section 3 the econometric methodology employed to get the estimates of the different components of the time series of technological progress is described. Section 4 shows the empirical results obtained in the analysis of Total Factor Productivity in the U.S. economy under this methodology. Finally, Section 5 presents the conclusions of the paper.

2. Theoretical Background

In the Growth Accounting literature, observed economic growth is partitioned into components associated with factor accumulation and a residual that reflects technical progress and other elements. This breakdown of the rate of growth of aggregate output into different components has its foundation in the pioneering work of Solow (1957). In this work, Solow derives a measure of technical progress, and shows how to employ it to correct the estimation of the production function. He starts with the Neoclassical production function[1]

$$Y(t) = F(K(t), L(t), A(t)) \tag{2.1}$$

where $Y(t)$ is the flow of output produced at time t, $K(t)$ is the physical capital stock accumulated at time t, and $L(t)$ is the labour input at time t. The production function also depends on $A(t)$, the level of technology, and the notation makes explicit that it varies with time. Taking total (logarithmic) differential of equation (2.1) and dividing through by Y yields,

$$\frac{\dot{Y}}{Y} = g + \left(\frac{F_K.K}{Y}\right).\left(\frac{\dot{K}}{K}\right) + \left(\frac{F_L.L}{Y}\right).\left(\frac{\dot{L}}{L}\right) \tag{2.2}$$

where F_K and F_L are the factor (social) marginal products, and g (technical progress) is given by

[1] By Neoclassical production function, we mean that the function is concave, twice continuously differentiable, satisfies the Inada (1964) conditions and that both factors are essential in production.

$$g \equiv \left(\frac{F_A.A}{Y}\right).\left(\frac{\dot{A}}{A}\right) \qquad (2.3)$$

Solow assumed technological change to be Hicks-Neutral, so that it could be factored out of the production function in the following way[2],

$$Y(t) = A(t)F(K(t), L(t)) \qquad (2.4)$$

In this particular case technological change would be given by

$$g = \frac{\dot{A}}{A} \qquad (2.5)$$

Equation (2.2) suggests that the rate of growth of real output can be decomposed into the growth rates of capital and labour, weighted by their output elasticities, and the rate of growth of technical progress. Consequently, the rate of technical progress can be obtained from this equation as a residual,

$$g = \frac{\dot{Y}}{Y} - \in_K .\frac{\dot{K}}{K} - \in_L \frac{\dot{L}}{L} \qquad (2.6)$$

where $\in_K$ is the output elasticity with respect to capital and $\in_L$ is the output elasticity with respect to labour. In practice, as these elasticities are not observable, to compute technical change researchers usually assume that each input is paid their (social) marginal products, so that $F_K = r$ (the rental price of capital) and $F_L = w$ (the wage rate). This substitution allows the rate of change of technical progress to be expressed in terms of observable income shares as

[2]By assuming Hicks-Neutral technological change, as stated by Solow (1957, p. 312), shifts in the production function "leave marginal rates of substitution untouched but simply increase or decrease the output attainable from given inputs".

$$\hat{g} = \frac{\dot{Y}}{Y} - s_K . \frac{\dot{K}}{K} - s_L . \frac{\dot{L}}{L} \tag{2.7}$$

where s_K and s_L are the respective shares of each factor payment in total output, and $\hat{g}$ is often described as an estimate of Total Factor Productivity (TFP) or the Solow residual.

Solow made it explicit that in applied work the residual would pick up any factor shifting the production function. However, he labelled it technical progress under the presumption that technological change would be the main influence being captured by it. He found some ground for this assertion in his estimates of the factor $A(t)$ for the US economy, which showed a strong upward trend during the period 1909-1949.[3]

The production function specified by Solow (1957) to measure technological progress is the same specification given to the production function in the Solow-Swan model or Neoclassical model of economic growth. In this model the factor $A(t)$ is introduced in the production function in order to enable the modelled economy to reproduce the observed pattern of some macroeconomic variables that register growth in per capita terms over the years. Therefore, the specification of the production function is intended to pick up those driving forces that bring about economic growth under the Neoclassical model of economic growth. It is important to notice, however, that such a specification for the production process does not provide an explicit account of any other forces that drive short-run fluctuations in the economy as those ones claimed by the Real Business Cycle approach. From this perspective, a more appropriate specification for the production process seems to be one that explicitly distinguishes those forces that drive economic growth from those associated with business cycles.

[3] A negative trend in A(t) would imply the unreasonable case of technical regress, something that would have discouraged Solow from writing his paper (see, Solow 1957, p.316).

In modern Macroeconomics the production function is specified in such terms that it is allowed to pick up forces that drive both economic growth and business cycles, and it is described as follows

$$Y(t) = \lambda(t)F(K(t), L(t), A(t)) \tag{2.8}$$

Here the production process is similar to that one specified in equation (2.1) except that there is an explicit account of *temporary* changes in the production function through a random variable $\lambda(t)$, while *secular* improvements in technology are measured by $A(t)$. Hence, the production function establishes a clear distinction between forces that drive economic growth from those that drive short-run fluctuations.[4]

In the economic growth literature the specification given to the production process ignores the term $\lambda(t)$, while in the business cycle literature growth is omitted or it is simply started with a transformed economy.[5] Therefore, $\lambda(t)$ and $A(t)$ stand for processes whose driving forces are completely different, and consequently they require different specifications. In the business cycle literature $\lambda(t)$ is commonly described as a stationary process, which displays considerable serial correlation, with first-differences nearly serially uncorrelated, while in the economic growth literature $A(t)$ is usually specified as a non-stationary process that can be expressed either as a trend-stationary process or a difference-stationary process. Even though economists have considered it appropriate to separate these different processes according to the subject of study (i.e. economic growth or short-run fluctuations), it seems clearly inappropriate to ignore them in an empirical estimation of technological progress. For that reason, if equation (2.8) is employed and the same reasoning is carried out as before, we arrive at an expression for TFP for the particular case of Hicks-Neutral technological change given by

[4] This specification is found in papers such as King, Plosser and Rebelo (1988) and King and Rebelo (1999).

