CIRANO

Centre interuniversitaire de recherche
en analyse des organisations

Série Scientifique
Scientific Series

95s-37

**DYNAMIC INCENTIVE
CONTRACTS WITH
UNCORRELATED PRIVATE
INFORMATION AND HISTORY
DEPENDENT OUTCOMES**

Gérard Gaudet, Pierre Lasserre, Ngo Van Long

Montréal
Septembre 1995

Dynamic Incentive Contracts with Uncorrelated Private Information and History Dependent Outcomes[*]

Gérard Gaudet[†], Pierre Lasserre[†], Ngo Van Long[‡]

Résumé / Abstract

La dynamique des modèles actuels de contrats incitatifs provient de la capacité du principal, à partir des actions observées dans le présent, d'apprendre quelque chose sur l'information qui ne lui sera pas directement accessible dans le futur. Nous étudions ici une autre source de dynamique, négligée dans la littérature sur l'information mais standard dans toutes les autres branches de la science économique, et qui résulte du fait que les actions courantes définissent les opportunités futures. C'est ce qui se passe lorsqu'il y a investissement, apprentissage, R-D, etc. Pour bien identifier les propriétés dynamiques résultant de ce type de situations dans les modèles de principal-agent avec information asymétrique, nous nous en tenons à des modèles où il n'y a aucune corrélation entre information présente et information future, si bien qu'aucun effet dynamique ne résulte directement de l'asymétrie d'information.

In existing papers on dynamic incentive contracts, the dynamic structure of the principal-agent relationship arises exclusively from the ability of the principal to learn about the hidden information over time. In this paper we deal with a different source of dynamics, which is considered standard in all areas of economics other than the information literature: we study situations where current opportunities depend on past and current actions, notwithstanding any information conveyed by the actions. Standard examples include investment, "Learning by doing", and R&D. In order to focus on this neglected source of dynamics, we restrict our attention to situations involving asymmetric information in each period, but without any intertemporal informational correlation, so that no dynamic effect arises from informational asymmetries directly. This makes comparisons with static results both easier and more interesting.

Key words: Incentive contracts, Dynamic, Asymmetric information, Principal-agent relationship, Investment, Learning by doing.

Mots clés :   Contrats incitatifs, Dynamique, Information asymétrique, Relation principal-agent, Investissement, Learning by doing.

Address. Please address all correspondence to Pierre Lasserre, Département des sciences économiques, UQAM, C.P. 8888, succ. A, Montréal (Qc), Canada, H3C 3P8. E-mail: pierre@uqam.ca.

---

1 Introduction

When studying dynamic incentive contracts, authors have focused their attention on a repeated relationship in which the principal was potentially able to become better informed as the relationship evolved over time. In that literature, in each period, once the hidden information has been revealed to the agent, the surplus to be shared between the principal and the agent depends on current actions only; thus the dynamic structure of the relationship arises exclusively from the ability of the principal to learn about the hidden information over time.

If there is no intertemporal correlation between realizations of the private information, the principal does not learn anything about the future from the revelation of current information; as a result the dynamic relationship is a repetition of static incentive contracts based on the revelation principle. If, instead, the current realization carries information about the probability distribution of future realizations, revelation by the agent of current information affects future relationships, opening the way to a ratchet effect. Principal-agent contractual relationships have been studied extensively in that context under various conditions regarding commitment, renegotiation, and the duration of the relationship. (see Besanko and Sappington, 1987; Laffont and Tirole, 1988; Baron, 1989). The major results sharply contrast with the static principal-agent theory. In particular, in the absence of commitment, the revelation principle breaks down; there does not exist any separating equilibria if there is a continuum of types; "good" types are not preserved from distorting contract requirements.

In this paper, we deal with a different source of dynamics, the one which is considered standard in all areas of economics other than the information literature. We envisage situations where current opportunities depend on past, as well as current, actions, notwithstanding any information conveyed by the actions. Standard examples include investment, "Learning by doing", exhaustible-resource extraction, and R & D.

Substantial theoretical and empirical work on these topics has been carried out over the years, including setups involving uncertainty and strategic opportunities. Surprisingly, the theory has never been extended to deal with information asymmetries. Although our paper may be viewed as a first step toward fulfilling this need, it is also a contribution to the principal-agent literature. Indeed our focus is on the robustness of some results of the static principal-agent literature to the introduction of the dynamic features mentioned above. While we use a simple, specific, model to help intuition, our points are fairly general. In particular we do not assume Markovian characteristics, although this is the prevalent approach in the literature on investment, resource extraction, etc., and we do not impose a fixed optimization period, as is usually done in dynamic principal-agent models.

In order to focus on the dynamics that result directly from observable actions, rather than from informational asymmetries, we assume uncorrelated realizations of the hidden factor over time; to facilitate interpretation we also assume that the hidden factor is a characteristic of the agent, i.e. we analyze adverse selection problems. Thus the principal does not learn anything about the future from the revelation of the current realization. Under such circumstances, it is obvious, as illustrated by Besanko and Sappington (1987, section 7), that the revelation principle remains effective. This makes comparisons with static results both easier and more interesting.

The following results apply to such a wide class of models that they can be taken to summarize conventional wisdom about static adverse-selection principal-agent models.

Fact 1 Under symmetric information about the agent's type, the agent obtains no rent and its activity is optimal.

Fact 2 The "worst" type that is willing to participate receives no rent.

Fact 3 There is no distortion to the "best" type.

Fact 4 Under standard technical assumptions that include a monotonic hazard rate for the distribution of types, the principal's problem has a unique interior solution; this solution is separating for all types that participate, distorting except for the best type, and such that the agent's action, and its rent, are monotonic functions of its type.

