



Série Scientifique
Scientific Series

95s-33

**REAL INVESTMENT
DECISIONS UNDER
INFORMATION CONSTRAINTS**

*Gérard Gaudet, Pierre Lasserre,
Ngo Van Long*

Montréal
Juillet 1995

CIRANO

Le CIRANO est une corporation privée à but non lucratif constituée en vertu de la Loi des compagnies du Québec. Le financement de son infrastructure et de ses activités de recherche provient des cotisations de ses organisations-membres, d'une subvention d'infrastructure du ministère de l'Industrie, du Commerce, de la Science et de la Technologie, de même que des subventions et mandats obtenus par ses équipes de recherche. La *Série Scientifique* est la réalisation d'une des missions que s'est données le CIRANO, soit de développer l'analyse scientifique des organisations et des comportements stratégiques.

CIRANO is a private non-profit organization incorporated under the Québec Companies Act. Its infrastructure and research activities are funded through fees paid by member organizations, an infrastructure grant from the Ministère de l'Industrie, du Commerce, de la Science et de la Technologie, and grants and research mandates obtained by its research teams. The Scientific Series fulfils one of the missions of CIRANO: to develop the scientific analysis of organizations and strategic behaviour.

Les organisations-partenaires / The Partner Organizations

- Ministère de l'Industrie, du Commerce, de la Science et de la Technologie.
- École des Hautes Études Commerciales.
- École Polytechnique.
- Université de Montréal.
- Université Laval.
- McGill University.
- Université du Québec à Montréal.
- Bell Québec.
- La Caisse de dépôt et de placement du Québec.
- Hydro-Québec.
- Banque Laurentienne du Canada.
- Fédération des caisses populaires de Montréal et de l'Ouest-du-Québec.
- Télé globe Canada.
- Société d'électrolyse et de chimie Alcan Ltée.
- Avenor.
- Service de développement économique de la ville de Montréal.

Ce document est publié dans l'intention de rendre accessible les résultats préliminaires de la recherche effectuée au CIRANO, afin de susciter des échanges et des suggestions. Les idées et les opinions émises sont sous l'unique responsabilité des auteurs, et ne représentent pas nécessairement les positions du CIRANO ou de ses partenaires.

This paper presents preliminary research carried out at CIRANO and aims to encourage discussion and comment. The observations and viewpoints expressed are the sole responsibility of the authors. They do not necessarily represent positions of CIRANO or its partners.

Real Investment Decisions Under Information Constraints*

G rard Gaudet[†], Pierre Lasserre[‡], Ngo Van Long^{*‡}

R sum  / Abstract

En pr sence d'asym trie d'information entre propri taires (le principal) et administrateurs (l'agent), la relation investissement co t du capital et la relation investissement valeur implicite du capital subissent une distorsion pour tous les types. Dans un mod le avec co t d'ajustement par ailleurs standard, il apparait notamment un r gime d'inaction pour une certaine gamme de co ts. Ce ph nom ne se pr sente sous une forme diff rente de ce qu'implique la pr sence de co ts fixes ou d'irr versibilit s, mais ressemble   ce qui survient lorsque le co t du capital est diff rent pour une hausse que pour une baisse du stock. L'incertitude, qui prend la forme d'un  largissement de la distribution des types, tend cependant   r duire l'investissement. Le mod le clarifie l'interpr tation du q de Tobin sous asym trie d'information et explique certains r sultats de la litt rature sur les fusions et acquisitions.

We investigate investment behavior when there is asymmetry of information between owners (the principal) and managers (the agent). The model accepts the standard cost-of-adjustment model as a particular case and is directly compared with it. For all types, information asymmetry distorts the relationship between investment and the cost of capital, and the relationship between investment and the shadow value of capital. In particular, a regime of inaction appears over a certain cost range, in an observationally different way than when fixed adjustment costs, or irreversibilities, cause a similar phenomenon. Uncertainty, in the form of an increase in the spread of agents' types, tends to reduce investment despite symmetric adjustment cost and perfect competition. The model clarifies the interpretation of Tobin's q under asymmetric information and explains some results of the mergers and acquisition literature.

Mots cl s : Contrats incitatifs ; Dynamique ; Information asym trique ; Relation principal-agent ; Investissement ; Incertitude.

Key words: Incentive contracts; Dynamic; Asymmetric information; Principal-agent relationship; Investment; Cost of adjustment; Uncertainty.

[†] The Authors are members of the CIRANO; Pierre Lasserre also belongs to the GREQAM, Universit  d'Aix-Marseille II. Financial support from the Fonds FCAR du Gouvernement du Qu bec is gratefully acknowledged.

Please address all correspondence to G rard Gaudet, D partement d' conomie, Universit  Laval, Qu bec, Canada G1K 7P4. E-mail: Gaudetg@cirano.umontreal.ca.

[‡] D partement d' conomie and GREEN, Universit  Laval, Qu bec.

[‡] D partement des sciences  conomiques, Universit  du Qu bec   Montr al.

* Department of Economics, McGill University.

1 Introduction

There is a huge literature on real business investment. It reflects the importance of investment both as a component of and as a determinant of economic activity. It also reflects the considerable empirical and theoretical difficulties associated with the analysis of capital spending. This literature is exemplified in the survey by Robert Chirinko (1993) or in the recent paper by Abel and Eberly (1994). In its branch evolving from the neoclassical tradition it still largely neglects important issues of agency and information.

Yet investment as perhaps the most important business decision is at the core of the corporate governance challenge. In the introduction of its 1994 survey of that subject The Economist wrote: ‘... managers have become insufficiently accountable to shareholders. From blatant thievery such as that by Robert Maxwell to lousy investments (Japanese firms’ purchases of overpriced American property or American Express’s attempt to become a financial conglomerate) to failures to tackle looming problems quickly enough as at IBM there is ample evidence of waste that might have been avoided had bosses been on a tighter rein.’

To be fair it should be noted that agency costs, asymmetric information and corporate control considerations have been extensively studied on the financial side of the investment equation (see Harris and Raviv 1991). However because of its emphasis on capital structure and financial markets the finance literature has not contributed directly to investigating the role of such considerations in the determination of real investment.

Our paper is a contribution toward filling this gap. We introduce asymmetric information into a standard cost-of-adjustment model of investment and we derive the resulting behavior of the investment function. Our model describes a principal-agent relationship with investors (inside or outside shareholders) as principal and managers as the agent. It could also apply to situations involving a regulator as principal and the

firm as agent.

Unlike what might have been expected, the incentive compatible investment function is not merely an attenuation of its full information counterpart, but may involve a qualitatively different behavior for some types. Several results appear surprising at first glance, both because they add some hitherto ignored considerations to real investment decisions and their determinants, and because they illustrate the implications of introducing dynamic considerations to standard principal-agent models. Thus we find that situations may occur where both the low cost and the high cost types are asked to carry out the same investment as they would under full information, if the principal faced the same shadow price of capital. However, because the shadow price of capital is affected by informational asymmetry, we find that the investment behavior of all types must in fact be modified relative to the full information situation. We also show that investment behavior under asymmetric information exhibits hysteresis, although for reasons quite different from those described by Dixit and Pindyck (1994). Of course, these results depend on the seriousness of informational asymmetries; in contrast with otherwise similar adjustment cost models (Abel 1983; Caballero 1991) an increase in uncertainty, taking the form of an increase in the spread of types, has a depressing effect on investment. Our model also casts a new shadow on the q theory of investment. We show that, if informational asymmetries matter, evaluating the shadow price of capital by market mechanisms raises a much more serious issue than the well understood problem of the identity between average and marginal capital value. The relevant rent actually is split between shareholders and managers, and the stock exchange only provides information on the share accruing to the formers.

We describe and further motivate the model in Section 2. Section 3 is devoted to solving the problem of the principal, which is to tell managers what investment to choose, given their informational advantage and conflicting objective. The similarity of

our basic model with the standard cost of adjustment model allows easy comparisons with the latter in particular with the extended model of Abel and Eberly. In Section 4 we identify and explain the qualitative differences and similarities in the predictions of our model and the standard cost of adjustment model. We also discuss the implications of information asymmetry on the q theory of investment and more generally the additional contribution of our model to major existing models of real business investment.

2 The model

As mentioned earlier there are several reasons why the objectives of managers and of shareholders may not coincide and why managers may have an informational advantage over shareholders. Harris and Raviv (1991) although with a focus on implications for capital structure survey most of them. Since many capital structure decisions aim at improving real investment decisions they are relevant to our discussion. The reader is referred to their paper for details and references.

Conflicts between shareholders and managers may arise because managers hold only part of the residual claim. Rather than devoting themselves entirely to profit enhancing objectives they may withhold information about the best investment prospects in order to promote decisions more favorable to their own personal benefits. Another type of conflict may arise because shareholders have an incentive to invest suboptimally under limited liability: if a project is successful they capture most of the gain but if the project fails they cannot lose more than their investment. On the other hand managers may lose their reputation in a bankruptcy (Diamond 1989; Hirshleifer and Thakor 1989). Similarly it has been shown by Myers and Majluf (1984) that if outside investors are less well-informed than managers about the value of the firm's assets then equity may be mispriced by the market. If an investment is to be financed by issuing new equity underpricing may be so severe that new investors capture more than the net present value

of the project resulting in a net loss' (Harris and Raviv p.306) to insiders including managers if they own equity.

