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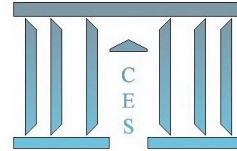
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Relevancy of the Cost-of-Capital Rate

for the Insurance Companies

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Abstract

For many assets and liabilities there exist deep and liquid markets so that the market value
are readily observed. However, for non-hedgeable risks, the market value of liabilities must
be estimated. The Draft Solvency II Directive suggests in article 75 that the valuation of
technical provisions (for non hedgeable risks) shall be the sum of a best estimate and a mar-
ket value margin measuring the cost of risk. The market value margin is calculated as the
present value of the cost of holding the solvency capital requirement for non-hedgeable risks
during the whole run-off period of the in-force portoflio. One of the major input of the mar-
ket value margin is the cost-of-capital rate which corresponds to the risk premium applied
on each unit of risk. According to European Commission (2007), European Insurance and
Reinsurance Federation (2008), and Chief Risk Officer Forum (2008), a single cost-of-capital
rate shall be used by all insurance undertakings and for all lines of business. This paper aims
at analyzing the cost-of-capital rate given by European Insurance and Reinsurance Federa-
tion (2008), and Chief Risk Officer Forum (2008). In particular, we highlight that it is very
difficult to assess a cost-of-capital rate by using either the frictional cost approach or the full
industry information beta methodology. Nevertheless, we highlight also that it seems to be
irrelevant to use only one risk premium for all the risks and all the companies. We show that
risk is not characterized by a fixed price. In fact, the price of risk depends on the basket of
risks at which it belongs, the risk level considered and the time period.

Keywords: market value margin, cost-of-capital rate, diversification effect, risk level

JEL classification: G12, G20, G22, G32.

1 Introduction

For ten years, the European Commission, actuarial associations and supervisory authorities have been focused on improving the efficiency and effectiveness of the oversight of insurance entities. In parallel, discussions in the field of financial accounting have developed an explicit framework whom one of the main component is the valuation of the financial securities to their fair value. In order to improve shareholder information, willingness to give a major rule to market prices seems to be obvious because they are considered as information vector.

For many assets and liabilities there exist deep and liquid markets so that the market value are readily observed. However, for non-hedgeable risks, the market value of liabilities must be estimated. The Draft Solvency II Directive suggests in article 75 that the valuation of technical provisions (for non hedgeable risks) shall be the sum of a best estimate and a market value margin. In order to determine it, a cost-of-capital methodology should be used. It bases the risk margin on the theoritical cost to third party to supply capital to the company in order to protect against risks to which it could be exposed. Under the Cost-of-Capital approach, the market value margin is determining by capturing the cost of providing an amount of eligible own fund equal to the solvency capital requirement (SCR) necessary to support insurance obligations over their life time. In other words, under a cost of capital approach, the market value margin is calculated as the present value of the cost of holding the solvency capital requirement for non-hedgeable risks during the whole run-off period of the in-force portoflio. Thus, one needs to estimate both the solvency capital requirement related to non-hedgeable risk and the annual cost of capital rate. The cost of capital in each year would be given by the product of the solvency capital requirement of each year and the underlying cost of capital rate. The market value margin is then obtained by discounting these amounts:

$$MVM = CoC \times \sum_t \frac{SCR_t}{(1 + r_t)^t},$$ (1.1)

where r_t stands for the risk-free rate.

According to European Commission (2007), European Insurance and Reinsurance Federation (2008), and Chief Risk Officer Forum (2008), a single cost-of-capital rate shall be used by all insurance undertakings and for all lines of business. For example, QIS 4 Technical Specifications stand that all participants should assume that this cost-of-capital rate is 6% (above the relevant risk-free interest rate), following the figure of the Swiss Solvency Test (see for example European Insurance and Reinsurance Federation (2008)). On the contrary Chief Risk Officer Forum (2008) suggests that the cost-of-capital rate should be in a range of 2.5%-4.5% per annum.

Nevertheless, with these results, some questions may be highlighted. How is it possible to calibrate such a cost-of-capital rate? Is it relevant to use the same cost-of-capital rate for all the insurance companies as it is assumed by the European Commission and the Chief Risk Officer forum? Is it relevant to use the same cost-of-capital rate for all the lines of business? Is it relevant to use the same cost-of-capital rate for all the risks?

The paper is organized as follows. Section 2 presents some methods which allow to capture the cost-of-capital rate and analyzes the assumptions of Chief Risk Officer Forum (2008). Section 3 highlights the main elements which allow to understand the differences in the betas previously identified. Section 4 introduces another method in order to calibrate the cost-of-capital rate and points out some other factors which have to be included in the valuation of the cost of risk. Lastly, section 5 concludes.

2 Different methods to capture the cost-of-capital rate

In the corporate finance literature, the cost of capital rate is often defined as the weighted average of the cost of equity capital and cost of debt capital. Such a methodology is based on the equity risk premium and on the cost of debt. Chief Risk Officer Forum (2008) describes two methods based on the capital asset pricing model and on the frictional cost approach in order to obtain an approximation of the equity risk premium.

2.1 Capital Asset Pricing Model and Fama-French Two Factors Model

2.1.1 Methodology

The Capital Asset Pricing Model and the Fama-French two factors models is a methodology used by Chief Risk Officer Forum (2008) to provide with some estimators for the equity risk premium. The CAPM of Sharpe and Linter, the most popular model to estimate the cost of equity of publicly traded companies, expresses the cost of capital rate as the sum of the risk-free rate and a market risk premium based upon the systematic risk. The Fama-French two factors model has been developped in order to improve the explanation power of the model by adding a second factor to the model. This factor is the ratio of the book value of equity to the market value of equity (BV/MV ratio). This ratio reflects the financial distress. The financially vulnerable firms have higher values of this ratio than stronger firms. This factor controls for the tendency of investors to require higher expected returns on stocks in financially vulnerable firms since these firms will perform particularly poorly exactly when individual investors' portfolio are experiencing overall losses.

In order to determine a pure cost of capital rate for life and non-life insurance, and to take into account the fact that most of the companies participate in both industries, it is necessary to reflect the relative proportion of their entire portfolio of businesses. Accordingly, Cummins and Phillips (2005) (after Ehrhardt and Weiss (1992) and Kaplan and Peterson (1998)) use the full-information industry beta which allows to decompose the individual company cost estimates into industry specific cost estimates. The underlying insight is that the observable beta for the overall firm is a weighted average of the unobservable betas of the underlying lines of business.

In other words, following the CAPM the cost of capital rate is as follows:

$$r_{it} - r_{ft} = \alpha_i + \beta_i \times (r_{mt} - r_{ft}) + e_{it}, \tag{2.1}$$

whereas for the FF2F model it can be expressed as:

$$r_{it} - r_{ft} = \alpha_i + \beta_{mi} \times (r_{mt} - r_{ft}) + \beta_{vi} \times \pi_{vi} + \eta_{it}, \tag{2.2}$$

where r_{it} is the monthly return on the stock of the firm i in month t, r_{ft} is the corresponding risk-free rate, r_{mt} is the monthly return on the value-weighted market portfolio , π_{vt} is the additional factor included to capture financial distress, β_i is the estimated CAPM beta, β_{mt} and β_{vt} are the estimated market and value risk coefficients, e_{it} and η_{it} are the error terms.

