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Competitive equilibria with consumption possibility depending on endowments : a global analysis

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Competitive equilibria with consumption possibility depending on endowments: a global analysis ⋆

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Abstract

In the spirit of Smale's work, we consider a pure exchange economy with general consumption sets. We consider the case in which the consumption set of each household is described in terms of an inequality on a function called *possibility function*. The possibility function represents the restricted consumption possibility on commodity markets. The main innovation comes from the dependency of the possibility function with respect to the individual initial endowment. We prove that, generically, equilibria are finite and they locally depend on the initial endowments in a smooth manner.

JEL classification: C62, D11, D50.

Key words: General economic equilibrium, consumption sets, regular economies.

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1 Introduction

The objective of the paper is to generalize a result of the Debreu (1970) type to exchange economies with general consumption sets. As in Smale (1974.b), the consumption sets are described by possibility functions (see also del Mercato, 2004). We consider the case where the possibility functions depend on individual initial endowments.

Under generic conditions, equilibria are finite and they locally depend on "fundamental" parameters of the economy in a smooth manner for rather general consumption sets, replacing the usual positive orthant as commodity space.[3]

In Smale (1974.b), the consumption sets are described in terms of functions, this idea is usual in the global approach of equilibrium analysis for production economies. We call *possibility function* a function describing a consumption set. Indeed, in Debreu (1959), each household has to choose a consumption in his consumption set defined as the set of all *possible* consumptions for him.

The possibility function represents the restricted consumption possibility on commodity markets. Many different authors have considered *restrictions* on the markets, Balasko, Cass and Siconolfi (1990), Polemarchakis and Siconolfi (1997), Cass, Siconolfi and Villanacci (2001). But, the difference with our approach is that in these papers the restriction is on portfolio markets but not on commodity markets.

Our analysis encompasses the case in which the possibility function of each household depends on his initial endowment. Indeed, if one considers the individual initial endowment as an *indicator of social class*, it affects our consumption possibility (but not necessarily our preferences). Our *social status* clearly affects our knowledge and our information on the existence of some available goods. Since the possibility function represents the restricted consumption possibility on commodity markets, it is natural to assume the dependency on the individual initial endowment. For example, the accessibility to the commodity markets is a *measure of the development degree* of a country which clearly depends on its resources.

Moreover, as Mas-Colell and Smale have pointed out (see Smale, 1974.b), the case in which the consumption set of each household depends on his initial endowment becomes important when consumptions are on the boundary of the consumption sets,

[3] For other works where households do not necessarily consume every positive quantity of every commodity, see Smale (1974.a), Villanacci (1993), and Bonnisseau and Rivera (2003).

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"Perhaps it would make more sense to allow grater latitude for the initial allocation in the definition of economy."

A simple example of consumption constraint depending on the individual initial endowment arises when there is no purchase in good 1 for household h. If x_h^1 denotes the consumption of good 1 of household h, and e_h^1 denotes the endowment of good 1 owned by household h, then the consumption set of household h is determined by the following constraint

$$x_h^1 \leq e_h^1 \tag{1}$$

In this paper, we consider a pure exchange economy with a finite number of households. Each household is characterized by an initial endowment of commodities, a *possibility function* and a utility function. The possibility function of each household depends on his initial endowment.

Taking prices and initial endowment as given, each household maximizes his utility function in his consumption set under his budget constraint. The definition of competitive equilibrium is the natural one.

Most related to our model, del Mercato (2004) proved the existence of equilibria for a pure exchange economy in which the possibility function of each household depends on his initial endowment and on the consumption choices of others.

The main contribution of the paper is that, generically, the economies are *regular*. As in Debreu (1970), Smale (1974.a, 1974.b), and Mas-Colell (1985), equilibria are generically finite and they locally depend on "fundamental" parameters of the economy in a smooth manner. To prove this result we follow the strategy laid out by Cass, Siconolfi and Villanacci (2001), and Villanacci et al. (2002).

To get existence of equilibria, in Assumptions 1 and 2, we take back the assumptions made by del Mercato (2004).

The regularity result holds when, at equilibrium, all agents are in the interior of their consumption sets, which is always true in the already studied case without possibility functions.

Since nothing prevents the equilibrium allocations to be on the boundary of the consumption sets, the proof of the generic regularity result becomes rather technical and difficult. Before, it shall be show that, generically, the equilibrium function is differentiable at each equilibrium allocation.

We want to encompass the case analyzed by Smale (1974.b) in which the possibility function does not depend on initial endowment, and the case in which

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it depends only on the net trade.[4] Then, we posit two abstract assumptions, namely Assumptions 12 and 14. It is easily shown that Assumption 12 covers the two situations mentioned above. Assumption 14 covers the second one when, at equilibrium, all consumption constraints are binding and all possibility functions depend on initial endowments.

In the very particular case in which, at equilibrium, all consumption constraints are binding, all possibility functions depend on initial endowments, and at most one agent does not have a possibility function depending on the net trade, Assumption 15 is needed. Observe that this assumption is in the same spirit of the Smale Assumption (see Remark 6). Actually, Assumption 15 deals with the case in which the allocation of a consumer does not satisfy strict complementarity. From Remark 16, we can deduce that Assumption 15 means that, at equilibrium, there exists a bilateral costless transfer on endowments such that the allocations lie in the interior of the consumption sets after transfers.

The paper is organized as follows. In Section 2, we describe the basic elements of our model. In Section 3, the concept of competitive equilibrium is further analyzed. We characterize equilibria in terms of equilibrium function, using first order conditions for household's maximization problem. Then, we present Theorem 9 which states the non-emptiness and compactness of the equilibrium set. In Section 4, we state the definition of regular economy and the main result of the paper, Theorem 17, which states that, generically in the initial endowments space, the economies are regular. In Subsection 4.1, first we give the definition of *border line case* (i.e., a situation in which, at equilibrium, a consumption is on the boundary of the consumption set and the associated Lagrange multiplier vanishes). Second, Proposition 21 establishes that, generically in the initial endowments space, border line cases do not occur. In Subsection 4.2, the strategy of the proof for Theorem 17 is detailed. Especially as a first step, we deduce from Proposition 21 that the equilibrium function is generically differentiable at each equilibrium allocation. All the proofs are gathered in Appendix.

2 The model and the assumptions

There are $C < \infty$ physical commodities or goods. Good c is denoted by superscript c. The commodity space is $\mathbb{R}^C_{++}$. There are $H < \infty$ households or consumers labeled by subscript $h \in \mathcal{H} := \{1, ..., H\}$. Each household $h \in \mathcal{H}$ is characterized by an initial endowment of goods, a consumption set described

[4] Observe that (1) is a simple example of consumption constraint depending on the net trade.

by a *possibility function*, and preferences described by a utility function. The possibility function of household h depends on his initial endowment.

The notations are summarized below.

- x_h^c is the consumption of commodity c by household h;
  $x_h := (x_h^1, .., x_h^c, .., x_h^C)$; $x := (x_h)_{h \in \mathcal{H}} \in \mathbb{R}_{++}^{CH}$.
- e_h^c is the endowment of commodity c owned by household h;
  $e_h := (e_h^1, .., e_h^c, .., e_h^C) \in \mathbb{R}_{++}^C$; $e := (e_h)_{h \in \mathcal{H}} \in \mathbb{R}_{++}^{CH}$.
- As in general equilibrium model à la Arrow–Debreu, each household $h \in \mathcal{H}$ has to choose a consumption in his consumption set, i.e. in the set of all *possible* consumption plans for him. In our paper, the consumption set of household h is described in terms of a function χ_h. Hence we call χ_h *possibility function*. Moreover, the possibility function of household h depends on his initial endowment. That is, given $e_h \in \mathbb{R}_{++}^C$, the consumption set of the household h is the following set,

$$X_h(e_h) = \left\{ x_h \in \mathbb{R}_{++}^C \ : \ \chi_h(x_h, e_h) \geq 0 \right\}$$

  where χ_h is a function from $\mathbb{R}_{++}^C \times \mathbb{R}_{++}^C$ to $\mathbb{R}$ called possibility function of household h.
- Each household $h \in \mathcal{H}$ has preferences described by a utility function u_h from $\mathbb{R}_{++}^C$ to $\mathbb{R}$, and $u_h(x_h) \in \mathbb{R}$ is the utility of household h associated with the consumption $x_h \in \mathbb{R}_{++}^C$.
- $\mathcal{E} := (e_h, \chi_h, u_h)_{h \in \mathcal{H}}$ is an economy.
- p^c is the price of one unit of commodity c; $p := (p^1, .., p^c, .., p^C) \in \mathbb{R}_{++}^C$.
- Given $w = (w^1, .., w^c, .., w^C) \in \mathbb{R}^C$, we denote

$$w^{\setminus} := (w^1, .., w^c, .., w^{C-1}) \in \mathbb{R}^{C-1}$$

We make the following assumptions on $(\chi_h, u_h)_{h \in \mathcal{H}}$. The assumptions on the utility functions are *quite* standard in "smooth" general equilibrium models. The most interesting innovation leads in the additional assumptions on the possibility functions, since nothing prevents the equilibrium allocations to be on the boundary of the consumption sets. [5]

Assumption 1 *For all $h \in \mathcal{H}$,*

(1) u_h is a C^2 function,
(2) u_h is differentiably strictly increasing, i.e.

$$\forall x'_h \in \mathbb{R}_{++}^C, D_{x_h} u_h(x'_h) \gg 0$$

[5] Observe that to get the existence and compactness results stated in Theorem 9, del Mercato (2004) requires each utility function to be C^1, and each possibility function to be C^1 with respect to the individual consumption. We need C^2 utility and possibility functions in Section 4.

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(3) u_h *is differentiably strictly quasi-concave, i.e.*

$$\forall y \in \mathbb{R}^C \backslash \{0\} \text{ and } \forall x'_h \in \mathbb{R}^C_{++} \text{ s.t.}$$
$$D_{x_h} u_h(x'_h) y = 0, \text{ we have } y^T D^2_{x_h} u_h(x'_h) y < 0, \text{ and}$$

(4) for each $u \in \operatorname{Im} u_h$, $cl_{\mathbb{R}^C}\{x_h \in \mathbb{R}^C_{++} : u_h(x_h) \geq u\} \subseteq \mathbb{R}^C_{++}$.

