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Stable pricing in monopoly and equilibrium-core of cost games

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Abstract

We prove the existence of subsidy free and sustainable pricing schedule in multiproduct contestable markets. We allow firms to discriminate the local markets that are composed by a set of the products line and a set of agents. Results are obtained under an assumption of fair sharing cost and under boundary condition of demand functions. The pricing problem is modelled in terms of equilibrium-core allocations of parameterized cost games. *Journal of Economic Literature* Classification Numbers: C71, L11, L12.

Keywords: cooperative games, contestable markets, sustainability, subsidy free, parameterized cost games.

Résumé

L'existence de tarifications sans subventions croisées et soutenable est prouvée dans un marché contestable multiproduit où les firmes ont la possibilité de discriminer les marchés locaux, composés d'une partie de la ligne commerciale et d'une partie d'agents. Les résultats sont obtenus sous une hypothèse de fonction de coût à partage équitable, et sous des conditions de bord des fonctions de demandes. Le problème de tarification est modélisé par des cœurs-équilibres de jeux de coût paramétrés. *Journal of Economic Literature* Classification Numbers : C71, L11, L12.

Mots-clés : jeux coopératifs, marchés contestables, soutenabilité, subventions croisées, jeux de coût paramétrés.

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1 Introduction

The paper lies in the continuation of the theory of contestable markets, paradigm developed in the seventies by Bailey, Baumol, Panzar and Willig [1, 9]. One of the main concern of the theory deals with stable pricing. In essence, the different definitions of stability all satisfy coalition proof properties, in other terms, the market is to be protected against entries from rivals. Such stability concepts shall therefore explain the emergence of industrial structure, and those price strategies may be expected in the process of market regulation. Indeed, it is likely that the incumbent firm is willing to protect its market against rivals through a stable pricing. Note however that, as quoted by Faulhaber [5, p. 967], stable pricing does not consist of *" welfare maximizing; nor are we entitled to assume that such prices are socially superior on grounds of social justice"*. Hence, even if normative questions are still open, the issue remains whether there exists such a pricing that explains a given industrial structure.

The literature on this topic can be divided into four different classes (following the terminology of contestable markets):

- Existence of subsidy free pricing, [2, 3, 5].
- Links between sustainable and subsidy free pricing, [2, 3, 8].
- Existence of anonymous equitable pricing, [6, 10, 12].
- Links between sustainable and anonymous equitable pricing, [8, 11, 12].

We address the two first problems and a new point gives an additional lighting on the extant literature. It deals with the assumption of price discrimination: firms maintain separated the local markets through individual prices.

Let us describe briefly the markets we analyze. A market is said to be a natural monopoly market if any production bundle can be produced at the lowest cost by one firm. Therefore, the incumbent firm has an advantage on any combination of firms committed in the production of the same bundle of goods. That kind of markets is thus characterized by the existence of sub-additive costs:

$$C(y) + C(y') \geq C(y + y')$$

When the incumbent and entrant firms share the same productive techniques given by a cost function, and the entry is free, the market is said to be contestable. It is well known that the advantage of the incumbent firm given by the existence of sub-additive costs is not sufficient to guarantee stable pricing. Our objective is to provide sufficient conditions for the existence of stable pricing in a contestable market.

We put our attention on the notions of subsidy free pricing and sustainable pricing. A pricing is said to be subsidy free if the incumbent achieves the budget balance condition for that pricing and the expenditures of any coalition is lower than the cost of that coalition .

The stronger definition of sustainability states that the incumbent firm can set a price schedule such that the budget is balanced on the market, and, no entrant with a non negative profit can attract a group of agents, or in a more general sense a sub-market of goods and agents. Attracting in the sense that the entrant price schedule is preferred by the chosen sub-market. We mention that in a companion paper, Iehlé [7] studies a more general definition of sustainability.

The existence of sustainable and subsidy free pricing is proved in discriminatory markets for elastic demand functions under assumptions of regular markets and fair sharing cost function. Roughly speaking, the regularity assumption states that above the zero profit price level, the profit remains non negative on a given set. Besides, all along the paper, the fair sharing cost property will play a central role. The property ensures that for any outputs levels, there exists a sharing of the cost such that no coalition will be charged more than the cost of serving that coalition alone. It shall turn out to be a necessary condition for the existence of stable pricing. In other terms, the property states a stable condition for any constant demands. Hence to go a step further towards elastic demands in the analysis of contestable markets, one needs to assume that stability holds for inelastic demands. It is shown that a sufficient condition to realize the fair sharing cost property is given by an assumption of increasing returns to coalitions on the cost function.

In our contestable market modelling, entrants have a large choice of strategies. Entrants can decide to enter on a subset of the products line and/or a subset of the agents. Henceforth, the entrants commit to supplying this sub-market. Thus, our formulation of contestable markets is sufficiently general to take into account, as special cases, contestable markets where there is: no discrimination and a global demand (one agent) as in Baumol et al. [2] and Faulhaber and Levinson [6]; no discrimination and a full product line supply as in Faulhaber and Levinson [6]; discrimination and single output firms as in Bendali et al. [3]. All along the paper, we state two different classes of results. First, results for the case of double articulation entries (i.e. sub-market composed by a subset of agents and goods); second, results within a single articulation entries where firms commit to providing, either, the full products line, or all the agents. These two classes of results are close but not comparable.

