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Katrin MILLOCK, Angels XABADIA, David ZILBERMAN

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Katrin Millock[a], Angels Xabadia[b] and David Zilberman[c]

a **Paris School of Economics, CNRS**
Université Paris 1 Panthéon-Sorbonne
Centre d'Economie de la Sorbonne
106/112 Boulevard de l'Hopital
75647 Paris Cedex 13
France
Email: millock@univ-paris1.fr
Phone: +33 1 44 07 82 60
Fax: +33 1 44 07 82 31

b **Department of Economics**
University of Girona
Campus de Montilivi
17071 Girona
Spain
Email: angels.xabadia@udg.edu

c **Department of Agricultural and Resource Economics**
207 Giannini Hall
University of California
Berkeley, California 94705
USA
Email: zilber@are.berkeley.edu

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Abstract

With the development of modern information technologies, relying on nanotechnologies and remote sensing, a number of systems can be envisaged that allow for monitoring of the negative externalities generated by producers, consumers or travelers – road pricing schemes or individual emission meters for automobiles are two examples. In the paper, we analyze a dynamic model of stock pollution when the regulator has incomplete information on emissions generated by heterogeneous agents. The paper's contribution is to explicitly study a decentralized policy for adoption of monitoring equipment over time. Each agent has to choose between paying a fixed fee or installing monitoring technology and paying a tax on actual emissions. We determine the second-best tax rates, the pattern of monitoring technology adoption, and identify conditions for the voluntary diffusion of monitoring technologies over time.

Résumé : Les nouvelles technologies de mesure et le contrôle des externalités de stock

Depuis longtemps, la régulation des externalités se heurte à des problèmes d'information. Avec le progrès technique il existe maintenant de nouvelles technologies qui se révèlent prometteuses pour la régulation des externalités : codes-barres digitaux (pouvant servir à tracer les lieux et méthodes de production), nanotechnologies permettant de tracer les molécules de produits chimiques dispersés dans l'eau ou de suivre des molécules transportés dans l'air, ou encore détecteurs de longue portée permettant de détecter les mouvements de voiture et de les faire payer selon leurs usages des routes en fonction de l'horaire et du flux de la circulation. Ce genre de technologies nécessite souvent un grand investissement dans l'infrastructure ainsi que des coûts variables afin de gérer chaque agent lié au système, comme c'est le cas pour les systèmes d'information géographique, par exemple. Le régulateur se trouve donc face au problème suivant : sous quelles conditions doit-il investir dans ces nouvelles technologies permettant une meilleure traçabilité des externalités, et, étant donné le progrès technique qui diminue les coûts d'investissements avec le temps, quand investir ? Comment la régulation doit-elle s'adapter pour prendre en compte ces nouvelles possibilités techniques ? Nous tentons de répondre à ces questions, en analysant un modèle d'optimisation dynamique d'un planificateur social face à un problème d'externalité de stock où le coût social augmente avec l'accumulation d'émissions. Nous utilisons des méthodes d'optimisation en deux étapes afin de résoudre le problème du planificateur : d'abord déterminer les émissions optimales selon les caractéristiques des agents, et ensuite déterminer la trajectoire optimale d'émissions dans le temps. Etant donné que le régulateur manque d'information sur les émissions des agents individuels nous proposons ensuite une politique volontaire de location de ce genre d'équipement une fois que l'investissement dans l'infrastructure aurait été effectué.

Keywords: externalities, environmental taxation, monitoring technology adoption, diffusion, nanotechnologies

JEL codes: D62, H23, L51, O33, Q58

1. Introduction

Some of the major environmental problems of our time are stock externality problems, including contamination of bodies of water by accumulating salt and chemicals, climate change, other air pollution problems where accumulating pollutants damage health or property, deforestation and loss of biodiversity, etc. Frequently, the activities contributing to these problems cannot easily be attributed to individual agents, which is a challenge to policy making. However, applications of new technologies including computers and the internet, wireless telephony, remote sensing, and geographic information systems, enable the introduction of increased numbers of monitoring systems to identify externality sources. In some cases, e.g., road pricing in Singapore, we already see instantaneous monitoring of road use that generate negative externalities (congestion and air pollution). The new technologies may require large investment in infrastructure, as well as in individual units of equipment. While in some cases, individual agents may need to invest in new technologies, in other situations, polluters may subscribe to third- party monitoring services. If, through learning by doing and accumulated knowledge, costs of the technologies are likely to decrease over time, then adoption becomes more likely.

The objective of this paper is to investigate the adoption of new monitoring technologies over time, as part of a policy to control stock externalities. What is the socially optimal time path of investment in monitoring technology? What policy should the regulator adopt to encourage investment in monitoring technologies? The paper's contribution is to explicitly study a decentralized policy for adoption of such new technology for monitoring over time. We thus focus on individual agents' incentives to adopt monitoring technology, and on the trade-off between a decentralized policy and a policy consisting of compulsory monitoring. The optimal policy suggested in the paper in cases when monitoring or auditing are feasible but costly is that each agent is assumed "Guilty until Proven Innocent" as proposed by

Swierzbinski (2002), i.e., agents are required to pay the maximum pollution fee, and it is up to them to prove that they are entitled to a refund. Several existing or proposed regulations of pollution or damages stipulate that the polluter is responsible to provide evidence or measurement of pollution, and without it the government uses a conservative default value. For example, The California Department of Pesticide Regulation establishes default inhalation rates for children and adults for assessing the exposure rates to chemicals and those are imposed in cases when the applicator cannot provide her own assessments. The new proposed concept outline for the California Low Carbon Fuel Standard Regulation suggests that the carbon content of fuels will be verified at the user's expense for every fuel category, otherwise a "conservative" default value is assumed.[1]

The scheme that we propose for delegation of the investment in monitoring has potentially important applications since it results in diffusion of monitoring technology over time through voluntary adoption of the new technology. Resources are thus saved since monitoring costs may decline over time as the technology develops if there are economies to scale in production. One potentially relevant application is the use of smart dust in monitoring and tracing pollution to its source (Warneke et al., 2001; Sailor and Link, 2005). Identity preservation is already part of current food safety policies that rely on the tracing of a faulty product towards its origin (source). The trend today is towards nano-level identity preservation and tracing, and identity preservation through the tagging of molecules of dirty input can now be envisaged using nanotechnologies. Identity preservation applied to polluting inputs such as pesticides and chemical fertilizers would enable the regulator to trace the source of pollution in case of environmental degradation. The possibility to use identity preservation as a means to mark polluting inputs thus has interesting applications for water

[1] Proposed Concept Outline for the California Low carbon Fuel Standard Regulation, accessed at http://www.arb.ca.gov/fuels/lcfs/032008lcfs_reg_outline.pdf.

quality policy, and will significantly improve on the information that is available to the regulator.

The paper relates to several bodies of literature: the literature on threshold models of technology adoption, the literature on stock externalities and the literature on pollution regulation with costly information. The threshold model of technology adoption was first introduced by David (1969); it assumes that agents are heterogeneous (for example in size) and that profit maximization implies a threshold in the heterogeneity parameter, after which it becomes profitable for the individual agent to adopt. Technology diffusion over time will then depend on the distribution and dynamics of the characteristic that determines heterogeneity among adopters (Stoneman, 1983; Sunding and Zilberman, 2001). Here we use a threshold model to study the diffusion of new monitoring technology for stock externalities. Major environmental problems including water pollution, soil erosion, buildup of pesticides resistance, and climate change are frequently stock externality problems (Farzin, 1996), and moreover they are usually caused by heterogeneous sources (Hoel and Karp, 2002; Xepapadeas 1992). Thus, for an efficient design of policies to control stock externalities both time and heterogeneity dimensions of these problems should be considered (Xabadia, Goetz and Zilberman, 2006). The buildup of the pollution stock can be modified through changes in production practices, by reducing input use, by decreasing the number of agents that operate in the economy, and through adoption of modern technologies (Khanna and Zilberman, 1997). The last body of research that we contribute to is the literature on pollution regulation with costly information. Many of today’s most important pollution problems are plagued by costly information on individual emissions. Examples include traffic emissions and agricultural runoff into water, such as nitrogen or pesticide leaching from fields. Carbon emissions from stoves and burners are another example. The diffuse pollution from many small sources whose individual emissions are unobservable constitutes a nonpoint source pollution problem. Following Holmstrom (1982), it has been suggested to levy a tax equal to the full social marginal cost on each polluter (Segerson, 1988; Xepapadeas, 1991; Herriges, Govindasamy and Shogren, 1994; Laffont, 1994; Hansen, 1998). In some cases, it can be difficult to do so, in particular when polluters do not realize their impact on the aggregate measure of pollution (Cabe and Herriges, 1992), or when the regulator cannot be certain about the level of cooperation within the group (Millock and Salanié, 2005). Investing resources in improving the monitoring of individual emissions may thus be worthwhile. Xepapadeas (1995) showed how risk-averse polluters may prefer to pay an emissions tax rather than a variable ambient tax. Millock, Sunding and Zilberman (2002) proposed discriminatory treatment for agents who invest in monitoring equipment and pay a tax proportional to the pollution they generate, while others will pay a fixed tax. Thus, the definition of nonpoint source pollution is not fixed but will evolve as the social cost of pollution changes and as the cost of monitoring technologies is reduced over time.