Under the class of dynamic setups just outlined, we will show that, with the trivial exception of *Fact 1*, these results have to be modified in general. In particular, we will emphasize what happens to *Facts 2* and *3*, and why. In Section 2, after underlining the scope of our discussion by providing a few examples, we will present a model and describe its main features under symmetric, and then under asymmetric, information. In particular, it will be shown that the absence of commitment requires the imposition of an additional participation constraint, not present in static models. Then Section 3 will provide a more thorough comparison of the static, and the dynamic, results, focusing in particular on the "best type". By then, it should be clear to the reader that *Facts 2-4* are specific to static situations, and why. The example provided in Section 4, therefore, is more than a simple illustration; it emphasizes that exceptions may arise under assumptions that might be considered benign in the asymmetric information literature.

2   The model

2.1   Setup

In order to help intuition let us think of a principal-agent relationship where the principal is a government seeking to appropriate itself a share of excess profits by its agent. The latter is a firm whose technology is such that current profits depend on past and current levels $q_1, q_2, \ldots, q_t$ of some control variable such as output, and on the current realization

of a parameter $\theta_t > 0$ which is observable by the firm only. Thus there is a problem of adverse selection but not of moral hazard. θ_t has range $\left[\theta^L, \theta^H\right]$ and is distributed according to the stationary density function f. Thus, if the firm is in business, its profit function in period t is $\Pi_t(q_1, q_2, \ldots, q_t, \theta_t)$ with

$$\partial^2 \Pi_t / \partial q_t^2 \leq 0; \quad \partial^2 \Pi_t / \partial q_t \partial \theta_t \leq 0; \quad \partial \Pi / \partial \theta_t < 0 \tag{1}$$

If the firm is not in business in t, it earns zero profit.

An example involving "learning-by-doing" is

$$\Pi_t(q_1, q_2, \ldots, q_t, \theta_t) = p_t q_t - \frac{1}{1 + \sum_{s=1}^{t-1} \gamma_s q_s} C(\theta_t, q_t) \tag{2}$$

where $q \geq 0$ is output, p is its price, and the second term on the right-hand side represents production costs that shift down as cumulative weighted output increases.

The case of a Ricardian non-renewable resource, with extraction costs increasing as cumulative extraction rises, would be represented as

$$\Pi_t(q_1, q_2, \ldots, q_t, \theta_t) = p_t q_t - \frac{1}{\beta + \gamma[\bar{X} - \sum_{s=1}^{t-1} q_s]} C(\theta_t, q_t) \tag{3}$$

where $\bar{X}$, $\bar{X} \geq \sum_{s=1}^{t-1} q_s$, is the initial amount of non-renewable resource, $q_t \geq 0$ is output, whose price is p, while β and γ are non negative parameters that specify how extraction costs shift up with cumulative extraction.

As a third example, consider a firm whose output is a function $g(K_t)$ of the stock of capital K_t, whose rate of depreciation is ρ, so that $K_t = \sum_{\tau=1}^{t} q_\tau (1 - \rho)^{-(t-\tau)}$. The firm may invest at rate $q_t \geq 0$ to build up capital subject to privately observed adjustment costs. This may be represented by

$$\Pi_t(q_1, q_2, \ldots, q_t, \theta_t) = p_t g(K_t) - C(q_t, K_t, \theta)$$

Problems involving other types of investments, reputation, etc., can be cast in this framework. Other technological constraints may be added as needed. For example, a Hotelling mining problem, involving the extraction of a finite stock of homogeneous mineral reserves, is solved in Gaudet et al. (1991).

In what follows, in order to avoid useless complications, we will assume that $q = 0$ and $\Pi = 0$ when the firm withdraws (shuts down, either temporarily or permanently). The objective of the firm is to maximize expected cumulative discounted profits, net of transfers R_t to the principal. The latter maximizes a weighted sum consisting of the expected discounted sum of her transfer income on the one hand, and the expected discounted sum of the net profits left to the firm on the other hand. Whether it be due to the preferences of the principal, or to the cost of transferring income from the firm to the principal, the latter term is given a lower weight, α, $0 < \alpha < 1$, than the sum of transfers.

The principal is able to commit to a one-period contract but not to any longer-term contract. As already discussed by several authors (see, for instance, Laffont and Tirole (1988)), this assumption can capture situations where the principal changes over time, and the current principal (e.g. government, shareholders) cannot commit future ones. It can also arise if long-run contracts are too costly to write, or, which is perhaps not very different, if future actions and contingencies, although observable upon their realization, are not easily foreseeable or described, so that they cannot be used as elements of an enforceable contract. Being unable to precommit, the principal cannot credibly give up her claim to a share in future rents in exchange for a lump-sum payment whose amount would be agreed upon before future cost conditions are revealed to the firm. Neither can she credibly commit herself never to resume a severed relationship if it is in her interest to do so[1].

---

[1] This last restriction would also follow from renegotiation proofness in a long run contract.

2.2   Symmetric information

Consider first the reference scenario of symmetric information. Hence, suppose that θ_t is observed by both the firm and the principal upon its realization. Then, at any date t, the principal is in a position to require the firm to set to q_t so as to maximize total surplus, the sum of current surplus and the discounted value of expected cumulative future surpluses , $^s_{t+1}(q_t, ..., q_1)$, and to appropriate herself all surpluses (the superscript 's' refers to symmetric information). Thus in effect, the principal chooses q_t that maximizes

$$\Pi_t(q_t, ..., q_1, \theta_t) \ + \ \delta, ^s_{t+1}(q_t, ..., q_1) \tag{4}$$

Since a total surplus of zero can always be achieved by shutting down forever, the maximum is necessarily non negative, and so is , $^s_{t+1}$. However Π_t may be negative. Future prices, as well as all functions C_t, are public knowledge; thus the expectation in , is taken over future values of θ. The conditions for the maximization of (4) must be satisfied at all dates. Thus, assuming that the price parameters, the range of θ, and the profit functions Π_t are such that the solution is interior

$$\frac{\partial \Pi_t(q_t, \ldots, q_1, \theta_t)}{\partial q_t} + \delta \frac{\partial, ^s_{t+1}(q_t, ..., q_1)}{\partial q_t} = 0 \quad \forall \theta_t \in [\theta^L, \theta^H] \text{ and } \forall t \tag{5}$$

