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Factor Analysis and Independent Component Analysis in Presence of High Idiosyncratic Risks*

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Résumé / Abstract

Cet article traite le cas d'un marché d'actions dont les rendements sont susceptibles d'être expliqués par une structure factorielle. Sur le marché américain, il est montré que des risques idiosyncratiques élevés existent pour la plupart des actions quelque soit le modèle d'évaluation utilisé (CAPM ou APT). La présence de ces risques idiosyncratiques élevés peut empêcher une évaluation correcte des facteurs générant les rendements, lorsqu'une méthode d'analyse factorielle classique est utilisée. Il est ici proposé d'utiliser la méthode de l'Analyse en Composantes Indépendantes (INCA), reposant sur les réseaux neuronaux, pour parvenir à une évaluation correcte des facteurs; cette méthode permet de prendre en compte la majeure partie de l'information contenue dans les distributions des rendements des actions, en utilisant les moments d'ordre élevé de ces distributions. À l'aide de simulations de marchés artificiels, pour lesquels différentes hypothèses des processus de générations des rendements sont retenus, il est montré que la méthode de l'INCA permet une amélioration significative de l'estimation de la structure factorielle, en particulier lorsque des composantes idiosyncratiques élevées sont présents dans les les rendements des actions. Dans ce dernier cas, une méthode classique d'analyse factorielle, comme l'Analyse en Composantes Principales, peut échouer totalement dans l'estimation des facteurs.

This paper addresses the case when stock market returns are assumed being generated through a factorial structure. High levels of idiosyncratic risk are shown to exist for most stocks on the US market, when CAPM or APT are used for the estimation of diversifiable risks. The presence of these high idiosyncratic risks may not allow a correct estimation of the generating factors when using a classic factor analysis method. The Independent Component Analysis is introduced as an adequate method for factor estimation; using neural networks, this method allows taking into account the information contained in higher moments. Through simulations of markets with various assumptions on the kind of processes followed by the generating factors, this method is shown to strongly improve the factors estimation, especially when high idiosyncratic risks are present. In the latter case,

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a traditional factor analysis, such as the Principal Component Analysis, may fail to estimate the generating factors.

Mots Clés : Analyse en composantes indépendantes, analyse en composantes principales, modèle d'évaluation par arbitrage, risques idiosyncratiques

Keywords: Independent component analysis, principal component analysis, arbitrage pricing theory, idiosyncratic risks

1 Introduction

Multifactorial analysis of stock returns shows an increasing interest since the development of the Arbitrage Pricing Theory (APT) by (Ross 1976) and extensions of this theory. If the parameters of the Capital Asset Pricing Model (CAPM) are well known (the market portfolio and the sensibilities of the stocks to this portfolio), one challenge of the APT is that the generating factors are *a priori* unknown. These factors are either to be assumed as derived from some observable series, following the work of (Chen, Roll, and Ross 1986) for instance, or must be estimated from the series of stock returns. As the generating factors constitute one of the *assumptions* of the model, empirical analysis of the APT hence depends on the correct preliminary estimation of the generating factors. Existing studies of APT show however evidence that the rightness of the factor estimation are subject to discussion. On a first hand, the estimated factors usually can not be identified to any known data series, except the market portfolio returns, and it could be expected from the model that all the estimated factors could more or less be identified. On a second hand, as it is shown for instance by (Dhrymes, Friend, and Gultekin 1984), the factors estimation is not stable when a smaller period or less assets are used: the number of factors increase when more assets are considered, which can reject the assumption of the existence of a generating structure. In this paper, it is studied in which conditions the estimation of factors can be successfully led. It is put in evidence that the presence of high-level risks is not innocent in the factors estimations process and may not allow a correct estimation of the factor series by classical factor analysis.

Review of APT principles and studies are briefly described in section 2. It is proposed in section 3 to use an alternative method to the classical factors analysis methods, the INdependent Component Analysis method (InCA), which was originally proposed by (Herault and Jutten 1991) and lays on neural networks techniques. InCA includes high-order correlations between stock returns in order to perform a better factor analysis.

Methods for estimating factors are usually methods that were originally developed for observable processes in which noises could be neglected or of a small effect. In section 4, evidence is shown on the US stocks market that high noises (or in financial terms, idiosyncratic risks) exist for most of the stocks, in the sense that these risks are of the same level or even greater than the diversifiable risks calculated by the CAPM or the APT. In order to test the impact of high-level residual risks on the factors estimation, and to perform comparisons between methods, stock markets are simulated in section 5, with various assumptions on the generating processes of the factors and noises (brownian motions, stochastic volatility processes and jump processes), and various levels of idiosyncratic risks (noises) are introduced. Section 6 tests the behavior and performance of the estimations for these various simulated markets when INdependent Component Analysis or Principal Component Analysis is used. The stability of the estimations is studied when only a few assets, or smaller periods are used. Finally section 7 concludes the paper.

2 Arbitrage Pricing Theory

The Arbitrage Pricing Theory was proposed by (Ross 1976) as an alternative to the Capital Asset Pricing Model developed by (Sharpe 1964), (Lintner 1965) and (Mossin 1966). The fundamental assumption of the model is that the $i = 1..N$ stocks returns are linearly generated by a small number of K common factors:

$$\tilde{r}_i = E[\tilde{r}_i] + b_{i1}\tilde{s}_1 + b_{i2}\tilde{s}_2 + ... + b_{iK}\tilde{s}_K + \tilde{\epsilon}_i, \tag{1}$$

where

- $\tilde{s}_j$ is a factor with no correlation to any other factor and b_{ij} is the loading of this factor j for the i-th asset;
- the expectations of the errors $\tilde{\epsilon}_i$ are zero, their variances are bounded and the errors are mutually independent and independent of factors; in mathematical terms: $E[\tilde{\epsilon}_i] = 0$, $\sigma^2(\tilde{\epsilon}_i) < \infty$, $E[\tilde{\epsilon}_i|\tilde{\epsilon}_j] = 0$, $\forall i \neq j$ and $E[\tilde{\epsilon}_i|\tilde{s}_k] = 0$, $\forall i, k$.

With the assumption that returns are generated through this factorial structure, (Ross 1976) derives from non arbitrage principles that there exists $(K+1)$ coefficients λ_0, λ_1, ..., λ_K such that for every asset i:

$$E[\tilde{r}_i] = \lambda_0 + \lambda_1 b_{i1} + ... + \lambda_K b_{iK}, \tag{2}$$

where λ_0 is the risk premium for an asset with no sensibility to any of the market factors (zero-beta portfolio or riskless asset) and λ_k is the risk premium for the k-th factor.

In Ross model, the APT relation is an exact relation if diversifiable risk can be eliminated. Ross presumes that the number of assets is large enough such that the covariance matrix of the errors can be considered as diagonal. (Huberman 1982) and (Ingersoll 1984) develop the Ross model by considering a sequence of economies with an increasing number of risky assets, which allows them to derive bounding errors for the relation. The APT as an equilibrium model is developed by (Connor 1984), (Chen and Ingersoll 1983), and (Grinblatt and Titman 1987), by adding the assumption that it is possible for one investor to diversify his portfolio without restraining his choices. With this hypothesis, for which conditions are studied by (Chen and Ingersoll 1983), (Grinblatt and Titman 1983), (Grinblatt and Titman 1987), (Wei 1987), the APT relation may actually be written as an exact relation and the APT as an equilibrium model. Derivations of the APT as an equilibrium model with a finite number of assets implies that the factorial structure is an approximative structure, which case is considered by (Chamberlain and Rothschild 1983).

As it was noticed before, the APT fundamental assumption is that the returns are generated by factors, which are mutually independent. In order to test the model, more assumptions must be added, for instance the factors are assumed to be stationary processes and the factors loadings are assumed to be constant. In this paper, conditions for a correct factor estimation are discussed and it is assumed that other assumptions on the generating structure are correct. With a matrix notation, the APT assumption is that the zero-meaned observed returns $\tilde{\mathbf{r}}(t)$ at time t are generated through the linear transform:

$$\tilde{\mathbf{r}}(t) = \mathbf{A}\tilde{\mathbf{s}}(t) + \tilde{\varepsilon}_t, \tag{3}$$

where $\mathbf{A} \in \mathbf{R}^{n \times m}$ is an unknown mixing matrix, and $\tilde{\varepsilon}_t$ is a noise term. The term $\tilde{\varepsilon}_t$ will here be omitted as it can not be separated from input signals; $\tilde{\mathbf{s}}$ and observed $\tilde{\mathbf{r}}$ are assumed to be of expectation zero. The first step is hence to estimate the factors $\tilde{\mathbf{s}}(t)$ (and the matrix $\mathbf{A}$), and to recover the original signals from the observations $\tilde{\mathbf{r}}(t)$, by a linear transform of the following kind:

$$\tilde{\mathbf{y}}(t) = \mathbf{W}\tilde{\mathbf{r}}(t), \tag{4}$$

where $\tilde{\mathbf{y}}(t)$ are the mutually independent recovered signals which approximate the actual input signals $\tilde{\mathbf{s}}(t)$, and $\mathbf{W} \in \mathbf{R}^{m \times n}$ is a de-mixing matrix. In a most general way, the estimation is a blind estimation as neither the actual input signals, nor the mixing characteristics, nor the signals characteristics (high or low bandwith, deterministic or not, etc.) are known, the only assumption being that sources are mutually independent and that the mixing is a linearly process. Two indeterminations are inherent to the problem: it is impossible to know neither the order of the signals nor their amplitude, hence the estimated signals $\mathbf{y}(t)$ are at best a transformation of actual input signals $\mathbf{s}(t)$:

$$\tilde{\mathbf{y}}(t) = \mathbf{DP}\tilde{\mathbf{s}}(t), \tag{5}$$

where $\mathbf{D}$ is a diagonal matrix, and $\mathbf{P}$ a permutation matrix (at best) or a rotation matrix. Both matrices $\mathbf{D}$ and $\mathbf{P}$ can not be determined; in practice, variances of signals (factors) are normalized to one ($\mathbf{D} = \mathbf{I}$).

Various methods exist in order to estimate the matrix $\mathbf{W}$. Two methods will here be used in order to compare the results of the Independent Components Analysis which is later proposed. The method of Principal Component Analysis consists in choosing the K first principal components in order to represent the maximum of the dispersion of the N returns. The classical PCA uses the $N \times N$ matrix of the covariance matrix of the assets of which the K eigenvectors corresponding to the highest eigenvalues are retained. The second method which will be used follows (Connor and Korajczyk 1986) who propose to use the $T \times T$ covariance matrix of the T observations rather than the $N \times N$ covariance matrix of assets, as a way to improve the estimation of the factors. One problem of this method is of course that it is much more computer-time expensive than a classical Principal Component Analysis[1].