[5] In the analysis of business cycles, models with steady state growth are transformed into stationary economies. This transformation is introduced to the Neoclassical growth model by scaling all the trending variables by the growth component $A(t)$.

$$\hat{g} = \frac{\dot{\lambda}}{\lambda} + \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} - s_K . \frac{\dot{K}}{K} - s_L . \frac{\dot{L}}{L} \qquad (2.9)$$

Equation (2.9) establishes an explicit distinction between fluctuations of the production function that occur in the short-run from those of a more permanent nature such as technological progress. This discrepancy between TFP and changes in technology, which is commonly ignored in the growth accounting literature, is the one that will be addressed in this paper by employing the structural time series approach.

3. Econometric Methodology

The econometric methodology employed in this paper is the structural time series approach developed by Harvey (1989) and Harvey and Shephard (1993), which builds on early work such as Nervole, Grether and Carlvalho (1979). The essence of this approach is to set up a model, which regards the observation as being made up of a trend (or permanent) component and an irregular (or temporary) component. Consequently, structural time series models are nothing more than regression models in which the explanatory variables are functions of time and the parameters are time varying. The estimation is conducted by setting the model in state space form, with the state of the system representing the various unobserved components. In the case of linear models, the Kalman filter is employed, which provides the means of updating the state as new observations become available.[6]

The simplest structural time series model, usually referred to as the *local level model*, is given by a trend component and an irregular term, which is a white noise process. The model can be written in the following way,

$$y_t = \mu_t + \varepsilon_t \qquad t = 1, 2, \ldots T \qquad (3.1)$$

[6] A thorough discussion of the methodological and technical ideas underlying this approach is found in Harvey, A. (1989).

where y_t is the observed value, μ_t is a trend and ε_t is a white noise disturbance term, that is, a sequence of serially uncorrelated random variables with constant mean, in this case zero, and constant variance, σ_ε^2. The trend component, μ_t, may take a variety of forms, the simplest being a level that fluctuates up and down according to a random walk

$$\mu_t = \mu_{t-1} + \eta_t \qquad t = \ldots -1, 0, 1, \ldots \qquad (3.2)$$

where η_t is a white noise disturbance with variance σ_η^2, which is uncorrelated with the stochastic term ε_t. No starting value needs to be specified for μ_t since it is assumed to have started at some point in the remote past.

An alternative specification for the trend component is the following

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t && (3.3) \\ \beta_t &= \beta_{t-1} + \varsigma_t && t = \ldots -1, 0, 1, \ldots \end{aligned}$$

where η_t and ς_t are mutually uncorrelated white noise disturbances with zero means and variances σ_η^2 and σ_ς^2, respectively. Together, (3.1) and (3.3) form what is often referred to as the *local linear trend model*. The effect of η_t is to allow the level of the trend to shift up and down, while ς_t allows the slope to change. The longer the variances the greater are the stochastic movements in the trend. We should notice that the trend specification given in (3.3) nests different processes such as, the random walk with drift trend ($\sigma_\varsigma^2 = 0$) and the deterministic linear trend ($\sigma_\eta^2 = \sigma_\varsigma^2 = 0$).

A cycle can be introduced to (3.1) in order to formulate a model more in line with economists' traditional view that the movements of an annually recorded time series for a macroeconomic variable are determined by a trend component, a cyclical component and a noise component. Formally,

$$y_t = \mu_t + \psi_t + \varepsilon_t \qquad\qquad t = 1, \ldots, T \tag{3.4}$$

where ψ_t is the cyclical component that is a function of time, and the other components have been specified above. Modelling the cyclical process takes the form

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix} \tag{3.5}$$

where ω_t and ω_t^* are uncorrelated white noise disturbance terms with variance σ_ω^2 and $\sigma_{\omega*}^2$, respectively, and ψ_t^* appears by construction in order to form ψ_t. The disturbance terms make the cycle stochastic rather than deterministic. The parameter $0 \le \lambda \le \pi$ is the frequency of the cycle, which is measured in radians. The period of a cycle corresponding to a frequency of λ is $2\pi / \lambda$ years. The coefficient $0 \le \rho \le 1$ is a damping factor on the amplitude of the cycle. If $0 < \rho < 1$ the process is a damped sine or cosine, wave. While if $\rho = 1$ the process is again a sine or cosine wave, but no damping movement is present. A single equation for ψ_t can be obtained by writing the model as

$$\psi_t = \frac{(1 - \rho\cos\lambda L)\omega_t + (\rho\sin\lambda L)\omega_t *}{1 - 2\rho\cos\lambda L + \rho^2 L^2} \tag{3.6}$$

where L is the lag operator. Equation (3.6) shows that the process described by ψ_t is an ARMA(2,1), which becomes an AR(2) whenever $\sigma_\omega^2 = 0$. A final point to note is that the stochastic cycle collapses to an AR(1) process when $\lambda = 0$ or π.

In the model described by equation (3.4) the cycle is introduced by adding it to a trend component and an irregular component. Such a model is usually referred to as the *trend plus cycle model*. An alternative way of introducing a cycle is by incorporating it into the trend. This specification is usually known as the *cyclical trend model*. In this case, trend and cycle are not separable, and the model can be formally written as