The paper follows the usual stream of cooperative game modelling of stable pricing in monopoly. Faulhaber [5] has first proposed existence results for subsidy free pricing within this formalism of TU game, by considering cost games. Our treatment is going further and consists of the construction of a family of parameterized cost games, where the games are parameterized by prices. The subsidy free pricing is restated as an equilibrium-core of the parameterized games. Then, the results are deduced from an existence result of Bonnisseau and Iehlé [4] that states the existence of an equilibrium-core in NTU games. We restate the result for the TU case. Contrasting with a usual strategy in the literature, our proof does not appeal Kakutani's fixed point theorem. The advantage stems from the fact that our strategy to show the existence of an equilibrium-core of parameterized cost games may be extended to a more general case of cost games where the transfers of wealth are limited.

Henceforth, our strategy to deduce sustainability is usual in the literature, see Mirman et al. [8] for instance. We first state an existence result for a subsidy free pricing, then a regular market condition leads to the stronger notion of sustainability.

All the results rely on boundary conditions on demand functions and/or cost function. In the literature, such conditions are always assumed and slightly differ from ours. Up to these technical boundary conditions, our results generalize the extant ones of the literature since our model is able to encompass different contestable markets of Baumol et al. [2], Bendali et al. [3], Faulhaber and Levinson [6].

The paper can be divided into two parts, it deals first with the notions stable pricing in contestable markets, second with the cooperative game modelling. In Section 2, the model of contestable market with discrimination and the stability concepts are defined. In Section 3, one states the existence of stable pricing. First, we state existence results, Theorems 3.1, 3.2 for a subsidy free pricing under an assumption of fair sharing cost function and boundary conditions. Secondly, we prove that any subsidy free pricing is sustainable under an assumption of regular market. We deduce from this fact the existence of a sustainable pricing, Theorems 3.3, 3.4. These results are all obtained as corollaries of technical results given in Section 4, where we define parameterized cost games and equilibrium-cores. Omitted proofs in the body of the paper are referred to Appendix.

2 The model

We consider a multiproduct contestable market with price discrimination.[1]

(C1) There are $\ell < \infty$ goods. Goods are denoted by script $b \in L := \{1...\ell\}$. The commodities space is $\mathbb{R}_+^L$. There are $n < \infty$ agents (or local markets). Agents are denoted by script $a \in N := \{1...n\}$.

(C2) Price space is $\mathbb{R}_+^{LN}$, that is, firms can set a price $p_a \in \mathbb{R}_+^L$ for each agent $a \in N$. Each agent $a \in N$ is endowed with a continuous demand function $D_a : \mathbb{R}_+^{LN} \to \mathbb{R}_+^L$.[2]

(C3) Incumbent and entrant firms share the same productive techniques given by a continuous cost function $C : \mathbb{R}_+^L \to \mathbb{R}_+$.

[1] We will use the following notations: for any set-valued mapping Γ, denote by Gr Γ its graph. $X \setminus A = \{x \in X \mid x \notin A\}$. Vector partial order: for all $x, y \in E$, Euclidean space of dimension L, $x \geq y$ if for all $k = 1...L$, $x_k \geq y_k$; $x > y$ if for all $k = 1...L$, $x_k \geq y_k$ and there exists j such that $x_j > y_j$; $x \gg y$ if for all $k = 1...L$, $x_k > y_k$. Let X be a finite set, we denote by $\mathbb{R}^X$ the Euclidean space whose components are indexed by the elements of X, $\mathbb{R}^{XY} := (\mathbb{R}^X)^Y$. For any $S \subset X$, E_S^X is the $\mid S \mid$-dimensional subspace of $\mathbb{R}^X$ defined as $\{x \in R^X \mid x_i = 0,\, i \notin S\}$. We denote by x^S the projection of an element of $\mathbb{R}^X$ on E_S^X. We denote by $<_S$ the partial order on E_S, i.e. $p' <_S p$ iff $p'_i \leq p_i$ for all $i \in S$, with at least one strict inequality.

[2] Note that demands for goods are inter-dependent between agents since they are defined on the whole price space.

(C4) Accessible sub-markets for entrants are the elements of the set $\{(A,B) \mid A \in \mathcal{N},\ B \in \mathcal{L}\}$, where $\mathcal{N}$ and $\mathcal{L}$ are respectively the non empty subsets of N and L.

In the remainder, the contestable markets will satisfy the above set of conditions (C1–4).