These are some of the reasons why shareholders' and managers' objectives may conflict. Our model will most directly reflect those where managers try to enhance their own well-being while shareholders try to keep the highest possible expected surplus to themselves. While the model would have to be altered if it was to focus on the other types of conflicts listed above the spirit of the exercise would remain the same. Several avenues may be available to (partially) resolve this conflict. We focus on one of them by modelling a principal-agent relationship where the shareholders act as a principal while managers as a group are their agent.

The objective of the agent is the maximization of managers' residual claim expected cumulative discounted revenues net of relevant costs minus transfers to the principal. The objective of the principal is to maximize a weighted sum of what is left to the agent and what is transferred to her. It is not crucial for the principal to assign a positive weight to the objective of the agent. In a crude way a positive weight may reflect the fact that shareholders often hold administrative positions themselves in other firms so that they may show concern for the well-being of managers as a profession and the fact that managers usually hold shares in their own company. Both the principal and the agent have the same discount rate.

Consider the simplest possible technology giving output at date t as a concave differentiable positively monotonic function of the stock of capital (time subscripts will be omitted where no ambiguity arises)

$$q_t = g(k_t)$$

Capital may be interpreted to include plant and equipment but also goodwill acquired through advertising or other marketing expenses knowhow the size and training level

of the labor force etc.. We assume that $g'(k)$ is finite and $g(k)$ is non negative for any non negative k . Capital evolves according to investment i_t . For simplicity assume that there is no depreciation so that

$$k_{t+1} = k_t + i_t, \quad k_t \geq 0 \quad (1)$$

The cost of investment is assumed to consist of the asset cost of equipment v plus a cost of adjustment which we take to be quadratic: $ai + \frac{1}{2}bi^2$. Thus in total the cost of an investment i is

$$[v + a]i + \frac{1}{2}bi^2$$

As argued above there are several reasons why the agent might prefer the principal not to know this cost with certainty. There are also several reasons why this cost may be private information. Consider v . Whether equipment is being sold or purchased the transaction price may differ from the posted price by an amount which depends on the relationship between the parties to the transaction. An extreme case would be transactions involving kickbacks. Such cost components are likely to be unknown to the principal whether it is a board of shareholders or a government. Also access to financing may depend on the relationship between managers and lenders; once financing costs are capitalized into the asset price personal differences imply that the asset cost component of the cost of investment is firm specific possibly unknown to shareholders. Similarly a may reflect firm or manager specific inconveniences associated with changes in size and organization; this is likely to be private information and as we argue further below of interest to the principal. More generally since expected investment returns determine the relative cost of investment assuming that the cost of investment is private information will serve to model the idea that managers are better informed about both returns and costs than shareholders.

Assuming for simplicity that the public information component of $v + a$ is zero and that b is common knowledge the cost of investment may be written

$$c(i_t, \theta_t) = \theta_t i_t + \frac{1}{2} b i_t^2 \quad (2)$$

where $\theta_t > 0$ is a privately observed parameter that varies from period to period. This formulation is also compatible with a focus on the cost of intermediation as in Bernanke and Gertler (1989) or Cooper (1994) where variations in the cost of capital accumulation reflect fluctuations in the frequency of monitoring a representative project. Managers may be better aware of such costs than shareholders¹.

In order to focus on the effect of asymmetric information on investment decisions we eliminate also any common uncertainty: future prices p_t are known with certainty and technology does not change over time. These assumptions are inconsequential for our purposes and have the advantage of simplicity². We also assume that the θ 's are uncorrelated over time so that θ has the same continuous distribution $f(\theta) > 0$ over the same interval $[\theta^L, \theta^H]$ at all dates. Thus knowing θ at any date t does not provide any information about its subsequent values. This assumption may not be very realistic; it clarifies the role of asymmetric information by ensuring that the sole source of dynamics in the model is the investment process. As in the standard cost of adjustment model of investment the sole reason why the decision maker must be forward looking is that it is infinitely costly to change the level of capital instantaneously and that current

¹Altering Cooper's formulation slightly in order to allow for capital to be durable we may write $k_{t+1} = k_t + \theta i_t$ where θ reflects the cost of intermediation: a given investment effort may affect the stock of capital differently according to the value of θ . Allowing for costs of adjustment the corresponding cost is $v i_t + \frac{1}{2} b [k_{t+1} - k_t]^2$. Consequently the cost of obtaining k_{t+1} given that the current stock is k_t is $\frac{v}{\theta} [k_{t+1} - k_t] + \frac{1}{2} b [k_{t+1} - k_t]^2$ which is analogous to (2).

²In the problem presented below it may alternatively be assumed that future output prices and technology are stochastic and combine in such a way that net revenues at date t are $R(k_t, \epsilon_t)$ where ϵ_t follows a Brownian motion. As long as there is no asymmetry in the observation of ϵ_t the standard dynamic programming solution approach will apply in that case and our qualitative results will not be affected.

investment decisions affect future capital levels.

The objective of the agent Π_t may be decomposed into current net profit π_t and expected cumulative discounted net future profit ψ_t from $t + 1$ on as evaluated at t . For the technology just described this means maximizing

$$\begin{aligned}\Pi_t &= \pi_t + \psi_t \\ &= p_t g(k_t) - \left[\theta_t i_t + \frac{1}{2} b i_t^2 \right] - R_t + \psi_t(k_{t+1})\end{aligned}\tag{3}$$

subject to (1) where R_t is the amount transferred to the principal. R may be thought of as dividends demanded by the principal.

As explained earlier we assume that one dollar left to the agent is worth α , $0 \leq \alpha < 1$ to the principal. Thus the principal maximizes

$$\alpha \left\{ p_t g(k_t) - \left[\theta_t i_t + \frac{1}{2} b i_t^2 \right] - R_t + \psi_t \right\} + R_t + \Gamma(k_{t+1})\tag{4}$$

subject to (1) and the rationality constraint $\Pi_t \geq 0$ where $\Gamma(k_{t+1})$ is cumulative discounted transfers to the principal from $t + 1$ on as expected at t .

In the rest of this Section we solve the symmetric information version of this problem. This solution will serve as a benchmark against which we will compare the solution of the asymmetric information problem. We will add the superscript s to refer to variables or functions that are defined or evaluated under symmetric information. Thus the use of Γ^s and ψ^s will indicate that the principal is aware that future decisions will be made in a symmetric information setup.

Under symmetric information θ is observed by both the principal and the agent upon its realization. Since the principal has the power to set R it is obvious that her best choice at all dates is to set R in such a way that $\Pi = 0$ leaving the agent indifferent between participating in the relationship or not. Consequently problem (4) is equivalent

to maximizing by choice of i

$$p_t g(k_t) - \left[\theta_t i_t + \frac{1}{2} b i_t^2 \right] + \Gamma_t^s(k_{t+1})$$

subject to (1). This is a simplified but standard version of the cost of adjustment investment model. The first-order condition for an interior solution ($i_t > -k_t$) is

$$i_t^s = \frac{\Gamma_t^{s'} - \theta_t}{b} \tag{5}$$

where $\Gamma_t^{s'}$ is the shadow price of capital – the discounted sum of expected future marginal revenue products. Thus if t happens to be the last period – T – and its derivatives vanish so that i is negative: since there is no use keeping any capital for future periods – it is desirable to sell as much of the remaining stock as possible – while keeping adjustment costs to an acceptable level. If instead t is the second last period – then keeping a marginal unit of capital for period $t + 1$ yields an advantage of $\Gamma^{s'} = \delta p_{t+1} g'(k_{t+1})$ in terms of increased future production – where δ is the discount factor $0 < \delta < 1$. This marginal value product of capital is non stochastic. However suppose now that there might be yet an extra period $t + 2$; then since θ_{t+1} is unknown i_{t+1} which will be given by (5) at $t + 1$ is unknown at t ; as a result – the marginal product of k_{t+2} , $g'(k_{t+2})$ is unknown at t – so that the marginal impact on future revenues of increasing the stock of capital at t is stochastic.

Define T as the first interruption in production – or equivalently – the first date at which k is zero³. In general T if finite is unknown and stochastic. Then the shadow

³Depending on the trajectory of p production may start again after an interruption. However marginal products in the new production phase will be independent of capital levels before the interruption which justifies defining T as the first interruption.

price of capital at t is under symmetric information

$$\Gamma_t^{s'} = E_t \left\{ \sum_{\tau=1}^{T^s-t} \delta^\tau p_{t+\tau} g' \left(k_{t+\tau}^s \right) \right\}$$

As (5) indicates when θ is high relative to $\Gamma^{s'}$ it is desirable to sell capital; otherwise it is desirable to buy. This depends on whether future output prices are high and on expected future levels of k as they develop when (5) is applied after successive realizations of θ . To avoid technical difficulties we rule out bubbles that is price trajectories that would cause $\Gamma^{s'}$ (and i^s) to be infinite.