Equation (5) yields q_t as a function of θ_t and of the history $(q_{t-1}, ..., q_1)$. If it exists, the terminal period T^s is stochastic; for any history, it may be defined as the first instance when , $^s_{t+1} = 0$. If, for some reason, before the realization of θ_t, it is known that $t = T^s$, then (5) reduces to the static condition

$$\frac{\partial \Pi_{T^s}(q_{T^s}, \ldots, q_1, \theta_{T^s})}{\partial q_{T^s}} = 0 \ \forall \ \theta_{T^s} \in [\theta^L, \theta^H] \tag{6}$$

6

Denote $q_{T^s}^s(\cdot, \theta_{T^s})$ the solution to (6). The corresponding surplus to be transferred to the principal, as expected just before the realization of θ_{T^s}, is

$$, {}_{T^s}^s (q_{T^s-1}, \ldots, q_1) = \int_{\theta^L}^{\bar{\theta}_{T^s}^s} [\Pi_{T^s}(q_{T^s}(\cdot, \theta_{T^s}), \ldots, q_1, \theta_{T^s})] f(\theta_{T^s}) d\theta_{T^s} \qquad (7)$$

where $f(\theta)$ is the density distribution of θ and $\bar{\theta}_{T^s}^s$ is the least efficient profitable type at T^s. In general, the next period is not necessarily expected to be the terminal one so that, in any period t, the surplus to be transferred to the principal, as expected just before the realization of θ_t, must include the expected discounted contribution of periods beyond t

$$, {}_t^s(q_{t-1}, \ldots, q_1) = \int_{\theta^L}^{\bar{\theta}_t^s} [\Pi_t(q_t^s(\cdot, \theta_t^s), \ldots, q_1, \theta_t) + \delta, {}_{t+1}^s (q_t^s(\cdot, \theta_t^s), \ldots, q_1)] f(\theta_t) d\theta_t \qquad (8)$$

where q_t^s denotes the solution to (5) at t and $\bar{\theta}_t^s$, the least efficient profitable type at t, is defined by the condition that expected cumulative discounted profits be at least as high as under the best alternative, which is either zero (permanent shutdown), or the expected discounted profit from shutting down in t and reopening later on

$$\bar{\theta}_t^s \equiv \max \left\{ \theta_t | \Pi(q_t, \ldots, q_1, \theta_t) + \delta, {}_{t+1}^s(q_t, \ldots, q_1) = \max \left[0, \delta, {}_{t+1}^s (0, q_{t-1}, \ldots, q_1) \right] \right\} \qquad (9)$$

2.3   Asymmetric information

Under asymmetric information the firm observes the realization of θ at any date, while the principal never does. If, as assumed, there is no correlation between the realizations of θ across periods, the firm does not lose any of its future informational advantage when it reveals current information to the principal. As a consequence there is no possibility of a ratchet effect, and the revelation principle applies as in static setups. Furthermore, since the principal cannot commit to long-term contracts, the asymmetric relationship

is repeated in each period, but conditioned on an up-dated history.

In the optimal incentive scheme, the firm chooses its output and the transfer payment to the principal from a menu offered by the principal and based on the firm's declaration of its type. The construction of such a scheme by standard methods is outlined in the Appendix; by the revelation principle, the scheme is such that it is not in the interest of the firm to lie about its type so that, in what follows, θ represents the true value, as revealed to the principal by the firm. For any history $\{q_{t-1}, \ldots, q_1\}$, let $\hat{\Pi}_t$ denote net cumulative discounted surplus to the firm

$$\hat{\Pi}_t(\theta_t; q_{t-1}, \ldots, q_1) \equiv \Pi_t(q_t(\theta_t; q_{t-1}, \ldots, q_1), q_{t-1}, \ldots, q_1, \theta_t) \Leftrightarrow R_t(\theta_t; q_{t-1}, \ldots, q_1)$$
$$+ \delta \Psi_{t+1}(q_t(\theta_t; q_{t-1}, \ldots, q_1), q_{t-1}, \ldots, q_1)$$

where $q_t(\theta_t; q_{t-1}, \ldots, q_1)$, $R_t(\theta_t; q_{t-1}, \ldots, q_1)$ respectively represent production and the transfer under the menu offered by the principal to the firm in period t, and $\Psi_{t+1}(q_t(\theta_t; q_{t-1}, \ldots, q_1), q_{t-1}, \ldots, q_1) \geq 0$ represents the value discounted to $t+1$, as expected at t, of cumulative future net profits to the firm if it continues its relationship with the principal when it is rational to do so (we specify below conditions under which any firm that stays in business decides to continue the relationship). In order to alleviate the notation, we shall write $\hat{\Pi}$ and q as functions of θ only except when more precision is needed. It is shown in the Appendix that

$$\frac{d\hat{\Pi}_t}{d\theta_t} = \frac{\partial \Pi_t}{\partial \theta_t} < 0 \tag{10}$$

and

$$\frac{dq_t}{d\theta_t} \leq 0 \tag{11}$$

When they correspond to the optimum incentive scheme, these functions, and other relevant functions, will be denoted with a superscript 'a' (for asymmetric information).

At any date t, facing the incentive compatibility conditions just outlined, firms will participate only if their net value is non negative

$$\hat{\Pi}_t(\theta_t) \geq 0 \quad \forall \theta_t \in [\theta^L, \bar{\theta}_t(q_{t-1}, \ldots, q_1)] \tag{12}$$

where $\bar{\theta}_t$ is the highest-cost firm which participates in period t. Note that, because of (10), any type $\theta_t < \bar{\theta}_t$ will participate, so that (12) may be replaced by

$$\hat{\Pi}_t(\bar{\theta}_t) \geq 0 \tag{13}$$

The assumed inability of the principal to commit herself not to resume a severed relationship if it is not her interest to do so implies that the participation decision is the agent's, and only the agent's. It is impossible for the principal to deny participation to an agent who finds it in his interest to participate. Consequently, if, in t, a firm is asked to make current profits that it judges too low, it can always withdraw, achieve $\Pi_t = 0$, and still participate in future periods if it so wishes. Although the interruption may last any number of periods, the firm only needs consider the current period, postponing its next participation decision until the next period. Thus **the inability to commit implies** a second rationality constraint

$$\hat{\Pi}_t(\theta_t; q_{t-1}, \ldots, q_1) \geq \delta \Psi_{t+1}(0, q_{t-1}, \ldots, q_1) \quad \forall \theta_t \in [\theta^L, \bar{\theta}(q_{t-1}, \ldots, q_1)] \tag{14}$$