In the case of the CAPM, the FIIB regression estimated is:

$$\beta_i = \sum_{j=1}^{J} \beta_{fj} \omega_{ij} + \xi_i, \tag{2.3}$$

with β_i = firm i's overall CAPM market systematic risk beta coefficient, β_{fj} = the full-information CAPM market systematic risk beta for industry j, ω_{ij} = firm i's industry participation weight for industry j, and ξ_i = random error term for firm i.

In the case of the FF2F systematic risk factor, the FIIB regression is exactly the same: $\beta_{mi} = \sum_{j=1}^{J} \beta_{fmj} \omega_{ij} + \nu_i$. Cummins and Phillips (2005) show that the relevant FIIB regression for the BV-to-MV beta is:

$$\beta_{vi} = \sum_{j=1}^{J} \beta_{f1vj} \omega_{ij} + \beta_{f2v} ln \frac{BV_i}{MV_i} \epsilon_i, \tag{2.4}$$

with $\beta_v i$ = BV-to-MV beta estimate firm i, β_{f1vj} = the full-information BV-to-MV beta intercept coefficient for industry j, β_{f2v} = the full-information BV-to-MV beta slope coefficient, ω_{ij} = firm i's industry participation weight for industry j and ϵ_i = random error term for firm i.

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2.1.2 Computation

The computation is only done for the European quoted insurance companies from 2000 to 2007. To make the results comparable, all the returns are converted in US dollars. The data relative to the individual returns come from Bloomberg. The risk premia relative to the financial distress come from the Fama-French database whereas the business mix were obtained from the Eurothesys database. The market index is country specific. The risk-free rate is the Libor 1-Month expressed in US dollars. Table 1 provides the different considered states and the number of companies per state and per year.

	2000	2001	2002	2003	2004	2005	2006	2007	Market Index
BE	3	3	3	2	2	2	2	2	BEL20 Index
CH	6	6	7	7	7	7	7	7	SMI Index
DE	16	15	15	14	14	16	14	14	HDAX INDEX
ES	1	1	1	1	1	1	1	1	MADX Index
FI	2	2	2	2	2	2	2	1	HEXP Index
FR	5	4	4	4	3	4	4	5	SBF250 Index
GB	15	15	18	17	19	19	18	17	NMX Index
IE	1	1	2	2	2	2	2	2	ISEQ Index
IT	12	12	9	9	10	11	11	10	SPMIB index
NL	4	3	3	3	3	2	2	2	AEX Index
NO	1	1	1	1	1	1	1	1	OSEAX Index
SE	3	4	4	5	5	5	5	4	SAX Index
Europe	69	67	69	67	69	72	69	66	

Table 1: Number of observations by country and by year - Relevant indices.

The length of the sample is quite stable over years. In addition to that, it appears clearly that the Great Britain, Germany and Italia are the more represented states in our sample. Even if the French market is a major European insurance market, just some companies are quoted, reflecting the fact that a lot of insurance activities are realized by mutual insurance companies. In this case, the cost of capital rate is not computed.

The estimation is done in three steps. Firstly, the individual beta of each insurance company is computed. Then, the beta of each line of business is estimated following the business mix of the insurance companies. Lastly, the cost of capital rate is calculated by using long run historical returns as presented in Chief Risk Officer Forum (2008):

- Excess market return : 7.81%,

- BV-MV (Value) risk premium : 4.92%.

The results for non-life and life segments are presented in table 2 and 3. The cost of capital is much higher for life insurance than for non-life. The β market risk is much lower for the non-life insurance than for the life insurance reflecting the fact that life insurance is more exposed to the

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Year	β Market.Risk	β Value.Risk	Risk Premium Market	Risk Premium Value	Total Premium
2000	0.772	0.293	6.030	1.670	7.700
2001	0.872	0.277	6.809	1.576	8.385
2002	0.907	0.310	7.083	1.766	8.848
2003	0.997	0.336	7.784	1.915	9.699
2004	0.959	0.328	7.489	1.867	9.356
2005	1.033	0.231	8.064	1.320	9.384
2006	1.111	0.200	8.677	1.142	9.819
2007	1.048	0.146	8.184	0.830	9.014

Table 2: Cost of capital rate for non-life insurance following the Full-Industry Information Beta methodology.

Year	β Market.Risk	β Value.Risk	Risk Premium Market	Risk Premium Value	Total Premium
2000	1.146	0.227	8.948	1.296	10.244
2001	1.231	0.179	9.615	1.021	10.636
2002	1.253	0.184	9.783	1.049	10.832
2003	1.677	0.292	13.098	1.663	14.761
2004	1.720	0.403	13.432	2.298	15.730
2005	1.849	0.380	14.441	2.165	16.606
2006	2.033	0.283	15.875	1.614	17.489
2007	2.078	0.353	16.225	2.010	18.236

Table 3: Cost of capital rate for life insurance following the Full-Industry Information Beta methodology.

market risk. Both are more exposed to the market risk than to the "value" risk. The market risk β is in a range 0.772-1.111 for life insurance whereas the value β is in a range 0.146-0.336. The results are quite similar for life insurance: 1.146-2.078 against 0.179-0.403. Thus, the total risk premium is mainly driven by the market risk premium. For life insurance the market risk premium explains at least 85% of the total risk premium. For the non-life insurance, the ratio is around 80%.

The results are comparable to the one get by Chief Risk Officer Forum (2008). However, they seem to be globaly lower and more stable, in particular for the non-life insurance. Indeed, our total risk premium is in a range 7.7%-9.8% whereas Chief Risk Officer Forum (2008) gets a total risk premium between 5.66% and 10.86% for the same time interval. These differences may be explained by some differences in the initial company sample and the use of country-specific indices instead of an global European index as in Chief Risk Officer Forum (2008).

2.1.3 Limitations

The Full-Information Industry Beta methodology provides with an estimate of the cost-of-capital rate. Nevertheless, compared to the CEIOPS (2008), such an estimate is not very relevant, both in terms of "risk premia" considered and in terms of underlying capital.

"Risk premia" issues   Firstly, as presented in Chief Risk Officer Forum (2008), the cost of capital for the market value margin is not equivalent to the total return required by shareholders. Not all the realized equity risk premium may be related to the pure risk premium due to risky liabilities. Part of the premium is for example due to the asset liability management policy or to the future profit expectations. Thus the cost of capital rate computed from the Full-Information Industry Beta methodology appears to be not completely relevant to valuate the market value margin of the in-force business.

