2001s-69

Dynamic Prevention in Short Term Insurance Contracts

M. Martin Boyer, Karine Gobert

Série Scientifique

Scientific Series

CIRANO

Centre interuniversitaire de recherche en analyse des organisations

Montréal

Décembre 2001

ISSN 1198-8177

Dynamic Prevention in Short Term Insurance Contracts*

M. Martin Boyer† and Karine Gobert‡

Résumé / *Abstract*

Le but de cet article est d'étudier les propriétés dynamiques des contrats d'assurance lorsque les assureurs détiennent une meilleure technologie de prévention des catastrophes que les assurés. Cette technologie est permanente au sens où elle ne se déprécie pas. Si les contrats de long terme ne sont pas possibles, les assurés font face à un problème d'engagement puisqu'ils voudraient renégocier le contrat ou changer d'assureur après que l'assureur initial a investi le montant optimal en prévention. À cause de ce problème de hold-up, nous montrons que l'investissement en prévention est retardé, et ce même si l'assuré demeure avec le même assureur sur tout l'horizon de fonctionnement. Nous montrons également que la dynamique des primes d'assurance diffère d'une suite de primes actuarielles.

This paper looks at the dynamic properties of insurance contracts when insurers have better technology at preventing catastrophic losses than the insured. The prevention technology is owned by the insurers and is permanent. If long-term contracts are not possible, the insured is faced with a commitment problem since he may want to renegotiate the contract or change insurer after his initial insurer has invested in prevention. Because of this hold-up problem, we find that the investment in prevention is delayed.

Mots clés : Assurance, Prévention, Engagement, Théorie des contrats, Aléa Moral

Keywords : *Insurance, Prevention, Commitment, Contract Theory, Moral Hazard*

JEL Classification : G

---

* We would like to thank SCSE, CEA and ARIA participants for comments, as well as the Georgia State University Risk Management and Insurance Department for its hospitality. This paper was first circulated under the title "Contracts, Catastrophes, Commitment and Capital". This research is financially supported by the Fonds pour la Formation des Chercheurs et d'Aide à la Recherche (FCAR - Québec) and by the Social Science and Humanities Research Council (SSHRC - Canada). The continuing support of Cirano is also gratefully acknowledged.

† Corresponding author : Martin Boyer is Associate Professor of Finance, École des Hautes Études Commerciales, Université de Montréal. 3000 Côte-Sainte-Catherine, Montréal, QC H3T 2A7 Canada; and Fellow Cirano, 2020 University Ave., 25th floor, Montréal, QC. martin.boyer@hec.ca.

‡ Karine Gobert is Assistant Professor of Economics, ConcordiaUniversity, 1455 W. de Maisonneuve Blvd., S-ER 100, Montréal, QC H3G 1M8 Canada; and Chercheure Cirano, 2020 University Ave., 25th floor, Montréal, QC.gobertk@vax2.concordia.ca.

1 Introduction

An assumption that is often implicit in the contract theory literature on prevention is that any agents for whom prevention is valuable are aware of the best practices available. This may not be the case, however. For example. in the case of pollution reducing technology a firm may not have the knowledge of the best available practices to reduce pollution (the firm produces golf carts, and emits in the process a toxic mercury by-product). The same rationale can also be used in the case of high-tech data management where a firm may not have the knowledge of the best available practices to manage computer hacker risk (the firm sells furniture and has all inventory and merchandise movement computerized). The core competence of the firm may be so different from the competence needed to reduce risk that the firm needs outside help to do it. There are therefore gains to trade if prevention technology specialists sell their expertise to firms that need it. The question then becomes Who are the specialists?

In some instances, the insurance industry is the best provider of prevention technology, especially if the potential loss is insured as the insurer also becomes the payer (i.e., insurance firms absorb the catastrophic loss when they insure the risk). Being faced with many risks of the same type in the same industry, insurers can learn a great deal more than firms about the most efficient way to reduce risk (see Mayers and Smith, 1982, and Doherty, 1997, for similar arguments). This is especially true for risks that are very rare, which means that only entities that deal with many possible exposures can correctly assess the quality of the prevention technology. Put another way, insurers have a comparative advantage in preventing catastrophes. Information technology management provides a good example. Firms generally have no special knowledge of their exposure to computer network and client database risk. Insurers on the other hand, given their long time presence on the markets and exposure to catastrophic risks, have acquired technological knowledge to prevent more efficiently catastrophes. Specialists working in those departments provide audits of the level of risk, consulting on solutions, and they can also implement and service the prevention technology for their corporate clients.[1]

A second important consideration for investments in prevention is that firms need to access the capital markets to finance the purchase of the prevention technology. This may be impossible, however, because of credit constraints. Constraints such as information asymmetries and imperfect

[1] One may also think of health insurance. Health providers (i.e., doctors) are usually better equipped to prevent diseases than their patients.

commitment prevent long term contracts and reduce the possibility of outside financing. It follows that firms need to finance a large portion of the cost of the technology internally, which means that they may not be able to invest an optimal level in prevention. Insurance companies with large cash holdings, on the other hand, can provide financing to their clients. According to Caillaud, Dionne and Jullien (2000), optimal financial contracts for firms with financial needs and exposure to value-reducing accident risk is a combination of a debt and an insurance contract. Financing and insurance do not have to be provided by separate entities. Insurers have an informational advantage to offer financing since they know the level of risk in the firm, which other market participants can ignore.

In this paper, we analyze the relationship between a firm and an insurer-provider of prevention, in a dynamic context. The firm faces an insurable catastrophic risk in each of a finite number of periods. It wants to invest in prevention, but, unfortunately, it does not have the technology to do so. Insurers on the other hand have the technology. The firm has to hire an insurer to invest in prevention and obtain insurance against the bankruptcy risk. Prevention investments cannot be observed by outside insurers. Hence, an insurer does not know the risk type of the firm unless he has worked with it in the previous period. A firm that wants to change insurer must make its risk type known, which we model as entering in a round of costly auditing. For example, the firm must hire an outside consultant who will assess the risk type of the firm.

Corporations that seek to manage potential catastrophic losses and/or environmental hazards can either attempt to reduce the event's impact on cash flows or reduce the likelihood that such an event will occur. This can be seen as an insurance problem where the insured firms must choose an optimal level of precaution: Insurance reduces a catastrophic event's impact on cash flows (severity) whereas investing in prevention technology reduces an event's frequency.

A catastrophic loss is defined in our model as a loss that will cause the firm to go bankrupt. In the separation of duties, our paper is similar to that of Caillaud, Dionne and Jullien (2000) where it is shown that although firms are risk neutral, financing requirements make it appear as if firms are risk averse. The firm needs insurance to cover bankruptcy risk. The insurer provides insurance and prevention technology.

The problem with investing in prevention technology is that its high specificity and complexity makes it non verifiable by parties outside the industry. This triggers potential opportunistic behaviors from the part of either the insurer managing the technology or the firm hosting it. This means that a firm cannot use debt contracts to invest in prevention since its risk type is unknown to financiers. On the other hand, the insurer may not want to finance the investment since he has no guarantee that the firm may not claim the investment has not been done and then contract with a competitor.

Moreover, the prevention technology is additive so that investment in prevention today reduces the likelihood of catastrophes not only today, but also in the future. Once the investment is in place, every other investor can benefit from it in the future since risk is now reduced. When there is only one period, the problem is quite trivial: The insurance contract is effective for the same exact time period as the prevention technology. This means that the insurer will invest in an optimal level of prevention on behalf of the firm. When more periods are involved, the insurer is faced with the dilemma that investing in prevention reduces the risk of the firm, but it reduces the risk of the firm no matter who insures the firm in the future.[2] Moreover, insurance contracts are typically short term in nature (renegotiated every year or so) whereas investment in prevention lasts a long time. The long-term investment benefits clash with short-term insurance contracts.

When investements are non-verifiable, but offer long-term benefits, insurers face a hold up problem: What prevents the firm from changing insurer? As is known from the literature on transaction-cost economy (see Williamson, 1979) the solution to the hold-up problem would be to bind both parties in a long term contract. The non verifiability of investment, however, generates imperfect commitment. Using a coal mining/electricity generation example Joskow (1987, 1988) shows that the hold-up problem may not be solved using long term contracts because commitment is imperfect and future uncertain. In this case, vertical integration can be the solution. We do not consider this possibility here since, even if the insurer's investment in the firm is specific, the relationship between an insurer and the firm is not. It may be profitable for an insurance company to have departments specialized in several highly specific risk. However, transaction costs are such that it is not profitable for the same insurance company to purchase all its clients since it would then have to change its activity completely.