In period t, as shown in the Appendix, the problem of the principal may be written as

$$\max_{q_t(\cdot), \bar{\theta}_t} \int_{\theta^L}^{\bar{\theta}_t} \{\Pi_t(q_t, q_{t-1}, \ldots, q_1, \theta_t) \Leftrightarrow [1 \Leftrightarrow \alpha] \hat{\Pi}_t(\theta_t)$$
$$+\delta[, {}^a_{t+1}(q_t(\theta_t); q_{t-1} \ldots, q_1) + \Psi^a_{t+1}(q_t(\theta_t); q_{t-1} \ldots, q_1)]\} f(\theta_t) d\theta_t \tag{15}$$

subject to (10), (13), and (14), where $, {}^a_{t+1}$ denotes cumulative future transfers to the

9

principal, discounted to $t + 1$, as expected at t, and the superscripts 'a' in , $_{t+1}^{a}$ and Ψ_{t+1}^{a} indicate that both the agent and the principal know that the principal will find it in her interest to use an incentive mechanism in all remaining periods. Treating $\hat{\Pi}_t(\theta_t)$ as a state variable and $q_t(\theta_t)$ as a control variable, (15) may be solved as an optimal control problem (see the Appendix). The solution $q_t^a(\theta_t; q_{t-1}, ..., q_1)$ must be such that

$$\frac{\partial \Pi_t(q_t, \ldots, q_1, \theta_t)}{\partial q_t} + [1 \Leftrightarrow \alpha]\frac{F(\theta_t)}{f(\theta_t)}\frac{\partial^2 \Pi_t(q_t, ..., q_1, \theta_t)}{\partial \theta_t \partial q_t} = \Leftrightarrow \delta \frac{\partial S_{t+1}^a(q_t, ..., q_1)}{\partial q_t} \qquad (16)$$

where $S_{t+1}^a \equiv , _{t+1}^a + \Psi_{t+1}^a$ represents the cumulative surplus, discounted to $t + 1$, as expected at t.

Also, at $\bar{\theta}_t^a$, constraints (13) and (14) must be satisfied. Suppose that the binding one is (13), then

$$\hat{\Pi}_t(\bar{\theta}_t^a; q_{t-1}, \ldots, q_1) = 0 \qquad (17)$$

This is the usual static result (*Fact 2*) that the marginal participating firm is left without any rent. However this result does not hold in general: suppose now that the binding participation constraint is (14); then

$$\hat{\Pi}_t(\bar{\theta}_t^a; q_{t-1}, \ldots, q_1) = \delta \Psi_{t+1}^a(0, q_{t-1}, ..., q_1) > 0 \qquad (18)$$

Here, unlike the static case, the marginal participating firm is left with a strictly positive rent; otherwise it would be better off temporarily withdrawing from its relationship with the principal. **While $\Psi_{t+1}^a(q_t^a, q_{t-1}, ..., q_1)$ is positive by definition, it is possible for $\Psi_{t+1}^a(0, q_{t-1}, ..., q_1)$ to be non positive; but (18) is not binding in that case. Since $\hat{\Pi}_t \Leftrightarrow \delta \Psi_{t+1}^a = \Pi_t \Leftrightarrow R_t$, (18) is binding whenever it would be desirable for the principal to ask the firm to experience a negative cash flow in the current period, in exchange for positive expected future profits. For example, in an investment context, whenever future output prices are high relative to the**

10

acquisition cost of equiment, it is desirable, in a full information situation, to build up capital, thus incurring negative current cash flows. In such a situation, the inability to commit requires the principal to set $R_t < 0$ if the firm is to agree to current investment expenditures involving a negative Π_t. Combining the above two possible terminal conditions, the participation margin under asymmetric information (analog to (9) under symmetry) is

$$\bar{\theta}_t^a = \max\left\{\theta_t | \hat{\Pi}(\theta_t; q_{t-1}, \ldots, q_1) = \max\left[0, \delta\Psi_{t+1}^a\left(0, q_{t-1}, \ldots, q_1\right)\right]\right\} \tag{19}$$

The foregoing discussion implies the proposition below

Proposition 1 *When* $\Psi_{t+1}^a\left(0, q_{t-1}, \ldots, q_1\right) > 0$, *the marginal participating firm enjoys a strictly positive rent.*

3 Comparing the static model with the dynamic model

We have just shown that *Fact 2* - zero rent to the marginal participating firm - no longer holds in our dynamic setup. We are now ready for a more thorough comparison of the solutions under symmetric, and under asymmetric, information.

Note first that the static solution obtains as a special case of the model studied in the previous section. Indeed, if $T = T^a = T^s$ is the last period, the solution under symmetric information is given by (6) taken at T, or

$$\frac{\partial \Pi_T(q_T, \ldots, q_1, \theta_T)}{\partial q_T} = 0 \quad \forall\, \theta_T \in [\theta^L, \theta^H] \tag{20}$$

11

The solution under asymmetric information is given by (16) taken at $t = T$, or

$$\frac{\partial \Pi_T(q_T, ..., q_1, \theta_T)}{\partial q_T} + [1 \Leftrightarrow \alpha] \frac{F(\theta_T)}{f(\theta_T)} \frac{\partial^2 \Pi_T(q_T, ..., q_1, \theta_T)}{\partial \theta_T \partial q_T} = 0 \qquad (21)$$

One notes that the term in S_t^a which appears in (16) has vanished since expected future surplus is identically zero in the last period. Comparing (21) with (20), one can verify that the solutions coincide for the 'best' types ($\theta_T = \theta^L$), provided output histories are identical up to $T \Leftrightarrow 1$. This illustrates *Fact 3*. Given (1), if the hazard rate $\frac{F(\theta)}{f(\theta)}$ is positively monotonous, the solution is clearly unique and strictly decreasing in θ_T, which implies that it is separating over $\left[\theta^L, \bar{\theta}_T\right]$. This, together with (10), illustrates *Fact 4*. Note that pooling trivially occurs (with $q_T = 0$) for $\theta_T \in \left[\bar{\theta}_T, \theta^H\right]$.