Almost all APT empirical studies use either factor analysis in order to estimate the portfolios replicating generating factors, like in the seminal study of (Roll and Ross 1980), or Principal Component Analysis (PCA), as in (Connor and Korajczyk 1988). One alternative method based on an autoregressive approach was proposed by (Mei 1993). Usual results of these studies is that the number of necessary factors is between three and five. These results are found for instance by (Roll and Ross 1980), (Lehmann and Modest 1988) or (Connor and Korajczyk 1988). Mei finds that at least seven factors are necessary. But, as found by (Trzcinka 1986) or (Dhrymes, Friend, and Gultekin 1984), the number of factors increases as the number of assets increase, and the estimated factor structures are not the same on different samples. This non stability of factors is a severe problem for APT, as it could reject the assumption of a generating factor. An other explanation of this unstability is that

[1] The third method which is commonly used is factor analysis. It is shown however by (Chamberlain and Rothschild 1983) that PCA and factor analysis are asymptotically equivalent. The PCA methods are used for comparison because they are the more similar to the InCA method which is proposed in the paper.

the methods used in order to estimate the factors lead to wrong estimations. This paper focuses on the latter explanation, by showing that existence of high-level idiosyncratic risks may bring severe problems in estimation of factors through traditional factor analysis.

3 Independent Component Analysis

One natural way to correct the problem of a wrong estimation is to add more information in the component analysis; the method of Independent Component Analysis developed in this section can bring a solution as the method includes high-order statistics to perform the analysis. The method uses neural networks techniques which have shown large interest in the late years in nonlinear analysis but also as a way to modelize and easily improve well-known classical linear analysis.

One first application of neural networks to data analysis was for the estimation of the principal components. (Oja 1982) first proposes to use a network allowing to estimate the first principal component and the algorithm is extended to the case of K principal components by (Oja and Karhunen 1985), and (Sanger 1989) who treats the case of local minima. Figure 1 represents the architecture of neural networks which are used in these methods: the network is composed of one input layer with N cells and one output layer with K cells. Observed signals (returns) are presented to the input layer, and the output layer gives, after convergence of the network, the estimated factor series.

Noting $\mathbf{x}(t)$ the zero-mean N-vector of input signals $\tilde{\mathbf{r}}(t)$ (the observed signals) and $\mathbf{y}(t)$ the K-vector of output signals (which are the generating signals to estimate), the response of the neural network is computed as: $\mathbf{y}(t) = \mathbf{W}\mathbf{x}(t)$, where $\mathbf{W}$ is a $K \times N$ mixing matrix. The goal of the algorithm is to minimize the costs function $e(\mathbf{W})$:

$$e(\mathbf{W}) = \frac{1}{2}\|\mathbf{e}\|^2, \tag{6}$$

where $\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}}$ and with $\hat{\mathbf{x}} = \mathbf{W}'\mathbf{y} = \mathbf{W}'\mathbf{W}\mathbf{x}$. Minimization of this function leads to the following learning algorithm at each step k:

$$\mathbf{W}(k+1) = \mathbf{W}(k) + \eta(k)[\mathbf{x}'(k) - \mathbf{y}'(k)\mathbf{W}(k)]. \tag{7}$$

Karhunen and Oja demonstrate that the algorithm converges to a linear combinaison of the K principal eigenvectors of Ω. Principal Component Analysis using neural networks has some subject-to-discussion advantages to the classic estimation method: it allows continuous learning from observed vectors, but adaptation of the mixing matrix is not always desirable as this tends to loose old informations; an other advantage of neural networks is that the variance-covariance matrix Ω has not to be estimated, but this advantage depends on the actual convergence time needed by the neural networks, which can be the same or greater that the time needed for one Ω estimation. Both algorithms lead however to the same results.

(Herault and Jutten 1991) propose to use an alternative approach to Principal Component Analysis, referred to as Independent Component Analysis (InCA). In order to estimate generating factors, InCA is designed not to search the principal components, which allow to represent the maximum of the returns dispersion, but the more independent factors which can linearly generate the returns. The algorithm includes higher order statistics than the second order moments (covariances) which are used by the PCA. Neural network used by (Herault and Jutten 1991) and following extensions are recurrent networks using nonlinear learning rules, which architecture is the same as the PCA neural networks architecture (figure 1). The goal of the algorithm is to find the mutually independent input signals $y_i(t)$ satisfying for every pair of signals (i,j) and for different values of parameters k and l:

$$E[y_i^{2k+1}(t)\, y_j^{2l+1}(t)] = 0, \quad \forall i, j. \tag{8}$$

Herault and Jutten propose to use the even functions as output nonlinear functions:

$$\begin{cases} \varphi(y) & = (\beta_1 y)^3 \\ \psi(y) & = \beta^3 \tanh(\beta_2 y) \end{cases} \tag{9}$$

where $\beta_1, \beta_2, \beta_3$ are strictly positive numbers. Taylor expansion of the product of these functions gives a sum of terms in y^{2k+1}, which all have to be zero to assure the convergence in a high-order statistics framework.

As Herault and Jutten algorithms treat only the case of two sources, and can not be easily extended to the case of more sources, various derivations have recently been done for this algorithm. For example, (Oja and Karhunen 1995) under the name of principal nonlinear component analysis, (Bell and Sejnowski 1995) through maximization of the entropy, (Cichoki, Unbehauen, Moszczynski, and Rummert 1994), or (Cardoso and Laheld 1996) under the name of equivariant adaptative algorithm. I will here use the (Amari, Cichocki, and Yang 1995) algorithm derivation, which lays on the *Kullback-Leiber* divergence measure and the Gram-Charlier expansion for stochastic variables. The divergence between signals y_i, $i = 1..K$, is measured by the *Kullback-Leiber* measure between joint distributions and product of marginal distributions, which is written as:

$$D(\mathbf{W}) = \int \frac{p(\mathbf{y}) \ln p(\mathbf{y})}{\prod_{i=1}^{K} p_i(y_i) \ln p_i(y_i)}. \tag{10}$$

The key of the algorithm consists in applying a 4-th order Gram-Charlier expansion to approximate the marginal distributions $p_i(y^i)$. It is beyond this paper to make this development, which can be found in (Amari, Cichocki, and Yang 1995). The Kullback-Leiber divergence measure $D(\mathbf{W})$ is used as the cost function of the network and developing the gradient descent algorithm leads to the general learning rule:

$$\mathbf{W}(k+1) = \mathbf{W}(k) + \eta(t)\left[\mathbf{I} - f(\mathbf{y})\mathbf{y}'\right]\mathbf{W}(k), \tag{11}$$

where the activation function $f(y)$ is defined by:

$$f(y) = \frac{29}{4}y^3 - \frac{47}{4}y^5 - \frac{14}{3}y^7 + \frac{25}{4}y^9 + \frac{3}{4}y^{11} \tag{12}$$

At each step, $\mathbf{y}_k$ is calculated as $\mathbf{y}_k = \mathbf{W}\mathbf{x}_k$, where $\mathbf{x}_k = \mathbf{W}'\mathbf{y}_k$.

The initialization of the network and the learning parameter η have a crucial importance for convergence performance. Here the initial value of $\mathbf{W}$ will be set as the value given by the PCA method; η is assigned a great value at the beginning of the algorithm (avoiding overflow traps), and is gradually decreased to 0. The algorithm converges to a unique solution with a correct choice for η and provided that the following conditions (14) are satisfied.

The choice of a 4-th order Gram-Charlier expansion is an arbitrary choice and the expansion could have been done at a higher level, but this do not give much improvement to the algorithm and computing time would be considerably increased. This 4-th order expansion allow to treat information included in dependencies of 3-th and 4-th order moments. As it is usually observed on financial markets, skewness and kurtosis is an important part of distributions and inclusion of these higher-order dependencies may hence allow better estimations.

The mathematical and statistical context of the InCA principles are first studied by (Common 1994). Study of the algorithm convergence and stability is led by (Cardoso and Laheld 1996) who consider sources with strong skewness components. (Amari, Chen, and

Cichocki 1997) study the algorithm stability in a more general framework when the learning rule of the algorithm is given by (11). Defining

$$\begin{cases} \sigma_i^2 = E[y_i^2], \\ k_i^2 = E[\dot{f}(y_i)], \\ m_i = E[y_i^2 \dot{f}(y_i)], \end{cases} \tag{13}$$

for the observed signals y_i, with $\dot{f}(y) = \frac{\partial f(y)}{\partial y}$, the learning algorithm is stable if the following conditions are verified :

$$\begin{cases} m_i + 1 > 0, \\ k_i > 0, \\ \sigma_i \sigma_j k_i k_j > 1, \end{cases} \tag{14}$$

for all signals y_i, y_j. With these conditions, the algorithm converges, and converges to a unique solution.

The use of Independent Component Analysis seems appropriated in the Arbitrage Pricing Theory framework. One default of PCA and generally of factor analysis is that the method do not search the independent components which generate the returns, but principal components. It will tend to isolate the components with the greater impact on the market, while taking into account only the two first moments. As it is noted by (Connor and Korajczyk 1988), this may tend to extract a sole principal factor. InCA tries to find a succession of components which are statistically the more independent possible; this point of view seems more in agreement with APT principles. The second advantage of the method is that it uses high-order statistics, hence the greatest part of the information which can be used considering only past returns. The inclusion of these high-order moments is likely to allow a more appropriate estimation. As it will be seen in the remaining of this paper, this is especially the case when high-level idiosyncratic risks exist on the market.

4 High idiosyncratic risks: Evidence from the US market

The ideal framework for factor analysis is when observed signals are linearly generated by a set of input signals and when noises are mainly due to observation errors and small errors in signal transmissions. The case of a financial market is slightly different. If it can be assumed that stock returns are dependent on a small set of macro or micro-economic factors, these assets represent nevertheless firms with various inner lives. Specific factors can not only be considered as negligible noises, but also as firms specific behaviors. The only assumption of classic pricing models is that the specific returns are mutually independent and can not be diversified, besides the fact that they are equal to zero in expectation. If these assumptions are accepted, idiosyncratic risks can however be important and it is here shown that their values can be as important as the part of risks which is diversifiable. As it will be seen later, the fact that these idiosyncratic risks are important can lead to inaccurate factors estimations when using traditional factor analysis.

To show the level of residual specific risks on factor estimation, a 4 year period was used going from January 1, 1994 through December 31, 1997. All the US stocks daily prices available from Datastream historical database in this period of time had been downloaded[2]. Returns were computed from the prices as logarithmic returns. Only stocks existing on the whole period were retained and among them only stocks with at least 60% of non-zero returns, in order to decrease the effect of a possible liquidity premium. Through this procedure, 1157 raws of stocks returns could be used. Two data sets were built. In the first set, assets were alphabetically sorted and grouped in 76 portfolios, each portfolio containing 15 or 16 stocks. In the second set, 120 stocks were randomly selected from the 1157 available stocks.