Let us now describe the scenario that takes place in the contestable market. There exists an incumbent firm that maintains separated the different sub-markets of agents $a \in N$, and provides the entire products line L to each of them with a technology described by the cost function C (for each $d \in \mathbb{R}^L_+$, $C(d)$ is the induced cost to produce the commodity bundle d). The firm is supposed to be full supplier, that is, it provides fully the demand induced by the price (set by itself). Therefore, $d_a = D_a(p)$ where d_a is the quantity of outputs produced for the agent a, and, $D_a(p) \in \mathbb{R}^L_+$ is the demand of agent a for a given price $p \in \mathbb{R}^{LN}_+$.[3]

On the side of entrants behavior, the entrants can choose a sub-market among a subset of agents and goods. As supposed before for the incumbent, the entrants are also full supplier on the chosen sub-market. Let AB be an element in $\mathcal{NL} := \{(S,T) \mid S \in \mathcal{N},\ T \in \mathcal{L}\}$, then the entrant, choosing AB, commits to providing fully the demands of agents $a \in A$ for the products line B: $D_a(p)^B \in E^L_{B+}$. Henceforth, the contestability takes place with respect to the prices.

2.1 Stable pricing

We are now in position to present formally two criteria of stability for a natural monopoly market. First, the notion of subsidy free pricing is defined, it can be seen as an internal stability concept where cross subsidies between sub-markets are made impossible. Second, the stronger notion of sustainability is introduced, it can be seen as an external criterion where no profitable entry, at any preferred pricing schedule, can be achieved.[4]

Definition 2.1 (Subsidy free) [5]

A pricing $p = (p_a)_{a\in N} \in \mathbb{R}^{LN}_+$ is subsidy free if:

1. $\sum_{a\in N} p_a \cdot d_a = C(\sum_{a\in N} d_a)$.
2. $d_a = D_a(p)$ *for each* $a \in N$.

[3] See Mirman et al [8] or Baumol et al. [2] for partial supplier analysis, i.e. markets where the firms can provide $d_a \leq D_a(p)$.

[4] The analysis is mostly focused on the two notions of subsidy free and sustainable pricing, we mention however that others close notions exist, as anonymous equitability and supportability, see [10].

[5] Faulhaber and Levinson [6] have studied a similar model without price discrimination and full products line supply, i.e. a price $p \in \mathbb{R}^L_+$ satisfying:

$$\begin{cases} p \cdot \sum_{a\in N} D_a(p) = C(\sum_{a\in N} D_a(p)) \\ p \cdot \sum_{a\in A} D_a(p) \leq C(\sum_{a\in A} D_a(p)) \text{ for each } A \in \mathcal{N}. \end{cases}$$

3. $\sum_{a \in A} p_a \cdot d_a^B \leq C(\sum_{a \in A} d_a^B)$ *for each* $AB \in \mathcal{NL}$.

We will refer hereafter to subsidy free on $\mathcal{N}$ *(resp.* $\mathcal{L}$*) if* $\mathcal{L}$ *(resp.* $\mathcal{N}$*) is fixed and equal to* $\{L\}$ *(resp.* $\{N\}$*) in the definition above.*

From the point of view of the firm, the no cross subsidies and budget balance conditions mean that there is no jurisdiction (a sub-market) achieving a positive profit and subsidizing another jurisdiction facing losses. In that way, subsidy free pricing is an internal criterion of stability, no jurisdiction is entitled to claim a reward for its participation.

Besides, subsidy free notion is in the heart of controversy for antitrust policies, the subsidy free pricing can stand for a first criterion to be prescribed to a natural monopoly.[6] Note also that the concept is appealing for the practice, it is an easy matter for the regulators to check whether or not the pricing is subsidy free. It consists of the verification of a finite number of inequalities. It is not the case for the second criterion of stability we introduce now.

When entrants are free to enter the market, one can additionally suppose that they will set their own pricing, unlikely to the previous definition of subsidy free pricing, where the pricing is fixed. This is the essence of sustainability, to enter a sub-market, the entrants set a pricing that is likely to be preferred by the agents of the chosen sub-market. Roughly, the pricing p' is preferred to p by agent a if $p'_a < p_a$.

Definition 2.2 (Sustainability) [7] *A price* $p = (p_a)_{a \in N} \in \mathbb{R}_+^{LN}$ *is sustainable if:*

1. $\sum_{a \in N} p_a \cdot d_a = C(\sum_{a \in N} d_a)$.
2. $d_a = D_a(p)$ *for all* $a \in N$.
3. *There is no* $(AB, p') \in \mathcal{NL} \times \mathbb{R}_+^{LN}$ *such that:*

$$\begin{cases} \sum_{a \in A} p'_a \cdot {d'}_a^B = C(\sum_{a \in A} {d'}_a^B) \\ d'_a = D_a(p') \ \textit{ for all } a \in A. \\ p'_a <_B p_a \textit{ for all } a \in A. \end{cases}$$

We will refer hereafter to sustainability on $\mathcal{N}$ *(resp.* $\mathcal{L}$*) if* $\mathcal{L}$ *(resp.* $\mathcal{N}$*) is fixed and equal to* $\{L\}$ *(resp.* $\{N\}$*) in the definition above.*

The first condition guarantees the budget balance, the second is the full supplier requirement. The third formalizes the stability concept: no firm can enter a sub-market AB, by setting a preferred pricing for agents in A and achieving the budget balance condition. The definition of sustainability is much stronger than one of subsidy free (it is proved in Lemma 3.1 that a sustainable pricing is a subsidy free pricing). Here, the market is fully protected against entry from rivals, and the incumbent is in invulnerable situation.