Turning to the dynamic case, we note that, since the right-hand side of (16) is independent of θ_t, the same condition (monotonous hazard rate) which ensures that the solution is fully separating for types $\theta_t \leq \bar{\theta}_t$ in the static case (*Fact 4*), implies separation here. Also, for $\theta_t \leq \bar{\theta}_t$, since (11) holds at arbitrary dates, the second feature of *Fact 4* (monotonicity of q and $\hat{\Pi}$) survives the extension to a dynamic setup. However, to the extent that $\bar{\theta}_t$ depends on future conditions and that firms which do not participate in t may participate again later on, it would be misleading to describe the solution as fully separating. Rather, the solution is separating for $\theta_t \leq \bar{\theta}_t$, and involves pooling at $q_t = 0$ for $\theta_t > \bar{\theta}_t$. Thus pooling occurs whenever $\bar{\theta}_t < \theta^H$. **This is summarized in the next proposition.**

Proposition 2 *If $\frac{F(\theta)}{f(\theta)}$ is positively monotonous, the solution is fully separating over $\left[\theta^L, \bar{\theta}_t\right]$ and such that $q_t^a(q_{t-1}, \ldots, q_1, \theta_t)$ and $\hat{\Pi}_t(q_{t-1}, \ldots, q_1, \theta_t)$ are strictly decreasing in θ_t. Over $\left[\bar{\theta}_t, \theta^H\right]$, there is pooling at $q_t^a(q_{t-1}, \ldots, q_1, \theta_t) = 0$.*

We now focus on the dynamic counterpart of *Fact 3*; thus we want to compare output under asymmetric information with output under symmetric information at arbitrary

dates, and determine whether the 'best' type selects the same, efficient, level of q_t under both informational setups. For interior solutions, this means that condition (16), namely

$$\frac{\partial \Pi_t(q_t, \ldots, q_1, \theta_t)}{\partial q_t} + [1 \Leftrightarrow \alpha]\frac{F(\theta_t)}{f(\theta_t)}\frac{\partial^2 \Pi_t(q_t, \ldots, q_1, \theta_t)}{\partial \theta_t \partial q_t} = \Leftrightarrow \delta \frac{\partial S_{t+1}^a(q_t, \ldots, q_1)}{\partial q_t}$$

has to be compared with condition (5), namely

$$\frac{\partial \Pi_t(q_t, \ldots, q_1, \theta_t)}{\partial q_t} = \Leftrightarrow \delta \frac{\partial \, \overset{s}{\underset{t+1}{}}(q_t, \ldots, q_1)}{\partial q_t}$$

Comparing these conditions for identical output histories, we see that a necessary and sufficient condition for types θ^L to choose the same output under symmetry as under asymmetry is

$$\frac{\partial S_{t+1}^a(q_t, \ldots, q_1)}{\partial q_t} = \frac{\partial \, \overset{s}{\underset{t+1}{}}(q_t, \ldots, q_1)}{\partial q_t} \tag{22}$$

The condition requires the marginal impact of increasing output on total expected future surpluses from continuing the relationship under asymmetric information to equal the marginal impact of increasing output on total expected future surpluses under symmetric information. **This result is stated in the following proposition.**

Proposition 3 *Firms of type θ^L choose a different output under asymmetric information than under symmetric information, unless $\frac{\partial S_{t+1}^a(q_t, \ldots, q_1)}{\partial q_t} = \frac{\partial \Gamma_{t+1}^s(q_t, \ldots, q_1)}{\partial q_t}$*

It is well known that the revelation of private information is achieved at a cost in efficiency, so that, generally, $S_{t+1}^a \neq \, \overset{s}{\underset{t+1}{}}$. Indeed, *Fact 3* identifies type θ^L as the one exception to that rule in a static setup. In a dynamic context, however, a firm of type $\theta_t = \theta^L$ will be of a higher-cost type with a positive probability in some future period, so that its expected future surplus under asymmetry is certainly lower than under symmetry. While it does not follow that (22) is necessarily violated, because (22) is a condition on marginal, rather than total, surpluses, the presumption is that it will

13

be violated in most circumstances.

We have shown that the static properties of static incentive contracts, in particular *Fact 2* (the marginal participating firm is left without any rent) and *Fact 3* (the 'best' firm is asked to operate efficiently) no longer hold in our dynamic setup. Unlike previously studied dynamic asymmetric information models, where the dynamic arises from intertemporal correlations between the privately observed parameter rather than through the impact of past decisions on current profits, this happens despite the fact that the revelation principle still applies. The optimum contracts now involve future expected surpluses. If these are high enough even when the firm does not participate in the current period, then the static rationality constraint (zero profit) no longer ensures participation: the firm faces a strictly positive best alternative, which explains why *Fact 2* no longer holds. The relevance of future expected surpluses to current output decisions also implies that the 'best' firm itself is affected by asymmetry in future information despite our assumption that there is no correlation between stochastic realizations from one period to the other: while the firm may be of type θ^L in the current period, its type in future periods will be worse with a positive probability, so that expected future surpluses associated with type θ^L will be lower than under symmetric information. This is why *Fact 3* no longer holds.