$$y_t = \mu_t + \varepsilon_t \qquad t = 1, 2, \ldots T \qquad (3.6)$$

$$\beta_t = \beta_{t-1} + \varsigma_t \qquad t = \ldots -1, 0, 1, \ldots$$

$$\mu_t = \mu_{t-1} + \psi_{t-1} + \beta_{t-1} + \eta_t$$

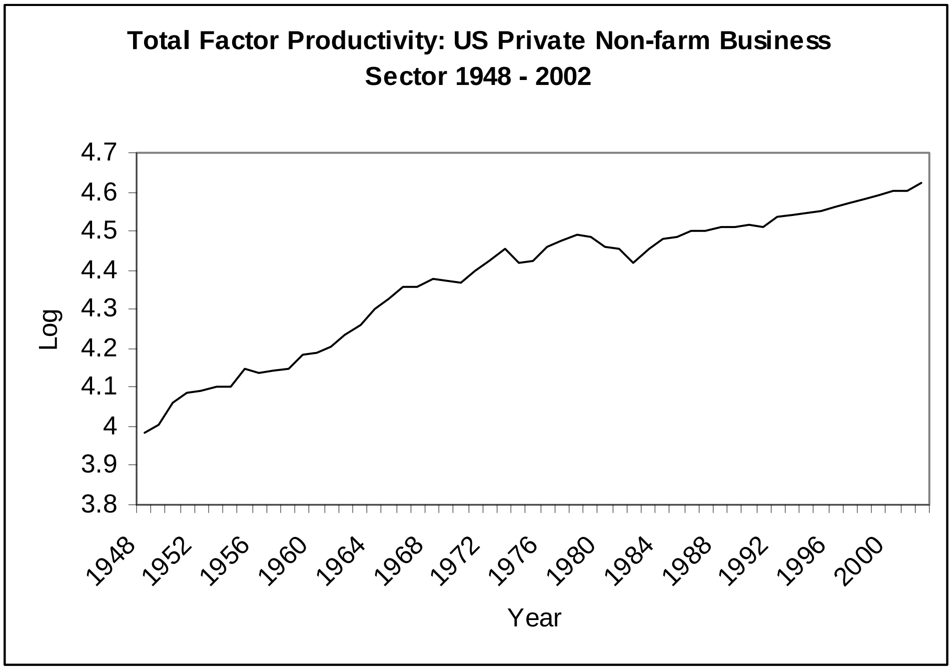
The trend plus cycle model (3.4) and the cyclical trend model (3.6) are the most important formulations of structural time series models that exhibit cyclical process.

4. Empirical Results

In this section the empirical results of the paper will be presented. The time series to be analysed is the widely cited measure of Total Factor Productivity for the U.S. economy produced by the Bureau of Labour Statistics (BLS).[7] Figure 1 shows the annually recorded TFP series in logarithmic terms for the period 1948-2002.

[7] Series Id: MPU750023 (K)

Figure 1



The series computed by the BLS uses for real output the national accounting data from the Bureau of Economic Analysis (BEA). The private non-farm business sector includes all of gross domestic product except the output of general government, government enterprises, non-profit institutions, the rental value of owner-occupied real estate, the output of paid employees of private households, and farms from the private business sector, but includes agricultural services. The output index, which is supplied by BEA, is computed as chained superlative index (Fisher Ideal Index) of components of real output, and then adjusted by the BLS. Labour input is obtained by Tornqvist-aggregation of the hours at work by all persons, classified by education, work experience, and gender with weight determined by their shares of labour compensation. Finally, the capital input measures the services derived from the stock of physical assets and software. The assets included are fixed business equipment, structures, inventories and land. The BLS produces an aggregate input measure obtained by Tornqvist aggregation of the capital stock of each asset type using estimated rental prices.[8]

[8] More detailed information on methods, limitations, and data sources is provided BLS Bulletin 2178 (September 1983), "Trends in Multifactor Productivity, 1948-81".

The U.S. TFP series has been widely analysed and a growing body of research has emerged around it. Among the most salient and well-known features of the series are the patterns of productivity slowdowns after 1973, which has been associated by some researchers with the oil price shocks of the 1970s, and rebounds after 1995. Additionally, it has been recognised that TFP tends to move pro-cyclically; in periods of economic expansion, TFP is unusually large, while during recessions, it is low or even negative.

In the economic literature there are very few cases of an explicit treatment of the presence of different components in the TFP series. An exception to this is found in King and Rebelo (1999), where the productivity series is specified in terms of two components; a trend which is assumed to be linear and deterministic, and a cyclical component which follows an first-order autoregressive process, AR(1). Employing quarterly data of TFP for the U.S. economy during the period 1947 (first quarter) to 1996 (fourth quarter) they fit a linear trend to the series, and then use the residuals to estimate an AR(1) model –the resulting point estimate of the persistence parameter is 0.979. It is this decomposition of the TFP series that is addressed in this work, but by employing a formal econometric methodology in the specification process in order to get estimates of the different components of the series and to determine their main characteristics.

In order to narrow down the number of suitable structural time series models for the U.S. TFP series some statistics have been computed, which provide additional information in relation to the main characteristics of the different components of the variable. In relation to the trend of the variable, unit root tests can provide a valuable insight into the presence of either a deterministic or stochastic secular component in the series.

To determine whether or not the U.S. TFP series is characterised by having a unit root in their autoregressive representations, a modified Augmented Dickey-Fuller test (hereafter ADF-GLS$^{\tau}$) developed by Elliot, Rothenberg and Stock (1996), which has difference-stationary [or I(1)] as the null hypothesis will be employed. An important property of this test is that it has more power than the original ADF tests, and is approximately uniformly most power invariant. Similarly, a second test that is a version of the Kwiatkowski, Phillips, Schmidt and Shin (1992) tests developed by Leybourne and McCabe (1994), which has trend-stationary [or I(0)] as the null hypothesis [hereafter KPSS(LM)] will be conducted.

The KPSS(LM) results will be used to corroborate the information obtained by applying the ADF-GLS$^{\tau}$ test, and vice versa. Consequently, if the ADF-GLS$^{\tau}$ test rejects the unit root hypothesis and the KPSS(LM) test fail to reject the stationary null hypothesis then, these results will be considered as strong evidence in favour of a trend-stationary process. By contrast, if the ADF-GLS$^{\tau}$ test fails to reject the null hypothesis but the KPSS(LM) rejects it, we will consider this as strong evidence supporting the view of the presence of a difference-stationary process. If both tests fail to reject their respective null hypothesis then, it will be considered that the data does not contain sufficient information to discriminate between these two kinds of processes.[9]

Null specific critical values for the ADF-GLS$^{\tau}$ tests using a preferred difference-stationary specification following the approach specified by Cheung and Chinn (1997) have been generated.[10] Similarly, for the KPSS(LM) tests null specific critical values using a preferred trend-stationary specification following the procedure suggested in Leybourne and McCabe (1996) have been computed.[11] In Table 1 the ADF-GLS$^{\tau}$ statistic and the KPSS(LM) statistic together with their associated 10%, 5% and 1% critical values for the U.S. TFP series are presented.

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[9] In cases where both tests reject their respective null hypothesis, as argued by Cheung and Chinn (1997), it might be an indication that the data generating mechanism is more complex than that captured by standard linear time series models.

