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some complementary results on complementarity

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Cournot Competition in Spatial Markets:

some complementary results on complementarity

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Abstract

We study in this paper location equilibria for a symmetrical two-store duopoly selling complementary varieties, both on the linear and the circular markets. In contrast to the existing literature, besides assuming that each affiliate produces a different complementary variety, we equally consider in turns complementarity among all varieties on the market and substitutability between rival varieties. On the segment market, the intuition of a single-plant entity behaviour is enough to obtain and justify the result of central agglomeration. On the circle, instead, we are able to check the multiple equilibria property, to the extent that besides the intuitive spatial pattern, we identified each time a second one, involving diametrical dispersion, viable though only for low degrees of complementarity.

Keywords: complementary products, multi-store competition, spatial Cournot model;

JEL: D43, L13, R32

Résumé

Cet article étudie le choix de localisation dans un modèle d'oligopole de Cournot où les firmes produisent des biens complémentaires. Pour la première fois dans la littérature, on fait l'hypothèse que chaque filiale produit des variétés distinctes. On analyse les implications spatiale d'un tel cadre (correspondant par exemple à des firmes produisant des biens-système) en considérant les cas de la complémentarité entre variétés rivales et puis de la substituabilité. L'agglomération totale est toujours un équilibre pour la ville linéaire, ce qui est bien intuitive suivant notre hypothèse de complémentarité intra-firme. Néanmoins, pour la ville circulaire, en plus de l'équilibre intuitif impliquant l'agglomération des propres filiales, on démontre à chaque fois l'existence d'un deuxième équilibre, qui exige la dispersion diamétrale des firmes. Ceci montre la robustesse de la multiplicité d'équilibres de localisation dans le cadre circulaire.

1. Introduction

The earliest contributions to the spatial competition topic endogenized location choice for single-store firms selling a homogenous product and competing in prices. It turned out that the location-price model cannot sustain spatial agglomeration, either on the linear (see d'Aspremont et al. (79)) or the circular (see Kats (1995)) markets. Later, models studying location choice for Cournot rivals proved that the intensity of competition was determinant for this result. Anderson and Neven (1991) and Hamilton et al. (1989) established thus the central agglomeration on the segment. More recently, Pal (1998), Matsushima (2001) and Gupta et al. (2004) showed in turn that the shape of the market was equally important for the location outcome, since Cournot competitors cannot completely agglomerate on the circular market, but instead disperse, although they may cluster at several distinct locations.

Given the appealing properties of the Cournot spatial models¹, such as overlapping firm areas or agglomeration at discrete points, several papers have soon modified various assumptions of the initial framework². Pal and Sarkar (2002) proved that competition among multi-store firms yields clustering of different firms' outlets on the segment³. Shimizu (2002) relaxed the product homogeneity assumption in a single-outlet duopoly framework and confirmed the central agglomeration result on the segment, but showed that the outcome on the circle depends on whether goods are complements or substitutes. Yu and Lai (2003) extended the analysis to a

¹Cournot shipping models can apply both on the production and the consumption side. Choosing locations approximates for firms the mechanism of flexible manufacturing, where the basic product (standing for the location of the firm) is then customized at a certain cost (here, the shipping cost) so as to make it available to consumers. At the same time, a set of consumers with preferences defined on a set of goods can normally be represented by such a location model, which actually was in the beginning the very purpose of the analogy between the spatial setting and the product differentiation framework.

²In addition of the articles we quote next, see also Mayer (2000) and Matsumura et al. (2005) for contributions dealing with the properties of the production or transport cost functions in spatial markets.

³The complexity of the model has prevented yet addressing the parallel outcome for the circular market - Chamorro-Rivas (2000) and Cosnita (2005) have nevertheless established results for particular cases.

two-store duopoly, and found that on the circle firms agglomerate but stores disperse when rival products are complementary and own products are substitutes.

We acknowledge Yu and Lai's 2003 article as the starting point of our analysis, since we further explore the implications of the product complementarity assumption for the location equilibrium pattern of a spatial oligopoly. We consider in turns the following two frameworks: first of all, the case of complementarity among all varieties on the market, which is at the same time an "extreme" extension of Yu and Lai's complementarity assumption, but also the complete reversal of the usual hypothesis of homogenous product in the spatial Cournot literature. Secondly, we tackle the opposite of Yu and Lai's framework, meaning we consider the case of an oligopoly characterized by intra-firm complementarity⁴ but inter-firm substitutability⁵. These two different settings allow us to alter one particular hypothesis which has not been yet formally analyzed in Cournot shipping models, namely the one variety per firm assumption. To facilitate the comparison with previous theoretical contributions, we stick with the multi-store symmetric duopoly framework, and also deal with both the linear and circular markets.

