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Comparison of experts in the non-additive case

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Résumé

Le modèle de comparaison d'experts proposés par E. Lehrer ("Comparison of experts JME 98") est adapté dans un contexte d'incertain non modélisable par l'espérance d'utilité. Nous examinons ce que deviennent les résultats de Lehrer dans ce nouveau contexte. Contrairement à l'espérance d'utilité, il y a plusieurs manières de définir les stratégies qui permettent de comparer les experts, nous en proposons quelques une qui assurent une valeur positive à l'information.

Mots clés: Préférences non additives, experts.

Abstract

We adapt the model of comparisons of experts initiated by Lehrer ("Comparison of experts JME 98") to a context of uncertainty which cannot be modelised by expected utility. We examine the robustness of Lehrer results in this new context. Unlike expected utility, there exist several ways to define the strategies allowing to compare the experts, we propose some of them which guarantee a positive value of information.

Keywords: Non-additive preferences, experts.

JEL Classification: D80, D82.

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Comparison of experts in the non-additive case

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1 Introduction

For a few years, using non-additive theories to represent uncertainty has been the matter of a growing interest. In economic theory, more and more issues which include uncertainty, so far modeled by probability measures, have been revisited with non-additive models.

In this paper we extend the work of Ehud Lehrer "Comparison of experts" [12] in a non-additive framework. More precisely, we adopt exactly the model of Lehrer, but instead of probability measures (i.e. additive functionals), we consider Knightian uncertainty represented by non-additive functionals.

As in Lehrer, a decision maker facing uncertainty is informed about some event containing the real state of nature. Based on this information, a rational decision maker selects a strategy which maximises his functional (if such a strategy exists, which we suppose).

In case where the real functional is unknown to the decision maker, he may resort to the advice of an expert. An expert suggests a certain functional, say I', as the real one. If the decision maker takes the advice of the expert, he chooses a policy which maximises his expected utility. It may happen that more than one expert is available. We compare expert I and expert II holding two functionals I' and I'' respectively. Expert I claims that he is more knowledgeable than expert II in the following sense: whatever the utility function and the action set are, it is always better, with respect to the real functional I, to choose the optimal policy according to I' rather than according to I''.

Our mathematical result is similar but generally not quite the same as Lehrer where expert I is better than expert II if and only if I' is a convex combination of I and I'': roughly speaking when I' is smaller than I it must be a convex combination of I and I'' and when it is greater it is also a convex combination but the two coefficients of convexity, one for greater and one for smaller, may be different.

A popular non-additive functional is the Choquet integral (see Schmeidler 89 [15]) which defines an integral according to a capacity v instead of a probability measure m. We pay special attention to what becomes our result in this special case where the non-additive functionals are Choquet integrals. It turns out that expert I can be more knowledgeable than expert II but v' is not a convex combination of v and v''.

We look for situations in which "more knowledgeable" means "convex combination": that is the case for capacities which are distortions of a given probability as for Yaari's model (see Yaari 87 [18]).

As for measures, the following situation may happen: to foster the advice of expert II might be better than to foster the one of expert I, although v' is closer, in a geometric sense, to v than v''. Lehrer refers to this phenomenon as "doing the right thing for the wrong reason". Not as for measures even for non-atomic capacities, being closer, as defined by Lehrer (see theorem 2 in Lehrer [12]), does not guarantee v' to be a convex combination of v and v''.

In the last part we deal with updating the preferences: the decision maker updates his functional according to the atom of the partition which he knows to have occured. We suggest three methods which are all equivalent to the maximisation of expectation in the additive case with the Bayes rule. Let us recall that updating in the non-additive case has been a major topic in non-additive measure theory (see e.g. Gilboa and Schmeidler 93 [9], Grant, Eichberger and Kelsey [4] or Wang [17] for Choquet integral, Hanany and Klibanoff [10] for maximin expected utility, Maccheroni, Marinacci and Rustichini [14] or Lehrer [13] in a very general framework).

We propose a first method, taking into account updating, in which the D.M associates to each atom of the partition an optimal action for the functional updated according to that atom. Then we define "more knowledgeable" such as the expert I's optimal action, for each atom of each partition, is always better than the one of expert II according to the real functional updated according to the atom. That is equivalent, for each atom of each partition, to : the updated functionals satisfy the same condition as described above for maximisation of expectation. With probability and Bayesian updating when the unconditional functionals satisfy that condition, so do the updated functionals. It is no more true in the non-additive case with most of the known updating rules. However there is an exception : f-bayesianism of Gilboa and Schmeidler 93 [9] provided one uses the same f for all the functionals.

That method (comparing each atom) does not provide a "global" value which could be considered as a value of information ; we propose a unique expression to maximise, for which "more knowledgeable" is equivalent to the method described above where we consider each atom.

However that expression does not necessarily give a positive value of information, so we look for an updating rule for which maximising each updated functional gives the same strategy as the one given by the maximisation of the functional without updating. Thus it does guarantee a positive value of information. We show that it is possible with f-bayesianism provided the "good" f is selected.

2 Model

Let Ω be the set of states of nature, Σ be the $\sigma-$algebra of events on Ω. Following Aumann (74) [1], the information is modeled by a partition P i.e. a finite set of pairwise disjoint elements of Σ, the union of which is Ω. $\mathcal{P}$ denotes the set of partitions of Ω with respect to Σ. If ω occurs, the D.M. knows the atom of the partition P that contains ω. He cannot distinguish two different states in the same atom of the partition P. Let C denote an atom of P. Let A be a set of actions. We assume there is a bounded utility $u : A \times \Omega \to \mathbb{R}$. A strategy is an element of $S = \{s : \Omega \to A\}$.
If the D.M. has the information P, his strategy would be measurable with respect to P. The P-measurable strategies are called P-strategies and denoted as a set by S_P. (P, A, u) is called an information structure for (Ω, Σ).

Example 1 $\Omega = \{R, Y, B\}$, $\Sigma = \mathcal{P}(\Omega)$, $A = \{a, b\}$
The utility is given in the following table:

u	a	b
R	0	1
Y	2	1
B	1	2

The information is the following partition: $P = \{R\} \cup \{Y, B\}$.