In addition to that, the market risk premia may also be discussed. In order to calibrate the betas and the risk premia we use, after Chief Risk Officer Forum (2008) and Cummins and Phillips (2005), a long run historical returns computed in the total market, with companies reflecting all the economy sectors. Nevertheless, we may be interesting to know if such a risk premium is relevant for insurance companies.
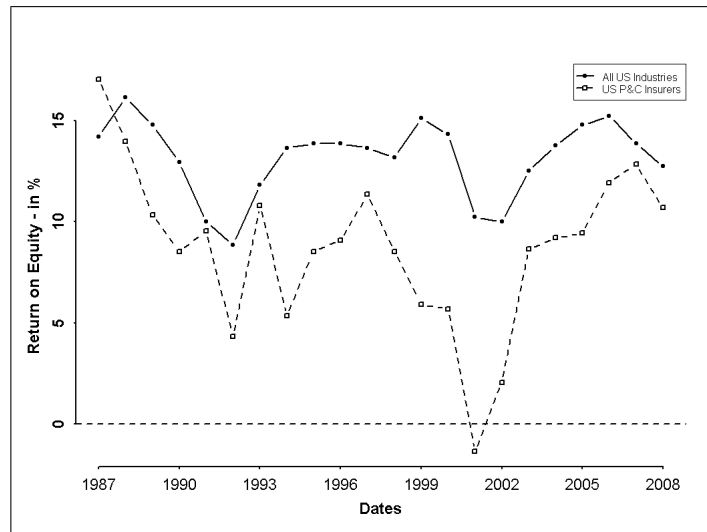

Figure 1: Return on Equity of all US Industries and US P&C Insurers.
source: Insurance Information Institute.

Figure 1 provides with the return on equity (ROE) of all US industries and US P&C Insurers from 1987 and 2008. It appears clearly that the ROE of US insurers is always, excepted in 1987, below the global ROE of all the industries. The gap may be so high that the ROE of US insurers is negative whereas the global economy remains profitable, like in 2001. Moreover, the ROE of US insurers is more volatile than the ROE of all the industries, reflecting the evolutions of the economy as a whole but also more particular issues. For example, all major troughs of the curve may be related to catastrophic events, like Andrew (in 1992), the Northridge earthquake (in 1994) or the WTC attack (in 2001). On the contrary the good results of 1997 are due to the lowest catastrophic events in 15 years.

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That means that the long run historical return calibrated in the total market does not reflect the return required by investors who purchase insurance assets. Two main interpretations are possible. Either the true required return for insurance assets is lower than the long run historical return calibrated in the total market in order to take into account the fact that the ROE for insurance industry is lower than the ROE for all industries. Or, the required return for insurance assets is higher in order to remunerate investors who purchase assets characterized by a higher volatility.

According to us, the risk premia for the insurance companies has to be lower than the risk premia of the other sectors. Firstly, the core insurance business is quite orthogonal to the rest of the economy. Indeed, catastrophe losses, legal changes, or car damages are not directly linked to the state of the economy. Thus, insurance companies introduces a diversification effect in the investor portfolio which has to be remunerated by a lower risk premium. In addition to that, insurance activity is characterized by a time difference between the premium collecting and the losses payment. Thus, premiums provide liquidity with to the insurance companies which allow them to develop highly profitable asset management actitivity. Accordingly, the cost of capital rate applied on pure insurance business must neither to include an additional return for asset management activity, and neither an additional return for the asset risk, because we are focusing on the pure insurance business.

Underlying capital issues Following 1.1, the market value margin is calculated as the present value of the cost of holding the solvency capital requirement during the whole run-off period of the in-force portfolio. Thus, according to us in such a framework, the cost-of-capital rate has to be consistent with the underlying capital. At this stage, the Fama-French methodology is characterized by two main drawbacks: first, premia do not reflect the allocated risk capital and second, the return appears to be constant whatever the return period.

A major issue in the Full-Information Beta methodology is the determination of the components of the betas of both the market and value risk premia. Indeed, these components are computed using the business mix of each insurance company. Nevertheless the business mix does not reflect exactly the part of each segment in the total allocated capital risk of the company. The annual financial report of the Allianz Group illustrates this point:

- in 2007, the statutory premiums were closed to €44 millions for the property-casualty insurance operations against €49 millions for life/health operations.

- nevertheless, the risk report shows that the allocated internal risk capital is mainly driven by property-casualty: the diversified risk capital is equal to €18 millions for these activities

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whereas is it only of €5 millions for life/health activities.

Thus, following the basic approach of the FIIB methodology, the weights ω_{if} would be respectively 47% and 53% for non-life and life insurance whereas it would be 78% and 22% following an approach based on the relevant exposed capital. This would cause some major differences in the computation of the betas and of the risk premia per lines of business. Unfortunately, such information is not available for all the companies of our sample. Thus, we are not able to assess the consequences of using the allocated risk capital instead of the business mix in the computation of the FIIB methodology.

Another limitation of the Full-Information Industry Beta methodology is that it does not allow the cost of capital rate be varying with the confidence interval at which capital requirements are calculated. Even if Solvency II consider only one cost of capital rate, such an approach is irrelevant in order to optimize business of an insurance company.

2.2 The frictional cost approach

2.2.1 Basic Results

Another method to compute the cost of capital rate is based on the frictional cost approach. This methodology allows to cope with the previous identified issues. Following Hancock et al. (2001), *the frictional capital costs represent the insurer's cost of taking insurance risk and capture the opportunity costs shareholders incur when investing capital via an insurance company rather than directly in the financial markets.* The double taxation costs, the financial distress costs, the agency costs and the costs of regulatory restrictions are the sources of the frictional costs. However, Chief Risk Officer Forum (2008) assumes that the agency and regulatory restriction costs have not to be taken into account. Thus, the cost of capital appears to be the sum of the double taxation costs and of the financial distress costs: CoC_{DT} and CoC_{FD}.

Let α be the confidence interval at which the firm is capitalized, $EC(\alpha)$ the actual capital of the firm and $CoC\left(EC\left(\alpha\right)\right)$ the cost of capital rate. In order to model it, the following assumptions are done:

- the P&L is normally distributed, with mean $EC(\alpha) \times Rf$ and volatility σ, where R_f is the risk-free rate;

- we denote by x the annual result of the firm;

- τ is the corporate tax rate;

- T_{tax} is the tax carry forward period in years.