4 An example

Do these findings have any importance for theoretical modeling of dynamic incentive contracts? Perhaps surprisingly, the example below will show that exceptions may arise under assumptions that are widely accepted in dynamic asymmetric information models. Consider the cost of adjustment model mentioned at the beginning of the paper. As is customary in the information literature, assume a fixed number of periods and a range of θ such that, considering prices, all types participate under both informational contexts:

14

$\bar{\theta}_t^s = \bar{\theta}_t^a = \theta^H$. Thus we have, for a firm in business, whose current stock of capital is $K_t = \sum_1^t q_\tau \left(1 \Leftrightarrow \rho\right)^{t-\tau}$

$$\Pi_t(q_1, q_2, \ldots, q_t, \theta_t) = p_t g\left(K_t\right) \Leftrightarrow C\left(q_t, K_t, \theta\right)$$

with $q_t \geq 0$, $K_t \geq 0$, $K_0 > 0$ given[2]. Assume further[3] that $C = \frac{1}{2}q_t^2 + \left(\theta_t + \phi_t\right)q_t + \theta_t$ and that

$$p_t = p \text{ for } t \leq T$$

$$p_t = 0 \text{ afterwards}$$

$$\theta_t \sim f\left(\theta_t\right), \ \theta_t \in \left[\theta^L, \theta^H\right] \text{ for } t \leq T$$

$$\theta_t = 0 \text{ for } t > T$$

$$\phi_t = \phi > 0 \text{ for } t \leq T$$

$$\phi_t = 0 \text{ afterwards}$$

The assumptions on p_t, θ_t, and ϕ_t ensure that no profitable production or investment activity may take place after T, so that the fixed horizon would in fact be endogenously chosen.

Let us consider a firm whose capital is K_t under symmetric, and asymmetric, information alternatively. Under symmetric information, if t is the second last period, so that $t + 1 = T$,

$$\Pi_{t+1}^s = \Pi_T^s = \int_{\theta^L}^{\bar{\theta}_{t+1}^s} \left\{ p_{t+1} g\left(K_{t+1}^s\right) \Leftrightarrow \frac{1}{2}\left[q_{t+1}^s\right]^2 \Leftrightarrow \left[\theta_{t+1} + \phi_{t+1}\right]q_{t+1}^s \Leftrightarrow \theta_{t+1} \right\} f\left(\theta_{t+1}\right) d\theta_{t+1}$$

---

[2] Using the convention that current investment is incorporated into current capital immediately rather than with a lag of one period allows us to illustrate our point more compactly, by considering the second last period rather than period $T - 2$.

[3] Allowing θ to enter as a fixed cost, besides affecting the marginal cost, is not customary in the principal-agent literature. We use this formulation as a simple way to ensure that assumption $\frac{\partial \pi}{\partial \theta} < 0$ is met. This assumption could itself be weakened to allow equality, but the additional mathematical complexities would only obscure the foregoing argument. In Gaudet et al. (1992), we solve a problem where q may take any real value, the fixed cost is observable, and $\frac{\partial \pi}{\partial \theta} \leq 0$.

The interior solution for q_t^s is given by (5), which becomes

$$\Leftrightarrow p_t + q_t^s + \theta_t + \phi_t = \delta p_{t+1} \int_{\theta^L}^{\bar{\theta}_{t+1}^s} \frac{\partial g\left(K_t\left[1 \Leftrightarrow \rho\right] + q_{t+1}^s\right)}{\partial q_t} f\left(\theta_{t+1}\right) d\theta_{t+1} \qquad (23)$$

Similarly, under asymmetric information, if t is the second last period

$$S_{t+1}^a = S_T^a = \int_{\theta^L}^{\bar{\theta}_{t+1}^a} \left\{ p_{t+1} g\left(K_{t+1}^a\right) \Leftrightarrow \frac{1}{2}\left[q_{t+1}^a\right]^2 \Leftrightarrow\left[\theta_{t+1} + \phi_{t+1}\right] q_{t+1}^a \Leftrightarrow \theta_{t+1} \right\} f\left(\theta_{t+1}\right) d\theta_{t+1}$$

The interior solution for q_t^a must satisfy (16), which gives

$$\Leftrightarrow p_t + q_t^a + \theta_t + \phi_t \Leftrightarrow [1 \Leftrightarrow \alpha] \frac{F(\theta_t)}{f(\theta_t)} = \delta p_{t+1} \int_{\theta^L}^{\bar{\theta}_{t+1}^a} \frac{\partial g\left(K_t\left[1 \Leftrightarrow \rho\right] + q_{t+1}^a\right)}{\partial q_t} f\left(\theta_{t+1}\right) d\theta_{t+1} \quad (24)$$

Thus, in period $t = T \Leftrightarrow 1$, for $\theta_t = \theta^L$, the left-hand sides of (23) and (24) are equal, except, possibly, for differences in q_t^a and q_t^s, so that

$$q_t^a < (\geq) q_t^s \Leftrightarrow$$
$$\int_{\theta^L}^{\bar{\theta}_{t+1}^a} \frac{\partial}{\partial q_t^a} g\left(K_t\left[1 \Leftrightarrow \rho\right] + q_{t+1}^a\right) f\left(\theta_{t+1}\right) d\theta_{t+1} < (\geq) \int_{\theta^L}^{\bar{\theta}_{t+1}^s} \frac{\partial}{\partial q_t^s} g\left(K_t\left[1 \Leftrightarrow \rho\right] + q_{t+1}^s\right) f\left(\theta_{t+1}\right) d\theta_{t+1}$$

The above integrals are likely to differ: first, $\bar{\theta}_{t+1}^a$ normally differs from $\bar{\theta}_{t+1}^s$; second, q_{t+1}^a will differ from q_{t+1}^s for all realizations of θ_{t+1} but θ^L. However, allowing for the assumption that $\bar{\theta}^a = \bar{\theta}^s = \theta^H$, and noting that $\frac{\partial K_t}{\partial q_t} = 1$, equality obtains if $\frac{\partial g\left(K_t[1-\rho]+q_{t+1}^a\right)}{\partial q_t} = \frac{\partial g\left(K_t[1-\rho]+q_{t+1}^s\right)}{\partial q_t}$ for all θ_{t+1}, which is true if $\frac{\partial g}{\partial K}$ is independent of K and q i.e. if the production technology exhibits constant returns.