[10] Cheung and Chinn (1997) generate null specific critical values using a selected difference-stationary specification, which is chosen from models with lag parameters p and q ranging from 0 to 5 using the BIC statistic.

[11] Leybourned and McCabe (1996) generate null specific critical values by fitting an ARIMA (p,1,1) model with p set initially at 5, and then reducing it to 4 if the statistic $z(p=5)=\left|T^{1/2}\hat{\varphi}_5\hat{\theta}\right|$<1.645, and so on. Once the value of p has been determined a preferred trend-stationary description is obtained by re-estimating an ARIMA (p,0,0) model with a time trend.

Table 1
ADF-GLS$^{\tau}$ and KPSS(LM) Tests: U.S. TFP (1948-2002)

Statistic	***Actual***	***Critical Values***		
		10%	***5%***	***1%***
ADF-GLS$^{\tau}$	-1.2697	-2.8583	-3.1873	-3.8360
KPSS(LM)	1.1635	0.8648	1.0005	1.1569

In the first row of Table 1 the results obtained from applying the ADF-GLS$^{\tau}$ test is shown. It is possible to see that the actual statistic is well below the rejection area of the null hypothesis of a unit root. Additionally, in the second row of the table the results of the KPSS(LM) tests is presented. According to this result there is a clear rejection of the null hypothesis of a trend-stationary process as it is rejected at a 1% significant level. Based on the results obtained in both the ADF-GLS$^{\tau}$ tests and the KPSS(LM) tests we find strong evidence to disregard the possibility of having a deterministic linear trend in the times series of the TFP series for the U.S. economy.

It is known that unit root tests are sensitive to the presence of structural breaks in a series. Perron (1989) demonstrated that when there are structural changes in a series the standard tests for unit root hypothesis against the trend-stationary alternatives are biased towards the non-rejection of a unit root. Considering this possibility structural change tests following the methodology suggested by Perron (1997) have been conducted.

Perron's technique consists of examining the likelihood of three different kinds of changes in the structure of a series: one that permits an exogenous change in the level of the series (Model A), one that allows an exogenous change in the slope (Model B), and finally one that considers changes in both level and slope (Model C).[12] Table 2 shows the results obtained by conducting structural break tests on the time series of the U.S. TFP.

[12] Perron's (1997) methodology involves estimating the regressions for the three models for all possible break points, and selecting that point where the t-statistic of the null hypothesis of a unit root is the highest in absolute value.

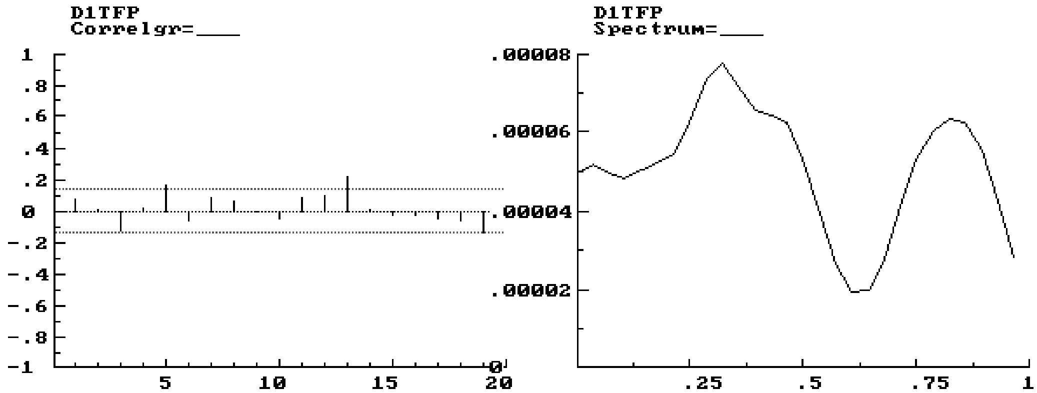
Table 2
Structural Break Tests: U.S. TFP (1948-2002)

Model	***Time Break***	***Statistic***	***Critical Values***		
			10%	***5%***	***1%***
Model A	1962	-4.232	-4.92	-5.23	-5.92
Model B	1970	-3.485	-4.44	-4.74	-5.41
Model C	1962	-4.011	-5.29	-5.59	-6.32

The table above shows those years in which the t-statistics of the null hypothesis of a unit root were found to be the highest in absolute value. For both models, the one that allows a change in level and the one that allows a change in level and slope, the suggested time break was at the early 1960s, while for the model with an exogenous change in slope the time break was at the beginning of the 1970s. The critical values were obtained from Perron's tables (1997) with a sample size selected according to the one that is closest to the size of the series under study. As can be seen from the table the tests fail to reject the null hypothesis of a unit root at 10% significant level for all the specifications. Consequently, these results seem to corroborate the absence of a deterministic linear trend in the time series of TFP in the U.S. economy.

In order to evaluate the possibility of the presence of a cyclical component in the U.S. TFP series some descriptive statistics such as the correlogram and the power spectrum can provide useful information. Figure 2 presents the estimates of these statistics for the series in first-differences (i.e. the U.S. TFP rate of growth).

Figure 2
U.S. Total Factor Productivity (First-Differences):
Correlogram and Power Spectrum



The correlogram shows small individual autocorrelations not providing strong evidence of the presence of cyclical movement in the series, although there seems to be some evidence of cyclical movement buried with noise. However, a much clearer message emerges from the examination of the power spectrum, which shows what appears to be a cycle with a period between 6 to 7 years, and the possibility of additional cyclical movements.[13]

Based on the information gathered by conducting unit root tests and the descriptive statistics employed to evaluate the presence of cyclical movements in the series, some likely specification for the trend and the cyclical components of a structural time series model for the data have been estimated.[14] Table 3 shows some basic diagnostic and goodness-of-fit statistics for these different structural time series models.

[13] On this graph the period is obtained as 2 divided by the frequency.
[14] Structural time series models were estimated using the econometric software Stamp 5.0.