In the first case, we confirm the intuition that complementarity between all varieties induces total agglomeration both on the segment and the circle. However, on the latter we equally identify an equilibrium pattern of intra-firm agglomeration and equidistant firm dispersion that is sustainable for low product complementarity. In the second case, by assuming instead that rival products are perfect substitutes, we still obtain complete central agglomeration on the linear market, as well as multiple equilibria on the circular one. We show that on the circle, spatial

⁴ Assuming intra-firm complementarity is realistic to the extent that real-life firms often produce simultaneously complementary goods, such as brick and cement, or operating systems and internet navigators. Moreover, intra-firm complementarity is the natural outcome of any conglomerate merger.

⁵ The same setting of affiliates producing complementary goods, and rival stores producing substitutes was retained by Tan and Yuan (2003) to examine the incentives to divisionalize of rival conglomerate firms, competing though in prices and in a non-spatial market.

Cournot competition between two-store firms producing substitutable system goods exhibits both intra-firm agglomeration with equidistant firm dispersion, but also inter-firm clustering with intra-firm diametrical dispersion.

The main findings of our paper are the following. On the one hand, we are able to extend the multiple equilibria property of the circular market to the case of multi-store competition with intra-firm complementarity, regardless of the assumption on inter-firm competition (i.e. substitutability or complementarity). This allows us to conclude that dealing with complementarity between own varieties can be tricky in a circular Cournot setting. Yu and Lai's bottom line idea was that intra-firm complementarity basically induces multi-plant firms to act as single-store ones⁶, and we confirm that this intuition is sufficient on the linear market. We show here in turn that directly assuming single-plant behaviour leaves out on the circle viable alternative equilibrium patterns for certain degrees of complementarity. Incidentally, this stresses that results obtained for single-plant competition do not necessarily extend to multi-plant settings.

On the other hand, we remind the relevance of assumptions on plant-level (rather than firm-level) competition, since the partial dispersion outcome with intra-firm agglomeration is entirely due to our hypothesis of intra-firm complementarity. Actually, we obtain the opposite result of Yu and Lai (2003) for the two-plant duopoly by making the contrary assumption on intra-firm competition, although their result and one of the patterns we identify are identical from the point of view of the spatial distribution of varieties.

The remainder of the paper is organized as follows. Section 2 outlines the model and presents the complete complementarity case, whereas section 3 deals with the intra-firm complementarity,

⁶This is due to the strategic agglomeration effect of intra-firm complementarity, which has each affiliate sell more at the other affiliate's location.

inter-firm substitutability case. Both the segment and the circle markets are studied under standard linearity assumptions on cost and demand functions, which are used throughout the paper. Profits' expressions, as well as First Order Conditions (FOCs) and Second Order Conditions (SOCs) are generally extremely complex and space-consuming, therefore more often than not they shall be omitted for brevity⁷. Section 4 concludes by summarizing results and comparing them with those available so far in the literature.

2. Complete Complementarity

Let there be two firms competing in quantities on the unit market (be it linear or circular). Each firm owns two stores and each store delivers only one good. Assume that all goods are symmetrically complementary among them. Individual market demands at any location x are linear and symmetric, and since each plant sells a different product, the inverse demand will typically be given by: $P_i(x) = a - q_i(x) + bq_j(x) + bq_k(x) + bq_h(x)$, where $i, j, k, h \in \{1, 2, 3, 4\}$ and $i \neq j \neq k \neq h$ with $a, b > 0$ and independent of x . Consumers are uniformly distributed along the unitary perimeter of the market and consume all products.

We assume that each good is produced with the same technology exhibiting constant marginal costs, normalized to zero. Each firm incurs the same linear transport cost to ship the product to consumers' locations: $t|x - z|$, where z is the location from which the product is shipped⁸. To keep things simple, let $t = 1$, or equivalently, let a be the transport-cost adjusted reservation price. Consumers have a prohibitive transport cost, preventing arbitrage, therefore firms can and

⁷A Technical Appendix containing them as well as all relevant computation is of course available on request from the author.