This exception to the general results presented above arises because, with a constant returns technology, the marginal impact of changing q_t on expected future surplus is independent of the future level of K provided the participation margin is unaffected. Indeed, constant returns is also necessary in this example: if g is not linear, the integral on the right-hand side of (24) will differ from the corresponding integral in (23) unless

$q_{t+1}^a(\theta_{t+1}) = q_{t+1}^s(\theta_{t+1})$ for all θ_{t+1}; but such equality itself requires g to be linear as can be verified by comparing the appropriate versions of (23) and (24). As our example clearly illustrates, however, the linearity of the current profit function in K_t is not sufficient to ensure the linearity of cumulative expected future surpluses in q_t. The participation margins $\bar{\theta}_t^a$ and $\bar{\theta}_t^s$ must also be identical and invariant; this is the case when they arise as corner solutions as we have assumed in our example [4].

We should note also that it is easy, although cumbersome, to extend the analysis of our example to earlier periods. Besides the conditions just identified, the survival of *Fact 3* will obtain if, for each supplementary period, the participation margin in the new period is a corner solution at θ^H. Consequently, for the cost of adjustment model, *Fact 3* may arise under apparently benign assumptions and, perhaps, be taken for what it is not: a generic property of the type of dynamic models studied in this paper. The reader can verify that no such possibility arises with the other two examples (learning by doing; resource extraction) mentioned earlier.

---

[4] The assumption that $\bar{\theta}_t^s = \bar{\theta}_t^a = \theta^H$ is satisfied if and only if, at any date

$\max\{\theta_t | \Pi(q_t, ..., q_1, \theta_t) + \delta \, {}_{t+1}^s(q_t, ..., q_1) \geq \max\{0, \delta, {}_{t+1}^s(0, q_{t-1}, ..., q_1)\}\} = \theta^H$

and

$\max\left\{\theta_t | \hat{\Pi}(\theta_t; q_{t-1}, \ldots, q_1) \geq \max\left[0, \delta \Psi_{t+1}^a(0, q_{t-1}, ..., q_1)\right]\right\} = \theta^H$

In particular, when $t = T - 1$ and g is linear (say $g(k) = K$), these assumptions reduce to

$\max\left\{\theta_t | pK_t - q_t^2 - (\theta_t + \phi) q_t - \theta_t \geq 0\right\} = \theta^H$

and

$\max\left\{\theta_t | pK_t - q_t^2 - (\theta_t + \phi) q_t - \theta_t - R_t \geq 0\right\} = \theta^H$

They are obviously satisfied when p and K_t are high enough relative to θ^H and ϕ.

Appendix: Asymmetric information

1 Incentive compatibility

Denote the net total value of the firm at t

$$\phi_t(\tilde{\theta}_t, \theta_t; q_{t-1}, ..., q_1) \equiv \Pi_t(q_t(\tilde{\theta}_t; q_{t-1}, ..., q_1), ..., q_1, \theta_t)$$

$$\Leftrightarrow R_t(\tilde{\theta}_t; q_{t-1}, ..., q_1) + \delta \Psi_{t+1}(q_t(\tilde{\theta}_t; q_{t-1}, ..., q_1), ..., q_1)$$

where $q_t(\tilde{\theta}_t; q_{t-1}, ..., q_1)$, $R_t(\tilde{\theta}_t; q_{t-1}, ..., q_1)$ respectively represent production and the transfer under the menu offered by the principal to the firm in period t, and $\Psi_{t+1}(q_t(\tilde{\theta}_t; q_{t-1}, ..., q_1), ..., q_1) \geq 0$ represents the value discounted to $t+1$, as expected at t, of cumulated future net profits to the firm if it continues its relationship with the principal (we will later on impose conditions under which it is in the interest of the firm to do so). When they correspond to the optimum incentive scheme, these functions will be denoted with a superscript ^{a}. We will also drop arguments $q_{t-1}, ..., q_1$ in order to alleviate notation.

Incentive compatibility is defined by the condition that the firm chooses to reveal its true type when selecting an output-transfer combination from the menu. Thus $\tilde{\theta}_t$ must equal θ_t when the firm chooses it so as to maximize ϕ_t. The first-order condition for period t, $\partial \phi_t / \partial \tilde{\theta}_t = 0$, must be satisfied when $\tilde{\theta}_t = \theta_t$ for all θ_t[5]

$$\left[\frac{\partial \Pi_t(q_t(\tilde{\theta}_t), ..., q_1, \theta_t)}{\partial q_t} + \delta \frac{\partial \Psi_{t+1}(q_t(\tilde{\theta}_t), ..., q_1)}{\partial q_t} \right] \frac{dq_t(\tilde{\theta}_t)}{d\tilde{\theta}_t} = \frac{dR(\tilde{\theta}_t)}{d\tilde{\theta}_t} \qquad (25)$$

when $\tilde{\theta}_t = \theta_t \ \forall \ \theta_t \in [\theta^L, \theta^H]$.

---

[5]Clearly existence and unicity require some additional conditions on Π which we assume to be satisfied.

For any variations $d\theta_t = d\tilde{\theta}_t$, (25) must continue to hold. This, together with the second-order condition $\partial^2 \phi_{t+1}/\partial \tilde{\theta}_t^2 \leq 0$ implies

$$\frac{\partial^2 \Pi_t}{\partial q_t \partial \theta_t} \frac{dq_t}{d\tilde{\theta}_t} \geq 0 \qquad (26)$$

Since $\partial^2 \Pi/\partial q_t \partial \theta_t \leq 0$ (abandoning from here on the distinction between $\tilde{\theta}_t$ and θ_t since they are equal), $\frac{dq_t}{d\theta_t} \leq 0$, which is (11). Also, since by definition $\hat{\Pi}_t(\theta_t) = \phi_t(\theta_t, \theta_t)$, applying the envelope theorem, we have (10): $\frac{d\hat{\Pi}_t}{d\theta_t} = \frac{\partial \Pi_t}{\partial \theta_t} < 0$.