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The double taxation costs are the following:

$$CoC_{DT} = \frac{\tau}{T_{tax}} \int_0^\infty x \cdot \varphi_{T_{tax} \times EC(\alpha) \times Rf, \sigma\sqrt{T_{tax}}}(x)dx, \tag{2.5}$$

Following the assumption that if the firm's equity drops below the SCR, the company needs to obtain new capital, the financial distress costs may be expressed by:

$$CoC_{FD} = \int_{-\infty}^0 (SCR - x) \times f^\star_{EC(\alpha)}(x)dx, \tag{2.6}$$

where $f^\star_{EC(\alpha)}(x)$ is the risk-neutral probability density function of the annual P&L, given by:

$$f^\star_{EC(\alpha)}(x) = \mathcal{Q}\left(\Phi^{-1}\left(f_{EC(\alpha)}(x) + \lambda\right)\right), \tag{2.7}$$

with $\mathcal{Q}$, the Student-t distribution with degree of freedom k and λ the Sharpe ratio.
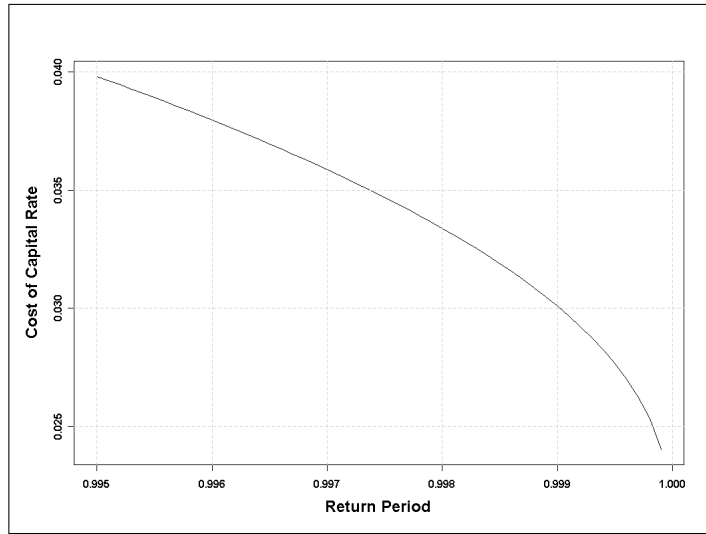
Figure 2: Cost of capital rate per return period following a frictional cost approach.

This calibration method for the cost of capital rate allows to obtain a capital charge in the range 2.5%-4.5%. Nevertheless, the method presents some limits which would highly increase the cost of capital rate.

2.2.2 Analysis of the Frictional Cost Approach

The frictional cost approach allow to compute a cost-of-capital rate more or less consistent with the Solvency II framework. Some limitations, however, characterize the model due to the frictional costs taken into account or to the technical assumptions.

Agency Costs The agency costs are due to the misalignment of the interests of senior management and shareholders. Chief Risk Officer Forum (2008) does not modelled them because it assumes that they are more related to the franchise value than to non-hedgeable risk on the in-force business. In addition to that, it believes that the supervisor scrutiny help management to meet its obligations, in particular in situation of financial distress. Even if Chief Risk Officer Forum (2008) highlights the fact that the misalignment should be reflected in a higher cost of capital rate, it decided that it was inappropriate to model them. This point of view is really difficult to interpret.

First of all, the uncertainty is a major component of the cost of capital rate. That may be highlighted by the insurance linked securities market (see for example Gatumel and Guégan (2008b)). Before Katrina, the spreads in the market are almost stable, slightly decreasing. The hurricane causes a major change in the market. The spreads blow up immediately after the catastrophe and they are characterized by an increasing trend during one year. We may explain this behaviour as follows. Before Katrina, investors were interested by the trade of this type of risk and quite confident in the issued risk. But Katrina, and the first default of a bond, changed the risk perception. And even if the traded risk was similar, investors required a higher return in order to buy it. We may easily assume that it would be quite similar for an insurance company. As long as it is not characterized by financial distress, investors would require a return driven by the financial perspectives. But an adverse signal would not only modify them but also wouldn't put them at ease relatively to the future. Thus, investors would require a higher return which is not captured by the methodology of Chief Risk Officer Forum (2008). And even regulatory oversight is likely to increase, it would affect future business and future profit opportunities of the investors. Indeed, we may easily expect that the regulatory oversight would be linked to a very prudential strategy and thus to a limited return on equity. That phenomen would reinforce the increase trend of the cost of capital at which the company would find money. That is for example clearly the case with the companies which have been restructurated following the 2008 financial crisis. Their share prices is very low reflecting the fact that the investors require a high risk premium in order to purchase them.

The argument of Chief Risk Officer Forum (2008) is that the agency costs are more or less taken into account through parameters λ and k of the risk-neutral distribution. We agree with that. However, it appears that the parameters chosen are irrelevant. Indeed, as explained in Wang (2004) the Student-t distribution is used in order to take into account the uncertainty in the parameter estimation. The uncertainty is due to the limited available data, to the uncertainty in the estimation of catastrophe losses and to the fear of the investors. Consistently, we may expect a decreacreasing link between the market price of risk and the degrees of freedom of the Student-t

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distribution. Higher would be k, more Gaussian would be the Student-t distribution, reflecting a lower uncertainty and thus, lower would be the risk premium required by the investors. On the contrary, with a high uncertainty, k would be small and the market price of risk would be high. That does not appear with the parameters chosen by Chief Risk Officer Forum (2008). The parameters (λ, k) are respectively $(0.75, 0.6)$ and $(0.45, 5)$. With the results presented in Gatumel and Guégan (2008b) we get some different results provided by Figure 3.
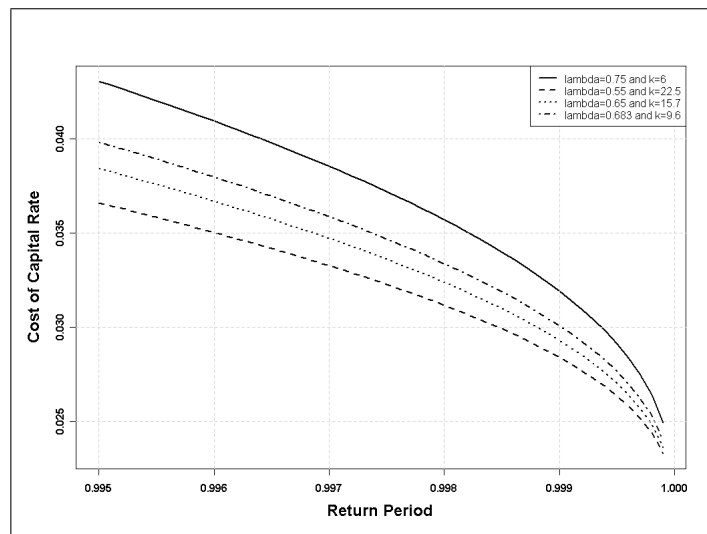
Figure 3: Cost of capital rate per return period following a frictional cost approach and for some λ and k.

It appears that the cost of capital rate is an increasing fonction of the parameters λ. Higher is the market price of risk in the insurance linked securities market, higher is the cost of capital rate, whatever the return period. Our results differ slightly from the results of Chief Risk Officer Forum (2008). We get a range 2.4%-4%, mainly due to the fact that our computation does not provide with a λ parameter as much high as 0.75.

Technical assumptions   The first technical issue of the frictional cost approach is the assumption of a P&L distributed following a Gaussian distribution. We understand that it is a traditional and practical assumption. Nevertheless, it seems to be not appropriate. We consider that if we want to capture the behaviour of the solvency capital requirement, we have to capture the real beaviour of the tails of the P&L distribution. A Gaussian distribution does not allow that because it does not allow to capture the fact that insurance industry is characterized by the fact some adverse events occur more often than expected. Thus, the tails of the P&L distribution would by ticker than those of the Gaussian distribution. It is for example the case in the insurance linked securities market (see Gatumel and Guégan (2008a)). We can propose

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to use a Normal Inverse Gaussian distribution computed on some P&L historical data in order to take into account this phenomenom.