[6] The aircraft industry provides a good example of pricing with cross subsidies.

[7] Definitions slightly differ with respect to authors, see Baumol et al. [9], Mirman et al. [8] or Ten Raa [11].

3 Existence of stable pricing

Using results of Appendix for equilibrium-cores in parameterized cost games, we state two existence results for subsidy free pricing and sustainability pricing. Firstly, we exhibit a subsidy free pricing, then we deduce the sustainable pricing under the restriction of regular markets. The subsidy free pricing can be restated as allocations of an equilibrium-core of parameterized cost games. To study the non-emptiness of the core of these games, we will suppose that the cost function satisfies a condition of fair sharing for any output structure:[8]

Definition 3.1 *The cost function C satisfies the property of fair sharing if for all $d_a \in \mathbb{R}^L_+$, $a \in N$, there exists $q \in \mathbb{R}^{LN}_+$ such that:*

$$\begin{cases} \sum_{ab \in NL} q_{ab} = C(\sum_{a \in N} d_a) \\ \sum_{ab \in AB} q_{ab} \leq C(\sum_{a \in A} d_a^B) \text{ for all } AB \in \mathcal{NL}. \end{cases} \tag{1}$$

We will refer hereafter to fair sharing on $\mathcal{N}$ (resp. $\mathcal{L}$) if $\mathcal{L}$ (resp. $\mathcal{N}$) is fixed and equal to $\{L\}$ (resp. $\{N\}$) in the definition above.

The above property states the existence of a fair sharing $(q_a)_{a\in N}$ among the agents for any structure of outputs $(d_a)_{a\in N}$. In other terms, no coalition can achieve a positive profit if its members decide to secede. We will obtain the subsidy free pricing from this property on the costs since the subsidy free property can be seen as the extension of the fair sharing cost property to the case of inelastic demand functions. Indeed, the two notions are linked up together, for any subsidy free pricing p, the vector $(q_{ab})_{ab\in NL}$ given by the expenditures of the agents at price p: $(p_{ab}D_a(p)_b)_{ab\in NL}$ satisfies the fair sharing property for the output level given by $(D_a(p))_{a\in N}$. Conversely, a fair sharing implementation $(\sum_{b\in L} q_{ab})_{a\in N}$ associated with a positive outputs level $(d_a)_{a\in N}$ can be seen as the expenditures of the agents with inelastic demand functions $(d_a)_{a\in N}$ at the subsidy free pricing: $p_a^b = \frac{q_a^b}{d_a^b}$.

Note also that the condition of fair sharing states equivalently that for all outputs $(d_a)_{a\in N}$: for all balanced family $\mathcal{B}$ and balancing weights $(\gamma_{AB})_{AB\in\mathcal{B}}$,

$$\sum_{AB\in\mathcal{B}} \gamma_{AB} C(\sum_{a\in A} d_a^B) \geq C(\sum_{a\in N} d_a).$$

Sharkey [13] and Baumol et al. [2] put a great attention on fair sharing cost functions, they exhibit technological properties such that the condition fair sharing always exists. For instance, they provide a sufficient condition on the cost function to be fair sharing.

(WC) $C(x+z)-C(x) \geq C(x+y+z)-C(x+y)$ for all $x, y, z \in \mathbb{R}^L_+$.

(WC) is called weak cost complementarities condition.

[8] In the literature, this evaluation of fairness, is also called the stand alone test, as in Faulhaber [5], Faulhaber and Levinson [6].

Proposition 3.1 (Baumol et al.) *Under* (WC)*, the cost function is fair sharing.*

For the proof, see [2, Proposition 8C10, p.203].

One can provide another sufficient condition that involves a notion of increasing returns to coalitions. Given an output structure and a coalition, the grand coalition is able to purchase the output at the lowest cost per capita. coalition.

(IRC) For each $(d_a)_{a\in N} \in \mathbb{R}_+^{LN}$, $\frac{C(\sum_{a\in A} d_a^B)}{|A|} \geq \frac{C(\sum_{a\in N} d_a)}{|N|}$ for all $AB \in \mathcal{NL}$.

Proposition 3.2 *Under* (IRC)*, the cost function is fair sharing.*[9]

Proof of Proposition 3.2. In order to show the implication it suffices to show equivalently that the following inequality holds true: for all $(d_a)_{a\in N}$ and all balanced family of coalition $\mathcal{B}$, with balancing weights $(\gamma_{AB})_{AB\in\mathcal{B}}$, $\sum_{A\in\mathcal{B}} \gamma_A C(\sum_{a\in A} d_a^B) \geq C(\sum_{a\in N} d_a)$. Indeed from (IRC),

$$\sum_{A\in\mathcal{B}} \gamma_A C(\sum_{a\in A} d_a^B) \geq \frac{C(\sum_{a\in N} d_a)}{|N|} \sum_{A\in\mathcal{B}} \gamma_A |A|$$

Furthermore by computations of the balancing weights, we deduce:

$$\sum_{A\in\mathcal{B}} \gamma_A |A| = \sum_{a\in N} \sum_{A\in\mathcal{B}, A\ni a} \gamma_A = \sum_{a\in N} 1 = |N|$$

□

In the remainder, the fair sharing property will play a central role, since the results are all stated under this condition. It shall turn out that the condition is also a necessary condition for the existence of stable pricing. The following proposition illustrates the point for the case of subsidy free pricing. Since any sustainable pricing is also a subsidy free pricing (see Lemma 3.1), the condition of fair sharing cost is consequently a necessary condition for sustainability.