Assumption 2 *For all* $h \in \mathcal{H}$,

(1) the function χ_h *is a* C^2 *function,*
(2) (Convexity of the consumption set) for each $e_h \in \mathbb{R}^C_{++}$, *the function* $\chi_h(\cdot, e_h)$ *is quasi-concave,*[6]
(3) (Non-empty intersection with the budget set) for each $e_h \in \mathbb{R}^C_{++}$, *there exists* $\widetilde{x}_h \in \mathbb{R}^C_{++}$ *such that* $\chi_h(\widetilde{x}_h, e_h) > 0$ *and* $\widetilde{x}_h \ll e_h$,
(4) (Non-satiation) for each $e_h \in \mathbb{R}^C_{++}$, $x'_h \in \mathbb{R}^C_{++}$ *and* $\chi_h(x'_h, e_h) = 0$ *imply*

$$a)\ D_{x_h}\chi_h(x'_h, e_h) \neq 0, \text{ and}$$
$$b)\ D_{x_h}\chi_h(x'_h, e_h) \notin -\mathbb{R}^C_{++}$$

(5) (Global desirability) $(x', e) \in \mathbb{R}^{CH}_{++} \times \mathbb{R}^{CH}_{++}$ *and* $\chi_h(x'_h, e_h) = 0$ *for every* $h \in \mathcal{H}$, *imply*

$$\forall\, c \in \{1, ..., C\},\ \exists\, h(c) \in \mathcal{H} \text{ s.t. } D_{x^c_{h(c)}}\chi_{h(c)}(x'_{h(c)}, e_{h(c)}) \geq 0$$

Since χ_h is C^1 and quasi-concave (see Assumptions 2.1 and 2.2), as a consequence of Assumptions 2.3 and 2.4 we get the following proposition. It will play a fundamental role in the result of generic regularity (in particular see the proof of Lemmas 20 and 23). In the following section, we just observe that, at optimum, Proposition 3 implies that an analogous condition to the Smale Assumption holds (see Remark 6).

Proposition 3 *Let* $(x'_h, e_h, p) \in \mathbb{R}^C_{++} \times \mathbb{R}^C_{++} \times \mathbb{R}^C_{++}$, *such that*

$$\chi_h(x'_h, e_h) = 0 \text{ and } p(x'_h - e_h) = 0$$

Then, p *and* $D_{x_h}\chi_h(x'_h, e_h)$ *are linearly independent.*

[6] Since χ_h is C^2, we have that for each $y \in \mathbb{R}^C$ and for each $x'_h \in \mathbb{R}^C_{++}$ such that $D_{x_h}\chi_h(x'_h)y = 0$, we have $yD^2_{x_h}\chi_h(x'_h, e_h)y^T \leq 0$.

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3 Competitive equilibria

The purpose of this section is to adapt to the previous model the standard competitive equilibrium concept usually defined for a pure exchange economy. First, we give the household h's maximization problem, market clearing conditions and the definition of competitive equilibrium. Then, we characterize the solution of household h's maximization problem in terms of first order necessary and sufficient conditions (see Proposition 5). Therefore, we restate equilibria in terms of solutions of equations (see Remark 8), from which we deduce the *equilibrium function* F (see (5)). Finally, we have the theorem stating the non-emptiness and the compactness of the equilibrium set (see Theorem 9).

Without loss of generality, we choose commodity C as the *numeraire good*. Then, given $p^{\backslash} \in \mathbb{R}^{C-1}_{++}$ with innocuous abuse of notation we denote

$$p := (p^{\backslash}, 1) \in \mathbb{R}^{C}_{++}$$

Household h's maximization problem is defined as follows. For any given $\mathcal{E}$ and $p^{\backslash} \in \mathbb{R}^{C-1}_{++}$,

$$\begin{aligned} &\max_{x_h \in \mathbb{R}^C_{++}} \; u_h(x_h) \\ &s.t. \\ &\chi_h(x_h, e_h) \geq 0 \\ &-p(x_h - e_h) \geq 0 \end{aligned} \tag{2}$$

We say that $x = (x_h)_{h \in \mathcal{H}}$ satisfies market clearing conditions if

$$\sum_{h \in \mathcal{H}} x_h = \sum_{h \in \mathcal{H}} e_h \tag{3}$$

That is, if the aggregate consumption is equal to the total resources associated with e.

Definition 4 *$(x, p^{\backslash}) \in \mathbb{R}^{CH}_{++} \times \mathbb{R}^{C-1}_{++}$ is a competitive equilibrium for $\mathcal{E}$ if*

- *for all $h \in \mathcal{H}$, x_h solves problem (2) at $p^{\backslash}$, and*
- *x satisfies market clearing conditions (3).*

Proposition 5 *For each economy $\mathcal{E}$ and $p^{\backslash} \in \mathbb{R}^{C-1}_{++}$, a solution to problem (2) exists and is unique. x_h is the solution to problem (2) if and only if there exists $(\lambda_h, \mu_h) \in \mathbb{R}_{++} \times \mathbb{R}$ such that (x_h, λ_h, μ_h) is the unique solution to the*

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following system at $p^\backslash$.

$$\begin{cases} (h.1)\ D_{x_h}u_h(x_h) - \lambda_h p + \mu_h D_{x_h}\chi_h(x_h, e_h) = 0 \\ (h.2)\ -p(x_h - e_h) = 0 \\ (h.3)\ \min\{\mu_h, \chi_h(x_h, e_h)\} = 0 \end{cases} \tag{4}$$

Proposition 5 is a particular case of results obtained by del Mercato (2004). Then, we omit the proofs.

In the following remark we just observe that at optimum an analogous condition to the Smale Assumption holds (see *NCP Hypothesis*, Smale, 1974.b).

Remark 6 *For a given economy $\mathcal{E}$ and $p^\backslash \in \mathbb{R}^{C-1}_{++}$, let be x_h the solution to problem (2). From Propositions 3 and 5, and Assumptions 1.2 and 2.4, the following property holds.*

(Smale's Assumption) $\chi_h(x_h, e_h) = 0$ implies that $D_{x_h}u_h(x_h)$ and $D_{x_h}\chi_h(x_h, e_h)$ are linearly independent.

Remark 7 *From now on, we take for fixed $(\chi_h, u_h)_{h\in\mathcal{H}}$. Then, an economy is completely described by an element $e = (e_h)_{h\in\mathcal{H}} \in \mathbb{R}^{CH}_{++}$.*

Define the set of endogenous variables as $\Xi := \left(\mathbb{R}^C_{++} \times \mathbb{R}_{++} \times \mathbb{R}\right)^H \times \mathbb{R}^{C-1}_{++}$, with generic element $\xi := (x, \lambda, \mu, p^\backslash) := ((x_h, \lambda_h, \mu_h)_{h\in\mathcal{H}}, p^\backslash)$. We can now describe *extended equilibria* using the system of Kuhn-Tucker conditions (4) and market clearing conditions (3). Observe that, from Definition 4 and Proposition 5, the market clearing condition for good C is "redundant" (see equations $(h.2)_{h\in\mathcal{H}}$ in (4)). Therefore, in the following remark we omit in (3) the condition for good C.

Remark 8 *$\xi = (x, \lambda, \mu, p^\backslash) \in \Xi$ is an extended competitive equilibrium for $e \in \mathbb{R}^{CH}_{++}$ if and only if*

- *for all $h \in \mathcal{H}$, (x_h, λ_h, μ_h) solves system (4) at $p^\backslash$, and*
- *x satisfies the following market clearing conditions,*

$$\sum_{h\in\mathcal{H}} (x_h^\backslash - e_h^\backslash) = 0$$

Define the following *equilibrium function* $F : \Xi \times \mathbb{R}^{CH}_{++} \to \mathbb{R}^{\dim \Xi}$,

$$F(\xi, e) := \left(\left(F^{(h.1)}(\xi, e), F^{(h.2)}(\xi, e), F^{(h.3)}(\xi, e)\right)_{h\in\mathcal{H}}, F^{(M)}(\xi, e)\right) \tag{5}$$

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where for each $h \in \mathcal{H}$,

$$F^{(h.1)}(\xi, e) := D_{x_h} u_h(x_h) - \lambda_h p + \mu_h D_{x_h} \chi_h(x_h, e_h)$$

$$F^{(h.2)}(\xi, e) := -p(x_h - e_h)$$

$$F^{(h.3)}(\xi, e) := \min \{\mu_h, \chi_h(x_h, e_h)\}$$

and

$$F^{(M)}(\xi, e) := \sum_{h \in \mathcal{H}} (x_h^{\setminus} - e_h^{\setminus})$$

Moreover, define the following function

$$F_e : \xi \in \Xi \to F_e(\xi) := F(\xi, e) \in \mathbb{R}^{\dim \Xi}$$

Therefore, from Remark 8 and the above definitions, we have that $\xi \in \Xi$ is an extended equilibrium for $e \in \mathbb{R}^{CH}_{++}$ if and only if $F(\xi, e) = F_e(\xi) = 0$. With innocuous abuse of terminology, we will call ξ simply an equilibrium.

The following theorem states the non-emptiness and the compactness of the equilibrium set. Theorem 9 is a particular case of the existence and compactness results obtained by del Mercato (2004). Then, we omit the proof.

Theorem 9 *For each $e \in \mathbb{R}^{CH}_{++}$, the set $F_e^{-1}(0)$ it is not empty and it is compact.*

4 Regular economies

As in Debreu (1970), Smale (1974.a, 1974.b), and Mas-Colell (1985), we show that, generically, equilibria are finite and they locally depend on exogenous variables in a smooth manner. To prove this result we follow the strategy laid out by Cass, Siconolfi and Villanacci (2001), and Villanacci et al. (2002).

Observe that for each $h \in \mathcal{H}$ the function

$$F^{(h.3)}(\xi, e) = \min \{\mu_h, \chi_h(x_h, e_h)\}$$

is not everywhere differentiable. We have to take in account this feature in the definition of *regular economy*.

Definition 10 *$e \in \mathbb{R}^{CH}_{++}$ is a regular economy if*

(1) for each $\xi^* \in F_e^{-1}(0)$, F_e is differentiable in ξ^*, and
(2) 0 is a regular value for F_e, which means that for each $\xi^* \in F_e^{-1}(0)$, $D_\xi F_e(\xi^*)$ is onto.

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From Theorem 9, Corollary 26 in the Appendix, and the implicit function theorem for boundaryless manifolds, we get the following properties of regular economies.

Proposition 11 *If $e \in \mathbb{R}^{CH}_{++}$ is a regular economy, then*

(1) $F_e^{-1}(0)$ is a finite set;
(2) for each $\xi^* \in F_e^{-1}(0)$, $D_\xi F_e(\xi^*)$ is onto;
(3) for each $\xi^* \in F_e^{-1}(0)$, there exists an open set $U \subseteq \mathbb{R}^{CH}_{++}$, an open set $V \subseteq \mathbb{R}^{\dim \Xi}$ and a unique function $g : U \to V$ such that $(\xi^*, e) \in V \times U$, g is C^1, $g(e) = \xi^*$ and for every $e' \in U$, $F\left(g(e'), e'\right) = 0$.

The already known results without possibility functions shows that the differentiability and regularity results hold when, at equilibrium, all agents are in the interior of their consumption sets. But, since nothing prevents the equilibrium allocations to be on the boundary of the consumption sets, from now on, we make additional assumptions on $(\chi_h, u_h)_{h \in \mathcal{H}}$. We want to encompass the case analyzed by Smale (1974.b) in which the possibility function does not depend on initial endowment, and the case in which it depends only on the net trade. Then, we posit two abstract assumptions, namely Assumptions 12 and 14.