⁸In the case of the circular market, the norm stands for the shorter distance of the two possible ways to ship goods along the circumference.

will price discriminate across the set of spatially differentiated markets. Given constant marginal delivery costs, a set of independent Cournot equilibria obtains for each location x . There are no set-up or location costs. The game we consider is two-stage, firms choosing locations first and then competing in quantities. The backwards induction will be used to find the SPNE.

Regardless of the shape of the market, one can compute the equilibrium quantities supplied at each market point by each firm at the second stage. Let the first firm (denoted "12") sell goods 1 and 2, and the second firm (denoted "34") sell goods 3 and 4. Firms' profits at each market point x write

$$\begin{cases} \Pi_{12}(x) = (P_1(x) - c_1(x)) \cdot q_1(x) + (P_2(x) - c_2(x)) \cdot q_2(x) \\ \Pi_{34}(x) = (P_3(x) - c_3(x)) \cdot q_3(x) + (P_4(x) - c_4(x)) \cdot q_4(x) \end{cases}$$

where $c_i(x)$, $i = 1, 2, 3, 4$ stands for the constant marginal delivery cost of product i to location x . Solving the simultaneous system of FOCs gives the equilibrium quantities supplied at each market point:

$$\begin{cases} q_1^*(x) = \frac{1}{4b+8b^2-4} (2c_1 - 2ab - 2a - 2bc_1 + 2bc_2 + bc_3 + bc_4 - b^2c_1 - b^2c_2 + b^2c_3 + b^2c_4) \\ q_2^*(x) = \frac{1}{4b+8b^2-4} (2c_2 - 2ab - 2a + 2bc_1 - 2bc_2 + bc_3 + bc_4 - b^2c_1 - b^2c_2 + b^2c_3 + b^2c_4) \\ q_3^*(x) = \frac{1}{4b+8b^2-4} (2c_3 - 2ab - 2a + bc_1 + bc_2 - 2bc_3 + 2bc_4 + b^2c_1 + b^2c_2 - b^2c_3 - b^2c_4) \\ q_4^*(x) = \frac{1}{4b+8b^2-4} (2c_4 - 2ab - 2a + bc_1 + bc_2 + 2bc_3 - 2bc_4 + b^2c_1 + b^2c_2 - b^2c_3 - b^2c_4) \end{cases}$$

To ensure positive quantities for each store throughout the market, let $a > 2$ and $b < 0.5$.

At the first stage, in order to optimally locate their outlets, the duopolists maximize their overall profits with respect to store locations denoted p_1 and p_2 , and r_1 and r_2 respectively:

$$\max_{p_1, p_2} \Pi_{12}(p_1, p_2; r_1, r_2; x) = \max_{p_1, p_2} \left(\int_0^1 (q_1^*(p_1, p_2; r_1, r_2; x))^2 dx + \int_0^1 (q_2^*(p_1, p_2; r_1, r_2; x))^2 dx \right)$$

and

$$\max_{r_1, r_2} \Pi_{34}(r_1, r_2; p_1, p_2; x) = \max_{r_1, r_2} \left(\int_0^1 (q_3^*(r_1, r_2; p_1, p_2; x))^2 dx + \int_0^1 (q_4^*(r_1, r_2; p_1, p_2; x))^2 dx \right).$$

2.1. The linear market

We begin our discussion by directly giving the outcome of the game on the linear market.

Result 1: *The only spatial equilibrium for a two-store duopoly characterized by intra- and inter-firm complementarity competing in quantities on the linear market is total central agglomeration.*

The intuition for the above result is actually enough to prove it. Because of the transport cost linearity, each outlet sells the most at its own location but less and less as the delivery location is farther away. Thus, minimizing transport costs induces each outlet to choose the central location, and therefore they cluster⁹. In addition, this agglomeration effect due to the existence of market borders is enhanced by the strategic effect due to product complementarity (and it is straightforward to check that here Best Reply functions are upward sloping). Intra-firm complementarity means it is optimal to match a higher quantity of the other affiliate by a higher quantity of its own, so the best two stores owned by the same firm can do is to share the same location¹⁰. By the same token, inter-firm complementarity gives firms incentive to agglomerate. To sum up, on the linear market the three agglomeration forces enhance one another, making all stores cluster at the market center.

⁹Basically, this is the "natural location" effect of the linear market, where cost-minimization always gives incentives towards the unique most preferred location, the mid-segment point.

