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**A Semi-Parametric Factor  
Model for Interest Rates**

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# A Semi-Parametric Factor Model for Interest Rates

*Eric Ghysels<sup>†</sup>, Serena Ng<sup>‡</sup>*

## Résumé / Abstract

Dans cette étude nous proposons des modèles à facteurs semi-paramétriques pour taux d'intérêts. Nous construisons les facteurs comme des fonctions linéaires de variables clés normales et réelles. L'estimateur de dérivée moyenne, proposé par Hardle et Stoker (1989) et Powell, Stock et Stoker (1989) nous permet d'estimer ces facteurs comme fonctions linéaires sans connaître leurs relations avec les taux d'intérêts. Une fois les facteurs identifiés et estimés nous estimons dans une deuxième étape cette dernière relation par méthodes non-paramétriques.

*Understanding the dynamics of interest rates and the term structure has important implications for issues as diverse as real economic activity, monetary policy, pricing of interest rate derivative securities and public debt financing. Our paper follows a longstanding tradition of using factor models of interest rates but proposes a semi-parametric procedure to model interest rates. In a semi-parametric approach one typically parameterizes the object of interest while leaving unspecified the rest of the model. We construct factors as linear functionals of key economic time series involving unknown parameters, but treat the response of interest rates to the factors in a nonparametric way. The Average Derivative Estimator, which is a semi-parametric procedure proposed by Hardle and Stoker (1989) and Powell, Stock and Stoker (1989), allows us to proceed in two steps, namely we first identify factors without assuming knowledge of the response function of interest rates to the factors. Once the factors are identified, we proceed with estimating the response function using nonparametric methods. We can view our semi-parametric approach as a prelude to a fullblown parametric formulation for a factor term structure model. Indeed, our empirical results suggest a short term rate specification which deviates from standard parametric models often considered in the literature.*

**Mots Clés :** Taux d'intérêts, Structure par terme, Modèles à facteurs, Méthodes semi-paramétriques, Estimateur de dérivée moyenne

**Keywords :** Interest Rates, Term Structure, Factor Models, Semi-Parametric Models, Average Derivative Estimator

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# 1 Introduction

Understanding the dynamics of interest rates and the term structure has important implications for issues as diverse as real economic activity, monetary policy, pricing of interest rate derivative securities and public debt financing. It is therefore not surprising that the study of interest rates occupies a prominent place in theoretical and empirical finance as well as macroeconomics. The continuous flow of research papers suggesting new ways to capture the complexity of the dynamics in the conditional mean and variance of interest rates reveals that the literature is still in search of an adequate theoretical and empirical set of models.<sup>1</sup> In response to this situation a number of recent papers have surfaced abandoning the traditional parametric models and proposing a non-parametric approach to study interest rates and the term structure. Examples of such work include Ait-Sahalia (1993), Bekdache and Baum (1994) and Gouriéroux and Scaillet (1994).

This paper proposes a semi-parametric procedure to model interest rates. In a semi-parametric approach one typically parameterizes the object of interest while leaving unspecified the rest of the model. The paper follows a longstanding tradition of using factor models of interest rates. We construct factors as linear functionals of key economic time series involving unknown parameters, but treat the response of interest rates to the factors in a nonparametric way. The Average Derivative Estimator, which is a semi-parametric procedure proposed by Hardle and Stoker (1989) and Powell, Stock and Stoker (1989), allows us to proceed in two steps, namely we first identify factors (also called indices as explained in Section 2) without assuming knowledge of the response function of interest rates to the factors. Once the factors are identified, we proceed with estimating the response function using non-parametric methods.

There is a certain appeal to this two step procedure. While estimation of a set of factors is not uncommon in parametric models following, for instance, the classical paper by Cox, Ingersoll and Ross (1985) and many others, the assumptions of linearity and normality are relaxed in our semi-parametric setting. Bansal and Viswanathan (1993) proposed a nonlinear *APT* which involves the pricing of assets through a nonlinear

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<sup>1</sup>It is impossible to reference the multitude of papers on the subject. Although there is no comprehensive recent survey of the finance and macroeconomics literature together one can rely on Chan, Karolyi, Longstaff and Sanders (1992) for a discussion of the continuous time models and their empirical support. Shiller (1990) on the other hand provides an excellent review of the macroeconomics literature.

pricing kernel. In their procedure the factors and response functions are estimated simultaneously where the response function is estimated via polynomial series expansions or neural networks. While their analysis is similar in some ways to ours, each has advantages but can also be criticized for certain shortcomings. On the one hand, estimation of a nonlinear *APT* model involves a large number of parameters, and many moment conditions must be imposed to achieve identification. Moreover, even when the pricing kernel is estimated, one does not yet have a prediction model for the interest rate. Indeed, the pricing kernel in the nonlinear *APT* only reflects the marginal rates of substitution rather than provides a prediction formula for the interest rate. On the other hand, their procedure is more apt to handle no-arbitrage conditions, involving non-negativity constraints on the pricing kernel. Imposing such conditions in our procedure is more difficult, which is the reason why we refrain from modeling the term structure and focus exclusively on the dynamics of the interest rate series. We can view our semi-parametric approach as a prelude to a fullblown parametric formulation for a factor term structure model. Indeed, our empirical results suggest a short term rate specification which deviates from standard parametric models often considered in the literature.

Following Chan et al.(1992) we start from a discretization of a standard continuous time diffusion factor model . However, we do not consider various parametric specifications for the drift and volatility functions. In section 2 we describe the details of the model specification and estimation and discuss the comparison between parametric, nonparametric and semi-parametric procedures. Using three interest rate series, a one-month T-bill, a five-year government bond and a ten-year one, we estimate the semi parametric factor models. The results are reported in section 3. Conclusions appear in section 4.

## 2 Econometric Specification and Estimation

The semi-parametric analysis of interest rates consists of i) using the average derivatives of interest rates with respect to a set of economically relevant variables to form factors, and ii) expressing interest rates as additive but not necessarily linear functions of these factors. The first subsection motivates the use of semi-parametric factor models. Then we briefly review the average derivative estimator and the general additive model. Implementation issues are deferred to the last part of the section. It should be clarified at the outset that what is often referred to as “factor models” in the finance literature is sometimes referred to as

“index models” in the semi-parametric and other literatures, and we will on occasions use the two terms interchangeably.

## 2.1 Interest Rate Models: Parametric, Non-parametric and Semi-parametric

It is quite common to use parametric factor models for interest rates and their term structure. Vasicek (1977), Courtadon (1982), and Cox, Ingersoll and Ross (1985) (henceforth CIR) presented various single factor continuous time models for the short term rate  $r_t$  (as well as the entire term structure) with the dynamics presented by a stochastic differential equation:

$$dr_t = \mu(z_t; \beta)dt + \sigma(z_t; \beta)dW_t \quad (1)$$

where  $W_t$  is a standard Brownian Motion,  $z_t$  is a state variable process and the functions  $\mu(z_t; \beta)$  and  $\sigma(z_t; \beta)$  are respectively the drift and diffusion functions parameterized by the vector  $\beta$ . Most interest rate models assume a linear mean-reverting drift such as  $\mu(z_t; \beta) = \beta_1(\beta_2 - r_t)$  where  $z_t = r_t$ . In this case the spot rate tends to its unconditional mean  $\beta_2$  at a rate  $\beta_1$ . The volatility functions differ more widely, though often a constant elasticity of variance (CEV) specification is adopted, i.e.  $\sigma^2(z_t; \beta) = \sigma^2 r_t^{\beta_3}$  where  $\beta_3 = 0$  for Vasicek’s model,  $\beta_3 = 2$  in the model proposed by Courtadon and finally  $\beta_3 = 1$  for CIR. Chan, Karolyi, Longstaff and Sanders (1992), henceforth CKLS, provide an extensive empirical study of such models for the short rate. In more recent work several multifactor extensions appeared in the literature aiming at modeling the entire term structure based on a selected set of maturities.<sup>2</sup> Following CKLS, and others, let us consider an Euler discretization of (2.1), namely:

$$\Delta r_t = r_t - r_{t-1} = \mu(z_t; \beta) + \sigma(z_t; \beta)\epsilon_t \quad (2)$$

where  $\epsilon_t$  is i.i.d.  $N(0,1)$ . Hence the conditional mean and variance for the interest rate process are respectively:

$$E(\Delta r_t | z_t) = \mu(z_t; \beta) \quad (3)$$

$$V(\Delta r_t | z_t) = \sigma(z_t; \beta). \quad (4)$$

As noted earlier standard interest rate models differ with regard to the choice of the functions  $\mu$  and  $\sigma$  as well as the choice of factors  $z_t$ . We

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<sup>2</sup>Examples include Chen and Scott (1992), Chen and Scott (1993), Duffie and Kan (1993), Frachot and Lesne (1993), Heath, Jarrow and Morton (1992), Pearson and Sun (1994), Pennachi (1991), Stambaugh (1988), among others.

will try to be agnostic about this by using a semi-parametric approach to modeling the factors  $z_t$ , and adopting functions of flexible form for both the conditional mean and the conditional variance.

To appreciate the use of semi-parametric models, we first need to discuss the strength and limitations of parametric and non-parametric models. Suppose we are interested in the relationship between  $y$  (the response variable, like  $\Delta r_t$ ) and  $x$  (a set of  $k$  dimensional predictor variables, like the factors  $z_t$ ). In parametric analysis, we would consider a model such as<sup>3</sup>

$$y_t = G(x_t'\beta) + u_t. \quad (5)$$

We would then make distributional assumptions (such as normality) about  $u_t$ , pick a convenient form for the link function  $G(\cdot)$ , like the functions  $\mu$  and  $\sigma$  mentioned above, and then proceed with least squares, method of moments, or maximum likelihood estimation. For obvious reasons, the linear functional  $G(\cdot)$  is by far the most popular since the model  $y = x\beta + u$  is easy to estimate. Moreover, the one to one relationship between  $y$  and  $x$  provides a convenient interpretation of  $\beta$  as “change in  $y$  per unit change in  $x$ ”. While this classical linear regression model is adequate for a variety of applications, there are many cases where  $y$  and  $x$  are related in some unknown and non-linear way and the normality of  $u_t$  breaks down. A case in point is financial time series which often exhibit non-linearity in both their conditional mean and variance.

At the other end of the spectrum are non-parametric models which assume only that the relationship between  $y$  and  $x$  obeys some smoothness and regularity conditions. As such, these models impose no assumptions about the form of the link function  $G(\cdot)$  or on the distributional properties of  $u_t$ . It is an extreme approach to letting the data speak for the relationship between  $y$  and  $x$ . An especially attractive feature of non-parametric regressions is that when  $x$  consists of a small number of variables, useful insights can be often gained just by graphing the function that relates  $y$  to  $x$ . Examples of non-parametric models include kernel regressions, general additive models, and polynomial or Fourier series expansions. [Hardle \(1990\)](#) provides a discussion of these models.

While non-parametric models are useful in many contexts, as tools of economic analyses, they have some drawbacks. The first is that statistical flexibility is achieved at the cost of not being able to incorporate

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<sup>3</sup>We use a generic function  $G(\cdot)$  here which in principle may stand for the conditional mean or conditional variance. The remainder of this section will focus mostly on the conditional mean, however. The necessary changes for the conditional variance are straightforward.

economic theory in the empirical analysis. For example, it would be difficult to impose or test for constant returns to scale if we estimate a production function by kernel regressions. Likewise, it would be difficult to impose no-arbitrage conditions on the interest rate model. Second, when the dimension of  $x$  is large, graphical analysis provides little intuition. Third, and perhaps more important from a practical point of view, is that the number of observations needed for the data to speak for the underlying relationship between  $y$  and  $x$  increases with the dimension of  $x$ . Given the sample size and the dimension of  $x$  typically encountered in economic analyses, there is rarely enough data to obtain satisfactory statistical precision in the non-parametric estimates. This problem is known as the "curse of dimensionality" in the statistics literature.

In between the parametric and non-parametric paradigms is a less extreme modeling strategy, semi-parametric models, whereby structural assumptions can be imposed on some but not all the parameters of interest. Index models belong to the semi-parametric paradigm. The simplest index model consists of a single index,  $x'\beta$ , and is of the form

$$\begin{aligned} y &= m(x) + u = \tilde{G}(x'\beta) + u, \\ E(u|x) &= E[y - \tilde{G}(x'\beta)|x] = 0. \end{aligned} \tag{6}$$

Several features of this model are noteworthy. First, no assumption is made on the distribution of  $u$ . As long as  $E(u|x) = 0$ ,  $E(y|x)$  is completely summarized by the function  $\tilde{G}(\cdot)$ , which can take on any smooth form provided it can be estimated non-parametrically. Second, while the contribution of  $x$  to the index is linear, as measured by  $\beta$ , the model permits a non-linear relationship between  $y$  and  $x$  through the non-linear link function  $\tilde{G}(x'\beta)$ . Third, since the index  $x'\beta$  is one dimensional, graphical tools can still be used to analyze the relationship between  $\tilde{G}(\cdot)$  and  $x'\beta$ . Additional insight can also be gained with a plot of  $\tilde{G}(\cdot)$  and  $x'\beta$ , which is the marginal response function.

To interpret  $\beta$  in  $\tilde{G}(x'\beta)$  when  $x$  is  $k$  dimensional, consider changing  $x_1$  to  $x_1 + \Delta x_1$ . Then  $\Delta E(y|x) = \partial E(y|x) / \partial x_1 \Delta x_1 = d\tilde{G} / d(x'\beta) \beta_1 \Delta x_1 = \tilde{G}' \beta_1 \Delta x_1$ . The coefficient  $\beta$  is proportional to the pointwise derivatives of  $y$ , with the proportionality factor being  $\tilde{G}'$ , which varies with the value of  $x$ . The effects of changing other components of  $x$  are similarly summarized by the corresponding  $\beta$  coefficient. Note, however, that the  $\tilde{G}(\cdot)$  estimated on  $x'\beta$  will not be invariant to common scale changes on the  $\beta$ 's. Thus, it is the relative, not the absolute, magnitude of  $\beta$  that matters. This suggests normalizing  $\beta$  by the mean effect such that

$$\delta = E(m') = E(\tilde{G}')\beta \tag{7}$$



and redefine  $\tilde{G}$  to  $G$ , imposing the normalization that  $E(G') = 1$ . The normalization implies that the average impact of a change in the index on the mean of  $y$  is 1. The result is the model

$$E(y|x) = m(x) = G(x'\delta). \quad (8)$$

The normalized model allows us to measure  $\delta$  in units of “ $y$ -changes” / “ $x$ -changes”, units comparable to coefficients in a classical linear model. Furthermore,  $y$  and  $x'\delta$  are, on average, related in a one-to-one manner. Viewed in this light, the interpretation of  $\delta$  as the “average derivative” of  $y$  on  $x$  is immediate.