With a non-Gaussian distribution, the double-taxation cost component of the fricitional cost approach may be discussed. Indeed, the cost of capital rate is a return applied on the solvency capital requirement in order to capture the market value margin. In others words, it is a return applied on a necessary capital to support insurance obligations over their life time. By definition, this capital would be used in order to cover some adverse events like a major deviation of the past reserves or some underwritting issues. Mispricing or unexpected major catastrophic events may explain such a result. Thus, the solvency capital requirement aims at covering a *negative* result, that means a result not directly concerned by tax issues. With a Gaussian P&L assumption, a tax carry forward period seems to be relevant because tax on past negative result may be used in order to reduce tax on future positive result. But, with a P&L distribution having tick tails, negative results of one year cannot be used to reduce future tax issues. We may illustrate that point through AIG bankruptcy. The negative results of the insurer does not create deferred tax because of the bankruptcy of the group.

Full-Information Industry Beta and frictional cost approaches provide with two very different cost-of-capital rate estimates, both in terms of result – we get ranges 7.7%-9.8% and 2.5%-4.5% – and in terms of consistency with the Solvency II framework.

3 Is it relevant to have the use the same cost-of-capital rate whatever the insurance company?

Previous methodologies allow to compute a cost of capital rate for the insurance companies, but without really considering them. Indeed, the frictional cost of capital approach is based on the major assumption that all the companies have the same characteristics which may be summarized by some elements. Among them, we may cite the for instance the volatility of the P&L or the corporate tax rate. Similarly, the Full-Information Industry Beta assumes that all the beta determinants are due to the business-mix of the insurance companies, without considering other elements: size of the companies, structure of the asset side of the balance sheet, origin country, etc.

Nevertheless, the individual cost of capital required by the investors varies from one company to the other. That is reflected in various individual betas relative both to the market risk premium and to the value risk premium. Table 4 provides with some estimates for a selection of insurance companies. The highest beta for the market risk premium is equal to 1.713 (Skandia Insurance Group) whereas the lowest is 0.778 (Mannheimer AG Holding). Similarly, the highest beta for

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the value risk premium is 0.762 (Baloise) whereas the lowest is -0.321 (Prudential Group). In other words, betas may vary a lot from one company to this other.

	β Market Risk	β Value Risk
ALLIANZ AG	0.924	-0.230
AXA	1.288	0.400
BALOISE HOLDING	1.439	0.762
MANNHEIMER AG HLDG	0.778	-0.035
PROVIDENT FINL GROUP	1.544	-0.321
PRUDENTIAL PLC	0.789	-0.139
SKANDIA	1.713	0.003
ST JAMES PLACE	1.318	0.203

Table 4: Market Risk and Value Risk β for some selected companies.

In order to study the reason why betas are different or what are the main factors which have to be taken into account when valuating an insurance company, we can use the traditional principal component analysis. Based on the companies previously used in our Full-Information Industry Beta computation, it aims at discriminating between identified factors:

- structure of the asset side of the balance sheet,

- structure of the liability side of the balanche sheet,

- registration country,

- business mix,

- market indicators.

The structure of the assets is captured through the percentage of variable yield securities and the percentage of fixed income securities in the company portfolio. These variables allow to determine the risk profile of the assets and the potential return they are able to provide. For instance, we may expect that higher will be the share of the variable yield securities, higher will be the expected return. On the contrary, the fixed income securities woul certainly give loward reward. But, obsviously, the volatility of the total result will be higher if the share of the variable yield securities is high. The structure of the liability side is given by the percentage of technical provisions in the balance sheet. Higher is this percentage, lower will be the capital available which aims at covering those liabilities. The business mix is exactly the same as previously. Lastly, the main market indicators are the price to book ratio and the current market capitalization.

Because the variable are in different units, the computation is done on centered and scaled data. The results are provided in table 5 and in Figure 4. Table 5 summarizes the position of each variable on each axis of analysis. Figure 4(a) is the diagramm relative to the eigen values.

Figures 4(b) to 4(d) provides with the representation of the variables and the projection of each company in the principal plan.

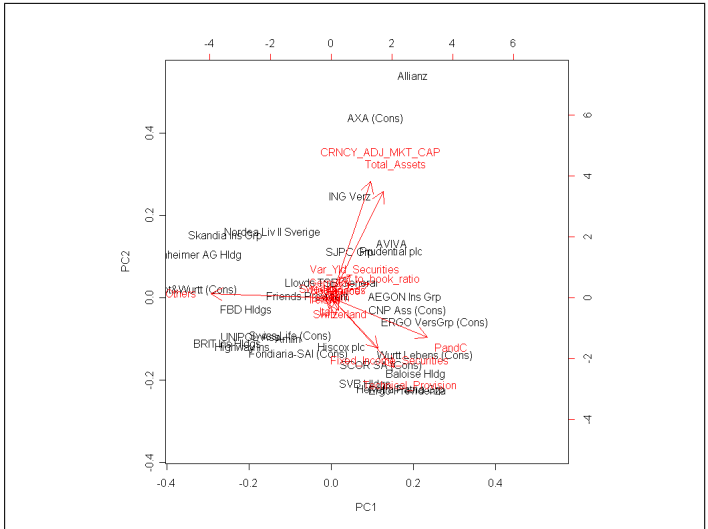|	PC1	PC2	PC3
Var. Yield Securities	0.097	0.118	-0.496
Fixed Income Securities	0.245	-0.270	0.422
Total Assets	0.272	0.567	0.266
Technical Provisions	0.330	-0.374	0.257
Belgium	0.000	0.000	0.000
France	0.055	0.032	0.063
Germany	-0.004	0.062	-0.071
Ireland	-0.027	-0.004	-0.014
Italy	-0.012	-0.062	0.057
Netherlands	0.030	0.034	0.038
Sweden	-0.053	0.042	-0.027
Switzerland	0.037	-0.068	0.037
P&C	0.504	-0.211	-0.063
Others	-0.634	0.020	0.199
Price to Book Ratio	0.200	0.078	-0.564
Market Capitalization	0.208	0.620	0.246

Table 5: Coordinates of the variables on the three axis.

According to the scree test, it seems to be relevant to retain three axis of analysis. They allow to explain 71% of the inertia. The extreme values of the axis 1 are given by the variables PandC and Others which are relative to the business mix of the insurance companies. Clearly, it appears that this axis distinguishes the type of business realized by the companies. In other words, axis 1 is relative to a diversification effect between source of insurance risks. For example, this axis highlights the differences of business realized by the Skandia Insurance Group (mainly in life and saving) and the Baloise company (mainly in private property). The second axis is characterized by very high values for the variables "total assets" and "market capitalization". The size of the companies is there highlighted. Thus, Figure 4(d) shows the difference between companies like Axa or Allianz, leaders of the insurance market, and the other. Finally, the third axis is characterized by high values for "Variable Yield Securities" and "Fixed Income Securities". In others words, the third variable to take into account in the analysis is the structure of the asset side of the insurance companies.