3 The model under asymmetric information

3.1 Preliminary remarks

Consider now the asymmetric information case. θ is observed upon its realization by the agent. Consequently the principal must rely on the information given to her by the agent in order to pursue her objective. Since there is no intertemporal correlation between the θ 's the agent does not lose any of his future informational advantage when he reveals current information to the principal. As a consequence if the principal chooses to use an incentive mechanism there is no possibility of a ratchet effect as in Laffont and Tirole (1988) and the revelation principle applies as in static setups.

We assume that the principal cannot credibly give up her claim to a share in any future rents in exchange for a lump-sum payment whose amount would be agreed upon before future cost conditions are revealed to the agent. This assumption is justified by wealth constraints on managers. Indeed almost by definition the existence of a publicly-held firm implies that managerial skills and wealth are held by separate groups of individuals. We do assume however that the principal is able to commit to one-

period contracts. Thus during any period shareholders will keep the managers if they receive the dividends and see the action (investment) that were agreed upon.

Under such circumstances the best the principal can do is to design a succession of one-period contracts or mechanisms in such a way as to maximize her objective subject to her informational disadvantage. By the revelation principle whenever the contracts discriminate between types they must induce the agent to reveal his private information which requires the properties described below.

3.2 The incentive contract

The incentive contract consists of a menu of investment-transfer pairs $\{(i_t(\hat{\theta}_t), R_t(\hat{\theta}_t))\}$ where $\hat{\theta}_t$ represents the level of θ_t announced by the agent when selecting a pair from the menu. In order to induce truthful revelation at t the menu must be such that it is in the interest of the agent to choose $\hat{\theta} = \theta \forall \theta \in [\theta^L, \theta^H]$. The objective of the agent is to maximize by choice of $\hat{\theta}$

$$\phi(\theta, \hat{\theta}) = pg(k) - \left[\theta i(\hat{\theta}) + \frac{1}{2} b i(\hat{\theta})^2 \right] - R(\hat{\theta}) + \psi(k + i(\hat{\theta})) \quad (6)$$

where unless otherwise mentioned variables (functions) are evaluated (defined) at t . Define the optimized value of ϕ as

$$\Pi(\theta) = \phi(\theta, \theta)$$

From the first and second order conditions for the maximization of ϕ we have (Guesn erie and Laffont 1984)

$$\frac{di}{d\theta} \leq 0 \quad (7)$$

and

$$\frac{d\Pi}{d\theta} = -i(\theta) \quad (8)$$

Furthermore as a rationality condition the contract must be such that for any participating agent

$$\Pi \geq 0 \quad (9)$$

Although it is clear from (6) that Π is made up of a current component plus a component corresponding to future profits where both components are net of transfers to shareholders (9) does not imply a commitment by the principal to keep the same manager in the future. It implies that if the agent is fired at t he gets a compensation of ψ . Our assumption is that shareholders are able to commit to one period contracts involving such golden parachutes⁴.

3.3 The problem of the principal

At any date t given k and under constraints (7) (8) and (9) the principal must choose functions $i(\theta)$ and $R(\theta)$ in such a way as to maximize

$$E \{ \alpha \Pi(\theta) + R(\theta) + \Gamma(k + i(\theta)) \}$$

Substituting the definition of Π using (6) and rearranging this is equivalent to choosing $i(\theta)$ in such a way as to maximize

$$\int_{\theta^L}^{\theta^H} \left\{ pg(k) - \theta i(\theta) - \frac{1}{2} bi(\theta)^2 - [1 - \alpha] \Pi(\theta) + S(k + i(\theta)) \right\} f(\theta) d\theta \quad (10)$$

⁴If golden parachutes were not available a second rationality constraint requiring current net cash flows π to be non negative would have to be satisfied. Since ψ is non negative and $\Pi = \pi + \psi\Gamma$ that constraint is at least as strict as (9). Thus golden parachutes allow shareholders to operate in a less constrained environment.

subject to (7) (8) and (9) where $S(k+i)$ defined as $\mathbb{P}(k+i) + \Psi^a(k+i)$ is the sum of surpluses to be shared by the agent and the principal at and beyond $t+1$ as expected at t and discounted to t . As indicated by the superscripts a it is understood that future decisions will be made in the same asymmetric information setup so that S reflects this knowledge.⁵

Problem (10) can be treated as an optimal control problem where i is the control variable and where Π the state variable is subject to a non negativity constraint. Define

$$L(\Pi, i, \mu, \lambda; k) = \left\{ pg(k) - \theta i(\theta) - \frac{1}{2} b i(\theta)^2 - [1 - \alpha] \Pi(\theta) + S(k + i(\theta)) \right\} f(\theta) - \mu(\theta) i(\theta) + \lambda(\theta) \Pi(\theta) \quad (11)$$

where $\lambda \geq 0$ and μ respectively are the Lagrangian multiplier associated with constraint (9) and the costate variable associated with Π . At this stage constraint (7) is being ignored; we will make use of it in the process of selecting a candidate solution and we will specify *ex post* conditions under which it is satisfied by the unconstrained solution. One necessary condition the maximum principle implies that at all θ where i is continuous and $\frac{d\mu}{d\theta}$ exists

$$i^a(\theta) = \frac{S' - \theta}{b} - \frac{\mu(\theta)}{bf(\theta)} \quad (12)$$

where S' the shadow value of capital represents the combined value to both the principal and the agent of expected discounted future marginal products. This rule differs from its symmetric-information counterpart (5) in two important ways. First since the share accruing to the principal under symmetric information is the total surplus \mathbb{P} in (5) is the analog of S in (12); however surpluses will normally differ under symmetric and under asymmetric information as the investment programs implied by (5) and (12) will

⁵We assume that $S(k_{+1})$ exists and is continuously differentiable and concave for any $k_{+1} \geq 0$; this implies that we rule out price trajectories that would cause S' to be infinite (as was done with $\Gamma^{s'}$ under symmetric information).

usually differ. Consequently the first term on the right-hand side of (5) will normally differ from the first term on the right-hand side of (12) although they measure the same concept. Second and more familiar is the presence of an extra term $\frac{\mu(\theta)}{bf(\theta)}$ in the expression. As in static principal-agent models this term causes a distortion to the operative decisions of the agent. As will be shown below unlike static asymmetric information production models where agents are typically induced to produce less than under full information this term may be positive or negative causing i to be either higher or lower than under full information. Perhaps more fundamental a difference will be the fact established further below that the solution is not fully separating under conventional assumptions on $f(\theta)$.

Another condition which must hold over intervals where $\frac{d\mu}{d\theta}$ exists is

$$\frac{d\mu}{d\theta} = [1 - \alpha] f(\theta) - \lambda(\theta) \quad (13)$$

Integrating gives

$$\mu(\theta) = [1 - \alpha] F(\theta) + A - \Lambda(\theta) \quad (14)$$

where F is the cumulative distribution of θ , A is a constant of integration and

$$\Lambda(\theta) \equiv \int_{\theta^L}^{\theta} \lambda(\tau) d\tau \quad (15)$$

measures the cumulative impact on the objective of the principal of meeting rationality constraints (9) for all types $\tilde{\theta} \leq \theta$.

3.4 Solution

In problems with constraints on the state variables such as (10) discontinuities in costate variables may occur only at junctions between an interval where the state constraint is

binding and an interval where it is not binding⁶. The continuity of μ elsewhere will be useful to characterize the solution. It is also useful to note that the first two terms on the right-hand side of (14) together are strictly increasing in θ while the last term goes the opposite way. Thus on intervals over which λ is positive ($\Pi = 0$) μ may be increasing or decreasing; when $\lambda = 0$ μ is strictly monotonic; at junctions μ may have a discontinuity. Similarly $\Lambda(\theta)$ is continuous as an integral of multipliers and the control variable i is continuous except possibly at junctions between intervals where $\Pi = 0$ and intervals where $\Pi > 0$. Furthermore given our assumption that S' is finite adjustment costs imply that i is finite for any θ ; it follows by (8) that Π is continuous over $[\theta^L, \theta^H]$.

Let $[\theta^-, \theta^+] \subseteq [\theta^L, \theta^H]$ be an interval over which (9) is binding; define θ^- by the condition that $\Pi(\theta^-) = 0$ and if $\theta^- \neq \theta^L$ then $\Pi(\theta) > 0$ for any $\theta < \theta^-$ in a neighborhood of θ^- ; define θ^+ by the condition that $\Pi(\theta^+) = 0$ and if $\theta^+ \neq \theta^H$ then $\Pi(\theta) > 0$ for any $\theta > \theta^+$ in a neighborhood of θ^+ . Thus locally $[\theta^-, \theta^+]$ is the largest possible interval over which (9) is binding; however making the definition local allows for the possibility that $[\theta^L, \theta^H]$ contains zero or several (disjoint) intervals satisfying the definition of $[\theta^-, \theta^+]$. By Lemma 1 (see Appendix) one and only one such interval exists; it must be different from $[\theta^L, \theta^H]$.