The first papers to study the dynamics of investment in monitoring have focused on the regulator's centralized decision of investment in her stock of knowledge about the pollution process (Xepapadeas, 1995; Dinar and Xepapadeas, 1998, 2002; Farzin and Kaplan, 2004). Dinar and Xepapadeas (2002) develop a model of the regulator's information acquisition for regulating groundwater in irrigated agriculture. Monitoring is treated as the regulator's stock of knowledge (information), which can be added to by investments in geographical information systems (GIS), or study of the soil conditions in the region and other factors that affect transport and fate. There is thus no individual decision to adopt a monitoring technology at each individual source. The model shows theoretically and empirically (Dinar and Xepapadeas, 1998) that it is more efficient to direct resources to investment in knowledge capital about the emissions process than trying to monitor input use in order to levy input taxes as a proxy to pollution taxes. Farzin and Kaplan (2004) also model monitoring as an effort on behalf of the regulator to improve a stock of knowledge capital, including knowledge of pollution transport and fate. They analyze the problem of a private or public manager that must target abatement resources in a National Park area with a fixed budget, and where the sediment load (pollution) is a function of unknown site-characteristics. Their simulations confirm that information acquisition improves the budget allocation of the National Park Manager and hence reduces expected damage compared with the case of an ex ante, uniform prior distribution of abatement effort.

The basic model is presented in Section 2 below. Section 3 studies the baseline, regulation without observing individual pollution levels, while Section 4 studies the solution with mandatory monitoring. We analyze the optimal timing for introducing compulsory monitoring technology in Section 5. In Section 6 we analyze the proposed decentralized policy for monitoring diffusion over time, give conditions for monitoring diffusion and compare policies. The last Section concludes.

2. The Model

We model a large number of agents that are heterogeneous with regard to a special parameter θ. The parameter θ should be interpreted as a characteristic that is unobservable to the regulator and affect efficiency. It could represent managerial ability, site-specific ecological conditions or properties of physical capital (bad insulation). In order to keep the model simple, we assume that θ is fixed and does not change over time. The regulator knows the overall distribution of θ, which is defined on a support $\left[\underline{\theta},\overline{\theta}\right]$, with a known continuous probability density function $g(\theta)$ and distribution function $G(\theta)$. The heterogeneity parameter θ combined with input use determines output of each agent. Profits are represented by a reduced form notation where pollution (z) is viewed as an input to production, and profits net of input costs are denoted by $\pi(z,\theta)$. The profit function $\pi(z,\theta)$ is assumed twice differentiable, increasing and concave in pollution and the efficiency parameter.[2] There are 2 different cases for $\frac{\partial^2 \pi}{\partial z \partial \theta}$. Either the efficiency parameter increases the marginal profitability of the polluting input and $\frac{\partial^2 \pi}{\partial z \partial \theta} > 0$, or increased efficiency implies reduced marginal profitability of the polluting input and $\frac{\partial^2 \pi}{\partial z \partial \theta} < 0$. Both cases can occur, but in different contexts. For example, if θ is a quality parameter and very productive agents also are very polluting, we would be in the first case; pollution from fertilizer use in agriculture would be an illustration. On the other hand, when the efficiency parameter measures efficiency in input use and pollution is created from unused residues from production, the second case applies. In the analysis, we will assume that the second condition holds and that more productive agents also have less pollution, as is the case with modern irrigation technologies that imply less water run-off and hence pollution from the field.

Pollution is a function of the heterogeneity parameter θ: $z=z(\theta)$. The technology adoption indicator at time t is $\delta_i(\theta,t)$, where i=0 indicates no monitoring, and i=1 indexes monitoring. The technology share $\delta_i(\theta,t)$ is between 0 and 1. For example, $\delta_1(\theta,t)=1$ if everyone of quality θ has invested in monitoring technology at time t. The following condition has to hold at each time t:

$$\sum_{i=0,1} \delta_i(\theta,t) \leq 1 \quad \forall t. \tag{1}$$

We thus take into account effects on the extensive margin, that is, the possibility that some agents will not produce.

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[2] We assume agents share a common production function (as well as pollution function) and that output differs only because of differences in θ and use of inputs. Later on, we will introduce an index i to indicate the difference in pollution when a firm is monitored or not.

We can now define pollution at time t as a function of θ and whether an agent is monitored or not: $z_i=z_i(\theta,t)$. Using the share of monitoring technology adoption, aggregate profits and pollution at time t are defined as:

$$\Pi(t)=\int_{\underline{\theta}}^{\overline{\theta}}\left(\sum_{i=0,1}\delta_i(\theta,t)\pi(z_i(\theta,t),\theta)\right)g(\theta)d\theta \tag{2}$$

$$Z(t)=\int_{\underline{\theta}}^{\overline{\theta}}\left(\sum_{i=0,1}\delta_i(\theta,t)z_i(\theta,t)\right)g(\theta)d\theta \tag{3}$$

Each agent pays a fixed annual unitary cost for monitoring, denoted v_i, with $v_0=0$ and $v_1=v$. It is assumed equal for all monitored agents, since it represents a cost connected with the technology of tagging the pollution and not the individual agent. One example is transponder technology in road traffic control. The annualized cost can also be interpreted as a fee for a certification agency. The aggregate monitoring technology cost borne by the agents is:

$$V(t)=\int_{\underline{\theta}}^{\overline{\theta}}\left(\sum_{i=0,1}\delta_i(\theta,t)v_i\right)g(\theta)d\theta \tag{4}$$

There is also a fixed cost to the regulator to develop the infrastructure that supports the monitoring system, denoted by $M(t)$. This cost is incurred at the moment of installing infrastructure. It is assumed that this cost decreases over time at a rate k, that is, $\frac{\dot{M}(t)}{M(t)}=-k$

The pollutant accumulates over time with emissions $Z(t)$, less the natural rate of decay, α, here assumed to be a simple linear function of the stock, $S(t)$:

$$\dot{S}(t)=Z(t)-\alpha S(t)=\int_{\underline{\theta}}^{\overline{\theta}}\left(\sum_{i=0,1}\delta_i(\theta,t)z_i(\theta,t)\right)g(\theta)d\theta-\alpha S(t) \tag{5}$$

Finally, the social cost of the stock of pollution at time *t* is an increasing convex function *C(S(t))*.

3. Optimal Regulation without Observing Pollution – The Case of a Franchise Fee

Assume first that monitoring technologies are not available or that their cost is prohibitive. In this case, it is not possible to observe the agent's emissions and only fixed fees are possible.[3] When an agent of quality θ faces a fixed fee, he solves the following problem:

$$\underset{z_0}{Max} \quad \pi(z_0(\theta,t),\theta) - F(t) \text{ s.t. } z_0 \geq 0 .$$

For every θ, either $z_0^*(\theta,t)$ solves $\frac{\partial \pi\left(z_0(\theta,t),\theta\right)}{\partial z} = 0$ or $z_0^*(\theta,t)$=0. This equation reflects the behavior of agents that cannot be observed individually. In this case, agents will produce at the private profit maximizing level as long as profits are positive.

The solution of the private problem leads to computing critical quality levels $\theta_0(F)$ such that agents with $\theta \geq \theta_0(F)$ will be operating while all the agents with $\theta < \theta_0(F)$ are not operating. The pollution level of an agent varies with the heterogeneity parameter, and we can derive the following result on the impact of θ on pollution:

$$\frac{dz^*}{d\theta} = \frac{-\frac{\partial^2 \pi}{\partial z \partial \theta}}{\frac{\partial^2 \pi}{\partial z^2}} < 0 \quad \text{iff} \qquad \frac{\partial^2 \pi}{\partial z \partial \theta} < 0.$$

The condition for a negative relation between pollution and the efficiency parameter holds when pollution originates from residues of production, and when the heterogeneity parameter is an indicator of input use efficiency. For example, in the case where irrigation is applied on

[3] We study the extreme case when only a fixed fee is possible. Extensions of the model may include alternative regulatory measures depending on the information available to the regulator in the initial situation: input taxes (if input use is observable at no cost) or best management practices (if technology or practices are observable at no cost). In all cases, the policy instrument will be a second-best one, imperfectly correlated with the variable of interest, in this case pollution.

lands with different water holding capacities, heavy soils will utilize a higher percentage of applied water yielding more output and less runoff from the field and less pollution.

Let $J_0(F) = \int_{\theta_0(F)}^{\bar{\theta}} \pi(z_0^*(\theta,t),\theta)\ g(\theta) d\theta$.