Assumption 12 *For all $h \in \mathcal{H}$, $(x_h^*, e_h^*) \in \mathbb{R}^C_{++} \times \mathbb{R}^C_{++}$ and $\chi_h(x_h^*, e_h^*) = 0$ imply that there exist an open neighborhood N^* of (x_h^*, e_h^*) in $\mathbb{R}^C_{++} \times \mathbb{R}^C_{++}$ and $\gamma_h^* \in \mathbb{R}$ such that*

$$D_{e_h} \chi_h(x'_h, e'_h) = \gamma_h^* D_{x_h} \chi_h(x'_h, e'_h),\ \forall\ (x'_h, e'_h) \in N^*$$

As consequence of Assumption 12, we get

$$D^2_{x_h e_h} \chi_h(x_h^*, e_h^*) = \gamma_h^* D^2_{x_h} \chi_h(x_h^*, e_h^*) \tag{6}$$

Remark 13 *Observe that*

- *if the possibility function χ_h does not depend on the individual initial endowment, then Assumption 12 is satisfied with $\gamma_h^* = 0$;*
- *if the possibility function χ_h depends on the net trade, i.e. $\chi_h(x_h, e_h) := \widetilde{\chi}_h(x_h - e_h)$ where $\widetilde{\chi}_h : \mathbb{R}^C \to \mathbb{R}$ is a differentiable function, then Assumption 12 is satisfied with $\gamma_h^* = -1$.*

From now on, γ_h^* is the real number obtained by Assumption 12.

Assumption 14 *Let $(x^*, e^*) \in \mathbb{R}^{CH}_{++} \times \mathbb{R}^{CH}_{++}$ such that $\chi_h(x_h^*, e_h^*) = 0$ for every $h \in \mathcal{H}$. If $\gamma_h^* \neq 0$ for each $h \in \mathcal{H}$, then $\gamma_h^* \geq -1$ for each $h \in \mathcal{H}$.*

When, at equilibrium, all agents are on the boundary of their consumption sets and all possibility functions depend on initial endowments, one easily checks that Assumption 14 holds true if χ_h depends only on the net trade.

In the very particular case in which, at equilibrium, all consumption constraints are binding, all possibility functions depend on initial endowments, and at most one agent does not have a possibility function depending on the net trade, Assumption 15 is needed. Observe that this assumption is in the same spirit of the Smale Assumption (see Remark 6). Actually, Assumption 15 deals with the case in which the allocation of a consumer does not satisfy strict complementarity.

Assumption 15 *$H \geq 2$ and $C > 2$. Let $(x^*, e^*) \in \mathbb{R}^{CH}_{++} \times \mathbb{R}^{CH}_{++}$ such that $\chi_h(x^*_h, e^*_h) = 0$ for every $h \in \mathcal{H}$. If*

(1) $\gamma^*_h \neq 0$ for each $h \in \mathcal{H}$, and
(2) there exists $\widetilde{h} \in \mathcal{H}$, such that $\gamma^*_h = -1$ for each $h \neq \widetilde{h}$,

then, there exist k and $\widetilde{k}$ in $\mathcal{H}$, $k \neq \widetilde{k}$, such that

$$D_{x_k} u_k(x^*_k), D_{x_k} \chi_k(x^*_k, e^*_k) \text{ and } D_{x_{\widetilde{k}}} \chi_{\widetilde{k}}(x^*_{\widetilde{k}}, e^*_{\widetilde{k}})$$

are linearly independent.

Remark 16 *$H \geq 2$ and $C > 2$. Let $(\xi^*, e^*) \in F^{-1}(0)$ such that $\chi_h(x^*_h, e^*_h) = 0$ for every $h \in \mathcal{H}$. If*

(1) $\gamma^*_h \neq 0$ for each $h \in \mathcal{H}$, and
(2) there exists $\widetilde{h} \in \mathcal{H}$, such that $\gamma^*_h = -1$ for each $h \neq \widetilde{h}$,

then, Assumptions 12 and 15 imply that there exist k and $\widetilde{k}$ in $\mathcal{H}$, $k \neq \widetilde{k}$, such that

$$p^*, \; D_{e_k} \chi_k(x^*_k, e^*_k) \text{ and } D_{e_{\widetilde{k}}} \chi_{\widetilde{k}}(x^*_{\widetilde{k}}, e^*_{\widetilde{k}})$$

are linearly independent.

Note that in Remark 16, differently from Assumption 15, the price is involved as well as the derivative with respect to the initial endowments. Then, we can deduce that Assumption 15 means that, at equilibrium, there exists a bilateral costless transfer on endowments such that the allocations lie in the interior of the consumption sets after transfers.

From now on, Assumptions 1, 2, 12, 14, and 15 hold true. The main result of this section is the following theorem which states the result of generic regularity.

Theorem 17 *The set $\mathcal{R}$ of regular economies is an open and full measure subset of $\mathbb{R}^{CH}_{++}$.*

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In Subsection 4.1, first we state the definition of *border line case*. Second, Proposition 21 establishes that border line cases occur outside an open and full measure subset E of the space $\mathbb{R}^{CH}_{++}$. The proof of Proposition 21 is built upon Lemmas 19 and 20. In Subsection 4.2, the strategy of the proof for Theorem 17 is detailed. Especially as a first step, we deduce from Proposition 21 that F is differentiable in $F^{-1}(0) \cap (\Xi \times E)$. Then, we show Theorem 17 using Lemma 23 and Proposition 24.

4.1 Border line cases

First, we give the definition of *border line case*.

Definition 18 *Given $(\xi, e) \in F^{-1}(0)$, household h is at a border line case if*

$$\mu_h = \chi_h(x_h, e_h) = 0$$

The main result of this subsection is Proposition 21 stating that border line cases occur outside an open and full measure subset E of the space $\mathbb{R}^{CH}_{++}$. To construct the set E and to prove that E is open and full measure subset of $\mathbb{R}^{CH}_{++}$ we need to introduce some preliminary definitions and lemmas.

Define

$$B_h := \left\{(\xi, e) \in F^{-1}(0) : \ \mu_h = \chi_h(x_h, e_h) = 0\right\} \text{ and } B := \bigcup_{h \in \mathcal{H}} B_h$$

Observe that B_h is closed in $F^{-1}(0)$ for each $h \in \mathcal{H}$, therefore B is closed in $F^{-1}(0)$. Define also the restriction to $F^{-1}(0)$ of the projection of $\Xi \times \mathbb{R}^{CH}_{++}$ onto $\mathbb{R}^{CH}_{++}$,

$$\Phi : (\xi, e) \in F^{-1}(0) \longrightarrow \Phi(\xi, e) := e \in \mathbb{R}^{CH}_{++}$$

and

$$E := \mathbb{R}^{CH}_{++} \setminus \Phi(B)$$

Observe that, by definition, for each $(\xi, e) \in F^{-1}(0) \cap (\Xi \times E)$ and for each $h \in \mathcal{H}$, either

$$\mu_h > 0 \text{ or } \chi_h(x_h, e_h) > 0$$

We have to prove that $\Phi(B)$ is closed and of measure zero in $\mathbb{R}^{CH}_{++}$.

The closedness of $\Phi(B)$ follows from the closedness of B in $F^{-1}(0)$ and the properness of Φ obtained by the following lemma.

Lemma 19 *The function Φ is proper.*

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To show that $\Phi(B)$ is of measure zero, define

$$\mathcal{P} := \left\{ \mathcal{J} = \{\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3\} \middle| \begin{array}{l} \mathcal{H}_i \subseteq \mathcal{H},\ \forall i = 1,2,3; \mathcal{H}_1 \cup \mathcal{H}_2 \cup \mathcal{H}_3 = \mathcal{H}; \\ \mathcal{H}_i \cap \mathcal{H}_j = \emptyset,\ \forall i,j = 1,2,3, i \neq j;\ \text{and } \mathcal{H}_3 \neq \emptyset \end{array} \right\}$$

Let $\mathcal{J} = \{\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3\} \in \mathcal{P}$, for each $i = 1,2,3$ denote by $\mathcal{H}_i(\mathcal{J})$ the set $\mathcal{H}_i$ in $\mathcal{J}$, and by $|\mathcal{H}_i(\mathcal{J})|$ the number of element of $\mathcal{H}_i(\mathcal{J})$. We define

$$\Xi_{\mathcal{J}} := \mathbb{R}^{(C+1)H}_{++} \times \left(\mathbb{R}^{|\mathcal{H}_1(\mathcal{J})|+|\mathcal{H}_3(\mathcal{J})|} \times \mathbb{R}^{|\mathcal{H}_2(\mathcal{J})|}_{++}\right) \times \mathbb{R}^{(C-1)}_{++} \tag{7}$$

Observe that $\dim \Xi_{\mathcal{J}} = \dim \Xi$. Let the function

$$F_{\mathcal{J}} : \Xi_{\mathcal{J}} \times \mathbb{R}^{CH}_{++} \to \mathbb{R}^{\dim \Xi_{\mathcal{J}}}$$

$$F_{\mathcal{J}}(\xi, e) := \left(\left(F^{(h.1)}(\xi, e), F^{(h.2)}(\xi, e), F^{(h.3)}_{\mathcal{J}}(\xi, e) \right)_{h \in \mathcal{H}}, F^{(M)}(\xi, e) \right)$$

where $F_{\mathcal{J}}$ differs from F defined in (5), for the domain and for the component $F^{(h.3)}_{\mathcal{J}}$ defined below

$$F^{(h.3)}_{\mathcal{J}}(\xi, e) := \begin{cases} \mu_h \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup \mathcal{H}_3(\mathcal{J}), \\ \chi_h(x_h, e_h) \text{ if } h \in \mathcal{H}_2(\mathcal{J}) \end{cases}$$

Moreover, given $\mathcal{J} \in \mathcal{P}$ define the set

$$E_{\mathcal{J}} := \{(\xi, e) \in F^{-1}_{\mathcal{J}}(0) : \chi_h(x_h, e_h) = 0,\ \forall h \in \mathcal{H}_3(\mathcal{J})\}$$

Given an arbitrary $(\xi, e) \in B$, we can define endogenously

$$\mathcal{J}(\xi, e) := \{\mathcal{H}_1(\xi, e), \mathcal{H}_2(\xi, e), \mathcal{H}_3(\xi, e)\} \in \mathcal{P}$$

where

$$\mathcal{H}_1(\xi, e) := \{h \in \mathcal{H} :\ \mu_h = 0 \textit{ and } \chi_h(x_h, e_h) > 0\}$$
$$\mathcal{H}_2(\xi, e) := \{h \in \mathcal{H} :\ \mu_h > 0 \textit{ and } \chi_h(x_h, e_h) = 0\}$$
$$\mathcal{H}_3(\xi, e) := \{h \in \mathcal{H} :\ \mu_h = \chi_h(x_h, e_h) = 0\}$$

Therefore $(\xi, e) \in E_{\mathcal{J}(\xi,e)}$, and then we get

$$\Phi(B) \subseteq \bigcup_{\mathcal{J} \in \mathcal{P}} \Phi(E_{\mathcal{J}}) \tag{8}$$

Since the number of sets involved in the above union in finite, to show that $\Phi(B)$ is of measure zero it is enough to show that $\Phi(E_{\mathcal{J}})$ is of measure zero, for each $\mathcal{J} \in \mathcal{P}$. To show that $\Phi(E_{\mathcal{J}})$ is of measure zero for each $\mathcal{J} \in \mathcal{P}$, we need the following definitions and the following key lemma.