With no binding contracts and no integration, we show that the hold-up situation results in suboptimal investment and delays in prevention investment under simple assumptions. We also show that in each period a firm contracts with the same insurer who may use this informational

[2] In that sense, our prevention technology has some basic characteristics of a public good because once an insurer makes the investment, every future insurer benefits from it.

advantage to charge a loading on the premium. Competition for this rent, however, drives an insurer's profit to zero in the initial period.[3]

We present the basic model in the following section. In section 3 we present the contract under full commitment on the part of the firm and the insurer. Section 4 focuses on the case where no long-term contracts can be signed. We introduce long term relationships with renegotiation in section 5 of the paper. It will then be clear that asymmetric information may not be the only problem reducing precaution in a dynamic context. In section 6, we discuss some assumptions underlying the resutls of our model to test its robustness. Section 7 concludes.

2 The basic model

Suppose N insurers[4] and 1 firm living $T+1$ periods denoted $t = 0 \cdots T$. In each period, the firm receives a non random revenue W. Also, the firm faces in each period a potential catastrophic loss $L > W$ for which it wants to (or has to) insure. We can also view this L as including a penalty imposed by government for environmental damages due to the firm's operations. The probability of such a catastrophic loss may be reduced through investment in some prevention technology. We denote x_t the amount invested in prevention in period t and $X_t = \sum_{\tau=0}^{t} x_\tau$ the accumulated amount invested in prevention from period 0 through t. The amount invested in prevention is known to the firm at all time.

A loss L occurs in period t with probability $\Pi(X_t)$, whereas no loss occurs with probability $(1 - \Pi(X_t))$. More prevention is better, which means that the probability is decreasing in the amount of prevention: $\Pi'(X_t) < 0$. The probability function is convex, $\Pi''(X_t) > 0$, so that the marginal decrease in the probability of loss is reduced with each additional dollar invested in prevention. Finally, some prevention is always desirable as $\Pi'(0) = -\infty$. We also assume that

[3] The standard literature on hold-up is static in the sense that the bargaining occurs only once. In our case, however, the insurance contract is resigned every period. We innovate on Rogerson, (1992) and Gul (2001), by presenting a hold-up problem where insurers offer standard insurance contracts that generate zero profit in expectation. This contrasts with the firm's impossibility to commit in the long run to the insurance relationship, which induces the insurer to invest only small amounts in each period. Other models on dynamic insurance have typically centered around the repetition of adverse selection (see Cooper and Hayes, 1987, and Dionne and Doherty, 1994) and moral hazard (see Rogerson, 1985, and Chiappori, Macho, Rey and Salanié, 1994) problems, whereas, in our case, the information problem is dealt with through verification. When commitment is impossible, however, ratchet effects arise that induce agents to delay the revelation of their true risk type (Laffont and Tirole, 1987). The key concern in our case is on the dynamics of prevention investments by consecutive insurers. More recently, the "Precautionary Principle" has shed light on problems of decision under dynamic uncertainty (see Briys and Schlesinger, 1990, Gollier, Jullien and Treich, 2000 and Immordino, 2000).

[4] N does not have to be large to entail competition among insurers.

there is no depreciation in self-protection so that any amount invested in period t is still in use in the subsequent periods.[5] It is not possible to remove any part of the technology (i.e. investment is irreversible). We can view this prevention technology as an organizational design that is costly to implement, but that can run fully without alterations; this organizational design cannot be undone, however.

The firm is faced with a potential loss L which could bankrupt it. Because of bankruptcy costs, we can view the firm as being risk averse and thus in need of insurance.[6] Define $V(W)$ as the per period value of the firm over final wealth, with $V'(.) > 0$ and $V''(.) < 0$. The intertemporal utility of the firm is given by

$$\mathcal{V} = \sum_{t=0}^{T} \delta^t \left(\prod_{\tau=0}^{t} (1 - \Pi(X_\tau)) V(W - x_t) \right),$$

where it is assumed that the firm can go bankrupt in any period t with probability $\Pi(X_t)$, and where each period is discounted at some rate $\delta \leq 1$.

Since the firm has a concave utility function each period, it wishes not only to purchase insurance against the realization of an accident, but it also wishes to smooth its income across time. There are therefore Pareto improving trades possible between the firm and a risk-neutral insurer, not only between states, but also across time. Purchasing insurance against the loss L eliminates the possibility of bankruptcy.

The accumulated amount invested in self-protection X_t cannot be observed costlessly by outsiders.[7] Hence, potential insurers do not know the firm's probability of accident unless a costly audit is performed. Auditing in period t costs a and reveals the firm's exact level of risk, $\Pi(X_t)$. If an audit is conducted, the firm's level of risk becomes public information.

At every period t, there is an insurance contract running between the firm and an insurer. A contract can last one period or more, this paper examines the path of investment in prevention under different contractual timings. The contract specifies a contingent transfer from the firm to the insurer, p_t^i, $i = n, l$ where l denotes the loss state and n the no-loss state.[8] The contract also

[5] We show in part 6.2 that the introduction of a depreciation rate does not alter the result.

[6] See Mayers and Smith (1982), MacMinn (1987), Smith and Stulz (1985) and Caillaud, Dionne and Jullien (2000) for similar arguments. Another reason why firms may be viewed as risk averse (and in need of insurance) is that the managers who run the firm are undiversified, and thus need protection from adverse shocks; see Stulz (1984), Campbell and Kracaw (1987) and DeMarzo and Duffie (1995).

[7] An outsider in period t is defined as any player (specifically any insurance company) that did not transact with the firm in period $t-1$.

[8] The investment in prevention, the insurance premium and the indemnity payment are embodied in p_t^i.

specifies an amount of investment in prevention x_t made prior to Nature's move. We denote d_t the final wealth of the firm in period t, d_t representing the firm's earnings W minus any transfer made to the insurer or as a payment for the audit.

We suppose that if an insurer remains with the firm for more than one period, it gains proprietary knowledge of the firm's level of risk. Since investment in self-protection is controlled by the insurers, the firm's stock of prevention technology at the beginning of period t, X_{t-1}, is known to the period $t-1$ insurer. In other words at the beginning of period t, the incumbent insurer knows the accumulated amount invested in prevention from period 0 till period $t-1$, which means that the incumbent insurer knows the risk type of the firm at the beginning of period t, $\Pi\left(X_{t-1}\right)$.

For the firm to contract with another insurer at this time, it has to provide an audit report to inform the newcomer on the level of risk. For any outside insurer to learn a firm's risk type an audit must be performed. The result of the audit is public information and is disseminated prior to the insurers making bids to insure the firm's risk. Although an audit makes the prevention technology known to all insurers, only $N-1$ insurers benefit from such an information dissemination, since the N^{th} insurer (the incumbent) already knows this information prior to the audit being conducted.

3 Full commitment

When the firm and the insurer can commit for the entire horizon, a long term optimal contract can be found in period 0 that prescribes transfers for all future contingencies. Such a contract stipulates a sequence of transfers $\{p_t^n, p_t^l\}_{t=0}^T$ and investments $\{x_t\}_{t=0}^T$ that maximizes the firm's value over the T periods, under the constraint that the insurer's expected discounted payoff is at least equal to zero. Since the contract provides insurance against catastrophic losses, the firm no longer faces states of the world in which it goes bankrupt. The problem then is:

$$
\begin{aligned}
\max_{\{X_t\},\{p_t^n\},\{p_t^l\}} \quad & \sum_{t=0}^{T} \delta^t \Big(\left(1-\Pi(X_t)\right) V(W-p_t^n) + \Pi(X_t) V(W-p_t^l) \Big) \\
\text{s.t.} \quad & X_t \geq X_{t-1} \qquad \forall\, t = 1 \cdots T & (1) \\
& -X_0 + \left(1-\Pi(X_0)\right) p_0^n + \Pi(X_0) \left(p_0^l - L\right) + \\
& \qquad \sum_{t=1}^{T} \delta^t \left\{ -(X_t - X_{t-1}) + \left(1-\Pi(X_t)\right) p_t^n + \Pi(X_t) \left(p_t^l - L\right) \right\} \geq 0 & (2)
\end{aligned}
$$

Note that the firm and the insurer have the same discount factor δ.

The set of constraints (1) prevents disinvestment in the technology, ensuring that the total amount invested is non-decreasing. This is a set of T irreversibility constraints. The last constraint, (2), is the insurer's participation constraint. It ensures that the insurer receives at least an expected payoff of zero over its lifetime. Note that we subtract the cost of investment in prevention from the insurer's payoff. The result is independent of the writing, however, considering that the insurer pays for the investment and charges it to the firm through the transfers p_t. By writing the problem this way, we allow the insurer to act as a financier for the firm, paying for whatever investment is necessary and being reimbursed by the firm through time.

Let us associate Lagrange multipliers λ_t, $t = 1 \cdots T$ to the irreversibility constraints and μ to the insurer's participation constraint. First order conditions for the problem are:

$$\mu = V'(W - p_t^n) = V'(W - p_t^l) \qquad \text{for all } \; t = 0, \cdots T \tag{3}$$

$$\delta^T \mu (1 + \Pi'(X_T)L) = \lambda_T \tag{4}$$

$$\delta^t \mu [1 - \delta + \Pi'(X_t)L] = \lambda_t - \lambda_{t+1} \quad \text{for all } \; t = 0, \cdots T-1, \quad \lambda_0 = 0 \tag{5}$$

Solving this system of equations yields the optimal long-term contract. The optimal contract is described in what follows.