So the D.M. has to choose among the strategies
$S_P = \{a, b, a_{/\{R\}} + b_{/\{Y,B\}}, b_{/\{R\}} + a_{/\{Y,B\}}\}$

$\mathcal{F} = \{X : \Omega \to \mathbb{R}\}$ is the set of functions measurable with respect to Σ from Ω to $\mathbb{R}$.
For $C \in \Sigma$ and $X \in \mathcal{F}$, $X_{/C}$ denotes the restriction of X to C.
Let $E \in \Sigma$, E^* denotes the indicator function of E ($\forall \omega \in \Omega$, $E^*(\omega) = 1$ if $\omega \in E$ and 0 else).
Instead of using Savage model of expected utility computed with a probability measure as Lehrer [12] did, we consider that the uncertainty is modeled by a functional I from $\mathcal{F}$ to $\mathbb{R}$. That functional is unknown to the decision maker. Note that to define a functional is equivalent to define a preference relation $\preceq$ on $\mathcal{F}$ with $X \preceq Y \Leftrightarrow I(X) \leq I(Y)$.
A well-known example, that we use, is the model of Schmeidler [15] which uses a capacity v instead of a measure m and the Choquet integral as functional.

3 Maximising the expectation

3.1 Generalities

This first method deals only with the unconditional preferences. We call it MK, for More Knowledgeable. When the D.M. gets a functional J, he selects a strategy $\overline{s}_P$ that maximises the expectation according to J:
$\overline{s}_P = \arg \max_{s_P \in S_P} J(u(s_P(\omega), \omega))$.
Expert I sells a functional I' that leads to a strategy $\overline{s}'_P$.
Expert II sells a functional I'' that leads to a strategy $\overline{s}''_P$.
The functional J will be used to denote any functional (i.e. I as well as I' or I'').

Definition 1 *Expert I is more knowledgeable than expert II if for all (P, u, A), $I(u(\overline{s}'_P(\omega), \omega)) \geq I(u(\overline{s}''_P(\omega), \omega))$ i.e. $\overline{s}'_P$ is always better than $\overline{s}''_P$ according to the real functional I for any information structure.*

Proposition 1 *The following assertions are equivalent:*
(i) Expert I is more knowledgeable than expert II.
(ii) $\forall X \in \mathcal{F}, \forall Y \in \mathcal{F}$, $I'(X) \geq I'(Y)$ and $I''(X) \leq I''(Y) \Rightarrow I(X) \geq I(Y)$.

Proof. Let us assume (i).
Let $X \in \mathcal{F}, Y \in \mathcal{F}$ such that $I'(X) \geq I'(Y)$ and $I''(X) \leq I''(Y)$.
We can consider any partition, e.g. the coarsest, $P = \{\Omega\}$.
Like in Gilboa and Lehrer, $A = \{X, Y\}$, $u(X, \omega) = X(\omega)$ and $u(Y, \omega) = Y(\omega)$. We get $\overline{s}'_P = X$ and $\overline{s}''_P = Y$. So from (i) we have $I(X) \geq I(Y)$.
Let us assume now (ii).
For any information structure (P, A, u), $X(\omega) = u\left(\overline{s}'_P(\omega), \omega\right)$ and $Y(\omega) = u\left(\overline{s}''_P(\omega), \omega\right)$. (ii) implies $I(X) \geq I(Y)$.
So expert I is more knowledgeable than expert II. ■

Remark 1 *When there are several optimal strategies, we do not precise how one is selected. We will see in the proof of proposition 2 why we do not consider it as a problem.*

Remark 2 *Proposition 1 is a kind of restatement of the model in terms of preferences. When, for two acts expert I and expert II disagree, the true functional must rank those two acts in the same order as expert I. The true functional represents preferences unknown to the D.M., the experts give to him functionals that represent preferences allowing to guess that unknown preference.*

Now we make assumptions on the functionals which will hold all along this paper.
We consider constant additive and positively homogenous functionals:
$\forall X \in \mathcal{F}$, $\forall \lambda \in \mathbb{R}^+$, $J(\lambda X) = \lambda J(X), \forall c \in \mathbb{R}$, $J(X + c) = J(X) + c$.
We have therefore:

Proposition 2 *The following assertions are equivalent:*
(i) Expert I is more knowledgeable than expert II.
(iii) $\exists \alpha \in [0,1], \exists \beta \in [0,1]$, $\forall X \in \mathcal{F}$, $I(X) \leq I'(X) \Rightarrow$
$I'(X) = \alpha I(X) + (1-\alpha) I''(X)$
and $I(X) \geq I'(X) \Rightarrow I'(X) = \beta I(X) + (1-\beta) I''(X)$.

Remark 3 *We will write* $I'(X) = \alpha I(X) + (1-\alpha) I''(X)$ *strictly if we don't have* $I'(X) = I(X) = I''(X)$.

Proof. Let assume (iii).
Let $X \in \mathcal{F}, Y \in \mathcal{F}$ such that $I'(X) \geq I'(Y)$ and $I''(X) \leq I''(Y)$ (1)
Let us prove $I(X) \geq I(Y)$.
1) Let us assume $I'(X) \geq I(X)$ (i.e. $I'(X) = \alpha I(X) + (1-\alpha)I''(X)$).
1st case: $I'(Y) \geq I(Y)$ (i.e. $I'(Y) = \alpha I(Y) + (1-\alpha)I''(Y)$).
Either $\alpha \neq 0$ and

$$I(X) = \frac{1}{\alpha}I'(X) - \frac{1-\alpha}{\alpha}I''(X) \geq \frac{1}{\alpha}I'(Y) - \frac{1-\alpha}{\alpha}I''(Y) = I(Y)$$

so $I(X) \geq I(Y)$.

Or $\alpha = 0$ and $I'(X) = I''(X)$, $I'(Y) = I''(Y)$,
so from (1) $I'(X) = I''(X) = I'(Y) = I''(Y)$.
Actually in this case we can say expert I has the same forecast than expert II, so we can say that expert I is more kowledgeable (as the "more" is not strict).
2nd case: $I'(Y) \leq I''(Y)$ (i.e. $I'(Y) = \beta I(Y) + (1-\beta)I''(Y)$).
We have $I''(Y) \leq I'(Y) \leq I(Y)$ and $I(X) \leq I'(X) \leq I''(X)$.
As $I'(X) \geq I'(Y)$ we have $I''(Y) \leq I'(Y) \leq I'(X) \leq I''(X)$.
So $I''(Y) \leq I''(X)$. And so $I''(Y) = I'(Y) = I'(X) = I''(X)$.
We are exactly in the situation as above and we can always say expert I is more knowledgeable.