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Given $\mathcal{J} \in \mathcal{P}$, for each $\overline{h} \in \mathcal{H}_3(\mathcal{J})$ define the function

$$F_{\mathcal{J},\overline{h}} : \Xi_{\mathcal{J}} \times \mathbb{R}^{CH}_{++} \to \mathbb{R}^{\dim \Xi_{\mathcal{J}}+1}$$

$$F_{\mathcal{J},\overline{h}}(\xi, e) := \left(F_{\mathcal{J}}(\xi, e), F^{(\overline{h}.4)}_{\mathcal{J}}(\xi, e)\right), \text{ where } F^{(\overline{h}.4)}_{\mathcal{J}}(\xi, e) := \chi_{\overline{h}}(x_{\overline{h}}, e_{\overline{h}})$$

Moreover, for each $e \in \mathbb{R}^{CH}_{++}$, define the following function

$$F_{\mathcal{J},\overline{h},e} : \xi \in \Xi_{\mathcal{J}} \longrightarrow F_{\mathcal{J},\overline{h},e}(\xi) := F_{\mathcal{J},\overline{h}}(\xi, e) \in \mathbb{R}^{\dim \Xi_{\mathcal{J}}+1}$$

The above definition allows us to conclude that, for each $\mathcal{J} \in \mathcal{P}$, for each $\overline{h} \in \mathcal{H}_3(\mathcal{J})$ and for each $e \in \mathbb{R}^{CH}_{++}$, $F_{\mathcal{J},\overline{h}}$ and $F_{\mathcal{J},\overline{h},e}$ are differentiable on all their domain.

Lemma 20 *For each $\mathcal{J} \in \mathcal{P}$ and for each $\overline{h} \in \mathcal{H}_3(\mathcal{J})$, 0 is a regular value for $F_{\mathcal{J},\overline{h}}$.*

Then, from results of differential topology (see Theorems 25 and 27 in Appendix), given $\mathcal{J} \in \mathcal{P}$, for each $\overline{h} \in \mathcal{H}_3(\mathcal{J})$ there exists a full measure subset $\Omega_{\mathcal{J},\overline{h}}$ of $\mathbb{R}^{CH}_{++}$ such that for each $e \in \Omega_{\mathcal{J},\overline{h}}$, $F^{-1}_{\mathcal{J},\overline{h},e}(0) = \emptyset$. Given $\mathcal{J} \in \mathcal{P}$, let

$$\Omega_{\mathcal{J}} := \bigcup_{\overline{h} \in \mathcal{H}_3(\mathcal{J})} \Omega_{\mathcal{J},\overline{h}}$$

$\Omega_{\mathcal{J}}$ is a full measure subset of $\mathbb{R}^{CH}_{++}$, and we have

$$\Omega_{\mathcal{J}} \subseteq \mathbb{R}^{CH}_{++} \setminus \Phi(E_{\mathcal{J}})$$

Indeed, let $e \in \Omega_{\mathcal{J}}$, by definition we have that there exists $\overline{h} \in \mathcal{H}_3(\mathcal{J})$ such that $F^{-1}_{\mathcal{J},\overline{h},e}(0) = \emptyset$. If $e \in \Phi(E_{\mathcal{J}})$, then there exists $\xi \in \Xi_{\mathcal{J}}$ such that $\xi \in F^{-1}_{\mathcal{J},h,e}(0)$ for each $h \in \mathcal{H}_3(\mathcal{J})$, and we get a contradiction.

Therefore,

$$\Phi(E_{\mathcal{J}}) \subseteq \mathbb{R}^{CH}_{++} \setminus \Omega_{\mathcal{J}}$$

Since $\mathbb{R}^{CH}_{++} \setminus \Omega_{\mathcal{J}}$ is of measure zero, $\Phi(E_{\mathcal{J}})$ is of measure zero as well. Then, $\Phi(B)$ is of measure zero (see (8)).

Therefore, from Lemmas 19 and 20 it follows the following proposition.

Proposition 21 *There exists an open and full measure subset E of $\mathbb{R}^{CH}_{++}$ such that for each $(\xi, e) \in F^{-1}(0) \cap (\Xi \times E)$ and for each $h \in \mathcal{H}$, either*

$$\mu_h > 0 \text{ or } \chi_h(x_h, e_h) > 0$$

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4.2 Generic regularity

In this subsection we prove that Theorem 17 holds.

From now on, the set E is the open and full measure subset of $\mathbb{R}^{CH}_{++}$ obtained in Proposition 21. Observe that F is differentiable in $F^{-1}(0)\cap(\Xi\times E)$. Indeed, given $(\xi^*, e^*) \in F^{-1}(0) \cap (\Xi \times E)$, define

$$\begin{aligned} \mathcal{H}_1(\xi^*, e^*) &:= \mathcal{H}_1^* := \{h \in \mathcal{H} : \mu_h^* = 0 \text{ and } \chi_h(x_h^*, e_h^*) > 0\} \\ \mathcal{H}_2(\xi^*, e^*) &:= \mathcal{H}_2^* := \{h \in \mathcal{H} : \mu_h^* > 0 \text{ and } \chi_h(x_h^*, e_h^*) = 0\} \end{aligned} \tag{9}$$

By Proposition 21, it follows that

$$\mathcal{H}_1^* \cup \mathcal{H}_2^* = \mathcal{H} \text{ and } \mathcal{H}_1^* \cap \mathcal{H}_2^* = \emptyset$$

Since the linear functions and the possibility functions are continuous, there exists an open neighborhood I^* of (ξ^*, e^*) in $\Xi \times E$ such that for each $(\xi, e) \in I^*$,

$$F^{(h.3)}(\xi, e) = \begin{cases} \mu_h \text{ if } h \in \mathcal{H}_1^* \\ \chi_h(x_h, e_h) \text{ if } h \in \mathcal{H}_2^* \end{cases}$$

Remark 22 *From now on, the domain of F will be $\Xi \times E$ instead of $\Xi \times \mathbb{R}^{CH}_{++}$.*

The main result of this subsection is Proposition 24. To prove Proposition 24, we need the following definitions and the following key lemma. Observe that there is a slight difference between the below definitions and the definitions given in Subsection 4.1. In Subsection 4.1, we considered the set $\mathcal{P}$ of appropriate $\{\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3\}$ with $\mathcal{H}_3 \neq \emptyset$. In this subsection, we are interested to describe the case in which $\mathcal{H}_3 = \emptyset$. Then, we define

$$\mathcal{A} := \{\mathcal{I} := \{\mathcal{H}_1, \mathcal{H}_2\} : \ \mathcal{H}_i \subseteq \mathcal{H}, \ \forall i = 1, 2; \mathcal{H}_1 \cup \mathcal{H}_2 = \mathcal{H} \text{ and } \mathcal{H}_1 \cap \mathcal{H}_2 = \emptyset\}$$

Let $\mathcal{I} = \{\mathcal{H}_1, \mathcal{H}_2\} \in \mathcal{A}$, for each $i = 1, 2$ denote by $\mathcal{H}_i(\mathcal{I})$ the set $\mathcal{H}_i$ in $\mathcal{I}$, and by $|\mathcal{H}_i(\mathcal{I})|$ the number of element of $\mathcal{H}_i(\mathcal{I})$. We define

$$\Xi_{\mathcal{I}} := \mathbb{R}^{(C+1)H}_{++} \times \left(\mathbb{R}^{|\mathcal{H}_1(\mathcal{I})|} \times \mathbb{R}^{|\mathcal{H}_2(\mathcal{I})|}_{++}\right) \times \mathbb{R}^{(C-1)}_{++} \tag{10}$$

and we observe that $\dim \Xi_{\mathcal{I}} = \dim \Xi$. We also define the following function

$$F_{\mathcal{I}} : \Xi_{\mathcal{I}} \times E \to \mathbb{R}^{\dim \Xi_{\mathcal{I}}}$$

$$F_{\mathcal{I}}(\xi, e) := \left(\left(F^{(h.1)}(\xi, e), F^{(h.2)}(\xi, e), F_{\mathcal{I}}^{(h.3)}(\xi, e)\right)_{h\in\mathcal{H}}, F^{(M)}(\xi, e)\right)$$

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where $F_{\mathcal{I}}$ differs from F defined in (5), for the domain and for the component $F_{\mathcal{I}}^{(h.3)}$ defined below

$$F_{\mathcal{I}}^{(h.3)}(\xi, e) := \begin{cases} \mu_h \text{ if } h \in \mathcal{H}_1(\mathcal{I}), \\ \chi_h(x_h, e_h) \text{ if } h \in \mathcal{H}_2(\mathcal{I}) \end{cases}$$

All the above definitions allows us to conclude that for each $\mathcal{I} \in \mathcal{A}$, $F_{\mathcal{I}}$ is differentiable on all its domain.

Lemma 23 *For each $\mathcal{I} \in \mathcal{A}$ and for each $(\xi^*, e^*) \in F_{\mathcal{I}}^{-1}(0)$, $D_{(\xi,e)}F_{\mathcal{I}}(\xi^*, e^*)$ has full row rank.*

Observe that from Remark 22, for each $(\xi^*, e^*) \in F^{-1}(0)$ we have that

- $\mathcal{I}^* := \{\mathcal{H}_1^*, \mathcal{H}_2^*\} \in \mathcal{A}$ (see 9),
- $(\xi^*, e^*) \in F_{\mathcal{I}^*}^{-1}(0)$, and
- $D_{(\xi,e)}F(\xi^*, e^*) = D_{(\xi,e)}F_{\mathcal{I}^*}(\xi^*, e^*)$.

Then, from Lemma 23 we get the following result.

Proposition 24 *$H \geq 2$ and $C > 2$. 0 is a regular value for F.*

From the above proposition and from the transversality theorem (see Theorem 27 in Appendix), there exists a full measure subset E^* of E such that for each $e \in E^*$, 0 is a regular value for F_e. Since E is a full measure subset of $\mathbb{R}_{++}^{CH}$, E^* is a full measure subset of $\mathbb{R}_{++}^{CH}$. Since the set $\mathcal{R}$ of regular economies contains E^*, $\mathcal{R}$ is a full measure subset of $\mathbb{R}_{++}^{CH}$. Moreover, from Lemma 19 and from Corollary 28 in the Appendix, it follows that $\mathcal{R}$ is an open subset of $\mathbb{R}_{++}^{CH}$.