¹⁰This cannot be the case with product substitutability, because downward sloping Best Replies push outlets to disperse and to serve distinct half-markets, so as to lessen intra-firm competition.

Before addressing the circular case, we make one final remark for the linear one. Although the system of FOCs is hard to solve explicitly, we can nevertheless venture to say that total agglomeration is the unique equilibrium pattern. For that, it is enough to realize that on the linear market there is no strategic effect whatsoever that may generate dispersion. Both cost-minimization and strategic complementarity give incentives to cluster at the most-preferred location, the market center, without any countervailing force.

Corollary: *In a model with complete complementarity (both at plant and firm level), total central agglomeration is the only equilibrium for a location - Cournot game on the linear market, regardless of the number of firms or plants.*

2.2. The circular market

Turning now to the circle market, one needs to realize that depending on the relative position of the different stores, several cases are to be discussed in order to determine the location equilibrium. Indeed, given the intra-firm complementarity assumption, one cannot assume that each firm will locate its two stores within distinct disjoint half-circles, as is typically done when own products are substitutes¹¹. The following cases were discussed¹²:

¹¹With homogenous products, to minimize transport costs firms supply to each location from the closest store only.

¹²The corresponding detailed computations are presented in the Technical Appendix available on request.

$$\left\{ \begin{array}{l} \text{case 1 : } 0 \leq p_1 \leq p_2 \leq r_1 \leq r_2 \leq 1/2 \leq p_1 + 1/2 \leq p_2 + 1/2 \leq r_1 + 1/2 \leq r_2 + 1/2 \leq 1 \\ \text{case 2 : } 0 \leq r_1 - 1/2 \leq p_1 \leq p_2 \leq r_2 - 1/2 \leq 1/2 \leq r_1 \leq p_1 + 1/2 \leq p_2 + 1/2 \leq r_2 \leq 1 \\ \text{case 3 : } 0 \leq p_1 \leq r_1 - 1/2 \leq r_2 - 1/2 \leq p_2 \leq 1/2 \leq p_1 + 1/2 \leq r_1 \leq r_2 \leq p_2 + 1/2 \leq 1 \\ \text{case 4 : } 0 \leq p_1 \leq r_1 \leq p_2 - 1/2 \leq r_2 - 1/2 \leq 1/2 \leq p_1 + 1/2 \leq r_1 + 1/2 \leq p_2 \leq r_2 \leq 1 \end{array} \right.$$

Analytical explicit solutions could not be obtained, due to the complexity of the FOCs' system, but we were able to identify each time one equilibrium pattern. For instance, cases 1 and 4 confirm the intuition of total agglomeration of stores. Indeed, this is merely the extension of Yu and Lai's result, since intra-firm complementarity induces own stores to cluster, and therefore firms cluster too, because they behave as single-plant entities.

In turn, cases 2 and 3 yield the interesting and rather unexpected outcome of intra-firm agglomeration but inter-firm diametrical dispersion, i.e. $(p_1^* = 0, p_2^* = 0, r_1^* = 1/2, r_2^* = 1/2)$, but only if product complementarity is low enough (i.e. $b < 0.275$). This allows us to give the following:

Result 2: *On the circular market, a two-store duopoly producing symmetrically complementary varieties and competing in quantities will exhibit total agglomeration of stores, but also partial agglomeration with equal distance dispersion. In the latter case, each firm will locate both outlets at the opposite ends of a diameter, provided that varieties be not too complementary ($b < 0.275$).*

Generally speaking, in spatial models with location choice it has long been established that the FOCs simply translate what is called the quantity median property, i.e. total quantity sold

by the production plant to the left of its location needs to equal that to the right, if this location is to be optimal. The crucial difference between the linear and the circular frameworks is that on the segment, a firm's quantity median is unique, whereas on the circle it is not. Actually, for given competitors' locations, if a point μ on the circle satisfies the quantity median property for a firm, it is straightforward to see that the diametrically opposite location $\mu + 1/2$ does it too.

As already mentioned, as long as one assumes that firms behave as single-unit, due to inter-firm complementarity, the total agglomeration result cannot be surprising. It is straightforward to see that sharing the same location is a pattern that necessarily satisfies the quantity median property for every firm. However, in the circular market, partial agglomeration with diametrical dispersion is another such pattern. In our framework with perfectly symmetric complementarity, both patterns survive the final test of the SOCs, although not for the same degree of complementarity.