2) Now, let us assume $I'(X) \leq I(X)$ (i.e. $I'(X) = \beta I(X) + (1-\beta)I''(X)$).
1st case: $I'(Y) \leq I(Y)$.
We are in the same case as above and so we have $I(X) \geq I(Y)$.
2nd case: $I'(Y) \geq I(Y)$ (i.e. $I'(Y) = \alpha I(Y) + (1-\alpha)I''(Y)$).
We have $I(Y) \leq I'(Y) \leq I''(Y)$, $I''(X) \leq I'(X) < I(X)$.
$I'(X) \geq I'(Y)$ so $I(Y) \leq I'(Y) \leq I'(X) < I(X)$.
So $I(Y) \leq I(X)$.
In any case we verify (ii).

To prove the converse, let us suppose $\overline{\text{(iii)}}$, the contrary of (iii).
1) Either $\exists Z \in \mathcal{F}, \exists \alpha \notin [0,1]$ such that $I'(Z) = \alpha I(Z) + (1-\alpha)I''(Z)$.
2) Or $\exists W \in \mathcal{F}, \exists Z \in \mathcal{F}, \exists \alpha \in [0,1]$,$\exists \beta \in [0,1]$ such that $I(Z) \leq I'(Z) \Longleftrightarrow I(W) \leq I'(W)$ and $I'(Z) = \alpha I(Z) + (1-\alpha)I''(Z)$ and
$I'(W) = \beta I(W) + (1-\beta)I''(W)$.

To prove $\overline{\text{(ii)}}$, let us construct $X \in \mathcal{F}$ and $Y \in \mathcal{F}$ such that
$I'(X) > I'(Y), I''(X) > I''(Y)$ and $I(X) < I(Y)$.

1st case: $\exists Z \in \mathcal{F}\ I'(Z) \notin [I(Z), I''(Z)]$.
Let us suppose $I'(Z)$, $I(Z)$ and $I''(Z)$ follow this ranking:
$I'(Z) < I(Z) \leq I''(Z)$.
Let f an affine function such that $f(I'(Z)) < 0$; $f(I(Z)) > 0$ and $f(I''(Z)) > 0$ with $f(x) = ax + b$.
Let $X = Z$ and $Y = (1 + a)Z + b$.
We obtain the desired inequalities: $I'(X) > I'(Y), I''(X) > I''(Y)$ and $I(X) < I(Y)$.
According to the ranking of $I'(Z)$, $I(Z)$ and $I''(Z)$ with the same method we can obtain X and Y as desired.
We obtain (ii).
2nd case: w.l.o.g. $\beta < \alpha$.
a) If $I(z) \leq I'(z) \leq I''(z)$ and so $I(w) \leq I'(w) \leq I''(w)$
Let $X = \dfrac{z - I(z)}{I''(z) - I(z)}$
$Y_1 = \dfrac{w - I(w)}{I''(w) - I(w)}$.
Note that we have necessarily $I(z) < I''(z)$ and $I(w) < I''(w)$
By straightforward computations:
$I(X) = 0$, $I''(X) = 1$ and $I'(X) = 1 - \alpha$
$I(Y_1) = 0$, $I''(Y_1) = 1$ and $I'(Y_1) = 1 - \alpha$.
$Y = Y_1 + \varepsilon$ with $0 < \varepsilon < \beta - \alpha$.
b) If $I''(z) \leq I'(z) \leq I(z)$, we make a similar reasoning.
Therefore we have the desired X and Y and so (iii) implies (ii). ■

3.2 The additive case

If all the functionals are additive and positive ($J(X + Y) = J(X) + J(Y)$ for all X and Y and $X \geq 0 \implies J(X) \geq 0$ so $J(X) = \int_\Omega X dm$ with m probability measure), we are in Lehrer's framework.
Thanks to proposition 2 we obtain an other proof of Lehrer's theorem 1

> (i) $\Longleftrightarrow m' \in [m, m'']$ where $m' \in [m, m'']$ means $\exists \alpha \in [0, 1], \forall E \in \Sigma, m'(E) = \alpha m(E) + (1 - \alpha) m''(E)$.

Proof. As we have (i) $\Longleftrightarrow$ (iii) we just have to prove that if $I(X) = \int X dm$ with m probability measure defined in (iii), $\alpha = \beta$.
Let $E \in \Sigma$, let us assume $m(E) \leq m'(E)$

then $m'(E) = \alpha m(E) + (1-\alpha)m''(E)$.
As $m(E^c) + m(E)$ and $m'(E^c) + m'(E) = 1$ then $m(E^c) \geq m'(E^c)$.
So $m'(E^c) = \beta m(E^c) + (1-\beta)m''(E^c)$
$1 = m'(\Omega) = m'(E) + m'(E^c)$
$1 = \alpha m(E) + \beta m(E^c) + (1-\alpha)m''(E) + (1-\beta)m''(E^c)$
$1 = \alpha m(E) + \beta m(E^c) + 1 - \alpha m''(E) - \beta m''(E^c)$
$0 = \alpha\,(m(E) - m''(E)) + \beta(m(E^c) - m''(E^c))$
$0 = (\alpha - \beta)(m(E) - m''(E))$
If $\alpha \neq \beta$ we obtain $m(E) = m''(E)$ and so $m(E) = m'(E) = m''(E)$. ■

Remark 4 *The theorem 1 of Lehrer [12] can be infered from proposition 1 and Farkas lemma with $n = 2$. One can consider the proposition 2 as a kind of Farkas lemma with $n = 2$ for non-additive functionals.*

3.3 The Choquet case

We suppose that all the functionals are comonotonic additive and monotone i.e. for all X and Y comonotonic $J(X+Y) = J(X) + J(Y)$ and $X \geq Y \Rightarrow J(X) \geq J(Y)$ so $J(X) = \int_\Omega X dv$ with v Choquet capacity (see Schmeidler [15] or, for an axiomatization in this framework, Chateauneuf [2]).