PC1 is driven by the business mix. Nevertheless, it appears clearly that companies with comparable business mix are characterized by very different betas. For example the betas of Axa and Prudential are pairwise very different. The β relative to the market risk premium are respectively equal to 1.288 and 0.789 whereas the β relative to the value risk premium are respectively equal to 0.400 and -0.139. These results clearly highlight that if the business-mix is a key characteristics of a firm, it is not a key determinant of its valuation. According to us, this result is
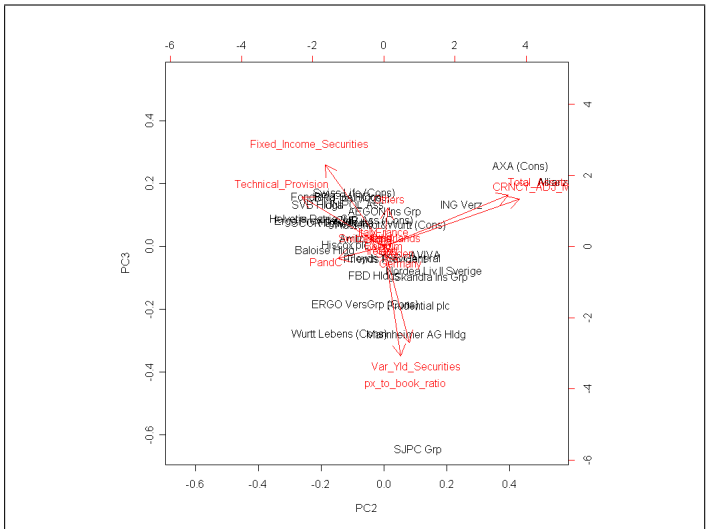
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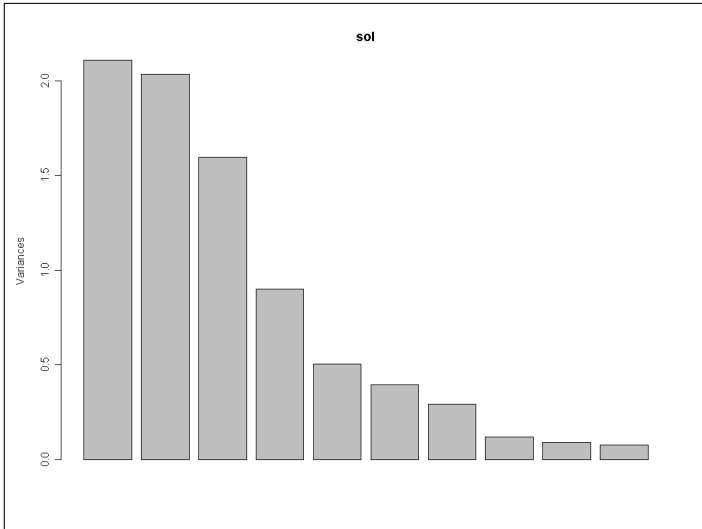
(a) Inertia explanation by the different eigen values.

(b) Representation of the variables in the plan PC1-PC2.
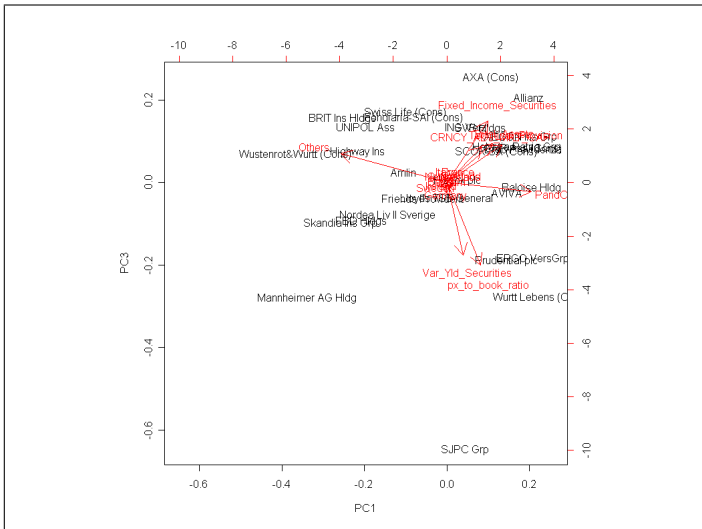
(c) Representation of the variables in the plan PC1-PC3.

(d) Representation of the variables in the plan PC2-PC3.

Figure 4: Principal Component Analysis - Main Results.

quite obvious. Indeed, the required return aims at remunerating the shareholder in counterparty of the risk purchased and as reflected in the annual report of Allianz, the risk of a line is not directly linked to its weight in the company portfolio.

Similarly, PC2 and PC3 do not allow to capture clear factors determining the cost-of-capital rate. For instance, Axa and Allianz are leaders in the market but characterized by very different β (see Figure 4).

According to these results, it appears that the main factors explaining the differences between the major insurance companies are different from those explaining the β relative to the market risk premium and the β relative to the value risk premium. Thus, the company itself has not to be integrated when valuating its risk. What is it important to capture is the characteristics of

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these risks.

4 Is it relevant to use the same cost-of-capital rate whatever the risk level?

Compared to the Fama-French Two Factors model, the frictional cost approach is not based on market data and depends on a lot of assumptions. For example, the components of the frictional costs, the way to take into account the agency costs or the way to model the P&L are crucial in the final cost-of-capital rate. On the contrary, it allows to determine a pure insurance cost of capital rate.

It possible to reconcile both approaches by using data of the insurance linked securities market. Indeed, because pure insurance risk are regularly traded in this market, it offers the way to understand how the investors valuate the insurance risks. Moreover, some models have been developped in order to capture the market price of risk in this market (see for example Gatumel and Guégan (2008b)).

For example the Fermat Capital Management model considers that the rate on line of the insurance linked security i has the following form:

$$Y_i = X_i + \beta \times MPR, \tag{4.1}$$

where: Y_i is the rate on line or, more commonly, the spread or, the cost of capital rate,

X_i is the Expected Loss,

$\beta = \frac{1}{\sqrt{m_i}}$, with m_i the weight of the risk covered by the issue i,

$MPR = \lambda \times \sqrt{X_i \times (1 - X_i)}$,

λ is the Sharpe Ratio, according to the Fermat model.