Table 3
U.S. Total Factor Productivity
Structural Time Series Models
Diagnostics and Goodness-of-Fit Statistics

Model	Log-Lik.	P.E.V.	H(h)	Q(p,q)	RSQ	AIC	BIC
Random Walk with Drift	211.03	3.07E-4	0.323	9.874	0.063	4.26E-4	5.91E-4
Smooth Trend	213.09	2.84E-4	0.332	7.848	0.132	3.94E-4	5.48E-4
Local Linear Trend	213.11	2.78E-4	0.321	8.475	0.151	4.00E-4	5.76E-4

Q(p,q) is Box-Ljung statistics based on first p residual autocorrelations and 6 degrees of freedom. H(h) is a heteroskedasticity test with 17,17 degrees of freedom. An asterisk indicates a significant value at 5% level.

All these models assume the presence of a trend, two cycles and an irregular component. The table shows diagnostic and goodness-of-fit statistics for three structural time series models with different specifications for the trend or secular component of the series. The first statistical specification assumes that the trend component follows a random walk with drift, which is specified by employing equation (3.3) and a deterministic slope (i.e. $\sigma_{\varsigma}^2 = 0$). The second statistical specification for the long-run component is a variant of the local linear trend model, which introduces a somewhat smoother trend by employing equation (3.3) with a deterministic level (i.e. $\sigma_{\eta}^2 = 0$) and a stochastic slope. Finally, the last specification for the long-run component is the local linear trend model, which stipulates the level and the slope to be stochastic (i.e. equation 3.3).

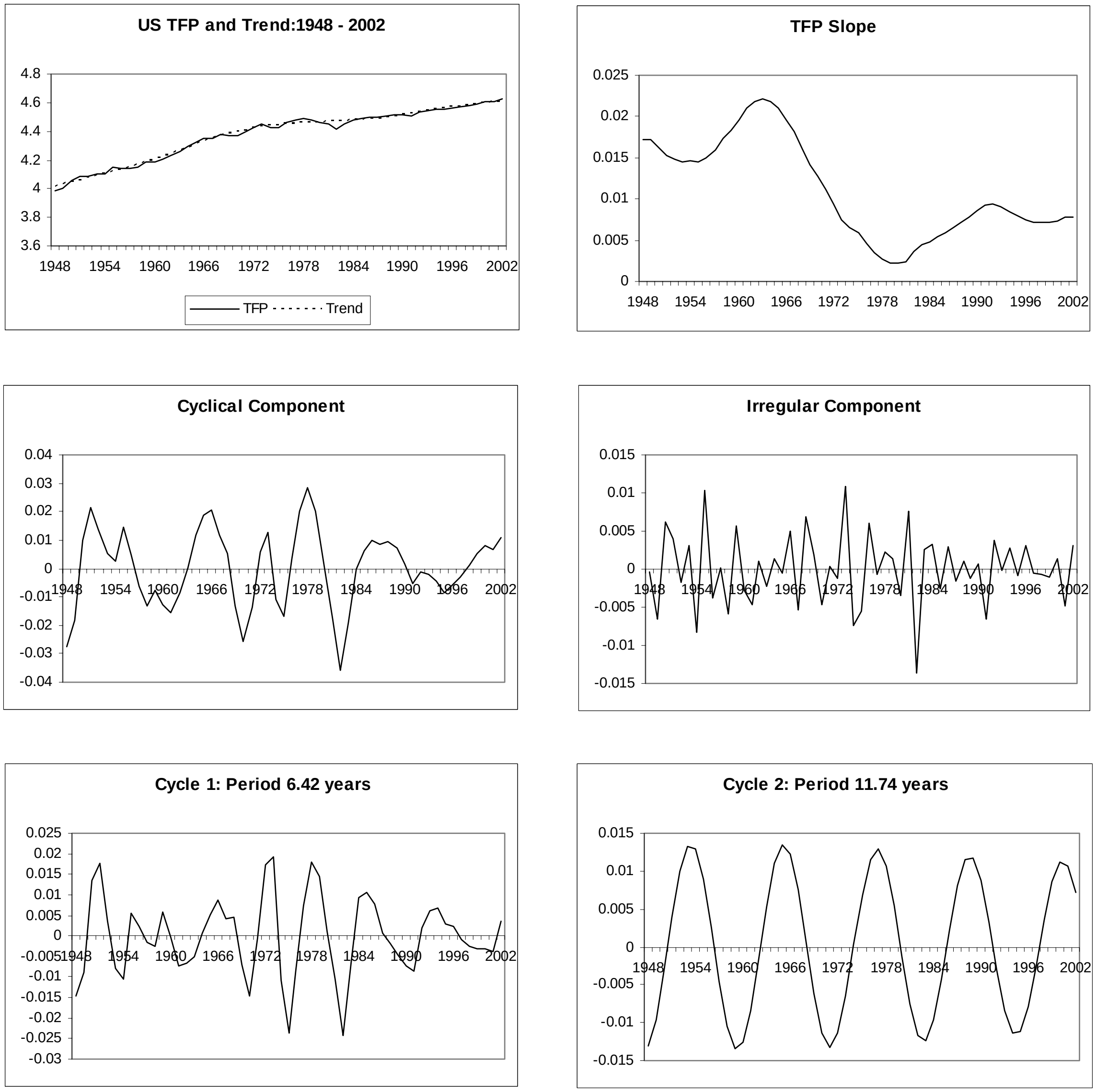
Diagnostic checking tests are conducted by computing the Box-Ljung Q(p,q) statistic for serial correlation, which is based on the first p residual autocorrelations and tested against a χ^2 distribution with q (i.e. p + 1 minus the number of estimated parameters) degree of freedom. A simple diagnostic test for heteroskedasticity H(h), which is the ratio of the squares of the last h residuals to the squares of the first h residuals, where h is set to the closer integer of T/3. This statistic is compared with the appropriate significant point of an F distribution with (h,h) degrees of freedom.

The Prediction Error Variance (P.E.V.), the coefficient of determination (R_D^2) and the information criteria (Akaike Information Criterion, AIC, and Bayesian Information Criterion, BIC) provide the goodness-of-fit statistics. The Prediction Error Variance is the variance of the one-step-ahead prediction errors in the steady state. These statistics have been employed to compute the information criteria, which are the appropriate statistics to compare models that have different numbers of parameters.[15] The coefficient of determination, R_D^2, is the statistic recommended by Harvey (1989, chapter 5), which enables the fit of the estimated model to be compared directly to a random walk with drift. For $0 < R_D^2 \leq 1$ the model is giving a better fit than the random walk with drift; for $R_D^2 = 0$ the fit is the same; while for $R_D^2 < 0$ the fit of the model is worse than the random walk with drift. Table 1 also presents information related to the Log-Likelihood.