It is assumed that the zero-meaned N-vector $\tilde{\mathbf{r}}_t$ of daily returns are generated through the factor structure:

$$\tilde{\mathbf{r}}_t = A\tilde{\mathbf{s}}_t + \tilde{\epsilon}_t. \tag{15}$$

where $\tilde{\mathbf{r}}_t$ is a K-vector of factors realisations, A the $N \times K$ of factors loadings, and $\tilde{\epsilon}_t = \tilde{\mathbf{r}}_t - A\tilde{\mathbf{s}}_t$ are noises or specific factors. It is assumed that the model is correct and that the only sources of risks are the factors $\tilde{\mathbf{s}}_t$ or the specific factors $\tilde{\epsilon}_t$. It is also assumed that the factors loadings are constant for every asset. Let us define the vector of global risk ξ_r on individual stocks as:

$$\xi_r = \text{diag}[\text{cov}(\tilde{\mathbf{r}}_t')], \tag{16}$$

the diversifiable component of risk ξ_d:

$$\xi_d = \text{diag}[\text{cov}((A\tilde{\mathbf{s}}_t)')], \tag{17}$$

and the remaining specific risks ξ_s:

$$\xi_s = \text{diag}[\text{cov}(\tilde{\epsilon}_t)'] = \text{diag}[\text{cov}((\tilde{\mathbf{r}}_t - A\tilde{\mathbf{s}}_t)')]. \tag{18}$$

[2]The data were retrieved from the Datastream connexion at University of Geneva, Switzerland.

It can be expected that $\xi_r \simeq \xi_d + \xi_s$, depending on the diagonality of the errors $\tilde{\epsilon}_t$.

On the two data sets, the values ξ_r, ξ_d and ξ_s were computed for three models: the CAPM and two versions of the APT for which respectively 5 and 10 generating factors are considered. For the CAPM, the unique factor is the market portfolio: $\tilde{\mathbf{s}}_t = \tilde{\mathbf{r}}_{mt}$; the Datastream Vontobel index has been used as a proxy for the market portfolio returns $\tilde{\mathbf{r}}_{mt}$ and A is the usual beta vector $\beta = (\beta_1, \beta_1, ..., \beta_N)$, where the β_i's, $i = 1..N$, are defined by $\beta_i = \sigma_{im}/\sigma_m$. For the APT, Principal Component Analysis or Connor-Korajczyk method are used to estimate the factor loadings A and the factor series $\mathbf{s}_t$, and the number of factors is set as a rather high number of 10.

For sets of portfolios and of individual stocks, levels of computed risks are sorted and displayed respectively in figures 2 and 3. The minimum and maximum values, means and standard deviations of ξ_r, ξ_d and ξ_s are reported for both sets in table 1. The analysis of residual risks shows that the level of these risks can be of the same order or greater than the diversifiable risks. When individual assets are considered (figure 3), the idiosyncratic risks are much higher than the diversifiable risks, for all the models. Using portfolios (figure 2) decreases the part of idiosyncratic risks, as it could be expected from the smoothing effect of building these portfolios. Residual risks stay however high even when portfolios are used. They are greater than the diversifiable risks calculated for the CAPM and can not be neglected even when a rather high number of factors are used in the APT. It can be argued that not enough factors are introduced in the structure. A number of ten factors is however greater than the number of factors which are considered by most studies of APT in US market.

Figures 2 and 3 give also a good indication of the way that models work when factors are added to the structure. CAPM catches an almost constant level of risks across assets or portfolios, whatever the global risk of individual assets or portfolios. When adding factors, the new model catches the risks of assets or portfolios containing the most important residual risks. This shows a rather not intuitive behaviour of the model in a financial market: the addition of one factor covers in preference the asset with the higher residual risk, and the method seems to behave as if this asset series was actually added into the structure. Finally, Connor-Korajczyk method seems to lead to a model with lower explanatory power as the diversifiable risks are always lower than the diversifiable risks obtained by the classical PCA method.

It can be argued that as long the idiosyncratic risks are not diversifiable, the estimation of factors do not suffer of their presence. This should be true if these risks were small. What is found is that idiosyncratic risks are not negligible but of the same level of diversifiable risks and moreover not *uniform*. When estimating the covariance matrix, idiosyncratic variances will mix with the diversifiable variances in an unknown way as these variances can be more or less important. This can be neglected if there are idiosyncratic risks are small or uniform; when this is not true, as it seems to be the case, estimation may become inaccurate. By simulating artificial stock markets in the following sections, it will be shown that classic factor analysis can dramatically fail to estimate the factorial structure when high idiosyncratic risks exist.

5 Simulated markets

In order to empirically test the factor estimations when stocks are generated through a factor structure, three types of markets were simulated with different assumptions on the generating stochastic processes and with different levels of idiosyncratic risks for each generated stock in the market. Returns $\tilde{r}_{it}$ of one stock i are generated following the model:

$$\tilde{r}_{it} = \lambda_0 + \sum_{k=1}^{K} b_{ik}\tilde{s}_{kt} + \rho\omega_i\tilde{\varepsilon}_{it}, \tag{19}$$

where $\tilde{s}_{kt}$ are the simulated generating factors, b_{ik} the loadings of the factors for asset i, $\tilde{\varepsilon}_{it}$ a blank noise with standard deviation ω_i, and ρ a parameter allowing to control the level of specific risks.

In a first step, factors $\tilde{s}_{kt}$ and noises $\tilde{\varepsilon}_{it}$ are generated. The K number of factors is set as $K = 2$. Three types of markets are generated, referred as A, B and C, depending on the assumptions which are made on the processes followed by factors and noises. Factors and noises, i.e. assets specific factors, are assumed to follow the same type of process.

- In the first market A, factors correspond to log-normal prices, generated through the geometric brownian motion:

$$dX(t) = \mu X(t)dt + \sigma X(t)dz_t, \tag{20}$$

  where dz_t is a Wiener process. For all series, σ is a constant equal to an annualized value of 0.14, and μ is set to zero.

- In the second market B, factors follow a stochastic volatility model. In this model, the factors X follow a stochastic process:

$$dX(t) = \mu X(t)dt + \sigma_t X(t)dz_1, \tag{21}$$

where the volatility σ_t is such that $\ln \sigma_t$ follows a mean-reverting process:

$$d \ln \sigma = \beta(\bar{a} - \ln \sigma)dt + \gamma dz_2. \tag{22}$$

The value of a has been set to -4.968, corresponding to a mean-reverting level of 0.11 on a annual basis. Values of β and γ have been set respectively to 0.0316 and 0.0949, corresponding to annualized values of 0.5 and 1.5.

- Finally, the third market C is generated through a jump-process:

$$dX(t) = [\alpha X(t) - \lambda X(t)\bar{y}(t)]dt + \sigma X(t)dz_t + y(t)dQ_t, \tag{23}$$

where α is the instantaneous expected change in X per unit of time, σ^2 is the instantaneous variance of the change in X, conditional on the change being a brownian motion outcome, λ is the probability per unit time that the change in X is a jump process outcome, $y(t)$ is the random variable outcome for the change in X, conditional on the change being a jump process outcome. The value of λ has been set to 0.05 (corresponding to a mean of one jump every 20 observations), and $y(t)$ has been taken as a normal random value with standard deviation equals to 3.5σ.

For each of the three markets, factors and noises have been generated with the same kinf of processes, leading to the generation of $(N+K)$ independent series. As series are randomly generated but finite, the covariances are not strictly zero; these covariances are small but as levels of noise is controlled in a second step, it would be impossible to assure reasonable independence between noises and factors. The independence at level two is assured by performing the following transformation: let us define the $(N+K) \times T$ matrix $\mathbf{X} = [\tilde{\mathbf{r}}; \tilde{\epsilon}]$, this matrix is transformed into a new matrix $\mathbf{X}^*$ by

$$\mathbf{X}^* = VD^{-1/2}V'\mathbf{X}, \tag{24}$$

where V is the matrix of eigenvectors of $\Omega = \mathbf{X}\mathbf{X}'$ and D is a diagonal matrix with eigenvalues of $\mathbf{X}$ on the diagonal. New factors $\tilde{\mathbf{r}}^*$ and noises $\tilde{\epsilon}^*$ are taken as $\mathbf{X}^* = [\tilde{\mathbf{r}}^*; \tilde{\epsilon}^*]$, which assures that covariances between new noises $\tilde{\epsilon}^*$ and factors $\tilde{\mathbf{r}}^*$, as long as the covariances of errors and of factors are zero. After this final transformation, the means of factors have been rescaled to annualized values of respectively 0.16 and 0.8 and their annualized standard deviation to 0.14. Simulation use a number of $N = 100$ stocks observed on $T = 1000$ times. This number of stocks is a realistic number on small markets (like the Swiss market for instance), and $T = 1000$ data for each stock would correspond to four years of daily data or twenty years of weekly data. Finally, factors loadings are randomly selected. As actual betas in financial markets are not uniformly distributed, the loadings are taken from a Student law with a d.f. of 10. The means of factors loadings are forced to be one.

Statistics of the generating factors for the three markets are reported in table 2 together with statistics on the market indices for seven stock markets from 1984 until 1997. This table gives an idea of the realism of the simulatged markets: the mean and standard deviation are compatible with what is observed on most international markets. The kurtosis of the C market (jump-process) is compatible with the actual observed kurtosis which are important in all the markets, indicating the existence of fat tails; the skewness of actual markets is not perfectly approached by the simulated markets: the simulated B market (stochastic volatility) has the most important component of skewness. Finally the market A (generated by geometric brownian motions) seems the less realistic in respect to the actual markets statistics.

After the generation of factors and of factors loadings, noises are added to the structure. For each market, different levels of noises are added. All noises have been standardized to have mean 0 and their standard deviations w_i have been randomly taken from a Student law with a d.f. of 10. The level of idiosyncratic risks compared to the level of diversifiable risks are controlled through a parameter δ which will vary from 0 (no noise) to 2 (idiosyncratic risks are twice as important as the diversifiable risks). This value of δ is obtained by setting the value of ρ in equation (19) in order that the mean of the standard deviations of $\rho\omega_i\epsilon_i$ (specific risks) divided by the mean of the standard deviations of $\Gamma_i = \sum_k b_{ik}(s_{kt} - \bar{s}_k)$ (diversifiable risks) is equal to δ:

$$\delta = \rho \cdot \frac{\sum_{i=1}^{N} \sqrt{\omega_i^2 \epsilon_i \epsilon_i'}}{\sum_{i=1}^{N} \sqrt{\Gamma_i \Gamma_i'}}. \tag{25}$$

For each type of markets A, B and C, 40 markets are generated with a value of δ varying from 0.05 to 2. Figure 4 displays the global and residual risks for the generated market of type A (gaussian processes). Comparing with the risks which were obtained on actual markets and displayed in figures 2 and 3, a value for δ around $\delta = 1$ seems the more realistic case when portfolios are used, but a value as high as $\delta = 2$ seems realistic when individual stocks are considered. The value of $\delta = 1$ corresponds to a case where the mean of the residual (isosyncratic) risks of assets is equal to the mean of the diversifiable risks. The realism of these values is confirmed when examining the twenty largest eigenvalues which are displayed in figure 6 for both the simulated markets with different values of δ, and for US market when individual assets or portfolios are considered: the shape of the eigenvalues function corresponds to a value of δ around 2 when individual assets are considered, and between $\delta = 0.75$ and $\delta = 1.25$ when portfolios are considered.