Depending on the position of $[\theta^-, \theta^+]$ in $[\theta^L, \theta^H]$ the trajectories of Π and λ over $[\theta^L, \theta^H]$ must conform to one of five cases. This is described in the following proposition.

Proposition 1 *If it exists, the candidate solution must fall under one of the following five cases:*

- *Case 1: $\theta^- = \theta^+ = \theta^L$; $\lambda(\theta) = 0 \forall \theta$; $\Pi(\theta) > 0 \forall \theta > \theta^L$; $\Pi(\theta^L) = 0$. In this case:*

1. *$\mu(\theta)$ is strictly positively monotonic; $\mu(\theta^H) = 0$;*

⁶See Seierstad and Sydsaeter (1986 Chapter 5). Their sufficiency conditions for a solution may be applied in this context. An important characteristic of these conditions is the possibility of discontinuities in the solution $i(\theta)$ and in $\mu(\theta)$ when $\Pi = 0$.

2. $i(\theta) < 0 \forall \theta$;
 3. $\Pi(\theta)$ is strictly positively monotonic.
- *Case 2:* $\theta^- = \theta^L < \theta^+ < \theta^H$; $\Pi(\theta) = 0 \forall \theta \in [\theta^L, \theta^+]$; $\lambda(\theta) = 0$ and $\Pi(\theta) > 0, \forall \theta > \theta^+$. In this case:
 1. $\mu(\theta)$ is strictly positively monotonic over $] \theta^+, \theta^H]$; $\mu(\theta^H) = 0$;
 2. $i(\theta) = 0 \forall \theta < \theta^+$; $i(\theta) < 0 \forall \theta \geq \theta^+$;
 3. $\Pi(\theta)$ is strictly positively monotonic $\forall \theta \geq \theta^+$.
 - *Case 3:* $\theta^L < \theta^- < \theta^+ \leq \theta^H$; $\lambda(\theta) = 0$ and $\Pi(\theta) > 0 \forall \theta < \theta^-$; $\Pi(\theta) = 0 \forall \theta \in [\theta^-, \theta^+]$; $\lambda(\theta) = 0$ and $\Pi(\theta) > 0 \forall \theta > \theta^+$. In this case:
 1. $\mu(\theta)$ is strictly positively monotonic over $[\theta^L, \theta^-[$ and over $] \theta^+, \theta^H]$; $\mu(\theta^H) = \mu(\theta^L) = 0$;
 2. $i(\theta) > 0 \forall \theta \leq \theta^-$; $i(\theta) = 0, \theta^- < \theta < \theta^+$; $i(\theta) < 0 \forall \theta \geq \theta^+$;
 3. $\Pi(\theta)$ is strictly positively (negatively) monotonic $\forall \theta \geq \theta^+$ ($\forall \theta \leq \theta^-$);
 4. $\theta^- \neq \theta^+$.
 - *Case 4:* $\theta^L < \theta^- \leq \theta^+ = \theta^H$; $\lambda(\theta) = 0$ and $\Pi(\theta) > 0 \forall \theta < \theta^-$; $\Pi(\theta) = 0, \forall \theta \in [\theta^-, \theta^H]$. In this case:
 1. $\mu(\theta)$ is strictly positively monotonic over $[\theta^L, \theta^-]$; $\mu(\theta^L) = 0$;
 2. $i(\theta) > 0 \forall \theta \leq \theta^-$; $i(\theta) = 0 \forall \theta > \theta^-$;
 3. $\Pi(\theta)$ is strictly negatively monotonic $\forall \theta \leq \theta^-$.
 - *Case 5:* $\theta^L < \theta^- = \theta^+ = \theta^H$; $\lambda(\theta) = 0 \forall \theta$; $\Pi(\theta) > 0 \forall \theta < \theta^H$; $\Pi(\theta^H) = 0$. In this case:

1. $\mu(\theta)$ is strictly positively monotonic; $\mu(\theta^L) = 0$;
2. $i(\theta) > 0 \forall \theta$;
3. $\Pi(\theta)$ is strictly negatively monotonic.

Proof.

1. In all cases Γ the monotonicity of μ follows from (13) when $\lambda = 0$ and the values of $\mu(\theta^H)$ and (or) $\mu(\theta^L)$ are transversality conditions. In Case 3 $\mu(\theta^-) > 0$ follows from the fact that μ strictly rises from $\mu(\theta^L) = 0$ to $\mu(\theta^-)$; similarly $\mu(\theta^+) < 0$ follows from the fact that μ strictly rises from $\mu(\theta^+)$ to $\mu(\theta^H) = 0$.
2. In all cases Γ $i(\theta) = 0$ whenever $\Pi(\theta) = 0$ because of (8). Now we prove that $i(\theta) < 0 \forall \theta$ in case 1. Starting from $\Pi(\theta^L) = 0$ Π is to become positive; given (8) Γ it follows that $i(\theta^L)$ must be negative; because of (7) Γ i will then remain negative over the rest of the interval. The sign of i in other cases is established in a similar way.
3. In all cases Γ the claimed monotonicity of Π is implied by (8) and the sign of i over the relevant interval.
4. To show that $\theta^- \neq \theta^+$ in case 3 Γ suppose otherwise. Given the monotonicity of μ over $[\theta^L, \theta^-[$ and over $]\theta^+, \theta^H]$ and the fact that $\mu(\theta^H) = \mu(\theta^L) = 0$ Γ there must be a discontinuity in μ at $\theta^+ = \theta^-$ Γ with $\mu(\theta^-) > 0$ and $\mu(\theta^+) < 0$; since $\frac{\partial i^a}{\partial \mu} < 0$ by (12) Γ it follows that $i(\theta^+) > 0$ Γ a contradiction. ■

Proposition 1 describes the qualitative properties of all possible solutions. The five cases are illustrated in Figure 1. Case 1 represents a situation where the interval $[\theta^-, \theta^+]$ is squeezed to the left of $[\theta^L, \theta^H]$ and is actually reduced to θ^L . In other cases Γ the interval is progressively shifted to the right Γ so that case 5 represents a situation where the

interval is reduced to θ^H . For each case Γ the figure gives the optimal investment Γ profit Γ and shadow price of profit satisfying the qualitative properties stated in Proposition 1.

As will be shown below Γ these situations occur according to the magnitude of S' . To complete the characterization Γ we pick a candidate solution that assumes the absence of any discontinuities in μ at θ^+ and θ^- Γ and we verify that it satisfies all other conditions in Seierstad and Sydsaeter's sufficiency theorem Γ allowing us to conclude that it solves problem (10). By Lemma 2 (see Appendix) Γ such a solution exists if f satisfies the following assumption.

Assumption 1 *f has the following properties:*

$$l(\theta) \equiv \frac{1 - F(\theta)}{f(\theta)} \text{ is non increasing}$$

$$h(\theta) \equiv \frac{F(\theta)}{f(\theta)} \text{ is non decreasing}$$

The solution is described in the following propositions Γ proven in the Appendix.

Proposition 2 *If $\mu(\theta)$ is continuous over $[\theta^L, \theta^H]$, it is given, in cases 1-3 and 4-5 respectively, by*

$$\mu(\theta) = [1 - \alpha] F(\theta) - [1 - \alpha] + \Lambda(\theta^+) - \Lambda(\theta) \quad (16)$$

and

$$\mu(\theta) = [1 - \alpha] F(\theta) - \Lambda(\theta) \quad (17)$$

where $\Lambda(\theta) = 0 \forall \theta$ in cases 1 and 5, while, in cases 2 and 3, $\Lambda(\theta) = 0 \forall \theta \leq \theta^-$, $\Lambda(\theta) = \Lambda(\theta^+)$, $\theta \geq \theta^+$, and, for $\theta^- < \theta < \theta^+$

$$\Lambda(\theta) = \left[[1 - \alpha] \frac{F(\theta) - 1}{f(\theta)} - [S' - \theta] + \frac{\Lambda(\theta^+)}{f(\theta)} \right] f(\theta) \quad (18)$$

with $\Lambda(\theta^+) = [S' - \theta^L] f(\theta^L) + [1 - \alpha]$ in case 2 and $\Lambda(\theta^+) = [1 - \alpha]$ in case 3. In

case 4

$$\Lambda(\theta) = \left[[1 - \alpha] \frac{F(\theta)}{f(\theta)} - [S' - \theta] \right] f(\theta) \quad (19)$$

The function Λ in Proposition 2 is well defined. However Γ by its definition (15) Γ it must also be increasing. Lemma 2 ensures that this is true.

Proposition 1 does not define θ^- and θ^+ when they are in the interior of $[\theta^L, \theta^H]$. This is done in Proposition 3.