The structural time series model that registers better goodness-of-fit based on both information criteria is the smooth trend plus cycle and irregular components. The diagnostic tests of this model indicate that the fit is fine. Figure 3 shows the different components of the structural time series model for the TFP series of the U.S. economy.

[15] The information criteria have been computed using the procedure suggested in Harvey (1989), pp.269-270.

Figure 3
U.S. TFP Unobserved Components 1948-2002



From the figure above it can be seen how the secular component of technological progress has evolved over the years. The estimates of this component suggest that technological progress slows down in the U.S. economy long before the oil price shocks of the 1970s. Technological progress seems to have reached a peak at the beginning of the 1960s when it starts to slow down until early 1980s to rebound then after. The estimated standard error of the disturbances driving the slope ($\hat{\sigma}_{\varsigma}$) is 0.0026. For the cyclical component the model suggests the presence of two cycles with frequencies

$\lambda_1 = 0.979$ (6.42 years period) and $\lambda_2 = 0.535$ (11.74 years period). The estimate of ρ for the first cycle is 0.810, while for the second cycle it is 0.998, which is very close to 1 indicating the presence of a deterministic cycle. The estimated standard errors of the disturbances driving these two cycles are 0.0068 and 0.0006, respectively. Finally, the irregular component seems to be the most volatile part of the model with an estimated standard deviation of 0.0075.

An important issue to address at this stage is to compare the results obtained in the study of the U.S. TFP series with those of the U.S. real output series. If business cycles are mainly driven by short-run fluctuations in the production function, as it is claimed by the Real Business Cycles approach, then we should expect close similarities between the cyclical movements shown by the TFP series with those shown by the real output series. Similarly, if the secular component of the TFP series drives economic growth, then it should be found that both the TFP series and the real output (per labour) series share a single common trend. In order to compute the correlogram and the spectrum of the real output series it is important to determine the main characteristic of the trend to conduct the proper de-trending procedure. In Table 4 the ADF-GLS$^{\tau}$ statistic and the KPSS(LM) statistic together with their associated 10%, 5% and 1% critical values for the U.S. real output series are presented.[16]

Table 4
ADF-GLS$^{\tau}$ and KPSS(LM) Tests: U.S. Real Output (1948-2002)

Statistic	*Actual*	*Critical Values*		
		10%	*5%*	*1%*
ADF-GLS$^{\tau}$	-2.8717	-2.8544	-3.1386	-3.7726
KPSS(LM)	0.0568	0.1674	0.3536	0.6278

In the table above it is possible to observe that the ADF-GLS$^{\tau}$ statistic is below the rejection area as it is not possible to reject the null hypothesis of the presence of a unit root in the real output series at 5% significant level. Additionally, the results obtained by conducting the KPSS(LM) test does not allow the rejection of the null hypothesis of a

[16] The real output series is the same employed by BLS in the computation of the U.S. TFP series.

trend stationary process either. Therefore, we should conclude that for the time series of real output in the U.S. economy the data does not contain sufficient information to discriminate between a difference-stationary process and a trend-stationary process.

As in the U.S. TFP series, structural change tests have been conducted on the U.S. real output series. Table 5 shows the results obtained from these tests.

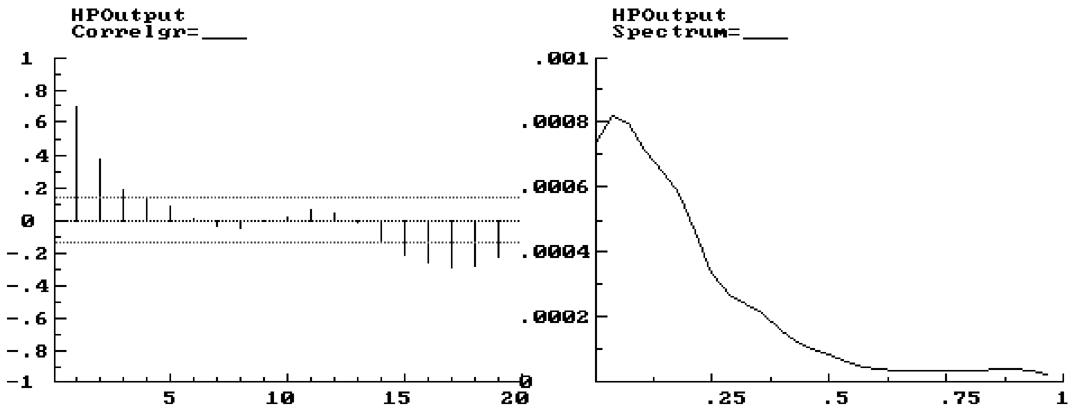
Table 5
Structural Break Tests: U.S. Real Output (1948-2002)

Model	***Time Break***	***Statistic***	***Critical Values***		
			10%	***5%***	***1%***
Model A	1962	-4.358	-4.92	-5.23	-5.92
Model B	1971	-3.554	-4.44	-4.74	-5.41
Model C	1962	-4.380	-5.29	-5.59	-6.32

Interestingly, the results shown by the table above indicate likely time breaks similar to those obtained in the examination of the U.S. TFP series. However, as in the case of the U.S. TFP series, the tests fail to reject the null hypothesis of a unit root at 10% significant level for all possible specifications.