2 The optimal contract

The principal must choose the functions $q_t(\cdot)$ and $R_t(\cdot)$, and the cutoff type $\bar{\theta}_t$ so as to maximize

$$\int_{\theta^L}^{\bar{\theta}_t} \{R_t(\theta_t) + \alpha[\hat{\Pi}_t(\theta_t) \Leftrightarrow \delta \Psi_{t+1}^a(q_t, \ldots, q_1)] + \delta[, {}_{t+1}^a(q_t, \ldots, q_1) + \alpha \Psi_{t+1}^a(q_t, \ldots, q_1)]\} f(\theta_t) d\theta_t$$

where $, {}_{t+1}$ denotes cumulative future transfers to the principal, discounted to $t+1$, as expected at t, and the superscripts in $, {}_{t+1}^a$ and Ψ_{t+1}^a indicate that the agent knows that the principal will find it in her interest to use an incentive mechanism in all remaining periods. Types $\theta_t > \bar{\theta}_t$ are not necessarily out of business; they are just pooled in period t.

Eliminating R_t using the definitions of ϕ_t and $\hat{\Pi}_t$, the problem becomes (15)

$$\max_{q_t(\cdot), \bar{\theta}_t} \int_{\theta^L}^{\bar{\theta}_t} \{\Pi_t(q_t(\theta_t), q_{t-1}, \ldots, q_1, \theta_t) \Leftrightarrow [1 \Leftrightarrow \alpha]\hat{\Pi}_t(\theta_t)$$

$$+\delta[, {}_{t+1}^a(q_t(\theta_t); q_{t-1} \ldots, q_1) + \Psi_{t+1}^a(q_t(\theta_t); q_{t-1} \ldots, q_1)]\} f(\theta_t) d\theta_t$$

19

subject to (10), (13), and (14). The Hamiltonian is

$$H = \{\Pi_t(q_t(\theta_t), ..., q_1, \theta_t) - [1 - \alpha]\hat{\Pi}_t(\theta_t) + \delta S^a_{t+1}(q_t(\theta_t), ..., q_1)\}f(\theta_t)$$
$$+ \mu_t \frac{\partial \Pi_t(q_t(\theta_t),...,q_1,\theta_t)}{\partial \theta_t} + \lambda_t\left[\hat{\Pi}_t(\theta_t) - \delta\Psi^a_{t+1}(0, q_{t-1}, ..., q_1)\right]$$

where μ_t is the costate variable associated with $\hat{\Pi}_t$, $\lambda_t \geq 0$ is a multiplier associated with (14), and S^a_{t+1} is the expected future surplus, defined as the sum of , $^a_{t+1}$ and Ψ^a_{t+1}.

The following necessary conditions must hold

$$\{\frac{\partial \Pi_t(q_t, ..., q_1, \theta_t)}{\partial q_t} + \delta\frac{\partial S^a_{t+1}(q_t, ..., q_1)}{\partial q_t}\}f(\theta_t) + \mu_t\frac{\partial^2 \Pi_t(q_t, ..., q_1, \theta_t)}{\partial \theta_t \partial q_t} = 0 \qquad (27)$$

$$\frac{d\mu_t}{d\theta_t} = [1 - \alpha]f(\theta_t) - \lambda_t(\theta_t) \qquad (28)$$

Also, by (10), $\hat{\Pi}_t(\theta^L)$ necessarily satisfies (13) if so does $\hat{\Pi}_t(\bar{\theta}_t)$; consequently, $\hat{\Pi}_t(\theta^L)$ is free, so that

$$\mu_t(\theta^L; q_{t-1}, \ldots, q_1) = 0 \qquad (29)$$

Note that, by (10), if (14) is binding at $\hat{\theta}_t$, then it is binding for any $\theta_t \geq \hat{\theta}_t$; but if (14) is binding over $\left[\hat{\theta}_t, \bar{\theta}_t\right]$, then $\hat{\Pi}$ is constant over that interval, contradicting (10). It follows that (14) may be binding only at $\theta_t = \bar{\theta}_t$ if at all. As a result, $\lambda_t = 0 \; \forall \; \theta_t < \bar{\theta}_t$ so that, integrating (28) considering (29), we have

$$\mu_t(\theta_t; q_{t-1}, \ldots, q_1) = [1 - \alpha]F(\theta_t) \qquad (30)$$

Substituting for μ_t in (27), we obtain (16)

$$\frac{\partial \Pi_t(q_t, \ldots, q_1, \theta_t)}{\partial q_t} + [1 - \alpha]\frac{F(\theta_t)}{f(\theta_t)}\frac{\partial^2 \Pi_t(q_t, ..., q_1, \theta_t)}{\partial \theta_t \partial q_t} = -\delta\frac{\partial S^a_{t+1}(q_t, ..., q_1)}{\partial q_t}$$

Finally, at $\bar{\theta}_t$, constraints (13) and (14) must be satisfied. Suppose that the binding

one is (13); then the transversality condition is $\mu_t(\bar{\theta}_t)\hat{\Pi}_t(\bar{\theta}_t; q_{t-1}, \dots, q_1) = 0$ so that, since $\mu_t\left(\bar{\theta}_t\right) > 0$, we obtain (17)

$$\hat{\Pi}_t(\bar{\theta}_t; q_{t-1}, \dots, q_1) = 0$$

Suppose now that the binding participation constraint is (14); then we have (18)

$$\hat{\Pi}_t(\bar{\theta}_t; q_{t-1}, \dots, q_1) = \delta \Psi_{t+1}^a(0, q_{t-1}, \dots, q_1) > 0$$

References

Baron, D. P. (1989) "Design of Regulatory Mechanisms and Institutions," in *Handbook of Industrial Organisation*, ed. R. Schmalensee and R. D. Willig. New York: North Holland, 1347–1447.

Besanko, D. and D. E. M. Sappington (1987) *Designing Regulatory Policy with Limited Information.* London: Harwood Academic Publishers.

Gaudet, G., P. Lasserre and N. V. Long (1995) "Optimal Resource Royalties with Unknown and Temporally Independent Extraction Cost Structures," *International Economic Review*, **forthcoming**, .

Laffont, J.-J. and J. Tirole (1988) "The Dynamics of Incentive Contracts," *Econometrica*, **56**, 1153–1175.