Then, the regulator's problem is to find the level of the fixed annual fee over time to maximize social welfare. Thus, the optimization problem is given by:

$$\underset{F(t)}{Max} \int_0^{\infty} \{J_o(F(t)) - C(S(t))\} e^{-rt} dt$$
$$s.t. \quad \dot{S}(t) = Z_0(F(t)) - \alpha S(t), \ \ S(0) = s_0$$

where $Z_0(F) = \int_{\theta_0(F)}^{\bar{\theta}} z_0^*(\theta,t)\ g(\theta) d\theta$, that is, the aggregate level of emissions, s_0 is the initial stock of pollution at time *0* and *r* is the social discount rate. We define the current value Hamiltonian by

$$H_0 \equiv J_0(F(t)) - C(S(t)) - \lambda_0 \left(Z_0(F(t)) - \alpha S(t) \right),$$

where λ_0 denotes the costate variable. It has been multiplied by minus one to facilitate the interpretations. The first-order conditions for an interior solution are given by

$$\frac{\partial H_0}{\partial F} \equiv \frac{\partial J_0(F(t))}{\partial F} - \lambda_0 \left(\frac{\partial Z_0(F(t))}{\partial F} \right) = 0 \tag{6}$$

$$\frac{\partial H_0}{\partial S} \equiv \lambda_0 \alpha - \frac{\partial C(S(t))}{\partial S} = \dot{\lambda}_0 - r\lambda_0 \tag{7}$$

$$\dot{S} = Z_0(F(t)) - \alpha S, \ \ S(0) = s_0 \tag{8}$$

Equation (6) states that the loss in private benefits (due to activity close-down) from a marginal increase in the fixed fee should equal the marginal decrease in aggregate emissions evaluated at the shadow price λ_0. Equation (7) explains the variation in the shadow cost of a delayed reduction of a marginal unit of the pollution stock from period *t* to period *t+1*. It establishes that the change is equal to the extra discounting and "decay" foregone paid on the shadow cost, $(\alpha + r)\lambda_0$, minus the social cost of the extra pollution associated with the delay, $\partial C / \partial s$.

For a sustainable environmental policy, the social planner is particularly interested in the achievement of a steady state, defined by equations (7) and (8) with $\dot{\lambda}_0 = \dot{S} = 0$. The following proposition describes the local stability properties of the steady state under the assumption of an interior solution for the control variable.

Proposition 1: The steady state equilibrium point of the system of equations (7) and (8) is characterized by a local saddle point, where the stable path leading to the steady state is upward sloping. Therefore, the pollution stock, *S*, and its shadow cost, λ_0, evolve over time in the same direction. Aggregate pollution, Z_0, varies negatively with respect to the shadow cost. Moreover, $\frac{\partial F}{\partial \lambda_0} > 0$, that is, the optimal fixed fee and the shadow cost also evolve in the same direction.

Proof: In appendix.

If the initial pollution stock is smaller (greater) than the steady-state stock of pollution, the optimal policy consists of choosing the fixed fee *F(t)* initially below (above) its steady-state value and progressively increasing (decreasing) *F(t)* until the steady-state is reached.

4. The Mandatory Monitoring Solution

In this section we present the benchmark when monitoring technologies are available from time 0 (*M*=0). In this case, the most common policy has been the imposition of monitoring on all sources of pollution. Then, the social planner has full information on each polluter's type, and the nonpoint problem converts to a point source pollution problem where Pigouvian taxes can be implemented. Thus, the problem of the agents is now given by:

$$\underset{z_1}{Max} \quad \pi(z_1(\theta,t),\theta) - \tau(t) z_1(\theta,t) - v \text{ subject to } z_1 \geq 0 .$$

Therefore, each agent chooses a level of pollution $z_1^*(\theta,t)$ that fulfills $\frac{\partial \pi(z_1^*(\theta,t),\theta)}{\partial z} = \tau$, or $z_1^*(\theta,t) = 0$ when $\pi(z_1(\theta,t),\theta) - \tau(t) z_1(\theta,t) - v < 0$. Thus, as in the former case, we can compute a critical theta $\theta_1(\tau)$ that will determine which agents that will operate in the economy. Like in the case of a pollution fee, the net social rents and the pollution level of an agent vary with the heterogeneity parameter. Net social rents, defined as $\pi(z_1(\theta,t),\theta) - v - \tau z_1(\theta,t)$, are non-decreasing in the efficiency parameter:

$$\frac{\partial \pi_i^*}{\partial \theta} = \frac{\partial \pi_i}{\partial \theta} - \tau \frac{\partial z_i}{\partial \theta} > 0 .$$

Since production profits increase and pollution decreases with the efficiency parameter, social net rents are strictly increasing in the efficiency parameter. It can easily be shown, for an operating unit, that $z_1^*(\theta,t) \leq z_0^*(\theta,t)$, since Pigouvian taxes affect the intensive margin, in addition to the extensive margin.

Denote $J_1(\tau) = \int_{\theta_1(\tau)}^{\bar{\theta}} \left(\pi(z_1^*(\theta,t),\theta) - v \right) \mathrm{g}(\theta) d\theta$.

The problem of the social planner is to find the evolution of the unit pollution tax over time that should be imposed on agents to maximize the discounted value of the net benefits of production less costs of monitoring and the social economic losses due to the accumulation of the pollutant:

$$\underset{\tau(t)}{Max} \int_0^{\infty} \{J_1(\tau(t)) - C(S(t))\} e^{-rt} \quad dt$$

s.t. $\dot{S}(t) = Z_1(\tau(t)) - \alpha S(t)$, $S(0) = s_0$, where $Z_1(\tau(t)) = \int_{\theta_1(\tau)}^{\bar{\theta}} z_1^*(\theta,t)g(\theta)d\theta$.

The current value Hamiltonian is defined as:

$$H_1 = J_1(\tau(t)) - C(S(t)) - \lambda_1 \left(Z_1(\tau(t)) - \alpha S(t) \right)$$

where λ_1 denotes the costate variable.[4] The first-order conditions for an interior solution are:

$$\frac{\partial H_1}{\partial \tau} \equiv \frac{\partial J_1}{\partial \tau} - \lambda_1 \frac{\partial Z_1}{\partial \tau} = 0 \quad (9)$$

$$\frac{\partial H_1}{\partial S} \equiv \lambda_1 \alpha - \frac{\partial C(S(t))}{\partial S} = \dot{\lambda}_1 - r\lambda_1 \quad (10)$$

$$\dot{S} = Z_1(\tau) - \alpha S, \; S(0) = s_0 \quad (11)$$

Equation (9) states that the marginal loss in private net benefits of increasing the Pigouvian tax should equal the temporal shadow value of the decrease in emissions. Equation (10) explains the variation in the shadow cost of a delayed reduction of a marginal unit of the pollution stock from period *t* to period *t+1*. It establishes that the change is equal to the extra discounting and "decay" forgone paid on the shadow cost, $(\alpha + r)\gamma$, minus the social cost of the extra pollution associated with the delay, $\partial C / \partial s$.

The steady state equilibrium of the system of equations (10) and (11) is qualitatively identical to the steady state defined in Proposition 1. Therefore, the pollution stock, *S*, and its shadow cost, λ_1, evolve over time in the same direction, and the optimal tax and the shadow cost also evolve in the same direction.

[4] Similarly to the previous case, the co-state variable has also been multiplied by minus one to facilitate the interpretations.

5. Optimal Timing of Introducing Compulsory Monitoring Technology

In the last section we only took into account the variable cost of monitoring (*v*) and assumed that monitoring was available at zero initial investment cost. However, this is far from reality. Usually, monitoring requires a large investment cost in infrastructure, e.g., satellites for GIS, and therefore, it may not be optimal to install monitoring technologies from the beginning of the time horizon. Thus, it is important to determine when monitoring should be installed. We start by assuming that once the investment cost is incurred by the government, monitoring will become mandatory for all agents in the economy.

The problem of the social planner, in addition to determine the level of the taxes to impose on the agents, is to decide at what time *T* to carry out the investment in monitoring infrastructure. Her decision problem is given by:

$$\underset{F(t),\tau(t),T}{Max} \int_0^T \{J_0(F(t)) - C(S(t))\} e^{-rt} dt - M(t)e^{-rT} + \int_T^\infty \{J_1(\tau(t)) - C(S(t))\} e^{-rt} dt$$

s.t.

$$\dot{S}(t) = Z_0(F(t)) - \alpha S(t), \qquad 0 \le t \le T$$
$$\dot{S}(t) = Z_1(\tau(t)) - \alpha S(t), \qquad T \le t < \infty$$
$$S(0) = s_0$$
$$\dot{M}(t) = -kM(t)$$

We derive necessary conditions for the optimization problem following Makris (2001) and Boucekkine, Saglam and Vallée (2004). Conditions (6-8) remain valid for $0 \le t \le T$ and conditions (9-11) for $T \le t < \infty$. However, there is one additional matching condition with respect to the social cost of pollution: $\lambda_0(T) = \lambda_1(T)$, as well as conditions with respect to the switching time *T*:

$$
\begin{aligned}
&H_0(T)+(r+k)M(T)=H_1(T) \qquad 0<T<\infty \\
&H_0(T)+(r+k)M(T)\leq H_1(T) \qquad T=0 \\
&H_0(T)+(r+k)M(T)\geq H_1(T) \qquad \text{then never introduce monitoring.}
\end{aligned}
\tag{12}
$$

Following Boucekkine, Saglam and Vallée (2004), since the objective function of the problem is strictly concave in the control variables and since it is discounted, the transversality condition is given by $\lim_{t\to\infty} S(t)\lambda_1(t)=0,\quad$ with $\lim_{t\to\infty}\lambda_1(t)\geq 0$.