6 Factor estimations on simulated markets

The three markets simulated in the previous section have been used to estimate the factors by considering only the generated stock returns. The results of Independent Component Analysis are compared to the results given by Principal Component Analysis and to the results given by the Connor-Korajczyk method. In a first try, small noises ($\delta = 0.05$) are added to each stock returns, that is a small part of the stock risk is due to its idiosyncratic risk. In a second try, high levels of noises are introduced; these risks will be of the same level or greater than the diversifiable risks due to the factors.

6.1 Markets with small noises components ($\delta = 0.05$)

In a first step, only small noises are added, and the value of δ is fixed to 0.05. This corresponds to an almost perfect case where almost all the stock risks are diversifiable, and the only variations from the diversifiable returns can be seen as information transmission errors. The $\hat{s}_{it}$ generating factors have been estimated by PCA and by InCA. In order to test the estimation performance, the following regressions were led for the two factors s_{1t} and s_{2t} which actually generated the returns:

$$\begin{cases} s_{1t} = a_{10} + a_{11}\hat{s}_{1t} + a_{12}\hat{s}_{2t} + \eta_{1t} \\ s_{2t} = a_{20} + a_{21}\hat{s}_{1t} + a_{22}\hat{s}_{2t} + \eta_{2t} \end{cases} \tag{26}$$

Results of these regressions are reported for the three analysis methods and for the three types of markets A, B and C in table 3.

In the case of the gaussian market (market A), Principal and Independent Component Analysis should theoretically show the same behaviour. Indeed, moments of 3-th order are null and 4-th order are not discriminant. In this case the algorithm is equivalent to the Oja and Karhunen algorithm for Principal Component Analysis. As expected, factors are correctly estimated for all the decompositions, with R^2 near to one, but have been rotated.

A remarkable fact is that the estimated factors are almost equal to the actual factors for Independent Component Analysis when considering market B or C, for which skewness and kurtosis exist. In this case, Independent Component Analysis allows finding factors which are very near to the actual factors, with a negligible rotation part. This is illustrated in figure 5 where estimated factors are compared to the corresponding actual factors. It must be noticed, however, that the order and the scale are not determined, and some factors may be of opposite sign to the real factors. InCA method can decrease rotation problems of factor estimation when there exist significant skewness and kurtosis components. As it is usually concluded, the rotation of factors is however not a fundamental problem in a financial framework as the main goal is to catch the generating space. Avoiding factor rotations can nevertheless make easier the identification of factors; moreover if the significativity of the factor premiums are considered as a criteria for deciding on the number of factors, the estimated factors should not be rotated during the components estimation.

6.2 Markets with various levels of noises

As noticed above, additional terms in the factorial structure should not be considered in financial markets as only measurements errors or needless information terms. In fact, strong volatilities may exist for one asset even if the specific factor is not diversifiable and can therefore not be included in a diversified allocation. In this section, strong additional noises are added to the returns generated by the factors. The value δ has been increased from 0.05 to 2 with steps of 0.05. For a value $\delta = 0.05$, almost no specific factors exist for stocks. When the value is 2, the specific risks of stocks are on average twice as large as the values of diversifiable risks.

In order to test the rightness of the decompositions, the same regressions were led as in the precedent section:

$$\begin{cases} s_{1t} = a_{10} + a_{11}\hat{s}_{1t} + a_{12}\hat{s}_{2t} + \eta_{1t} \\ s_{2t} = a_{20} + a_{21}\hat{s}_{1t} + a_{22}\hat{s}_{2t} + \eta_{2t} \end{cases} \tag{27}$$

where s_{it} $(i = 1..2)$ is the actual i-th factor used to generate the returns, and $\hat{s}_{1t}$ and $\hat{s}_{2t}$ are the estimated factors. The results of these regressions for the three types of markets A, B and C and for three component analysis methods are reported in table 4, with values of δ equal to 0.05, 0.5, 1 and 2. The lowest R^2 values of the two regressions and for δ increasing from 0.05 to 2 are presented in figure 7.

Introduction of high-level risks appear to conduct to a wrong estimation of factors by classical methods as soon as $\delta > 0.75$. For $\delta = 1$, which has been seen as a realistic value for δ on a actual financial market, both PCA and Connor-Korajczyk method fail to estimate the second factor; indeed the R^2 of the regression is near to zero for one of the actual factor, what means that this factor can not be retrieved by the estimated factors. The behavior of Connor-Korajczyk method seems even less robust than PCA to the introduction of these residual risks, as both factors are wrongly estimated, and not only one as it is the case with PCA. If high-residual risks exist, what seems to be the case in real markets, this has great consequences on the estimation of factors: Connor-Korajczyk method may fail to estimate all the factors; PCA may fail to estimate all the factors except one, and this one factor is likely to be the portfolio market which can be expected as having the most influence in the market stocks returns. As the factor structure is the main assumption of APT, a wrong estimation of these factors will of course lead to unpredictable and no pertinent test of the APT itself.

The InCA, INdependent Component Analysis, which is here proposed succeeds in estimating these factors, and appears to be robust to the introduction of high-level risks. Even with a level of residual risks twice larger than the diversifiable risks ($\delta = 2$), the factor estimation with InCA is still correct. The introduction of these risks do not even modify the behaviour of InCA in terms of rotation, as the method estimates the factors with a minimum rotation, provided that skewness and kurtosis are present in the stock returns. One remarkable behaviour of InCA method is that even in the case of a gaussian market, where moments of 3rd and 4th order are not pertinent information, the method allows a correct estimation of the factors.

6.3 Estimations through subgroups of assets

An important aspect of factor analysis is the stability of the decomposition when only a part of the data is observed or when the model is tested on subgroups of assets [see for instance (Grinblatt and Titman 1987)]. In order to test the stability on subgroups of assets, the sample was divided into two subsets of assets, each subset containing the same number of assets ($N_1 = N_2 = 50$), and the following regression was led for $i = 1, 2$:

$$\hat{g}_{it} = a_{i0} + a_{i1}\hat{s}_{1t} + a_{i2}\hat{s}_{2t} + \epsilon_{it}, \tag{28}$$

where $\hat{g}_{it}$ is the i-th factor estimated with data of the first group, and $\hat{s}_{1t}$ and $\hat{s}_{2t}$ are factor series estimated on the second group. Results of the regression for the three types of markets are reported in table 5 for $\delta = 0.05, 1, 2$. The figure 8 reports the lowest R^2 obtained for the regressions.

In the case of Principal Component Analysis methods, the factor structure obtained by using PCA is not stable when the assets are divided into two subgroups, even when the estimation seemed previously correct for values of δ smaller than 1. Estimated factors are different from one subgroup to an other, which indicates that factors are wrongly estimated in one of the samples or in both. An intuitive conclusion is that if high residual risks exist on the market, a greater number of assets must be used in order to correctly estimate the factor structure. Reciprocally, the chance to get a wrong estimation of factors when high-level idiosyncratic risks exist strongly increase when the number of assets is smaller. Grouping assets into portfolios has hence an unknown impact on the rightness of the estimation: if idiosyncratic risks decrease for portfolios, which can lead to improve the estimation, the size of the sample which can be used also decreases, which process can lead to annihilates the expected improvement of the estimation. In many studies of the APT, stocks are grouped into subsamples, and the test of the model is led independently in each subgroup; for instance, the seminal study of (Roll and Ross 1980) constitute groups containing 60 stocks each. The existence of high idiosyncratic risks apparently indicates that a correct estimation of factors is not possible with such a small number of stocks.

The Independent Component Analysis gives good results for all three markets, even if the performance of the estimation appears to decrease when less assets are used. The R^2 values decrease from almost 1 to values around 0.7 or 0.8 when high-levels of residual risks are introduced, but the estimation remains pertinent. Between the two groups, the factors are retrieved without rotation, except permutation or scale change (here by an opposite sign); this is even the case for the gaussian markets where the estimated factors were rotated.

6.4 Estimations through subperiods

The stability over time is *a priori* less important for financial markets as it is not excluded by the APT model that one factor appears or disappears during a long period. As a known structure was used to generate the returns, it is however important to look at the properties of the estimations when a shorter period of time is used, especially when the PCA gives poor results; indeed, it could be argued that a too short period of time was used in this case. The $T = 1000$ times period is divided into two equal $T_1 = T_2 = 500$ times subperiods, and the following regression is run:

$$\hat{g}_{it} = a_{0i} + a_{1i}\hat{s}_{1t} + a_{2i}\hat{s}_{2t} + \epsilon_{it}, \tag{29}$$

where $\hat{g}_{it}$ is the factor estimated in the subperiod, $\hat{s}_{1t}$ and $\hat{s}_{2t}$ are factors estimated using the whole period of time, of which realisations are retained only in the subperiod where the $\hat{g}_{it}$ factors are estimated.

Results for the regressions are reported in table 6 only for a value $\delta = 2$, for which all methods expect InCA fail to correctly estimate the generating factors. Connor-Korajczyk method leads to inconsistent estimations what is not surprising as both factors were wrongly estimated with this method; but strangely enough the other methods lead to the same estimations even if the number of observations decrease. This is a good thing when InCA is used as this method already estimated correctly the factors, and it means that the method is robust to the decreasing of the number of observations (providing that this number is however significant). Most importantly, the stability over periods is found for PCA method even when this method was doing a poor job. This apparently means that PCA has no chance to find the correct result even if the period was much longer, and that the failure of PCA is due to the method itself and not to an estimation problem which could occur if the number of observations was too small.

7 Conclusion

This paper addresses two problems: What is the impact of high idiosyncratic risks on the factor estimations in a multifactorial model? Which method in such a case may perform a correct estimation of factor series?

The first part of this paper puts in evidence that high idiosyncratic risks exist for the stocks of the US market, when the CAPM is assumed to be the correct model but also when the APT is used with a rather large number of 10 factors. The magnitude of these idiosyncratic risks is of the same level as the diversifiable risks calculated by considering a CAPM or an APT when portfolios are considered, and greater when individual stocks are considered.

In case of high levels of idiosyncratic risks, a classic factor analysis like PCA may fail to estimate the factors. This had been shown by simulating markets with different assumptions on the generating processes. As soon as the level of idiosyncratic risks is around the level of diversifiable risks given by the generating factor structure, only one factor of two is correctly estimated by this method. The critical point is around $\delta = 1$, that is when idiosyncratic and diversifiable risks are of the same magnitude. As this value seems consistent with what is observed on actual markets, PCA has a great chance to lead to wrong factor structure estimation in a real financial market. The method seems very sensible to the number of stocks that are included in the sample, and a number of 50 portfolios or 100 individual assets are shown to likely lead to wrong estimations.