## 2.2 The Single Index Model and the Average Derivative Estimator

Usefulness of the single index model rests on the ability to estimate the average derivatives non-parametrically and without suffering from the curse of dimensionality. Ichimura (1993) developed a semi-parametric least squares estimator of  $\delta$  that is  $\sqrt{N}$  consistent and asymptotically normal under regularity conditions, but the estimator requires optimizing a non-linear objective function that is not necessarily concave or unimodal. An alternative is to estimate  $\delta$  by what is appropriately known as the “average derivative estimator”. The estimator is also  $\sqrt{N}$  consistent and asymptotically normal under regularity conditions but does not require solving iterative non-linear optimization problems. The estimator relies on kernel density and regression estimators, and takes as starting point  $E(y|x) = m(x) = G(x'\delta)$ . A natural estimator of the average derivatives is the “direct” average derivative estimator:

$$\hat{\delta} = N^{-1} \sum_{i=1}^N \hat{m}'(x_i) \hat{1}_i, \quad (9)$$

where  $\hat{1}_i = 1[\hat{f}(x_i) \geq b]$  is an indicator function that drops the observation when the estimated density,  $\hat{f}(x)$ , is smaller than some value  $b$ , a procedure that is sometimes referred to as “trimming”.

Alternative average derivative estimators have been proposed by Härdle and Stoker (1989) and Powell, Stock and Stocker (1989) with the same asymptotic properties as the direct average derivative estimator. The estimator we use is the “indirect slope estimator” defined as

$$\hat{d} = S_{lx}^{-1} S_{ly} \quad (10)$$

where  $S_{lz} = N^{-1} \sum l(x_i) \hat{1}_i (z_i - \bar{z})$ ,  $\hat{l}(x) = \frac{\hat{f}'(x)}{\hat{f}(x)}$  is the score. The estimator is motivated by the fact that  $E(m') = E(l_y)$  upon integration by

parts, and that  $(\partial x' / \partial x) = I_d = E(lx') = cov(l, x)$ . Thus,

$$d = E(m') = \left[ E \left( \frac{\partial x'}{\partial x} \right) \right]^{-1} E(m'). \quad (11)$$

The indirect slope estimator is then constructed from the sample moments of the appropriate quantities of (??). The advantage of the indirect slope estimator over the direct average derivative estimator is that the smoothing required on both the numerator and denominator of  $\hat{d}$  reduces the smoothing bias that arise in finite samples. See Stoker (1993). Some intuition on this estimator can be gained by noting that it is an instrumental variable estimator using the scores as instruments. The scores reduce to the matrix  $x$  if the true relationship between  $y$  and  $x$  is indeed linear, in which case,  $\hat{d}$  reduces to the least squares estimator. Further details on this estimator are provided in the Appendix. Given the estimates of the average derivatives, an index  $z = x'\hat{d}$  can be formed.

### 2.3 The Generalized Additive Model

It is simple to extend the single index model to multiple indices. To anticipate what is to follow in the empirical section, we want to explain interest rates movements with factors that are based on industrial output, money growth, and inflation. Use of these variables can be motivated by many macroeconomic paradigms (e.g. the IS-LM model). More specifically, we envision a factor model consisting of two basic indices; one comprising of nominal variables and one comprising of just real economic variables. Partitioning the matrix  $x$  into  $x_1$  and  $x_2$ , we have two indexes  $z_1 = x_1'\delta_1$  and  $z_2 = x_2'\delta_2$ . Given that we have in mind such an explicit economic structure, we do not therefore expect a single function  $G(z_1, z_2)$  to provide a satisfactory link to both indices. This leads us to consider a more general model such as

$$\theta(y) = \alpha + \sum_{i=1}^{m_0} \phi_i(z_i) + \epsilon, \quad (12)$$

where  $z_i$  is our  $i^{th}$  index. The above model is the Alternating Conditional Expectations (ACE) model of Brieman and Friedman (1985) and aims to maximize the correlation between  $\hat{\theta}(y)$  and  $\hat{\phi}(z)$ . For example, in an analysis of excess returns by Foresi and Perachi (1995)  $\theta(y)$  is the log-odds ratio. A shortcoming of the ACE is that it can produce anomalous results if  $\epsilon$  and  $\phi_i(z_i)$  fail the independence and normality assumptions, an issue of concern given the application in question.

The model we will use is the Generalized Additive Model (GAM) introduced by Hastie and Tibshirani (1990). It is a restrictive ACE model with  $\theta(y) = y$  since it is of the form

$$y = \alpha + \sum_{i=1}^{m_0} \phi_i(z_i) + \epsilon. \quad (13)$$

The general additive model with indices formed from the average derivatives is in many ways similar to the Projection Pursuit Regression (PPR) of Friedman and Stuetzle (1981). In *PPR* models, the predictor variables  $x_i, i = 1 \dots k$  are “projected” onto the direction vectors  $a_j, j = 1 \dots m_0$  to get lengths  $a'x$ , and optimization is carried out in “pursuit” of good direction vectors. More formally, the objective of *PPR* is to minimize  $E[y - \sum_{i=1}^{m_0} \beta_m \phi_i(a'_i x)]^2$  over all possible values of  $\beta_i, \phi_i$ , and  $a_i$ . The direction vectors have interpretations analogous to the average derivatives, but a *PPR* chooses  $\beta$  simultaneously with  $\phi$ , whereas the multiple index model does this in two steps. Computationally, the index model is simpler to estimate since it does not require any non-linear optimization. From our point of view, the index approach allows us to focus on combinations of variables that provide meaningful interpretation to interest rate dynamics. For example, we would discard an index which is a linear combination of housing starts and the exchange rate since the index has little economic meaning. One can therefore say that our index model is a *PPR* with restrictions based upon economic reasoning.

If  $m_0 = 1$ , one can apply non-parametric regression techniques such as kernels and splines to estimate  $\hat{\phi}(z_i)$  since it is the only component function in the GAM. It is worth noting here that even though  $z_i$  is a generated regressor it achieves pointwise consistency at rate  $N^{2/5}$  as though  $\hat{\delta}$  is known.<sup>4</sup>

The estimation of a GAM is slightly more involved in higher dimensions. A method commonly used to estimate  $\hat{\phi}_i$  when  $m_0 > 1$  is the “backfitting algorithm” discussed in Hastie and Tibshirani (1990), where  $\hat{\phi}$  is obtained by a spline smoother. This is implemented in software such as Splus.<sup>5</sup> A drawback of this approach, from our point of view, is that it is difficult to give economic interpretation to resulting spline regressions. We therefore estimate  $\hat{\phi}$  by a polynomial regressions. Polynomial regressions provide non-parametric approximations to the

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<sup>4</sup>See Theorem 10.4.2 of Härdle (1990).

<sup>5</sup>The alternative “easy” method is to use LOESS (locally weighted regression smoothing) of Cleveland (1979) which seems to give results similar to splines.

true regression functions, as controlled by the order of the polynomials, but can be estimated by least squares method.

