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Résumé / Abstract

Dans cet article, nous avons examiné la relation entre la formation d'habitudes et le concept de prudence de Kimball. En utilisant d'abord le modèle de deux périodes de Kimball, nous avons démontré que la formation d'habitudes mène à une prime de prudence plus élevée et une plus grande épargne précautionnelle, pourvu que l'individu ait une prudence absolue décroissante. Nous avons ensuite développé le modèle afin d'investiguer la relation entre prudence et formation d'habitudes dans un sytème à multiples périodes. Nous avons démontré que, même s'il n'y a pas de formation d'habitudes, la prime de prudence n'est pas positive sauf si la propension marginale à la richesse est constante. Par la suite, nous avons trouvé qu'il n'est pas possible de conclure, même dans le cas d'une fonction d'utilité avec une simple formation d'habitudes, que les habitudes augmentent ou diminuent la prime précutionnelle à la Kimball quand il y a de multiples périodes.

In this paper we have examined the relationship between habit formation and Kimball's concept of prudence. Using first, Kimball's two-period model we have shown that habit formation leads to a larger prudence premium and greater precautionary saving, provided that the individual has decreasing absolute prudence. Then, we have extended the model to investigate the relationship between prudence and habit formation in a multi-period framework. We have shown that, even when there is no habit formation, the prudence premium is not unambiguously positive unless the marginal propensity out of wealth is constant. Then we have found that it is not possible to conclude, even with the utility function exhibiting a simple form of habit formation, that habits increase or decrease the precautionary premium "in the sense of Kimball" when there exists multiple periods.

Mots Clés : Formation d'habitudes, prudence

Keywords: Habit formation, prudence

JEL: D81, D91

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1. INTRODUCTION

It has been argued by Deaton [4] that habit formation leads to a behavior akin to prudence. The validity of this notion depends on the definition of concept of prudence. This was Kimball [11] who first introduced the concept of prudence to characterize the "sensitivity of a decision variable to income risk". He argues that income risk leads to precautionary saving when the individual has non-increasing absolute risk aversion. Kimball and Weil [12] extended Kimball's analysis to Kreps-Porteus preferences.[1]

The early literature on individual precautionary saving is associated with Leland [13]. In his analysis of the theory of precautionary saving, he has shown that whenever we depart from the certainty equivalence (CE) framework by allowing the utility function to be time-separable and have a positive third derivative, an increase in labor income uncertainty will reduce current consumption and change the slope of the consumption function when insurance markets are not complete. In a two-period model with a deterministic interest rate, Sandmo [15] shows that if current consumption is a normal good and temporal risk aversion is decreasing, saving is higher under uncertainty.[2]However, if the interest rate is stochastic but income is deterministic he shows that the response of saving to risk is ambiguous. The subsequent literature associated with Miller [14], Sibley [17] and Schechtman [16] generalizes the earlier two-period models' results to arbitrary multiple periods model.

In this paper, we will modify Kimball's model by introducing habit formation in consumption choices, a type of non-separability in preferences, to examine the impact of such an assumption on the prudence measure and mathematically verify the validity of Deaton's argument. In a two-period model, we will show that habit formation leads to a larger prudence premium and greater precautionary saving, provided that the individual has decreasing absolute prudence. Then, we will investigate the relationship between prudence and habit formation in a multi-period framework. We will show that, even when there is no habit formation, the prudence premium is not unambiguously positive unless the marginal propensity out of wealth is constant. Next, we will prove that it is not possible to conclude, even with the utility function exhibiting a simple form of habit formation, that habits increase or decrease the precautionary premium "in the sense of Kimball" when there exists multiple periods.

The implications of habit formation were first discussed in Duesenberry [5]'s work. His proposition was that families are willing to sacrifice saving in order to protect their living standards. In the event of a fall in income, consumption will not fall proportionately, producing a ratchet effect.

Whereas time-separable preferences imply that current utility depends only on current consumption, time-non-separable preferences with habit formation imply that past real consumption patterns and levels form consumer habits which persist long enough to reduce the effects of current income changes on current consumption. For a given level of current expenditures, past purchases contribute to a habit stock. Hence, it is an increase of current consumption over and above the habit stock which raises current utility.