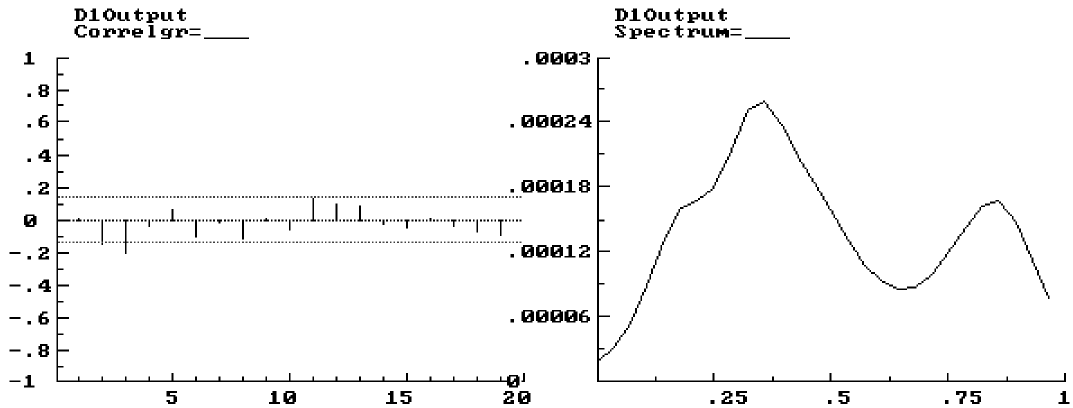
Based on the previous results it is necessary to establish an assumption in relation to the kind of process described by the trend of the series in order to render stationarity in the series and compute both the correlogram and the spectrum. In Figure 4 estimates of these descriptive statistics for the de-trended U.S. real output series under the assumption of a trend-stationary process are shown.

Figure 4
U.S. Real Output (Linear De-Trending):
Correlogram and Power Spectrum



The information provided by the correlogram shows clear cyclical movements in the stationary component of the series. The data generating mechanism seems to be that of a second order autoregressive process, AR(2), with complex roots. Nevertheless, the message given by the power spectrum suggests the presence of a cycle with a very long period (λ is close to cero), which is not in accordance with the evidence of cyclical fluctuations observed in the economy. By contrast, under the assumption of a difference-stationary process for the U.S. output series the results are more in accordance with the empirical evidence on business cycles. In Figure 5 the correlogram and the power spectrum for the U.S. real output growth are shown.

Figure 5
U.S. Real Output (First-Differences):
Correlogram and Power Spectrum



The figure above shows the correlogram and power spectrum for the first-differences of the U.S. real output (i.e. the growth rate of real output). Similarly to the case of the TFP series, the autocorrelations are small providing weak evidence of cyclical movement in the series. However, an examination of the spectrum indicates a clear cycle with a period between 5 to 6 years, and the possibility of an additional cycle of longer periodicity. It is interesting to notice the close similarity between the power spectrum of the first-differences of TFP and the one obtained for the real output series. Based on these results, it seems reasonable to disregard the presence of a deterministic linear trend in the U.S. real output series. Table 6 shows some basic diagnostic and goodness-of-fit statistics for suitable structural time series models for the U.S. real output series.

Table 6
U.S. Real Output
Structural Time Series Models
Diagnostics and Goodness-of-Fit Statistics

Model	Log-Lik.	P.E.V.	H(h)	Q(p,q)	RSQ	AIC	BIC
Random Walk with Drift	188.28	6.64E-4	0.117	6.356	0.256	9.21E-4	12.8E-4
Smooth Trend	186.51	6.91E-4	0.201	6.279	0.225	9.59E-4	13.3E-4
Local Linear Trend	188.28	6.64E-4	0.117	6.364	0.256	9.55E-4	13.8E-4

Q(p,q) is Box-Ljung statistics based on first p residual autocorrelations and 6 degrees of freedom. H(h) is a heteroskedasticity test with 17,17 degrees of freedom. An asterisk indicates a significant value at 5% level.

As in the case of the U.S. TFP series all these models assume the presence of a trend, two cycles and an irregular component. The structural time series model with the best goodness-of-fit based on both information criteria is the random walk with drift plus cycle and irregular components. The diagnostic tests indicate no problem with the fit of the model. Figure 6 displays the different components of the structural time series model for the real output series of the U.S. economy.

Figure 6
U.S. Real Output Unobserved Components (1948-2002)

