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**Seasonal Adjustment and  
Volatility Dynamics**

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# Seasonal Adjustment and Volatility Dynamics\*

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## Résumé / Abstract

Nous étudions l'effet de filtre sur l'estimation de processus de type GARCH. Le cas du filtre linéaire est analysé dans un contexte général pour des processus GARCH faibles. Plusieurs cas spéciaux sont discutés, notamment celui du filtre d'ajustement X-11 pour les effets saisonniers. Nous trouvons que ce filtre produit un effet de persistance saisonnière au niveau de la volatilité. Nous abordons ensuite le filtrage non linéaire dans le cas du filtre X-11. Une étude de Monte Carlo démontre qu'il y a des différences très importantes entre la représentation linéaire du filtre et le programme non linéaire appliqué aux données réelles.

*In this paper we try to enhance our understanding of the effect of filtering, particularly seasonal adjustment filtering, on the estimation of volatility models. We focus exclusively on ARCH models as a specific class of models and examine the effect of both linear and nonlinear filters on (seasonal) volatility dynamics. The case of linear filters is treated in a general abstract setting applicable to seasonal adjustment as well as various other linear filters often applied to transform raw data. Next we focus on specific cases like the first and seasonal differencing filters as well as the X-11 filter, both its linear representation and the (nonlinear) procedure implemented in practice. We uncover surprising features regarding the linear X-11 filter, e.g. it introduces a small seasonal pattern in volatility. More interestingly, we show that the linear X-11 and the actual procedure produce serious downward biases in ARCH effects and their persistence. Finally, we uncover important differences between the linear version of X-11 and the actual procedure.*

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# 1 Introduction

Many effects produced by seasonal adjustment filters are still not well understood. We have a fairly good grasp of what happens to *linear* time series models or *linear* regression models when the data are filtered with a *linear* filter. However, in the case of (1) nonlinear models, (2) nonlinear features of data, or when (3) nonlinear filtering is applied to the data, the implications are to a large extent still unknown and unexplored.

This paper tries to shed light on the effect of filtering and in particular seasonal adjustment filters on the volatility dynamics of time series. Obviously, standard procedures like X-11 are not designed to deal with time series exhibiting conditional (seasonal) heteroskedasticity, despite the fact that there is evidence that quite a few macroeconomic time series feature seasonality in the conditional variance (see for instance Burrige and Wallis (1990), Fiorentini and Maravall (1996), Jaditz (1996) and Racine (1997)). We assume first that the filter is linear and examine the effect of filtering on a regression model with GARCH errors. We study the impact of linear filtering on the volatility dynamics characterized by the autocovariance function of the squared residuals. The characterization of the linear filtering effects are restricted to the weak definition of GARCH (see Drost and Nijman (1993)). We provide explicit analytic results for general weak GARCH(p,q) processes. In the general case it is difficult to appraise the effect of filtering unless we make either some specific assumptions about the filter weights or about the process. We focus on cases of specific interest like first and seasonal differencing filters as well as linear seasonal adjustment filters, such as the linear approximation to the Census X-11. We uncover surprising features regarding the linear X-11 filter, namely it introduces a small seasonal pattern in volatility.

We also focus on the specific case of weak GARCH(1,1) processes. First analytic results are obtained for specific filters. Next, we conduct a Monte Carlo study involving GARCH(1,1) and seasonal GARCH processes where the linear X-11 filter and the actual X-11 procedure are examined side-by-side. We show that the linear X-11 and the actual procedure produce serious downward biases in ARCH effects and their persistence. The linear filter also differs from the actual program in terms of its effect on seasonal ARCH. The latter completely erases seasonal autocorrelations in volatility while the former doesn't. We attribute these differences to the outlier correction routine in the X-11 program. We also investigate the volatility dynamics of several key macroeconomic time series before and after filtering.

Section 2 covers the case of a general linear filter and the several spe-

cial cases such as the linear X-11 approximation. In section 3 we cover the case of GARCH(1,1) processes. The Monte Carlo study which investigates the nonlinear features of the X-11 procedure and its impact on volatility dynamics also appears in section 3 while in section 4 we present some empirical evidence based on a set of widely used macroeconomic time series. The paper concludes with section 5.

## 2 Linear filters and volatility

For the purpose of our presentation we consider a linear regression model with GARCH(p,q) residuals:

$$y_t = x_t b + \varepsilon_t \quad (2.1)$$

$$\varepsilon_t^2 = \omega + \sum_{j=1}^{\max(p,q)} (\alpha_j + \beta_j) \varepsilon_{t-j}^2 + \nu_t - \sum_{j=1}^q \beta_j \nu_{t-j} \quad (2.2)$$

Before proceeding some comments regarding (2.2) are in order. The effect of filtering raises issues quite similar to those encountered with temporal aggregation. Indeed, linear filtering and temporal aggregation both involve combinations of observations pertaining to different time periods. The class of ARCH processes as introduced by Engle (1982), and generalized by Bollerslev (1986), is not closed under temporal aggregation as noted by Drost and Nijman (1993). For temporal aggregation to work one has to weaken the original definition of the process. Therefore, the GARCH(p,q) model appearing in (2.2) is assumed to be a *weak* GARCH process as defined by Drost and Nijman. This implies that  $\sigma_t^2 = \mathbb{E}_{L_t}(\varepsilon_{t+1}^2)$  with  $\varepsilon_{t+1}^2 = \sigma_t^2 + \nu_{t+1}$  where  $\mathbb{E}_{L_t}(\cdot)$  is defined as the linear projection on the space spanned by  $\{1, (\varepsilon_{t-j}, \varepsilon_{t-j}^2) : j \geq 0\}$ .

Following Sims (1974) and Wallis (1974) we assume that the regressors  $x_t$  are strictly exogenous. Moreover, both  $y_t$  and  $x_t$  have nonseasonal (NS) and seasonal (S) components and so do the residuals, namely:

$$z_t = z_t^{NS} + z_t^S \quad z = x, y, \varepsilon \quad (2.3)$$

It is assumed that all the data (and hence residuals) are filtered by the *same* linear filter, i.e.

$$\hat{z}_t^{NS} = \theta(L) z_t = \sum_{k=-\infty}^{+\infty} \theta_k L^k z_t \quad z = x, y \quad (2.4)$$

where  $\hat{z}^S = z_t - \hat{z}_t^{NS}$ ,  $z = x, y$  and  $L^k z_t = z_{t-k}$ . Since a uniform filter is used across all data we do not expect any bias in the OLS estimator  $\hat{b}_{OLS}$  using  $\hat{y}_{NS}$  and  $\hat{x}_{NS}$  as Sims and Wallis showed in their seminal papers. To facilitate our presentation we will assume  $\hat{b}_{OLS} \equiv b$  and ignore all estimation uncertainty in order to focus on the properties of  $\varepsilon_t$ , in particular its volatility dynamics. Hence, we are interested in studying the properties of  $\varepsilon_t^F \equiv \theta(L)\varepsilon_t$  namely the filtered residual process. Let us first consider the autocovariance structure of the squared unfiltered series:

$$\gamma_2(j) = \mathbb{E}_L \varepsilon_t^2 \varepsilon_{t-j}^2 \quad (2.5)$$

where  $\mathbb{E}_L(\cdot)$  as noted before represents the linear unconditional projection associated with the  $L^2$  representation of the  $\{\varepsilon_t^2\}$  process. Its filtered counterpart can be written as:

$$\gamma_2^F(j) = \mathbb{E}_L (\varepsilon_t^F)^2 (\varepsilon_{t-j}^F)^2 = \mathbb{E}_L (\theta(L)\varepsilon_t)^2 (\theta(L)\varepsilon_{t-j})^2. \quad (2.6)$$

To proceed we formulate an assumption which is implied by the weak GARCH definition put forward by Drost and Nijman (1983). In particular:

*Assumption 2.1:* The GARCH process in (2.2) satisfies the conditions of a weak GARCH. It implies that  $\mathbb{E} \varepsilon_\tau^2 \varepsilon_{\tau'} \varepsilon_{\tau''} = \mathbb{E}_L \varepsilon_\tau^2 \varepsilon_{\tau'} \varepsilon_{\tau''} = 0$  for  $\tau \neq \tau' \neq \tau''$ .

It will also be convenient to introduce the following notation:

$$\theta_2(L) \equiv \sum_{k=-\infty}^{+\infty} \theta_k^2 L^k.$$

In a first subsection we will deal with the general linear filtering case without being specific about the particular features of the filter weights. In the next subsection we will treat some specific special cases.

## 2.1 The general case of linear filters

Using Assumption 2.1 we can rewrite (2.6) and obtain a first general result.

*Proposition 2.1:* Under Assumption 2.1 the autocovariance function of the squares of the filtered process  $\varepsilon_t^F \equiv \theta(L)\varepsilon_t$  satisfies:

$$\gamma_2^F(j) = \mathbb{E}_L(\theta_2(L)\varepsilon_t^2)(\theta_2(L)\varepsilon_{t-j}^2) + 4 \sum_{k=-\infty}^{+\infty} \sum_{i < k} \theta_i \theta_k \theta_{i+j} \theta_{k+j} \mathbb{E}_L \varepsilon_{t-i}^2 \varepsilon_{t-k}^2 \quad (2.7)$$

Proof: See Appendix.

It should be noted that the formula in (2.7) is difficult to appraise unless we make either some specific assumptions about the filter weights or else about the process. The easiest case is one where there are no GARCH features, i.e.  $\alpha_j$  and  $\beta_j$  are both zero. In this special case of homoskedastic errors one obtains:

$$\gamma_2^F(j) = \left[ \sum_{k=-\infty}^{+\infty} \theta_k^2 \theta_{k+j}^2 \right] \gamma_2(0) \quad (2.8)$$

which means that filtering a homoskedastic residual process with a general linear filter will yield, not surprisingly, ARCH-type effects determined by the squared filter weights, indeed from equation (2.8) we can also formulate the autocorrelation function as follows:

$$\rho_2^F(j) = \frac{\left[ \sum_{k=-\infty}^{+\infty} \theta_k^2 \theta_{k+j}^2 \right]}{\left[ \sum_{k=-\infty}^{+\infty} \theta_k^4 \right]} \quad (2.9)$$

where  $\rho_2^F(j)$  is the autocorrelation of the squared filtered residuals. One would like to use some specific values for the filter weights of course. This will be treated in the next subsection. In general one can say that the autocovariance structure of the squared residuals before and after (linear) filtering resembles somewhat that of linearly filtered ARMA models. Ghysels and Perron (1993) examined in detail the effect of linear filtering on the autocovariance structure of ARMA, ARIMA and seasonal unobserved component ARIMA models. One could transplant these results to the case of weak GARCH(p,q) models, provided two important modifications are made. The first is that the second term in (2.7) needs to be negligible, which is in some cases valid as will be discussed later. Second, unlike in the case of linear ARMA models we no longer need to investigate the actual filter weights but rather the squared weights. These have never before been the focus of attention of course, even for frequently used filters like the X-11 filter.