An increase in current consumption in response to an increase in wealth or permanent income has two effects: it increases current utility, holding habit stock fixed; but (everything else equal) decreases utility at $t+1$. Since increasing consumption today generates a future externality, the rational consumer will respond to an increase in wealth or permanent income with a more moderate increase in consumption. In the presence of habit formation, an increase in current consumption increases the marginal utility of future consumption. There is thus, an adjacent complementarity in consumption.

Recent empirical papers in the consumption literature have argued for the role of habits in determining consumption. Constantinides [3], Ferson and Constantinides [8], Dynan [6], Carroll, Overland and Weil [2], Heaton [10], Fuhrer and Klein [9] are among others.

In section 2, we will first introduce habit formation into the Kimball's two-period model of optimal consumption-saving, in which we examine the prudence measure. In section 3, we will extend the relationship between prudence and habit formation in a multi-period framework by using induction method. We will first examine the last two periods and calculate the prudence premium for this specific period. We then repeat the analysis for the initial period t to finally prove for any period j. Section 4 concludes the paper.

2. KIMBALL'S MODEL WITH HABIT FORMATION

In Kimball's model, the consumer makes an optimal consumption-saving decision subject to a risky second period income. Since the preferences are time-non-separable in consumption, the current utility will depend not only on current consumption but also on the habit stock, x_t. The habit formation parameter α is the degree to which the habit stock affects current utility and it is between zero and one. Habit stock x_t is a weighted average of all past consumptions and can be defined as $x_t \equiv (1-\zeta)\sum_{j=0}^{\infty} \zeta^j c_{t-1-j}$, where weights add to one with $(1-\zeta)$ being the depreciation parameter of habits, $0 \leqslant \zeta \leqslant 1$. When the depreciation of habits is equal to one, ($\zeta = 0$), i.e., the case where past values of consumption before c_{t-1} do not affect the habit stock, we have a model which reflects one-period habit formation, i.e. $x_t = c_{t-1}$. For simplicity, we will assume this one-period habit formation in this paper. When we introduce habit formation in this model, the optimization problem becomes:

$$Max_{c_1} \;\; v(c_1 - \alpha c_0) + \beta E v(\widetilde{c}_2 - \alpha c_1) \tag{2.1}$$

$$\begin{aligned} \text{s.t. } c_1 &= y_1 - s_1 \\ \widetilde{c}_2 &= s_1 R + \widetilde{y}_2 \end{aligned}$$

Define c_t as time t consumption, y_t as time t income, s_t as time t saving, $R \equiv 1 + r$ where r is the real interest rate, α is the habit formation parameter, $0 < \alpha \leqslant 1$; a tilde indicates a random variable. $E(.)$ denotes expectations conditional on the information available at period one. Consider a two-period utility function $v(c_1) + \beta v(c_2)$ where v is increasing and concave and β is the discount factor.

The first order condition is:

$$-v'(y_1 - s_1^* - \alpha c_0) + \beta\,(R + \alpha)\,E v'(s_1^* R + \widetilde{y}_2 - \alpha y_1 + \alpha s_1^*) = 0 \tag{2.2}$$

Definition (1) χ is the precautionary premium, i.e., the amount of income such that if the individual has $E\widetilde{y}_2 - \chi$ with certainty he or she chooses the same level of optimal saving s_1^* when facing the risk. That is:

$$\begin{aligned} & -v'(y_1 - s_1^* - \alpha c_0) + (R+\alpha)\,\beta E v'(s_1^* R + \widetilde{y}_2 - \alpha y_1 + \alpha s_1^*) \qquad (2.3) \\ = \;& -v'(y_1 - s_1^* - \alpha c_0) + (R+\alpha)\,\beta v'(s_1^* R + E\widetilde{y}_2 - \chi - \alpha y_1 + \alpha s_1^*) \end{aligned}$$

Taking a second order approximation of the left-hand side (LHS) and a first order approximation of the right-hand side (RHS) around $E\widetilde{y}_2$, and solving for the prudence premium χ, we get:

$$\chi \cong \frac{1}{2}\sigma_y^2 \eta \qquad (2.4)$$

where σ_y^2 is the variance of income and η is the prudence measure:

$$\eta \cong -\frac{v'''(s_1^* R + E\widetilde{y}_2 - \alpha(y_1 - s_1^*))}{v''(s_1^* R + E\widetilde{y}_2 - \alpha(y_1 - s_1^*))} \qquad (2.5)$$