I propose here to use the Independent Component Analysis as a new method for estimating the generating factor series. Although this method is based on neural networks techniques, its advantage is to be easily understandable and applicable. Moreover, it can be viewed as an extension of PCA in which high-order statistics information is included. Empirical tests put in evidence the robustness of the method with respect to the inclusion of high idiosyncratic risks. Even when very high idiosyncratic risks are added, twice as large as the diversifiable risks, InCA succeeds in retrieving the correct factor structure. The structure is stable when assets are divided into subgroups. This stability of the structure is important in the APT framework as an unstability of the structure prevents testing the model in a subgroup of assets as an equilibrium model; this is the main advantage of APT, as noted by (Grinblatt and Titman 1983). The non stability of the factor structure obtained by classic factor analyis is the main critic which can be addressed to the use of these methods, as shown for instance by (Chen 1983). Besides, one empirical advantage of InCA is that the method allows rebuilding the input signals without rotation when significantly high-order dependence exists between these input factors. The only indetermination remains the order and the scale of factors, which is unavoidable. Of course, rotation may still exist, but its smallness may allow an easier identification of the actual generating factors.

The use of InCA seems moreover well adapted to the case of factor estimation in a financial multifactorial framework such as the APT, by avoiding the fact that factor analysis tends to isolate the components which have the greatest impact on the market. The second advantage of InCA is to allow considering higher-moments in the distributions, especially skewness or kurtosis, that is the greatest part of information when only past returns are considered. In that sense, the advantage for InCA is that no assumption is made about the returns distributions, which is one feature of APT. Empirical studies using factor analysis or PCA take usually only the second moments (variances and covariances) into account, and implicitly assume that the risk can be represented in a mean-variance world. A final interesting feature of the Independent Component Analysis is that the method explicitly considers that the factor structure is not strict, but an approximative one. InCA indeed uses the four first moments for decomposition and the fact that residual covariances are not necessarily zero.

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		CAPM		PCA5		PCA10		CK10	
Portfolios	ξ_r	ξ_d	ξ_s	ξ_d	ξ_s	ξ_d	ξ_s	ξ_d	ξ_s
Mean	0.139	0.085	0.105	0.105	0.088	0.112	0.079	0.076	0.111
StdDev	0.032	0.022	0.041	0.028	0.027	0.035	0.021	0.045	0.015
Min	0.097	0.037	0.059	0.068	0.027	0.069	0.020	0.022	0.080
Max	0.229	0.140	0.220	0.228	0.152	0.228	0.123	0.213	0.145
Individual	ξ_r	ξ_d	ξ_s	ξ_d	ξ_s	ξ_d	ξ_s	ξ_d	ξ_s
Mean	0.321	0.095	0.303	0.130	0.278	0.157	0.254	0.135	0.263
StdDev	0.139	0.051	0.137	0.135	0.099	0.162	0.084	0.169	0.080
Min	0.133	0.008	0.132	0.012	0.100	0.018	0.042	0.010	0.089
Max	0.868	0.229	0.860	0.862	0.600	0.867	0.459	0.863	0.457

Table 1: Global risks and residual (idiosyncratic) risks

Global and residual risks on the US market are presented for the period going from January 1, 1994 through December 31, 1997. ξ_r represent the total risks of assets; ξ_d the diversifiable risks calculated by the model and ξ_s represents the specific remaining risks. The risks are calculated for the CAPM, for the APT where the 5 (PCA5) or the 10 factors (PCA10) are estimated using Principal Component Analysis, and for the APT with 10 factors computed through the Connor-Korajczyk method (CK10). The risks are computed when all the stocks available during the whole period are grouped into 76 portfolios, or when a random selection of 120 individual stocks are considered.

Market indices

Country	Mean	StDev	Skewness	Kurtosis	KS	ProbaKS
Germany	0.102	0.168	-1.053	10.808	0.081	0.000
Canada	0.084	0.117	-1.824	34.418	0.092	0.000
France	0.128	0.169	-0.688	8.333	0.066	0.000
Japan	0.018	0.183	-0.343	15.111	0.083	0.000
Switzerland	0.147	0.151	-1.743	21.413	0.100	0.000
United Kingdom	0.110	0.137	-1.475	21.879	0.053	0.000
United States	0.143	0.150	-3.370	70.577	0.097	0.000

Simulated markets

Market A	Mean	StDev	Skewness	Kurtosis	KS	ProbaKS
Factor 1	0.160	0.140	0.014	0.015	0.026	0.492
Factor 2	0.080	0.140	0.119	-0.097	0.021	0.777
Market B	Mean	StDev	Skewness	Kurtosis	KS	ProbaKS
Factor 1	0.160	0.140	-0.246	1.440	0.040	0.076
Factor 2	0.080	0.140	-0.163	0.911	0.039	0.097
Market C	Mean	StDev	Skewness	Kurtosis	KS	ProbaKS
Factor 1	0.160	0.140	0.105	17.927	0.105	0.000
Factor 2	0.080	0.140	-0.008	16.308	0.088	0.000

Table 2: Statistics on generated factors and on market indices
The first panel reports the statistics of market indices (daily returns) for seven markets during the period going from January 1, 1984 through December 31, 1997, corresponding to $N = 3652$ returns observation. The second panel reports statistics of factors used to artificially generate stock returns (1000 observations). The reference values for the normal distribution are $\sqrt{15/N}$ for skewness (0.0641 for the actual markets and 0.1224 for the simulated markets) and $\sqrt{96/N}$ for kurtosis (0.1621 for the actual markets and 0.3098 for the simulated markets). Means and standard deviations are annualized. The last columns give the scores of Kolmogorov-Smirnov and the corresponding probabilities that the factors distributions are normal.

Market A

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.687	0.726	1.000	s_1	0.322	0.946	0.999	s_1	-0.941	0.339	1.000
	s_2	0.727	0.687	1.000	s_2	0.947	-0.322	1.000	s_2	0.335	0.942	1.000

Market B

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.722	0.692	1.000	s_1	0.962	0.271	1.000	s_1	-1.000	0.003	1.000
	s_2	0.692	0.722	1.000	s_2	-0.271	0.963	1.000	s_2	0.042	0.999	1.000

Market C

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.704	0.710	1.000	s_1	0.630	-0.776	1.000	s_1	-1.000	-0.012	1.000
	s_2	0.710	0.704	1.000	s_2	0.776	0.630	1.000	s_2	-0.006	1.000	1.000

Table 3: Comparison of factors in markets with low noises
The three panels report for each actual generating factor s_{it}, $i = 1, 2$, the coefficients of the regression $s_{it} = a_{i0} + a_{i1}\hat{s}_{1t} + a_{i2}\hat{s}_{2t} + \eta_{it}$, where $\hat{s}_{kt}$ is the k-th factor estimated either by Principal Component Analysis (PCA), by the Connor-Korajczyk method (CK) or by Independent Component Analysis (InCA).

Market A

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.687	0.726	1.000	s_1	0.322	0.946	0.999	s_1	-0.941	0.339	1.000
	s_2	0.727	0.687	1.000	s_2	0.947	-0.322	1.000	s_2	0.335	0.942	1.000
0.5	s_1	-0.725	0.665	0.968	s_1	0.981	-0.100	0.973	s_1	-0.929	0.353	0.988
	s_2	0.675	0.735	0.996	s_2	0.100	0.980	0.970	s_2	0.358	0.931	0.995
1.0	s_1	0.010	0.024	0.001	s_1	0.152	-0.219	0.071	s_1	0.952	-0.271	0.980
	s_2	0.658	0.740	0.980	s_2	-0.010	0.942	0.887	s_2	0.275	0.954	0.986
1.5	s_1	0.201	-0.100	0.050	s_1	-0.610	0.334	0.483	s_1	0.951	-0.245	0.965
	s_2	0.699	0.685	0.958	s_2	0.004	-0.232	0.054	s_2	0.265	0.952	0.977
2.0	s_1	0.128	0.007	0.016	s_1	-0.257	-0.024	0.067	s_1	0.872	-0.401	0.922
	s_2	0.679	0.677	0.919	s_2	-0.082	0.037	0.008	s_2	0.430	0.895	0.986

Market B

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.722	0.692	1.000	s_1	0.962	0.271	1.000	s_1	-1.000	0.003	1.000
	s_2	0.692	0.722	1.000	s_2	-0.271	0.963	1.000	s_2	0.042	0.999	1.000
0.5	s_1	-0.701	0.697	0.977	s_1	0.740	0.648	0.968	s_1	-0.996	0.026	0.993
	s_2	0.704	0.707	0.996	s_2	-0.656	0.739	0.977	s_2	0.063	0.994	0.993
1.0	s_1	0.661	-0.668	0.883	s_1	0.731	-0.589	0.881	s_1	0.990	0.006	0.981
	s_2	0.699	0.703	0.983	s_2	-0.571	-0.730	0.859	s_2	0.035	0.992	0.986
1.5	s_1	0.136	0.010	0.018	s_1	-0.147	0.209	0.065	s_1	0.009	-0.984	0.969
	s_2	0.674	0.704	0.951	s_2	0.009	-0.132	0.018	s_2	0.983	0.073	0.972
2.0	s_1	-0.131	0.071	0.022	s_1	0.001	0.021	0.000	s_1	-0.973	-0.024	0.948
	s_2	0.701	0.665	0.934	s_2	-0.174	0.032	0.031	s_2	0.033	0.983	0.967

Market C

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.704	0.710	1.000	s_1	0.630	-0.776	1.000	s_1	-1.000	-0.012	1.000
	s_2	0.710	0.704	1.000	s_2	0.776	0.630	1.000	s_2	-0.006	1.000	1.000
0.5	s_1	0.693	-0.695	0.964	s_1	0.585	-0.788	0.963	s_1	0.008	-0.992	0.985
	s_2	0.710	0.702	0.997	s_2	0.796	0.589	0.979	s_2	0.994	0.022	0.988
1.0	s_1	-0.591	0.566	0.669	s_1	0.586	0.066	0.348	s_1	-0.004	0.987	0.975
	s_2	0.704	0.698	0.983	s_2	-0.612	0.547	0.674	s_2	0.987	0.016	0.975
1.5	s_1	-0.364	0.377	0.275	s_1	0.378	0.395	0.299	s_1	0.976	0.028	0.953
	s_2	0.701	0.686	0.962	s_2	-0.573	0.136	0.347	s_2	0.002	0.980	0.960
2.0	s_1	0.123	-0.004	0.015	s_1	0.256	-0.118	0.079	s_1	-0.015	0.982	0.965
	s_2	0.690	0.678	0.937	s_2	0.030	0.140	0.020	s_2	0.982	0.027	0.965

Table 4: Comparison of estimated factors with different values of noises

The panels report for each actual generating factor s_{it}, $i = 1, 2$, the coefficients of the regression $s_{it} = a_{i0} + a_{i1}\hat{s}_{1t} + a_{i2}\hat{s}_{2t} + \eta_{it}$, where $\hat{s}_{kt}$ is the k-th factor estimated either by Principal Component Analysis (PCA), by the Connor-Korajczyk method (CK) or by Independent Component Analysis (InCA). The parameter δ controls the average level of noises which are added to the returns of the stock.