Proposition 3 *If $\mu(\theta)$ is continuous over $[\theta^L, \theta^H]$, in cases 3 and 4, θ^- is defined by the condition*

$$0 = [S' - \theta^-] f(\theta^-) - [1 - \alpha] F(\theta^-) \quad (20)$$

In cases 2 and 3, θ^+ is defined by the condition

$$0 = [S' - \theta^+] f(\theta^+) - [1 - \alpha] F(\theta^+) + 1 - \alpha \quad (21)$$

Proposition 3 defines θ^- and θ^+ implicitly. By Lemma 2 Γ when they exist within the interval $[\theta^L, \theta^H]$ Γ they also satisfy $\theta^- < \theta^+$. The next proposition specifies the values of the shadow price of capital that cause each of the five cases to arise.

Proposition 4 *Let $S'_1 < \theta^L$ and $S'_4 > \theta^H$ be respectively defined by*

$$0 = [S'_1 - \theta^L] f(\theta^L) + 1 - \alpha \quad (22)$$

and

$$0 = [S'_4 - \theta^H] f(\theta^H) - [1 - \alpha] \quad (23)$$

Then cases 1-5 arise according to the value of S' relative to S'_1 , θ^L , θ^H and S'_4 :

- *Case 1 corresponds to: $S' \leq S'_1 < \theta^L$;*

- *Case 2 corresponds to: $S'_1 < S' \leq \theta^L$;*
- *Case 3 corresponds to: $\theta^L \leq \theta^- \leq S' \leq \theta^+ \leq \theta^H$ where the left two inequalities are strict unless $S' = \theta^L$ and the right two inequalities are strict unless $S' = \theta^H$;*
- *Case 4 corresponds to: $\theta^H < S' < S'_4$;*
- *Case 5 corresponds to: $S' \geq S'_4 > \theta^H$.*

The candidate solution described in propositions 1-4 exists since it was obtained by construction. It satisfies all conditions for an optimum except that (7) was not imposed that the monotonicity of Λ was not verified and that the condition $\theta^- < \theta^+$ was not verified. By Lemma 2 Assumption 1 is sufficient for these properties to be satisfied.

4 Discussion

The optimal program under asymmetric information differs notably from its full information counterpart. The most visible difference is the pooling phase (at $i = 0$) that occurs between negative and positive investment regimes under asymmetric information. Under symmetric information as (5) clearly shows investment is positive if $\Gamma^{s'}$ the marginal value of capital in terms of expected discounted future surpluses is higher than θ and vice versa; there is no interval of θ between the two regimes over which $i = 0$. Informational asymmetry usually introduces inefficiency under assumptions similar to those made here but without causing such pooling. What happens here is that there is a conflict between two incentives to misrepresent θ . A manager whose θ would place him in the category of capital buyers under full information has an incentive to overstate θ in order to overstate his cost of buying; but too big an overstatement might place him in a high θ group of types who should normally sell capital under full information. However for sellers the incentive to misrepresent goes in the opposite direction: they

should understate θ to understate their revenues from selling. Managers whose θ is close to S' the marginal value of capital at which investment would switch from negative to positive under full information would face such a dilemma. Precisely all types in $[\theta^-, \theta^+]$ face that dilemma which is resolved by asking them not to invest. This situation involving conflicting incentives is an example of ‘Inflexible Rules in Incentive Problems’ as analyzed by Lewis and Sappington (1989). It arises here naturally rather than being engineered by a principal in order to alleviate an incentive problem as in their case.

For types whose cost of capital is close to the S' the loss from setting $i = 0$ is low relative to the saving that such inefficiency allows in the cost of inducing more profitable types to reveal their true θ . Such profitable types may occur at both ends of the θ spectrum with profits being generated either by buying (low θ) or selling (high θ) capital. Taking Case 3 as an example this appears more clearly if (12) is written (using Proposition 2) in the following form

$$\theta + bi(\theta) = \begin{cases} S' + [1 - \alpha]l(\theta), & \theta \geq \theta^+ \\ S' - [1 - \alpha]h(\theta), & \theta \leq \theta^- \end{cases} \quad (24)$$

The left-hand side is the cost of the marginal unit of capital inclusive of its adjustment cost component. For firms at either end of $[\theta^L, \theta^H]$ the second term on the right-hand side of the appropriate line vanishes implying that marginal cost is set equal to S' . This is indeed the same rule as under full information (misleadingly) suggesting that types θ^L and θ^H are asked to behave efficiently. Since $S' \in [\theta^-, \theta^+]$ (Proposition 4) $\theta^L < S'$ and $\theta^H > S'$ so that this rule requires low θ types to buy capital and high θ types to sell. In contrast types whose θ is closer to S' are being asked to deviate from that rule by an amount proportional to l or h . Since l is non increasing and non positive and h is non decreasing and non negative the wedge is wider the closer θ is to the relevant switching value (θ^- or θ^+). This is illustrated in Figure 2 for a uniform distribution of θ over $[1, 2]$

with $b = 1$. Applying rule (24) beyond type θ^+ (θ^-) would imply requesting a positive (negative) investment from a type that would be asked to sell (buy) capital under full information at identical marginal value of capital. The optimal contract reflects the fact that it is less costly in terms of efficiency loss to choose pooling instead in the vicinity of S' . Thus there is a third instance of (apparent) efficient behavior: when θ is equal to S' the agent invests zero as would be the case under full information.

As hinted above one would be mistaken to believe that types θ^L θ^H and S' are asked to invest as under full information. Although the rule is the same as under full information for these types there is inefficiency arising from the fact that at any given k S' differs from its full information counterpart $\Gamma^{s'}$. This difference appears because whatever its current type there is a strictly positive probability (unless T is known to be within two periods) that in the future an agent will be asked to invest a different amount than would be warranted under symmetric information.

The distinction between S' and $\Gamma^{s'}$ also has implications for interpreting Tobin's q . Under symmetric information according to the q theory of investment the stock exchange valuation V of a firm provides a measure of the total value of its assets. Under appropriate assumptions (Hayashi 1982; Abel and Eberly 1994) this also applies at the margin so that $\frac{V}{k}$ provides a measure of the contribution to V of the marginal (and average) unit of capital as it is perceived by the market⁷. In our notation $\frac{V}{k}$ would then provide a measure of $\Gamma^{s'}$. According to the q theory investment should be chosen in such a way as to equate the marginal cost of acquiring one unit of capital with $\Gamma^{s'}$ and the unobserved $\Gamma^{s'}$ should be replaced with $\frac{V}{k}$ in the investment equation: $bi_i^s + \theta = \frac{V}{k}$ where θ is interpreted to include the market purchase price p_K . Under asymmetric information the shadow price of capital S' cannot be measured in the same way: the reason is that S is a surplus *to be shared* between the principal and the agent ($S \equiv \Gamma^a + \psi^a$) while

⁷The identification of marginal q with average q is a restriction that can be circumvented; see Abel and Blanchard (1986). This does not affect our argument.

the stock exchange measures only the portion Γ^a that accrues to the principal under asymmetric information. Furthermore it is clear in our simple specification of the asymmetric information model that the relative size of Γ^a and ψ^a is sensitive to the parameters of the problem. Consequently the proportional error made by measuring S using stock exchange valuation is likely to be highly variable both across firms across sectors and over time⁸. This might be yet another explanation for the lack of reliability of q models of investment. In fact there is evidence that Tobin's q as measured by the ratio of Γ^a over the replacement cost of tangible assets might be sensitive to information asymmetries. The literature on diversification and performance provides evidence that Tobin's q is negatively correlated with the degree of diversification when diversification is into unrelated businesses. This evidence is discussed and complemented by Lang and Stulz (1994). It may imply that agency costs are higher in firms that diversify into unrelated business than in more focused firms (i.e. $S < \Gamma^s$) but also that the share of S appropriable by shareholders P which is what markets measure is lower in unfocused firms than in focused ones.

As was mentioned earlier the qualitative nature of our results is unaffected if θ is defined to include besides an idiosyncratic component the observable asset price of equipment p_K . The pooling that occurs under asymmetric information provides a new explanation for investment to be insensitive to variations in the cost of capital and in the shadow value of capital over certain ranges. Abel and Eberly (1994) summarize conditions under which the standard cost of adjustment model of investment involves $i = 0$ over an asset price region. In the absence of any non negativity constraint one possibility is the presence of a fixed cost for any non zero level of investment; in that case however there are discontinuities in the optimal investment function which do not arise in our model. Another possibility is the presence of a kink in the cost

⁸There is evidence of such variability in the recent empirical literature. See Demers et al. (1994).

of investment function at $i = 0$ either due to the different nature of negative versus positive investment or to a difference between purchase and resale prices. Here there is no discontinuity in the optimal investment function so that its qualitative properties as a relationship between investment and its cost are similar to the properties of i^a . Although not distinguishable in that respect the two models differ in the investment uncertainty relationship. This is discussed further below.

Besides the standard neoclassical theory of investment the optimality of not reacting to a changing environment over a certain range has been identified and discussed in a growing literature on hysteresis. As far as investment theory is concerned this literature is best presented in Dixit and Pindyck (1994). Faced with an irreversible decision to take under uncertainty firms value positively the option of waiting for more information. This introduces a wedge the option value between the cost of investment and its expected marginal product value. This option value explains why over a certain range firms optimally choose not to react to variations in the cost of investment or in the expected marginal product value. As when fixed costs are associated with non null investment levels which indeed makes the investment decision irreversible there are discontinuities between inaction regimes and regimes of active investment as opposed to the asymmetric information model.