## 2.2 Some specific cases of linear filters

The case of a general linear filter applies to many different situations such as first and seasonal differencing filters, linear versions of the X-11 filter, to optimal linear signal extraction filters (see e.g. Pierce (1979), Bell (1984), Maravall (1988), among others) or to filtering procedures often encountered in empirical macro such as the Hodrick and Prescott (1997) and Baxter and King (1995) high-pass filters. In the remainder of this section we will focus our attention on some specific filters in order to derive theoretical results which are easier to interpret than equation (2.7). Two types of frequently encountered linear filters will be considered. The first class of filters are of the type  $\theta(L) \equiv (1 - L^S)$  where  $S$  can take any positive integer value, i.e. this class includes first differencing ( $S = 1$ ) as well as seasonal differencing filters ( $S > 1$ ). These cases cover situations where the regressors are (seasonally) differenced which would occur when the regression model in (2.1) involves nonstationary regressors with GARCH residuals and the data are filtered before estimating the volatility dynamics. The second class of filters is surely the most common and most interesting. It involves the linear version of the X-11 filter.<sup>1</sup> The first proposition covers the filters  $\theta(L) \equiv (1 - L^S)$ .

*Proposition 2.2:* Let  $\theta(L) \equiv (1 - L^S)$  then under Assumption 2.1 the autocovariance function of the squares of the filtered process satisfies:

$$\gamma_2^F(0) = 2\gamma_2(0) + 6\gamma_2(S) \quad (2.10)$$

while for  $j > 0$ :

$$\gamma_2^F(j) = 2\gamma_2(j) + \gamma_2(j+1) + \gamma_2(j-S) \quad (2.11)$$

Proof: See Appendix.

A fairly simple case of interest is again homoskedastic residuals, *i.e.* the unfiltered process features  $\gamma_2(j) = 0$ , for  $j > 0$ . In such a case  $\gamma_2^F(S) = \gamma_2(0) \neq 0$  and we can write the autocorrelation function as:  $\rho_2^F(j) = 1$  for  $j = 0, S$  and zero otherwise. Hence, first differencing introduces ARCH(1) effects in homoskedastic residuals while seasonal differencing produces seasonal ARCH. Next we turn to the linear X-11 filter. The filter is two-sided and involves over 80 leads and lags, which makes the derivation of explicit analytical results, such as in Proposition 2.2, much more difficult. Fortunately we can investigate the features of

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<sup>1</sup>The linear X-11 procedure is discussed in Young (1968), Wallis (1974), Bell (1992) and Ghysels and Perron (1993).

the linear X-11 filter via other means. First we should note that the second term in (2.7) becomes negligible as the product of adjacent X-11 filter weights is small, a useful feature of the filter. A consequence of this feature indeed is that we can simply focus on the features of the  $\theta_2(L)$  filter, *i.e.* a filter with the squared weights of the linear X-11 filter. Since we focus on the effect of filtering it is worthwhile to consider first a spectral domain approach.<sup>2</sup> In Figure 2.1 we plotted the squared gain (or transfer function) implied by the squared weights of the monthly linear X-11 filter. For the purpose of comparison we also plotted the linear X-11 filter transfer functions. Hence, Figure 2.1 shows the transfer functions of  $\theta(L)$  and  $\theta_2(L)$  in the case of the linear X-11 monthly filter.

[Insert Figure 2.1 somewhere here]

The transfer functions appearing in Figure 2.1 are quite revealing. The X-11 filter has the familiar pattern which retains the spectral power at all but the seasonal frequency and its harmonics. The filter with squared weights has very different properties. First, as we expect from a smoothing filter, we observe the variance reduction effect. Indeed, the transfer function takes values between roughly .4 and .62. Another feature to note is that the filter weights of the linear X-11 filter sum to one (a feature important for leaving constants and linear trends unaffected as stressed by Ghysels and Perron (1993) in the context of unit root testing). The sum of the squared weights is less than one, more specifically .7852, which yields a zero frequency squared gain of  $(.7852)^2$  or .6165, which is the value appearing at the zero frequency in Figure 2.1. The most remarkable feature, however, is that  $\theta_2(L)$  does *not* have troughs at the seasonal frequency and its harmonics. Instead, it actually has small peaks. Consequently, the X-11 filter, while reducing the overall variance of  $\varepsilon_t$  will in fact *slightly amplify instead of remove* seasonal correlation in the conditional variance dynamics.

### 3 The case of weak GARCH(1,1)

In the last subsection we devoted our attention to some specific filters without explicit assumptions regarding the unfiltered volatility structure.

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<sup>2</sup>Since we restrict ourselves to linear projections we can rely on the equivalent time domain and spectral domain representations provided that the innovation process has finite variance. In many empirical applications one assumes that  $\nu_t$  has a Student distribution. The estimated degrees of freedom are always larger than two, which justifies our assumption about the innovation variance.

We will now assume a specific autocorrelation structure and investigate the effect of linear filtering. The simplest case one can consider is the weak GARCH(1,1) model. Simplicity is not the only reason to elaborate on this particular case. Indeed, it is also appealing to digress on GARCH(1,1) processes because they appear as quite relevant in many empirical applications. Obviously, while the GARCH(1,1) model will figure prominently in our analysis we need also to examine, given the context of seasonality, GARCH models which exhibit seasonal features. This will be treated in the next section. From Bollerslev (1986) we know that we can rely on standard results of ARMA models (see e.g. Box and Jenkins (1970, p. 76) or Fuller (1996, p. 72)). Hence, for the weak GARCH(1,1) process we have that:

$$\gamma_2(0) = \frac{(1 + \beta^2 + 2(\alpha + \beta)\beta)}{(1 - (\alpha + \beta)^2)} \quad (3.1)$$

for lag zero and for  $j \neq 0$ :

$$\gamma_2(j) = \frac{(1 + \beta(\alpha + \beta))(\alpha + 2\beta)}{(1 - (\alpha + \beta)^2)} (\alpha + \beta)^{j-1} \quad (3.2)$$

whereas the autocorrelation function is:

$$\gamma_2(j) = \frac{(1 + \beta(\alpha + \beta))(\alpha + 2\beta)}{(1 + \beta^2 + 2(\alpha + \beta)\beta)} (\alpha + \beta)^{j-1} \quad (3.3)$$

We can take advantage of these specific autocovariances to obtain more explicit formulas which describe explicitly the effect of certain linear filters on the volatility dynamics. Proposition 3.1 states a first such result, namely:

*Proposition 3.1:* Let  $\theta(L) \equiv (1 - L^S)$ . Moreover, let us denote  $\lambda \equiv (\alpha + \beta)$  and  $\xi \equiv (\alpha + 2\beta)$ . Then under Assumption 2.1 the autocorrelation function of the squares of a filtered GARCH(1,1) process satisfies:

$$\rho_2^F(j) \equiv \frac{[2\lambda^S + \lambda^{S+1} + 1][1 + \beta^2 + \lambda\beta]}{2\lambda^S(1 + \beta^2 + 2\lambda\beta) + 6\lambda^{2S-1}(1 + \beta\lambda\xi)} \rho_2(j) \quad (3.4)$$

and the autocorrelation function is unbiased if the parameters  $\alpha$  and  $\beta$  solve the following equation:

$$\beta\xi\lambda^{2S} + \lambda^{2S-1} + (1 + 2\beta^2 + 2\alpha\beta)\lambda^{S+1} + [1 + \beta(2\alpha + 3\beta)] = 0. \quad (3.5)$$

Proof: See Appendix

It is interesting to note there will be a downward bias in the autocorrelation function after filtering if equation (3.5) holds with  $<$  instead of equality. Finally, replacing equality in equation (3.5) by  $>$  will describe parameter settings for the GARCH(1,1) which feature an upward bias in the autocorrelation function induced by filtering. Three special cases of (3.5) are most relevant in practical applications. They are the cases  $S = 1$ ,  $S = 4$  and  $S = 12$ . For these cases, equation (3.5) specialize respectively to:

$$f_1(\alpha, \beta) \equiv (1 + 4\beta^2 + 3\alpha\beta)(\alpha + \beta)^2 + (1 + 2\alpha + 3\beta)\beta + \alpha + 1 \quad (3.6)$$

$$f_4(\alpha, \beta) \equiv \beta(\alpha + 2\beta)(\alpha + \beta)^8 + (\alpha + \beta)^7 + (1 + 2\beta^2 + 2\alpha\beta)(\alpha + \beta)^5 + [1 + \beta(2\alpha + 3\beta)] \quad (3.7)$$

$$f_{12}(\alpha, \beta) \equiv \beta(\alpha + 2\beta)(\alpha + \beta)^{24} + (\alpha + \beta)^{23} + (1 + 2\beta^2 + 2\alpha\beta)(\alpha + \beta)^{13} + [1 + \beta(2\alpha + 3\beta)] \quad (3.8)$$

We relied on numerical computations to characterize those three equations. The three plots appearing in Figure 3.1 show the functions  $f_1(\alpha, \beta)$ ,  $f_4(\alpha, \beta)$  and  $f_{12}(\alpha, \beta)$  for the parameter range  $\alpha, \beta \in (-1, 1)$ . In all three cases the functions take positive values, meaning that filtering by  $(1 - L)$ ,  $(1 - L^4)$  and  $(1 - L^{12})$  will yield an *upward* bias in the autocorrelations of squared residuals generated by weak GARCH(1,1) processes.<sup>3</sup>

## 4 A simulation study

Having explored so far the effect of filtering analytically we turn now to simulations to address several issues which were difficult to handle via explicit solutions. To obtain the analytic results discussed in the previous two sections we had to make several simplifying assumptions.

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<sup>3</sup>The function  $f_{12}(\alpha, \beta)$  takes values equal to one for a large range of the parameter space as appears from Figure 3.1(c).

Indeed, we ignored all the potential nonlinearities of seasonal adjustment procedures and their impact on volatility dynamics. In this section we carry our analysis a step further in different directions. We investigate the actual X-11 program with all its potential sources of nonlinearities, as discussed in detail by Ghysels, Granger and Siklos (1996), and compare it with the linear filter results. Moreover, we consider in addition to GARCH(1,1) also seasonal GARCH processes. Finally, we also compare finite sample properties with the asymptotic ones. Unlike the approach taken in the previous sections we no longer rely on analytic methods but (have to) rely on Monte Carlo simulations. We will describe the design in a first subsection before reporting the findings.