Proposition (1) *The impact of habit formation on prudence is determined according to whether the individual has decreasing, constant or increasing absolute prudence. If absolute prudence, η, is decreasing with an increase in the second* *period income then labor income uncertainty will raise the marginal propensity to consume at a given level of consumption. Under the hypothesis of decreasing absolute prudence (DAP), habit formation makes the individual more prudent: since the prudence premium is higher, precautionary saving increases with habit formation. On the other hand, if there is increasing absolute prudence, then labor income uncertainty will lower the marginal propensity to consume out of wealth at a given level of first period consumption. Under the hypothesis of increasing absolute prudence (IAP), habit formation makes the individual less prudent towards the uncertainty of income, and thus the precautionary savings against income uncertainty will be lower.*

Proof.

Taking the derivative of the prudence measure with respect to the habit formation parameter α, we obtain:

$$\frac{d\eta}{d\alpha} = c_1^* \frac{[v''v'''' - (v''')^2]}{[v'']^2} \tag{2.6}$$

We can also express (**??**) as the change in the precautionary measure with the change in the net expected second period consumption:

$$\frac{d\eta}{d\alpha} = -c_1^* \frac{d\eta}{d\widehat{c}_2}$$

where $\widehat{c}_2$ is the net expected consumption in the second period,

$$\widehat{c}_2 = E(\widetilde{c}_2 - \alpha c_1)$$

Therefore,

$$\frac{d\eta}{d\widehat{c}_2} \leqslant (\geqslant)\ 0 \Rightarrow \frac{d\eta}{d\alpha} \geqslant (\leqslant)\ 0. \blacksquare$$

An important example of DAP is the exponential utility function. Note that with a quadratic utility function $v''' = 0$ so that the prudence premium is zero. Therefore, we conclude that under the hypothesis of decreasing absolute prudence, habit formation increases precautionary saving.

3. PRUDENCE IN AN INFINITE HORIZON PROBLEM

In this section we extend the analysis to an infinite horizon problem. The individual faces a stochastic income stream $\{y_t\}$ which is *i.i.d.* The sequence of real interest rate is known with certainty. We assume that current utility depends on lagged consumption in a general form, as captured by the non-separable utility function $U(c_t, c_{t-1})$. The utility function is continuous, concave in its arguments and has a positive third derivative:

$$U(0) = \infty,\ U' > 0,\ U'' < 0,\ U''' > 0.$$

The value function of the infinite horizon problem is the limit function of a sequence of value functions $\{V^n(W, c)\}$. The dynamic programming problem has become:

$$\begin{aligned} V(W_t, c_{t-1}) &= Max_{W_{t+1}, c_t} \{U(c_t, c_{t-1}) + \beta E_t V(W_{t+1}, c_t)\} \\ &\text{s.t.} : W_{t+1} = R_{t+1}(W_t - c_t) + \widetilde{y}_{t+1} \end{aligned} \tag{3.1}$$

Solving for the prudence premium χ as in the two-period case:

$$\chi \cong \frac{1}{2}\sigma_y^2 \underbrace{[(-\frac{V_{www}R_{t+1}}{V_{ww}R_{t+1} - V_{cw}}) + (\frac{V_{cww}}{V_{ww}R_{t+1} - V_{cw}})]}_{\eta,} \tag{3.2}$$

where η is the measure of prudence. Hence, habit formation introduces a second term in the precautionary premium. It also modifies the denominator of the first term. In what follows we will attempt to determine the sign of each term. In order to determine the signs of the derivatives of the value function V_{ww}, V_{cw}, V_{www}, and V_{cww} we proceed by induction. We first examine the last two periods of a finite horizon T period problem. We then make the induction assumption and finally consider the initial period t.

3.1. THE LAST TWO PERIODS

Definition (2) $V_T(W_T, c_{T-1})$ as the value with zero periods to go (i.e. for the last period) and identical to the last period utility level, $U(c_T, c_{T-1})$, since last period consumption is equal to the wealth available in the last period.