Proposition 3.3 *If the cost function is not fair sharing, there exist demand functions such that there is no subsidy free pricing.*

Proof of Proposition 3.3. The proof is straightforward. Suppose that there exists $(d_a)_{a\in N} \in \mathbb{R}_+^{LN}$ such that (1) is not satisfied for any $q \in \mathbb{R}_+^{LN}$. It suffices to consider the constant demand functions $D_a = d_a$ for all $a \in N$. These demand functions cannot be supported by a subsidy free pricing, otherwise set $q_{ab} = p_{ab} D_a(p)_b$ that leads to a contradiction. □

We posit the following assumption on the demand functions.

[9] One can state the analogue of Propositions 3.1 and 3.2 for the cases of full products line supply or all agents supply.

(D) Demand functions D_a are bounded above, and there exists a minimal threshold $\epsilon > 0$ in the consumption, i.e. $D_a \geq \epsilon \mathbf{1}$ for each $a \in N$.[10]

One can state the first main results of the paper:

Theorem 3.1 *Under (D), any contestable market with fair sharing cost admits a subsidy free pricing.*

Theorem 3.2 *Under (D),*

- *Any contestable market admits a subsidy free pricing on $\mathcal{N}$ if firms commit to providing entirely the products line, $\mathcal{L} = \{L\}$, and the cost function is fair sharing on $\mathcal{N}$.*
- *Any contestable market admits a subsidy free pricing on $\mathcal{L}$ if firms commit to providing all the agents, $\mathcal{N} = \{N\}$, and the cost function is fair sharing on $\mathcal{L}$.*

The statements above are very close. Firstly, it is worth pointing out that Theorem 3.2 is not a corollary of Theorem 3.1. Obviously, a subsidy free pricing in the sense of Theorem 3.1 is also subsidy free for the specific cases of Proposition 3.2, but remark that the fair sharing assumption is weaker in these cases. Thus, the two results are not comparable. We exhibit the detailed proof for Theorem 3.1, which is a direct corollary of Proposition 4.1, whereas the proof of Theorem 3.2 is omitted (it follows the same lines of arguments, see also Remark 1 in Section 5).

We state our results under a condition of fair sharing cost function. Additionally, we need boundary conditions (D) on demand functions and/or cost function. In the literature, such conditions are always assumed and slightly differ from ours. Up to these technical boundary conditions, our results generalize the extant ones of the literature since our model is able to encompass different contestable markets of Baumol et al. [2], Bendali et al. [3], Faulhaber and Levinson [6].

Let us turn to the notion of sustainability. We make the link with the sustainability by introducing the assumption of regular market, this way of doing is usual in the literature, see Bendali et al. [3], Mirman et al. [8] or Ten Raa [11]. The condition of regularity implies that the subsidy free pricing is not dominated by another pricing to get eventually the notion of sustainability. The regularity condition holds on the profit function:

For any sub-market $AB \in \mathcal{NL}$, the profit function is given by:

$$\Pi_{AB} : p \longrightarrow \sum_{a \in A} p_a \cdot D_a(p)^B \; - \; C(\sum_{a \in A} D_a(p)^B)$$

[10] Ten Raa [10] uses the condition of threshold in the consumption. See Remark 5 in Section 5.

From (D) and the continuity of the function C, we know that there exists an upper bound $B \in \mathbb{R}_+$ such that $C(D_a(p)_b) \leq B$ for any $p \in \mathbb{R}_+^{LN}$ and $ab \in NL$. The market is said to be regular if it satisfies the following condition:

(R) Let $\mathcal{K}$ be the multidimensional cube $\prod_{ab \in NL}[0, \frac{B}{\epsilon}]$, for any submarket AB it holds that: $\mathcal{K} \cap \{\Pi_{AB} > 0\} = \mathcal{K} \cap \{\{\Pi_{AB} = 0\} + \mathbb{R}_{++}^{LN}\}$.

We will say that (R) is satisfied on $\mathcal{N}$ (resp. $\mathcal{L}$) if $\mathcal{L}$ (resp. $\mathcal{N}$) is fixed and equal to $\{L\}$ (resp. $\{N\}$) in the definition above.

The condition can be seen as a notion of positive comprehensiveness from above. Above zero profit level, profits remain positive on a compact set $\mathcal{K}$.

In Bendali et al. [3], the authors suppose that the expenditures functions are strictly increasing with respect to prices. In Mirman et al. [8], a weaker assumption is at stake, the profits are strictly increasing with respect to prices. Regularity is the weakest version, note also that it is consistent with a bell curve profit mapping.