A polynomial regression of order  $p$  for each of the two index takes the form:

$$y_i = \alpha_0 + \sum_{i=1}^p \alpha_{1i} z_1^i + \sum_{i=1}^p \alpha_{2i} z_2^i + \epsilon. \quad (14)$$

Evidently,  $\phi_1(z_1)$  is approximated by  $\sum_{i=1}^p \alpha_{1i} z_1^i$ . This has the distinct advantage over a spline approximation to  $\phi_i(z_i)$  in that the marginal effect of  $x_i$  on  $y$  can be calculated immediately. More importantly, if  $z_1 = x'_1 \delta_1$  and  $z_2 = x'_2 \delta_2$  are index variables based on a set of variables  $x$ ,  $\partial y / \partial x_j$  can be calculated as  $\sum_{i=1}^p (\alpha_{1i} \delta_{1j}^i) + \sum_{i=1}^p (\alpha_{2i} \delta_{2j}^i)$ , where  $\delta_{1j} = \partial z_1 / \partial x_j$  are the weights of  $x_j$  in index  $z_1$  as determined by the estimates of the average derivatives. A similar interpretation holds for  $\delta_{2j}$ . It is worth emphasizing that while  $z_i$  is linear  $x_j$ ,  $y$  is non-linear in  $z_1$  and  $z_2$ , which in turn implies that  $y$  is also non-linear in  $x_j$ .

## 2.4 Estimation Issues

Estimation of General Additive index models for interest rates and spreads raises several issues not previously analyzed in the literature. In our applications,  $y$  is changes in interest rates and  $x$  are economic time series. While the theory on the average derivative estimator just described was developed under the assumptions that  $x$  and  $y$  are stationary and that  $u$  is *i.i.d.*, some of the variables involved in the analysis of interest rates are non-stationary, while others exhibit a strong degree of serial correlation.

The problem of non-stationarity is overcome by differencing the non-stationary variables to achieve stationarity. If the noise component of the differenced variable is *i.i.d.*, then consistency of the average derivative estimator follows from the proof of Hardle and Stocker (1989). In cases when the *i.i.d.* assumption fails, we appeal to results for consistency of density and kernel estimators for  $\alpha$  mixing observations (see Robinson (1983) and Singh and Ullah (1985)). As discussed in Chen and Tsay (1993), estimation of general additive autoregressive models is still asymptotically valid when time series data are used, although some additional care must be taken to avoid spurious fitting of additive autoregressive models in finite samples.<sup>6</sup>

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<sup>6</sup>The problem arises because a bad fit on  $(\hat{x}_{t-1})$  has a direct impact on the dependent variable in the next step of the backfitting algorithm. For this reason, we make no attempt to fit additive autoregressive models in this analysis.

The average derivative estimator can be shown to be valid even when the data are serially correlated as consistent estimation of the densities is not affected by the presence of serial correlation. However, in such a case, we need an estimate of the covariance matrix of the average derivative regression residuals, denoted  $r_u$ , that takes into account the fact that  $u$  is not *i.i.d.*. Thus, in our analysis,  $S_{r_u r_u}$  defined in the Appendix, is the heteroskedastic-autocorrelation consistent variance covariance matrix using the Parzen window with automatic selection of the bandwidth as discussed in Andrews (1991). Since the  $r_{ui}$  are prewhitened and recolored by a first order VAR, it amounts to using the procedure proposed by Andrews and Monahan (1992).

We also need to take into consideration the possibility that  $E(u|x)$  may not be zero since the variables underlying the factors (or index) are likely to be contemporaneously correlated with shocks to interest rates. To circumvent this problem, we use lags of  $x$  in the estimation of the average derivatives. This can be seen as an instrumental variables implementation of the average derivative estimator.

The final issue concerns the choice of bandwidth in estimating the average derivatives. We standardize all the variables to have a mean of zero and a unit variance. The same bandwidth  $h$  can be used to evaluate the multidimensional kernel function because it is invariant to the scale of the variables:

$$K(u_1 \dots u_k) = \prod_{i=1}^k \kappa(u_i) \quad (15)$$

$$\text{where } \kappa(u_i) = \frac{1}{\sqrt{2\pi}} \exp(-u_i^2/2).$$

The bandwidth is obtained as the plug-in value based on equation (4.14) of Powell and Stocker (1992). For the sample size and number of regressors used in the analysis, we settle for a bandwidth of 0.7.

### 3 Empirical Results

We construct multiple index models for the first difference of three interest rates: a one-month T-bill, a five-year government bond and a ten-year one. The data are monthly yields from 1964 to 1990 and are taken from McCulloch (1990), for the 1964 to 1983 part of the sample, and Kwon (1992), who extended the data set from 1983 to 1990. Hence the sample contains 384 monthly observations for the three different interest rate series.

Since interest rates provide the link between the real and the financial side of the economy we expect interest rates to be affected by real and monetary factors. This being the case, our goal is to model the first differences of interest rates as a function of two indices: one comprising of real economic variables, and another comprising of nominal ones. The estimation of average derivatives involves seven variables (all lagged one period): money growth, the first difference of the (log) exchange rate between the U.S. and the U.K., the rate of inflation, changes in industrial production, changes in housing starts, changes in retail sales and finally changes in finished goods inventories. All the series are seasonally adjusted and were retrieved from Citibase. Hence, they are standard series used in US empirical macroeconomic studies. The nominal index factor is constructed using the first three series, while the real factor is based on a combination of the last four.

We present in Table 1 the estimates for the three interest rates. We report the ADE parameter estimates with two types of  $t$  statistics. These are based on two sets of standard errors estimates, the first are valid under i.i.d. while the second involves a heteroskedasticity and autocorrelation consistent estimation procedure as described in section 2.4. As a matter of comparison we also report the OLS estimates and their  $t$  statistics for the index parameter estimates. The results in Table 1 show that the effect of money growth is positive, as expected, but is not well determined for two of the three maturities. Its impact is largest, and statistically most significant, for the five-year rate. The foreign exchange variable has a significant effect both on the short-term and long-term maturities. It is interesting to note that its largest effect is also on the five-year bond while its impact is much smaller for the one-month and ten-year interest rate changes. The last component of the nominal factor appearing in the model is the rate of inflation. Its impact is highly significant for all maturities and roughly flat across the one-month and five-year bonds. Interestingly, the estimate is quantitatively and statistically less important in the equation for the five-year rate, in contrast to the estimates for money growth and the exchange rate.

The next four parameter estimates form the real index factor. The first three variables underlying the real index series all have a positive impact. The impact of retail sales changes on the short rate appears not to be significant, however. Housing starts and changes in inventories on the other hand seem to have a significant short term impact which becomes less significant at the longer maturities. The impact of inventory changes is negative on interest rate changes, as expected. However, the effect of inventories on the short-term rate is insignificant. These estimates reveal the interesting fact that real economic variables found

to have explanatory power for the short term rate do not necessarily have explanatory power for the longer term maturities and vice versa. Indeed, the real and nominal variables used here are more capable of explaining the longer term maturities. While the search is not exhaustive, experimentation with other explanatory variables lead to the same general conclusion that the average derivatives are better determined in the equations for the longer term maturities than the short term rate.

Table 1: Average Derivative and OLS estimates of one-month, five-year and ten-year interest rate model factor indexes

	One-month		Five-year		Ten-year	
	ADE	OLS	ADE	OLS	ADE	OLS
Money	.00669 (1.093) [1.079]	.01285 (1.408)	.00703 (1.521) [2.818]	.00978 (1.780)	.00562 (1.224) [1.246]	.00598 (1.340)
Exch.	.00175 (1.470) [1.315]	.00311 (2.039)	.00254 (2.673) [2.466]	.00223 (2.432)	.00161 (1.987) [1.963]	.00122 (1.640)
Infl.	.02857 (3.129) [3.602]	.00230 (.228)	.01799 (2.987) [3.044]	.01207 (1.992)	.01813 (3.820) [3.700]	.01220 (2.480)
Ind. Pr.	.00854 (2.660) [2.497]	.01381 (3.358)	.00389 (1.706) [1.549]	.00451 (1.823)	.00359 (1.899) [1.693]	.00389 (1.936)
Hous. st.	.00068 (2.266) [2.060]	.00081 (1.984)	.00035 (1.750) [1.521]	.00020 (.807)	.00028 (1.555) [1.400]	.00006 (.285)
Sales	.00158 (.827) [.814]	.00199 (.817)	.00467 (3.288) [3.554]	.00300 (2.050)	.00350 (2.941) [2.941]	.00256 (2.154)
Invent.	-.00815 (-.486) [-.518]	-.03183 (-1.502)	-.02424 (-1.990) [-2.095]	-.01727 (-1.354)	-.02885 (2.779) [2.896]	-.01597 (-1.543)

Notes:  $t$  statistics are in parenthesis. The standard errors for the average derivatives corrected for heteroskedasticity using the Andrews and Monahan (1992) procedure.