Market A

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.997	-0.074	0.999	s_1	-0.023	0.999	0.999	s_1	-0.999	-0.004	0.998
	s_2	-0.074	0.997	1.000	s_2	0.998	0.023	0.997	s_2	-0.004	1.000	0.999
0.5	s_1	0.921	-0.084	0.855	s_1	-0.871	-0.349	0.880	s_1	0.974	0.027	0.950
	s_2	0.079	0.988	0.982	s_2	-0.341	0.857	0.851	s_2	-0.056	0.989	0.981
1.0	s_1	0.003	0.052	0.003	s_1	0.397	-0.572	0.485	s_1	-0.947	-0.200	0.934
	s_2	0.062	0.957	0.920	s_2	0.061	-0.106	0.015	s_2	-0.151	0.953	0.934
1.5	s_1	0.026	0.186	0.035	s_1	0.039	0.000	0.002	s_1	0.923	0.157	0.874
	s_2	0.052	0.897	0.807	s_2	0.034	0.007	0.001	s_2	-0.051	0.945	0.897
2.0	s_1	0.020	0.628	0.395	s_1	-0.018	0.030	0.001	s_1	-0.805	-0.209	0.689
	s_2	0.026	0.525	0.276	s_2	0.006	-0.009	0.000	s_2	-0.135	0.955	0.931

Market B

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.984	-0.173	0.999	s_1	-0.640	0.768	0.999	s_1	-0.999	-0.003	0.999
	s_2	-0.173	0.985	1.000	s_2	0.768	0.640	0.999	s_2	-0.003	0.999	0.999
0.5	s_1	-0.949	0.089	0.908	s_1	0.836	-0.406	0.864	s_1	-0.984	-0.007	0.968
	s_2	0.090	0.987	0.983	s_2	0.386	0.867	0.901	s_2	-0.031	0.984	0.972
1.0	s_1	0.706	-0.060	0.502	s_1	-0.685	-0.192	0.506	s_1	0.959	0.069	0.928
	s_2	0.032	0.962	0.927	s_2	0.174	-0.595	0.385	s_2	-0.022	0.970	0.940
1.5	s_1	0.050	0.324	0.108	s_1	-0.036	-0.020	0.002	s_1	-0.932	0.099	0.884
	s_2	0.149	0.847	0.739	s_2	0.014	-0.008	0.000	s_2	-0.002	0.944	0.891
2.0	s_1	0.080	0.710	0.511	s_1	0.000	0.001	0.000	s_1	-0.019	-0.890	0.794
	s_2	0.069	0.437	0.196	s_2	0.018	-0.021	0.001	s_2	0.926	0.107	0.874

Market C

δ	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
0.1	s_1	-0.999	0.005	0.998	s_1	-0.132	0.991	0.999	s_1	-0.999	0.002	0.998
	s_2	0.005	1.000	1.000	s_2	0.990	0.132	0.999	s_2	-0.001	0.999	0.998
0.5	s_1	-0.903	0.122	0.829	s_1	-0.449	-0.803	0.847	s_1	0.034	0.969	0.940
	s_2	0.110	0.986	0.984	s_2	0.798	-0.492	0.879	s_2	0.963	0.007	0.928
1.0	s_1	0.137	-0.064	0.023	s_1	0.514	-0.108	0.276	s_1	-0.948	-0.024	0.899
	s_2	0.092	0.963	0.937	s_2	0.315	-0.114	0.112	s_2	0.032	0.950	0.903
1.5	s_1	-0.037	0.037	0.003	s_1	0.044	-0.133	0.020	s_1	-0.908	-0.077	0.824
	s_2	0.022	0.900	0.811	s_2	0.102	0.075	0.016	s_2	0.075	0.922	0.850
2.0	s_1	0.030	0.223	0.051	s_1	-0.003	-0.004	0.000	s_1	-0.055	0.919	0.844
	s_2	0.079	0.845	0.720	s_2	-0.009	-0.006	0.000	s_2	-0.919	0.063	0.843

Table 5: Comparison of factors estimated on subgroups of assets

The table reports the regression values for $\hat{g}_{it} = a_{0i} + a_{1i}\hat{s}_{1t} + a_{2i}\hat{s}_{2t} + \epsilon_{it}$, where $\hat{g}_{it}$ is the factor estimated on the first subgroup of assets, and $\hat{s}_{1t}$, $\hat{s}_{2t}$ are the factors estimated on the second subgroup. Factors are estimated either by Principal Component Analysis (PCA), by the Connor-Korajczyk method (CK) or by Independent Component Analysis (InCA).

Market A

$\delta = 2$	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
Period 1	$\hat{g}_1$	-1.029	0.073	0.988	$\hat{g}_1$	0.934	0.044	0.868	$\hat{g}_1$	0.974	0.125	0.995
	$\hat{g}_2$	0.041	1.019	0.998	$\hat{g}_2$	0.028	-1.030	0.988	$\hat{g}_2$	-0.119	1.011	0.999
Period 2	$\hat{g}_1$	0.963	0.058	0.991	$\hat{g}_1$	-0.119	0.052	0.017	$\hat{g}_1$	0.985	-0.241	0.996
	$\hat{g}_2$	-0.029	0.976	0.998	$\hat{g}_2$	0.029	0.961	0.992	$\hat{g}_2$	0.269	0.945	0.999

Market B

$\delta = 2$	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
Period 1	$\hat{g}_1$	-1.009	0.089	0.992	$\hat{g}_1$	0.511	-0.012	0.231	$\hat{g}_1$	0.184	0.989	0.992
	$\hat{g}_2$	0.048	0.998	0.998	$\hat{g}_2$	-0.051	-1.010	0.995	$\hat{g}_2$	-0.944	0.235	0.997
Period 2	$\hat{g}_1$	0.982	0.107	0.994	$\hat{g}_1$	0.937	0.101	0.983	$\hat{g}_1$	1.020	0.085	0.992
	$\hat{g}_2$	-0.062	0.991	0.998	$\hat{g}_2$	-0.063	0.977	0.995	$\hat{g}_2$	-0.086	0.982	0.994

Market C

$\delta = 2$	PCA	$\hat{s}_1$	$\hat{s}_2$	R^2	CK	$\hat{s}_1$	$\hat{s}_2$	R^2	InCA	$\hat{s}_1$	$\hat{s}_2$	R^2
Period 1	$\hat{g}_1$	1.006	-0.033	0.856	$\hat{g}_1$	-0.377	-0.189	0.157	$\hat{g}_1$	0.929	0.043	0.995
	$\hat{g}_2$	0.010	0.953	0.997	$\hat{g}_2$	-0.296	1.009	0.965	$\hat{g}_2$	0.012	0.929	0.997
Period 2	$\hat{g}_1$	0.923	0.048	0.984	$\hat{g}_1$	-0.947	0.101	0.959	$\hat{g}_1$	1.084	-0.069	0.993
	$\hat{g}_2$	-0.017	1.048	0.996	$\hat{g}_2$	0.075	0.920	0.993	$\hat{g}_2$	-0.022	1.085	0.994

Table 6: Comparison of factors estimated in subperiods
For each subperiod, the regression is $\hat{g}_{it} = a_{0i} + a_{1i}\hat{s}_{1t} + a_{2i}\hat{s}_{2t} + \epsilon_{it}$, where $\hat{g}_{it}$ is the factor estimated in the subperiod one or two, and $\hat{s}_{1t}$, $\hat{s}_{2t}$ the factors estimated using the whole period, but with taking only the realisations in the subperiod where the $\hat{g}_{it}$ factors are estimated. Factors are estimated either by Principal Component Analysis (PCA), by the Connor-Korajczyk method (CK) or by Independent Component Analysis (InCA).

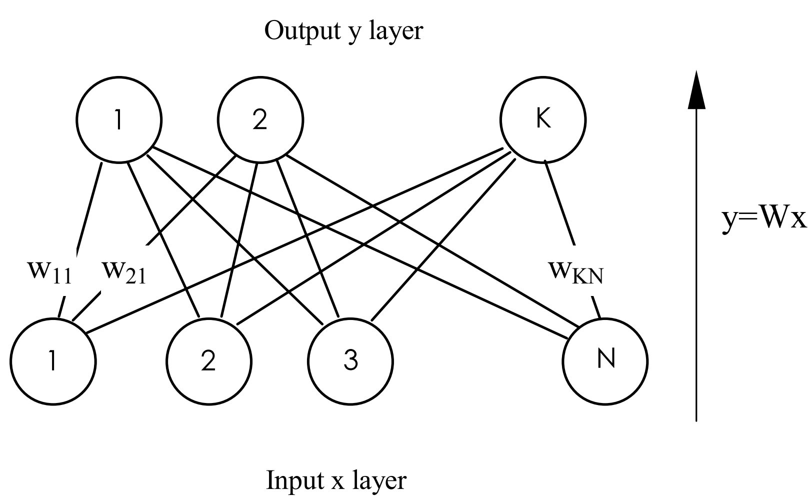


Figure 1: Neural networks architecture for factor analysis
Factor analysis with neural networks generally use an architecture without a hidden layer. The input observed signals **r** are propagated to the output layer **y** through the transformation $\mathbf{y} = \mathbf{Wr}$. The $K \times N$ weights w_{ij} are recursively updated by applying an adequate learning rule depending on an energy function $e(\mathbf{r}, \mathbf{y})$.