Theorem 3.3 *Under (D) and (R), any contestable market with fair sharing cost admits a sustainable pricing.*

Proof of Theorem 3.3. From Theorem 3.1, there exists a subsidy free pricing p^*. Firstly, note that $p^* \in \mathcal{K}$. Indeed, consider the definition of subsidy free pricing. As a special case one has: for all $ab \in NL$,

$$p_{ab} D_a(p)_b \leq C(D_a(p)_b)$$

Using the boundedness and threshold conditions of Theorem 3.1, one gets the result. The following lemma will end up the proof.

Lemma 3.1 *Under Assumptions of Theorem 3.3, the following two propositions are equivalent:*

1. *p is subsidy free.*
2. *p is sustainable.*

Proof of Lemma 3.1. Suppose that the regular contestable market admits a subsidy free pricing p. Since $\sum_{a \in A} p_a \cdot D_a(p)^B \leq C(\sum_{a \in A} D_a(p)^B)$ and $p \in \mathcal{K}$, it follows from regularity that for all p' such that $p_a <_B p'_a$ for all $a \in A$, it holds that $\Pi_{AB} < 0$. Hence, the pricing p is sustainable. Conversely, suppose that the regular contestable market admits a subsidy free pricing p, then for all $p' \in \mathbb{R}_+^{LN}$ such that $p_a >_B p'_a$ for all $a \in A$, it holds that $\Pi_{AB}(p') < 0$. From the continuity of the mappings C and $(D_a)_{a \in N}$, by taking the limit, it is clear that for $p' = p$, one gets $\Pi_{AB}(p') \leq 0$. Hence, the pricing p is subsidy free. $\square$

We deduce that the subsidy free pricing p^* is sustainable. $\square$

Symmetrically, one states results for the case of fixed product line or fixed number of agents in the provision of the goods. The proof is omitted (the result can be deduced straightforwardly from Theorem 3.2).

Theorem 3.4 *Under (D),*

- *Any contestable market admits a sustainable pricing on $\mathcal{N}$ if $\mathcal{L} = \{L\}$, the cost is fair sharing on $\mathcal{N}$ and (R) is satisfied on $\mathcal{N}$, or*
- *Any contestable market admits a sustainable pricing on $\mathcal{L}$ if $\mathcal{N} = \{N\}$, the cost function is fair sharing on $\mathcal{L}$ and (R) is satisfied on $\mathcal{L}$.*

4 Equilibrium-core of parameterized cost games

In this Section, the notion of parameterized TU games is defined. We state a non-emptiness result for the associated concept of core, called equilibrium-core. Then, this abstract result is appealed to show the non-emptiness of a generalized core of cost games, where additionally to the core stability requirement, the core allocation must satisfy a consistency property with respect to a parameter.[11] We state Proposition 4.1 from which we deduce Theorem 3.1, related to subsidy free pricing.

Let us first define a TU cooperative game. Consider a finite set of players X, $\mathcal{X}$ the set of non empty subsets of X. The characteristic function is $v : \mathcal{X} \to \mathbb{R}_+$. Thus, for each $S \in \mathcal{X}$, v_S is the payoff of the coalition S. Let $(v_S, S \in \mathcal{X})$ denote a TU game.

An imputation in S is a vector $q = (q_x)_{x\in S}$ such that $\sum_{x\in S} q_x \le v_S$. Consider an imputation q in N, it is dominated by q' in S if q' is an imputation in S and $q'_x > q_x$ for each $x \in S$. The core of a game is the set of all imputations that are undominated in any coalition. A family of coalitions $B \subset \mathcal{X}$ is balanced if there exists $\lambda_S \in \mathbb{R}_+$ for each $S \in \mathcal{B}$ such that:

$$\sum_{S\in\mathcal{B}} \lambda_S \mathbf{1}^S = \mathbf{1}^X$$

A TU game is said to be balanced if for any balanced family of coalitions $\mathcal{B}$ with balancing coefficients $(\lambda_S)_{S\in\mathcal{B}}$, it holds that:

$$\sum_{S\in\mathcal{B}} \lambda_S v_S \ge v_N$$

We need to consider now a more general model of TU games called parameterized TU games.[12] The parameter set is Θ and, a game is associated to each $\theta \in \Theta$, that is, one has a mapping v_S from Θ to $\mathbb{R}$ for each coalition S. Let $((v_S)_{S\in\mathcal{X}}, \Theta)$ denote a parameterized TU game.

Finally, the equilibrium condition on the parameters is represented by a set-valued mapping G from $\Theta \times \mathbb{R}^X$ to Θ.

[11] In the literature, the results are usually obtained directly from an application of a fixed point theorem, we adopt a different strategy, which is closely related, see comments in Section 5.

[12] The reader is referred to [4] for further details.

(PH0) Θ is a non-empty, convex, compact subset of an Euclidean space. G is an upper semi-continuous set-valued mapping with non-empty and convex values.

(PH1) v_S, $S \in \mathcal{X}$, are continuous mappings on Θ.