#### 4.1 The design

The data samples we generate are drawn from two types of processes, the first is a GARCH(1,1) process, while the second is a seasonal GARCH which will be presented momentarily. The former is defined as:

$$y_t = \varepsilon_t \quad \text{with} \quad \varepsilon_t^2 = \omega + (\alpha + \beta) \varepsilon_{t-1}^2 + \nu_t + \beta \nu_{t-1} \quad (4.1)$$

It should be noted that we do *not* consider a weak GARCH, hence  $\sigma_t^2 = E_t(\varepsilon_{t+1}^2)$ , which means that it is based on a conditional expectation instead of a linear projection. The technical aspects of the simulation design are quite similar to those described in section 2.4.3 of Ghysels, Granger and Siklos (1996) (henceforth referred to GGS). We used the PROC X-11 procedure of SAS version 6.01. The number of replications was 1000, which is a larger number than in GGS yet small relative to the usual standards. As discussed in detail in that paper, using the actual X-11 procedure is computationally intensive and therefore forces one to consider a relatively small number of replications. Because the linear filter is two-sided it requires pre- and post- sample data. To generate such data points we took 10 years of monthly pre-sample and an equal number of post-sample points. Starting values are less of an issue here than in GGS since we do not model the mean but instead generate zero mean processes which are uncorrelated. We consider two sample sizes, one is called “small” and amounts to 10 years of monthly data, i.e. 120 observations, and the second is called “large”, or roughly 1000 observations. As in GGS we took 83 years or 996 observations to be more precise. Before describing the second data generating process and the parameter values for both processes let us also point out that we only consider the so-called additive version of the X-11 program (see GGS section 1.2 for a description of their differences). The additive version is

directly comparable to the linear filter version described in the previous section.

The second process is quite similar to that described in (3.1). Indeed, the only difference is that it involves a seasonal AR lag.<sup>4</sup> Namely, the second process is defined as:

$$y_t = \varepsilon_t \quad \text{with} \quad \varepsilon_t^2 = \omega + (\alpha + \beta) \varepsilon_{t-12}^2 + \nu_t + \beta \nu_{t-1} \quad (4.2)$$

We have to choose three parameters for both processes since (3.2) does not entail any additional parameters. We know that  $E\varepsilon_t^2 = \omega / (1 - (\alpha + \beta))$  for the process in (3.1). We set  $E\varepsilon_t^2 = 1$ , i.e. generate a zero mean uncorrelated process with unit variance. This setup allows us to eliminate  $\omega$  and substitute it by  $1 - (\alpha + \beta)$  for the GARCH(1,1) and the seasonal GARCH. Hence we are left with the choice of the parameters  $\alpha$  and  $\beta$ . We first take  $\alpha = \beta = 0$ , and ask if there is no heteroskedasticity of any kind to start with, does X-11 produce some? For the processes defined by (3.1) we have many empirical examples suggesting that  $\alpha + \beta \approx 1$ . In view of this evidence we also took  $\alpha + \beta = 1$ . with  $\alpha = .2$  and  $.1$  since the latter is usually estimated as roughly between  $.1$  and  $.2$ . The third and final specification for the GARCH(1,1) involves parameters  $\alpha = \beta = .4$ , and hence covers a non-unit root case. For the seasonal GARCH we have somewhat less empirical guidance, yet analogous to the GARCH(1,1) case we took the same parameter settings.

To conclude the description of the design we have to discuss what features of the simulated series will be retrieved and studied. In line with the results presented in section 2 we study the autocorrelation function (ACF) of the linear filter seasonally adjusted and the additive X-11 adjusted series. For the ACF's we computed twenty six lags.

## 4.2 Results

Tables A.1 through A.5, which appear in the Appendix, report the simulation results. To keep the number of tables to a reasonable minimum we do not report all the simulation results as certain patterns emerged which became repetitive. Table A.1 covers the case of a white process (i.e.  $\alpha = \beta = 0$ ). Each of the tables reporting simulation results have

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<sup>4</sup>Seasonality in ARCH can be obtained either through seasonal lags (i.e. p and/or q equal to the seasonal lag), through unobserved components, as in Fiorentini and Maravall (1996) or through periodic structures as studied by Bollerslev and Ghysels (1996). We focus here only on the former.

the same structure. The top panel of five rows covers the large sample results while the lower panel pertains to the small sample sizes. Each panel contains the ACF of the unfiltered squared simulated processes, the linearly filtered with their bias as well as the X-11 filtered cases and their bias. In the case of white noise we observe a slight *downward* bias in the ACF when the series are adjusted with the linear X-11 filter except at the seasonal lag where the autocorrelation attains a small positive value of 0.0569. The actual X-11 program also produces biases which are typically of comparable size but opposite sign at nonseasonal lags. At the seasonal lag the X-11 procedure produces a downward bias, namely -0.0917. The behavior in small samples is similar to that in large samples. In summary, from the white noise case reported in Table A.1 we learn that the linear filter introduces some small biases at nonseasonal lags and a more serious upward bias at the seasonal lag. The latter was expected since the transfer function plotted in Figure 2.1 showed small peaks at the seasonal frequency and its harmonics. The X-11 program also produces a considerable bias at the seasonal frequency, double in size compared to the linear filter, but here the bias is downward. Some differences between the linear X-11 and actual procedure start to appear, yet more significant ones will be revealed by the simulation results involving genuine ARCH processes. It should also be noted that the magnitude of the biases found so far are small.<sup>5</sup>

We turn our attention now to several classes of ARCH models, the first appearing in Table A.2 with  $\alpha = \beta = .4$  and GARCH(1,1) dynamics. We observe a very serious downward bias. For instance the first order autocorrelation of the volatility for the unfiltered series is roughly on the order of .769 while after filtering the raw series with the linear X-11 filter this autocorrelation drops dramatically to .334 and with the actual X-11 procedure even further to .209. Hence the smoothing effect of seasonal adjustment has a serious impact on the volatility dynamics in terms of persistence. What happens at the seasonal frequencies is the opposite of what happens elsewhere. Indeed, the linear version produces an upward bias in the ACF at lag 12, again due to the peak in the transfer function. According to the results in Table A.2 the unfiltered series volatility has .015 autocorrelation at lag 12 while it attains 0.066 with linearly filtered series. For the actual X-11 procedure the bias remains negative, however.

The most remarkable differences between the X-11 procedure and its linear counterpart are revealed when we examine seasonal volatility

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<sup>5</sup>If we think of  $\rho_2^F(j) \sim N(0, 1/\sqrt{n})$  under the null, we would not very often find significant correlations in the case of filtered white noise.

dynamics. The first such case appears in Table A.3 where a seasonal GARCH process is simulated with parameters  $\alpha = \beta = .4$ . First, it should be noted that we observe again a downward bias at the non-seasonal frequencies. With the linear filter the downward bias becomes more serious in small samples. The interesting results appear at the seasonal lags. In large samples as well as small ones the autocorrelations are roughly cut in size by 56 % (down from 0.6550 to 0.3284 in large samples for instance) with the linear X-11 filter. It should also parenthetically be noted that the biases reported at lag 12 also extend to the 24th lag in this as well as all the other cases we report. The actual X-11 procedure has a far more devastating impact as it completely erases the seasonal volatility dynamics. This result is quite typical as we found it to appear in all our simulations. An explanation for this phenomenon must be sought in the differences between the linear X-11 filter and the actual procedure. Ghysels, Granger and Siklos (1996) describe the different steps that are involved in the X-11 program. These steps remove trends, seasonal means and correct for outliers. The series which we simulated have neither a trend nor a seasonal in the mean. Hence, we can think of our simulations as generating an irregular component. At the end of the section we will elaborate further on this issue but first we will move on to the last case which involves IGARCH processes.

The results in Tables A.4 and A.5 report the unit root and seasonal unit root cases, i.e.  $\alpha + \beta = 1$ . We only report simulations based on setting  $\alpha = .1$ . We observe an enormous downward bias with the linear X-11 filter which tends to be even bigger in small samples and larger when the actual X-11 procedure is used. This is a very significant result as it shows that IGARCH is erased by seasonal adjustment filtering. The seasonal case reported in Table A.5 shows again the complete elimination of seasonal heteroskedasticity by the X-11 program. Since the case  $\alpha + \beta = 1$  is quite relevant from an empirical point of view we must conclude from these simulations that whenever seasonal adjustment is applied to (monthly) data we expect to find little evidence of GARCH and in particular persistence in volatility as well as seasonality in volatility left after filtering.

To conclude the section we will examine closer the reason why the actual X-11 differs so much from the linear filter. The X-11 program involves an outlier detection procedure applied to the irregular component, which in our case corresponds to the raw  $\varepsilon_t$  series. First, a moving sample window five-year standard deviation  $\hat{\sigma}_t$  of the  $\varepsilon_t$  series is computed. Implicitly, it is assumed that volatility is either constant or changes slowly to justify a five-year (i.e. 60 monthly observations) rolling



window estimate of the volatility. A first estimate is denoted  $\sigma_t^{(1)}$ . After eliminating observations with  $|\varepsilon_t| \geq 2.5 \sigma_t^{(1)}$  the standard error is recomputed, yielding  $\sigma_t^{(2)}$  which is based on a sample with a random number of observations less or equal 60. This second estimate is used to purge influential observations from  $\varepsilon_t$  and replace them by smoothed nearest neighbor estimates using a weighting scheme described by equations (1.3) and (1.4) in GGS (1996). This data replacement scheme intervenes except when  $0 \leq |\varepsilon_t| \leq 1.5 \sigma_t^{(2)}$ , which is an extremely tight margin. The 1.5 value can be changed by the X-11 program user. To verify whether it is indeed the outlier detection scheme which has such a devastating impact we ran our simulation setting extremely wide margins on the outlier detection scheme, i.e. it intervenes only when  $|\varepsilon_t| > 9.9 \sigma_t^{(2)}$  which is the maximum allowable in the X-11 program. For all practical purposes such a wide margin means that so-called outliers are rarely corrected. The results with the outlier corrections turned off were the same as those which were obtained with the linear filter. Clearly, the outlier detection scheme can have devastating effects when volatility is highly persistent, particularly when the persistence is seasonal. For nonseasonal GARCH dynamics the sixty-observations rolling window scheme producing  $\sigma_t^{(1)}$  is very much like historical volatility computation applied to (daily) financial time series. One way to check the effect of the outlier corrections is to simulate data and examine the frequency of interventions by the procedure. In Table A.6 we report Monte Carlo simulation distributions of X-11 outlier intervention frequencies for two types of processes, the white noise process and the IGARCH process (nonseasonal and seasonal). The simulation setup is exactly the same as in the previous tables, i.e. we consider two sample sizes, small and large. The simulations are based on 10000 replications and the entries to the table are percentiles of the percentage of the sample affected by the outlier procedure. There is one caveat we should note regarding the simulations. They are based only on  $\sigma_t^{(1)}$ , i.e. the standard error was not reestimated. This slight deviation from the actual procedure results in conservative estimates of the outlier intervention procedure since most often  $\sigma_t^{(2)} \leq \sigma_t^{(1)}$ . The results in Table A.6 show that an average of 5.12 percent or roughly fifty observations are affected in large samples (i.e. 996 observations) when the data are white noise Gaussian. The distribution is rather concentrated, as it ranges from 2.61 to 7.73 percent. In small samples containing 120 observations the distribution is more spread out but has the same mean. This first case shows that the outlier intervention is too invasive as it should not affect a Gaussian process. When we examine the nonseasonal IGARCH process we observe actually

a drop in the mean to 2.63 percent in large samples, yet the distribution is entirely shifted, often no corrections occur, but when the outlier procedure intervenes it does affect more data, up to 13.15 percent in large samples and even 43.33 percent in small samples which is roughly 50 out of 120 observations. The seasonal IGARCH case has a much higher mean in both large and small samples. The large sample distribution is the more shifted to the right compared to the two other large sample distributions, while the small sample distribution also attains high values, though not as extreme as the nonseasonal IGARCH case. From these results we must conclude that the X-11 outlier corrections scheme can seriously affect data and perhaps somewhat unintentionally seems to erase all seasonality in the conditional variance, something which the linear version of X-11 does not accomplish. More importantly, the outlier correction scheme reduces significantly the persistence in volatility. Last but not least we also need to observe that the linear filter is not a good approximation of what actually happens to seasonal volatility dynamics when data are passed through the X-11 program. Clearly, if one wants to remove seasonality both in mean and variance, the actual X-11 procedure does it remarkably well, but at some cost elsewhere in the analysis because the outlier correction procedure is responsible for the remarkable performance.