Proposition 1 *The full commitment contract offers full insurance and complete smoothing through a constant transfer p independent of the state, an amount X^{**} is invested in the initial period ($X_0 = X^{**}$) such that*

$$\left(1 + \left(\sum_{i=0}^{T} \delta^i \right) \Pi'(X^{**})L \right) = 0$$

and no investment is done in the subsequent periods.

Proof: See appendix.

Full insurance and complete smoothing is implied by the set of conditions (3). Marginal utilities are equalized through time and across states with a constant transfer $p = p_t^n = p_t^l$ for all t.

The amount X^{**} invested in period 0 equates the marginal cost of investment with its marginal profitability. Since investing today helps decrease the accident probability in each subsequent period, the marginal benefit of precaution takes into account the decrease of the expected loss in each future period, discounted by factor δ. Hence, the longer the horizon (the higher T), the greater the optimal investment X^{**} made in period 0.

After period 0, however, the irreversibility constraints are binding since the marginal benefit of investment is shrinking with the number of periods left until the end of the horizon. The firm and the insurer would like to sell back the technology when the weight of future profitability decreases. Irreversibility constraints forbid that, which means that no investment occurs in periods 1 through T; i.e., $x_t = 0 \; \forall \; t = 1, ..., T$.

The full-commitment long-term insurance contract is such that the insurer bears the cost of optimal investment X^{**} in period 0 and spreads out evenly the reimbursement by the firm over the future periods, through an increase in the premium. The risk neutral insurer then offers financing to the firm and makes smoothing optimal when contracts are perfect.

4 No long-term contracts

Section 3 deals with the idealistic case where there are no contract imperfections, so that a long term contract always exist and does at least as well as a sequence of short term contracts.[9] In order to be able to find the dynamic insurance relationship when parties can renegotiate, we first study the extreme opposite of a full commitment long term contract. Hence, we determine in this section the equilibrium sequence of competitive contracts when the firm cannot remain in relation with the same insurer more than one period.

If the players do not have access to long term contracts, they may rely on a sequence of competitive short-term insurance contracts. When an audit is conducted every period every insurer becomes informed about the firm's risk. We suppose for now that there are no attempts from the insurers to try and keep the contract for longer than one period.

4.1 The short-sighted firm

First, let us concentrate on the level of precaution achieved by a myopic firm that does not consider the future benefits of precaution.[10] The firm buys insurance and precautionary investment to maximize the per period firm's value subject to the insurer's participation constraint for the period. In each period t, the contract solves:

$$\max_{p_t^n, p_t^l, X_t} \quad [1 - \Pi(X_t)]V(W - a - p_t^n) + \Pi(X_t)V(W - a - p_t^l)$$

[9] Note that in such an idealistic environment, financing X^{**} is possible in period 0 through a debt contract. We address this issue in section 6.3.

[10] We can view this case as letting $\delta = 0$.

$$X_t \geq X_{t-1}$$
$$-X_t + X_{t-1} + (1 - \Pi(X_t))\, p_t^n + \Pi(X_t) \left(p_t^l - L\right) \geq 0$$

The first order conditions for that problem entail full insurance in each period t through unconditional transfers p_t^* such that $V'(W - a - p_t^n) = V'(W - a - p_t^l) = V'(W - a - p_t^*)$. It also entails the one-period optimal level of care X^* such that $1 + \Pi'(X^*)L = 0$. Since the prevention technology does not depreciate, there is no investment after the first period: $X_t = X_{t-1}$ for $t \geq 1$. Since there is no investment after period 0, the periodic contracts are identical in each period $t > 0$ and transfers p_t^* are equal to the fair insurance premium: $p^* = \Pi(X^*)L$. However, the amount X^* is invested in period 0 and $p_0^* = p^* + X^*$.

This has many implications. First, the firm must be able to pay for the investment level X^* in the first period: $W - a - X^* - p^* \geq 0$. Second, even if the firm can pay for it, the concavity of V implies that the firm would like to spread the cost of investment on many periods. Moreover, the firm should be aware that period 0 investment in prevention brings benefits for more than one period and should then want to invest an amount greater than X^*. The rational firm should then choose investment in care and insurance so as to maximize a lifetime utility. We describe this problem in the next subsection.

4.2 The long-sighted firm

A rational firm considers its entire lifetime when solving for periodic insurance contracts and prevention. The problem is then the following:

$$\max_{p_t^n, p_t^l, X_t} \quad \sum_{t=0}^{T} \delta^t \left\{(1 - \Pi(X_t))\, V(W - a - p_t^n) + \Pi(X_t) V(W - a - p_t^l)\right\}$$
$$X_t \geq X_{t-1} \quad \text{for all } t$$
$$-X_t + X_{t-1} + (1 - \Pi(X_t))\, p_t^n + \Pi(X_t) \left(p_t^l - L\right) \geq 0 \quad \text{for all } t$$

The firm chooses the sequence of contracts that maximizes its intertemporal utility over its lifetime. Since an audit is conducted every period, the firm is able to contract with any insurer every period, which means that a periodic participation constraint must be included for insurers every period. Hence, we have $T + 1$ participation constraints imposing that the insurer's per period expected payoff is at least zero.

Applying multipliers λ_t, $t = 1, \cdots T$ to the T irreversibility constraints and μ_t, $t = 0, \cdots, T$ to the insurer's $T + 1$ participation constraints we find the following system of first order conditions:

$$\delta^t V'(W - a - p_t^n) = \delta^t V'(W - a - p_t^l) = \mu_t \tag{6}$$

$$-\left(1 + \Pi'(X_t)L\right)\mu_t + \mu_{t+1} + \lambda_t - \lambda_{t+1} = 0 \tag{7}$$

$$-\left(1 + \Pi'(X_T)L\right)\mu_T + \lambda_T = 0 \tag{8}$$

Condition (6) implies that the firm chooses full insurance in each period. Smoothing is not perfect, however, since the marginal utility of earnings evolves over time with μ_t. Solving for this system of equations allows us to state the following proposition.

Proposition 2 *When the firm must audit and sign a short term competitive insurance contract in each period, the path of investments and transfers are such that there is a date $\hat{t} \in \{1, ..., T\}$ such that*

- *for $t < \hat{t}$, a positive investment x_t is made in each period (multipliers λ_t are zero), the ratio of marginal utilities is increasing through time and always less than 1. Transfers in these periods are $p_t = x_t + \Pi(X_t)L$ so that the firm's earnings are given by $d_t = W - a - x_t - \Pi(X_t)L$.*
- *In period $\hat{t}$, there is no investment since condition $1 + \left(\sum_{i=0}^{T-\hat{t}} \delta^i\right) \Pi'(X_{\hat{t}-1})L \geq 0$ holds.*
- *For $t \geq \hat{t}$, the ratio of marginal utilities is 1 and the irreversibility constraints are binding ($\lambda_t > 0$), so that no investment occurs. The firm's earnings are then given by $d_t = W - a - \Pi(X_{\hat{t}})L$.*

The firm has two opposite needs. First, it must invest as soon as possible in order to decrease the expected loss and, hence, the insurance premium to be paid in each period. The need to smooth income delays the investment, however. The size of the delay depends on the fixed income W, the cost of auditing a and the one-period optimal investment X^*. Hence, the irreversibility multiplier λ is zero for the first $\hat{t}$ periods and the irreversibility constraint becomes binding when the number of future periods is not high enough to make a further investment valuable.

To prove this proposition, we need to present seven claims that are obtained from the first order conditions. Together they lead to the characterization of the sequence of transfers and investments obtained through a sequence of competitive contracts.

Claim 1 *In each period t, the transfer is $p_t = X_t - X_{t-1} + \Pi(X_t)L$ whatever the state of nature. That means, the firm chooses full insurance in each period but may not achieve perfect smoothing.*

Proof: The proof of all seven claims are relegated to the appendix.

The optimal contract gives full insurance to the firm since condition (6) implies $p_t^n = p_t^l = p_t$ for all t. Let us denote d_t the firm's income in period t in the contract, $d_t = W - a - p_t$. Since there is perfect competition on the insurance market, the transfer p_t is equal to the fair price of insurance plus the cost of investment: $p_t = X_t - X_{t-1} + \Pi(X_t)L$ so that $d_t = W - a - \Pi(X_t)L - (X_t - X_{t-1})$. Hence, condition (6) now writes:

$$\delta^t V'(W - a - p_t) = \delta^t V'(d_t) = \mu_t.$$

Claim 2 *Investment in period 0 is never smaller than the one period optimal investment, that is,* $X_0 geq X^*$*.*

Note that the level of precaution X^* maximizes the firm's earnings in one period when the insurance premium is fair: $X^* = \arg\max_X \{W - X - \Pi(X)L\}$. So, the firm wants to invest at least that level in the first period. Since investment in period 0 brings benefits in period 1, there may be an incentive to invest beyond X^* in $t = 0$.