5 Appendix

Proof of Proposition 3. Otherwise, suppose that $D_{x_h}\chi_h(x'_h, e_h) = \beta p$ with $\beta \neq 0$. Since $p \gg 0$ and $\chi_h(x'_h, e_h) = 0$, we have that $\beta > 0$ (see Assumption 2.4). Therefore, $D_{x_h}\chi_h(x'_h, e_h) \gg 0$. From Assumption 2.3, $\widetilde{x}_h \in \mathbb{R}_{++}^C$ satisfies

$$\chi_h(\widetilde{x}_h, e_h) > 0 \text{ and } \widetilde{x}_h \ll e_h \tag{11}$$

Since χ_h is C^1 and quasi-concave (see Assumptions 2.1 and 2.2),

$$\chi_h(\widetilde{x}_h, e_h) - \chi_h(x'_h, e_h) > 0$$

implies that

$$D_{x_h}\chi_h(x'_h, e_h)(\widetilde{x}_h - x'_h) \geq 0$$

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Then, from $D_{x_h}\chi_h(x'_h, e_h) \gg 0$ and (11) we get

$$D_{x_h}\chi_h(x'_h, e_h)(e_h - x'_h) > 0$$

that is $\beta p(e_h - x'_h) > 0$, contradicting $p(x'_h - e_h) = 0$. ■

Proof of Lemma 19. The proof follows the same steps as the compactness result obtained in Theorem 9. ■

Proof of Lemma 20. We have to show that for each $(\xi^*, e^*) \in F^{-1}_{\mathcal{J},\overline{h}}(0)$, $D_{(\xi,e)}F_{\mathcal{J},\overline{h}}(\xi^*, e^*)$ has full row rank.

Let $\Delta := \Big((\Delta x_h, \Delta\lambda_h, \Delta\mu_h)_{h\in\mathcal{H}}, \Delta p^\setminus, \Delta v\Big) \in \mathbb{R}^{(C+2)H} \times \mathbb{R}^{C-1} \times \mathbb{R}$. It is enough to show that

$$\Delta \cdot D_{(\xi,e)}F_{\mathcal{J},\overline{h}}(\xi^*, e^*) = 0 \Longrightarrow \Delta = 0$$

We consider two cases: Case 1. $\mathcal{H}_1(\mathcal{J}) \neq \emptyset$, and Case 2. $\mathcal{H}_1(\mathcal{J}) = \emptyset$.

Case 1. $\mathcal{H}_1(\mathcal{J}) \neq \emptyset$. Without loosing of generality we suppose $1 \in \mathcal{H}_1(\mathcal{J})$.

The computation of the partial jacobian matrix with respect to

$$\Big((x_h, \lambda_h, \mu_h)_{h\in\mathcal{H}}, e_1, e_{\overline{h}}, p^\setminus\Big)$$

is described below. To simplify the lecture of the matrices, we use the following simplified notations

- $D^2_{x_h}u_h := D^2_{x_h}u_h(x^*_h)$;
- $D_{x_h}\chi_h := D_{x_h}\chi_h(x^*_h, e^*_h)$ and $D_{e_h}\chi_h := D_{e_h}\chi_h(x^*_h, e^*_h)$;
- $D^2_{x_h}u_h, \chi_h := D^2_{x_h}u_h(x^*_h) + \mu^*_h D^2_{x_h}\chi_h(x^*_h, e^*_h)$ and $D^2_{x_h e_h}\chi_h := D^2_{x_h e_h}\chi_h(x^*_h, e^*_h)$.

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	x_1	λ_1	μ_1	$x_{\overline{h}}$	$\lambda_{\overline{h}}$	$\mu_{\overline{h}}$	$x_{h'}$	$\lambda_{h'}$	$\mu_{h'}$	e_1	$e_{\overline{h}}$	$p^{\backslash}$
$F^{(1.1)}$	$D^2_{x_1}u_1$	$-p^{*T}$	$D_{x_1}\chi_1^T$									$-\lambda_1^*[I_{C-1}\|0]^T$
$F^{(1.2)}$	$-p^*$									p^*		$-\left(x_1^{*\backslash}-e_1^{*\backslash}\right)$
$F^{(1.3)}_{\mathcal{J}}$			1									
$F^{(\overline{h}.1)}$				$D^2_{x_{\overline{h}}}u_{\overline{h}}$	$-p^{*T}$	$D_{x_{\overline{h}}}\chi_{\overline{h}}^T$						$-\lambda_{\overline{h}}^*[I_{C-1}\|0]^T$
$F^{(\overline{h}.2)}$				$-p^*$							p^*	$-\left(x_{\overline{h}}^{*\backslash}-e_{\overline{h}}^{*\backslash}\right)$
$F^{(\overline{h}.3)}_{\mathcal{J}}$						1						
$F^{(h'.1)}$							$D^2_{x_{h'}}u_{h'},\chi_{h'}$	$-p^{*T}$	$D_{x_{h'}}\chi_{h'}^T$			$-\lambda_{h'}^*[I_{C-1}\|0]^T$
$F^{(h'.2)}$							$-p^*$					$-\left(x_{h'}^{*\backslash}-e_{h'}^{*\backslash}\right)$
$F^{(h'.3)}_{\mathcal{J}}$							$D_{x_{h'}}\chi_{h'}$					
F^M	$[I_{C-1}\|0]$			$[I_{C-1}\|0]$			$[I_{C-1}\|0]$			$-[I_{C-1}\|0]$	$-[I_{C-1}\|0]$	
$F^{(\overline{h}.4)}_{\mathcal{J}}$				$D_{x_{\overline{h}}}\chi_{\overline{h}}$							$D_{e_{\overline{h}}}\chi_{\overline{h}}$	

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The partial system $\Delta \cdot D_{(\xi,e)} F_{\mathcal{J},\overline{h}}(\xi^*, e^*) = 0$ is written in detail below.

$$\begin{cases} \Delta x_h D^2_{x_h} u_h(x^*_h) - \Delta\lambda_h p^* + \Delta p^{\setminus}[I_{C-1}|0] = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup (\mathcal{H}_3(\mathcal{J}) \setminus \{\overline{h}\}) \\ \Delta x_{\overline{h}} D^2_{x_{\overline{h}}} u_{\overline{h}}(x^*_{\overline{h}}) - \Delta\lambda_{\overline{h}} p^* + \Delta p^{\setminus}[I_{C-1}|0] + \Delta v D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \Delta x_{h'} \left[D^2_{x_{h'}} u_{h'}(x^*_{h'}) + \mu^*_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})\right] - \Delta\lambda_{h'} p^* + \\ \Delta\mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) + \Delta p^{\setminus}[I_{C-1}|0] = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ -\Delta x_h p^{*T} = 0 \text{ for each } h \in \mathcal{H} \\ \Delta x_h D_{x_h} \chi_h(x^*_h, e^*_h)^T + \Delta\mu_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup \mathcal{H}_3(\mathcal{J}) \\ \Delta x_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})^T = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ \Delta\lambda_1 p^* - \Delta p^{\setminus}[I_{C-1}|0] = 0 \\ \Delta\lambda_{\overline{h}} p^* - \Delta p^{\setminus}[I_{C-1}|0] + \Delta v D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \sum_{h\in\mathcal{H}} \lambda^*_h \Delta x^{\setminus}_h + \sum_{h\in\mathcal{H}} \Delta\lambda_h (x^{*\setminus}_h - e^{*\setminus}_h) = 0 \end{cases}$$

Since $p^{*C} = 1$, we get $\Delta\lambda_1 = 0$ and $\Delta p^{\setminus} = 0$. Therefore, the above system becomes the following one

$$\begin{cases} \Delta x_h D^2_{x_h} u_h(x^*_h) - \Delta\lambda_h p^* = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup (\mathcal{H}_3(\mathcal{J}) \setminus \{\overline{h}\}) \\ \Delta x_{\overline{h}} D^2_{x_{\overline{h}}} u_{\overline{h}}(x^*_{\overline{h}}) - \Delta\lambda_{\overline{h}} p^* + \Delta v D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \Delta x_{h'} \left[D^2_{x_{h'}} u_{h'}(x^*_{h'}) + \mu^*_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})\right] - \Delta\lambda_{h'} p^* + \\ \Delta\mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ -\Delta x_h p^{*T} = 0 \text{ for each } h \in \mathcal{H} \\ \Delta x_h D_{x_h} \chi_h(x^*_h, e^*_h)^T + \Delta\mu_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup \mathcal{H}_3(\mathcal{J}) \\ \Delta x_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})^T = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ \Delta\lambda_{\overline{h}} p^* + \Delta v D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \sum_{h\in\mathcal{H}} \lambda^*_h \Delta x^{\setminus}_h + \sum_{h\in\mathcal{H}} \Delta\lambda_h (x^{*\setminus}_h - e^{*\setminus}_h) = 0 \\ \Delta\lambda_1 = \Delta p^{\setminus} = 0 \end{cases} \quad (12)$$

From system (12), we get

$$\Delta x_h D^2_{x_h} u_h(x^*_h) \Delta x^T_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup (\mathcal{H}_3(\mathcal{J}) \setminus \{\overline{h}\}) \quad (13)$$

and

$$\Delta x_{h'} D^2_{x_{h'}} u_{h'}(x^*_{h'}) \Delta x^T_{h'} = -\mu^*_{h'} \Delta x_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) \Delta x^T_{h'} \text{ if } h' \in \mathcal{H}_2(\mathcal{J})$$

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Then, Assumption 2.2 and $\mu^*_{h'} > 0$ for each $h' \in \mathcal{H}_2(\mathcal{J})$ imply that

$$\Delta x_{h'} D^2_{x_{h'}} u_{h'}(x^*_{h'}) \Delta x^T_{h'} \geq 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \tag{14}$$

Observe that from $F_{\mathcal{J},\overline{h}}(\xi^*, e^*) = 0$ and system (12), we get

$$D_{x_h} u_h(x^*_h) \Delta x^T_h = \lambda^*_h p^* \Delta x^T_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup \mathcal{H}_3(\mathcal{J})$$

and

$$D_{x_{h'}} u_{h'}(x^*_{h'}) \Delta x^T_{h'} = \lambda^*_{h'} p^* \Delta x^T_{h'} + \mu^*_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) \Delta x^T_{h'} = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \tag{15}$$

Then, (13), (14) and Assumption 1.3 imply that

$$\Delta x_h = 0 \text{ for each } h \neq \overline{h}$$

Therefore, the relevant equations of system (12) become

$$\begin{cases} \Delta \lambda_h p^* = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup (\mathcal{H}_3(\mathcal{J}) \setminus \{\overline{h}\}) \\ \Delta x_{\overline{h}} D^2_{x_{\overline{h}}} u_{\overline{h}}(x^*_{\overline{h}}) - \Delta \lambda_{\overline{h}} p^* + \Delta v D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \Delta \lambda_{h'} p^* - \Delta \mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ \Delta x_{\overline{h}} p^{*T} = 0 \\ \Delta \mu_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{J}) \cup (\mathcal{H}_3(\mathcal{J}) \setminus \{\overline{h}\}) \\ \Delta x_{\overline{h}} D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}})^T + \Delta \mu_{\overline{h}} = 0 \\ \Delta \lambda_{\overline{h}} p^* + \Delta v D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \lambda^*_{\overline{h}} \Delta x^{\backslash}_{\overline{h}} + \sum_{h \in \mathcal{H}} \Delta \lambda_h (x^{*\backslash}_h - e^{*\backslash}_h) = 0 \\ \Delta \lambda_1 = \Delta p^{\backslash} = 0 \\ \Delta x_h = 0 \text{ for each } h \neq \overline{h} \end{cases}$$