## 5 Empirical evidence

We initially considered a total of 28 monthly and quarterly time series covering a span of 20 or more years of data. A list is provided in Table A.7. In comparing actual time series with the simulation experiment conducted above we face several difficulties. First, for the sake of simplicity, it was assumed in the simulations that all series are additively seasonally adjusted when in practice this is not always the case (see GGS 1996). Second, we assumed that no mean regression model was fitted while in practice a model for the conditional mean is necessary. Obviously, any source of misspecification in the mean will affect the residuals and therefore potentially interfere with ARCH effects. Despite these considerations, the simulations suggest potentially large biases in the ACF's of the artificial series filtered in a number of ways and one would expect this phenomenon should be broadly replicated with actual time series. Space limitations prevent us from showing all the ACF's for the various combinations tested. Instead we report results for a set of key macroeconomic time series at the monthly frequency. They are as follows: the Consumer Price Index (CPI), the CPI excluding energy prices, the CPI

excluding food and energy prices, the adjusted Monetary Base, and the M1 measure of the money stock. The latter two series are additively adjusted since they themselves are aggregated from a variety of components (i.e. currency outside banks, reserves of the banking system, checkable deposits) and are known to display considerable seasonality. The CPI series are of obvious policy interest and food and energy prices are also known to have a seasonal component.

Testing proceeded as follows. In the first instance, we fit an AR(6) model to the first log difference of the officially seasonally adjusted series. In the case of unadjusted data, that is, the "unfiltered" series, two types of regression models were considered. First, a seasonal differencing filter was applied to the log levels of the series. A second approach was to estimate a first log difference AR specification involving twenty four lags and also projected on centered deterministic seasonal dummy variables. Both of these specifications are commonly used in applied work. We examined the ACF of the squared residuals, as we did in the simulations, and a GARCH(1,1) model was also fitted to all of the above specifications. The results of the ACF's appear in Table A.8. The ACF's of squared residuals of the regressions for M1 and the adjusted monetary base display features which resemble very much those of the simulation results. Indeed, we find that the persistence in volatility is greatly reduced after seasonal adjustment and the seasonal dependence in the autocorrelations was also erased. There are some differences between the two ACF's for the unadjusted series, i.e. the specification of the mean regression had an impact on the volatility dynamics of the residuals as expected but fortunately those differences did not alter the interpretation of the results. The situation is quite different with the price indices, however. Here the results are not so much in line with the simulation evidence and moreover they are also very much affected by the specification of the mean regression. For the CPI we find more persistence with the seasonally adjusted data if we look at the first lags of the ACF's. We still find that the seasonal autocorrelation has been eliminated, however, which is in line with the simulation results we obtained. Similar results were found for the other price indices, except that they show still a fair amount of seasonality in volatility after filtering with X-11.

In a final Table A.9 we report the parameter estimates for GARCH(1,1) models fitted to the residuals. For the sake of presentation we only report the case of seasonal differencing with NSA data. The monetary base data appear first in the table. When we use the sum of  $\alpha$  and  $\beta$  as a measure of persistence we observe that for the SA data we find roughly .34 while for the seasonal differencing specification we find .76. For M1

the difference in persistence is rather small, .92 (SA) versus .99 (NSA). This is also the case with the price index series considered.

## 6 Conclusions

This paper is a first towards understanding the effect of filtering on nonlinear time series models. The class of models we examined were GARCH-type processes. We explored a neglected dimension of the impact of seasonal adjustment filters such as X-11. While previous research has focused on the distortions, such as non-linearities, introduced by the application of seasonal filters we examine the impact of seasonal adjustment and other filters on the volatility dynamics. Our analysis reveals that filters such as X-11 (or linear X-11) introduce substantial biases in the volatility dynamics. Volatility is modelled via GARCH-type processes with allowances made for differing degrees of persistence. Focusing on the autocorrelation function of the squared residuals from various GARCH processes we find that X-11 tends to reduce, and in some cases, completely eliminates seasonal volatility dynamics and substantially reduce the overall persistence. We also found substantial differences between the linear X-11 and actual X-11 filter, showing the significant impact outlier corrections have in practice. Our results also showed that the linear X-11 filter in fact introduces a small seasonal dependence in volatility which appears most clearly in the case of white noise residuals. We also examined  $(1 - L^S)$  filters for  $S \geq 1$ . In the case of GARCH(1,1) we showed that such filters always introduce upward biases in the ACF of squared residuals. Despite the inherent problems that exist when one moves from the comfort of the simulations to the use of actual time series we were able to find similar types of biases uncovered in the Monte Carlo experiments. It is clear from our paper that much is still to be learned about the effects seasonal adjustment filters such as X-11 have on the (nonlinear) time series properties of data. The case of ARCH-type features is only a first small step on a very relevant and practical subject.

## References

- [1] Baxter, M. and R.G. King (1995), “Measuring Business Cycles: Approximate Band Pass Filters for Economic Time Series”, *American Economic Review*.
- [2] Bell, W.R. (1984), “Signal Extraction for Nonstationary Time Series”, *The Annals of Statistics*, 13, 646-664.
- [3] Bollerslev, T. (1986), “Generalized Autoregressive Conditional Heteroskedasticity”, *Journal of Econometrics* 31, 307-327.
- [4] Bollerslev, T, R.F. Engle and D. Nelson (1994), “ARCH Models”, in R.F. Engle and D.L. McFadden (ed.), *Handbook of Econometrics*, Vol. IV, (North-Holland, Amsterdam).
- [5] Bollerslev, T. and E. Ghysels (1996), “Periodic Autoregressive Conditional Heteroskedasticity”, *Journal of Business and Economic Statistics* 14, 139-152.
- [6] Box, G.E.P. and G.M. Jenkins (1970), *Time Series Analysis, Forecasting and Control*, (Holden-Day, San Francisco).
- [7] Burridge, P. and K.J. Wallis (1990), “Seasonal Adjustment and Kalman Filtering: Extension to Periodic Variances”, *Journal of Forecasting* 9, 109-118.
- [8] Drost, F. and T.E. Nijman (1993), “Temporal Aggregation of GARCH Processes”, *Econometrica* 60, 909-927.
- [9] Engle, R.F. (1982), “Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation” *Econometrica* 50, 987-1008.
- [10] Fiorentini, G. and A. Maravall (1996), “Unobserved Components in ARCH Models”, *Journal of Forecasting* 15, 175-201.
- [11] Fuller, W. (1996), *Introduction to Statistical Time Series*, 2nd Ed. (John Wiley, New York).
- [12] Ghysels, E. C.W.J. Granger and P.L. Siklos (1996), “Is Seasonal Adjustment a Linear or Nonlinear Data-Filtering Process? *Journal of Business and Economics Statistics* 14, 374-386 (with Discussion).

- [13] Ghysels, E. and P. Perron (1983), “The Effect of Seasonal Adjustment Filters on Tests for a Unit Root”, *Journal of Econometrics* 55, 57-98.
- [14] Hodrick, R.J. and E.C. Prescott (1997), “Postwar U.S. Business Cycles: An Empirical Investigation”, *Journal of Money, Credit and Banking* 29, 1-16.
- [15] Jaditz, T. (1996), “Seasonality in Variances is Common in Macro Time Series”, Discussion Paper U.S. Bureau of Labor Statistics.
- [16] Maravall, A. (1988), “A Note on Minimum Mean Squared Error Estimation of Signals With Unit Roots”, *Journal of Economic Dynamics and Control*, 12, 589-593.
- [17] Pierce, D.A. (1979), “Signal Extraction Error in Nonstationary Time Series”, *The Annals of Statistics*, 7, 1303-1320.
- [18] Racine, M.D. (1997), “Modelling Seasonality in U.S. Stock Market Returns”, Discussion Paper, Wilfrid Laurier University.
- [19] Sims, C.A. (1974), “Seasonality in Regression”, *Journal of the American Statistical Association* 69, 618-626.
- [20] Wallis, K.F. (1974), “Seasonal Adjustment and Relations between Variables”, *Journal of the American Statistical Association* 69, 618-32

## Appendix

Proof of Proposition 2.1: From the definition of the filtered autocovariance function we have that:

$$\begin{aligned}
\gamma_2^F(j) &= \mathbf{E}_L \left( \sum_{k=-\infty}^{+\infty} \theta_k \varepsilon_{t-k} \right)^2 \left( \sum_{k=-\infty}^{+\infty} \theta_k \varepsilon_{t-k-j} \right)^2 & (A.1) \\
&= \mathbf{E}_L \left( \sum_{k=-\infty}^{+\infty} \theta_k^2 \varepsilon_{t-k}^2 + 2 \sum_{k=-\infty}^{+\infty} \sum_{i < k} \theta_k \theta_i \varepsilon_{t-k} \varepsilon_{t-i} \right) \\
&\quad + \left( \sum_{k=-\infty}^{+\infty} \theta_k^2 \varepsilon_{t-k-j}^2 + 2 \sum_{k=-\infty}^{+\infty} \sum_{i < k} \theta_i \theta_k \varepsilon_{t-k} \varepsilon_{t-i-j} \right) \\
&= \mathbf{E}_L \left( \sum_{k=-\infty}^{+\infty} \theta_k^2 \varepsilon_{t-k}^2 \right) \left( \sum_{k=-\infty}^{+\infty} \theta_k^2 \varepsilon_{t-k-j}^2 \right) \\
&\quad + 4 \mathbf{E}_L \left( \sum_{k=-\infty}^{+\infty} \sum_{i < k} \theta_k \theta_i \varepsilon_{t-k} \varepsilon_{t-i} \right) \\
&\quad + \left( \sum_{k=-\infty}^{+\infty} \sum_{i < k} \theta_k \theta_i \varepsilon_{t-k-j} \varepsilon_{t-i-j} \right)
\end{aligned}$$

where the latter expression follows from the weak GARCH Assumption 2.1. Moreover, using the same assumption we can show that the second term in (A.1) specializes to that appearing in (2.7).