Thus our model implies an observationally different investment behavior than the most well-known alternatives. One apparent exception is the cost-of-adjustment model when there is a kink in the cost of adjustment at $i = 0$. Both models imply inaction over some range in the cost of capital or its shadow value and a progressive departure without discontinuity from that situation at values outside the inaction range. However the two models differ in the way investment is affected by uncertainty. Comparisons are not straightforward though because the types of uncertainties envisaged in both models are somewhat different: uncertainty about future capital productivity in the

standard cost of adjustment model; uncertainty about the current cost of capital in the asymmetric information model. In the asymmetric information model an increase in uncertainty taking the form of a wider spread in the distribution of θ will normally reduce the absolute value of investment as illustrated in Figure 3 for the uniform distribution. In contrast in the kinked cost-of-adjustment model the result of Abel (1983) and Caballero (1991) apply: they find that increased uncertainty increases the investment of competitive firms with constant returns to scale at least when the random shocks are idiosyncratic to individual firms as they are in our model. Such reversals in the positive correlation between uncertainty and investment as implied by our asymmetric information model have been encountered in other contexts and discussed extensively by Caballero. He observes that adjustment-cost asymmetry combined with imperfect competition produce this reversal in symmetric information models underlining that imperfect competition “is also the paramount factor”. Our model exhibits this property without adjustment-cost asymmetry or imperfect competition.

5 Conclusion

Investment theory especially the body of literature underlying the study of business fixed investment spending largely neglects issues of agency and information. In this paper we have introduced asymmetry of information between shareholders and managers into an otherwise standard cost of adjustment model of investment.

This produces an investment equation with clearly distinguishable features. The most remarkable one is a new form of hysteresis which results from conflicting incentives to misrepresent costs for certain types. Hysteresis arises when the conflict is resolved by the use of an inflexible rule as in Lewis and Sappington (1989). Departures from the inaction regime are smooth as in the model of Abel and Eberley (1994) involving adjustment costs with a kink at $i = 0$. However their model can be distinguished from

ours by the nature and role of uncertainty.

Our model also casts a new light on the q theory of investment. It shows that if information asymmetries are present the shadow value of capital should be defined to include rents accruing to managers. Failing that q is poorly and inconsistently measured.

From the point of view of agency theory the investment model turns out to have interesting peculiarities. First our model introduces a form of dynamics which has been neglected so far in principal-agent models although it is standard in other fields of economics. The intertemporal link is provided by capital and investment but might as well involve learning by doing or R&D. At first sight this type of dynamics does not appear to affect the result pervasive in static agency theory according to which the behavior of 'good' types is the same as under full information. Thus the lowest-cost manager is asked to choose i so as to equate marginal cost to the shadow price of capital as under full information. However since there is a positive probability of not being lowest-cost in some future period distortions will occur in the future almost certainly so that the shadow price of capital is different than under full information: the same investment rule yields a different investment level.

Second depending on the cost of capital positive or negative investment may be desirable to shareholders and managers. As a result 'good' types to whom a full information investment rule applies may coexist at both ends of the type range.

LEMMAS AND PROOFS⁹

A Lemma 1

Lemma 1 *There exists one and only one interval $[\theta^-, \theta^+]$, possibly reduced to a single point but necessarily different from $[\theta^L, \theta^H]$.*

Proof. In order to show that there exists at least one interval Γ we show that $\Pi > 0 \forall \theta$ is impossible. Suppose that $\Pi > 0 \forall \theta$; then μ is continuous over $[\theta^L, \theta^H]$ and Γ by (14) Γ is strictly monotonic. However the transversality conditions corresponding to $\Pi(\theta^L) > 0$ and $\Pi(\theta^H) > 0$ Γ $\mu(\theta^L) = 0$ and $\mu(\theta^H) = 0$ respectively Γ cannot be both satisfied by a monotonic Γ continuous trajectory. Now suppose that there is more than one interval satisfying the definition of $[\theta^-, \theta^+]$; by definition the intervals must be separated by intervals over which $\Pi > 0$; thus there exists $\theta_1 < \theta_2 < \theta_3$ such that $\Pi(\theta_1) = 0$ Γ $\Pi(\theta_2) > 0$ Γ and $\Pi(\theta_3) = 0$. Consequently Γ as θ increases from θ_1 to θ_3 Γ the continuous function $\Pi(\theta)$ must first rise Γ which requires $i < 0$ by (8) Γ then diminish Γ which requires $i > 0$. This violates (7). It remains to show that $[\theta^-, \theta^+]$ is different from $[\theta^L, \theta^H]$. Suppose otherwise; then Γ by (8) Γ in order to maintain $\Pi = 0$ over the whole interval Γ $i = 0 \forall \theta$. Since the objective to maximize is $\int_{\theta^L}^{\theta^H} \left\{ pg(k) - \theta i(\theta) - \frac{1}{2} bi(\theta)^2 - [1 - \alpha] \Pi(\theta) + S(k + i(\theta)) \right\} f(\theta) d\theta$ Γ and since either $S'(k + i(\theta)) > \theta_L$ Γ or $S'(k + i(\theta)) < \theta_H$ Γ or both Γ the program Γ $\Pi = 0 \forall \theta$ may be strictly improved Γ either (a) by setting $i > 0$ over a neighborhood of θ^L Γ or (b) by setting $i < 0$ over a neighborhood of θ^H Γ or both. Let us show that this is feasible under the constraints imposed by asymmetric information. Thus suppose (a) applies and choose

⁹Propositions or Lemmas are stated in the Appendix only if they are not stated in the main text.

some $i_1 > 0$, $\Delta > 0$ and Θ such that for $\theta \in [\theta^L, \Theta_1]$ the difference between the contribution of the perturbed program and the original candidate satisfies¹⁰

$$pg(k) - \theta i_1 - \frac{1}{2} b i_1^2 - [1 - \alpha] \Pi^1(\theta) + S(k + i_1) - pg(k) - S(k) > \Delta > 0$$

where $\Pi^1(\theta)$ is such that (8) is satisfied and $\Pi^1(\Theta_1) = 0$: $\Pi^1(\theta) = \Pi_L - \theta i_1$. The principal may ask agents of type $\theta < \Theta_1$ to set $i = i_1$ offering them $\Pi^1(\theta)$ rather than asking them to set $i = 0$ and offering them $\Pi = 0$. They will find it in their interest to accept each yielding an increment of at least Δ to the objective of the principal. Thus $i = 0 = \Pi \forall \theta$ cannot be optimal implying that $[\theta^-, \theta^+] \subset [\theta^L, \theta^H]$. ■

B Proposition 2

We start from (14) and use the transversality conditions corresponding to each case in order to eliminate the constant of integration A . The assumed continuity of μ implies that there is only one such constant of integration in each case. In cases 1-3 $\Pi(\theta^H)$ is free so that $\mu(\theta^H) = 0$; (14) implies

$$A = \Lambda(\theta^H) - [1 - \alpha]$$

Substituting into (14) recognizing that $\Lambda(\theta^H) = \Lambda(\theta^+)$ gives (16). In cases 4-5 $\Pi(\theta^L)$ is free so that $\mu(\theta^L) = 0$; it follows from (14) that $A = 0$ which in turn implies (17).

$\Lambda(\theta) = 0$ by definition in cases 1 and 5. In cases 2 and 3 $\theta^+ < \theta^H$ so that by definition $\Lambda(\theta) = \Lambda(\theta^+) \forall \theta \geq \theta^+$. For $\theta \leq \theta^-$ by definition $\Lambda(\theta) = 0$. For

¹⁰if no such triplet (i^1, Θ_1, Δ) may be found then it is certain that a similar triplet corresponding to (b) $i^2 < 0$, $\Theta_2, \Delta > 0$ can be found such that for $\theta \in [\Theta_2, \theta^H]$

$$pg(k) - \theta i_2 - \frac{1}{2} b i_2^2 - [1 - \alpha] \Pi^2(\theta) + S(k + i_2) - pg(k) - S(k) > \Delta > 0$$

$\theta^- < \theta < \theta^+$ by Proposition 1 $i = 0$ which implies substituting (16) into (12)

$$0 = \frac{1}{b} \left\{ [S' - \theta] - \frac{[1 - \alpha][F(\theta) - 1] + \Lambda(\theta^+) - \Lambda(\theta)}{f(\theta)} \right\}$$

from which (18) follows. $\Lambda(\theta^+)$ is obtained as follows. In case 2 writing (18) at $\theta^- = \theta^L$ with $\Lambda(\theta^-) = 0$ by definition gives $\Lambda(\theta^+) = [S' - \theta^L] f(\theta^L) + [1 - \alpha]$. In case 3 since $\Pi(\theta^L)$ is free we further have as a transversality condition that $\mu(\theta^L) = 0$. Writing (16) at θ^L with $\Lambda(\theta^L) = \Lambda(\theta^-) = 0$ by definition yields $\Lambda(\theta^+) = [1 - \alpha]$.