Definition 4.1 *Let $((v_S)_{S\in\mathcal{X}}, \Theta)$ be a parameterized game. An equilibrium-core allocation is a pair $(q \in \mathbb{R}^X, \theta \in \Theta)$ such that q belongs to the core of the game $(v_S(\theta), S \in \mathcal{X})$ and $\theta \in G(\theta, q)$.*

We can state now a weak version of the abstract result of Bonnisseau and Iehlé [4]. The original version holds in NTU games. We let the reader check that Theorem 4.1 coincides to the TU case. Moreover, we point out that the result can be obtained directly as an application of Kakutani's fixed point theorem for this specific case of TU games.

Theorem 4.1 *Let $((v_S)_{S\in\mathcal{X}}, \Theta)$ be a parameterized TU game satisfying Assumptions (PH0) and (PH1) and such that, for each $\theta \in \Theta$, the TU game $(v_S(\theta), S \in \mathcal{X})$ is balanced, then there exists an equilibrium-core allocation.*

It is well known that the core of a TU game can be restated as a set of vectors $(q_x)_{x\in X}$ satisfying a finite number of inequalities, as follows:

$$\begin{cases} \sum_{x\in X} q_x = v_X \\ \sum_{x\in S} q_x \geq v_S \text{ for all } S \in \mathcal{X}. \end{cases}$$

We introduce now a generalized notion of the core for cost games. In cost games the worth of a coalition is supposed to model the cost associated to the coalition, so that we use converse inequalities. Consider the following families of mappings: $\psi_{AB} : \mathbb{R}^{LN}_+ \to \mathbb{R}_+$ for all $AB \in \mathcal{NL}$, $\phi_a : \mathbb{R}^{LN}_+ \to \mathbb{R}^L_+$ for all $a \in N$.

Definition 4.2 *The generalized core $\mathcal{M}$ is the set of vectors $p \in \mathbb{R}^{LN}_+$ such that:*

$$\begin{cases} \sum_{a\in N} p_a \cdot \phi_a(p) = \psi_{LN}(p) \\ \sum_{a\in A} p_a \cdot \phi_a(p)^B \leq \psi_{AB}(p) \text{ for all } AB \in \mathcal{NL}. \end{cases}$$

For each $p \in \mathbb{R}^{LN}_+$, let $\mathcal{M}^p$ denote the core of the TU game $(-\psi_S(p), S \in \mathcal{NL})$. The following result is deduced from Theorem 4.1, we prove that the generalized core is exactly an equilibrium-core of parameterized cost games. The definition below of generalized core is no more than a general formulation of the subsidy free problem.

Proposition 4.1 *Under Assumptions:*

1. *There exists $\epsilon > 0$, such that, for all $a \in N$, $\phi_a \geq \epsilon \mathbf{1}^L$. For all $ab \in NL$, $\psi_{\{ab\}}$ is bounded above by $B \in \mathbb{R}_+$.*

2. *ψ_{AB}, $AB \in \mathcal{NL}$, and ϕ_a, $a \in N$, are continuous.*

3. For $p \in \mathbb{R}_+^{LN}$, $\mathcal{M}^p$ is non empty.

$$\mathcal{M} \neq \emptyset.$$

Proof of Proposition 4.1.[13] We want to prove that $\mathcal{M}$ is non empty, it amounts to show that there exists a vector p^* satisfying: $(p_a^* \cdot \phi_a(p^*))_{a\in N} \in \mathcal{M}^{p^*}$. We use Theorem 4.1 to deduce the existence of such a vector from the existence of an equilibrium-core allocation.

Let Θ be the compact and convex subset of $\mathbb{R}^{NL}$, defined by $\Theta = \prod_{ab\in NL} \Theta_{ab}$ where: for each $ab \in NL$, $\Theta_{ab} = [0, \frac{B}{\epsilon}]$. Define the parameterized game as follows: given $S \in \mathcal{NL}$ and $\theta \in \Theta$, $v_S(\theta) = -\psi_S(\theta)$.

Consider now the following set-valued mappings. Firstly, let V be the comprehensive hull of the individually rational payoffs for the grand coalition.

$$V(\theta) := \left\{ x \in \mathbb{R}^{NL} \mid \sum_{i\in NL} x_i = v_N(\theta),\ x_i \geq v_{\{i\}}(\theta) \text{ for all } i \in NL \right\} - \mathbb{R}_+^{NL}$$

And, let $\bar{V}_N(\theta)$ be the non dominated payoff of $V(\theta)$:

$$\bar{V}(\theta) = \{x \in V(\theta) \mid \nexists y > x,\ y \in V(\theta)\}$$

Note that, for each $\theta \in \Theta$, $V(\theta)$ is a comprehensive from below and closed set of $\mathbb{R}^{NL}$. From the continuity of the mappings v_N and $v_{\{i\}}, i \in NL$, one can define a continuous mapping $p_V :\ \Theta \times \mathbb{R}^{NL} \to \partial V$, such that $p_V(\theta, q) = q$ for all $(q, \theta) \in \mathrm{Gr}\ \partial V$.