Proof of Proposition 2.2: In the special case of  $\theta_L \equiv (1 - L^S)$  we have that

$$\begin{aligned}
\mathbf{E}_L \left( (1 - L^S) \varepsilon_t \right)^2 \left( (1 - L^S) \varepsilon_{t-j} \right)^2 &= \mathbf{E}_L \left( \varepsilon_t^2 + \varepsilon_{t-S}^2 - 2\varepsilon_t \varepsilon_{t-S} \right) & (A.2) \\
\left( \varepsilon_{t-j}^2 + \varepsilon_{t-j-S}^2 - 2\varepsilon_{t-j} \varepsilon_{t-j-S} \right) &= \mathbf{E}_L \varepsilon_t^2 \varepsilon_{t-j}^2 + \mathbf{E}_L \varepsilon_t^2 \varepsilon_{t-j-S}^2 \\
&\quad - 2\mathbf{E}_L \varepsilon_t^2 \varepsilon_{t-j} \varepsilon_{t-S} + \mathbf{E}_L \varepsilon_{t-S}^2 \varepsilon_{t-j}^2 \\
&\quad + \mathbf{E}_L \varepsilon_{t-S}^2 \varepsilon_{t-j-S}^2 - 2\mathbf{E}_L \varepsilon_{t-S}^2 \varepsilon_{t-j} \varepsilon_{t-j-S} \\
&\quad - 2\mathbf{E}_L \varepsilon_t \varepsilon_{t-S} \varepsilon_{t-j}^2 - 2\mathbf{E}_L \varepsilon_t \varepsilon_{t-S} \varepsilon_{t-j-S}^2 \\
&\quad + 4\mathbf{E}_L \varepsilon_t \varepsilon_{t-S} \varepsilon_{t-j} \varepsilon_{t-j-S}
\end{aligned}$$

Equation (A.2) then yields for  $j=0$ , the autocovariance appearing in (2.10), provided Assumption 2.1 holds. With  $j>0$  we also obtain (2.11) under the same Assumption.

Proof of Proposition 3.1: From Proposition 2.2 we know that:

$$\gamma_2^F(j) = 2\gamma_2(j) + \gamma_2(j+1) + \gamma_2(j-S)$$

Using the GARCH(1,1) formula (3.1) through (3.3) we have

$$\gamma_2^F(0) = 2\gamma_2(0) + 6(\alpha + \beta)^{S-1} \frac{[(1 + \beta(\alpha + \beta)(\alpha + 2\beta))]}{[1 + \beta^2 + 2(\alpha + \beta)\beta]} \gamma_2(0)$$

$$\gamma_2^F(j) = [2 + (\alpha + \beta) + (\alpha + \beta)^{-S}] \gamma_2(j) \quad \text{for } j \neq 0$$

Therefore

$$\begin{aligned} \rho_2^F(j) &= \frac{[2 + (\alpha + \beta) + (\alpha + \beta)^{-S}] [(1 + \beta^2 + 2(\alpha + \beta)\beta)]}{\{2[1 + \beta^2 + 2(\alpha + \beta)\beta] + 6(\alpha + \beta)^{S-1}[1 + \beta(\alpha + \beta)(\alpha + 2\beta)]\}} \rho_2(j) \\ &= \frac{(\alpha + \beta)^{-S} [2(\alpha + \beta)^S + (\alpha + \beta)^{S+1} + 1] [1 + \beta^2 + 2(\alpha + \beta)\beta]}{2[1 + \beta^2 + 2(\alpha + \beta)\beta] + 6(\alpha + \beta)^{S-1}[1 + \beta(\alpha + \beta)(\alpha + 2\beta)]} \rho_2(j) \\ &= \frac{[2(\alpha + \beta)^S + (\alpha + \beta)^{S+1} + 1] [1 + \beta^2 + 2(\alpha + \beta)\beta]}{2(\alpha + \beta)^S [1 + \beta^2 + 2(\alpha + \beta)\beta] + 6(\alpha + \beta)^{2S-1} [(1 + \beta(\alpha + \beta)(\alpha + 2\beta))]} \rho_2(j) \end{aligned}$$

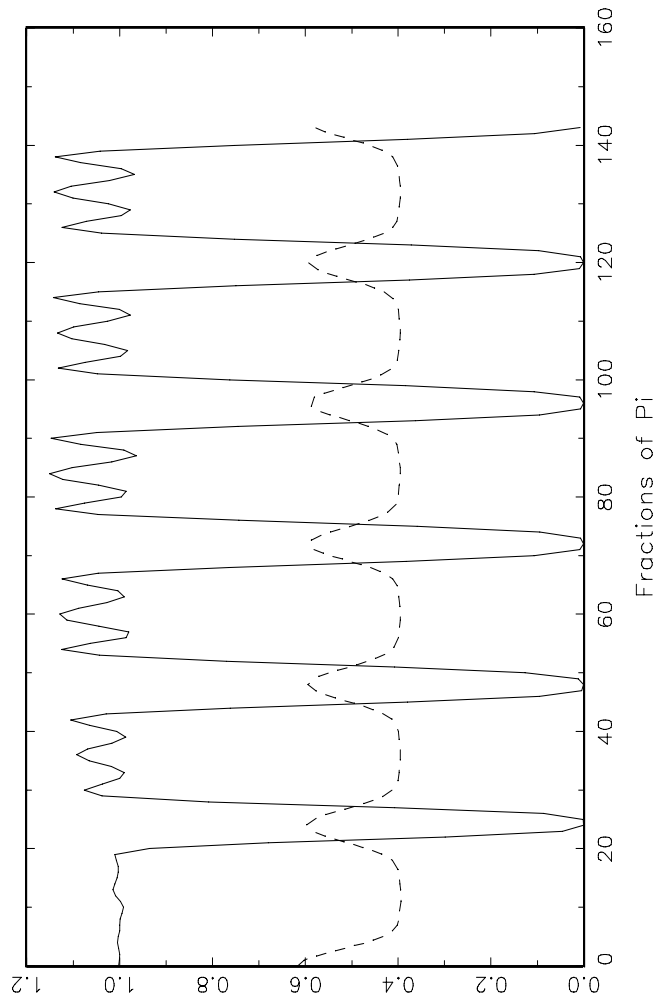
Substituting  $\lambda = (\alpha + \beta)$  and  $\xi = (\alpha + 2\beta)$  yields (3.4). The bias in the autocorrelation is zero when

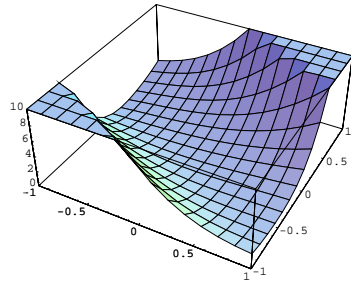
$$(2\lambda^S + \lambda^{S+1} + 1) (1 + \beta^2 + \lambda\beta) = 2\lambda^S (1 + \beta^2 + 2\lambda\beta) + 6\lambda^{2S-1} (1 + \beta\lambda\xi)$$

Algebraic simplification yields equation (3.5).

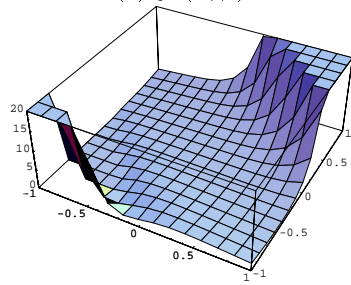


Figure 2.1: Transfer functions linear X-11 and Squared Filter

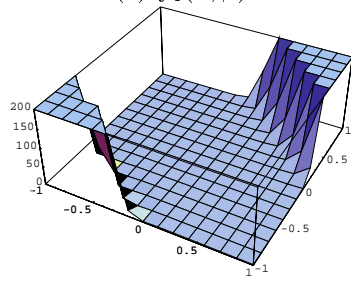




(a)  $f_1(\alpha, \beta)$



(b)  $f_4(\alpha, \beta)$



(c)  $f_{12}(\alpha, \beta)$

Figure 3.1 : Plots of functions  $f_i(\alpha, \beta)$   $i = 1, 4, 12$

**Table A.1: Biases in Volatility Autocorrelation Functions: GARCH(1,1) Model with  $\alpha = \beta = 0$**

| Lags                 |       | 1       | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       | 10      |
|----------------------|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Large Sample Results |       |         |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  |
|                      | Fil   | -0.0010 | -0.0009 | -0.0008 | -0.0007 | -0.0006 | -0.0005 | -0.0005 | -0.0005 | -0.0006 | -0.0009 |
|                      | Bias  | 0.0010  | 0.0009  | 0.0008  | 0.0007  | 0.0006  | 0.0005  | 0.0005  | 0.0005  | 0.0006  | 0.0009  |
| X-11                 | Fil   | 0.0150  | 0.0159  | 0.0129  | 0.0127  | 0.0107  | 0.0005  | -0.0049 | -0.0068 | -0.0060 | -0.0073 |
|                      | Bias  | -0.0150 | -0.0159 | -0.0129 | -0.0127 | -0.0107 | -0.0005 | 0.0049  | 0.0068  | 0.0060  | 0.0073  |
| Small Sample Results |       |         |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | -0.0084 | -0.0072 | -0.0086 | -0.0076 | -0.0081 | -0.0062 | -0.0084 | -0.0076 | -0.0069 | -0.0082 |
|                      | Fil   | 0.0104  | 0.0096  | 0.0091  | 0.0094  | 0.0083  | 0.0082  | 0.0088  | 0.0073  | 0.0074  | 0.0086  |
|                      | Bias  | -0.0188 | -0.0170 | -0.0177 | -0.0170 | -0.0164 | -0.0144 | -0.0172 | -0.0151 | -0.0143 | -0.0169 |
| X-11                 | Fil   | 0.0089  | 0.0086  | 0.0098  | 0.0105  | 0.0047  | -0.0012 | -0.0066 | -0.0107 | -0.0108 | -0.0048 |
|                      | Bias  | -0.0173 | -0.0158 | -0.0184 | -0.0181 | -0.0128 | -0.0050 | -0.0018 | -0.0029 | 0.0039  | -0.0034 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.1 (cont'd)**