We have then,

Proposition 3 *The following assertions are equivalent:*
(i) Expert I is more knowledgeable than expert II.
(iv) Either $\exists \alpha \in [0,1]$, $v' = \alpha v + (1-\alpha)v''$
or $\exists(\alpha,\beta) \in [0,1]^2$, $\alpha \neq \beta$ *such that for all chain* $\mathcal{C}$ *of* Σ
$\forall C \in \mathcal{C}, v'(C) \geq v(C)$ *and* $v'(C) = \alpha v(C) + (1-\alpha)v''(C)$
or $\forall C \in \mathcal{C}, v'(C) \leq v(C)$ *and* $v'(C) = \beta v(C) + (1-\beta)v''(C)$
(v) $\exists(\alpha,\beta) \in [0,1]^2$ $\forall E \in \Sigma$ *we have*
$v'(E) \geq v(E) \Rightarrow v'(E) = \alpha v(E) + (1-\alpha)v''(E)$
and $v'(E) \leq v(E) \Rightarrow v'(E) = \beta v(E) + (1-\beta)v''(E)$
And if $\alpha \neq \beta$ $\forall A \in \Sigma, B \in \Sigma$ *such that* $v'(A) = \alpha v(A) + (1-\alpha)v''(A)$ *and* $v'(B) = \beta v(B) + (1-\beta)v''(B)$, *we have* $\forall F \in \Sigma, F \subset A \cap B$ *or* $A \cup B \subset F$, $v(F) = v'(F) = v''(F)$.

Proof. Let us assume (iv)

Let $X \in \mathcal{F}$ finite ranged $X = \sum_{i=1}^{n} a_i A_i^*$, $a_1 \leq ... \leq a_n$

$$\begin{aligned} I(X) &= a_1 v(\Omega) + \sum_{i=1}^{n-1} (a_{i+1} - a_i) v(A_{i+1} \cup ... \cup A_n) \\ &= \sum_{i=1}^{n} x_i v(C_i) \ C_i \text{ is a chain} \end{aligned}$$

Thank to (iv) we get (iii) in that case.
If X is not finite ranged, there exists (X_n) finite ranged such that:
$I(X_n) \to I(X)$, $I'(X_n) \to I'(X)$ and $I''(X_n) \to I''(X)$
(see Denneberg 97 [3]).
Therefore we obtain (iii).
Let us assume (iii).
Let $E \in \Sigma$ with $X = E^*$ we get the first part of (v):
$v'(E) \geq v(E) \Rightarrow v'(E) = \alpha v(E) + (1 - \alpha) v''(E)$
and $v'(E) \leq v(E) \Rightarrow v'(E) = \beta v(E) + (1 - \beta) v''(E)$

Let us assume $\alpha \neq \beta$.
Let $A \in \Sigma$, $B \in \Sigma$ with $v'(A) \geq v(A)$ and $v'(B) \leq v(B)$.
Let $F \subset A \cap B$, $F \in \Sigma$.
Let us assume $v(F) \neq v'(F)$ and without loss of generality $v(F) < v'(F)$.
Then $v'(F) = \alpha v(F) + (1 - \alpha) v''(F)$.
Let $X = B^* + tF^*$, $t \in \mathbb{R}_+^*$.
$I(X) = v(B) + tv(F)$.
We can choose t such that X does not respect (iii):
$I'(X) = \beta v(B) + \alpha tv(F) + (1 - \beta) v''(B) + (1 - \alpha tv''(F))$
What we write
$I'(X) = xI(X) + (1 - x) I''(X)$
$x = \dfrac{\beta\,(v(B) - v''(B)) + \alpha t(v(F) - v''(F))}{v(B) - v''(B) + t(v(F) - v''(F))}$
As $v(F) \neq v''(F)$ (else $v'(F) = \alpha v(F) + (1 - \alpha) v''(F)$ implies $v(F) = v'(F)$),
x is a non-constant real function of t.
So we can choose t to have $x \neq \alpha$ and $x \neq \beta$
So we have $I(X) = I'(X)$
We have $I'(X) = \alpha I(X) + (1 - \alpha) I''(X)$
and $I'(X) = \beta I(X) + (1 - \beta) I''(X)$ $\alpha \neq \beta$
which imply $I'(X) = I(X) = I''(X)$. So we get (v).

Let us assume (v).
If $\alpha = \beta$ we get (iv): $v' = \alpha v + (1-\alpha)v''$.
If $\alpha \neq \beta$ let $\mathcal{C}$ be a chain.
Let us assume there exists $c \in \mathcal{C}$ without $v'(c) = v(c) = v''(c)$.
W.l.o.g. $v'(c) = \alpha v(c) + (1-\alpha)v''(c)$
then $\forall B \in \Sigma$ such that $B \subset C$ or $C \subset B$ we cannot have
$v'(B) = \beta v(B) + (1-\beta)v''(B)$ without $v'(B) = v(B) = v''(B)$
because it would not respect (v).
So we get (iv). ■

In proposition 3 (iv) means that expert I is more knowledgeable than expert II if and only if $v' \in [v, v'']$ or there are two coefficients of convexity, according to wether v is greater or smaller than v' and on any chain v' is always smaller or greater than v; (v) means that if $\alpha \neq \beta$, for two elements of Σ, one combination with α and the other with β, the three capacities must agree for all the sets contained in their intersection or containing their union.

The example below illustrates that expert I may be more knowledgeable than II and $v' \notin [v, v'']$.

Example 2

	R	Y	B	RY	RB	YB	Ω
v	0.1	0.2	0.3	0.4	0.5	0.6	1
v'	0.2	0.3	0.15	0.6	0.5	0.6	1
v''	0.3	0.4	0.1	0.8	0.5	0.6	1

v, v' and v'' are convex capacities and verify (iv) without being convex combinations.

In the next proposition we give assumptions that lead to "expert I is more knowledgeable than II" if and only if $v' \in [v, v'']$. That is to say capacities behave like probability measures when they are distortions of the same probability i.e. if the experts fulfil Yaari's model.