The value function with one period to go is then:

$$\begin{aligned} V_{T-1}(W_{T-1}, c_{T-2}) &= Max_{\{W_T, c_{T-1}\}}\, U(c_{T-1}, c_{T-2}) + \beta E_{T-1} V(W_T, c_{T-1}) \\ \text{s.t.} &: \quad W_T = R_{T-1}(W_{T-1} - c_{T-1}) + y_T \end{aligned} \tag{3.3}$$

Definition (3) The optimal decision rule for consumption is, c_{T-1}, is a function of current wealth and lagged consumption, $c_{T-1} \equiv g(W_{T-1,}c_{T-2})$.

In order to determine the signs of the derivatives of the value function V_{ww}, V_{cw}, V_{www}, and V_{cww} , we take the derivatives of the value function for the period

$T-1$ with respect to its state variables W_{T-1},wealth level in period $T-1$ and c_{T-2}, consumption in period $T-2$.

Proposition (2) *Even in the time-separable preferences case, the prudence premium for the last two period is only unambiguously positive if the marginal propensity to consume out of wealth is constant.*

Proof.

First, we take the second derivative of the value function for period $T-1$ with respect W_{T-1}:

$$\frac{\partial^2 V_{T-1}(W_{T-1})}{\partial W_{T-1}^2} = \beta E_{T-1}[(R_{T-1})^2 \frac{\partial^2 U(c_T)}{\partial c_T^2}(1 - \frac{\partial g}{\partial W_{T-1}})] \tag{3.4}$$

This derivative is negative by the concavity of the utility function and since the marginal propensity to consume out of wealth, $\frac{\partial g}{\partial W_{T-1}}$, is less than one.

Taking the third derivative of the value function for period $T-1$ with respect W_{T-1}:

$$\begin{aligned} \frac{\partial^3 V_{T-1}(W_{T-1})}{\partial W_{T-1}^3} &= \beta E_{T-1}[(R_{T-1})^3 \frac{\partial^3 U(c_T)}{\partial c_T^3}(1 - \frac{\partial g}{\partial W_{T-1}})^2 \\ &\quad -(R_{T-1})^2 \frac{\partial^2 U(c_T)}{\partial c_T^2}(\frac{\partial^2 g}{\partial W_{T-1}^2})] \end{aligned} \tag{3.5}$$

The infinite horizon prudence measure without habit formation, η', will be the ratio of these two derivatives:

$$\eta' = -\frac{E_{T-1}\left\{R_{T-1}U'''(1-\frac{\partial g}{\partial W_{T-1}})^2 - U''(\frac{\partial^2 g}{\partial W_{T-1}^2})\right\}}{E_{T-1}U''(1-\frac{\partial g}{\partial W_{T-1}})}$$

η' will be greater than zero if the following condition holds:

$$R_{T-1}(-\frac{U'''}{U''}) > \frac{-(\frac{\partial^2 g}{\partial W_{T-1}^2})}{(1-\frac{\partial g}{\partial W_{T-1}})^2} \tag{3.6}$$

Since the third derivative of the utility function is positive by assumption, $U''' > 0$, under decreasing or constant absolute risk aversion and since the concavity of the utility function implies that $U'' < 0$, the sign of the third derivative of the utility function with respect to wealth will be positive. Then the prudence premium is only unambiguously positive if the marginal propensity to consume out of wealth is constant, i.e. if $\frac{\partial^2 g}{\partial W_{T-1}^2} = 0$. This completes the proof.[3]■

Therefore, the individual must be sufficiently prudent with respect to period T consumption risk for him or her to be prudent with respect to wealth risk. This comes from the fact that the prudence premium is only unambiguously positive if the marginal propensity to consume out of wealth is constant. Then, for the last two periods case, we can write the sufficient condition for the third derivative of the value function with respect to wealth to be positive under no habit formation specification is the following:

$$-\frac{\frac{\partial^3 U}{\partial c_T^3}}{\frac{\partial^2 U}{\partial c_T^2}} > \frac{-(\frac{\partial^2 g}{\partial W_{T-1}^2})}{(1-\frac{\partial g}{\partial W_{T-1}})^2}(\frac{1}{R_{T-1}}).$$

3.1.1. ONE-PERIOD HABIT FORMATION

Now, let us consider the specific functional form that we have used in the section 2, namely $U(c_t - \alpha c_{t-1})$ and repeat the same procedure to find the signs of the derivatives of the value function with respect to its state variables.