| Lags                 |       | 11      | 12      | 13      | 14      | 22      | 23      | 24      | 25      | 26      |
|----------------------|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Large Sample Results |       |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  | 0.0000  |
|                      | Fil   | -0.0011 | 0.0569  | -0.0003 | 0.0000  | -0.0014 | -0.0015 | 0.0052  | -0.0011 | -0.0010 |
|                      | Bias  | 0.0011  | -0.0569 | -0.0003 | 0.0000  | 0.0014  | 0.0015  | 0.0052  | 0.0011  | 0.0010  |
| X-11                 | Fil   | -0.0070 | -0.0917 | -0.0071 | -0.0090 | -0.0088 | -0.0071 | -0.0660 | -0.0070 | -0.0091 |
|                      | Bias  | 0.0070  | 0.0917  | 0.0071  | 0.0090  | 0.0088  | 0.0071  | 0.0660  | 0.0070  | 0.0091  |
| Small Sample Results |       |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | -0.0080 | -0.0084 | -0.0092 | -0.0078 | -0.0066 | -0.0072 | -0.0082 | -0.0064 | -0.0052 |
|                      | Fil   | 0.0096  | 0.0464  | -0.0079 | -0.0087 | -0.0085 | -0.0079 | -0.0024 | -0.0071 | -0.0060 |
|                      | Bias  | -0.0176 | -0.0548 | -0.0013 | 0.0009  | 0.0019  | 0.0005  | -0.0058 | 0.0007  | 0.0008  |
| X-11                 | Fil   | -0.0102 | -0.0904 | -0.0102 | -0.0132 | -0.0099 | -0.0047 | -0.0633 | -0.0069 | -0.0041 |
|                      | Bias  | 0.0022  | 0.0820  | 0.0010  | 0.0054  | 0.0033  | 0.0025  | 0.0551  | 0.0005  | -0.0011 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.2: Biases in Volatility Autocorrelation Functions: GARCH(1,1) Model with  $\alpha = \beta = .4$**

| Lags                 |       | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8       | 9       | 10      |
|----------------------|-------|--------|--------|--------|--------|--------|--------|--------|---------|---------|---------|
| Large Sample Results |       |        |        |        |        |        |        |        |         |         |         |
| Lin                  | Unfil | 0.7692 | 0.5104 | 0.3474 | 0.2405 | 0.1686 | 0.1194 | 0.0852 | 0.0609  | 0.0435  | 0.0309  |
|                      | Fil   | 0.3339 | 0.2227 | 0.1529 | 0.1073 | 0.0777 | 0.0574 | 0.0440 | 0.0373  | 0.0342  | 0.0356  |
|                      | Bias  | 0.4532 | 0.2877 | 0.1945 | 0.1333 | 0.0909 | 0.0620 | 0.0412 | 0.0236  | 0.0093  | -0.0047 |
| X-11                 | Fil   | 0.2093 | 0.1488 | 0.1053 | 0.0810 | 0.0603 | 0.0340 | 0.0213 | 0.0149  | 0.0122  | 0.0123  |
|                      | Bias  | 0.5599 | 0.3616 | 0.2421 | 0.1595 | 0.1083 | 0.0854 | 0.0639 | 0.0460  | 0.0313  | 0.0186  |
| Small Sample Results |       |        |        |        |        |        |        |        |         |         |         |
| Lin                  | Unfil | 0.7342 | 0.4495 | 0.2784 | 0.1715 | 0.1036 | 0.0590 | 0.0292 | 0.0094  | -0.0046 | -0.0149 |
|                      | Fil   | 0.3022 | 0.1830 | 0.1114 | 0.0682 | 0.0398 | 0.0221 | 0.0097 | 0.0045  | 0.0014  | 0.0029  |
|                      | Bias  | 0.4320 | 0.2665 | 0.1670 | 0.1033 | 0.0638 | 0.0369 | 0.0195 | 0.0049  | -0.0060 | -0.0178 |
| X-11                 | Fil   | 0.1912 | 0.1304 | 0.0852 | 0.0639 | 0.0394 | 0.0156 | 0.0037 | -0.0034 | -0.0018 | -0.0045 |
|                      | Bias  | 0.5430 | 0.3191 | 0.1932 | 0.1076 | 0.0762 | 0.0434 | 0.0255 | 0.0128  | -0.0028 | -0.0104 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.2 (cont'd)**

| Lags                 |       | 11      | 12      | 13      | 14      | 22      | 23      | 24      | 25      | 26      |
|----------------------|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Large Sample Results |       |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | 0.0217  | 0.0150  | 0.0100  | 0.0062  | -0.0048 | -0.0051 | -0.0050 | -0.0047 | -0.0044 |
|                      | Fil   | 0.0422  | 0.0661  | 0.0370  | 0.0252  | 0.0089  | 0.0137  | 0.0060  | 0.0134  | 0.0079  |
|                      | Bias  | -0.0205 | -0.0510 | -0.0270 | -0.0189 | -0.0136 | -0.0188 | -0.0110 | -0.0180 | -0.0124 |
| X-11                 | Fil   | 0.0162  | -0.0798 | 0.2090  | 0.1474  | 0.0113  | 0.0160  | -0.0803 | -0.0512 | -0.0333 |
|                      | Bias  | 0.0055  | 0.0948  | -0.1990 | -0.1412 | -0.0161 | -0.0211 | 0.0753  | 0.0665  | 0.0289  |
| Small Sample Results |       |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | -0.0217 | -0.0271 | -0.0312 | -0.0337 | -0.0357 | -0.0358 | -0.0360 | -0.0365 | -0.0370 |
|                      | Fil   | 0.0097  | 0.0349  | 0.0049  | -0.0064 | -0.0156 | -0.0128 | -0.0162 | -0.0135 | -0.0178 |
|                      | Bias  | -0.0314 | -0.0620 | -0.0361 | -0.0274 | -0.0201 | -0.0230 | -0.0198 | -0.0230 | -0.0192 |
| X-11                 | Fil   | 0.0001  | -0.0643 | 0.1872  | 0.1286  | -0.0047 | -0.0010 | -0.0845 | -0.0021 | -0.0033 |
|                      | Bias  | -0.0218 | 0.0616  | -0.2184 | -0.1623 | -0.0310 | -0.0348 | 0.0485  | -0.0344 | -0.0337 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.3: Biases in Volatility Autocorrelation Functions: Seasonal GARCH(1,1) Model with  $\alpha = \beta = .4$**

| Lags                 |       | 1      | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       | 10      |
|----------------------|-------|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Large Sample Results |       |        |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | 0.1830 | -0.0089 | -0.0096 | -0.0083 | -0.0087 | -0.0097 | -0.0086 | -0.0083 | -0.0094 | -0.0085 |
|                      | Fil   | 0.0768 | -0.0039 | -0.0036 | -0.0038 | -0.0037 | -0.0041 | -0.0045 | -0.0039 | -0.0036 | -0.0032 |
|                      | Bias  | 0.1062 | -0.0050 | -0.0060 | -0.0045 | -0.0050 | -0.0056 | -0.0041 | -0.0044 | -0.0058 | -0.0053 |
| X-11                 | Fil   | 0.0448 | 0.0058  | 0.0043  | 0.0064  | 0.0023  | -0.0056 | -0.0073 | -0.0068 | -0.0065 | -0.0079 |
|                      | Bias  | 0.1382 | -0.0145 | -0.0139 | -0.0147 | -0.0110 | -0.0041 | -0.0013 | -0.0015 | -0.0029 | -0.0006 |
| Small Sample Results |       |        |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | 0.1359 | -0.0500 | -0.0458 | -0.0460 | -0.0452 | -0.0459 | -0.0441 | -0.0454 | -0.0451 | -0.0467 |
|                      | Fil   | 0.0489 | -0.0278 | -0.0252 | -0.0267 | -0.0247 | -0.0256 | -0.0247 | -0.0254 | -0.0255 | -0.0250 |
|                      | Bias  | 0.0870 | -0.0222 | -0.0206 | -0.0193 | -0.0205 | -0.0203 | -0.0194 | -0.0196 | -0.0196 | -0.0217 |
| X-11                 | Fil   | 0.0317 | -0.0052 | -0.0070 | 0.0002  | -0.0074 | -0.0150 | -0.0144 | -0.0194 | -0.0126 | -0.0184 |
|                      | Bias  | 0.1042 | -0.0448 | -0.0388 | -0.0452 | -0.0478 | -0.0309 | -0.0297 | -0.0260 | 0.0325  | -0.0283 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.3 (cont'd)**

| Lags                 |       | 11     | 12     | 13     | 14      | 22      | 23     | 24     | 25     | 26      |
|----------------------|-------|--------|--------|--------|---------|---------|--------|--------|--------|---------|
| Large Sample Results |       |        |        |        |         |         |        |        |        |         |
| Lin                  | Unfil | 0.1325 | 0.6550 | 0.1320 | -0.0087 | -0.0081 | 0.0963 | 0.4480 | 0.0960 | -0.0840 |
|                      | Fil   | 0.0781 | 0.3284 | 0.0600 | -0.0035 | -0.0035 | 0.0449 | 0.1981 | 0.0452 | -0.0037 |
|                      | Bias  | 0.0544 | 0.3266 | 0.0720 | -0.0087 | -0.0046 | 0.0514 | 0.2499 | 0.0508 | -0.0803 |
| X-11                 | Fil   | 0.0245 | 0.0194 | 0.0449 | 0.0052  | -0.0074 | 0.0242 | 0.0216 | 0.0189 | 0.0101  |
|                      | Bias  | 0.1080 | 0.6356 | 0.0871 | -0.0139 | -0.0070 | 0.0721 | 0.4624 | 0.0771 | -0.0941 |
| Small Sample Results |       |        |        |        |         |         |        |        |        |         |
| Lin                  | Unfil | 0.0762 | 0.5613 | 0.0723 | -0.0467 | -0.0419 | 0.0386 | 0.3229 | 0.0342 | -0.0423 |
|                      | Fil   | 0.0411 | 0.2826 | 0.0276 | -0.0247 | -0.0233 | 0.0139 | 0.1397 | 0.0114 | -0.0235 |
|                      | Bias  | 0.0351 | 0.2787 | 0.0447 | -0.0220 | -0.0186 | 0.0247 | 0.1832 | 0.0228 | -0.0188 |
| X-11                 | Fil   | 0.0090 | 0.0176 | 0.0297 | -0.0070 | -0.0176 | 0.0123 | 0.0176 | 0.0161 | 0.0098  |
|                      | Bias  | 0.0672 | 0.5435 | 0.0426 | -0.0390 | -0.0243 | 0.0263 | 0.3033 | 0.0181 | -0.0521 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.