Since $p^* \gg 0$, $\Delta \lambda_h = 0$ for each $h \in \mathcal{H}_1(\mathcal{J}) \cup (\mathcal{H}_3(\mathcal{J}) \setminus \{\overline{h}\})$. Moreover, $F_{\mathcal{J},\overline{h}}(\xi^*, e^*) = 0$ and Proposition 3 imply that $\Delta \lambda_{h'} = \Delta \mu_{h'} = 0$ for each

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$h' \in \mathcal{H}_2(\mathcal{J})$. Therefore, the above system becomes the following one

$$\begin{cases} \Delta x_{\overline{h}} D^2_{x_{\overline{h}}} u_{\overline{h}}(x^*_{\overline{h}}) - \Delta\lambda_{\overline{h}} p^* + \Delta v D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \Delta x_{\overline{h}} p^{*T} = 0 \\ \Delta x_{\overline{h}} D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}})^T + \Delta\mu_{\overline{h}} = 0 \\ \Delta\lambda_{\overline{h}} p^* + \Delta v D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \lambda^*_{\overline{h}} \Delta x^{\setminus}_{\overline{h}} + \Delta\lambda_{\overline{h}}(x^{*\setminus}_{\overline{h}} - e^{*\setminus}_{\overline{h}}) = 0 \\ \Delta p^{\setminus} = 0 \\ \Delta x_h = \Delta\lambda_h = \Delta\mu_h = 0 \text{ for each } h \neq \overline{h} \end{cases} \tag{16}$$

From Assumption 12, we know that $D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = \gamma^*_{\overline{h}} D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}})$ with $\gamma^*_{\overline{h}} \in \mathbb{R}$.

If $\gamma^*_{\overline{h}} = 0$, since $p^* \gg 0$ we get $\Delta\lambda_{\overline{h}} = 0$. Then, $\Delta x_{\overline{h}} = 0$ since $\lambda_{\overline{h}} > 0$ and $\Delta x_{\overline{h}} p^{*T} = 0$. Finally, $\Delta\mu_{\overline{h}} = 0$, and $\Delta v = 0$ since $D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) \neq 0$ (see Assumption 2.4.a).

If $\gamma^*_{\overline{h}} \neq 0$, from Proposition 3 we know that p^* and $D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}})$ are linearly independent. Then, we get $\Delta\lambda_{\overline{h}} = \Delta v = 0$, and $\Delta x_{\overline{h}} = 0$ since $\lambda_{\overline{h}} > 0$ and $\Delta x_{\overline{h}} p^{*T} = 0$. Finally, $\Delta\mu_{\overline{h}} = 0$.

Therefore, $\Delta = 0$.

Case 2. $\mathcal{H}_1(\mathcal{J}) = \emptyset$. If $\mathcal{H}_3(\mathcal{J}) \setminus \{\overline{h}\} \neq \emptyset$, without loosing of generality we can suppose $1 \in \mathcal{H}_3(\mathcal{J})$ and $1 \neq \overline{h}$. Then, the result follows as in Case 1. Therefore, we analyze the case in which $\mathcal{H}_3(\mathcal{J}) = \{\overline{h}\}$.

The computation of the partial jacobian matrix with respect to

$$\left((x_h, \lambda_h, \mu_h, e_h)_{h\in\mathcal{H}}, p^{\setminus}\right)$$

is described below.

	$x_{\overline{h}}$	$\lambda_{\overline{h}}$	$\mu_{\overline{h}}$	$x_{h'}$	$\lambda_{h'}$	$\mu_{h'}$	$e_{\overline{h}}$	$e_{h'}$	$p^{\backslash}$
$F^{(\overline{h}.1)}$	$D^2_{x_{\overline{h}}} u_{\overline{h}}$	$-p^{*T}$	$D_{x_{\overline{h}}} \chi^T_{\overline{h}}$						$-\lambda^*_{\overline{h}} [I_{C-1}\|0]^T$
$F^{(\overline{h}.2)}$	$-p^*$						p^*		$-\left(x^{*\backslash}_{\overline{h}} - e^{*\backslash}_{\overline{h}}\right)$
$F^{(\overline{h}.3)}_{\mathcal{J}}$			1						
$F^{(h'.1)}$				$D^2_{x_{h'}} u_{h'}, \chi_{h'}$	$-p^{*T}$	$D_{x_{h'}} \chi^T_{h'}$		$\mu^*_{h'} D^2_{x_{h'} e_{h'}} \chi_{h'}$	$-\lambda^*_{h'} [I_{C-1}\|0]^T$
$F^{(h'.2)}$				$-p^*$				p^*	$-\left(x^{*\backslash}_{h'} - e^{*\backslash}_{h'}\right)$
$F^{(h'.3)}_{\mathcal{J}}$				$D_{x_{h'}} \chi_{h'}$				$D_{e_{h'}} \chi_{h'}$	
F^M	$[I_{C-1}\|0]$			$[I_{C-1}\|0]$			$-[I_{C-1}\|0]$	$-[I_{C-1}\|0]$	
$F^{(\overline{h}.4)}_{\mathcal{J}}$	$D_{x_{\overline{h}}} \chi_{\overline{h}}$						$D_{e_{\overline{h}}} \chi_{\overline{h}}$		

The system $\Delta \cdot D_{(\xi,e)} F_{\mathcal{J},\overline{h}} (\xi^*, e^*) = 0$ is written in detail below.

$$\begin{cases} \Delta x_{\overline{h}} D^2_{x_{\overline{h}}} u_{\overline{h}}(x^*_{\overline{h}}) - \Delta\lambda_{\overline{h}} p^* + \Delta p^{\backslash} \left[I_{C-1}|0\right] + \Delta v D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \Delta x_{h'} \left[D^2_{x_{h'}} u_{h'}(x^*_{h'}) + \mu^*_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})\right] - \Delta\lambda_{h'} p^* + \\ \Delta\mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) + \Delta p^{\backslash} \left[I_{C-1}|0\right] = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ -\Delta x_h p^{*T} = 0 \text{ for each } h \in \mathcal{H} \\ \Delta x_{\overline{h}} D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}})^T + \Delta\mu_{\overline{h}} = 0 \\ \Delta x_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})^T = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ \Delta\lambda_{\overline{h}} p^* - \Delta p^{\backslash} \left[I_{C-1}|0\right] + \Delta v D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \Delta x_{h'} \mu^*_{h'} D^2_{x_{h'} e_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) + \Delta\lambda_{h'} p^* + \\ \Delta\mu_{h'} D_{e_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) - \Delta p^{\backslash} \left[I_{C-1}|0\right] = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ -\sum_{h\in\mathcal{H}} \lambda^*_h \Delta x^{\backslash}_h - \sum_{h\in\mathcal{H}} \Delta\lambda_h (x^{*\backslash}_h - e^{*\backslash}_h) = 0 \end{cases} \tag{17}$$

First, observe that we are in the case where, at equilibrium, all households are on the boundary of their consumption sets. Then, from Assumptions 12 and (6), for each $h \in \mathcal{H}$ we have that

$$D_{e_h} \chi_h(x^*_h, e^*_h) = \gamma^*_h D_{x_h} \chi_h(x^*_h, e^*_h) \text{ and } D^2_{x_h e_h} \chi_h(x^*_h, e^*_h) = \gamma^*_h D^2_{x_h} \chi_h(x^*_h, e^*_h) \tag{18}$$

for some $\gamma^*_h \in \mathbb{R}$. Now, we consider two possible sub-cases: in Case 2.1, we

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suppose that

$$\exists\, h \in \mathcal{H} : \gamma_h^* = 0$$

and in Case 2.2 we have that

$$\gamma_h^* \neq 0,\ \forall h \in \mathcal{H}$$

Case 2.1. If $h = \overline{h}$, $\gamma_{\overline{h}}^* = 0$. From system (17) we get $\Delta\lambda_{\overline{h}} = 0$ and $\Delta p^{\setminus} = 0$, since $p^{*C} = 1$. Then, the relevant equations of system (17) become

$$\begin{cases} \Delta x_{\overline{h}} D^2_{x_{\overline{h}}} u_{\overline{h}}(x^*_{\overline{h}}) + \Delta v D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \Delta x_{h'} \left[D^2_{x_{h'}} u_{h'}(x^*_{h'}) + \mu^*_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) \right] - \Delta\lambda_{h'} p^* + \\ \Delta\mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ -\Delta x_h p^{*T} = 0 \text{ for each } h \in \mathcal{H} \\ \Delta x_{\overline{h}} D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}})^T + \Delta\mu_{\overline{h}} = 0 \\ \Delta x_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})^T = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\ \sum_{h \in \mathcal{H}} \lambda^*_h \Delta x^{\setminus}_h + \sum_{h \neq \overline{h}} \Delta\lambda_h (x^{*\setminus}_h - e^{*\setminus}_h) = 0 \end{cases}$$

Since (14) and (15) hold, Assumption 1.3 implies that $\Delta x_{h'} = 0$ for each $h' \neq \overline{h}$. Then, Proposition 3 implies that $\Delta\lambda_{h'} = \Delta\mu_{h'} = 0$ for each $h' \neq \overline{h}$, and we get $\Delta x_{\overline{h}} = 0$ since $\lambda_{\overline{h}} > 0$ and $\Delta x_{\overline{h}} p^{*T} = 0$. Finally, $\Delta\mu_{\overline{h}} = 0$, and $\Delta v = 0$ since $D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) \neq 0$ (see Assumption 2.4.a). That is, $\Delta = 0$.

If $h \neq \overline{h}$, from (17) and (18), we get $\Delta\lambda_h = 0$ and $\Delta p^{\setminus} = 0$, since $p^{*C} = 1$. Then, using the above arguments, we get $\Delta x_{h'} = 0$ and $\Delta\lambda_{h'} = \Delta\mu_{h'} = 0$ for each $h' \neq \overline{h}$. Therefore, system (17) becomes system (16), and we get $\Delta = 0$ using the same arguments as in Case 1.