Moreover, in order for T^* to be a maximum, the condition $\frac{\partial H_0(T^*)}{\partial T}-k(r+k)M(0)e^{-kt}-\frac{\partial H_1(T^*)}{\partial T}<0$ has to be satisfied. The second-order condition holds here since the two Hamiltonians in our problem are time autonomous.

A postponement of the investment in monitoring technology saves the interest rate and the reduction in the cost of the technology. Substituting for *M(T)* into equation (12) leads to:

$$
\begin{aligned}
&H_0(T^*)+(r+k)M(0)e^{-kT^*}=H_1(T^*) \qquad 0<T^*<\infty \\
&H_0(T^*)+(r+k)M(0)e^{-kT^*}\leq H_1(T^*) \qquad T^*=0 \\
&H_0(T^*)+(r+k)M(0)e^{-kT^*}\geq H_1(T^*) \qquad \text{then never introduce monitoring}
\end{aligned}
\tag{13}
$$

Therefore, $\frac{\partial T^*}{\partial M(0)}>0$. If the initial investment is high, investment is delayed. On the other hand, the impact of a change in the rate of reduction of the investment cost (*k*) is ambiguous. We have $\frac{\partial T^*}{\partial k}=\frac{\left\{1-T^*(r+k)\right\}}{k(r+k)}\ {}^{>}_{<}\ 0$. There are two countervailing effects. An increase in *k* directly increases the rewards of waiting since it is cheaper to adopt later on. But at the same time it decreases the absolute investment cost $M(0)e^{-kT}$, so that equation (13) may be fulfilled earlier. The rewards of waiting dominate if $T^*<\frac{1}{(r+k)}$.

6. A Decentralized Policy for Monitoring Diffusion over Time.

Monitoring is costly ($v > 0$) and therefore it may not be optimal to monitor all sources of pollution at the time the fixed cost of investment in monitoring infrastructure becomes comparable to the economic efficiency gains from monitoring. This section presents a decentralized policy for adoption of monitoring equipment over time. The aim is to study the diffusion of monitoring technologies once the investment in the infrastructure has taken place. When agents are monitored, the regulator can levy a charge on each unit of pollution; but for unmonitored agents only fixed fees are possible that do not depend on the agent's pollution, nor type. We propose to analyse a simple scheme consisting of a fixed fee F_0 on agents that are not monitored and an emission tax τ for monitored agents combined with a fixed payment F_1 (that could be negative). Hence, polluters are assumed guilty until proven innocent (as in Swierzbinski, 2002), i.e., they pay a fixed fee unless they install monitoring equipment to prove their actual pollution.

The model relies upon two important assumptions – reversible investment in monitoring and that the regulator cannot exploit information in previous time periods to regulate individual agents in the current time period. These are important assumptions and we will thus spend some time on their justification and rationality for the question at hand. Apart from simplifying the solution of the model, it is important to allow for reversibility if the regulator wishes to maximize social welfare. Imagine a case where the marginal social damage cost decreases over time; then an optimal solution may involve increasing pollution and hence decreasing monitoring over time. It is thus necessary to allow for reversibility in the monitoring investment decision. The way we choose to model the monitoring investment decision is that the agents can choose whether to rent monitoring equipment or not in each time period. This implicitly means assuming that the regulator does not retain information on agents' quality. Although representing an extreme case, the assumption is not unreasonable.

Keeping track on the activity of every agent is costly, especially if the data is to be used for tax assessment in justifying and protecting against costly lawsuits. While our model is deterministic, a more realistic model would include randomness that could affect individual agents and their actions. The overall distribution of θ may be constant over time, but the θ of individual agents may vary. Income taxes, for example, require new reporting every year and rely little on past actions to justify present decisions. We can assume that the regulator has the analytical capacity and good sampling techniques that allow her to calculate sophisticated tax formulas, but that both monitoring and memory of individual behavior are costly.

6.1. The general problem

We follow Millock, Sunding and Zilberman (2002) and suggest the following simple linear taxation scheme to delegate adoption of monitoring: Agents that adopt monitoring equipment will pay an emission tax τ per unit of pollution, and a fixed amount, F_1, per operating agent, which is a tax when F_1 is positive and a subsidy when F_1 is negative. The non-adopters whose pollution is unobservable will pay the same fixed fee, F_0. In each period, the individual agent takes the regulatory instruments $F_0(t)$, $F_1(t)$ and $\tau(t)$ as given and solves the following problem:

$$Max \quad \{\pi(z_0(\theta,t),\theta) - F_0(t),\, \pi(z_1(\theta,t),\theta) - \tau(t)z_1(\theta,t) - F_1(t) - v\}.$$

For a given θ, the agent chooses to rent monitoring equipment ($\delta_1(\theta,t)=1$) iff

$$\pi(z_1(\theta,t),\theta) - \pi(z_0(\theta,t),\theta) - \tau(t)z_1(\theta,t) - F_1(t) + F_0(t) - v \geq 0.$$

Define the critical level θ_c by the level of the heterogeneity parameter for which an agent is just indifferent between renting monitoring equipment or not:

$$\pi(z_1(\theta_c,t),\theta_c) - \pi(z_0(\theta_c,t),\theta_c) - \tau(t) z_1(\theta_c,t) - F_1(t) + F_0(t) - v = 0 \quad \forall t \tag{14}$$

The following comparative statics describe the impact of the regulatory policy instruments on the critical level for adoption of monitoring:

$$\frac{\partial \theta_c}{\partial F_0} = -\frac{1}{\left(\frac{\partial \pi(z_1(\theta,t),\theta)}{\partial \theta_c} - \frac{\partial \pi(z_0(\theta,t),\theta)}{\partial \theta_c}\right)} < 0 \tag{15}$$

$$\frac{\partial \theta_c}{\partial F_1} = \frac{1}{\left(\frac{\partial \pi(z_1(\theta,t),\theta)}{\partial \theta_c} - \frac{\partial \pi(z_0(\theta,t),\theta)}{\partial \theta_c}\right)} > 0 \tag{16}$$

$$\frac{\partial \theta_c}{\partial \tau} = \frac{z_1(\theta_c,t)}{\left(\frac{\partial \pi(z_1(\theta,t),\theta)}{\partial \theta_c} - \frac{\partial \pi(z_0(\theta,t),\theta)}{\partial \theta_c}\right)} > 0 \tag{17}$$

since we assume that $\frac{\partial^2 \pi}{\partial z \partial \theta} < 0$.

The regulator's problem is:

$$\underset{F_0(t),F_1(t),\tau(t)}{Max} \int_0^\infty e^{-rt} \left(\int_{\theta_0(T_0)}^{\theta_c(F_0,F_1,\tau;v)} \pi(z_0(\theta,t),\theta)\, dG(\theta) + \int_{\theta_c(F_0,F_1,\tau;v)}^{\bar{\theta}} \pi(z_1(\theta,t),\theta)\, dG(\theta) - V - C(S(t)) \right) dt$$

s.t. $\dot{S}(t) = \int_{\theta_0(F_0)}^{\theta_c(F_0,F_1,\tau;v)} z_0(\theta,t)\, dG(\theta) + \int_{\theta_c(F_0,F_1,\tau;v)}^{\bar{\theta}} z_1(\theta,t)\, dG(\theta) - \alpha S$, $S(0) = s_0$

Following Xabadia, Goetz and Zilberman (2006), we will use a two-stage procedure to solve the problem. First, the regulator will choose the optimal allocation of emissions over quality, θ, given an aggregate level of pollution *Z*. Next, he will optimize the value of *Z* over time.

In the first stage, the regulator's solution is given by the value function *J(Z)* defined as

$$J(Z) = \underset{F_0, F_1, \tau}{Max} \int_{\theta_0(F_0)}^{\theta_c(F_0, F_1, \tau; v)} \pi(z_0(\theta), \theta)\, dG(\theta) + \int_{\theta_c(F_0, F_1, \tau; v)}^{\bar{\theta}} \{\pi(z_1(\theta), \theta) - v\} dG(\theta)$$

$$\text{s.t.} \qquad \int_{\theta_0(F_0)}^{\theta_c(F_0, F_1, \tau; v)} z_0(\theta)\, dG(\theta) + \int_{\theta_c(F_0, F_1, \tau; v)}^{\bar{\theta}} z_1(\theta)\, dG(\theta) = Z\,.$$

Since *Z* does not depend on the heterogeneity parameter (θ) the shadow cost of the pre-specified level of emissions (λ) is constant over θ. The optimal level of monitoring adoption can be attained by the policy described in the following proposition.

Proposition 2: Given aggregate pollution *Z*, optimal adoption over quality can be obtained with the combination of the following instruments:

A fixed fee on non-monitored agents: $F_0 = \lambda z_0(\theta_0)$

A fixed fee on monitored agents: $F_1 = \lambda\left(z_0(\theta_0) - z_0(\theta_c)\right)$

A unit emission tax on monitored agents: $\tau = \lambda$.

Proof: In appendix.