Then the value function for one period before the last period is as follows:

$$\begin{aligned} V_{T-1}(W_{T-1}, c_{T-2}) &= Max_{W_T, c_{T-1}}\ U(c_{T-1} - \alpha c_{T-2}) + \beta E_{T-1} V(W_T, c_{T-1}) \\ \text{s.t.} &: \quad W_T = R_{T-1}(W_{T-1} - c_{T-1}) + y_T \end{aligned} \tag{3.7}$$

Taking the derivative of the value function for period $T-1$ with respect to W_{T-1}, wealth in period $T-1$:

$$\frac{\partial V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}} = \beta E_{T-1} R_{T-1} \frac{\partial U(W_T, c_{T-1})}{\partial c_T} \tag{3.8}$$

where we have the identity of:

$$\frac{\partial U}{\partial c_T} \equiv \frac{\partial V(W_T, c_{T-1})}{\partial W_T}$$

Now recall that the next period's wealth is defined as:

$$W_T = (R_{T-1}(W_{T-1} - g(W_{T-1,}c_{T-2}) + y_T)$$

and the optimal decision rule c_{T-1} being a function of current wealth and lagged consumption:

$$c_{T-1} = g(W_{T-1,}c_{T-2}).$$

Next, we take the derivative of the left-hand side of (**??**) with respect to its state variables c_{T-2} and W_{T-1} respectively to obtain:

$$\frac{\partial^2 V_{T-1}(W_{T-1,}c_{T-2})}{\partial W_{T-1}\partial c_{T-2}} = \beta E_{T-1} R_{T-1} \frac{\partial g}{\partial c_{T-2}} \left[-\frac{\partial^2 U}{\partial c_T^2} R_{T-1} + \frac{\partial^2 U}{\partial c_T \partial c_{T-1}} \right] \qquad (3.9)$$

This derivative is positive since the term $\frac{\partial g}{\partial c_{T-2}}$, which shows to what extent current consumption is affected by lagged consumption, is positive. Habit formation implies that the higher the previous period's consumption, the higher has to be the current period's consumption level in order to guarantee a positive utility level.

Then taking the derivative of (??) with respect to W_{T-1} :

$$\frac{\partial^2 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2} = \beta E_{T-1} R_{T-1} U''[R_{T-1} - (R_{T-1} + \alpha)(\frac{\partial g}{\partial W_{T-1}})] \tag{3.10}$$

(??) is less than zero and the value function is concave in wealth if the following holds:

$$\frac{\partial g}{\partial W_{T-1}} < \frac{R_{T-1}}{R_{T-1} + \alpha} \tag{3.11}$$

i.e. if the marginal propensity to consume is not too large. Since lagged consumption increases current consumption due to habit formation, the denominator of the precautionary premium χ is also negative.

Recall that the precautionary premium with habit formation is equal to:

$$\chi \cong \frac{1}{2}\sigma_y^2(-\frac{V_{www}R_{t+1} + V_{cww}}{V_{ww}R_{t+1} - V_{cw}})$$

Now, in order to determine the magnitude and the sign of the numerator of the precautionary premium under habit formation, we take the derivative of (??) with respect to W_{T-1} to obtain:

$$\frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^3} = \beta E_{T-1} R_{T-1} \{[(R_{T-1})^2 U'''(1 - \frac{\partial g}{\partial W_{T-1}})^2]$$
$$-[R_{T-1} U'' \frac{\partial^2 g}{\partial W_{T-1}^2}] + [R_{T-1}(-\alpha U''')(1 - \frac{\partial g}{\partial W_{T-1}})(\frac{\partial g}{\partial W_{T-1}})]$$
$$+[\alpha^2 U'''(\frac{\partial g}{\partial W_{T-1}})^2] + [(-\alpha U'')(\frac{\partial^2 g}{\partial W_{T-1}^2})]\} \tag{3.12}$$

Then, this derivative is positive if the two conditions hold:

- Condition 1

$$\left(1 - \frac{\partial g}{\partial W_{T-1}}\right)^2 R_{T-1}^2 + \alpha^2 (\frac{\partial g}{\partial W_{T-1}})^2$$
$$>$$
$$(\frac{\partial^2 g}{\partial W_{T-1}^2}) \frac{U''}{U'''} [R_{T-1} + \alpha] + \alpha \left(1 - \frac{\partial g}{\partial W_{T-1}}\right) R_{T-1}$$

- Condition 2

$$(\frac{\partial g}{\partial W_{T-1}}) < \frac{R_{T-1}}{[R_{T-1} + \alpha]}$$

Condition 2 implies that the marginal propensity to consume has to be not too large for the third derivative of the value function to be unambiguously positive with habit formation.