Proposition 4 *Let p be a non-atomic probability measure. v, v' and v'' are capacities following Yaari's model i.e. $v = f_0 \circ p$, $v' = f_1 \circ p$ and $v'' = f_2 \circ p$ with f_0, f_1 and f_2 increasing continuous functions from [0,1] onto [0,1] therefore $(i) \Longleftrightarrow v' \in [v, v'']$.*

Proof. We just have to prove $(i) \Rightarrow v' \in [v, v'']$.
Let us suppose:

$\exists A \in \Sigma$, $v'(A) = \alpha v(A) + (1-\alpha)v''(A)$ strictly.
$\exists B \in \Sigma$, $v'(B) = \beta v(B) + (1-\beta)v''(B)$ strictly.
W.l.o.g. $p(B) \leq p(A)$.
As p is non atomic, there exists $C \subset A$, $p(C) = p(B)$.
(iv) implies $v'(C) = \alpha v(C) + (1-\alpha)v''(C)$.
So $f_1(p(B)) = \alpha\ f_0(p(B)) + (1-\alpha)\ f_2(p(B))$.
A contradiction with $\exists B \in \Sigma$, $v'(B) = \beta v(B) + (1-\beta)v''(B)$ strictly. ■

Even with very "regular" capacities the proposition 4 does not hold anymore:

Example 3 *Let λ and μ be two different σ-additive measures, let $v = \lambda$ and $v'' = \mu$.*
Let us define v' in the following way: if $\lambda < \mu$ then $v' = 0.3\lambda + 0.7\mu$, if $\lambda > \mu$ then $v' = 0.7\lambda + 0.3\mu$ and if $\lambda = \mu$ then $v' = \lambda$.
Those three capacities are σ-continuous; they satisfy (ii) and $v' \notin [v, v'']$.
Let us also notice that although the truth is a probability measure expert I which proposes a capacity is more knowledgeable than expert II which proposes a probability measure.
Moreover let us notice that those three capacities have the form: $f(\lambda, \mu)$.

Not as for measures, even for non-atomic capacities, being closer as defined by Lehrer (let us recall that definition: v' is closer to v than v'' if $\forall A \in \Sigma$ either $0 \leq v'(A) - v(A) \leq v''(A) - v(A)$ Or $0 \geq v'(A) - v(A) \geq v''(A) - v(A)$) does not garantee v' to be a convex combination of v and v'' nor expert I to be more knowledgeable than expert II.
Actually theorem 2 of Lehrer is no more true for capacities. Here is an example with distortions of probabilities:
let us take $v = f_0 \circ \lambda, v' = f_1 \circ \lambda, v'' = f_2 \circ \lambda$ with
$f_0 : [0,1] \rightarrow [0,1], x \longmapsto x^2$,
$f_1 : [0,1] \rightarrow [0,1], x \longmapsto x^3$,
$f_2 : [0,1] \rightarrow [0,1], x \longmapsto x^4$
and λ be the Lebesgue measure.
One obtains v, v' and v'' as in theorem 2 of Lehrer but v' is not a convex combination of v and v''.

4 Updating the preferences

We propose other ways for the D.M. to choose a strategy which take care of updating the beliefs.

4.1 First approach: comparing each atom

We will call MK_1 the first of the methods with updating that we describe. When the D.M. knows that $C \in P$ has occured, he updates his functional J according to C and, eventually, to P; then he obtains a functional $J_{C,P}$ which defines a preference relation $\preceq_{C,P}$.
Then he chooses an action $\overline{a}_{C,P}$ that maximises $J_{C,P}(u(a_{C,P}, \omega))$.
Expert I sells a functional I' that leads to an action $\overline{a}'_{C,P}$ $\forall C \in \Sigma, \forall P \in \mathcal{P}$.
Expert II sells a functional I'' that leads to an action $\overline{a}''_{C,P}$ $\forall C \in \Sigma, \forall P \in \mathcal{P}$.
With that method, we compare the actions selected for each atom $C \in P$. When the updating rule does not take into account the partition, we focus on the σ-algebra Σ and no more on the partition of $\mathcal{P}$.

Definition 2 *Expert I is MK_1-more knowledgeable than expert II iff* $\forall (P, C, u, A)$, $I_{C,P}(u(\overline{a}'_{C,P}, \omega)) \geq I_{C,P}(u(\overline{a}''_{C,P}, \omega))$ *i.e. $\overline{a}'_{C,P}$ is always better than $\overline{a}''_{C,P}$ according to the real functional updated according to C and P.*

We consider constant additive positively homogenous functionals which, when updated, remain constant additive and positively homogenous. We have the following proposition which is an avatar of proposition 2.

Proposition 5 *(i)' Expert I is MK_1-more knowledgeable than II.*
(ii)' $\forall P \in \mathcal{P}, \forall C \in P, \forall X \in \mathcal{F}, \forall Y \in \mathcal{F}$, $I'_{C,P}(X) \geq I'_{C,P}(Y)$
and $I''_{C,P}(X) \leq I''_{C,P}(Y) \Rightarrow I_{C,P}(X) \geq I_{C,P}(Y)$.
(iii)' $\forall P \in \mathcal{P}, \forall C \in P, \exists \alpha \in [0,1], \exists \beta \in [0,1]$ $\forall X \in F$
$I_{C,P}(X) \leq I'_{C,P}(X) \Rightarrow I'_{C,P}(X) = \alpha I_{C,P}(X) + (1-\alpha) I''_{C,P}(X)$
$I_{C,P}(X) \geq I'_{C,P}(X) \Rightarrow I'_{C,P}(X) = \beta I_{C,P}(X) + (1-\beta) I''_{C,P}(X)$.

Proof. Let us assume (i).
Let $X \in \mathcal{F}, Y \in \mathcal{F}$ such that $I'_{C,P}(X) \geq I'_{C,P}(Y)$ and $I''_{C,P}(X) \leq I''_{C,P}(Y)$
Like in Gilboa and Lehrer, let $A = \{X, Y\}$, $u(X, \omega) = X(\omega)$
and $u(Y, \omega) = Y(\omega)$.
We get $\overline{s}'_P = X$ and $\overline{s}''_P = Y$. So from (i) we have $I_{C,P}(X) \geq I_{C,P}(Y)$.

Let us assume now (ii).
For any information structure (P, C, A, u), let $X(\omega) = u\left(\overline{s}'_{C,P}(\omega), \omega\right)$ and $Y(\omega) = u\left(\overline{s}''_{C,P}(\omega), \omega\right)$.
(ii) implies $I_{C,P}(X) \geq I_{C,P}(Y)$.
So expert I is more knowledgeable than expert II. ■

That proposition is an adpation of propositions 1 and 2 for MK_1. Its proof is just an adaption in this new framework.

Now we consider that all the functionals are additive, therefore represented by probability measures, updated with the Bayes rule (which means we do not care about the partition in the updating rule).
In the proposition below we prove this method is an extension of Lehrer [12] because MK and MK_1 are equivalent in the additive case.