This policy scheme can be interpreted as a fixed fee on all agents $F_0 = \lambda z_0(\theta_0)$, and a subsidy $F_1 = -\lambda z_0(\theta_c)$ on agents that decide to adopt monitoring. The monitored agents also pay a unit emission tax which is equal to the shadow cost of emissions. In this way, monitored agents are taxed according to the pollution they generate, but subsidized for part of the overestimate of pollution before the installation of the monitoring system.

In the second stage, the value function *J(Z)* from the first stage is maximized over time:

$$\underset{Z(t)}{Max} \int_0^{\infty} \{J(Z(t)) - C(S(t))\} e^{-rt} dt$$

s.t. $\dot{S}(t) = Z(t) - \alpha S(t)$, $S(0) = s_0$, $Z(t) \geq 0$.

The parameter denoting aggregate emissions over the entire range of θ now becomes the decision variable in the second stage. The current value Hamiltonian of the second stage is defined as:

$$H = J(Z(t)) - C(S(t)) - \gamma\big(Z(t) - \alpha S(t)\big)$$

where γ denotes the costate variable. It has been multiplied by minus one to facilitate the interpretations. The first-order conditions for an interior solution are:

$$\frac{\partial H}{\partial Z} \equiv \frac{\partial J}{\partial Z} - \gamma = 0 \tag{18}$$

$$\frac{\partial H}{\partial S} \equiv \gamma\alpha - \frac{\partial C(S(t))}{\partial S} = \dot{\gamma} - r\gamma \tag{19}$$

$$\dot{S} = Z - \alpha S\,,\ S(0) = s_0 \tag{20}$$

Equation (18) states that the marginal value of aggregate emissions should equal the temporal shadow cost of the pollution stock γ. By the Envelope Theorem, a change in the value function as a result of a change in aggregate pollution Z is equal to λ. From (18), we then see that the shadow values of aggregate pollution in the first and second stages of the optimization are identical, i.e., $\lambda = \gamma$. Equation (19) explains the variation in the shadow cost of a delayed reduction of a marginal unit of the pollution stock from period t to period $t+1$. It establishes that the change is equal to the extra discounting and "decay" forgone paid on the shadow cost, $(\alpha + r)\gamma$, minus the social cost of the extra pollution associated with the delay $\partial C / \partial s$.

The steady state equilibrium of the system of equations (19) and (20) is qualitatively identical to the steady state defined in Proposition 1, so it is characterized by a saddle point.

Therefore, like in the cases of compulsory monitoring or no monitoring at all, the pollution stock, S, and its shadow cost, γ, evolve over time in the same direction.

The incentives for adoption of monitoring equipment over time can be summarized in the following proposition:

Proposition 3: Given that the initial pollution stock, S_0, is greater (smaller) than the steady-state stock of pollution, S^∞, the optimal decentralized policy for monitoring diffusion over time consists of:

a) choosing the unit emissions tax, τ, above (below) its steady-state value and gradually decreasing (increasing) the tax over time.

b) choosing the fixed fee, F_0, above (below) its steady-state value and gradually decreasing (increasing) it over time.

c) choosing the subsidy $|F_1|$, below (above) its steady-state value and gradually increasing (decreasing) it over time $\left(|\dot{F}_1| > 0, \text{ i.e.,} \frac{\partial F_1}{\partial Z} < 0\right)$ iff

$$1 < \frac{\tau\left|\frac{\partial z_0}{\partial \theta_c}\right|}{\frac{\partial \pi(z_1)}{\partial \theta_c} - \frac{\partial \pi(z_0)}{\partial \theta_c}} < \frac{z_0}{z_1}$$

Corollary: When $S_0 > S^\infty$, there will be diffusion of monitoring equipment over time, that is, $\dot{\theta}_c < 0$, if $\tau\left|\frac{\partial z_0(\theta_c)}{\partial \theta_c}\right| > \left(\frac{\partial \pi(z_1(\theta,t),\theta)}{\partial \theta_c} - \frac{\partial \pi(z_0(\theta,t),\theta)}{\partial \theta_c}\right)$. When $S_0 < S^\infty$, the contrary holds.

Proof: In appendix.

The Corollary determines the adoption of monitoring over time ($\dot{\theta}_c$). Since the decision on monitoring depends on both the evolution of the emission tax and the fixed fees, we define two scenarios:

A) The new externality case, where $S_0 < S^{\infty}$: $\dot{\tau} > 0$

B) The restoration case, where $S_0 > S^{\infty}$: $\dot{\tau} < 0$

In Case A, that we label the new externality case, there is early awareness of pollution and as pollution grows towards its steady-state level, the shadow cost of pollution increases over time causing aggregate emissions in each time period to decrease to allow the stock of pollution to reach its steady-state value. Aggregate emissions can be modified either by changing the proportion of monitored agents, as indicated by θ_c, or by changing the external margin, that is, by changing the number of agents that operate. As shown in the appendix, $\frac{\partial F_0}{\partial Z} < 0$, and therefore the optimal fixed fee increases over time causing the number of polluting agents to decrease until the steady state is reached. Moreover, since the shadow cost of pollution grows over time, the unit emissions tax also increases over time. In this way, the intensive and extensive margins can be considered as complementary. The direct effect of the increasing emission tax discourages monitoring adoption. On the other hand, the subsidy given to monitored agents, F_1, may increase or decrease over time and therefore, the diffusion of monitoring depends on the sign of $\tau\left|\frac{\partial z_0(\theta_c)}{\partial \theta_c}\right| - \left(\frac{\partial \pi(z_1(\theta,t),\theta)}{\partial \theta_c} - \frac{\partial \pi(z_0(\theta,t),\theta)}{\partial \theta_c}\right)$. If the marginal gain in profitability from the reduction in pollution brought about by monitoring adoption exceeds the marginal change in the pollution tax payment, then there will be full diffusion of monitoring over time.

The more interesting case may be Case B - the restoration case. For the restoration case, when monitoring has not been implemented or was not feasible to start with because of the costs of compulsory monitoring, the regulator may implement the second-best policy defined in Proposition 3. As pollution is reduced towards its optimal steady-state level, the shadow cost of pollution decreases over time, hence initial policy needs to be harsh to reduce sufficiently the aggregate emissions, and it softens till the steady-state is reached. In this case, $\dot{\tau} < 0$, causing a direct positive effect on the diffusion of monitoring equipment, that is, the reduction of the pollution tax encourages more agents to adopt monitoring equipment over time. The initial fixed fee F_0 needs also to be high to deter the more polluting agents and it decreases over time as the pollution problem diminishes. The final impact on adoption will depend, as before, on the evolution of the subsidy. We know that the evolution of the emission tax encourages monitoring adoption in this case, but that the evolution over time of the fixed fee has a discouraging effect on adoption. In the end, the condition for diffusion, stated in the Corollary of Proposition 3, implies that the marginal gain in the variable tax payment has to exceed the marginal change in profitability from the reduction in pollution caused by adoption of monitoring.

In order to compare the effects of mandatory and voluntary policies on the diffusion of monitoring over time we will focus on the case when the marginal unit of production is fixed.

6.2. Technology diffusion and policy comparison

In the special case of no effects on the extensive margin the optimal linear policy consists of a fixed fee on all agents equal to the social cost of pollution from the last unit to adopt monitoring equipment, and a per unit emission tax on monitored agents:

Proposition 4: With no effects on the extensive margin, optimal adoption of monitoring can be obtained with a fixed fee $F_0(t)$ on non-monitored units and a unit emission tax $\tau(t)$ on monitored units:

$\tau(t) = \gamma(t)$, and

$F(t) = \gamma(t) z_0(\theta_c, t)$.

Proof: In appendix.

In this case, how do the two policies of mandatory introduction of new monitoring technology versus voluntary adoption compare? First note that the voluntary policy leads to full adoption iff the lowest quality agent prefers to adopt monitoring technology given its costs and the values of the policy parameters in the steady-state:

$\pi(z_1(\underline{\theta}, t), \underline{\theta}) - \pi(z_0(\underline{\theta}, t), \underline{\theta}) - \tau(t) z_1(\underline{\theta}, t) + F(t) > v$ when $t \to \infty$. When full adoption occurs in the steady-state, the two policies differ only in the time path to obtain the steady-state:

Proposition 5: With no effects on the extensive margin, pollution build-up is faster under a policy of mandatory monitoring between $0 < t < T$ in the new externality case.

In the restoration case, the reduction of the existing pollution stock takes place at a smaller pace between $0 < t < T$ under a policy of mandatory monitoring.

Proof: The Proposition follows directly from the fact that $Z_0(F(t)) > Z_1(\tau(t))$, hence $\dot{S}_0(t) > \dot{S}_1(t)$.

FIGURE 1a ABOUT HERE

Figure 1a illustrates the pollution time path under the two policies for the new externality case. The pollution stock builds up faster initially under the mandatory monitoring policy, since the regulator can use only a franchise fee between time 0 and *T*, the date of the investment in the technology and introduction of mandatory monitoring. The voluntary policy yields a slower build-up of pollution over the entire time period, but will ultimately converge towards the same steady-state, if the conditions for full adoption of the new technology are satisfied. For the restoration case, the initial stock is reduced at a smaller pace with mandatory monitoring than under a policy of voluntary adoption of monitoring technology between time 0 and T (as illustrated in Figure 1b).