Since (**??**) is positive because lagged consumption increases current consumption, $\frac{\partial g}{\partial c_{T-2}} > 0$, the first term in the precautionary premium which is equal to

$$\frac{\frac{\partial^3 V_{T-1}}{\partial W_{T-1}^3}}{\frac{\partial^2 V_{T-1}}{\partial W_{T-1}^2} R_{T-1} - \frac{\partial^2 V_{T-1}}{\partial W_{T-1} \partial c_{T-2}}}$$

is positive.

Next, to find out the sign of V_{wwc}, we take the derivative of (**??**) with respect to c_{T-2} :

$$\frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2 \partial c_{T-2}} = (\frac{\partial g}{\partial c_{T-2}}) \beta E_{T-1} R_{T-1} [R_{T-1}(-\alpha U''')(1 - 2\frac{\partial g}{\partial W_{T-1}})$$
$$-(R_{T-1})^2 U'''(1 - \frac{\partial g}{\partial W_{T-1}}) + \alpha^2 U'''(\frac{\partial g}{\partial W_{T-1}})]$$
$$-\{\beta E_{T-1} \frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}} [U''(R_{T-1})^2 - R_{T-1}(-\alpha U'')]\} \quad (3.13)$$

First, note that the term $\frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}}$ is negative and shows the extent to which the marginal propensity to consume out of wealth is affected by lagged consumption. Since lagged consumption increases current consumption, the first term in braces is negative if :

- Condition 1: The third derivative of the utility function is positive. This holds by assumption.

- Condition 2: The marginal propensity to consume out of wealth, $\frac{\partial g}{\partial W_{T-1}}$, is not too large and being less than $\frac{R_{T-1}}{R_{T-1}+\alpha}$.

Since the term $\frac{\partial^2 g}{\partial c_{T-2} \partial W_{T-1}}$ is negative, $\frac{\partial^3 V_{T-1}(W_{T-1}, c_{T-2})}{\partial W_{T-1}^2 \partial c_{T-2}}$ is negative too. Therefore, the second term in the precautionary premium which is equal to:

$$\frac{\frac{\partial^3 V_{T-1}}{\partial W_{T-1}^2 \partial c_{T-2}}}{\frac{\partial^2 V_{T-1}}{\partial W_{T-1}^2} R_{T-1} - \frac{\partial^2 V_{T-1}}{\partial W_{T-1} \partial c_{T-2}}}$$

is also positive. The first step of the induction proof indicates that under the assumed utility function $U(c_t - \alpha c_{t-1})$, habit formation results in an additional precautionary premium.

3.2. THE INITIAL PERIOD t

After proving that the prudence premium is positive with habit formation for the last two periods, we will proceed the analysis by the induction method.

Proposition (3) *Under the induction assumption, the following inequalities hold:*

$$\frac{\partial^2 V_t}{\partial W_t \partial c_{t-1}} > 0; \frac{\partial^2 V_t}{\partial W_t^2} < 0; \frac{\partial^3 V_t}{\partial W_t^3} > 0; \frac{\partial^3 V_t}{\partial W_t^2 \partial c_{t-1}} < 0; and \frac{\partial^3 V_t}{\partial W_t \partial c_{t-1}^2} > 0.$$

Proof.

Writing the value function as for period t:

$$
\begin{aligned}
V_t(c_{t-1}, W_t) &= Max_{W_{t+1}, c_t} \quad U(c_t - \alpha c_{t-1}) + \beta E_t V_{t+1}(W_{t+1}, c_t) \\
\text{s.t.} &: \quad W_{t+1} = R_t(W_t - c_t) + y_{t+1}
\end{aligned}
$$

Calculating the first order condition with respect to c_{t-1}, and taking its complete differentiation with respect to c_t, c_{t-1} and W_t gives us:

$$
\begin{aligned}
&\left[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t\right] dc_t + \left[\frac{\partial^2 U}{\partial c_t \partial c_{t-1}}\right] dc_{t-1} \\
&\left[-\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t\right] dW_t = 0 \qquad (3.14)
\end{aligned}
$$

From (**??**) we can calculate the effect of lagged consumption on optimal consumption rule $\frac{\partial g}{\partial c_{t-1}}$ as:

$$
\frac{\partial g}{\partial c_{t-1}} = \frac{\alpha U''}{[U'' + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t]} \qquad (3.15)
$$

The effect of lagged consumption on optimal consumption is positive by the habit formation assumption.