We turn to establishing (19): this is done by substituting (17) into (12) and setting $i^a = 0$. ■

C Proposition 3

Since θ^- is the lowest level of θ at which constraint (9) is binding $\Lambda(\theta^-) = 0$. By Proposition 1 at θ^- $i = 0$. Writing (12) at θ^- while substituting the formulas for μ and Λ given in Proposition 2 for cases 3 and 4 implies in both cases that θ^- must satisfy

$$0 = \frac{1}{b} \left\{ [S' - \theta^-] - \frac{[1 - \alpha]F(\theta^-)}{f(\theta^-)} \right\}$$

which reduces to (20). Similarly by Proposition 1 at θ^+ $i = 0$. Writing (12) at θ^+ while substituting the formulas for μ and Λ given in Proposition 2 for cases 2 and 3 implies in both cases that θ^+ must satisfy

$$0 = \frac{1}{b} \left\{ [S' - \theta^+] - \frac{[1 - \alpha]F(\theta^+) - [1 - \alpha]}{f(\theta^+)} \right\}$$

which reduces to (21). ■

D Proposition 4

It is useful to refer to Figure 1 in order to see how the various cases are related to each other. The switch from Case 1 to Case 2 occurs when the value S'_1 of S' is such that θ^L solves the definition of θ^+ i.e. when

$$0 = [S'_1 - \theta^L] f(\theta^L) + 1 - \alpha$$

This implies $S'_1 < \theta^L$. The switch between Case 2 and Case 3 occurs when the value S'_2 of S' is such that θ^L solves the definition of θ^-

$$0 = [S'_2 - \theta^L] f(\theta^L) \tag{25}$$

which implies $S'_2 = \theta^L$. The switch between Case 3 and Case 4 occurs when the value S'_3 of S' is such that θ^H solves the definition of θ^+

$$0 = [S'_3 - \theta^H] f(\theta^H) \tag{26}$$

which implies $S'_3 = \theta^H$. The switch between Case 4 and Case 5 occurs when value S'_4 of S' is such that θ^H solves the definition of θ^-

$$0 = [S'_4 - \theta^H] f(\theta^H) - [1 - \alpha]$$

which implies $S'_4 > \theta^H$.

In cases 2 and 3 Γ (21) applies; it follows that $S' \leq \theta^+$ and that the inequality is strict unless $\theta^+ = \theta^H$; similarly Γ in cases 3 and 4 Γ (20) implies $S' \geq \theta^-$ and the inequality is strict unless $\theta^- = \theta^L$. ■

E Lemma 2

Lemma 2 *A sufficient condition for the candidate solution described in propositions 1-4 to solve the principal's problem is for f to satisfy Assumption 1.*

Proof. The candidate solution described in propositions 1-4 exists since it was obtained by construction. It satisfies all conditions for an optimum except that (7) was not imposed that the monotonicity of Λ was not verified and that the condition $\theta^- < \theta^+$ was not verified. We have to show that these last three properties are verified. When i is constant at zero (7) is satisfied. Let us consider other situations. We start with Case 1 as well as cases 2 and 3 for $\theta \geq \theta^+$. Substituting the appropriate values of μ and Λ from Proposition 2 into (12)

$$i(\theta) = \frac{1}{b} \left[S' - \theta + [1 - \alpha] \frac{1 - F(\theta)}{f(\theta)} \right]$$

Since $\frac{1-F(\theta)}{f(\theta)} \equiv l(\theta)$, a sufficient condition for $\frac{di}{d\theta} \leq 0$ is l to be non increasing. The other cases where i is non constant are cases 3 and 4 for $\theta \leq \theta^-$ and Case 5. After appropriate substitutions (12) gives

$$i(\theta) = \frac{1}{b} \left[S' - \theta - [1 - \alpha] \frac{F(\theta)}{f(\theta)} \right]$$

Since $\frac{F(\theta)}{f(\theta)} \equiv h(\theta)$ a sufficient condition for $\frac{di}{d\theta} \leq 0$ is f to be non decreasing.

It is immediate to verify using (8) that the monotonicity of Λ is implied by the monotonicity of l in cases 2 and 3 while in case 4 it is implied by (19) and the monotonicity of h . Similarly it can be verified using (20) or (21) that the monotonicity of h implies $\theta^- < \theta^+$. ■

References

- Abel A. B. (1983) "Optimal Investment Under Uncertainty" *American Economic Review* **73(1)** 228-33.
- Abel A. B. and O. J. Blanchard (1986) "The Present Value of Profits and Cyclical Movements in Investment" *Econometrica* **54(2)** 249-73.
- Abel A. B. and J. C. Eberly (1994) "A Unified Model of Investment Under Uncertainty" *American Economic Review* **84(5)** 1369-84 .
- Baron D. P. (1989) "Design of Regulatory Mechanisms and Institutions" in *Handbook of Industrial Organisation* ed. R. Schmalensee and R. D. Willig. New York: North Holland 1347-1447.
- Bernanke B. and M. Gertler (1989) "Agency Costs, New Growth and Business Fluctuations" *American Economic Review* **79** 14-31.
- Caballero R. (1991) "On the Sign of the Investment-Uncertainty Relationship" *American Economic Review* **81(1)** 279-88.
- Chirinko R. S. (1993) "Business Fixed Investment Spending: Modeling Strategies, Empirical Results, and Policy Implications" *Journal of Economic Literature* **31(December)** 1875-1911.
- Cooper R. (1994) "Financial Intermediation and Aggregate Fluctuations: A Quantitative Analysis" mimeo. Department of Economics, Boston University, Boston.
- Demers F. M. Demers and H. Shaller (1994) "Irreversible Investment and Costs of Adjustment" No 9416 CEPREMAP Paris .
- Dixit A. and R. S. Pindyck (1994) *Investment under Uncertainty*. Princeton: Princeton University Press.
- Diamond D. (1984) "Financial Intermediation and Delegated Monitoring" *Review of Economic Studies* **51** 393-414.
- Diamond D. (1989) "Reputation Acquisition in the Debt Market" *Journal of Political Economy* **97** 828-62.
- Guesn erie R. and J.-J. Laffont (1984) "A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm" *Journal of Public Economics* **25(December)** 329-69.
- Harris M. and A. Raviv (1990) "Capital Structure and the Informational Role of Debt" *Journal of Finance* **45** 321-49.
- Harris M. and A. Raviv (1991) "The Theory of Capital Structure" *Journal of Finance* **46(1)** 297-355.
- Hayashi F. (1982) "Finn's marginal q and Average q : a Neoclassical Interpretation" *Econometrica* **50(1)** 213-24.

- Holmstrom, B. and J. Tirole (1994) "Financial Intermediation, Loanable Funds and the Real Sector" mimeo, M.I.T. and Université de Toulouse .
- Jensen, M. C. and W. Meckling (1976) "Theory of the Firm: Managerial Behavior, Agency Cost, and Capital Structure" *Journal of Financial Economics* **3** 305-60.
- Laffont, J.-J. and J. Tirole (1988) "The Dynamics of Incentive Contracts" *Econometrica* **56** 1153-1175.
- Lang, L. H. P. and R. M. Stulz (1994) "Tobin's q , Corporate Diversification, and Firm Performance" *Journal of Political Economy* **102(6)** 1248-80 .
- Lewis, T. R. and D. E. Sappington (1989) "Inflexible Rules in Incentive Problems" *American Economic Review* **79(1)** 69-84.
- Myers, S. C. and N. S. Majluf (1984) "Corporate Financing and Investment Decisions when Firms Have Information that Investors do not Have" *Journal of Financial Economics* **13** 187-221.
- Seierstad, A. and K. Sydsaeter (1987) *Optimal Control Theory with Economic Applications* . Amsterdam : North Holland .
- Stulz, R. (1990) "Managerial Discretion and Optimal Financing Policies" *Journal of Financial Economics* **26** 3-27.
- The Economist (1994) "Watching the Boss: A Survey of Corporate Governance" January 29.