In other words, the Fermat model may be written as:

$$Y_i = X_i + \lambda \times \frac{\sqrt{X_i \times (1 - X_i)}}{\sqrt{m_i}}. \tag{4.2}$$

Following the fact that the bonds are regularly traded, λ evolves over time, reflecting the fair value of the risk in the insurance linked securities market:

$$Y_{it} = X_i + \lambda_t \times \frac{\sqrt{X_i \times (1 - X_i)}}{\sqrt{m_i}}. \tag{4.3}$$

By using the data relative to the secondary market at date t, it is possible to capture the Sharpe Ratio at each date. Moreover, by considering some classes of risk, it is possible to determine a
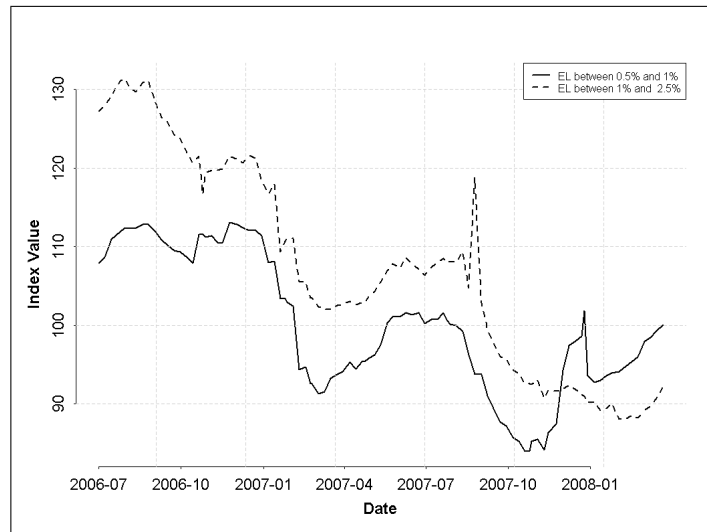
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Figure 5: Evolution of the Cat. Bond Spread Index since August 1st, 2006.

Sharpe Ratio and a cost of capital rate for different level of risk.

Following the available data we consider two risk levels: bonds having an expected loss between 0.5% and 1% and bonds having an expected loss between 1% and 2.5%. We use data of the pure catastrophe bond market, that means that we do not consider the insurance linked securities covering mortality risk, embedded value or industrial risk.

Figure 5 provides the evolution of a spread index tracking the spread behaviour between August 1st, 2006 and March 31st, 2008 for each risk level. Each index has the value 100 on January 1st, 2004. First of all, it is possible to highlight that the different levels of risk are characterized by different levels of the index. Riskier are the bonds, higher is the index. That reflects the fact that the insurance linked securities having the highest expected loss were characterized by the more increasing trend of their spreads between 2004 and August 2006. Then, between August 2006 and March 2008, all the risks have some similar evolutions, even if the bonds having an expected loss between 1% and 2.5% have a more increasing trend than the riskier one, at the end of the considered time period. Lastly, as it is explained in Gatumel and Guégan (2008b), the underlying risk, both real and perceived, allows to explain the evolution of the indices. For example, we can easely highlight the seasonality of the US hurricanes. Indeed, for each risk level, the spreads are increasing from March to the end of August.

Figures 6 and 7 provide with the evolution of the estimated Sharpe Ratio and of the Cost of Capital rate between August 1st, 2006 and March 31st, 2008 for three levels of risk. We consider the bonds having an expected loss under 0.5%, between 0.5% and 1%, and between 1% and 2.5%. As it is the case for the evolution of the insurance linked securities index, the Sharpe Ratio depends on the risk level. The lower the risk, the higher is the required return per unit of risk.
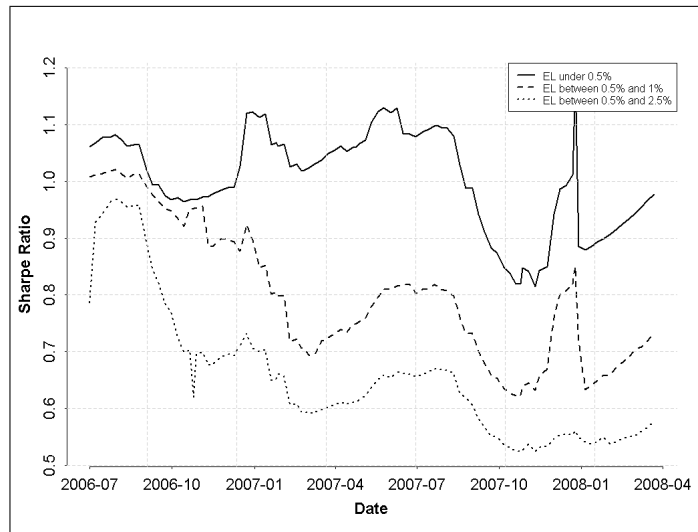
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Figure 6: Evolution of the Sharpe Ratio.

That reflects the fact that the investors are relatively more risk averse for the bonds having a lower return period. We may explain that by assuming that this type of bonds are characterized by a big uncertainty, even if it is not captured by the expected loss. Thus the investors require a relative higher return in order to accept the risk. In addition to that, the Sharpe Ratio is globally decreasing between August 2006 and March 2008 capturing the decreasing trend of the insurance linked securities spread index over the same period. No major catastrohic event, the independance between the "classical" financial markets and the catastrophe bond market, the increasing demand for this type of risks explain this phenomenon. The peak of the Sharpe Ratio for the bonds having an expected loss above 2% during the summer 2007 may be explained both by the season and by the financial crisis. Obviously, at the end of the hurricane season and with the independance of the catastrophe bond sphere, the Sharpe Ratio is decreasing.

A major element between on the one hand the cost of capital rate and on the other hand the expected loss and the market price of risk is the diversification factor m. This factor aims to capture the diversification effect introduced by the risk covered by the issue i in the portfolio. Higher will be the diversification effect, lower will be the ratio of the market price of risk by the diversification effect and thus lower will be the cost of capital rate. In the model, m depends on the global exposure which is underlying of the risk traded. For example, if the global exposure on the US hurricanes is $120 billion for a centenary event, if the exposure for the same return period but for the Californian earthquake is $90 billion and if it is %70 billion for the European windstorm, m is equal to 1 for bonds covering the US hurricanes, 2 for the Californian earthquake and 3 for the European windstorm. In other words, the exotic risks, like the Australian windstorm, are characterized by a lower risk premium because they allow to reduce the global risk of the portfolio. For a given expected loss, the cost of capital rate will be

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a decreasing function of the diversification factor. Figure 7 points out this phenomena.

(a) Expected Loss = 0.5%.