Similarly calculating the exact term for the marginal propensity to consume out of wealth, $\frac{\partial g}{\partial W_t}$, we obtain:

$$\frac{\partial g}{\partial W_t} \equiv \frac{dc_t}{dW_t} = -\frac{[-\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t]}{[U'' + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t]} \tag{3.16}$$

The marginal propensity to consume out of wealth is positive based on the assumptions about the utility function.

Taking the derivative of the current period value function with respect to current wealth, we find:

$$\frac{\partial V_t(W_{t,} c_{t-1})}{\partial W_t} = \beta E_t R_t \left[\frac{\partial V_{t+1}(R_t(W_t - c_t) + y_{t+1}, c_t)}{\partial W_{t+1}} \right] \tag{3.17}$$

Then, taking the derivative of (**??**) with respect to c_{t-1} :

$$\frac{\partial^2 V_t(W_{t,} c_{t-1})}{\partial W_t \partial c_{t-1}} = \frac{\partial g}{\partial c_{t-1}} \beta E_t \{ -\frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 + \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t \} \tag{3.18}$$

(**??**) is greater than zero since the effect of lagged consumption on optimal consumption is positive by the habit formation assumption, i.e. $\frac{\partial g}{\partial c_{t-1}} > 0$.

Unfortunately, the signs of:

$$\frac{\partial^2 g}{\partial W_t \partial c_{t-1}}, \quad \frac{\partial^2 g}{\partial W_t^2}, \frac{\partial^2 V_t}{\partial W_t^2}, \frac{\partial^3 V_t}{\partial W_t^2 \partial c_{t-1}}, \frac{\partial^3 V_t}{\partial W_t^3} \quad \text{and} \quad \frac{\partial^2 g}{\partial c_{t-1}^2}$$

are ambiguous and cannot be determined on the basis of the induction assumption. (See Appendix for the exact expressions).■

It is therefore not possible to conclude even by restricting the utility function to the form $U(c_t - \alpha c_{t-1})$ that habit formation increases the precautionary premium "in the sense of Kimball" when there exists multiple periods.

On the other hand, if there were no habit formation, the precautionary premium will be unambiguously positive if and only if:

$$R_t E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3}\left(1 - \frac{\partial g}{\partial W_t}\right)^2 > E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2 \partial c_t}\left(\frac{\partial^2 g}{\partial W_t^2}\right)$$

confirming the result that has been proven for the last two periods. That is, the prudence premium is unambiguously positive if and only if the marginal propensity to consume out of wealth, $\frac{\partial^2 g}{\partial W_t^2} = 0$, is constant. Thus, the individual must be sufficiently prudent with respect to period $t+1$ consumption risk in order to have a positive precautionary premium.

4. CONCLUSION

In this paper we have examined the relationship between habit formation and Kimball's concept of prudence. Using first, Kimball's two-period model we have shown that habit formation leads to a larger prudence premium and greater precautionary saving, provided that the individual has decreasing absolute prudence. Then, we have extended the model to investigate the relationship between prudence and habit formation in a multi-period framework. We have shown that, even when there is no habit formation, the prudence premium is not unambiguously positive unless the marginal propensity out of wealth is constant. Then we have found that it is not possible to conclude, even with the utility function exhibiting a simple form of habit formation, that habits increase or decrease the precautionary premium "in the sense of Kimball" when there exists multiple periods.

Notes:

1.Subsequently, Eeckhoudt and Schlesinger [7] further explored the implications of the link between risk aversion and prudence.

2.Let $U(c_1, c_2)$ denote the two-period utility function where c_t is consumption at time t=1,2. Sandmo assumes that $\frac{-\partial^2 U/\partial c_2^2}{\partial U/\partial c_2}$ is decreasing in c_2 and increasing in c_1. He defines this condition as decreasing temporal risk aversion.