Claim 3 *There is no investment in the last period. The firm's income in period* T *is then* $d_T = W - a - \Pi(X_{T-1})L$*.*

This, of course, results from our choice of a finite horizon. Since the firm no longer exists after period T, the optimal investment level is equal to the investment in the short sighted firm: X^* such that $1 + \Pi'(X^*)L = 0$. This no-investment result depends on the fact that: 1- investment is positive in period 0 since $\Pi'(0) = -\infty$; and that 2- from Claim 2, the minimum investment realized in period 0 is X^*. Hence, in period T, at least X^* has already been invested in prevention so that there is no need for further prevention in this last period.

Claim 4 *If there is investment in the first period only, the amount invested is less than the full commitment level* X^{**}*.*

When full commitment in long term insurance contract is not possible, the insurer never invests as much as X^{**} in the first period. Thus, there is underinvestment if repeated competitive contracts are imposed to the firm. This is due to the fact that with a sequence of short term, competitive contracts, the cost of investment cannot be financed by the first insurer who would spread the reimbursement of X_0 over the $T-1$ next periods. For period 1 through T, insurance must be priced at the expected loss on the competitive markets. Hence, the firm has to pay for X_0 in the first period. Since it is concerned with the smoothing of its revenue, it would decrease the amount invested in $t=0$. The next claim shows that underinvestment in the first period is indeed the consequence of investment delays.

Claim 5 *If for some $t<T$, $X_t = X_{t-1}$, then $X_\tau = X_{t-1}$ for all $\tau > t$*

Claim 5 means that if there is no investment performed in period t, then there will be none in the future. The proof of this claim teaches us a lot about the optimal pattern of investment. In each period, the marginal cost of investing an additional dollar in protection is compared to the marginal benefit of that investment (each of these amounts being, of course, weighted by the marginal utility in the period). Although the marginal cost of investment is equal to 1 in each period, the marginal benefit decreases over time because the discounted sum of future expected losses decreases as time passes whereas precaution increases.

To see why, note that in a full commitment contract, investment is done in period zero such that the marginal benefit of protection for the T periods to come is made equal to the marginal cost. The condition is then

$$1+\left(\sum_{i=0}^{T}\delta^i\right)\Pi'(X_0)L=0.$$

When long term contracts are not allowed, investment is delayed because the firm wants to smooth its revenue and distribute the burden of the investment over many periods. However, as time passes, the optimal amount of investment decreases since the number of future periods during which the firm benefits is reduced. Hence, given there is no further investment after period t (and given perfect income smoothing after t), the optimal investment rule is

$$1+\left(\sum_{i=0}^{T-t}\delta^i\right)\Pi'(X_t)L=0.$$

It follows that as soon as the above expression is nonnegative in one period, the existing amount of protection is sufficient for the remaining periods, even if it may not have been for the period before. This indicates delay in investment. Since investment only occurs in the first few periods, it follows that if at some point in time investment is not profitable, then it cannot be profitable afterwards.

Claim 6 *There is a period $\hat{t}$, $0 < \hat{t} \leq T$, in which no investment is made. The level of precaution reached before this period, $\hat{X}$, is such that $X^* < \hat{X} < X^{**}$.*

The optimal path of investments depends on the periodic comparison between marginal cost and benefits of further investments as well as on the trade-off between smoothing and protection. Optimal decisions are given by the first order conditions (7) that can be written for all t (with $\lambda_0 = 0$):

$$(1 + \Pi'(X_t)L) = \frac{\delta V'(d_{t+1})}{V'(d_t)} + \frac{\lambda_t - \lambda_{t+1}}{\delta^t V'(d_t)}$$

Claim 7 *The smoothing obtained in the contract is represented by the ratio $V'(d_{t+1})/V'(d_t)$, which is smaller than 1 as long as an investment is made, but increasing over time. After period $\hat{t}$, $V'(d_{t+1})/V'(d_t) = 1$.*

Intuitively, since there is some investment in period 1, the irreversibility constraint does not bind ($\lambda_1 = 0$). When there is no further investment (after period $\hat{t}$), competition between insurers imposes the same premium in each period. The firms's earnings are then constant after $\hat{t}$, implying a ratio of marginal utilities equal to 1.

Claims 1 to 7 contribute to prove Proposition 2 and help visualize the path of investments. This section makes clear that there is a trade off between early investment and smoothing. The firm is better off delaying investment on the first periods at the cost of facing a higher probability of loss in every period thereafter. As the number of periods remaining decreases, the marginal benefit of investment decreases, until the marginal benefit is zero. The level of technology at that time is lower than the level of technology invested in period 0 when long-term contracts were possible.

5 The non-binding long term contract

In section 4 we imposed that only one-period contracts are available and that the firm had to incur the audit cost every period. Since audits are costly, conducting audits every period is obviously sub-optimal. The incumbent insurer should take advantage of the information he has at the end of period $t-1$ to propose a contract that would be cheaper than conducting a costly audit in period t. It then follows that the transfer and investment schedule described in the previous section cannot be optimal if agents behave rationally.

We present in this section the optimal non-binding long term contract between a firm and its insurer taking into account that parties can renegotiate. The firm can still exit the relation in each period and contract with a competitor, provided it incurs the audit cost. The incumbent insurer can propose a better price to the firm since audits are unnecessary when the firm does not change insurer. We find the optimal arrangement starting from the sequence of competitive short term contracts and improving on the solution by allowing for offers by the incumbent insurer in each period. The setup is a repeated game where the incumbent insurer offers a contract in each period, taking into account the outside opportunities the firm can find on competitive markets.

We approach the non-binding long term contract in two steps. First, we assume that the schedule of investment is the same as in the previous section. This allows to show that the long-sighted sequence of short-term contracts may be improved upon by letting the incumbent insurer offer a contract so that the firm does not want to pay for an audit. We then describe the optimal sequence of investment in prevention.

5.1 Transfers

The solution for the optimal non-binding long term insurance relationship is not trivial. Contracts offered in each period depend on the value of the relationship in the future as well as on the amount invested in precaution in the past. Suppose initially that the stream of investments is the one obtained using the sequence of competitive short term contracts (Proposition 2). By allowing the incumbent insurer to offer a contract before the firm decides to enter a round of costly auditing (i.e., make its type public), the sequence of payments is altered. This result is presented as our third proposition.

Proposition 3 *Assuming the sequence of investments is the same as for a long-sighted firm facing short term contracts, the solution to the repeated insurance relationship is such that insurance is provided in every period by the same insurer. Transfers are then:*

- $p_0 = x_0 + \Pi(X_0)L - \delta a$.

- $p_t = x_t + \Pi(X_t)L + (1-\delta)a$ *for all* $0 < t < T$.

- *and* $p_T = \Pi(X_T)L + a$.

Proof: See appendix.

In the proof of Proposition 3 we show that the outcome of the sequence of short term contracts presented in section 4 can be recursively improved. This result obtains by the observation that, *for a given schedule of investments*, an insurer can make periodic positive profits and still offer the same lifetime utility to the firm. Keeping the same insurer all along allows the firm to economize on audits since the incumbent in period $t-1$ knows the amount of precaution at the beginning of period t. Given that the amount saved by the firm on an audit is a, the insurer makes a positive profit each period by offering a contract that is in every point the same as the one the firm would obtain after an audit has been performed.

Since the insurance market is competitive, every insurer in period t is willing to give the firm a rebate (of δa) to receive the contract and secure a rent (of a) the next period. In the first period, an initial audit has to be conducted before the firm chooses an insurer so that the initial insurer cannot receive rent a. Since the insurer is certain to keep the contract until period T, however, he is willing to offer a rebate in period 0 equal to the discounted value of the future profits he will be able to receive as the informed insurer. It follows that the initial period's insurer makes a zero net expected profit over its entire relationship.[11]

Hence for the same sequence of investments, transfers are equal to those when contracts are short-term, with the addition of a loading $(1-\delta)\,a$. All that is left to find now is the optimal sequence of investment. We have shown what the sequence of premia looks like for a given sequence of investment. We must now show that this sequence of investment may not be the one that obtains in the optimal relationship where the same insurer can keep the contract forever.

5.2 Investments

Since the firm can change insurer each period, investment is delayed. We have already established that it is not possible for an insurer to dissociate the investment in precaution from its payment by the firm. Optimally, the insurer would invest everything in period 0 and offer financing to the firm through future insurance premia. However, this is impossible since we know that, once the invest-

[11] To see why, note that if insurers are willing to give the firm a rebate of δa every period to secure a rent of a the next period, we have that for every period from $t=0$ to $t=T-1$, a rebate of δa is offered. At the same time rent a is pocketed by the insurer every period from $t=1$ to $t=T$. Given discount rate δ, we see that at $t=0$ the discounted payoff to the insurer is

$$\begin{aligned}\text{Payoff} &= -(\delta a)-\delta(\delta a)-\delta^2(\delta a)-...-\delta^{T-1}(\delta a)\\ &\quad +\delta(a)+\delta^2(a)+\delta^3(a)+...+\delta^{T-1}(a)+\delta^T(a)\end{aligned}$$

which is clearly equal to zero.

ment is performed, any investor can propose another contract with the profile p_t described above, that is accepted by the firm and leaves the first insurer uncompensated for its initial investment. There is, then, delay in the investment in prevention.