**Table A.4: Biases in Volatility Autocorrelation Functions: GARCH(1,1) Model with  $\alpha = .1$  and  $\beta = .9$**

| Lags                 |       | 1      | 2      | 3      | 4      | 5      | 6      | 7       | 8       | 9       | 10      |
|----------------------|-------|--------|--------|--------|--------|--------|--------|---------|---------|---------|---------|
| Large Sample Results |       |        |        |        |        |        |        |         |         |         |         |
| Lin                  | Unfil | 0.9902 | 0.9799 | 0.9699 | 0.9600 | 0.9503 | 0.9408 | 0.9313  | 0.9220  | 0.9128  | 0.9037  |
|                      | Fil   | 0.2935 | 0.2914 | 0.2886 | 0.2865 | 0.2833 | 0.2821 | 0.2793  | 0.2779  | 0.2752  | 0.2735  |
|                      | Bias  | 0.6967 | 0.6885 | 0.6813 | 0.6735 | 0.6670 | 0.6583 | 0.6520  | 0.6441  | 0.6376  | 0.6302  |
| X-11                 | Fil   | 0.2083 | 0.1465 | 0.1073 | 0.0802 | 0.0595 | 0.0357 | 0.0217  | 0.0149  | 0.0104  | 0.0126  |
|                      | Bias  | 0.7819 | 0.8334 | 0.8626 | 0.8798 | 0.8908 | 0.9051 | 0.9096  | 0.9071  | 0.9124  | 0.8911  |
| Small Sample Results |       |        |        |        |        |        |        |         |         |         |         |
| Lin                  | Unfil | 0.9374 | 0.8714 | 0.8098 | 0.7522 | 0.6981 | 0.6472 | 0.5991  | 0.5537  | 0.5109  | 0.4702  |
|                      | Fil   | 0.1828 | 0.1684 | 0.1576 | 0.1479 | 0.1389 | 0.1291 | 0.1231  | 0.1142  | 0.1047  | 0.0987  |
|                      | Bias  | 0.7546 | 0.7031 | 0.6522 | 0.6043 | 0.5592 | 0.5181 | 0.4760  | 0.4395  | 0.4062  | 0.3715  |
| X-11                 | Fil   | 0.1894 | 0.1286 | 0.0874 | 0.0622 | 0.0400 | 0.0180 | -0.0008 | -0.0028 | -0.0098 | -0.0050 |
|                      | Bias  | 0.7480 | 0.7428 | 0.7224 | 0.6900 | 0.6581 | 0.6292 | 0.5999  | 0.5565  | 0.5207  | 0.4752  |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.4 (cont'd)**

| Lags                 |       | 11      | 12      | 13     | 14     | 22     | 23     | 24      | 25     | 26     |
|----------------------|-------|---------|---------|--------|--------|--------|--------|---------|--------|--------|
| Large Sample Results |       |         |         |        |        |        |        |         |        |        |
| Lin                  | Unfil | 0.8946  | 0.8857  | 0.8769 | 0.8682 | 0.8017 | 0.7938 | 0.7859  | 0.7781 | 0.7704 |
|                      | Fil   | 0.2711  | 0.2943  | 0.2661 | 0.2632 | 0.2451 | 0.2425 | 0.2267  | 0.2377 | 0.2356 |
|                      | Bias  | 0.6235  | 0.5914  | 0.6108 | 0.6050 | 0.5567 | 0.5513 | 0.5592  | 0.5405 | 0.5348 |
| X-11                 | Fil   | 0.0147  | -0.0794 | 0.1599 | 0.1554 | 0.1175 | 0.1164 | -0.0151 | 0.0112 | 0.0312 |
|                      | Bias  | 0.8799  | 0.9651  | 0.7170 | 0.7128 | 0.6842 | 0.6774 | 0.7910  | 0.7893 | 0.7392 |
| Small Sample Results |       |         |         |        |        |        |        |         |        |        |
| Lin                  | Unfil | 0.4318  | 0.3954  | 0.3610 | 0.3284 | 0.1226 | 0.1028 | 0.0839  | 0.0658 | 0.0487 |
|                      | Fil   | 0.0909  | 0.1188  | 0.0793 | 0.0793 | 0.0714 | 0.0227 | 0.0073  | 0.0163 | 0.0116 |
|                      | Bias  | 0.3402  | 0.2766  | 0.2818 | 0.2571 | 0.0961 | 0.0801 | 0.0766  | 0.0495 | 0.0371 |
| X-11                 | Fil   | -0.0042 | -0.0643 | 0.1183 | 0.1060 | 0.0381 | 0.0335 | -0.0798 | 0.0234 | 0.0501 |
|                      | Bias  | 0.4360  | 0.4797  | 0.2427 | 0.2224 | 0.0845 | 0.0693 | 0.1647  | 0.0424 | 0.0186 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.5: Biases in Volatility Autocorrelation Functions: Seasonal GARCH(1,1) Model with  $\alpha = .1$  and  $\beta = .9$**

| Lags                 |       | 1       | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       | 10      |
|----------------------|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Large Sample Results |       |         |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | 0.0407  | 0.0318  | 0.0300  | 0.0314  | 0.0337  | 0.0302  | 0.0333  | 0.0304  | 0.0286  | 0.0298  |
|                      | Fil   | 0.0169  | 0.0136  | 0.0126  | 0.0130  | 0.0138  | 0.0123  | 0.0137  | 0.0127  | 0.0120  | 0.0129  |
|                      | Bias  | 0.0238  | 0.0182  | 0.0174  | 0.0184  | 0.0199  | 0.0179  | 0.0196  | 0.0177  | 0.0166  | 0.0169  |
| X-11                 | Fil   | 0.0450  | 0.0057  | 0.0051  | 0.0047  | 0.0044  | -0.0028 | -0.0070 | -0.0086 | -0.0075 | -0.0085 |
|                      | Bias  | -0.0043 | 0.0261  | 0.0263  | 0.0267  | 0.0293  | 0.0330  | 0.0403  | 0.0390  | 0.0361  | 0.0383  |
| Small Sample Results |       |         |         |         |         |         |         |         |         |         |         |
| Lin                  | Unfil | -0.0575 | -0.0637 | -0.0618 | -0.0592 | -0.0581 | -0.0583 | -0.0579 | -0.0583 | -0.0597 | -0.0614 |
|                      | Fil   | -0.0286 | -0.0321 | -0.0311 | -0.0287 | -0.0285 | -0.0283 | -0.0283 | -0.0282 | -0.0302 | -0.0302 |
|                      | Bias  | -0.0289 | -0.0316 | -0.0307 | -0.0305 | -0.0296 | -0.0300 | -0.0296 | -0.0301 | -0.0295 | -0.0312 |
| X-11                 | Fil   | 0.0273  | -0.0143 | -0.0030 | -0.0087 | -0.0010 | -0.0155 | -0.0229 | -0.0193 | -0.0146 | -0.0147 |
|                      | Bias  | -0.0848 | -0.0494 | -0.0588 | -0.0494 | -0.0573 | -0.0424 | -0.0350 | -0.0390 | -0.0451 | -0.0467 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.5 (cont'd)**

| Lags                 |       | 11      | 12     | 13      | 14      | 22      | 23      | 24     | 25      | 26      |
|----------------------|-------|---------|--------|---------|---------|---------|---------|--------|---------|---------|
| Large Sample Results |       |         |        |         |         |         |         |        |         |         |
| Lin                  | Unfil | 0.0382  | 0.9596 | 0.0377  | 0.0290  | 0.0269  | 0.0352  | 0.9223 | 0.0347  | 0.0264  |
|                      | Fil   | 0.0114  | 0.4298 | 0.0161  | 0.0120  | 0.0116  | 0.0149  | 0.3784 | 0.0146  | 0.0111  |
|                      | Bias  | 0.0268  | 0.5298 | 0.0216  | 0.0170  | 0.0153  | 0.0203  | 0.5239 | 0.0201  | 0.0153  |
| X-11                 | Fil   | 0.0223  | 0.0210 | 0.0488  | 0.0088  | -0.0017 | 0.0350  | 0.0547 | -0.0110 | -0.0010 |
|                      | Bias  | 0.0159  | 0.9386 | -0.0189 | 0.0202  | 0.0286  | 0.0002  | 0.8676 | 0.0457  | 0.0274  |
| Small Sample Results |       |         |        |         |         |         |         |        |         |         |
| Lin                  | Unfil | -0.0553 | 0.8259 | -0.0553 | 0.0606  | -0.0568 | -0.0521 | 0.6765 | -0.0518 | -0.0556 |
|                      | Fil   | -0.0144 | 0.3623 | -0.0260 | -0.0296 | -0.0280 | -0.0251 | 0.2650 | -0.0235 | -0.0274 |
|                      | Bias  | -0.0409 | 0.4636 | -0.0293 | -0.0306 | -0.0288 | -0.0270 | 0.4115 | -0.0283 | -0.0282 |
| X-11                 | Fil   | 0.0066  | 0.0201 | 0.0251  | -0.0198 | -0.0247 | 0.0095  | 0.0490 | 0.0099  | -0.0001 |
|                      | Bias  | -0.0619 | 0.8058 | -0.0804 | -0.0404 | -0.0321 | -0.0616 | 0.6275 | -0.0619 | -0.0555 |

Notes: All computations are based on 1000 Monte Carlo Simulations using the linear approximation to the X-11 filter (denoted Lin) and the SAS Proc X11 procedure (denoted X-11). The bias is defined as in eq. (2.9). The large sample configuration is based on 996 data points while the small sample reflects 120 observations. Details of the simulation design appear in Section 3.1.

**Table A.6: Monte Carlo Simulation Distributions of X-11 Program Outlier Intervention Frequencies**

| Percentiles   | Min  | 5%   | 10%  | 25%  | 50%  | 75%   | 90%   | 95%   | More  | Mean |
|---|------|------|------|------|------|-------|-------|-------|-------|------|
| <b>White Noise</b>  |      |      |      |      |      |       |       |       |       |      |
| Large Sample  | 2.61 | 4.12 | 4.32 | 4.72 | 5.12 | 5.52  | 5.92  | 6.13  | 7.73  | 5.12 |
| Small Sample  | 0.00 | 2.50 | 2.50 | 3.34 | 5.00 | 6.67  | 7.50  | 10.00 | 13.34 | 5.11 |
| <b>IGARCH <math>\alpha = .1</math> and <math>\beta = .9</math></b>          |      |      |      |      |      |       |       |       |       |      |
| Large Sample  | 0.00 | 0.00 | 0.00 | 0.00 | 2.21 | 4.22  | 6.22  | 7.33  | 13.15 | 2.63 |
| Small Sample  | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 6.67  | 15.83 | 20.00 | 43.33 | 4.32 |
| <b>Seasonal IGARCH <math>\alpha = .1</math> and <math>\beta = .2</math></b> |      |      |      |      |      |       |       |       |       |      |
| Large Sample  | 0.00 | 2.21 | 3.01 | 4.72 | 6.73 | 8.73  | 10.64 | 11.85 | 19.78 | 6.83 |
| Small Sample  | 0.00 | 0.83 | 1.67 | 5.00 | 8.33 | 10.83 | 14.17 | 15.83 | 27.50 | 8.04 |

Notes: Entries to this table are Monte Carlo simulation distributions of percentage of the sample observations (large is 996 observations and small is 190) affected by the X-11 program outlier selection procedure.