So far we have not commented on the differences between the ADE and OLS parameter estimates. The ADE estimates should be identical to the OLS estimates if the true model is linear and the assumption of normality holds. For the nominal variables, the OLS estimates are larger for money growth and generally smaller for the exchange rate compared to the ADE estimates. The ADE estimates for inflation are much larger than the OLS estimates. More importantly, they are statistically better determined than the OLS estimates. For the real variables, we also observe some noticeable differences. The ADE estimates are generally better determined. In the case of inventory changes, while the OLS estimates fail to find any statistically significant effects, the ADE estimates are significant for the two longer term rates.

Using the average derivatives as weights, we then construct a real index as a function of changes in industrial production, retail sales, housing starts, and inventories. Likewise we construct a nominal index using the ADE weights on the changes in money supply, the exchange rate and the inflation rate. A total of six indexes are thus constructed, as there are three different maturities of interest rates being modelled. In Figure 1

we plot the six index series. We note a remarkable similarity in the time series patterns of the three nominal and the three real series. A closer look reveals though that the real index for the one-month rate has some notable differences, particularly around the oil price shock of 1974 and the mid-eighties.

Let us return now to equation (1) in section 2.1. The typical continuous time interest factor diffusion models require the specification of a drift and volatility function. The Euler discretizations in equations (3) and (4) made this more explicit and workable in the current setting. So far we have identified a pair of factors without requiring the specification of the functional forms in (3) and (4). The next step of the analysis is to use polynomial regressions of order  $p$  estimated by OLS as in equation (14) to approximate the functionals for the conditional mean and conditional variance, which are of unknown form. The empirical results appear in Table 2. There are two parts to Table 2. The top part reports the polynomials for the conditional means. The residuals of the latter are squared and used in the next set of polynomial regressions for the conditional variance. Those estimates appear in the bottom part of Table 2. The conditional mean and variance regressions for each of the three interest rates reported in Table 2 involve the same real and nominal factors.

According to the results appearing in Table 2 we observe that we need a polynomial of degree 2, respectively 3, in the real and nominal index for the conditional mean regressions for all three interest rates. Note that the coefficient for the second order term of the nominal index is not significant, but the third order term is statistically well determined especially the longer the maturity of the bond. In contrast, the quadratic term for the real index is better determined the shorter the maturity. Evidently, the two indexes affect the various interest rates in non-linear, albeit different ways.



Table 2: Polynomial regressions of nominal and real indexes - Conditional Mean and Conditional Variance Models for one-month, five-year and ten-year interest rates

Conditional Mean

	One-month		Five-year		Ten-year	
	real	nominal	real	nominal	real	nominal
Degree 0	.00727 (.184)	-	.0109 (.458)	-	.01202 (.623)	-
1	3.4803 (4.877)	1.3242 (1.863)	1.4959 (3.479)	.9960 (2.3204)	1.1539 (3.3149)	.4940 (1.421)
2	-1.9366 (-2.727)	.0176 (.0248)	-.8490 (-1.982)	.0648 (.1512)	-.7817 (-2.253)	-.0995 (-.286)
3	-	-1.258 (-1.774)	-	-1.2775 (-2.9833)	-	-1.1216 (-3.231)

Conditional Variance

	One-month		Five-year		Ten-year	
	real	nominal	real	nominal	real	nominal
Degree 0	.4928 (6.971)	-	.1799 (8.704)	-	.1180 (9.766)	-
1	-4.7343 (-3.710)	-2.8155 (-2.206)	-.5636 (-1.508)	-.2439 (-.653)	-.2985 (-1.367)	-.2199 (-1.008)
2	2.5869 (2.032)	-	-	.8708 (2.336)	-	.5114 (2.348)

Notes:  $t$  statistics in parenthesis.

For the conditional variance models the specifications of the degrees of the polynomials are more diverse across the different rates. The short term rate evidently requires a functional form that is different from the longer term rates. The real index has weak explanatory power for the conditional variance of the longer term rates but has a well-determined and non-linear effect on the conditional variance of the short term rate. For the two long term rates, movements in the conditional variance are in large part picked up by the quadratic term of the nominal index.

To visualize better the results we plotted the implied response functions for the conditional mean in Figure 2. We find rather interesting nonlinear shapes both for the nominal indexes (appearing on the left) and the real ones. The latter has an increasing response function which levels off. For the nominal index the response functions are decreasing at very low levels of the index, are upward sloping for most part, but again slope downward for extreme large values of the nominal index. Of course the curvature at the extreme ends is not supported by the bulk of the data, nevertheless the tilted S-shape covers a large part of the

support of the data indicated by the ticks on the vertical axes in Figure 2.

The plots of the response functions of the conditional variance appears in Figure 3, which has the same structure as Figure 2. Contrary to the mean regressions we observe a very different shape for the short rate in comparison to the long rates. Clearly the volatility of short rates responds negatively to the real and nominal indexes. But for the long rates we observe that high values of the nominal index have an increasing effect on the volatility. These results are economically significant and reflect the different responses of the interest rates through the interest rate term structure to the economic factors. At the beginning of section 2 it was noted that most parametric specifications, such as those considered by CKLS, are assumed to be linear mean-reverting in the drift and linear in the variance. The results in Figure 2 show quite clearly that for empirically constructed factors we do not find a simple set of functions corresponding to what typical parametric specifications assume.

Finally, we need to make observations about the ACF (autocorrelation functions) of the residuals of the mean polynomial regressions. These appear in Figure 4. We plotted side-by-side the ACF of the residuals of the three interest rate models as well as the ACF of the squared residuals. The latter is the dependent variables in the conditional variance models.

There is something quite remarkable about the ACF's of the residuals. Indeed, they are uncorrelated, that is all temporal linear dependence was removed despite the fact that no lagged interest rate was put into the polynomial regressions. Hence the residuals were whitened by a combination of the nonlinearity and the factors. We conclude with Figure 5, which displays the time series plot of the squared residuals of the conditional mean regression. It clearly shows the volatility clustering effect so often encountered in financial series. Again, we note a remarkable difference between the short rate and the other rates. Moreover, we clearly observe a significant change in volatility since October 1979 when the Federal Reserve changed its operating rules. In fact we learn from Figure 5 that the volatility in the one-month T-bill seems to have returned to its pre-October 1979 level. In contrast, both the five-year and ten-year rate volatilities appear to have adopted very different volatility patterns. Such persistent changes are a contributing factor to the complexity and nonlinearity found in modelling interest rate dynamics.