FIGURE 1b ABOUT HERE

When full adoption of the new monitoring technology is not obtained, the steady-state pollution stock will be higher under the policy of voluntary adoption compared to the policy of mandatory monitoring, but since monitoring costs are saved and agents' private surplus is larger, this is socially optimal.

7. Conclusions

The contribution of this paper is to develop a dynamic model for the management of a stock pollution problem through gradual diffusion of monitoring equipment when the regulator has incomplete information on emissions generated by heterogeneous agents.

The model developed here could be applied to a wide variety of externalities, but the analysis focused on the problem of stock pollution. Most analyses of nonpoint source pollution problems assume that the regulator is able to fully observe some variables correlated with individual emissions, such as input use or technology choice. In practice, however, very often the regulator cannot observe individual input or output and there is no proxy variable available to the regulator to build a policy based on agents' observable practices. For example, the regulator may observe quantity of output but not quality and increases in quality may require use of chemicals or other assets. In case of indoor production observation is particularly difficult. Thus, it may be worthwhile to invest resources in improving the monitoring of individual emissions directly. We observe the emergence of third party observers that monitor externalities, taking advantage of new technologies (remote sensing, nanotechnologies). The use of such services is becoming part of the regulatory process. Alternatively, regulation may demand that new monitoring technologies are installed (as was the case with Continuous Emissions Monitoring for the US sulfur dioxide emissions market). In both cases, regulators require monitoring with accurate reporting of externalities. The burden of proof is on the polluter who should provide estimation of damages, or otherwise is taxed at a conservative default.

The paper has analyzed two different types of regulation when it comes to the introduction of new information technologies enabling monitoring of the activities of individual agents. First, we studied the case of mandatory monitoring when all agents are forced to adopt a monitoring technology that enables the regulator to perfectly assess each polluter's emissions. In that case first-best externality-correcting taxes can be imposed. We derived the optimal timing of mandatory regulation and compared the outcome with the optimal solution at the steady-state. However, mandatory monitoring is not necessarily the optimal policy, since new monitoring technologies require costly investment and the investment cost may decrease over time. Thus, we also considered the case of voluntary monitoring. The proposed regulatory scheme differentiates taxation according to the installation of individual monitoring equipment. Each agent has to choose between paying a fixed fee, or installing monitoring technology and paying a tax on the actual generation of externalities. The highest quality agents will have incentives to adopt a monitoring technology if the gain from direct taxation is large enough to outweigh the profit impact and the direct monitoring costs. Since the taxes will evolve over time as the social shadow price of the externality changes, adoption of monitoring technology will also change accordingly. We analyzed the adoption pattern and identified conditions for the diffusion of monitoring technologies over time.

APPENDIX

Proof of Proposition 1.

A linearization of the canonical system of differential equations around the steady-state values of λ_0 and S results in:

$$\begin{pmatrix} \dot{\lambda}_0 \\ \dot{S} \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{\lambda}_0}{\partial \lambda_0} & \frac{\partial \dot{\lambda}_0}{\partial S} \\ \frac{\partial \dot{S}}{\partial \lambda_0} & \frac{\partial \dot{S}}{\partial S} \end{pmatrix} \begin{pmatrix} \lambda_0 - \lambda_0^{\infty} \\ S - S^{\infty} \end{pmatrix} = \begin{pmatrix} \alpha + r > 0 & -\frac{\partial^2 C}{\partial S^2} < 0 \\ \frac{\partial Z_0}{\partial \lambda_0} \leq 0 & -\alpha < 0 \end{pmatrix} \begin{pmatrix} \lambda_0 - \lambda_0^{\infty} \\ S - S^{\infty} \end{pmatrix}$$

Since the trace of the Jacobian matrix is equal to r>0, employing the fact that it equals the sum of its eigenvalues assures that at least one eigenvalue is positive. Additionally, the determinant of the Jacobian matrix is negative and thus, the eigenvalues have opposite signs and the steady state equilibrium is locally characterized by a saddle point. For any initial value of S within the neighborhood of S^{∞}, where the superscript ∞ indicates the steady state equilibrium value, it is possible to find a corresponding value of the shadow cost which assures that the optimal environmental policy leads towards the long-run optimum.

The slopes of the $\dot{\lambda}_0 = 0$ and the $\dot{S} = 0$ isoclines are:

$$\lambda_{0S}\big|_{\dot{\lambda}_0=0} = -\frac{-C_{SS}}{\alpha + r} > 0, \qquad \lambda_{0S}\big|_{\dot{S}=0} = -\frac{-\alpha}{Z_{\lambda_0}} < 0$$

A phase diagram of the system might look like the one depicted below.

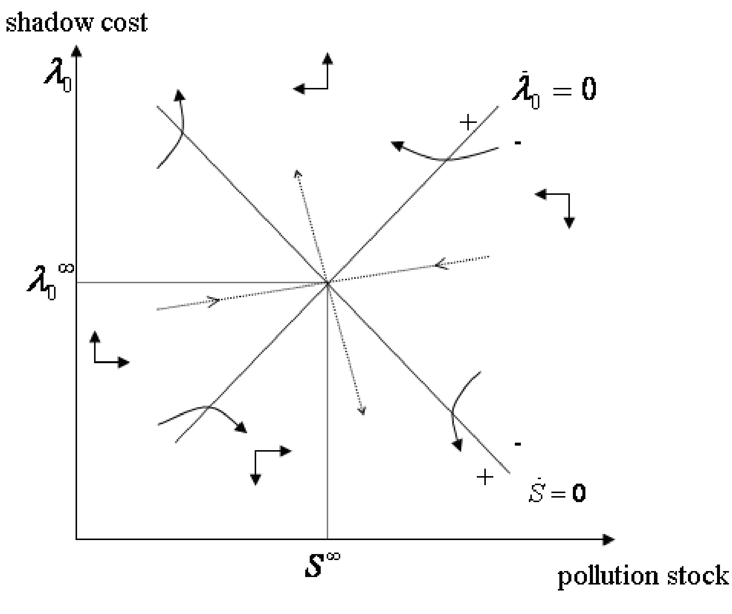

Figure: The phase diagram in the (S, λ_0) space.

It also follows that

$$\frac{\partial Z}{\partial t} = \frac{\partial Z}{\partial \lambda_0}\dot{\lambda}_0 + \frac{\partial Z}{\partial S}\dot{S} = \frac{\partial Z}{\partial \lambda_0}\dot{\lambda}_0 .$$

In addition, we have that $\frac{\partial F}{\partial \lambda_0} = \frac{\frac{\partial Z_0}{\partial F}}{\left[\frac{\partial^2 J_0}{\partial F^2} - \lambda_0 \frac{\partial^2 Z_0}{\partial F^2}\right]}$, with $\frac{\partial Z_0}{\partial F} < 0$.

$$\frac{\partial^2 J_0}{\partial F^2} = -\frac{\partial^2 \theta_0}{\partial F^2}\pi(\theta_0(F))g(\theta_0) - \left(\frac{\partial \theta_0}{\partial F}\right)^2 \frac{\partial \pi}{\partial \theta_0} g(\theta_0)$$

$$\frac{\partial^2 Z_0}{\partial F^2} = -\frac{\partial^2 \theta_0}{\partial F^2} z_0(\theta_0)g(\theta_0) - \left(\frac{\partial \theta_0}{\partial F}\right)^2 \frac{\partial z_0}{\partial \theta_0} g(\theta_0)$$

and

$$\frac{\partial^2 J_0}{\partial F^2} - \lambda_0 \frac{\partial^2 Z_0}{\partial F^2} = -\frac{\partial^2 \theta_0}{\partial F^2}\pi(\theta_0(F))g(\theta_0) - \left(\frac{\partial \theta_0}{\partial F}\right)^2 \frac{\partial \pi}{\partial \theta_0} g(\theta_0) + \lambda_0 \frac{\partial^2 \theta_0}{\partial F^2} z_0(\theta_0)g(\theta_0) + \lambda_0 \left(\frac{\partial \theta_0}{\partial F}\right)^2 \frac{\partial z_0}{\partial \theta_0} g(\theta_0)$$

$$\Leftrightarrow -\frac{\partial^2 \theta_0}{\partial F^2}\underbrace{\left(\pi(\theta_0(F)) - \lambda_0 z_0(\theta_0)\right)}_{A} g(\theta_0) - \left(\frac{\partial \theta_0}{\partial F}\right)^2 \underbrace{\left(\frac{\partial \pi}{\partial \theta_0} - \lambda_0 \frac{\partial z_0}{\partial \theta_0}\right)}_{B} g(\theta_0),$$

where term A is zero by the definition of the extensive margin under the optimal policy, and term B is positive since $\frac{\partial \pi}{\partial \theta_0} > 0$ and $\frac{\partial z_0}{\partial \theta_0} < 0$. Hence, $\frac{\partial F}{\partial \lambda_0} > 0$. Q.E.D.

Proof of Proposition 2.