(b) Expected Loss = 1%.
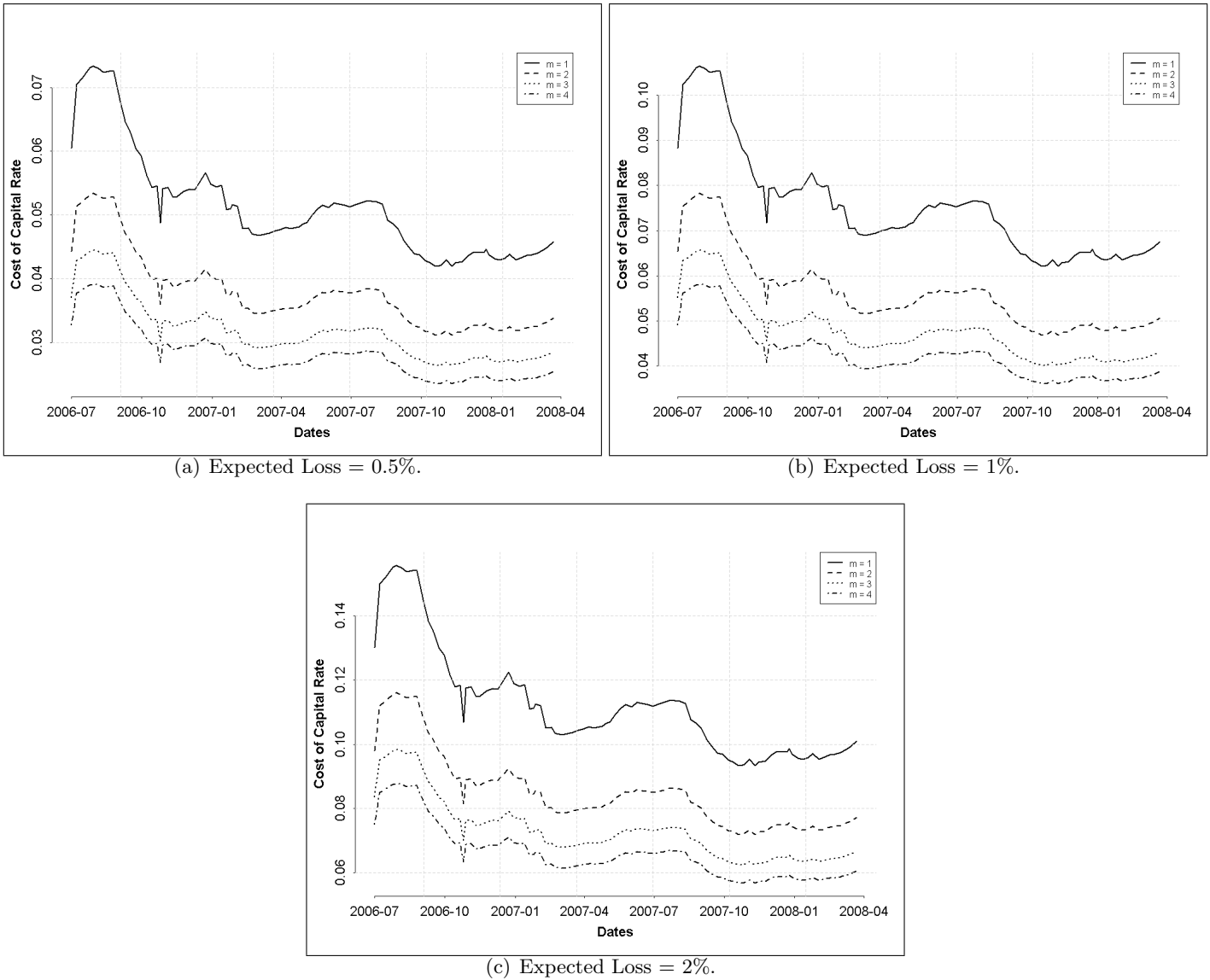

(c) Expected Loss = 2%.

Figure 7: Evolution of the Cost of Capital Rate for different risk levels and diversification factors.

Each curve represents the evolution of the cost of capital rate for a given return period, a given diversification factor and the market price of risk previously estimated. It appears clearly that higher is the expected loss, higher is the cost of capital rate. In addition to that, higher is the diversification factor, lower is the cost of capital rate. Indeed, for a given diversification factor, for example $m = 3$, it appears that the cost of capital is between 4-4.5%, when the expected loss is equal to 0.5%, and 7-8% for an expected loss equal to 2%. But for a same expected loss, for example 0.5%, it appears that the cost of capital may vary in a range 4-8%, depending on the diversification factor.

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To summarize, it appears clearly that the cost of capital rate has to be depending on the considered return period and on the allocated risk capital of the underlying segment. Higher will be the part of a segment in the global risk capital, higher will be the cost of capital applied on the required capital. The segment may interpreted in two ways. For example the segments may be considered as the sources of risk. In the 2007 annual report of Allianz AG, it appears (page 91) that the property-casualty actuarial risks may be divided into the Premium CAT risks (27%), the Premium non-CAT risks (38%) and the Reserve risks (37%). Following the fact that Allianz calculates the allocated risk capital with a Value-at-Risk at 99.97% as risk measure, the cost of capital rate for each segment would be the following: 7-8% for the Premium non-CAT risks, 6% for the Reserve risks and 4.5% for the Premium CAT risks.

But, the segments may also be considered as the different lines of business of the company: personal property, commercial property, motor, liability, etc. In this case, there won't be a cost of capital rate by source of risk but a cost of capital rate by line of business $\times$ source of risk. It will allow not only to capture the current exposure of each line of business but also the portfolio evolution by considering the different diversification factors used for example both for the Premium risk and for the Reserve risk.

It is very important to point out that, following such an approach, all the insurance companies wouldn't be characterized by the same cost of capital rate. Indeed, the cost of capital rate would depend on the business structure of each company. Moreover, this study is based on the example of the insurance linked securities market, because it is the more transparent exchange market of insurance risks. Thus, the cost of capital rates that we have determinated do not include a high liquidity premium. Because all the insurance portfolios are not traded in this market, the cost of risk applied on the allocated capital risk of the global portfolio of an insurance company should capture this additional risk. Thus, the cost of capital rate have to be higher than what we have previously determinated.

5   Conclusion on the Solvency II framework relevancy

This paper aims at analysing the positions of both CEIOPS (2008) and Chief Risk Officer Forum (2008) on the cost of capital rate for the insurance companies. We point out that estimate a cost of capital rate for the insurance companies is a difficult job. Obviously, there is no true or false price. Nevertheless, by using the insurance linked securities market, we point our that it is irrelevant to use only one risk premium for the risks of a company, as proposed by CEIOPS (2008). We show that each risk is not characterize by a price. In fact, the price of risk depends on the basket of risks at which it belongs, the risk level considered and the time period.

We propose to modify the equation 1.1 as follows:

$$MVM = \sum_t \frac{\int_{\alpha=0}^{1} CoC\left(\alpha\right) \times SCR\left(\alpha\right)_t d\alpha}{\left(1 + r_t\right)^t}, \tag{5.1}$$

with α the confidence interval at which the solvency capital requirement is computed. Our approach is consistent with the Solvency II framework because it bases the market value of insurance risks on the market value margin. However, accordingly with the insurance linked securities market, we consider that the market value margin has to depend on the risk level.

Unfortunately, the insurance linked securities market does not provide a cost-of-capital rate for all the insurance risks and all the risk levels. Thus, it would be more difficult to compute a market value margin following our approach than following a "simplified" Solvency II approach because it requires some additional inputs. Two solutions may be used. First, using the 6% of CEIOPS (2008) or the 2.5-4.5% of Chief Risk Officer Forum (2008) as reference price or consensus for a "reference" risk. The prices of other risks and other return period would be expressed as a function of this cost of risk. For instance, risk creating major diversification effect would be characterized by a lower price. It would be also the case for higher return period. Second, using market data like the insurance linked securities market for some return period (for example 100y and 200y) and interpolating.

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