Proposition 6 *For an additive Bayesian functional, being more knowldgeable for MK or MK_1 is equivalent.*

Proof. Expert I (whith probability m') is MK_0-more knowledgeable than expert II (with probability m''). The true functional is computed with the probability m.
Let $m' = \alpha m + (1-\alpha)m''$.
Let us prove $m'_C = \alpha_C m_C + (1-\alpha_C)m''_C$.
$\forall E \in \Sigma \; m'_C(E) = x m_C(E) + (1-x)m''_C(E)$ with

$$
\begin{aligned}
x &= \frac{m'_C(E) - m''_C(E)}{m_C(E) - m''_C(E)} = \frac{\frac{m'(E)m''(C) - m''(E)m'(C)}{m'(C)m''(C)}}{\frac{m(E)m''(E) - m(C)m''(E)}{m(C)m''(C)}} \\
x &= \frac{[m'(E)m''(C) - m''(E)m'(C)]m(C)}{[m(E)m''(C) - m(C)m''(E)]m'(C)} \\
x &= \frac{[(\alpha m(E) + (1-\alpha)m''(E))m''(C) - m''(E)(\alpha m(C) + (1-\alpha)m''(C))]m(C)}{[m(E)m''(C) - m(C)m''(E)]m'(C)} \\
x &= \frac{[\alpha m(E)m''(C) - \alpha m''(E)m(C)]m(C)}{[m(E)m''(C) - m(C)m''(E)]m'(C)} \\
x &= \frac{\alpha m(C)}{m'(C)} = \frac{\alpha m(C)}{[\alpha m(C) - (1-\alpha)m''(C)]} = \alpha_C \in [0,1].
\end{aligned}
$$

Thanks to the proposition 5 one can deduce expert I is MK-more knowledgeable than expert II.

Moreover the strategy $\overline{s}_p$ optimal for MK is formed with the $\overline{a}_{C,P}$ optimal for MK_1 by additivity of the functionals:

$$\begin{aligned}\max_{s_P \in S_P} J(u(s_P(\omega),\omega)) &= \max_{s_P \in S_P} \int u(s_P(\omega),\omega)dp \\ &= \sum_{C\in P} \max_{a\in A} \int_C u(a,\omega)dp = \sum_{C\in P} \max_{a\in A} J_C(u(a,\omega))\end{aligned}$$

■

According to proposition 5, in order to be coherent for our problem, if an expert is more knowledgeable with the initial functionals, it is always true for the updated functionals as it is for additive functionals with Bayes rule. It means the updating rule, when the updated functionals are also constant additive, positively homogenous and positive, should satisfy this condition:
$\exists\alpha \in [0,1], \exists\beta \in [0,1], \forall X \in \mathcal{F}\ I(X) \leq I'(X) \Rightarrow$
$I'(X) = \alpha I(X) + (1-\alpha)I''(X)$
and $I(X) \geq I'(X) \Rightarrow I'(X) = \beta I(X) + (1-\beta)I''(X)$
implies
$\forall P \in \mathcal{P}, \forall C \in P, \exists\alpha_{C,P} \in [0,1], \exists\beta_{C,P} \in [0,1], \forall X \in F$
$I_{C,P}(X) \leq I'_{C,P}(X) \Rightarrow I'_{C,P}(X) = \alpha_{C,P} I_{C,P}(X) + (1-\alpha_{C,P})I''_{C,P}(X)$
$I_{C,P}(X) \geq I'_{C,P}(X) \Rightarrow I'_{C,P}(X) = \beta_{C,P} I_{C,P}(X) + (1-\beta_{C,P})I''_{C,P}(X)$.

Among the many ways to update non-additive functionals, most of them do not fulfil that condition.

Example 4 *Full Bayesian updating (see Jaffray [11])*
All the functionals are Choquet integrals computed according to convex capacities. They are updated with the Full Bayesian rule $J_C(X) = \min\limits_{m\in C(v)} \int_C X dm_C$
where $C(v) = \{m;$ *m is a probability measure such that* $m(\Omega) = v(\Omega)$ *and for all A in* Σ,$m(A) \geq v(A)\}$
and m_C *denotes m, updated according to the Bayes rule. We can consider capacities satisfying (iv), when updated according to the Full Bayesian rule, they are very unlikely to still satisfy (iv).*
Let us consider the capacities of example 1. First we compute all the extreme points of the core of those capacities:

	m_1	m_2	m_3	m_4	m_5	m_6
R	0.1	0.1	0.4	0.2	0.2	0.4
Y	0.3	0.5	0.2	0.2	0.5	0.3
B	0.6	0.4	0.4	0.6	0.3	0.3

	m'_1	m'_2	m'_3	m'_4	m'_5	m'_6
R	0.2	0.2	0.3	0.4	0.35	0.4
Y	0.4	0.5	0.3	0.3	5	0.45
B	0.4	0.3	0.4	0.3	0.15	0.15

	m''_1	m''_2	m''_3	m''_4	m''_5	m''_6
R	0.3	0.3	0.4	0.4	0.4	0.4
Y	0.5	0.5	0.4	0.4	0.5	0.5
B	0.2	0.2	0.2	0.2	0.1	0.1

Let us suppose $\{R, Y\}$ has occured then the updated extremal measures of the core are:

	$m_{1\{R,Y\}}$	$m_{2\{R,Y\}}$	$m_{3\{R,Y\}}$	$m_{4\{R,Y\}}$	$m_{5\{R,Y\}}$	$m_{6\{R,Y\}}$
R	1/4	1/6	2/3	1/2	2/7	4/7
Y	3/4	5/6	1/3	1/2	5/7	3/7

	$m'_{1\{R,Y\}}$	$m'_{2\{R,Y\}}$	$m'_{3\{R,Y\}}$	$m'_{4\{R,Y\}}$	$m'_{5\{R,Y\}}$	$m'_{6\{R,Y\}}$
R	1/3	2/7	1/2	4/7	7/17	8/17
Y	2/3	5/7	1/2	3/7	10/17	9/17

	$m''_{1\{R,Y\}}$	$m''_{2\{R,Y\}}$	$m''_{3\{R,Y\}}$	$m''_{4\{R,Y\}}$	$m''_{5\{R,Y\}}$	$m''_{6\{R,Y\}}$
R	3/8	3/8	1/2	1/2	4/9	4/9
Y	5/8	5/8	1/2	1/2	5/9	5/9

Then the capacities updated according to the Full Bayesian rule are:

	$v_{\{R,Y\}}$	$v'_{\{R,Y\}}$	$v''_{\{R,Y\}}$
R	1/4	1/2	3/8
Y	1/3	1/2	1/2

$v_{\{R,Y\}}$, $v'_{\{R,Y\}}$ *and* $v''_{\{R,Y\}}$ *do not fulfil condition (iv).*

The next proposition presents a class of updating rules for which it is sufficient that expert I is more knowledgeable for the unconditional functional to guarantee it holds with the updated functionals: f-bayesianism updating proposed by Gilboa-Schmeidler 93 [9].
$\forall P \in \mathcal{P}, \forall C \in P$, $\exists\ f \in F$ which allows to define the preference relation $\preceq_{C,P}$ *as* follows:
$X \preceq_{C,P} Y \Leftrightarrow J(X_{/C} + f_{/\overline{C}}) \leq J(Y_{/C} + f_{/\overline{C}})$

In the next proposition let us notice that, knowing $P \in \mathcal{P}$ and $C \in P$, we use the same f to update the three functionals even if f may depend on C and P.

Proposition 7 *If each functional is updated according to f-bayesanism and if I', I and I'' verify (iii) then they verify (ii)'.*

Proof. If we suppose $\overline{\text{(ii)'}}$ we have $Y \prec'_{C,P} X$; $X \prec''_{C,P} Y$ and $X \prec_{C,P} Y$ which means $I'(Y_{/C} + f_{/\overline{C}}) < I'(X_{/C} + f_{/\overline{C}})$; $I''(X_{/C} + f_{/\overline{C}}) < I''(Y_{/C} + f_{/\overline{C}})$ and $I(X_{/C} + f_{/\overline{C}}) < I(Y_{/C} + f_{/\overline{C}})$.
That would be a violation of (ii) which is impossible thanks to proposition 2. ■

Let us recall the properties of f-bayesianism exposed by Gilboa and Schmeidler [9]. A D.M. who updates according to the f-bayesianism knowing E has occured, considers that f would have been chosen outside E, the event he knows to have occured.
If a D.M. considers that the best value of the utility is attained outside the event which has occured (i.e. in our problem M is attained for every $\omega \notin E$), that D.M. is said to be pessimistic and that updating rule is the well-known Dempster-Shaffer rule.
Similarly if a D.M.considers that the worst value of the utility is attained outside of the event which has occured (i.e. in our problem 0 is attained for every $\omega \notin E$), that D.M. is said to be optimistic and that updating rule is similar to the Bayes rule $\left(i.e. v_{/E}(F) = \frac{v(F \cap E)}{v(E)}\right)$.
Besides, for the Choquet integral, we are sure that the updated functional is still a Choquet integral if f has for only values the best and the worst.

4.2 Maximising one expression

For an additive functional J, it is sufficient to maximise $J(u(s_P(\omega), \omega))$ to get the strategy such that, for any atom of the partition, the action selected for that atom maximise the functional updated according to that atom. Thus we are allowed to define a value of information (as $\max_{s_P \in S_P} J(u(s_P(\omega), \omega))$). With the method proposed for MK_1, it is no more possible because the strategy formed with the actions that maximise $J_{C,P}(u(a_{C,P}, \omega))$ does not maximise $J(u(s_P(\omega), \omega))$.
However the method defined below gives us a way to maximise only one

expression and, if the functionals are monotone, it gives a definition of "more knowledgeable" equivalent to the one of MK_1. We will call that method MK_2, it is constructed as follows:
for $P \in \mathcal{P}, \forall C \in P$ the D.M. chooses the $\overline{a}_{C,P}$ that maximise
$J(\sum\limits_{C^*} J_{C,P}(u(a_{C,P}, \omega))C^*)$.
With that method we compute according to the unconditional functional the P-measurable function which has for value on each atom the value computed for that atom with the method MK_1.

Definition 3 *Expert I is MK_2-more knowledgeable than expert II iff $\forall(P, u, A)$, $I(\sum\limits_{C^*} I_{C,P}(u(a'_{C,P}, \omega))C^*) \geq I(\sum\limits_{C^*} I_{C,P}(u(a''_{C,P}, \omega))C^*)$ i.e. the strategy indicated by expert I is always better than the one of expert II according to the real functional I for any information structure.*

That definition is coherent since it coincides with the additive case.
Proof. We assume here that the the functional J is Bayesian additive. As the same $\overline{a}_C$ are optimal for MK and MK_1, the strategies are the same for MK and MK_2. MK_0 and MK_2 lead to the same computations because for an additive bayesian functional J we have $J(u(\overline{s}_P, \omega)) = J(\sum\limits_{C^*} J_C(u(\overline{a}_C, \omega))C^*)$.

$$
\begin{aligned}
J(J_C(u(\overline{a}_C, \omega))C^*) &= \int_\Omega \left(\sum_{C^*} \int_C (u(\overline{a}_C, \omega)dm)C^* \right) \\
&= \int_\Omega \left(\sum \int_C (u(\overline{a}_C, \omega)dm)C^* \right) \\
&= \int_\Omega (u(\overline{s}_P, \omega)dm = J(u(\overline{s}_P, \omega))
\end{aligned}
$$

■

Therefore MK and MK_2 are equivalent in the additive case. A functional J is said to be monotone if and only if for all f and g in $\mathcal{F}$, $f(\omega) \geq g(\omega)$ for all ω in Ω implies $J(f) \geq J(g)$. When the real functional is monotone, which is a very usual assumption, leading to a better strategy for any information structure is equivalent with the method MK_1 or the method MK_2.

Proposition 8 *If the "real" functional, I is monotone, MK_1 more knowledgeable is equivalent to MK_2 more knowledgeable.*

Proof. If expert I is MK_1 more knowledgeable than expert II, let us recall that the strategy optimal for MK_2 is formed with the optimal actions for $MK_{1.}$

$I(\sum_{C^*} I_{C,P}(u(a'_{C,P},\omega))C^*) \geq I(\sum_{C^*} I_{C,P}(u(a''_{C,P},\omega))C^*)$ because I is monotone and expert I is MK_1 more knowledgeable than expert II

(so $I_{C,P}(u(\overline{a}'_{C,P},\omega)) \geq I_{C,P}(u(\overline{a}''_{C,P},\omega)))$.

Reciprocally let (P,A,u) be an information structure, $\overline{a}'_{C,P}$ optimal for expert I and $\overline{a}''_{C,P}$ optimal for expert II.