## 4 Conclusions

In this paper we proposed to use the average derivative estimator framework applicable to factor index interest rate models. The appeal of this framework is that it does not require much a priori knowledge of the factors and their responses. The drawback is that in this framework it is difficult to impose arbitrage type conditions across the term structure or other a priori economic restrictions. The ADE estimates, the constructed indexes and the emerging empirical response functions are a lead to more structural factor models. The response functions we fitted for the conditional mean of the different maturities seem to suggest some clear similarities, though for volatility there were important differences in the response functions across maturities. However, the response functions we fitted were not in line with many of the drift and volatility functions that have been suggested in parametric factor models, such as linear drift and volatility functions mentioned in the beginning of section 2.1. In that regard the approach in this paper serves its purpose, it shows that many parametric models suggested so far must be significantly misspecified unless their factors differ from the indexes we recovered empirically. The latter is plausible, yet quite unlikely. In the introduction of the paper it was noted that many papers have been written on the subject of interest rate movements but with rather limited success so far. Our paper provides some guidance on what parametric models should try to mimic and aim for improvement through arbitrage and other structural restrictions.

## Appendix

In this Appendix we provide some details of the Average Derivative estimators discussed in section 2.3. First we start with the “direct” Average Derivative estimator. As noted in section 2.3 the estimator relies on kernel density and regression estimators, and takes as starting point  $E(y|x) = m(x) = G(x'\delta)$ . Using a kernel regression estimator to estimate  $m(x)$  with bandwidth  $h$ , we can write the non-parametric regression with as  $k$  regressor as

$$\hat{m}(x) = \frac{\hat{g}(x)}{\hat{f}(x)} = \frac{1/Nh^k \sum_{j=1}^n K((x_i - x_j)/h)y_j}{1/Nh^k \sum_{j=1}^n K((x_i - x_j)/h)}.$$

It follows that

$$m'(x) = \frac{\hat{g}'(x)}{\hat{f}(x)} - \hat{m}(x)\hat{l}(x),$$

where  $\hat{l}(x) = \frac{\hat{f}'(x)}{\hat{f}(x)}$  is the score. The “direct” average derivative estimator is given by

$$\hat{\delta} = N^{-1} \sum_{i=1}^N \hat{m}'(x_i)\hat{1}_i,$$

where  $\hat{1}_i = 1[\hat{f}(x_i) \geq b]$  is a indicator function that drops the observation when the estimated density is smaller than some value  $b$ , a procedure that is sometimes referred to as trimming.

The  $\sqrt{N}$  consistency of the direct average derivative estimator is an interesting result in its own right given that the pointwise derivatives,  $m'$ , have been shown to achieve consistency at a rate slower than  $\sqrt{N}$ . The accelerated rate of convergence of the average derivative estimator comes from the fact that the estimator is essentially constructed from a sum of  $m'$  over  $N$ , and hence a double sum (over  $N$ ) of terms involving  $f'(x)$ . A consequence of the double sum is that the window for smoothing  $f'(x_i)$  and  $f'(x_j)$  ( $i \neq j$ ) overlaps. An analysis of the sample variation of  $\hat{\delta}$  suggests this overlapping requires asymptotic “undersmoothing” which in turn speeds up the rate of convergence to rate  $\sqrt{N}$ . Formal conditions for  $\sqrt{N}$  asymptotic normality of  $\hat{\delta}$  can be found in Stoker (1991).

The variance of  $(\hat{d} - d)$  further simplifies to that of the classical linear model under the additional assumption about normality of  $x$  and  $u$ . More generally, the variance-covariance matrix of  $(\hat{d} - d)$  is computed from

$$\begin{aligned} \hat{u}_i &= (y_i - \bar{y}) - (x_i - \bar{x})' \hat{d} \\ r_{ui} &= \hat{l}(x_i) \hat{l}_i \hat{u}_i + N^{-1} h^{-k} \sum_{j=1}^N \left[ h^{-1} K' \left( \frac{x_i - x_j}{h} \right) - K \left( \frac{x_i - x_j}{h} \right) \hat{l}(x_j) \right] \frac{\hat{l}_j \hat{u}_j}{\hat{f}(x_j)} \\ \hat{\Omega} &= S_{lx}^{-1} S_{r_u r_u} S_{lx}^{-1}. \end{aligned}$$

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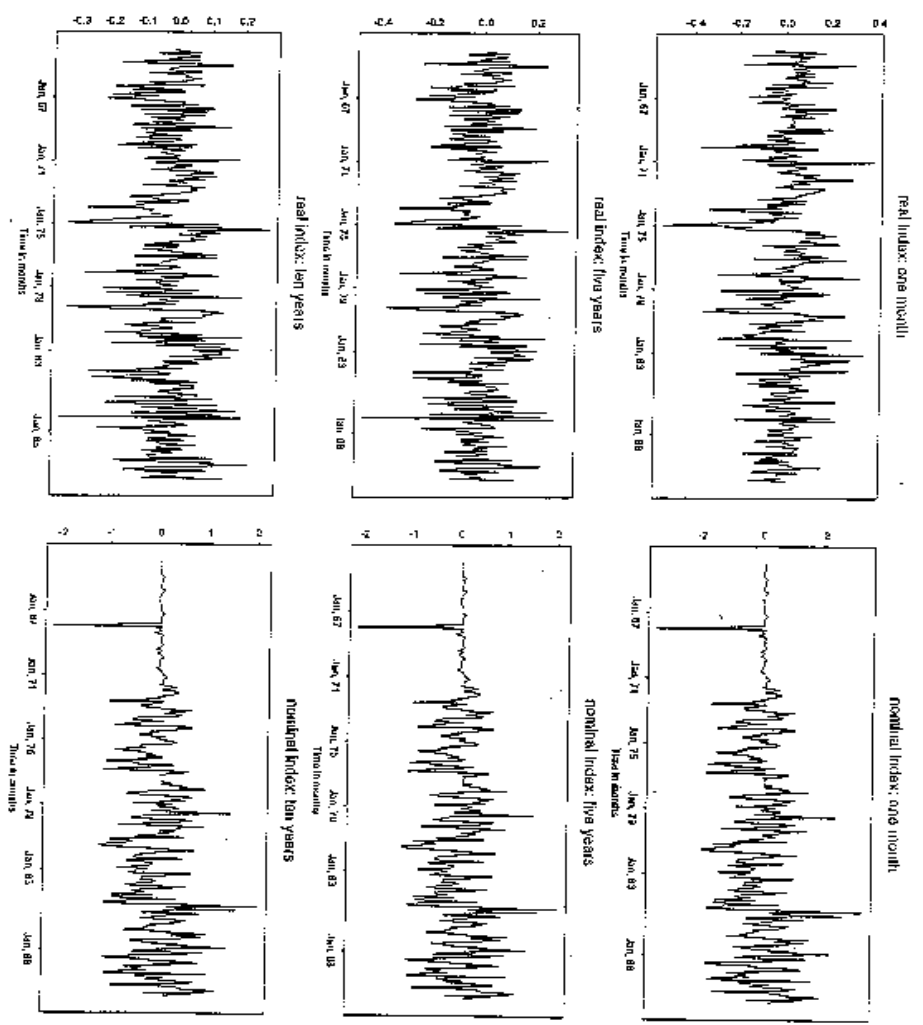