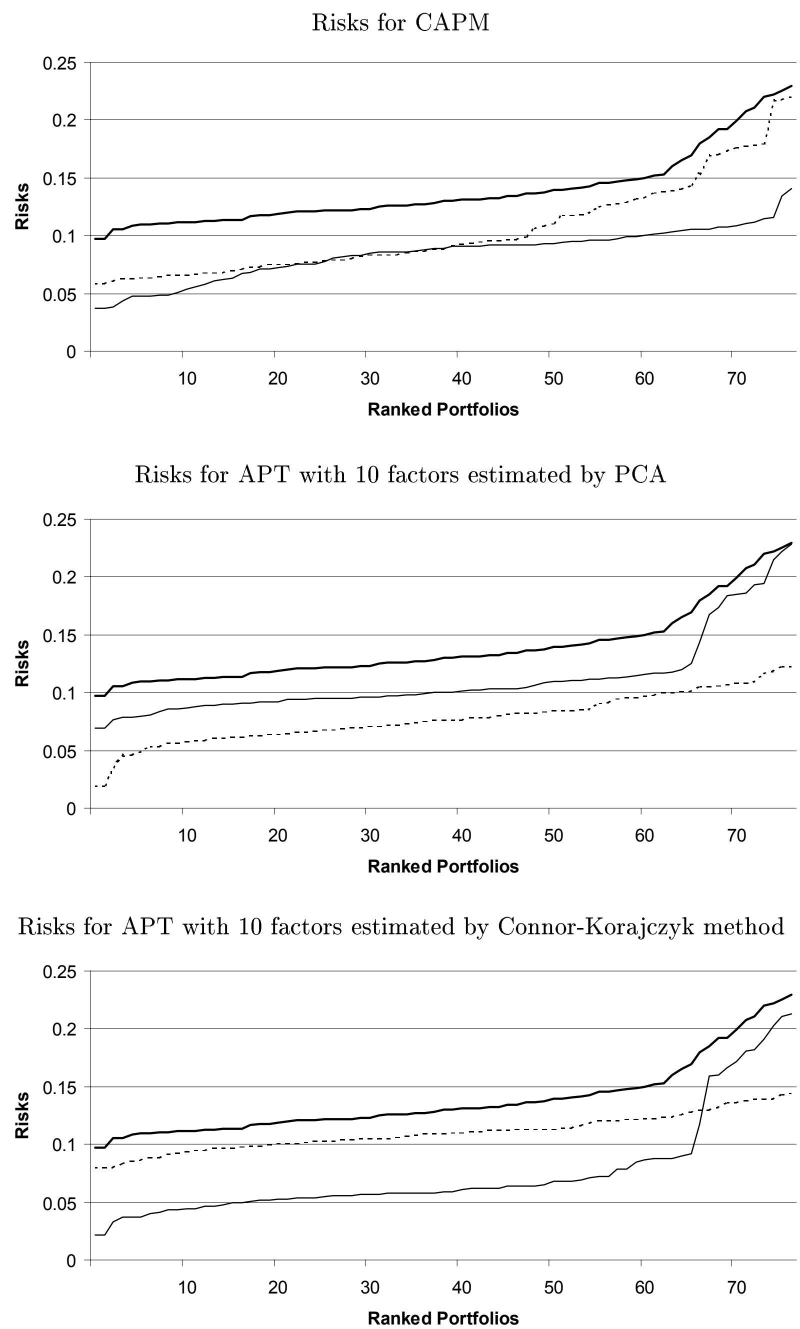


Figure 2: Global and idiosyncratic risks for portfolios on the US market

Global and residual risks are presented for the US market when stocks are grouped into portfolios, for the CAPM and for the APT with ten factors estimated by Principal Component Analysis or by Connor-Korajczyk method. The period goes from January 1, 1994 through December 31, 1997. The global risks (top curve) and diversifiable risks (lower curves) are represented by plain lines; the residual risks are represented by dash lines.

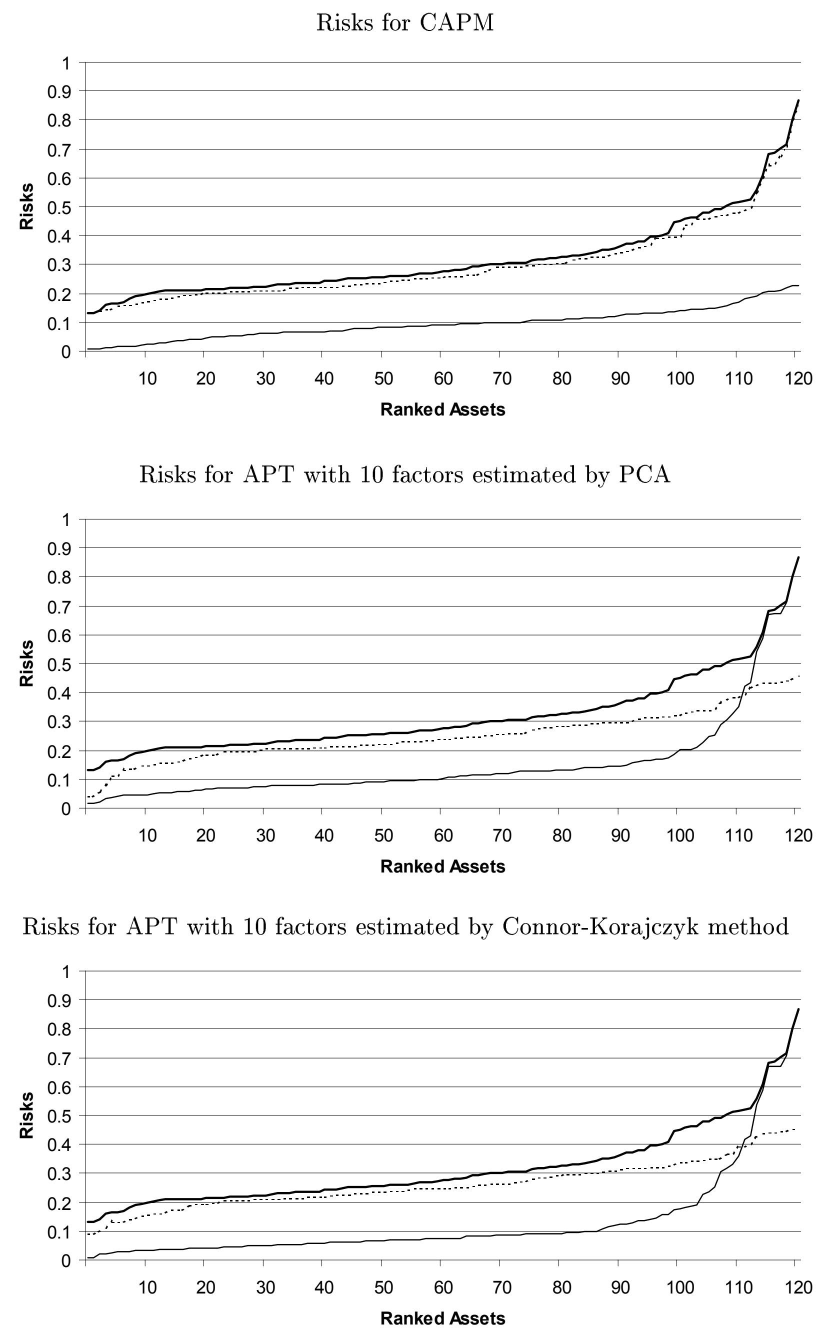


Figure 3: Global and idiosyncratic risks for single stocks on the US market

Global and residual risks are presented for the US market when individual stocks are considered, for the CAPM and for the APT with ten factors estimated by Principal Component Analysis or by Connor-Korajczyk method. The period goes from January 1, 1994 through December 31, 1997. The global risks (top curve) and diversifiable risks (lower curves) are represented by plain lines; the residual risks are represented by dash lines.

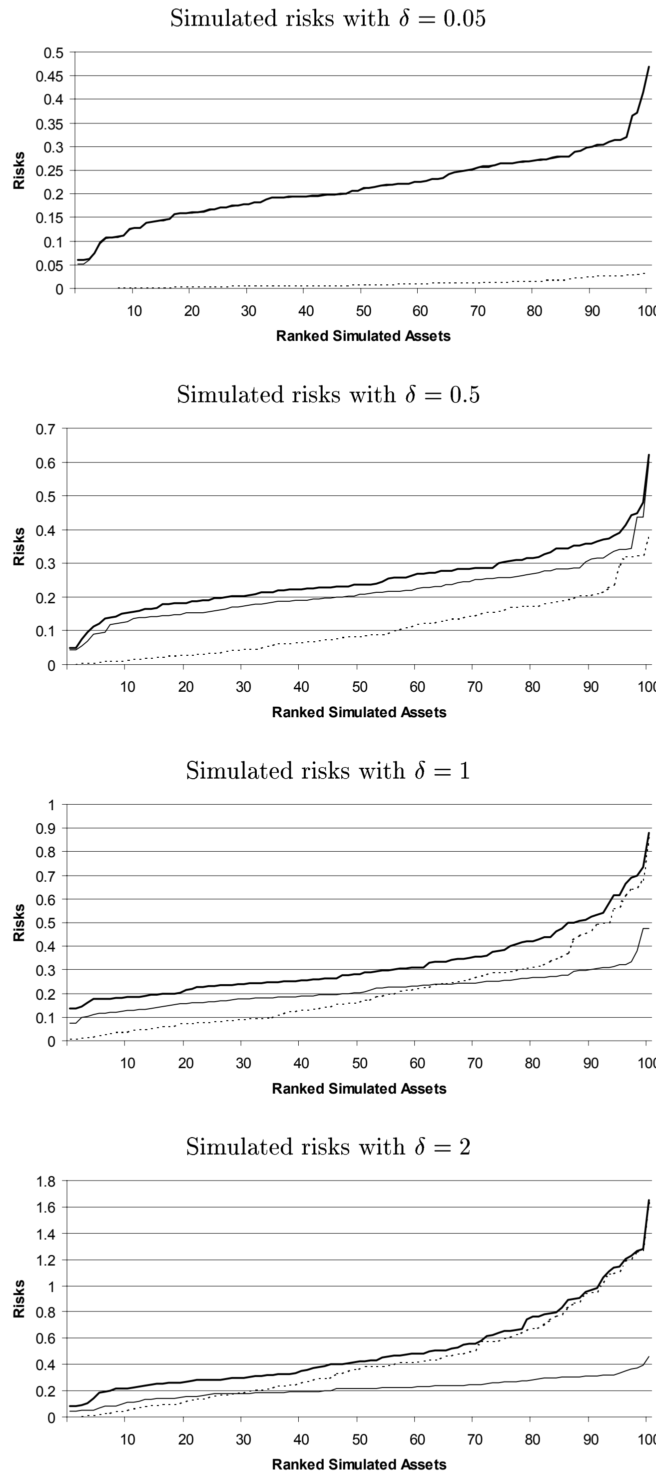


Figure 4: Plots of risks and residual risks on simulated markets

Global and residual risks are presented for simulated markets of type A (gaussian factors and noises) for different levels δ of idiosyncratic risks. The global risks (top curve) and diversifiable risks (lower curves) are represented by plain lines; the residual risks are represented by dash lines.

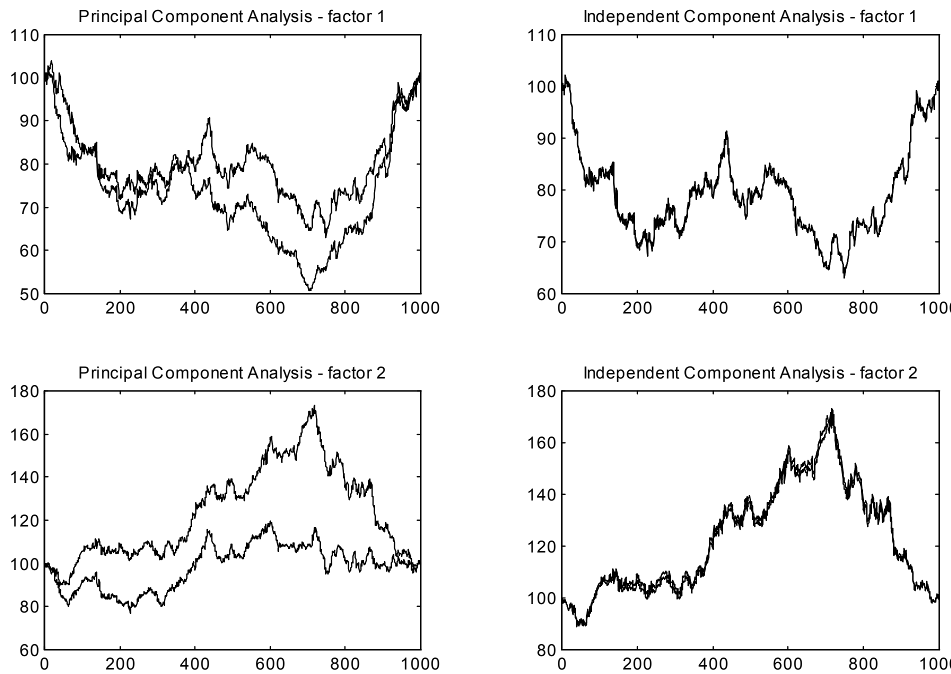


Figure 5: Plots of actual and estimated cumulated factors
The estimated factors are here presented for the second market B where factors are built following a stochastic volatility model. The factors are cumulated for PCA and InCA, with the transformation: $g_{it} = g_{i(t-1)}(1 + 10^{-2}s_{it})$, and $g_{i0} = 100$. In case of Independent Component Analysis, estimated factors can not be distinguished from the actual factors; in the case of estimation by Principal Component Analysis, the estimated factors are rotated and can not be identified with the actual generating factors.