Let us consider the following information structure $(P,A,\widetilde{u})$ with $\widetilde{u}(\omega) = u(\omega)$ if $\omega \in C$ and 0 else.

"Expert I MK_2 more knowledgeable than expert II" gives us

$I(I_{C,P}(u(\overline{a}'_{C,P},\omega))C^*) \geq I(I_{C,P}(u(\overline{a}''_{C,P},\omega))C^*)$

so $I_{C,P}(u(\overline{a}'_{C,P},\omega)) \geq I_{C,P}(u(\overline{a}''_{C,P},\omega))$ and we get the equivalence of MK_1 and MK_2 more knowledgeable. ■

A problem of positive value of information appears. A partition can lead to a worse value than a coarser one with updating rules like Full Bayesian rule or Dempster Shafer rule. However with the optimistic rule described by Gilboa and Schmeidler [9], the monotonicity of the real functional guarantees positive value of information. That rule is not very realistic since it means the D.M. considers the worst would have occured outside the atom which has occured. Searching for a more realistic updating rule is the point in the next section.

4.3 Positive value of information

As the expression computed in MK_2 does not guarantee a positive value of information, we give up that method. We could look for a special updating rule that would make us sure that the strategy formed with the actions that maximise $J_{C,P}(u(a_{C,P},\omega))$ maximises $J(u(s_P(\omega),\omega))$. It means that the strategy must be chosen in this way.

First the D.M. chooses an action $\overline{a}_{C,P}$ that maximises $J_{C,P}(u(a_{C,P},\omega))$.

Considering the strategy $\underline{s}_P$ formed with the $\overline{a}_{C,P}, \forall C \in P$, we get

$J(u(\underline{s}_P(\omega),\omega)) = \max_{s_P \in S_P} J(u(s_P(\omega),\omega))$.

Of course we shall not obtain the last equality for any updating rule: for a usual (i.e. Choquet and maximin expected utility) non-additive functional it will not hold with the usual updating rules except the one we'll put forward in the following paragraph.

First let us observe it holds in the additive case with the Bayes rule. Actually as observed at the end of the proof of proposition 6, "the strategy $\overline{s}_p$ optimal for MK is formed with the $\overline{a}_C$ optimal for MK_1 by additivity of the functionals" so in the additive case that method is equivalent to the one for MK. In the non-additive case we propose the following way to update.
We will use f-bayesianism with a $\overline{f}$ that satisfies:
$\exists t_P \in S_P, \overline{f}(\omega) = u(t_P(\omega), \omega)$, $t_{P/C} = \arg\max_{a \in A} J(u(a, \omega)_{/C} + \overline{f}_{/\overline{C}})$.
Such a $\overline{f}$ does always exist, we just have to take $\overline{f}(\omega) = u(t_P(\omega), \omega)$ with $t_P = \overline{s}_P = \arg\max_{s_P \in S_P} J(u(s_P(\omega), \omega))$.
That updating rule and that way to choose the strategy has two advantages: it provides a positive value of information and as it leads to the same computation as for MK, it provides a framework in which MK can be considered as a method taking care of the updating of the preferences. We call that rule f-max-Bayes rule.
Using that rule can be justified by the interpretation of Gilboa and Schmeidler; they argue the f chosen represents what the D.M. supposes would have happened outside C. In our problem the D.M. considers he would have maximised for each atom of the partition and so chooses the function $\overline{f}$ to update according to f-bayesianism.
However that method does not satisfy condition (iii)' because we update with a different $\overline{f}$ for each functional so, even if the strategies chosen are the same, we don't have the equivalence of more knowledgeable for MK and MK_1.

As in remark 2, we can restate that updating rule in a preference setup. We have got a set of act $\mathcal{A} \subset \mathcal{F}$ with a worst and best element. The functional J is updated according to the atom C of the partition P on the following way (f-max-Bayes rule): $\forall X \in \mathcal{A}$, $J_{C,P}(X) = \frac{J(X_{\diagup C} + f_{\diagup \overline{C}})}{J(C^* + f_{\diagup \overline{C}})}$ with $f = argmax(J(\sum_{C_i \in P, X_i \in \mathcal{A}} X_{i \diagup C_i}))$ where the C_i are the atoms of the partition P.

We define $\overline{X}_P = \sum_{C_i \in P} \overline{X}_{C_i, P \diagup C_i}$ with $\overline{X}_{C_i,P}$ that maximises $J_{C_i,P}(Y_{iC_i,P})$ for all $Y_i \in \mathcal{A}$, we can define a value of information $IV(P) = J(\overline{X}_P)$.

That value of information is positive since if a partition is finer than another one then its value of information is greater than the one of the other partition. Besides, as for an additive functional, $\overline{X}_P = \sum_{C_i \in P} \overline{X}_{C_i, P \diagup C_i}$ with $\overline{X}_{C_i,P}$ that maximises $J_{C_i,P}(Y_i)$ also maximises $J(\sum_{C_i \in P} Y_{i \diagup C_i})$ for all $Y_i \in \mathcal{A}$.

5 Concluding remarks

1) We could consider other ways to evaluate experts, which would not be directly a comparison. Example given, with the notation of section 2 we could define expert I (I') to aproach the real functional (I) at ε if

$\overline{s}_P = \arg\max\limits_{s_P \in S_P} I(u(s_P(\omega), \omega))$, $\overline{s}'_P = \arg\max\limits_{s_P \in S_P} I'(u(s_P(\omega), \omega))$ and

$\mid I(u(\overline{s}_P(\omega), \omega)) - I(u(\overline{s}'_P(\omega), \omega)) \mid \leq \varepsilon M$. Let us recall that M is the maximal value of u.

2) Proposition 1 is a restatement of our problem as "guessing the preferences". Here the D.M. just rank functions from Ω to $\mathbb{R}$, we could consider a more general setting with preferences on acts, we would then need a utility function to evaluate those acts according to a functional. However we think our result would not be different.

3) Example 2 can also be used for maximin expected utility. As a Choquet integral with a convex capacity is a minimum on a convex set of probabilities, it shows that, with maximin, one can get "more knowledgeable" without "convex combination". Besides a study of maximin expected utility would require to give a representation of the set of probabilities on which the minimum is computed as a convex closure of extreme points. Those extreme points are the ones for which the minimum is attained on a function. The condition of "more knowledgeable" can then be expressed on those extreme points.

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