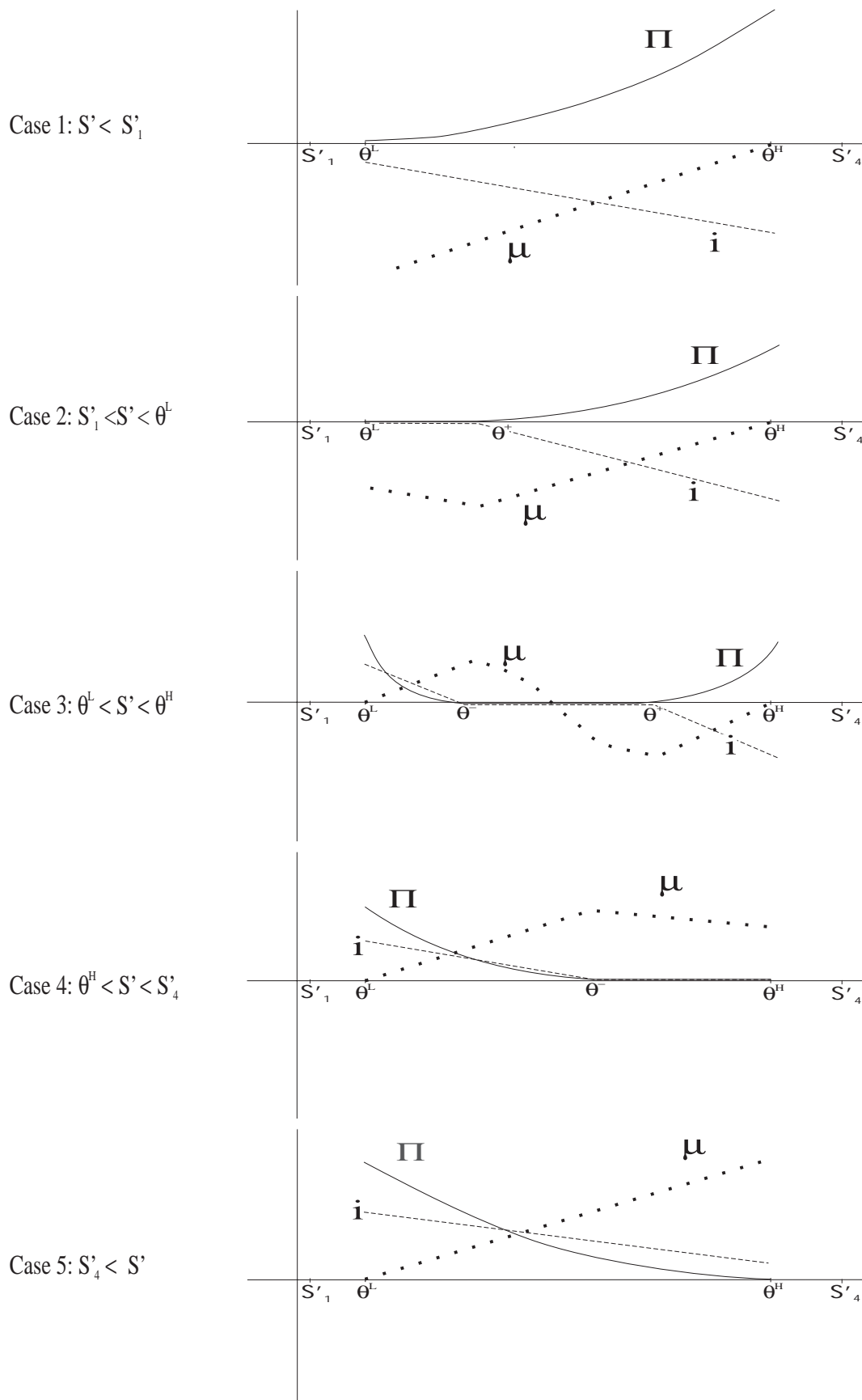


Figure 1: Optimal investment, profit, and shadow price of profit

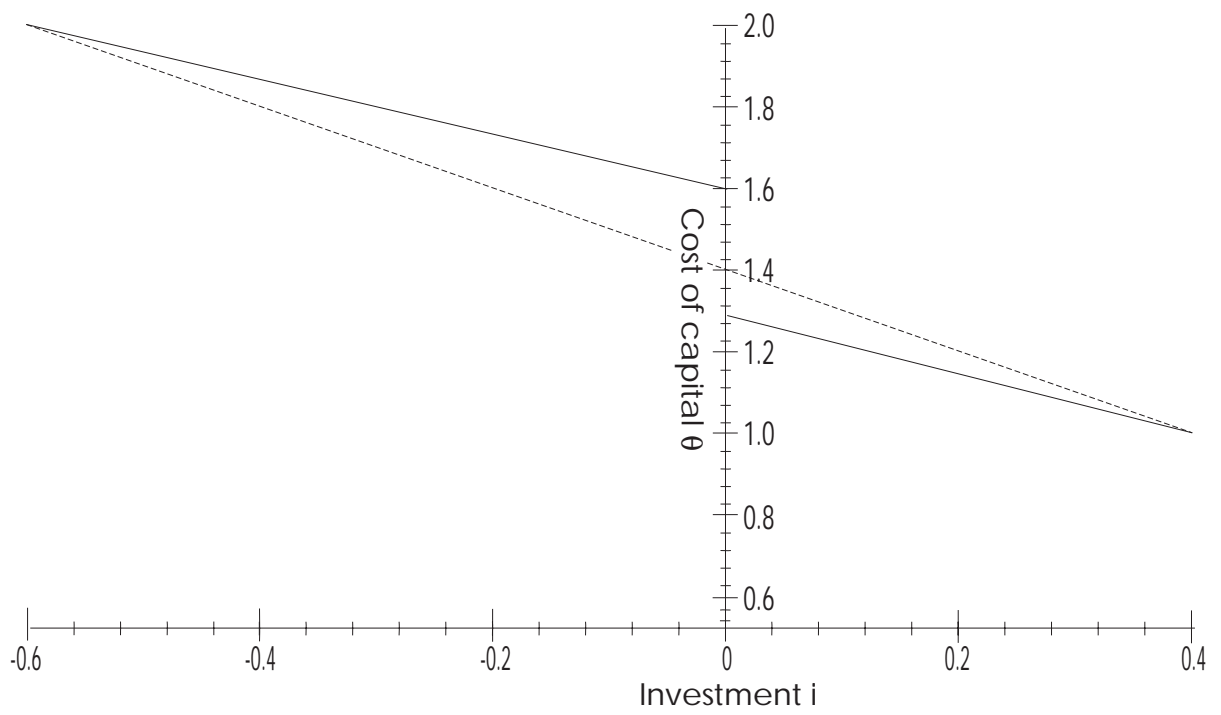


Figure 2: Inverse investment demand curve: asymmetric information (continuous line) and symmetric information (dotted line), $S' = 1.4$

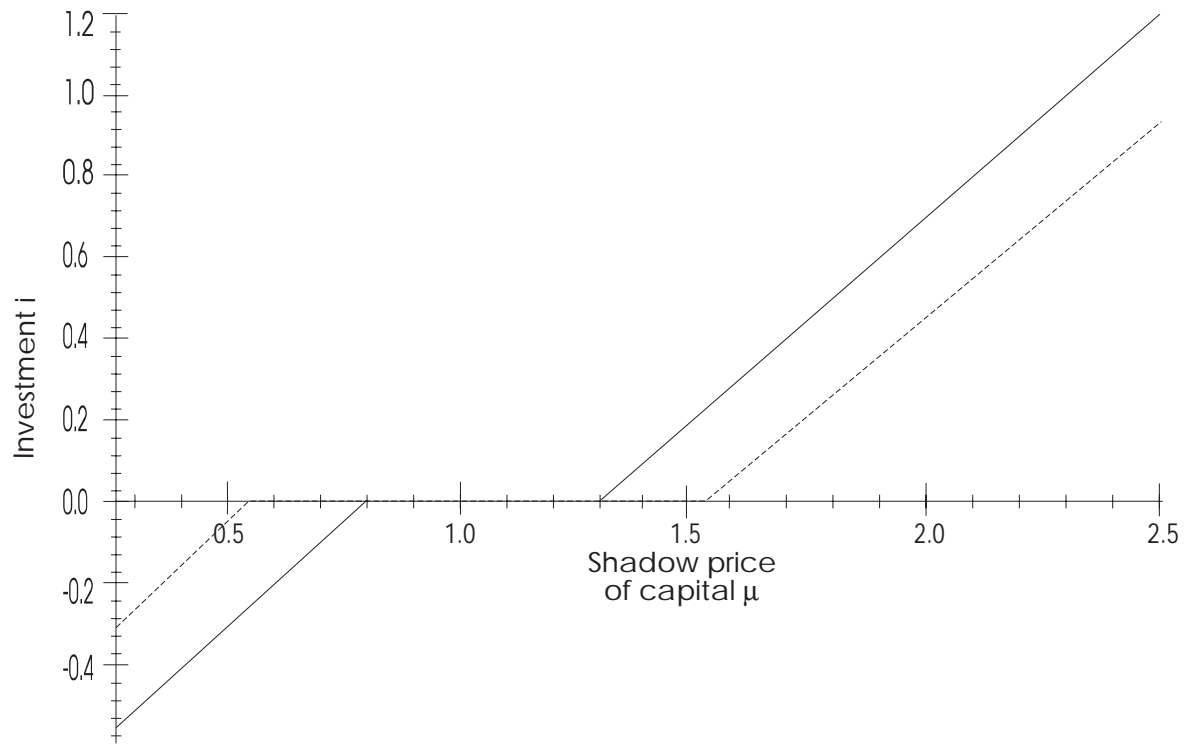


Figure 3: Investment as a function of S' under asymmetric information: high uncertainty (dotted line) and low uncertainty (continuous line), $q = 1.2$