Case 2.2. In this case, $\gamma_h^* \neq 0$ for each $h \in \mathcal{H}$. From system (17), for each $h' \neq \overline{h}$ we get

$$\Delta x_{h'} \left[D^2_{x_{h'}} u_{h'}(x^*_{h'}) + \mu^*_{h'} \left(D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) + D^2_{x_{h'} e_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) \right) \right] =$$
$$-\Delta\mu_{h'} \left[D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) + D_{e_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) \right]$$

Then, from (18) we have that for each $h' \neq \overline{h}$

$$\Delta x_{h'} \left[D^2_{x_{h'}} u_{h'}(x^*_{h'}) + \mu^*_{h'}(1 + \gamma^*_{h'}) D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) \right] =$$
$$-\Delta\mu_{h'}(1 + \gamma^*_{h'}) D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})$$

By system (17), it follows that for each $h' \neq \overline{h}$

$$\Delta x_{h'} D^2_{x_{h'}} u_{h'}(x^*_{h'}) \Delta x^T_{h'} = -\mu^*_{h'}(1 + \gamma^*_{h'}) \Delta x_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) \Delta x^T_{h'}$$

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Then, Assumptions 2.2 and 14, and $\mu^*_{h'} > 0$ for each $h' \neq \overline{h}$, imply that

$$\Delta x_{h'} D^2_{x_{h'}} u_{h'}(x^*_{h'}) \Delta x^T_{h'} \geq 0 \text{ for each } h' \neq \overline{h} \tag{19}$$

Since (15) holds, (19) and Assumption 1.3 imply that

$$\Delta x_{h'} = 0 \text{ for each } h' \neq \overline{h}$$

Therefore, system (17) becomes the following one.

$$\begin{cases} \Delta x_{\overline{h}} D^2_{x_{\overline{h}}} u_{\overline{h}}(x^*_{\overline{h}}) - \Delta\lambda_{\overline{h}} p^* + \Delta p^{\setminus} [I_{C-1}|0] + \Delta v D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ -\Delta\lambda_{h'} p^* + \Delta\mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) + \Delta p^{\setminus} [I_{C-1}|0] = 0 \text{ if } h' \neq \overline{h} \\ \Delta x_{\overline{h}} p^{*T} = 0 \\ \Delta x_{\overline{h}} D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}})^T + \Delta\mu_{\overline{h}} = 0 \\ \Delta\lambda_{\overline{h}} p^* - \Delta p^{\setminus} [I_{C-1}|0] + \Delta v D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) = 0 \\ \Delta\lambda_{h'} p^* + \Delta\mu_{h'} D_{e_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) - \Delta p^{\setminus} [I_{C-1}|0] = 0 \text{ if } h' \neq \overline{h} \\ \lambda^*_{\overline{h}} \Delta x^{\setminus}_{\overline{h}} + \sum_{h \in \mathcal{H}} \Delta\lambda_h (x^{*\setminus}_h - e^{*\setminus}_h) = 0 \end{cases} \tag{20}$$

Now, we consider again two possible sub-cases: in Case 2.2.1, we suppose that

$$\exists\, (h,k) \in \mathcal{H} \times \mathcal{H},\ h \neq k : \gamma^*_h \neq -1 \text{ and } \gamma^*_k \neq -1$$

and in Case 2.2.2, we have that

$$\exists\, \widetilde{h} \in \mathcal{H} : \gamma^*_h = -1,\ \forall\, h \neq \widetilde{h}$$

Case 2.2.1. In this case, we can suppose that there exists $h \neq \overline{h}$ such that $\gamma^*_h \neq -1$. From (20) and (18), we get

$$0 = \Delta\mu_h \left[D_{x_h} \chi_h(x^*_h, e^*_h) + D_{e_h} \chi_h(x^*_h, e^*_h)\right] = \Delta\mu_h (1 + \gamma^*_h) D_{x_h} \chi_h(x^*_h, e^*_h)$$

Since $D_{x_h} \chi_h(x^*_h, e^*_h) \neq 0$ (see Assumption 2.4.a), we get $\Delta\mu_h = 0$. Then, from system (20) we get $\Delta\lambda_h = 0$ and $\Delta p^{\setminus} = 0$, since $p^{*C} = 1$. Proposition 3 implies that $\Delta\lambda_{h'} = \Delta\mu_{h'} = 0$ for each $h' \neq \overline{h}$. Therefore, system (20) becomes system (16), and we get $\Delta = 0$ using the same arguments as in Case 1.

Case 2.2.2. Since $F_{\mathcal{J},\overline{h}}(\xi^*, e^*) = 0$, from Remark 16 there exist k and $\widetilde{k}$ in $\mathcal{H}$, $k \neq \widetilde{k}$, such that

$$p^*,\ D_{e_k} \chi_k(x^*_k, e^*_k) \text{ and } D_{e_{\widetilde{k}}} \chi_{\widetilde{k}}(x^*_{\widetilde{k}}, e^*_{\widetilde{k}})$$

are linearly independent.

If k and $\widetilde{k}$ are in $\mathcal{H}_2(\mathcal{J})$, from system (20) we get

$$\Delta\lambda_k p^* + \Delta\mu_k D_{e_k}\chi_k(x_k^*, e_k^*) = \Delta\lambda_{\widetilde{k}} p^* + \Delta\mu_{\widetilde{k}} D_{e_{\widetilde{k}}}\chi_{\widetilde{k}}(x_{\widetilde{k}}^*, e_{\widetilde{k}}^*)$$

that is

$$(\Delta\lambda_k - \Delta\lambda_{\widetilde{k}})p^* + \Delta\mu_k D_{e_k}\chi_k(x_k^*, e_k^*) - \Delta\mu_{\widetilde{k}} D_{e_{\widetilde{k}}}\chi_{\widetilde{k}}(x_{\widetilde{k}}^*, e_{\widetilde{k}}^*) = 0$$

which implies $\Delta\mu_k = \Delta\mu_{\widetilde{k}} = 0$ and $\Delta\lambda_k = \Delta\lambda_{\widetilde{k}}$. Then, from system (20) we get $\Delta\lambda_k = 0$ and $\Delta p^{\setminus} = 0$, since $p^{*C} = 1$. Proposition 3 implies that $\Delta\lambda_{h'} = \Delta\mu_{h'} = 0$ for each $h' \neq \overline{h}$. Therefore, system (20) becomes system (16), and we get $\Delta = 0$ using the same arguments as in Case 1.

If $k = \overline{h}$ or $\widetilde{k} = \overline{h}$, using similar arguments, we get $\Delta = 0$. ∎

Proof of Lemma 23. Let $\Delta := \Big((\Delta x_h, \Delta\lambda_h, \Delta\mu_h)_{h\in\mathcal{H}}, \Delta p^{\setminus}\Big) \in \mathbb{R}^{(C+2)H} \times \mathbb{R}^{C-1}$. It is enough to show that

$$\Delta \cdot D_{(\xi,e)} F_{\mathcal{I}}\left(\xi^*, e^*\right) = 0 \Longrightarrow \Delta = 0$$

We consider two cases: 1. $\mathcal{H}_1(\mathcal{I}) \neq \emptyset$, and 2. $\mathcal{H}_1(\mathcal{I}) = \emptyset$.

Case 1. $\mathcal{H}_1(\mathcal{I}) \neq \emptyset$. Without loosing of generality we suppose $1 \in \mathcal{H}_1(\mathcal{I})$.

The computation of the partial jacobian matrix with respect to

$$((x_h, \lambda_h, \mu_h)_{h\in\mathcal{H}}, e_1)$$

is described below.

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	x_1	λ_1	μ_1	x_h	λ_h	μ_h	$x_{h'}$	$\lambda_{h'}$	$\mu_{h'}$	e_1
$F^{(1.1)}$	$D^2_{x_1} u_1$	$-p^{*T}$	$D_{x_1} \chi_1^T$							
$F^{(1.2)}$	$-p^*$									p^*
$F_{\mathcal{I}}^{(1.3)}$			1							
$F^{(h.1)}$				$D^2_{x_h} u_h$	$-p^{*T}$	$D_{x_h} \chi_h^T$				
$F^{(h.2)}$				$-p^*$						
$F_{\mathcal{I}}^{(h.3)}$						1				
$F^{(h'.1)}$							$D^2_{x_{h'}} u_{h'}, \chi_{h'}$	$-p^{*T}$	$D_{x_{h'}} \chi_{h'}^T$	
$F^{(h'.2)}$							$-p^*$			
$F_{\mathcal{I}}^{(h'.3)}$							$D_{x_{h'}} \chi_{h'}$			
F^M	$[I_{C-1}\|0]$			$[I_{C-1}\|0]$			$[I_{C-1}\|0]$			$-[I_{C-1}\|0]$

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The partial system $\Delta \cdot D_{(\xi,e)} F_{\mathcal{I}} \left(\xi^*, e^*\right) = 0$ is written in detail below.

$$
\begin{cases}
\Delta x_h D^2_{x_h} u_h(x^*_h) - \Delta\lambda_h p^* + \Delta p^{\backslash} \left[I_{C-1}|0\right] = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{I}) \\
\Delta x_{h'} \left[D^2_{x_{h'}} u_{h'}(x^*_{h'}) + \mu^*_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})\right] - \Delta\lambda_{h'} p^* + \\
\Delta\mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) + \Delta p^{\backslash} \left[I_{C-1}|0\right] = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{I}) \\
-\Delta x_h p^{*T} = 0 \text{ for each } h \in \mathcal{H} \\
\Delta x_h D_{x_h} \chi_h(x^*_h, e^*_h)^T + \Delta\mu_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{I}) \\
\Delta x_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})^T = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{I}) \\
\Delta\lambda_1 p^* - \Delta p^{\backslash} \left[I_{C-1}|0\right] = 0
\end{cases}
$$

Since $p^{*C} = 1$, we get $\Delta\lambda_1 = 0$ and $\Delta p^{\backslash} = 0$. Therefore, the above system becomes the following one

$$
\begin{cases}
\Delta x_h D^2_{x_h} u_h(x^*_h) - \Delta\lambda_h p^* = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{I}) \\
\Delta x_{h'} \left[D^2_{x_{h'}} u_{h'}(x^*_{h'}) + \mu^*_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})\right] - \Delta\lambda_{h'} p^* + \\
\Delta\mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{I}) \\
-\Delta x_h p^{*T} = 0 \text{ for each } h \in \mathcal{H} \\
\Delta x_h D_{x_h} \chi_h(x^*_h, e^*_h)^T + \Delta\mu_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{I}) \\
\Delta x_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})^T = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{J}) \\
\Delta\lambda_1 = \Delta p^{\backslash} = 0
\end{cases}
\tag{21}
$$

From system (21), we get

$$
\Delta x_h D^2_{x_h} u_h(x^*_h) \Delta x^T_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{I}) \tag{22}
$$

and

$$
\Delta x_{h'} D^2_{x_{h'}} u_{h'}(x^*_{h'}) \Delta x^T_{h'} = -\mu^*_{h'} \Delta x_{h'} D^2_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) \Delta x^T_{h'} \text{ if } h' \in \mathcal{H}_2(\mathcal{I})
$$

Then, Assumption 2.2 and $\mu^*_{h'} > 0$ for each $h' \in \mathcal{H}_2(\mathcal{I})$ imply that

$$
\Delta x_{h'} D^2_{x_{h'}} u_{h'}(x^*_{h'}) \Delta x^T_{h'} \geq 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{I}) \tag{23}
$$

Observe that from $F_{\mathcal{I}}(\xi^*, e^*) = 0$ and system (21), we get

$$
D_{x_h} u_h(x^*_h) \Delta x^T_h = \lambda^*_h p^* \Delta x^T_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{I})
$$

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and

$$D_{x_{h'}} u_{h'}(x^*_{h'})\Delta x^T_{h'} = \lambda^*_{h'} p^* \Delta x^T_{h'} + \mu^*_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'})\Delta x^T_{h'} = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{I}) \tag{24}$$

Then, (22), (23) and Assumption 1.3 imply that

$$\Delta x_h = 0 \text{ for each } h \in \mathcal{H}$$

Therefore, the relevant equations of system (21) become

$$\begin{cases} \Delta\lambda_h p^* = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{I}) \\ \Delta\lambda_{h'} p^* - \Delta\mu_{h'} D_{x_{h'}} \chi_{h'}(x^*_{h'}, e^*_{h'}) = 0 \text{ if } h' \in \mathcal{H}_2(\mathcal{I}) \\ \Delta\mu_h = 0 \text{ if } h \in \mathcal{H}_1(\mathcal{I}) \\ \Delta\lambda_1 = \Delta p^\backslash = 0 \\ \Delta x_h = 0 \text{ for each } h \in \mathcal{H} \end{cases}$$

Since $p^* \gg 0$, $\Delta\lambda_h = 0$ for each $h \in \mathcal{H}_1(\mathcal{I})$. Moreover, $F_{\mathcal{I}}(\xi^*, e^*) = 0$ and Proposition 3 imply that $\Delta\lambda_{h'} = \Delta\mu_{h'} = 0$ for each $h' \in \mathcal{H}_2(\mathcal{I})$. Therefore, $\Delta = 0$.