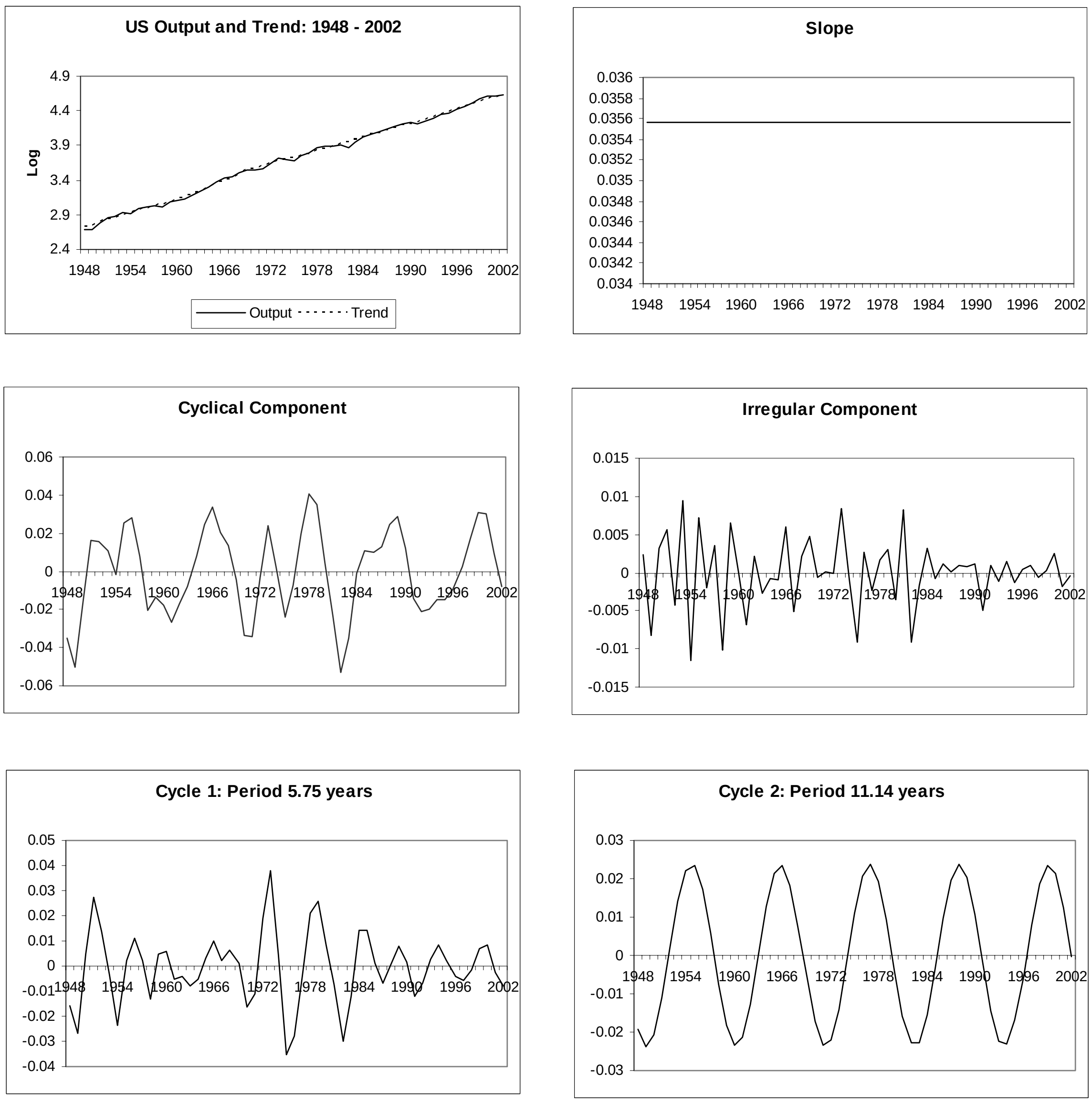


Figure 6 shows the significant differences that exist between the long-run components of the TFP series and the real output series of the U.S. economy. For the latter the trend is better described as a random walk with a drift of 0.036. The standard error of the disturbances of the level (σ_η) is 0.0149 making this component the most volatile part of the model. The cyclical component, on the other hand, shows strong similarities with those found for the U.S. TFP series. The model suggests the presence of two cycles with frequencies $\lambda_1 = 1.093$ (5.75 years period) and $\lambda_2 = 0.564$ (11.14 years period). The estimate of ρ for the first cycle is 0.817, while for the second cycle it is 1, indicating the presence of a deterministic cycle. The estimated standard deviation of the disturbances for the first cycle is 0.0103. The correlation between the cyclical component of the U.S. TFP and the cyclical component of the real output series is 0.86. Finally, the irregular component shows an estimated standard error of 0.0095.

In order to evaluate the existence of a single common trend between the time series of TFP and real output (per labour) for the U.S. economy, as suggested by the Neoclassical growth model, cointegration tests have been conducted.[17] The econometric investigation of this topic is based on the concept of cointegration introduced by Engle and Granger (1987). Its aim is to determine the number and shape of stationary linear combinations -named cointegrating relations- of time series which are themselves non-stationary. In order to conduct the cointegration tests the methodology developed by Johansen (1988, 1991, 1995) will be employed, which is based on maximum-likelihood estimation within a Gaussian vector autoregression. Table 5 shows the results of applying Johansen cointegration tests for the series under study.

Table 5
Johansen Cointegration Tests

Null	*Alternative*	*Statistic*	*95% C.V.*	*90% C.V*
λ_{trace}				
r = 0	r ≥ 1	21.03	25.77	23.08
r ≤ 1	r ≥ 2	5.03	12.39	10.55
λ_{max}				
r = 0	r = 1	16.00	19.22	17.18
r ≤ 1	r = 2	5.03	12.39	10.55

The eigenvalues in descending order are 0.26058 and 0.090453. Superscripts * indicates that the test statistic is significant at 10%.

[17] The BLS series Id number is MPU750021.

The unrestricted vector autoregressive (VAR) model for the variables in level was set with two lags as suggested by the BIC. The diagnostic tests for this model did not show problems of autocorrelation, heteroscedasticity or normality in the residuals. The specification given to the deterministic components of the model was that of unrestricted intercepts and restricted trend in the cointegration space. The results show that both statistics, the λ_{trace} and the λ_{max} statistic, fall in the non-rejection area of the null-hypothesis of no cointegration. Consequently, the results obtained do not provide evidence of the presence of a single common trend for the series of TFP and real output of the U.S. economy as it is suggested by economic theory. Although, it should be said that both statistics are relatively close to the 10% significant level suggesting the presence of one cointegrating relation.

5. Conclusions

In this work the presence of unobserved components in the time series of Total Factor Productivity is considered. This idea is central to modern Macroeconomics as the main approach in both the study of economic growth and the business cycle relies on certain features of the different components belonging to the time series of this variable. The econometric methodology employed in order to get the estimates of the different components of Total Factor Productivity is the structural time series approach developed by Harvey (1989) and Harvey and Shephard (1993) that build on early works such as Nervole, Grether and Carlvalho (1979).

In the examination of the 1948-2002 annually recorded U.S. Total Factor Productivity series computed by the Bureau of Labour Statistics the results indicate the presence of different unobserved components (i.e. trend, cycle and irregular component) as economic theory suggests. The secular component of the series seems to be better represented as a smooth trend, that is, a process given by a deterministic level and a stochastic slope. The estimates of this component suggest that technical progress in the U.S. economy reached a peak at the beginning of the 1960s when it started to decline until the early 1980s, to rebound afterward. This result contradicts the idea that technology in the U.S. economy slowed down in the 1970s as a result of the oil price shocks during this decade. Similarly, evidence supporting the view of the presence of a deterministic linear trend as it is sometimes assumed in the business cycles literature was strongly rejected. In relation to the cyclical component of the series, it seems to be best represented by two cycles with a period of 6.42 years and 11.74 years, respectively.

The results obtained in the analysis of the Total Factor Productivity series were compared with those obtained from a similar analysis of the U.S. real output series. Economic theory suggests that both the secular component of Total Factor Productivity and real output (per labour) should be the same. In addition, if shifts in the production function are the main driving forces generating short-run fluctuations in the economy, then the cyclical components of Total Factor Productivity and real output should share some of their main characteristics. The empirical results for the U.S. economy seem to suggest different secular components for the two series, as there is no evidence of the existence of cointegration between the series, although it should be mentioned that the actual statistics are relatively close to the 10% critical values. By contrast, the results were more in accordance with economic theory for the case related to short-run fluctuations. The cyclical component of the U.S. real output is better represented by two cyclical movements with periods 5.75 years and 11 years, respectively. Consequently, it has been found that the periodicity of the cyclical component of the two series is very similar one to another.

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