However, the path of investments may not be the one observed when the firm changes insurer in each period. Depending on the size of income effects, the sequence of investment can be different when the firm keeps the same insurer and then economize on audit costs compared to what obtains when the firm recontracts each period on the market. Indeed, the optimal smoothing rule is still given by

$$(1+\Pi'(X_t)\,L)=\frac{\delta V'(d_{t+1})}{V'(d_t)}+\frac{\lambda_t-\lambda_{t+1}}{\delta^t V'(d_t)},$$

with $d_t = W-(1-\delta)a-x_t-\Pi(X_t)L$ for $t=1,\cdots,T-1$. Since the firm does not have to pay for an audit in every period, earnings are higher than in the sequence of competitive short-term contracts. Depending on the function V, the ratio $\frac{\delta V'(d_{t+1})}{V'(d_t)}$ may be affected by the level of earnings d_t and d_{t+1}. This income effect changes the way investment is delayed from period to period.[12]

The sequence $x_0, x_1, \cdots, x_T$ that satisfies the smoothing condition may not be the one we obtained in the previous section. However, it is still such that there is a $\bar{t}>1$ such that $x_t=0$ for all $t\geq\bar{t}$. The possibility for the firm to renegotiate in each period makes the insurance relationship look much like as in a short term competitive environment. Because the firm has the possibility to renegotiate with other insurers, premia p_t cannot be used to spread the cost of investment through periods. The premium has to include the payment for the investment made in the period. Since the firm needs to smooth its earnings through time, the investment has to be delayed. The probability of accident is then decreased slowly in time while it could be made optimal in the first period if the firm could commit in a long term relationship with the insurer.

6 Other Issues

6.1 Perfect Memory

Robustness in these results should be tested for a change in the information setting. We supposed here that only the insurer operating the precaution technology in one period knows the level of risk in the beginning of the next period. This is a sensible assumption since outsiders may not be able to observe changes inside the firm. We also implicitly assumed that an insurer forgets this

[12] Note that if $V(d)=-\exp(-rd)$, there is no income effect and the profile of investments is exactly the one we find with the sequence of competitive short term contracts.

information as soon as he no longer insures the firm, and that all the market participants forget the outcome of the public audit after one period. Relaxing this hypothesis and supposing perfect memory instead does not affect the results: Investment is delayed and suboptimal, and follows the same sequence as shown in Proposition 3.

The reasoning is as follows. We know that the firm incurs the audit cost in the initial period so as to be able to obtain insurance initially. The initial insurer then invests some amount x_0 in prevention technology, an amount that is not known to outsiders. This means that outsiders, although they recall what was the firm's risk initially, do not know the firm's risk at the beginning of period 1. Suppose that a second round of investment is needed. If the firm wants to change insurer, it will need to incur the audit cost a. The incumbent insurer, knowing that the firm can incur the audit cost to signal its new risk characteristics, can make the audit unprofitable by offering the firm a rebate for not incurring the cost. And given that further investment is needed, the firm would have to incur the audit cost a third time to signal to the market its new risk characteristics after the two first rounds of investment.

Suppose on the other hand that no more investment is necessary. The firm then knows if it were to incur the audit cost that it would always pay a premium equal to the expected loss thereafter since every insurer would know, and recall, that the risk characteristics of the firm are such that no investment in prevention is warranted anymore. This means that the firm's sequence of payments until period T would be $-a - \Pi(X)L$, $-\Pi(X)L$, ..., $-\Pi(X)L$. On the other hand, if the incumbent insurer keeps the same contract structure as that of Proposition 3, the firm's sequence of payments until period T is $-(1-\delta)a - \Pi(X)L$, $-(1-\delta)a - \Pi(X)L$, ..., $-(1-\delta)a - \Pi(X)L$, $-a - \Pi(X)L$. To remain with the incumbent insurer, it then has to be that the second sequence of payments is less expensive than the first. Given the discount rate of δ, it is easy to show that the present values of the two sequences are equal. By concavity of the utility function, it follows that the firm prefers the second sequence of payments for any discount rate between zero and one. Hence, relaxing the assumption on imperfect memory does not change the results stated in Proposition 3.

6.2 Depreciation

Our prevention technology is such that any investment made in period t is fully in place in period $t+n$. In other words, the prevention technology does not depreciate nor become obsolete. This may be a strong assumption; investments made twenty years ago in prevention technology can hardly be perceived as pertinent today. Introducing a depreciation rate on the prevention technology, however, would not alter our results. Suppose that prevention capital depreciates at rate $(1-\gamma)$, so that at the beginning of period t, the level of prevention technology is γX_{t-1}. With depreciation of capital, one unit of capital today is no longer worth one unit tomorrow, but rather γ. This means that the marginal benefit of investing in one unit of prevention is reduced. The way it is reduced is only through a change in the discount rate: The discount rate is no longer δ, but rather $\gamma\delta$. The rest of the analysis is the same.

6.3 Financial Markets

Our paper assumes that financial markets are imperfect in the sense that the firm cannot finance its investment in prevention using a standard debt contract. Obviously, if the firm can have access to financial markets and borrow funds through a standard debt contract, the delay of precaution is not an issue anymore. If debt contracts are available, the firm could borrow the necessary amount to invest in an optimal level of prevention in period 0 and smooth its cost over the entire horizon. The insurer then only needs to offer technical expertise as to the prevention technology and offer full insurance at a fair price. This supposes, however, that long term debt contracts are available. If that is the case, where would be the loss in generality in saying that the initial insurer (i.e., the insurer who obtains the insurance contract in period zero after the first audit) purchases the entire debt of the firm, and then makes the optimal investment? It would not matter for the insurer that the firm changes insurers after period zero since it receives interest payments for the investment in prevention over the entire life of the firm. For our model, assuming that long-term insurance contracts are not possible is the same as assuming that standard debt contracts are not possible.

6.4 Predatory Pricing

One final issue that may warrant some discussion is the fact that the audit cost is assumed by the insured firm. We could imagine instead that the audit (which costs a) is paid by an outside insurer who wants to enter the bidding war for the insured's contract. Suppose the sequence of investment is the same as the one presented in section 4 and in Proposition 3. Suppose the incumbent insurer charges loading $a+\varepsilon_T$ in the last period. An outsider could then offer the firm to conduct an audit (which costs the outsider a) in exchange for which the firm signs a contract with the outsider that stipulates a premium load equal to $a + \frac{\varepsilon_T}{2}$. Clearly the firm is better off signing a contract with the outsider since it pays a lower total premium while receiving the same benefit.

To counter the behavior of the outsider, the incumbent insurer cannot charge a loading that is greater than the cost for an outsider to conduct an audit (i.e., a). The incumbent is therefore guaranteed to receive a profit equal to the outsider's audit cost (a) in the last period of the game. In period $T-1$, an outsider knows if he were to invest in an audit that he would be able to extract a rent of a in period T. This rent is valued at δa in period $T-1$. The outsider is then willing to invest in an audit if and only if $-a+\varepsilon_{T-1}+\delta a \geq 0$, where ε_{T-1} is the premium loading that can be charged in period $T-1$. The outsider will not invest in an audit in period $T-1$ if $\varepsilon_{T-1} \leq (1-\delta)\, a$. This means that the incumbent insurer cannot charge a loading greater than $(1-\delta)\, a$ in period $T-1$ to be certain to keep the contract and pocket profit a in period T.

In period $T-2$, an outsider would be willing to invest in an audit because he knows that if he gets the contract he would be able to pocket profit $(1-\delta)\, a$ in period $T-1$ and profit a in period T. In period $T-2$ these future profits are worth $\delta(1-\delta)\, a + \delta^2 a = \delta a$. An investment in an audit in period $T-2$ is then profitable if and only if $-a+\varepsilon_{T-2}+\delta a \geq 0$. This means that the incumbent insurer is still able to charge a premium load equal to $(1-\delta)\, a$ in period $T-2$ and still be able to keep the contract for the next two periods while pocketing profits of $(1-\delta)\, a$ in period $T-1$ and a in period T.

Applying the reasoning recursively, we find exactly the same sequence of transfers as in Proposition 3. There is therefore no loss in generality in assuming that the firm pays for the audit instead of an outside insurer.

7 Conclusion

In this paper, we have shown when firms do not have access to long term contracts that investment in prevention technology not only is delayed, but also less than the social optimum. This is true whether we force firms to recontract every period (after they pay for a costly audit), or whether we let the firm choose to renegotiate at any point in time or not. This investment path does not have any incidence on the amount of insurance purchased in each period as the firm is always fully insured. This does not mean, however, that the firm is able to perfectly smooth its income across time. Given that only short term contracts are available, firms end up paying each period for that period's marginal investment in prevention. This is in contrast with the full commitment case where the socially optimal level of investment is done in the initial period, and where the firm pays for this investment over its entire life, thus perfectly smoothing its income over time.