Indeed, we are left to question why this diametrical pattern involves inter-firm dispersion, and especially why this is no longer possible for the whole range of the complementarity parameter. The answer is provided by the Best Reply functions. At plant level, it is straightforward to notice that own output increases more with the other affiliate's output than with the quantity of a rival outlet: for instance, $BR_1 = \frac{a-c_1}{2} + bq_2 + \frac{b}{2}q_3 + \frac{b}{2}q_4$. At this point, it is clear that a firm's outlet values more the intra-firm complementarity than the inter-firm one. Nevertheless, it should be obvious that the latter can only be neglected for quite low values of b , since overlooking this supplementary agglomeration force lowers the total maximum profit of the firm¹³. This can

¹³It is straightforward to check that intra-firm agglomeration with inter-firm dispersion is only a local maximum (outlet profit of $\frac{1}{48}(2b-1)^{-2}(12a^2-b-6a+b^2+1)$), as compared with total agglomeration, which is the global maximum (individual outlet profit of $\frac{1}{48}(2b-1)^{-2}(12a^2-6a+1)$).

The fact that this two equilibrium patterns are not equivalent in terms of firms' profits is due on the one hand to the homogeneity of the circular space (in contrast, the market border effect on the segment makes

also be seen at firm level, by considering a firm's aggregate Best Reply: $BR^{12} = \frac{a - \frac{c_1 + c_2}{2}}{1 - b} + \frac{b}{1 - b} (q_3 + q_4)$. Inter-firm complementarity can be neglected only if this complementarity is not too strong ($b < 0.27$), since the coefficient $\frac{b}{1 - b}$ is increasing with b , so can only be approximated with 0 for low enough values of b .

We go on next to look into the case of complementarity between own varieties and substitutability between rival ones, so as to further explore the implications of the intra-firm complementarity assumption.

3. Intra-firm Complementarity with Inter-firm Substitutability

In this section, we deal with the opposite framework to that considered by Yu and Lai (2003). More precisely, instead of considering two complementary varieties produced each by one rival firm in its own two stores, we assume instead that firms produce two substitutable system goods, meaning that a firm's own plants produce complementary products, and that the firm's couple of varieties is substitute for the rival's ones. Keeping the same linearity assumptions and notations as before, let firm "12" ship varieties 1 and 2, complements, and let firm "34" ship products 3 and 4, also complements. However, the couples 1 and 3, and 2 and 4 are perfect substitutes respectively. Therefore, the system of linear market demands is the following:

$$\begin{cases} P_{13}(x) = a - (q_1(x) + q_3(x)) + b(q_2(x) + q_4(x)) \\ P_{24}(x) = a - (q_2(x) + q_4(x)) + b(q_1(x) + q_3(x)) \end{cases}$$

profit functions be single-peaked). On the other hand, total agglomeration naturally allows for higher profits in equilibrium thanks to the maximum impact of the symmetric complementarity assumption. The parallel outcome is obtained in the case of perfect product homogeneity for a triopoly on the circular market - total equidistant dispersion gives higher equilibrium profits than diametrical dispersion with partial clustering - see Cosnita (2005).

At the second stage of the game, firms' profits at each market point x write now

$$\begin{cases} \Pi_{12}(x) = (P_{13}(x) - c_1(x)) \cdot q_1(x) + (P_{24}(x) - c_2(x)) \cdot q_2(x) \\ \Pi_{34}(x) = (P_{13}(x) - c_3(x)) \cdot q_3(x) + (P_{24}(x) - c_4(x)) \cdot q_4(x) \end{cases}$$

where $c_i(x)$, $i = 1, 2, 3, 4$ stands for the constant marginal delivery cost of product i to location x . Solving the simultaneous system of FOCs gives the equilibrium quantities supplied at each market point:

$$\begin{cases} q_1^*(x) = \frac{1}{3b^2-3} (2c_1 - ab - a - c_3 + 2bc_2 - bc_4) \\ q_2^*(x) = \frac{1}{3b^2-3} (2c_2 - ab - a - c_4 + 2bc_1 - bc_3) \\ q_3^*(x) = \frac{1}{3b^2-3} (2c_3 - ab - c_1 - a - bc_2 + 2bc_4) \\ q_4^*(x) = \frac{1}{3b^2-3} (2c_4 - ab - c_2 - a - bc_1 + 2bc_3) \end{cases}$$

where $a > 2$ and $b \in (0, 1)$ to ensure positive quantities throughout the market.