Denoting the Lagrange multiplier by λ, the Lagrangian for the problem is:

$$L_1 = \int_{\theta_0(F_0)}^{\theta_c(F_0,F_1,\tau;v)} \left(\pi(z_0(\theta),\theta)\right) g(\theta)d\theta + \int_{\theta_c(F_0,F_1,\tau;v)}^{\bar{\theta}} \left(\pi(z_1(\theta),\theta) - v\right) g(\theta)d\theta +$$

$$+\lambda\left(Z - \int_{\theta_0(F_0)}^{\theta_c(F_0,F_1,\tau;v)} z_0(\theta)g(\theta)d\theta - \int_{\theta_c(F_0,F_1,\tau;v)}^{\bar{\theta}} z_1(\theta)g(\theta)d\theta\right)$$

The solution to the problem has to satisfy the following necessary conditions:

$$\begin{aligned}\frac{\partial L_1}{\partial F_0} = &-\frac{\partial \theta_0}{\partial F_0}\left\{\pi(z_0(\theta_0),\theta_0) - \lambda z_0(\theta_0)\right\} g(\theta_0) + \\ &+\frac{\partial \theta_c}{\partial F_0}\left\{\pi(z_0(\theta_c),\theta_c) - \pi(z_1(\theta_c),\theta_c) + v - \lambda(z_0(\theta_c) - z_1(\theta_c))\right\} g(\theta_c) = 0\end{aligned} \qquad \text{(A1)}$$

$$\frac{\partial L_1}{\partial F_1} = \frac{\partial \theta_c}{\partial F_1}\left\{\pi(z_0(\theta_c),\theta_c) - \pi(z_1(\theta_c),\theta_c) + v - \lambda(z_0(\theta_c) - z_1(\theta_c))\right\} g(\theta_c) = 0 \qquad \text{(A2)}$$

$$\frac{\partial L_1}{\partial \tau} = \frac{\partial \theta_c}{\partial \tau}\left\{\pi(z_0(\theta_c),\theta_c) - \pi(z_1(\theta_c),\theta_c) + v - \lambda(z_0(\theta_c) - z_1(\theta_c))\right\} g(\theta_c) +$$
$$+ \int_{\theta_c}^{\bar{\theta}} \left\{ \frac{\partial \pi(z_1(\theta),\theta)}{\partial z_1} - \lambda \right\} \frac{\partial z_1(\theta)}{\partial \tau} g(\theta) d\theta = 0 \qquad \text{(A3)}$$

$$\frac{\partial L_1}{\partial \lambda} = Z - \int_{\theta_0(F_0)}^{\theta_c(F_0,F_1,\tau;v)} z_0(\theta) g(\theta) d\theta + \int_{\theta_c(F_0,F_1,\tau;v)}^{\bar{\theta}} z_1(\theta) g(\theta) d\theta = 0 \qquad \text{(A4)}$$

Equations (15)-(17) in the article show that $\frac{\partial \theta_c}{\partial F_0}$, $\frac{\partial \theta_c}{\partial F_1}$ and $\frac{\partial \theta_c}{\partial \tau}$ are non-zero. Thus, for the necessary condition (A2) to hold we need to have

$\pi(z_0(\theta_c,t),\theta_c) - \pi(z_1(\theta_c,t),\theta_c) + v - \lambda\left(z_0(\theta_c,t) - z_1(\theta_c,t)\right) = 0 \quad \forall t$, and, by substitution in (A1); $\pi(z_0(\theta_0,t),\theta_0) - \lambda z_0(\theta_0,t) = 0$. By definition of the marginal quality, $\pi(z_0(\theta_0,t),\theta_0) - F_0(t) = 0$, and hence $F_0(t) = \lambda z_0(\theta_0,t)$. Given that $\frac{\partial z_1}{\partial \tau} < 0$ in the interval $\theta_c \le \theta < \bar{\theta}$, for (A3) to hold we need to have that $\frac{\partial \pi(z_1^*(\theta,t),\theta)}{\partial z} - \lambda = 0$. Private profit maximization leads to $\frac{\partial \pi(z_1^*(\theta,t),\theta)}{\partial z} = \tau$ and so, $\tau = \lambda$. Private profit-maximizing behavior further has that adoption is given by equation (14):

$\pi(z_1(\theta_c,t),\theta_c) - \pi(z_0(\theta_c,t),\theta_c) - \tau(t) z_1(\theta_c,t) - F_1(t) + F_0(t) - v = 0 \quad \forall t$

Comparing equation (14) with equation (A2) it can be observed that the regulator's optimal adoption decision coincides with the private agents iff

$F_0(t) - F_1(t) - \tau(t) z_1(\theta_c,t) - \lambda\left(z_0(\theta_c,t) - z_1(\theta_c,t)\right) = 0 \quad \forall t$. We thus have that $F_1(t) = \lambda\left(z_0(\theta_0,t) - z_0(\theta_c,t)\right)$. Q.E.D.

Proof of Proposition 3.

Prop. 3a) follows directly from the fact that $\tau = \lambda$ and that λ and S evolve over time in the same direction, hence $\dot{\tau} < 0$ iff $S_0 > S^\infty$.

Prop. 3b) and 3c): Let us redefine the policy scheme as a fixed fee on all agents $F_0 = \lambda z_0(\theta_0)$, and a subsidy (negative fee) $F_1 = -\lambda z_0(\theta_c)$ on agents that decide to adopt monitoring. In this case, applying the chain rule we obtain:

$\frac{\partial F_0}{\partial Z} = \frac{\partial \tau}{\partial Z} z_0(\theta_0) + \tau \frac{\partial z_0(\theta_0)}{\partial \theta_0}\frac{\partial \theta_0}{\partial F_0}\frac{\partial F_0}{\partial Z}$ and $\frac{\partial F_1}{\partial Z} = -\frac{\partial \tau}{\partial Z} z_0(\theta_c) - \tau \frac{\partial z_0(\theta_c)}{\partial \theta_c}\left(\frac{\partial \theta_c}{\partial \tau}\frac{\partial \tau}{\partial Z} + \frac{\partial \theta_c}{\partial F_1}\frac{\partial F_1}{\partial Z}\right)$

respectively. After some operations it gives:

$$\frac{\partial F_0}{\partial Z} = \frac{\frac{\partial \tau}{\partial Z} z_0(\theta_0)}{\left(1 - \tau \frac{\partial z_0(\theta_0)}{\partial \theta_0}\frac{\partial \theta_0}{\partial F_0}\right)} < 0, \qquad \text{(A5)}$$

$$\frac{\partial F_1}{\partial Z} = -\frac{\partial \tau}{\partial Z}\frac{\left(z_0(\theta_c) + \tau \frac{\partial z_0(\theta_c)}{\partial \theta_c}\frac{\partial \theta_c}{\partial \tau}\right)}{\left(1 + \tau \frac{\partial z_0(\theta_c)}{\partial \theta_c}\frac{\partial \theta_c}{\partial F_1}\right)} <> 0 . \qquad \text{(A6)}$$

since $-\frac{\partial \tau}{\partial Z} > 0$, $\frac{\partial F_1}{\partial Z} < 0$ if the nominator and denominator have opposite signs. In the case where $z_0(\theta_c) + \tau \frac{\partial z_0(\theta_c)}{\partial \theta_c}\frac{\partial \theta_c}{\partial \tau} > 0$ and $1 + \tau \frac{\partial z_0(\theta_c)}{\partial \theta_c}\frac{\partial \theta_c}{\partial F_1} < 0$, substituting the derivatives of θ_c leads, after some transformations, to:

$$\left.\begin{array}{l} \tau\left|\frac{\partial z_0(\theta_c)}{\partial \theta_c}\right|\frac{1}{\left(\frac{\partial \pi(z_1)}{\partial \theta_c} - \frac{\partial \pi(z_0)}{\partial \theta_c}\right)} < \frac{z_0(\theta_c, t)}{z_1(\theta_c, t)} \\ \tau\left|\frac{\partial z_0(\theta_c)}{\partial \theta_c}\right|\frac{1}{\left(\frac{\partial \pi(z_1)}{\partial \theta_c} - \frac{\partial \pi(z_0)}{\partial \theta_c}\right)} > 1 \end{array}\right\}$$

Since $\frac{z_0(\theta_c, t)}{z_1(\theta_c, t)} > 1$, $\frac{\partial F_1}{\partial Z} < 0$ iff $1 < \frac{\tau\left|\frac{\partial z_0(\theta_c)}{\partial \theta_c}\right|}{\left(\frac{\partial \pi(z_1)}{\partial \theta_c} - \frac{\partial \pi(z_0)}{\partial \theta_c}\right)} < \frac{z_0(\theta_c, t)}{z_1(\theta_c, t)}$.[5] Q.E.D.

Proof of Corollary.

Applying the chain rule, the change in θ_c is given by:

$$\frac{\partial \theta_c}{\partial Z} = \frac{\partial \theta_c}{\partial F_1}\frac{\partial F_1}{\partial Z} + \frac{\partial \theta_c}{\partial \tau}\frac{\partial \tau}{\partial Z} \qquad \text{(A7)}$$

since F_0 has been redefined as a fixed fee on all agents, and thus $\frac{\partial \theta_c}{\partial F_0} = 0$.