This result obtains for a given sequence of investments. As shown in section 3 of the paper, it is clear that an insurer who knows that it can hold the contract on the entire horizon will want to invest an amount X^{**} in the first period. The implementability of this policy depends on the enforceability of long-term relationships with the firm. If the firm can renegotiate with competitive insurers when the investment has been paid by the initial insurer, then the initial insurer will not invest X^{**} in the first period; instead the initial insurer will delay investment until some amount $X < X^{**}$ is invested.

The model we developed herein applies to the case of environmental hazard/insurance where the government sets the penalty/clean up fee of environmental damages to L. This loss which the firm must bear in case of an accident is catastrophic in the sense that the firm is better off insuring it than remaining in autarky. We did not address how the size of the penalty is chosen by the government. We can only presume (or hope?) that the government sets the penalty efficiently to cover all social costs related to environmental damages.

Another direct application is to the amount of prevention offered to patients by medical providers. If patients are able to switch from one health insurer to another, we would possibly observe a delay in the amount of preventive medicine offered by health providers. This then raises the question of whether the US health system, where individuals may not always have the same health insurer over their lifetime, produces socially inefficient health prevention incentives. On the other hand, a health system like in Canada, France and Germany provides efficient long term incentive to invest a socially optimal amount in prevention.

An aspect of the problem we did not touch is the possibility of moral hazard on the part of the firm in maintaining the prevention technology at its optimal level. Because the firm is insured and because the firm may be the only one to know whether regular maintenance is done, the insurer may no longer be willing to invest as much in prevention technology as before. This may delay investment in prevention even more.

8 References

1. Briys, E. and H Schlesinger (1990). Risk Aversion and the Propensity for Self-Insurance and Self-Protection. *Southern Economic Journal*, 57: 458–467.

2. Caillaud, B, G. Dionne and B. Jullien (2000). Corporate Insurance with Optimal Financial Contracting. *Economic Theory*, 16: 77–105.

3. Campbell T.S. and W.A. Kracaw (1987). Optimal Managerial Incentive Contracts and the Value of Corporate Insurance. *Journal of Financial and Quantitative Analysis*, 22: 315-328.

4. Chiappori, P.-A, I. Macho, P. Rey and B. Salanié (1994). Repeated Moral Hazard : The Role of Memory, Commitment, and the Access to Credit Markets. *European Economic Review*, 38: 1527–53.

5. Cooper, R. and B. Hayes (1987). Multi-period Insurance Contracts. *International Journal of Industrial Organization*, 5: 211–31.

6. DeMarzo, P.M. and D. Duffie (1995). Corporate Incentives for Hedging and Hedge Accounting. *Review of Financial Studies*, 8: 743-71.

7. Dionne, G. and N.A. Doherty (1994). Adverse Selection, Commitment, and Renegotiation: Extension to and Evidence from Insurance Markets. *Journal of Political Economy*, 102: 209–235.

8. Dionne G. and S. Spaeter (1998). Environmental Risk and Extended Liability: the Case of Green Technologies. Chaire de Gestion des Risques, École de Hautes Études Commerciales, Montréal, Working Paper 98-12.

9. Doherty, N.A. (1997). Corporate Insurance: Competition from Capital Markets and Financial Institutions. *Assurances*, 65: 63-94.

10. Gollier, C., B. Jullien and N. Treich (2000). Scientific Progress and Irreversibility: An Economic Interpretation of the Precautionary Principle. *Journal of Public Economics*, 75: 229–253.

11. Gul, F. (2001). Unobservable Investment and the Hold-Up Problem. *Econometrica*, 69: 343–376.

12. Immordino, G. (2000). Self-Protection, Information and the Precautionary Principle. *Geneva Papers in Risk and Insurance Theory*, 25: 179–187.

13. Joskow, P.L. (1985). Vertical Integration and Long-term Contracts: The Case of Coal-burning Electric Generating Plants. *Journal of Law, Economics and Organization*, 1: 33–79.

14. Joskow, P.L. (1987). Contract Duration and Relationship Specific Investments: Empirical Evidence from Coal Markets. *American Economic Review*, 77: 168–185.

15. Joskow, P.L. (1988). Asset Specificity and the Structure of Vertical Relationships: Empirical Evidence. *Journal of Law, Economics, and Organization*, 4: 95–117.

16. Laffont, J.-J. and J. Tirole (1987). Comparative Statics of the Optimal Dynamic Incentives Contract. *European Economic Review*, 31: 901–926.

17. MacMinn, R. (1987). Insurance and Corporate Risk Management. *Journal of Risk and Insurance*, 54: 658-77.

18. Mayers, D. and C.W. Smith, (1982). On the Corporate Demand for Insurance. *Journal of Business*, 55: 281-96.

19. Rogerson, W.P. (1985). Repeated Moral Hazard. *Econometrica*, 53: 69–76.

20. Rogerson, W.P. (1992). Contractual Solutions to the Hold-Up Problem. *Review of Economic Studies*, 59: 777–794.

21. Smith, C.W., and R.M. Stulz (1985). The Determinants of Firms' Hedging Policies. *Journal of Financial and Quantitative Analysis*, 20: 391-405.

22. Stulz, R.M. (1984). Optimal Hedging Policies. *Journal of Financial and Quantitative Analysis,* 19: 127-40.

23. Williamson, O.E. (1979). Transaction-Cost Economics: The Governance of Contractual Relations. *Journal of Law and Economics*, 22: 233–261.

24. Williamson, O.E. (1983). Credible Commitments: Using Hostages to Support Exchange. *American Economic Review*, 73: 519–540.

9 Appendix

Proof of Proposition 1. We show here that $x_0 = X^{**}$ and $x_t = 0$ for all $t \geq 1$. Using the system of first order equations (3)-(5) we apply the following recursive proof.

- Suppose $\lambda_T = 0$. Then, by (4) $1 + \Pi'(X_T)L = 0$ and $X_T \geq X_{t-1}$, that is,

  $$1 + \Pi'(X_{T-1})L \leq 1 + \Pi'(X_T)L = 0.$$

  But (5) for $t = T - 1$ implies $\delta^{T-1}\mu[1 - \delta + \Pi'(X_{T-1})] = \lambda_{T-1} \geq 0$ and then $1 + \Pi'(X_{T-1})L \geq \delta$. This is a contradiction.

  Hence, $\lambda_T > 0$ and $X_T = X_{T-1}$, that is $1 + \Pi'(X_T)L = 1 + \Pi'(X_{T-1})L > 0$

- Suppose $\lambda_{t+1} > 0$ and $\lambda_t = 0$ for any $t = 1 \cdots, T-1$. Then, $X_{t+1} = X_t \geq X_{t-1}$. Conditions (5) on investment imply:

  $$\delta^t \mu[1 - \delta + \Pi'(X_t)L] = -\lambda_{t+1} < 0$$
  $$\delta^{t-1}\mu[1 - \delta + \Pi'(X_{t-1})L] = \lambda_{t-1} \geq 0$$

  But this is a contradiction since $\lambda_t = 0$ implies that $1 - \delta + \Pi'(X_t)L \geq 1 - \delta + \Pi'(X_{t-1})L$.

- Since $\lambda_{t+1} > 0$ implies $\lambda_t > 0$ for all t and since $\lambda_T > 0$, $\lambda_t > 0$ for all $t > 0$ at the solution. This means that no investment is performed after period 0, so that $X_t = X_0$.

- Using $\mu\delta^T (1 + \Pi'(X_0)L) = \lambda_T$ and $\lambda_{T-1} = \mu\delta^{T-1}(1 - \delta + \Pi'(X_0)L) + \lambda_T$, we can recursively compute the multipliers λ_t to find:

  $$\lambda_t = \mu\delta^t \left[1 + \left(\sum_{i=0}^{T-t} \delta^i\right) \Pi'(X_0)L\right].$$

- The condition for investment in period $t = 0$ is $\mu(1 - \delta + \Pi'(X_0)L) = -\lambda_1$. Replacing λ_1 with the preceding formula, this can be rewritten as

  $$\mu \left(1 + \left(\sum_{i=0}^{T} \delta^i\right) \Pi'(X_0)L\right) = 0 \qquad \Rightarrow \qquad X_0 = X^{**}.$$

  Note that X^{**} is the level of precaution that minimizes the cost of investment plus the expected discounted value of potential losses. ∎

Proof of Claim 1. Condition (6) and usual regularity conditions on the function V imply that $p_t^n = p_t^l = p_t$. In each given period, the firm gets the same revenue in both states of nature, which means it is fully insured. The per period insurer's participation constraints impose $p_t = X_t - X_{t-1} + \Pi(X_t)L$. ∎

Proof of Claim 2. Let us write the firm's value for any given sequence of investments $X = (X_0, X_1, \cdots, X_T)$.