**Table A.7: Data Sources and Descriptions**

| Series                 | Description   |
|------------------------|---|
| Industrial Production  | A monthly index of the output of manufacturing, mining, and electric and gas utilities. Source: Federal Reserve Board, Statistical Release G.17. Available on the Internet at <a href="http://www.stls.frb.org">http://www.stls.frb.org</a> . |
| Money Supply           | M1, monetary aggregate. Source: Federal Reserve Board, Statistical Release H.6. Available on the Internet at <a href="http://www.stls.frb.org">http://www.stls.frb.org</a> .  |
| Nominal Interest Rates | Interest rate on 3-month Treasury bills. Source: Federal Reserve Board, Statistical Release H15. Available on the Internet at <a href="http://www.stls.frb.org">http://www.stls.frb.org</a> .   |
| Unemployment Rate      | Civilian labor force unemployment rate. Source: Bureau of Labor Statistics. Available on the Internet at <a href="http://stats.bls.gov">http://stats.bls.gov</a> .  |
| CPI                    | Consumer price index, all urban consumers. Source: Bureau of Labor Statistics. Available on the Internet at <a href="http://stats.bls.gov">http://stats.bls.gov</a> .   |
| PPI-Crude Material     | Crude materials for further processing. Source: Bureau of Labor Statistics. Available on the Internet at <a href="http://stats.bls.gov">http://stats.bls.gov</a> .  |
| PPI-Finished Goods     | Goods ready for sale to final demand. Source: Bureau of Labor Statistics. Available on the Internet at <a href="http://stats.bls.gov">http://stats.bls.gov</a> .  |
| Real Interest Rate     | Three-month treasury bill yield less CPI inflation.   |
| Real Earnings          | Earnings deflated by the CPI. I use the average hourly earnings series from the Bureau of Labor Statistics, available on the Internet at <a href="http://stats.bls.gov">http://stats.bls.gov</a> .  |

**Table A.8: Autocorrelations of Squared Residuals Empirical Data**

**Adjusted Monetary Base**

| <b>Lags</b> | <b>SA</b> | <b>NSA - Seas. Diff.</b> | <b>NSA - Seas. Dum.</b> |
|-------------|-----------|--------------------------|-------------------------|
| 1           | 0.071     | 0.330                    | 0.214                   |
| 2           | -0.016    | 0.155                    | 0.198                   |
| 3           | 0.109     | 0.048                    | 0.032                   |
| 4           | -0.010    | 0.069                    | 0.098                   |
| 5           | 0.063     | 0.082                    | 0.084                   |
| 6           | 0.095     | 0.092                    | 0.100                   |
| 7           | 0.038     | 0.087                    | 0.052                   |
| 8           | 0.027     | 0.100                    | 0.124                   |
| 9           | 0.159     | 0.180                    | 0.112                   |
| 10          | 0.038     | 0.104                    | 0.186                   |
| 11          | 0.023     | 0.259                    | 0.192                   |
| 12          | 0.036     | 0.262                    | 0.398                   |
| 13          | 0.054     | 0.127                    | 0.199                   |
| 14          | 0.006     | 0.123                    | 0.079                   |
| 22          | 0.084     | 0.100                    | 0.087                   |
| 23          | 0.078     | 0.090                    | 0.138                   |
| 24          | 0.081     | 0.126                    | 0.131                   |
| 25          | -0.021    | 0.138                    | 0.225                   |
| 26          | -0.002    | 0.150                    | 0.169                   |

**Table A.8 (cont'd)**

| <b>M1 Money Stock</b> |           |                          |                         |
|-----------------------|-----------|--------------------------|-------------------------|
| <b>Lags</b>           | <b>SA</b> | <b>NSA - Seas. Diff.</b> | <b>NSA - Seas. Dum.</b> |
| 1                     | 0.155     | 0.259                    | 0.241                   |
| 2                     | 0.184     | 0.052                    | 0.025                   |
| 3                     | 0.051     | 0.044                    | 0.059                   |
| 4                     | 0.169     | 0.091                    | 0.070                   |
| 5                     | 0.177     | 0.043                    | 0.108                   |
| 6                     | 0.127     | 0.020                    | 0.117                   |
| 7                     | 0.052     | 0.029                    | 0.097                   |
| 8                     | 0.250     | 0.060                    | 0.183                   |
| 9                     | 0.175     | 0.108                    | 0.047                   |
| 10                    | 0.069     | 0.042                    | 0.046                   |
| 11                    | 0.148     | 0.135                    | 0.149                   |
| 12                    | 0.087     | 0.223                    | 0.323                   |
| 13                    | 0.160     | 0.111                    | 0.106                   |
| 14                    | 0.071     | 0.005                    | 0.084                   |
| 22                    | 0.155     | 0.087                    | 0.181                   |
| 23                    | 0.032     | 0.128                    | 0.088                   |
| 24                    | -0.002    | 0.176                    | 0.008                   |
| 25                    | -0.016    | 0.062                    | 0.043                   |
| 26                    | 0.013     | 0.004                    | 0.005                   |



**Table A.8 (cont'd)**

**CPI all items**

| <b>Lags</b> | <b>SA</b> | <b>NSA - Seas. Diff.</b> | <b>NSA - Seas. Dum.</b> |
|-------------|-----------|--------------------------|-------------------------|
| 1           | 0.398     | 0.275                    | 0.173                   |
| 2           | 0.188     | 0.145                    | 0.348                   |
| 3           | 0.126     | 0.145                    | 0.237                   |
| 4           | 0.109     | 0.081                    | 0.173                   |
| 5           | 0.164     | 0.198                    | 0.239                   |
| 6           | 0.274     | 0.258                    | 0.352                   |
| 7           | 0.210     | 0.145                    | 0.169                   |
| 8           | 0.116     | 0.216                    | 0.385                   |
| 9           | 0.094     | 0.191                    | 0.383                   |
| 10          | 0.093     | 0.094                    | 0.125                   |
| 11          | 0.098     | 0.161                    | 0.158                   |
| 12          | 0.083     | 0.155                    | 0.229                   |
| 13          | 0.104     | 0.164                    | 0.217                   |
| 14          | 0.072     | 0.175                    | 0.441                   |
| 22          | 0.126     | 0.104                    | 0.188                   |
| 23          | 0.087     | 0.149                    | 0.091                   |
| 24          | 0.055     | 0.043                    | 0.392                   |
| 25          | 0.050     | 0.081                    | 0.103                   |
| 26          | 0.061     | 0.106                    | 0.104                   |

**Table A.8 (cont'd)**

| <b>CPI all items less energy</b> |           |                          |                         |
|----------------------------------|-----------|--------------------------|-------------------------|
| <b>Lags</b>                      | <b>SA</b> | <b>NSA - Seas. Diff.</b> | <b>NSA - Seas. Dum.</b> |
| 1                                | 0.165     | 0.062                    | 0.057                   |
| 2                                | 0.012     | 0.011                    | 0.019                   |
| 3                                | 0.018     | -0.017                   | 0.091                   |
| 4                                | 0.002     | -0.003                   | 0.068                   |
| 5                                | 0.166     | 0.189                    | 0.116                   |
| 6                                | 0.069     | 0.039                    | 0.038                   |
| 7                                | 0.096     | 0.034                    | 0.014                   |
| 8                                | 0.048     | 0.048                    | 0.025                   |
| 9                                | 0.013     | 0.050                    | 0.092                   |
| 10                               | 0.033     | 0.014                    | 0.056                   |
| 11                               | 0.018     | 0.001                    | 0.069                   |
| 12                               | 0.168     | 0.066                    | 0.248                   |
| 13                               | 0.084     | 0.041                    | 0.143                   |
| 14                               | 0.030     | 0.025                    | 0.105                   |
| 22                               | 0.581     | -0.012                   | -0.001                  |
| 23                               | 0.102     | 0.063                    | 0.094                   |
| 24                               | 0.042     | 0.139                    | 0.165                   |
| 25                               | 0.019     | -0.015                   | -0.023                  |
| 26                               | -0.013    | -0.003                   | 0.045                   |

Table A.8 (cont'd)

| CPI all items less food and energy |        |                   |                  |
|------------------------------------|--------|-------------------|------------------|
| Lags                               | SA     | NSA - Seas. Diff. | NSA - Seas. Dum. |
| 1                                  | 0.040  | 0.052             | 0.009            |
| 2                                  | 0.165  | 0.093             | 0.062            |
| 3                                  | 0.168  | 0.072             | 0.120            |
| 4                                  | 0.075  | 0.053             | 0.089            |
| 5                                  | 0.076  | 0.073             | 0.020            |
| 6                                  | 0.100  | 0.091             | 0.087            |
| 7                                  | 0.089  | 0.034             | 0.005            |
| 8                                  | 0.081  | 0.144             | 0.024            |
| 9                                  | 0.041  | 0.042             | 0.075            |
| 10                                 | 0.062  | 0.094             | 0.075            |
| 11                                 | 0.072  | 0.020             | 0.025            |
| 12                                 | 0.160  | 0.149             | 0.415            |
| 13                                 | 0.044  | 0.009             | -0.024           |
| 14                                 | 0.068  | 0.069             | 0.136            |
| 22                                 | 0.040  | 0.004             | 0.071            |
| 23                                 | 0.026  | 0.002             | 0.059            |
| 24                                 | -0.016 | -0.005            | 0.071            |
| 25                                 | 0.018  | -0.028            | -0.023           |
| 26                                 | 0.075  | 0.035             | 0.071            |

**Table A.9: GARCH(1,1) Parameter Estimates**

| <i>Series - Sample</i>              | $\alpha$ |                     | $\beta$ |                     |
|-------------------------------------|----------|---------------------|---------|---------------------|
| Monetary Base SA <b>59:08-95:12</b> | .002     | (.03)               | .34     | (1.39)              |
| Monetary Base seas. Diff            | -.01     | (.02)               | .77     | (.35) <sup>@</sup>  |
| M1 SA <b>59:01-95:12</b>            | .24      | (.06) <sup>*</sup>  | .68     | (.07) <sup>*</sup>  |
| M1 seas. Diff                       | .09      | (.03) <sup>*</sup>  | .90     | (.03) <sup>*</sup>  |
| CPI <b>46:01-95:12</b>              | .14      | (.019) <sup>*</sup> | .83     | (.022) <sup>*</sup> |
| CPI seas. Diff                      | .13      | (.82) <sup>*</sup>  | .82     | (.04) <sup>*</sup>  |
| CPI ex energy SA <b>57:01-95:12</b> | .10      | (.02) <sup>*</sup>  | .90     | (.02) <sup>*</sup>  |
| CPI ex energy seas. Diff            | .10      | (.02) <sup>*</sup>  | .91     | (.02) <sup>*</sup>  |
| CPI ex food & energy SA             | .14      | (.04) <sup>*</sup>  | .86     | (.04) <sup>*</sup>  |
| CPI ex food & energy seas. Diff     | .30      | (.05) <sup>*</sup>  | .73     | (.03) <sup>*</sup>  |

**Notes:** \* significant at the 1%, @ at the 5%, + at the 10% level.  
 Seas. Diff means the seasonal difference operator was applied.

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