The substitution of equation (A6) into (A7) and simplifying the resulting equation leads to

$$\frac{\partial \theta_c}{\partial Z} = \frac{-\frac{\partial \tau}{\partial Z}\left(\frac{\partial \theta_c}{\partial F_1} z_0(\theta_c) - \frac{\partial \theta_c}{\partial \tau}\right)}{\left(1 + \tau \frac{\partial z_0(\theta_c)}{\partial \theta_c}\frac{\partial \theta_c}{\partial F_1}\right)} \qquad \text{(A8)}$$

Substituting in equations (16) and (17) from the text for $\frac{\partial \theta_c}{\partial F_1}$ and $\frac{\partial \theta_c}{\partial \tau}$ gives:

[5] Note that the case where the numerator is negative and the denominator is positive is not feasible.

$$\frac{\partial\theta_c}{\partial Z}=\frac{-\frac{\partial\tau}{\partial Z}\left(\frac{z_0(\theta_c,t)}{\left(\frac{\partial\pi(z_1(\theta,t),\theta)}{\partial\theta_c}-\frac{\partial\pi(z_0(\theta,t),\theta)}{\partial\theta_c}\right)}-\frac{z_1(\theta_c,t)}{\left(\frac{\partial\pi(z_1(\theta,t),\theta)}{\partial\theta_c}-\frac{\partial\pi(z_0(\theta,t),\theta)}{\partial\theta_c}\right)}\right)}{\left(1+\tau\frac{\partial z_0(\theta_c)}{\partial\theta_c}\frac{\partial\theta_c}{\partial F_1}\right)} \quad \text{(A9)}$$

Given that the numerator in (A9) is positive, for $\frac{\partial\theta_c}{\partial Z}$ to be greater than zero, the denominator needs also to be positive. Substituting the value of $\frac{\partial\theta_c}{\partial F_1}$ into the expression $\left(1+\tau\frac{\partial z_0(\theta_c)}{\partial\theta_c}\frac{\partial\theta_c}{\partial F_1}\right)$, we obtain that it will be positive iff

$$1+\frac{\tau\frac{\partial z_0(\theta_c)}{\partial\theta_c}}{\left(\frac{\partial\pi(z_1,\theta)}{\partial\theta_c}-\frac{\partial\pi(z_0,\theta)}{\partial\theta_c}\right)}>0\text{, i.e., iff }\tau\left|\frac{\partial z_0(\theta_c)}{\partial\theta_c}\right|<\left(\frac{\partial\pi(z_1(\theta,t),\theta)}{\partial\theta_c}-\frac{\partial\pi(z_0(\theta,t),\theta)}{\partial\theta_c}\right).$$

Q.E.D.

Proof of Proposition 4.

With a fixed extensive margin, the regulator's problem is:

$$\underset{F(t),\tau(t)}{Max}\int_0^{\infty}e^{-rt}\left(\int_{\underline{\theta}}^{\theta_c(\tau,F;v)}\pi(z_0(\theta,t),\theta)\,dG(\theta)+\int_{\theta_c(\tau,F;v)}^{\bar{\theta}}\pi(z_1(\theta,t),\theta)\,dG(\theta)-V-C(S(t))\right)dt$$

$$\text{s.t. }\dot{S}(t)=\int_{\underline{\theta}}^{\theta_c(\tau,F;v)}z_0(\theta,t)\,dG(\theta)+\int_{\theta_c(\tau,F;v)}^{\bar{\theta}}z_1(\theta,t)\,dG(\theta)-\alpha S\,.$$

The current-value Hamiltonian is

$$H=\int_{\underline{\theta}}^{\theta_c(\tau,F;v)}\pi(z_0(\theta,t),\theta)\,dG(\theta)+\int_{\theta_c(\tau,F;v)}^{\bar{\theta}}\pi(z_1(\theta,t),\theta)\,dG(\theta)-V\left(G(\bar{\theta})-G(\theta_c)\right)-C(S(t))-$$
$$-\gamma\left(\int_{\underline{\theta}}^{\theta_c(\tau,F;v)}z_0(\theta,t)dG(\theta)+\int_{\theta_c(\tau,F;v)}^{\bar{\theta}}z_1(\theta,t)dG(\theta)-\alpha S\right)$$

where γ represents the shadow value of the pollution accumulation constraint. By ignoring the non-negativity constraints initially and simply writing the first-order conditions for the Hamiltonian we obtain:

$-\frac{\partial C}{\partial S}+\gamma\alpha=\dot{\gamma}-r\gamma$ for the costate variable.

$$\frac{\partial \theta_c}{\partial F}\{\pi(z_0(\theta_c,t),\theta_c)-\pi(z_1(\theta_c,t),\theta_c)+v-\gamma(z_0(\theta_c,t)-z_1(\theta_c,t))\}g(\theta_c)=0 \quad \forall t$$

$$\frac{\partial \theta_c}{\partial \tau}\{\pi(z_0(\theta_c,t),\theta_c)-\pi(z_1(\theta_c,t),\theta_c)+v-\gamma(z_0(\theta_c,t)-z_1(\theta_c,t))\}g(\theta_c)+$$

$$+\int_{\theta_c}^{\bar{\theta}}\left\{\frac{\partial \pi(z_1(\theta_c,t),\theta_c)}{\partial z_1}-\gamma\right\}\frac{\partial z(\theta_c,t)}{\partial \tau}g(\theta)d\theta=0 \quad \forall t$$

Rewriting equation (14) to define the critical level of quality (θ_c) for which the agent is indifferent between renting monitoring equipment or not gives:

$$\pi(z_0(\theta_c,t),\theta_c)-\pi(z_1(\theta_c,t),\theta_c)+\tau(t)z_1(\theta_c,t)-F(t)+v=0 \quad \forall t$$

$$\frac{\partial \theta_c}{\partial F}=-\frac{1}{\left(\frac{\partial \pi(z_1(\theta,t),\theta)}{\partial \theta_c}-\frac{\partial \pi(z_0(\theta,t),\theta)}{\partial \theta_c}\right)}<0 \qquad \text{(A10)}$$

$$\frac{\partial \theta_c}{\partial \tau}=\frac{z_1(\theta_c,t)}{\left(\frac{\partial \pi(z_1(\theta,t),\theta)}{\partial \theta_c}-\frac{\partial \pi(z_0(\theta,t),\theta)}{\partial \theta_c}\right)}>0 \qquad \text{(A11)}$$

Since equations (A10) and (A11) show that $\frac{\partial \theta_c}{\partial F}$ and $\frac{\partial \theta_c}{\partial \tau}$ are non-zero, we have that the regulator's optimal choice of monitoring adoption is characterized by $\pi(z_0(\theta_c,t),\theta_c)-\pi(z_1(\theta_c,t),\theta_c)+v-\gamma\left(z_0(\theta_c,t)-z_1(\theta_c,t)\right)=0 \quad \forall t$ and can deduce that if $\tau(t)=\gamma(t)$ and $F(t)=\gamma(t)z_0(\theta_c,t)$, then the agents' choices will coincide with the regulator's. Q.E.D.

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Figure 1a. The new externality case

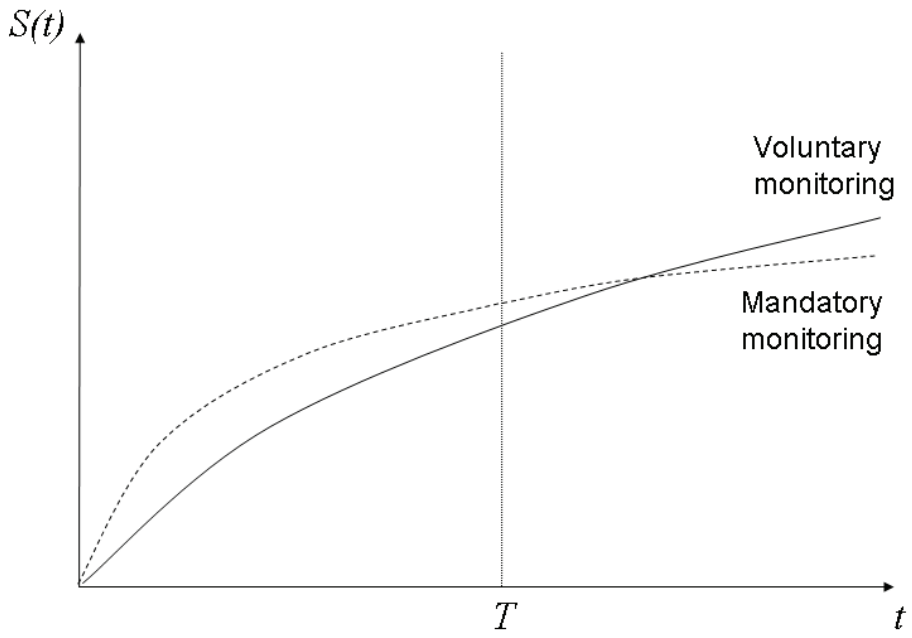

Figure 1b. The restoration case