Figure 1: Nominal and Real Index Series

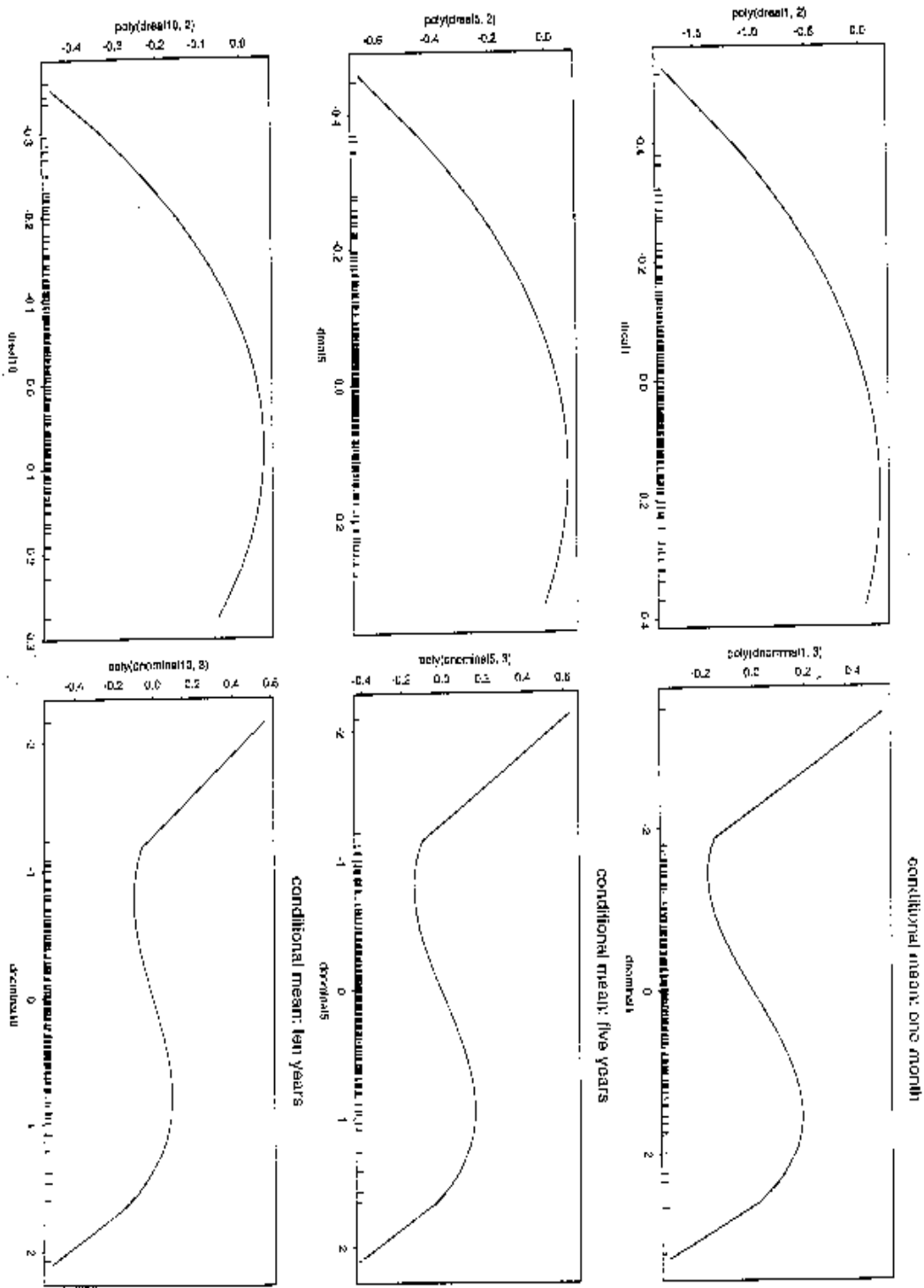


Figure 2: Implied Response Functions in Conditional Mean

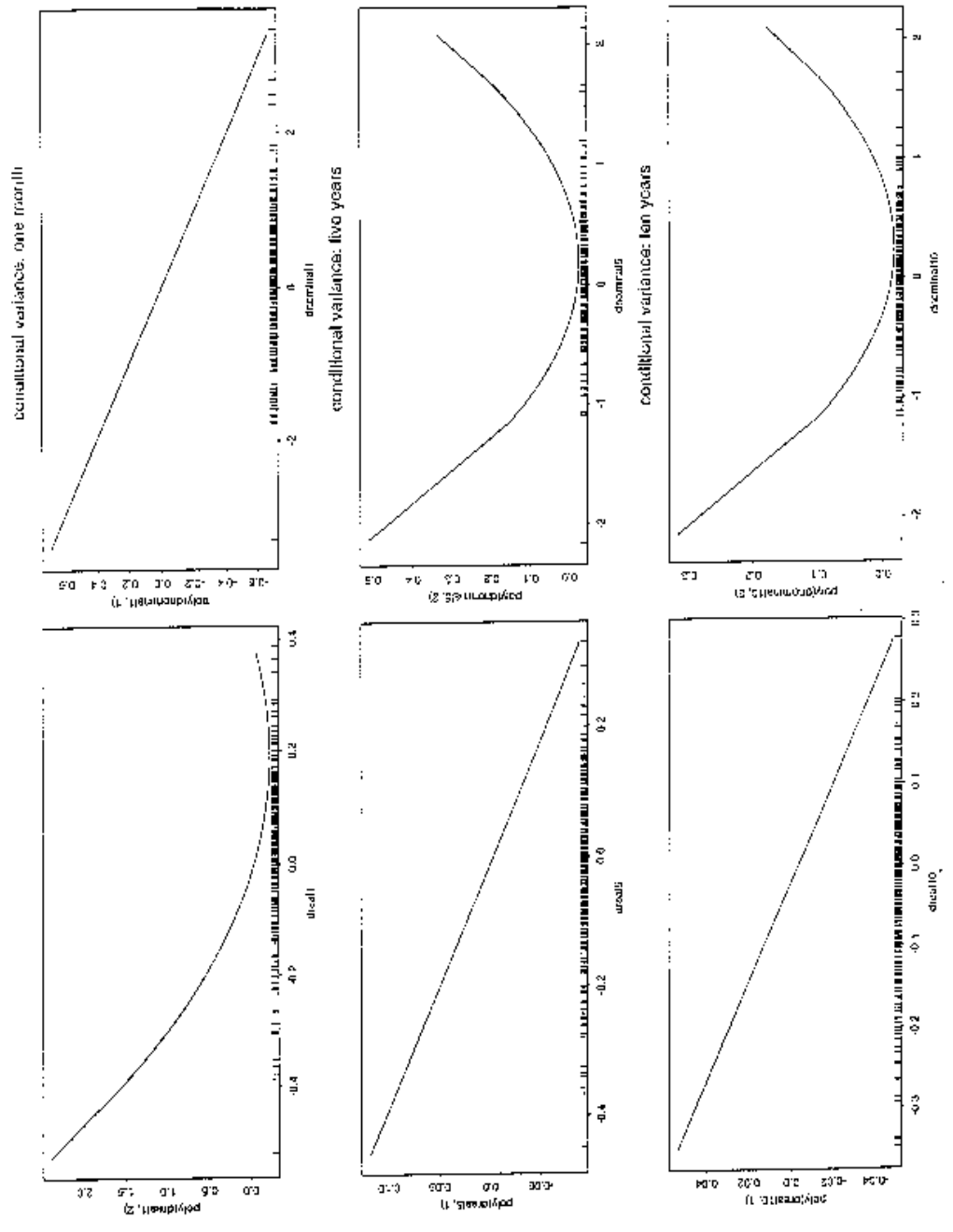


Figure 3: Implied Response Functions in Conditional Variance