Case 2. $\mathcal{H}_1(\mathcal{I}) = \emptyset$. Then, $\mathcal{H}_2(\mathcal{I}) = \mathcal{H}$.

The computation of the partial jacobian matrix with respect to

$$(x_h, \lambda_h, \mu_h, e_h)_{h \in \mathcal{H}}$$

is described below.

	x_h	λ_h	μ_h	e_h
$F^{(h.1)}$	$D^2_{x_h} u_h, \chi_h$	$-p^{*T}$	$D_{x_h} \chi^T_h$	$\mu^*_h D^2_{x_h e_h} \chi_h$
$F^{(h.2)}$	$-p^*$			p^*
$F^{(h.3)}_{\mathcal{I}}$	$D_{x_h} \chi_h$			$D_{e_h} \chi_h$
F^M	$[I_{C-1}\|0]$			$-[I_{C-1}\|0]$

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The partial system $\Delta \cdot D_{(\xi,e)} F_{\mathcal{I}}\left(\xi^*, e^*\right) = 0$ is written in detail below.

$$\begin{cases} \Delta x_h \left[D^2_{x_h} u_h(x^*_h) + \mu^*_h D^2_{x_h} \chi_h(x^*_h, e^*_h) \right] - \Delta \lambda_h p^* + \\ \Delta \mu_h D_{x_h} \chi_h(x^*_h, e^*_h) + \Delta p^{\setminus} \left[I_{C-1} | 0 \right] = 0 \text{ for each } h \in \mathcal{H} \\ -\Delta x_h p^{*T} = 0 \text{ for each } h \in \mathcal{H} \\ \Delta x_h D_{x_h} \chi_h(x^*_h, e^*_h)^T = 0 \text{ for each } h \in \mathcal{H} \\ \Delta x_h \mu^*_h D^2_{x_h e_h} \chi_h(x^*_h, e^*_h) + \Delta \lambda_h p^* + \\ \Delta \mu_h D_{e_h} \chi_h(x^*_h, e^*_h) - \Delta p^{\setminus} \left[I_{C-1} | 0 \right] = 0 \text{ for each } h \in \mathcal{H} \end{cases}$$

We are in the case where, at equilibrium, all households are on the boundary of their consumption sets. Then, from Assumptions 12 and (6), for each $h \in \mathcal{H}$ we have that

$$D_{e_h} \chi_h(x^*_h, e^*_h) = \gamma^*_h D_{x_h} \chi_h(x^*_h, e^*_h) \text{ and } D^2_{x_h e_h} \chi_h(x^*_h, e^*_h) = \gamma^*_h D^2_{x_h} \chi_h(x^*_h, e^*_h)$$

for some $\gamma^*_h \in \mathbb{R}$.

If there is $h \in \mathcal{H}$ such that $\gamma^*_h = 0$, we get $\Delta \lambda_h = 0$ and $\Delta p^{\setminus} = 0$, since $p^{*C} = 1$. Then, (23) and (24) hold, and Assumption 1.3 implies that $\Delta x_{h'} = 0$ for each $h' \in \mathcal{H}$. Moreover, $F_{\mathcal{I}}(\xi^*, e^*) = 0$ and Proposition 3 imply that $\Delta \lambda_{h'} = \Delta \mu_{h'} = 0$ for each $h' \in \mathcal{H}$. Therefore, $\Delta = 0$.

If $\gamma^*_h \neq 0$ for each $h \in \mathcal{H}$, since

$$\Delta x_h D^2_{x_h} u_h(x^*_h) \Delta x^T_h = -\mu^*_h (1 + \gamma^*_h) \Delta x_h D^2_{x_h} \chi_h(x^*_h, e^*_h) \Delta x^T_h \text{ for each } h \in \mathcal{H}$$

Assumptions 2.2 and 14, and $\mu^*_h > 0$ imply that

$$\Delta x_h D^2_{x_h} u_h(x^*_h) \Delta x^T_h \geq 0 \text{ for each } h \in \mathcal{H}$$

Then, (24) and Assumption 1.3 implies that $\Delta x_h = 0$ for each $h \in \mathcal{H}$. Therefore, the above system becomes the following one.

$$\begin{cases} \Delta x_h = 0 \text{ for each } h \in \mathcal{H} \\ -\Delta \lambda_h p^* + \Delta \mu_h D_{x_h} \chi_h(x^*_h, e^*_h) + \Delta p^{\setminus} \left[I_{C-1} | 0 \right] = 0 \text{ for each } h \in \mathcal{H} \\ \Delta \lambda_h p^* + \Delta \mu_h D_{e_h} \chi_h(x^*_h, e^*_h) - \Delta p^{\setminus} \left[I_{C-1} | 0 \right] = 0 \text{ for each } h \in \mathcal{H} \end{cases} \quad (25)$$

If there is $\overline{h} \in \mathcal{H}$ such that $\gamma^*_{\overline{h}} \neq -1$, since

$$0 = \Delta \mu_{\overline{h}} \left[D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) + D_{e_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) \right] = \Delta \mu_{\overline{h}} (1 + \gamma^*_{\overline{h}}) D_{x_{\overline{h}}} \chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}})$$

and $D_{x_{\overline{h}}}\chi_{\overline{h}}(x^*_{\overline{h}}, e^*_{\overline{h}}) \neq 0$ (see Assumption 2.4.a), we get $\Delta\mu_{\overline{h}} = 0$. Then, from system (25) we get $\Delta\lambda_{\overline{h}} = 0$ and $\Delta p^{\setminus} = 0$, since $p^{*C} = 1$. Proposition 3 implies that $\Delta\lambda_h = \Delta\mu_h = 0$ for each $h \neq \overline{h}$. Therefore, $\Delta = 0$.

If $\gamma^*_h = -1$ for each $h \in \mathcal{H}$, since $F_{\mathcal{I}}(\xi^*, e^*) = 0$, from Remark 16 there exist k and $\widetilde{k}$ in $\mathcal{H}$, $k \neq \widetilde{k}$, such that

$$p^*,\ D_{e_k}\chi_k(x^*_k, e^*_k) \text{ and } D_{e_{\widetilde{k}}}\chi_{\widetilde{k}}(x^*_{\widetilde{k}}, e^*_{\widetilde{k}})$$

are linearly independent. From system (25) we get

$$\Delta\lambda_k p^* + \Delta\mu_k D_{e_k}\chi_k(x^*_k, e^*_k) = \Delta\lambda_{\widetilde{k}} p^* + \Delta\mu_{\widetilde{k}} D_{e_{\widetilde{k}}}\chi_{\widetilde{k}}(x^*_{\widetilde{k}}, e^*_{\widetilde{k}})$$

that is

$$(\Delta\lambda_k - \Delta\lambda_{\widetilde{k}})p^* + \Delta\mu_k D_{e_k}\chi_k(x^*_k, e^*_k) - \Delta\mu_{\widetilde{k}} D_{e_{\widetilde{k}}}\chi_{\widetilde{k}}(x^*_{\widetilde{k}}, e^*_{\widetilde{k}}) = 0$$

which implies $\Delta\mu_k = \Delta\mu_{\widetilde{k}} = 0$ and $\Delta\lambda_k = \Delta\lambda_{\widetilde{k}}$. Then, from system (25) we get $\Delta\lambda_k = 0$ and $\Delta p^{\setminus} = 0$, since $p^{*C} = 1$. Once again, Proposition 3 implies that $\Delta\lambda_h = \Delta\mu_h = 0$ for each $h \in \mathcal{H}$. Therefore, $\Delta = 0$. ■

Theorem 25 *(Regular value theorem) Let M, N be C^r manifolds of dimensions m and n, respectively. Let $f : M \longrightarrow N$ be a C^r function. Assume $r > \max\{m - n, 0\}$. If $y \in N$ is a regular value for f, then*

(1) if $m < n$, $f^{-1}(y) = \emptyset$,
(2) if $m \geq n$, either $f^{-1}(y) = \emptyset$, or $f^{-1}(y)$ is an $(m-n)$-dimensional submanifold of M.

Corollary 26 *Let M, N be C^r manifolds of the same dimension. Let $f : M \longrightarrow N$ be a C^r function. Assume $r \geq 1$. Let $y \in N$ a regular value for f such that $f^{-1}(y)$ is non-empty and compact. Then, $f^{-1}(y)$ is a finite subset of M.*

The following results is a consequence of the Sard Theorem for manifolds. See, for example Villanacci et al. (2002).

Theorem 27 *Let M, Ω and N be C^r manifolds of dimensions m, p and n, respectively. Let $f : M \times \Omega \longrightarrow N$ be a C^r function. Assume $r > \max\{m - n, 0\}$. If $y \in N$ is a regular value for f, then there exists a full measure subset Ω^* of Ω such that for any $\omega \in \Omega^*$, $y \in N$ is a regular value for f_ω, where*

$$f_\omega : \xi \in M \longrightarrow f_\omega(\xi) := f(\xi, \omega) \in N$$

Corollary 28 *Let M, Ω and N be C^r manifolds of dimensions m, p and n, respectively. Let $f : M \times \Omega \longrightarrow N$ be a C^r function. Assume $r > \max\{m -$*

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$n, 0\}$. *Let* Γ *be a full measure subset of* Ω *such that for any* $\omega \in \Gamma$, $y \in N$ *is a regular value for* f_ω. *If the projection* $\pi_\Omega : (\xi, \omega) \in f^{-1}(y) \longrightarrow \pi_\Omega(\xi, \omega) := \omega \in \Omega$ *is proper, then* Γ *is open in* Ω.

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