As before, at the first stage, to optimally locate their outlets, the duopolists maximize their overall profits with respect to store locations denoted p_1 and p_2 , and r_1 and r_2 respectively:

$$\max_{p_1, p_2} \Pi_{12}(p_1, p_2; r_1, r_2; x) = \max_{p_1, p_2} \left(\int_0^1 (q_1^*(p_1, p_2; r_1, r_2; x))^2 dx + \int_0^1 (q_2^*(p_1, p_2; r_1, r_2; x))^2 dx \right)$$

and

$$\max_{r_1, r_2} \Pi_{34}(r_1, r_2; p_1, p_2; x) = \max_{r_1, r_2} \left(\int_0^1 (q_3^*(r_1, r_2; p_1, p_2; x))^2 dx + \int_0^1 (q_4^*(r_1, r_2; p_1, p_2; x))^2 dx \right).$$

3.1. The linear market

Not being able to postulate directly that affiliates necessarily locate within distinct-half-markets, we discussed the following three cases, depending on the relative position of stores:

$$\left\{ \begin{array}{l} \text{case 1 : } 0 \leq p_1 \leq p_2 \leq 1/2 \leq r_1 \leq r_2 \leq 1 \\ \text{case 2 : } 0 \leq p_1 \leq r_1 \leq 1/2 \leq r_2 \leq p_2 \leq 1 \\ \text{case 3 : } 0 \leq p_1 \leq r_1 \leq 1/2 \leq p_2 \leq r_2 \leq 1 \end{array} \right.$$

Explicit analytical solutions were not available, but each time central total agglomeration at $(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}; \frac{1}{2})$ was obtained as the location equilibrium, which should not come as a surprise.

Result 3: *A symmetric two-plant duopoly selling substitutable system goods and competing in quantities locates all stores at the center of the linear market.*

Given that complementarity within each firm makes affiliate share the same location, one is left with the easy problem of determining the spatial equilibrium for a duopoly selling substitutes on the linear market - the answer has long been provided by Anderson and Neven (1991), and is obviously central agglomeration. In other words, the intra-firm competition (complementarity) enhances the cost minimizing agglomeration effect which has firms on the segment take up the central location, despite the dispersion force generated by the inter-firm competition (substitutability).

Next we analyze the circular market, which is basically a "neutral" framework to the extent that it is free from any intrinsic agglomeration force.

3.2. The circular market

As expected, we start by a short¹⁴ list of cases discussed, such as:

$$\left\{ \begin{array}{l} \text{case 1 : } 0 \leq p_1 \leq p_2 \leq r_1 \leq r_2 \leq 1/2 \leq p_1 + 1/2 \leq p_2 + 1/2 \leq r_1 + 1/2 \leq r_2 + 1/2 \leq 1 \\ \text{case 2 : } 0 \leq r_1 - 1/2 \leq p_1 \leq p_2 \leq r_2 - 1/2 \leq 1/2 \leq r_1 \leq p_1 + 1/2 \leq p_2 + 1/2 \leq r_2 \leq 1 \\ \text{case 3 : } 0 \leq p_1 \leq p_2 - 1/2 \leq r_2 - 1/2 \leq r_1 \leq 1/2 \leq p_1 + 1/2 \leq p_2 \leq r_2 \leq r_1 + 1/2 \leq 1 \\ \text{case 4 : } 0 \leq p_1 \leq r_2 - 1/2 \leq p_2 \leq r_1 \leq 1/2 \leq p_1 + 1/2 \leq r_2 \leq p_2 + 1/2 \leq r_1 + 1/2 \leq 1 \end{array} \right.$$

Despite the analytical complexity of the system of FOCs w.r.t. the stores' locations, we were able to identify two distinct equilibrium patterns, and we equally checked that others could not be obtained (such as complete store and firm agglomeration or complete store and firm dispersion).

Result 4: *A symmetric two-plant duopoly selling substitutable system goods and competing in quantities on the circular market yields intra-firm agglomeration and inter-firm equidistant dispersion for all levels of intra-firm complementarity, but also intra-firm diametrical dispersion and intra-firm agglomeration if the above complementarity is low enough ($b < 0.171$).*

Following the linear case intuition, that firms behave as single-plant entities because of the internal complementarity, and given Pal's (1998) result of equidistant dispersion for firms selling substitutes on the circle, intra-firm agglomeration with equidistant diametrical firm dispersion is necessarily obtained in equilibrium (cases 1, 2 and 4 yield $(0; 0; \frac{1}{2}; \frac{1}{2})$).