Eigenvalues from actual US market　　Eigenvalues from simulated markets

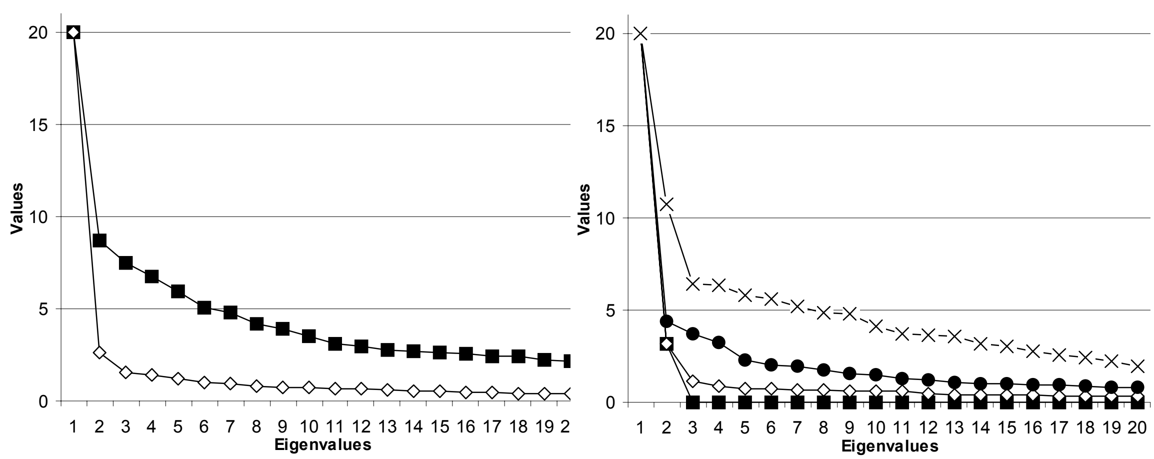
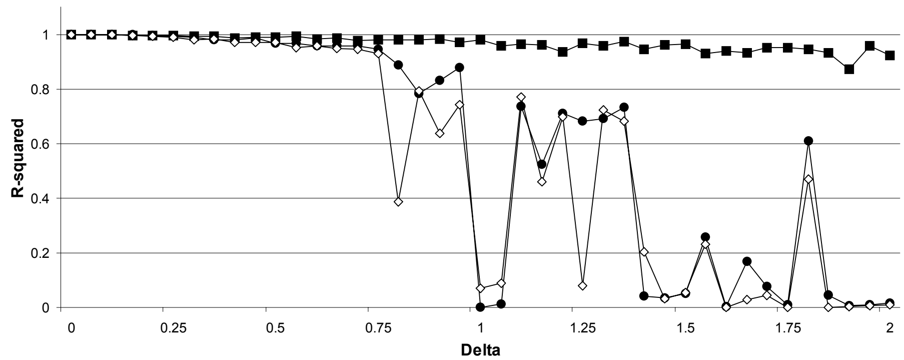


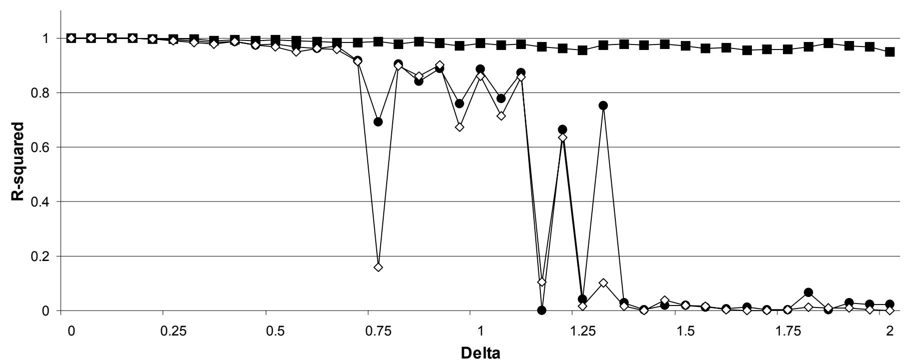
Figure 6: Plots of eigenvalues for actual and simulated markets

Eigenvalues are plotted on the left for the US market when portfolios are considered (lower curve) or when a random selection of 120 assets are considered. The period goes from January 1, 1994 through December 31, 1997. On the right are displayed the 20 highest eigenvalues for gaussian generated markets with values for δ equal to 0.05 (squares), 0.75 (diamonds), 1.25 (circles) and 2 (crosses). Eigenvalues have been standardized such that the greatest eigenvalue is equal to 20.

Lowest R^2 obtained for market A



Lowest R^2 obtained for market B



Lowest R^2 obtained for market C

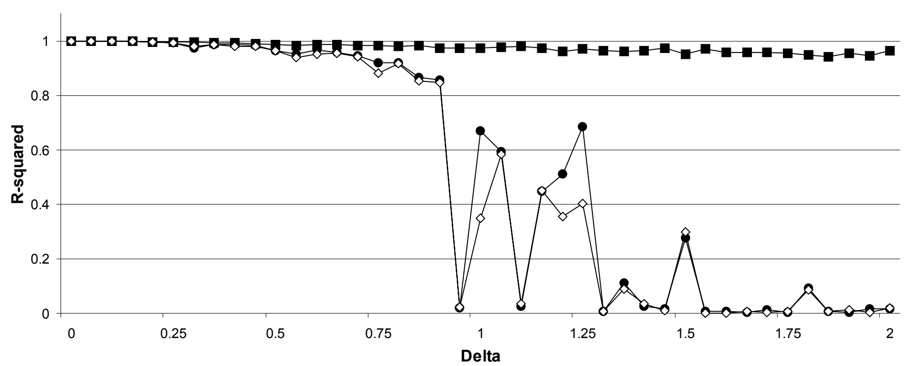
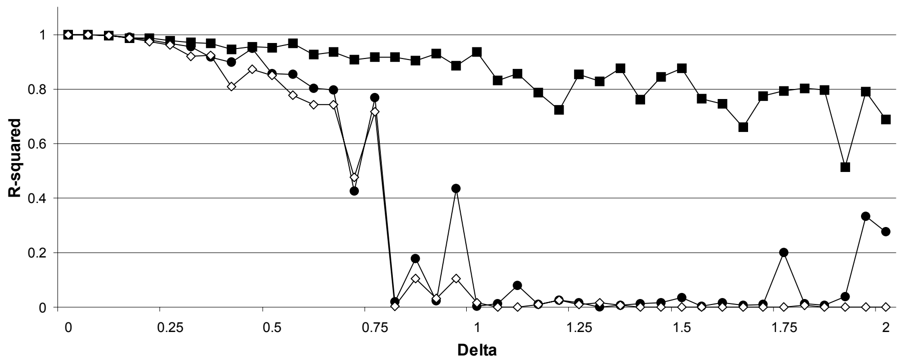


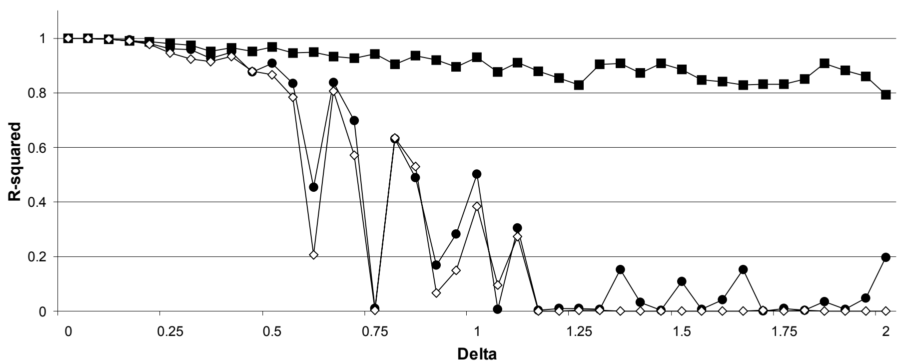
Figure 7: Smallest explanations of actual factors

The lowest R^2 values of the actual generating factors regressions above the estimated factors are displayed for values of δ increasing from 0.05 to 2 with steps of 0.05. Squares refer to the InCA method, plain circles to the PCA method and white circles to the Connor-Korajczyk method.

Lowest R^2 obtained for market A



Lowest R^2 obtained for market B



Lowest R^2 obtained for market C

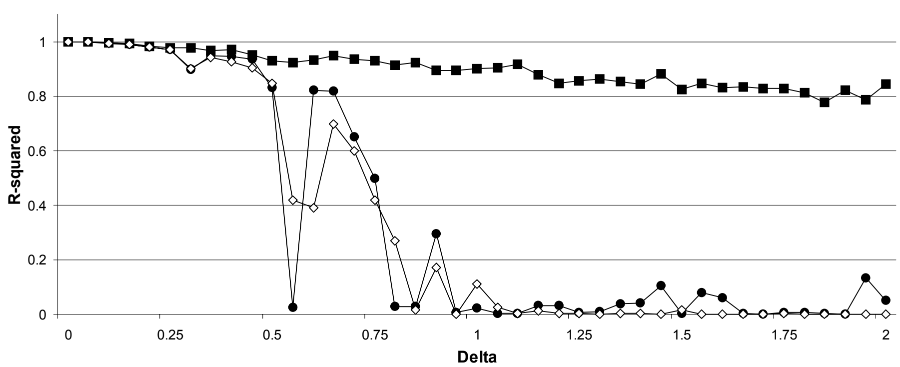


Figure 8: Smallest explanations through subgroups

The lowest R^2 values of the estimated factors on one subgroup of assets above the estimated factors on the other subgroup are displayed for a δ value increasing from 0.05 to 2 with a 0.05 step. Squares refer to the InCA method, plain circles to the PCA method and white circles to the Connor-Korajczyk method.

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