$$\begin{aligned}\mathcal{V}(X) &= V(W - a - X_0 - \Pi(X_0)L) + \delta V(W - a - X_1 + X_0 - \Pi(X_1)L) \\ &\quad + \sum_{\tau=2}^{T} \delta^\tau V(W - a - X_\tau + X_{\tau-1} - \Pi(X_\tau)L)\end{aligned}$$

Then, a change in the first period investment X_0 that leaves the other periods' levels of precaution unchanged entails a change in intertemporal value equal to

$$\frac{d\mathcal{V}(X)}{dX_0} = -V'(d_0)(1 + \Pi'(X_0)L) + \delta V'(d_1).$$

Having $X_0 \leq X^*$ implies $(1 + \Pi'(X_0)L) \leq 0$ by definition of X^*. Hence, if $X_0 \leq X^*$, then $d\mathcal{V}(X)/dX_0 \geq 0$, and the firm's value could be increased with $X_0 \geq X^*$. ∎

Proof of Claim 3. The irreversibility constraint is always binding in the last period: $\lambda_T > 0$ so that $X_T = X_{T-1}$. This is easy to show since if $\lambda_T = 0$, then $X_T \geq X_{T-1}$ and $1 + \Pi'(X_T)L = 0$. But $\lambda_T = 0$ in condition (7) for $t = T-1$ implies that $(1 + \Pi'(X_{T-1})L)\, \mu_{T-1} = \mu_T + \lambda_{T-1} > 0$ and, hence, that $X_{T-1} > X_T$, which is a contradiction. In the last period, the firm would always like to disinvest if this was possible. Hence, Claim 1 implies that the premium is $p_T = \Pi(X_T)L = \Pi(X_{T-1})L$. ∎

Proof of Claim 4. For this claim, we need to show the following corollary to claim 3.

Corollary 1 *In the problem of the long-sighted firm and the sequence of short term competitive contracts, the Lagrange multipliers for irreversibility constraints write*

$$\lambda_t = (1 + \Pi'(X_t)L)\, \delta^t V'(d_t) + \sum_{\tau=t+1}^{T} \delta^\tau V'(d_\tau)\Pi'(X_\tau)L.$$

Proof of Corollary 1. Claim 3 implies that $X_T = X_{T-1}$ and $\lambda_T = (1 + \Pi'(X_{T-1})L)\, \delta^T V'(d_T) > 0$ with $d_T = W - a - \Pi(X_{T-1})L$. The condition for investment (7) in the preceding period writes

$$\begin{aligned}\lambda_{T-1} &= (1 + \Pi'(X_{T-1})L)\, \delta^{T-1} V'(d_{T-1}) - \delta^T V'(d_T) + (1 + \Pi'(X_{T-1})L)\, \delta^T V'(d_T) \\ &= (1 + \Pi'(X_{T-1})L)\, \delta^{T-1} V'(d_{T-1}) + \Pi'(X_{T-1})L\delta^T V'(d_T)\end{aligned}$$

and then

$$\lambda_{T-2} = (1+\Pi'(X_{T-2})L)\,\delta^{T-2}V'(d_{T-2}) + \Pi'(X_{T-1})L\delta^{T-1}V'(d_{T-1}) + \Pi'(X_{T-1})L\delta^{T}V'(d_T)$$

so that computing each λ_t recursively we obtain:

$$\lambda_t = (1+\Pi'(X_t)L)\,\delta^t V'(d_t) + \sum_{\tau=t+1}^{T} \delta^\tau V'(d_\tau)\Pi'(X_\tau)L,$$

which completes the proof of the corollary.

Suppose the investment is done in the first period only, such that $x_0 = X_0 > 0$ and $x_t = 0$ for $t \geq 1$. Then, $X_t = X_0$ for all $t > 0$ and the firm's revenue is the same in each of these periods since the competitive insurers set the premia equal to $p_t = \Pi(X_0)L$ for all $t > 0$. Since we then have $V'(d_t) = V'(d_T)$ for all $t > 0$, Corollary 1 gives:

$$\lambda_1 = \delta V'(d_T)\left(1 + \left(\sum_{i=0}^{T-1}\delta^i\right)\Pi'(X_0)L\right)$$

The optimal condition for period 0 investment is:

$$V'(d_0)\,(1+\Pi'(X_0)L) - \delta V'(d_T) = -\lambda_1 = -\delta V'(d_T)\left(1 + \left(\sum_{i=0}^{T-1}\delta^i\right)\Pi'(X_0)L\right)$$

$$V'(d_0)\,(1+\Pi'(X_0)L) + \delta V'(d_T)\left(\sum_{i=0}^{T-1}\delta^i\right)\Pi'(X_0)L = 0$$

This can be rewritten as

$$V'(d_0)\,(1+\Pi'(X_0)L) - V'(d_T)\,(1+\Pi'(X_0)L) + \left\{V'(d_T)\left(1+\sum_{i=0}^{T}\delta^i\Pi'(X_0)L\right)\right\} = 0$$

The full commitment level of investment would be such that the last term is equal to 0:

$$\left(1+\sum_{i=0}^{T}\delta^i\Pi'(X^{**})L\right) = 0,$$

but this would impose $V'(d_0) = V'(d_T)$ which is impossible since the investment X_0 has to be paid for by the firm in period 0. Hence, $V'(d_0) > V'(d_T)$ and $\left(1+\sum_{i=0}^{T}\delta^i\Pi'(X_0)L\right) < 0$. This implies that there is underinvestment in precaution if everything has to be invested in the first period. ∎

<u>**Proof of Claim 5**</u>. This proof is made recursively.

- We know there is no investment in period T.

- Let us suppose there is no investment in $T-1$ neither and measure the welfare effect of a small increase of investment in period $T-1$. If there is no investment in $T-1$ the firm's dividend is the same in the last two periods: $d_T = d_{T-1} = W - a - \Pi(X_{T-2})L$. The discounted value of the firm for the last two periods writes:

$$\mathcal{V}_{T-1}(X_{T-2}) = V(d_{T-1}) + \delta V(d_T)$$

A small positive dx_{T-1} then entails a change $d\mathcal{V}_{T-1}(X_{t-2})$ in the firm's welfare:

$$d\mathcal{V}_{T-1}(X_{T-2}) = \{-V'(d_{T-1})[1 + \Pi'(X_{T-2})L] - \delta V'(d_T)\Pi'(X_{T-2})L\}\, dx_{T-1}$$

Hence, $d\mathcal{V}_{T-1}(X_{T-2})/dx_{T-1} > 0$ at $x_{T-1}=0$ if $1+\Pi'(X_{T-2})L + \delta\Pi'(X_{T-2})L < 0$.

- Let us suppose there is no investment in periods t through T, the firm's revenue then being the same $d_t = d_T$ in all these periods. A small increase of investment in period t has a positive impact on the firm's utility, $d\mathcal{V}_t(X_{t-1}) > 0$, if

$$1 + \sum_{i=0}^{T-t} \delta^i \Pi'(X_{t-1})L < 0.$$

Note that if $1 + \sum_{i=0}^{T-t} \delta^i \Pi'(X_{t-1})L \geq 0$ in one period t, then $1 + \sum_{i=0}^{T-(t+\tau)} \delta^i \Pi'(X_{t-1})L \geq 0$ for all $\tau = 1 \cdots T-t$. Hence, if the condition for no investment (given there is no investment in $t+1, \cdots T$) holds in t, it holds in all subsequent periods (given there is no investment after these periods). Since there is no investment in T, the condition is valid in $T-1$ and then in any preceding period and the result holds. ∎

Proof of Claim 6. From Claim 5, if no investment is made at date $\hat{t}$, then no investment is ever performed after $\hat{t}$. Since there is always a positive investment in $t = 0$, $\hat{t} > 0$. Since there is no investment performed in $t = T$, the first period with no investment made is any period $t > 0$. Period $\hat{t}$ is such that the condition for no investment established in the preceding Claim is verified in $\hat{t}$ and not in $\hat{t} - 1$:

$$1 + \left(\sum_{i=0}^{T-(\hat{t}-1)} \delta^i\right) \Pi'(X_{\hat{t}-2})L < 0 \quad \text{and} \quad 1 + \left(\sum_{i=0}^{T-\hat{t}} \delta^i\right) \Pi'(X_{\hat{t}-1})L \geq 0$$

Denote $\hat{X}$ the minimum precaution level such that no investment is performed in $\hat{t}$. From Claim 2 it has to be that $\hat{X} \geq X_0 > X^*$. This level is such that $1 + \left(\sum_{i=0}^{T-\hat{t}} \delta^i\right) \Pi'(\hat{X})L = 0$ while the full commitment level is such that $1+\left(\sum_{i=0}^{T}\delta^{i}\right)\Pi'(X^{**})L=0$. Hence, $\left(\sum_{i=0}^{T-\hat{t}}\delta^{i}\right)\Pi'(\hat{X})L=\left(\sum_{i=0}^{T}\delta^{i}\right)\Pi'(X^{**})L$ and $\left(\sum_{i=0}^{T-\hat{t}}\delta^{i}\right)\Pi'(\hat{X})L<\left(\sum_{i=0}^{T-\hat{t}}\delta^{i}\right)\Pi'(X^{**})L$. This implies $\hat{X}<X^{**}$. ∎

Proof of Claim 7. First order conditions for our problem write

$$(1+\Pi'(X_t)L)=\frac{\delta V'(d_{t+1})}{V'(d_t)}+\frac{\lambda_t-\lambda_{t+1}}{\delta^t V'(d_t)}$$

For $t<\hat{t}$ a positive investment is made each period so that $X_t>X_{t-1}$ and $\lambda_t=0$. We then have

$$1>(1+\Pi'(X_t)L)=\frac{\delta V'(d_{t+1})}{V'(d_t)},\quad t=0,\cdots,\hat{t}-1.$$

This ratio increases in time since $\Pi'(.)$ is increasing and X_t increases in time.