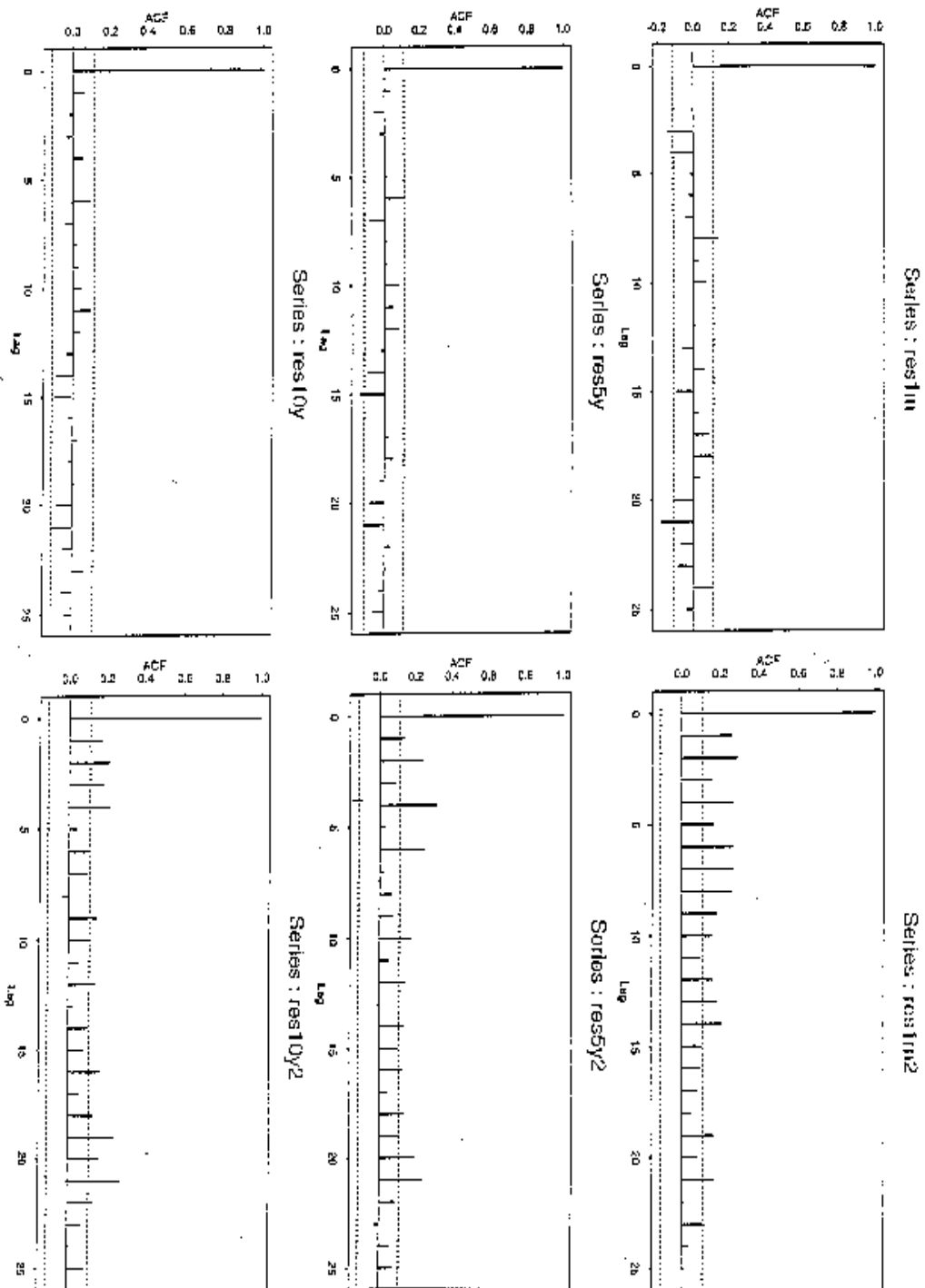


Figure 4: Autocorrelation Functions Condition Mean and Variance Regression

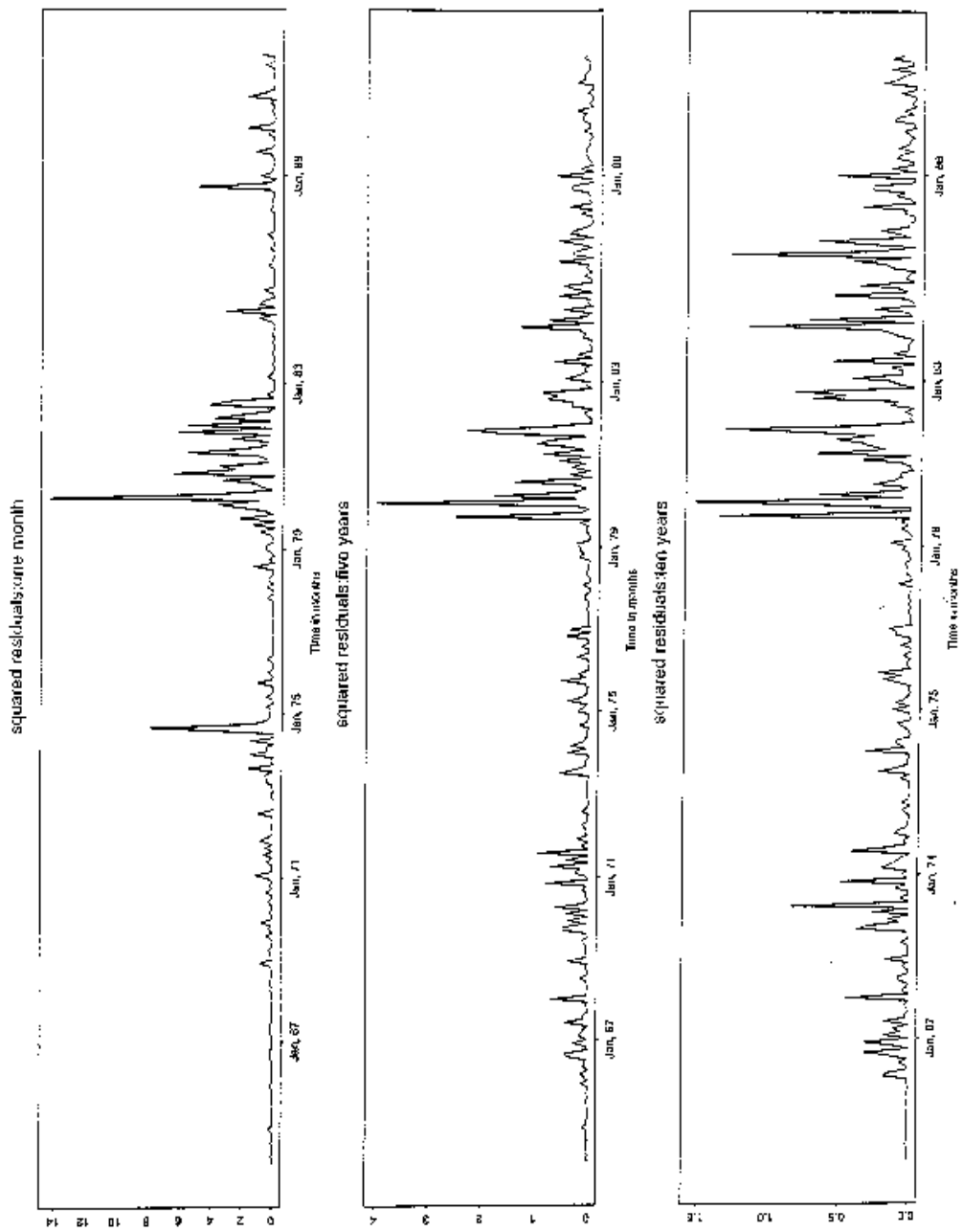


Figure 5: Squared Residuals of Mean Regression Models

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