3.However, according to Carroll and Kimball [1], consumption function is generally a concave function of wealth. This implies that the marginal propensity to consume out of wealth decreases in wealth, that is the term $\frac{\partial^2 g}{\partial W_{T-1}^2}$ is negative in general. Two exceptions include the exponential utility function when the interest rate is deterministic but income is random, and the constant relative risk aversion utility function when income is deterministic while the interest rate is random.

APPENDIX

Taking the derivative of (**??**)with respect to c_{t-1} we find:

$$\frac{\partial^2 g}{\partial c_{t-1}\partial W_t} = \{\frac{-\frac{\partial^2 U}{\partial c_t^2 \partial c_{t-1}}\frac{\partial g}{\partial W_t}[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]^2}\}$$
$$+\{\frac{\frac{\partial[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]}{\partial W_t}\frac{\partial^2 U}{\partial c_t \partial c_{t-1}}}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]^2}\}$$

Taking the derivative of (**??**)with respect to W_t we find:

$$\frac{\partial^2 g}{\partial W_t^2} = \{\frac{\frac{\partial[-\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]}{\partial W_t}}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]^2}\}$$
$$\times[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]$$
$$-\{\frac{\frac{\partial[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]}{\partial W_t}}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]^2}\}$$
$$\times[-\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}{R_t}^2 + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}R_t]$$

By taking the derivative of (**??**) with respect to c_{t-1} we find:

$$
\begin{aligned}
\frac{\partial^2 g}{\partial c_{t-1}^2} &= \{\frac{-\frac{\partial(\frac{\partial^2 U}{\partial c_t \partial c_{t-1}})}{\partial c_{t-1}}[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t]}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t]^2}\} \\
&+\{\frac{\frac{\partial[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t]}{\partial c_{t-1}} \frac{\partial^2 U}{\partial c_t \partial c_{t-1}}}{[\frac{\partial^2 U}{\partial c_t^2} + \beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2}(R_t)^2 - 2\beta E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t} R_t]^2}\}
\end{aligned}
$$

Taking the derivative of (**??**) with respect to W_t and c_{t-1}, we find:

$$
\frac{\partial^2 V_t(W_{t,}c_{t-1})}{\partial W_t^2} = \beta E_t R_t [\frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} R_t (1 - \frac{\partial g}{\partial W_t}) + \frac{\partial^2 V_{t+1}}{\partial W_{t+1}\partial c_t}\frac{\partial g}{\partial W_t}]
$$

and

$$
\begin{aligned}
\frac{\partial^3 V_t(W_{t,}c_{t-1})}{\partial W_t^2 \partial c_{t-1}} &= \frac{\partial g}{\partial c_{t-1}} R_t \{R_t \beta E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^2 \partial c_t}(1 - 2\frac{\partial g}{\partial W_t}) \\
&- R_t^2 E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3}(1 - \frac{\partial g}{\partial W_t}) + \beta E_t \frac{\partial^3 V_{t+1}}{\partial W_{t+1}\partial c_t^2}\frac{\partial g}{\partial W_t}\} \\
&- \beta(\frac{\partial^2 g}{\partial W_t \partial c_{t-1}}) R_t [R_t E_t \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} - E_t \frac{\partial^2 V_{t+1}}{\partial c_t \partial W_{t+1}}]
\end{aligned}
$$

respectively.

The third derivative of the value function for period t with respect to its state variable W_t is the following:

$$\frac{\partial^3 V_t}{\partial W_t^3} = \frac{\partial g}{\partial W_t}\{\beta E_t(R_t)^2 \frac{\partial^3 V_{t+1}}{\partial c_t \partial W_{t+1}^2}(1 - \frac{\partial g}{\partial W_t}) \quad + R_t \beta E_t \frac{\partial^3 V_{t+1}}{\partial c_t^2 \partial W_{t+1}}(\frac{\partial g}{\partial W_t})\}$$
$$-\beta[E_t(R_t)^2 \frac{\partial^2 V_{t+1}}{\partial W_{t+1}^2} - E_t R_t \frac{\partial^2 V_{t+1}}{\partial c_t \partial W_{t+1}}](\frac{\partial^2 g}{\partial W_t^2}) \quad + \beta E_t(R_t)^3 \frac{\partial^3 V_{t+1}}{\partial W_{t+1}^3}(1 - \frac{\partial g}{\partial W_t})^2$$

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