For $t>\hat{t}$, $\lambda_t>0$: no investment is made in those periods so $d_t=W-a-\Pi(X_{\hat{t}})L$, $\forall t>\hat{t}$ and $V'(d_t)/V'(d_{t+1})=1$, which completes the proofs of the seven claims. ∎

Proof of Proposition 3. Let us still denote $\hat{t}$ as the cut-off period after which no investment is performed when there are no long term contracts. $\hat{X}=X_{\hat{t}}$ is the amount invested in protection up until time $\hat{t}$. With a sequence of short term contracts, $\Pi\left(\hat{X}\right)$ is the probability of accident for the remaining $T-\hat{t}$ periods. Let A_t represent the incumbent insurer at the start of period t.

- At the beginning of period T, insurer A_T (that had the contract in $T-1$) knows the firm's level of risk. By accepting the incumbent insurer's contract for period T, the firm saves the audit cost. Everything else being equal, the incumbent insurer can charge a above the fair premium, that is, $p_T=\Pi\left(\hat{X}\right)L+a$ and still maintain his relationship with the firm. The firm is indifferent between this contract and the one it could obtain on the competitive market after an audit.

- The insurer who obtains the contract in period $T-1$ will be able to retain the contract in period T, and thus secure a payoff of a in the end period. The market equilibrium premium, $\tilde{p}_{T-1}$,[13] in $T-1$ is then such that

$$\tilde{p}_{T-1}-\Pi\left(\hat{X}\right)L+\delta a=0,$$

meaning that competitive insurers offer a rebate of δa to get the contract. Competition in period $T-1$ then gives the firm an intertemporal utility of

$$\mathcal{V}_{T-1}=V\left(W-a-\Pi\left(\hat{X}\right)L+\delta a\right)+\delta V\left(W-a-\Pi\left(\hat{X}\right)L\right).$$

[13] We identify by the tilde ($\tilde{}$) the market competitive premium.

- The incumbent insurer in period $T-1$, A_{T-1}, can offer a contract that allows the firm to save on the audit cost in $T-1$. This contract must give the firm an expected utility at least as high as what it can get with competitors, $\mathcal{V}_{T-1}$. The insurance premium (p_{T-1}) must then be such that

$$V\left(W-p_{T-1}\right)+\delta V\left(W-a-\Pi\left(\hat{X}\right)L\right)=V\left(W-a-\Pi\left(\hat{X}\right)L+\delta a\right)+\delta V\left(W-a-\Pi\left(\hat{X}\right)L\right)$$

That is, insurer A_{T-1} can keep the contract until the end of the horizon with the sequence of premia $p_{T-1}=a-\delta a+\Pi(\hat{X})L$ and $p_T=a+\Pi(\hat{X})L$. In period $T-1$, A_{T-1} obtains a rent of a since the firm does not have to perform an audit, but he has to offer a rebate of δa in order to keep the contract from competitors.

- Suppose the incumbent insurer in period $\hat{t}+1$ can keep the contract until T and secure a sequence of transfers $p_{\hat{t}+1}, p_{\hat{t}+2}, \cdots, p_T$ with $p_t=a-\delta a+\Pi(\hat{X})L$ for $t=\hat{t}+1,\cdots,T-1$ and $P_T=a+\Pi(\hat{X})L$. The intertemporal utility for the firm with such a sequence is $\mathcal{V}_{\hat{t}+1}$. In period $\hat{t}$, the competitive premium $\tilde{p}_{\hat{t}}$ is such that

$$\tilde{p}_{\hat{t}}-\Pi\left(\hat{X}\right)L+\sum_{t=\hat{t}+1}^{T-t-1}\delta^t\left(a-\delta a\right)+\delta^{T-t}a=0.$$

We then have $\tilde{p}_{\hat{t}}=\Pi(\hat{X})L-\delta a$, which yields utility $\mathcal{V}_{\hat{t}}$ to the firm equal to

$$\mathcal{V}_{\hat{t}}=V\left(W-a-\Pi\left(\hat{X}\right)L+\delta a\right)+\delta\mathcal{V}_{\hat{t}+1}$$

- Insurer $A_{\hat{t}}$ will take advantage of his private information to propose a premium $p_{\hat{t}}$ that gives the firm this exact expected utility. We thus have to find the $p_{\hat{t}}$ that solves

$$V\left(W-p_{\hat{t}}\right)+\delta\mathcal{V}_{\hat{t}+1}=V\left(W-a-\Pi\left(\hat{X}\right)L+\delta a\right)+\delta\mathcal{V}_{\hat{t}+1}$$

This is obtained with $p_{\hat{t}}=a-\delta a+\Pi\left(\hat{X}\right)L$.

Insurer $A_{\hat{t}}$ can continue its relationship with the firm for every period from then on. The total discounted payoff to the incumbent insurer who remains in this relationship from period $\hat{t}$ onward is given by $\sum_{t=0}^{T-\hat{t}-1}\delta^t\left(1-\delta\right)a+\delta^{T-\hat{t}}a=a$.

- In period $\hat{t}-1$ an investment $x_{\hat{t}-1}$ is made for which the firm has to pay. The insurer in period $\hat{t}-1$ would optimally invest $x_{\hat{t}-1}$ but spread the cost over the following periods. This would increase the firm's utility and allow the insurer to charge a higher premium in each period thereafter. However, insurer $A_{\hat{t}}$ cannot spread the cost of investment $x_{\hat{t}-1}$ on the following periods since competitors would then still offer the profile p_t from $\hat{t}$ on and obtain the contract. The investment $x_{\hat{t}-1}$ then has to be paid by the firm in period $\hat{t}-1$.

The competitive premium would then be such that

$$\tilde{p}_{\hat{t}-1} - x_{\hat{t}-1} - \Pi\left(X_{\hat{t}-2} + x_{\hat{t}-1}\right) L + \delta a = 0,$$

which gives the firm the utility

$$\mathcal{V}_{\hat{t}-1} = V\left(W - a - x_{\hat{t}-1} - \Pi\left(X_{\hat{t}-1}\right) L + \delta a\right) + \delta \mathcal{V}_{\hat{t}}$$

- The incumbent insurer $A_{\hat{t}-1}$ can make sure he keeps the contract in $\hat{t}-1$ if he performs the investment $x_{\hat{t}-1}$ and charges a transfer $p_{\hat{t}-1}$ such that

$$V\left(W - p_{\hat{t}-1}\right) + \delta \mathcal{V}_{\hat{t}} = V\left(W - a - x_{\hat{t}-1} - \Pi\left(X_{\hat{t}-1}\right) L + \delta a\right) + \delta \mathcal{V}_{\hat{t}},$$

That is

$$p_{\hat{t}-1} = (1-\delta)a + x_{\hat{t}-1} + \Pi\left(X_{\hat{t}-1}\right) L.$$

- Applying this reasoning recursively from $t = \hat{t}-1$ to $t = 1$ with investment x_t in each of those periods, we find that the payment in period $t < \hat{t}$ must be

$$p_t = \Pi\left(X_t\right) L + x_t + a - \delta a,$$

and the discounted payoff for the insurer getting the contract in t is $a - \delta a + \delta(a - \delta a) + \cdots + \delta^{T-t-1}(a - \delta a) + \delta^{T-t} a = a$

- In period 0, an audit must be conducted since there is no incumbent insurer with the information on the firm's risk. Insurer, then, cannot charge a and demand no audit. Since the insurer that receives the contract in $t = 0$ can remain in relation with the firm until $t = T$ and secure a payoff valued δa in period 0, competition pushes all investors to propose a rebate of δa in period 0. The transfer in $t = 0$ is then

$$p_0 = x_0 + \Pi\left(X_0\right) L - \delta a,$$

and the discounted total payoff from the contract for the insurer is 0. ∎

Liste des publications au CIRANO*

Série Scientifique / *Scientific Series* (ISSN 1198-8177)

---

* Consultez la liste complète des publications du CIRANO et les publications elles-mêmes sur notre site Internet :

http://www.cirano.qc.ca/publication/documents.html