However, we are once more able to prove the multiple equilibria property of the circular

¹⁴The complete case list and all the computations of this section are contained in the Technical Appendix available on request.

framework, since case 2, 3 and 4 equally yielded the clustering of rival stores and diametrical dispersion of affiliates (i.e. $(0; \frac{1}{2}; \frac{1}{2}; 1)$) as an alternative equilibrium pattern, although viable only for a low enough complementarity parameter ($b < 0.171$). In other words, among the four patterns that possibly satisfy the quantity median property on the circle, only two are obtained in equilibrium when $b < 0.171$, whereas neither total dispersion, nor total agglomeration are equilibria here.

In order to answer the question why affiliates can disperse and firms agglomerate when own products are complements but rival ones are substitutes, we turn to the Best reply functions. For instance, $BR_1 = \frac{a-c_1}{2} - \frac{1}{2}q_3 + bq_2 + \frac{b}{2}q_4$. Note that the only dispersion force is generated by the direct substitutability between varieties 1 and 3, and that its intensity is constant. That is why, in both equilibrium patterns obtained, direct substitutes are always diametrically opposite (see $p_1^* = 0$ and $r_1^* = 1/2$). However, when b is very close to 0, the direct and indirect complementarity effects (such as between 1 and 2 and 1 and 4) are roughly equal, thereby yielding the two alternative equilibria, exhibiting either intra-firm agglomeration (complementary varieties 1 and 2 share the same location, $p_1^* = p_2^* = 0$) or inter-firm agglomeration (complementary varieties 1 and 4 sharing the same location, $p_1^* = 0/1 = r_2^*$).

Although both this framework and Yu and Lai's similarly yield agglomeration between complementary varieties and equidistant dispersion of substitutable ones, it is the different ownership pattern that explains the fact that multiple equilibria obtain in our setting. Indeed, a quick look at the Best reply function in Yu and Lai's case - take variety 1 for the sake of an easy comparison ($BR_1 = \frac{a-c_1}{2} - q_3 + \frac{b}{2}q_2 + \frac{b}{2}q_4$) reveals that the direct intra-firm substitutability is dominant, so the equilibrium pattern necessarily involves intra-firm dispersion of own affiliates. Both the direct and the indirect complementarity (between 1 and 2 or 1 and 4) weigh equally within the

Best Reply function, but the two possible patterns resulting are basically the same, since they boil down to rival stores agglomerating (either clustering between 1 and 2, or between 1 and 4). In contrast, in our setting, once substitutable varieties disperse, the two alternatives left are not equivalent, since the former involves clustering of strongly complementary products, whereas the latter involves clustering of weakly complementary rival varieties.

4. Conclusion

Most of the papers on spatial competition with location choice have dealt with homogenous products, one of the reasons being that the spatial framework naturally yields product differentiation. Homogeneity is particularly important for Cournot spatial competition, since added to the strategic substitutability typically generates dispersion. We contribute to the literature by tackling the location choice of multi-store firms, and allowing each store to deliver a different product, which gives rise to intra-firm complementarity. This assumption was "combined" in turns with that of complementarity between rival varieties and substitutability between them. In both settings, we obtain the intuitive outcome of intra-firm agglomeration on both the linear and circular markets, but also the property of multiple equilibria for the latter. In the Appendix, we present a table summarizing our results and comparing them with the others results obtained so far.

To sum up, this paper basically reminds that the shape of the market and assumptions on plant-level rather than firm-level competition may be essential for determining the equilibrium pattern in a spatial model with location choice and various types of product differentiation.

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Appendix

Denote by A_1 and A_2 , and B_1 and B_2 respectively the locations of affiliates belonging to the same firm.

	Linear market	Circular market
<p>Intra-firm substitutability + inter-firm complementarity</p>	<p>(Yu and Lai (2003))</p>	<p>(Yu and Lai (2003))</p>
<p>Intra-firm complementarity + inter-firm substitutability <i>(this article)</i></p>		
<p>Intra-firm + Inter-firm complementarity <i>(this article)</i></p>		
<p>Intra-firm + inter-firm substitutability</p>	<p>(Pal and Sarkar (2002))</p>	<p>(Chamorro-Rivas (2000))</p>