The following mapping selects, for each pair (θ, q), in the non dominated set of individual rational payoff that are above the pseudo projection of q in the game parameterized by θ.

$$I(\theta, q) = \left\{ t \in \bar{V}(\theta) \mid t \geq p_V(\theta, q) \right\}$$

Lastly, one is led to the definition of the mapping $G :\ \Theta \times \mathbb{R}^{NL} \to \Theta$. Given $(\theta, q) \in \Theta \times \mathbb{R}_+^{NL}$:

$$G(\theta, q) = \{u \in \Theta \mid -(u_i\phi_i(\theta))_{i\in NL} \in I(\theta, q)\}$$

Now, suppose that Assumptions of Theorem 4.1 hold true for the parameterized game $((v_S)_{S\in\mathcal{NL}}, \Theta)$ and the set-valued mapping G, then there exists (θ^*, q^*) such that q^* belongs to the core of the game $(v_S(\theta^*), S \in \mathcal{NL})$ and $\theta^* \in G(\theta^*, q^*)$. Thus $I(\theta^*, q^*) = \{q^*\}$, that is, we have $-(\theta_i^*\phi_i(\theta^*))_{i\in NL} = q^*$ and $-q^* \in \mathcal{M}^{\theta^*}$. Hence $(\theta_i^*\phi_i(\theta^*))_{i\in NL} \in \mathcal{M}^{\theta^*}$ and we are done.

Lemma 4.1 *The parameterized game $((v_S)_{S\in\mathcal{N}}, \Theta)$ and the set-valued mapping G meet Assumptions of Theorem 4.1.*

This ends up the proof of Proposition 4.1. □

[13] Note that the proof also holds true for a more general model, where the induced game is not a transferable utility game. Indeed, we do not use the fact the core is polyhedral in the proof. In NTU games, the core of the game has not such a structure.

5 Concluding remarks

1. In Section 4, analog results are obtained if we consider now slightly different statements of the generalized core where $\mathcal{L}$ is set to $\{L\}$ or $\mathcal{N}$ is set to $\{N\}$. We omit the statements of these propositions. To prove the results, we shall need to consider a similar game where the strategy space is restricted to $\mathbb{R}^L$ for the case $\mathcal{N} = \{N\}$ and to $\mathbb{R}^N$ for the case $\mathcal{L} = \{L\}$. For instance, for the latter case: the generalized core $\mathcal{M}_N$ is the set of vectors $p \in \mathbb{R}_+^{LN}$ such that:

$$\begin{cases} \sum_{a\in N} p_a \cdot \phi_a(p) = \psi_N(p) \\ \sum_{a\in A} p_a \cdot \phi_a(p) \leq \psi_A(p) \text{ for all } A \in \mathcal{N}. \end{cases}$$

With respect to the families of mappings $\psi_A : \mathbb{R}_+^{LN} \to \mathbb{R}_+$ for all $A \in \mathcal{N}$, $\phi_a : \mathbb{R}_+^{LN} \to \mathbb{R}_+^L$ for all $a \in N$. Then, define the parameterized game as follows: given $S \in \mathcal{N}$ and $\theta \in \Theta$, $v_S(\theta) = -\psi_S(\theta)$. And,

$$V(\theta) := \left\{ x \in \mathbb{R}^N \mid \sum_{i\in N} x_i = v_N(\theta),\ x_i \geq v_{\{i\}}(\theta) \text{ for all } i \in N \right\} - \mathbb{R}_+^N$$

Henceforth, the lines of arguments are similar to Proof of Proposition 4.1.

2. If $n = 1$, Definition 2.2 can be restated as the set of prices $p \in \mathbb{R}_+^L$ such that:

1. $p \cdot D(p) = C(D(p))$.
2. There is no $(B \in \mathcal{L}, p' \in \mathbb{R}_+^L)$ such that:

$$\begin{cases} p' \cdot D(p')^B = C(D(p')^B) \\ p' <_B p. \end{cases}$$

Hence, it leads back to the definition of Baumol et al. [2] for a contestable market with global demands, these authors have not exhibited general conditions for sustainability existence. Theorem 3.3 provides an existence result for that family of contestable market.

3. Sustainability conditions have been largely studied by Mirman et al. [8], Panzar and Willig [9] and Ten Raa [11]. The most achieved results are given in Mirman [8] and Ten Raa [11]. In Ten Raa, the sustainability is proved under assumptions of complementarities costs and independent goods, also it is assumed that any output levels can be supplied at a profit. The strategy differs from ours, since the sustainability is deduced from an anonymous equity pricing, the proof does not involve a regularity condition.

4. The above results can be obtained as a direct application of Kakutani's fixed point theorem. The set-valued is the following:

$$\Gamma^*(p) = \left\{ u \in \mathbb{R}_+^{LN} \mid (u_{ab} D_a(p)_b)_{ab\in NL} \in \Gamma(p) \right\}$$

Where $\Gamma(p)$ is the core of the cost game associated to the pricing p. This idea is also at stake in Bendali et al. [3]. The authors deduce a subsidy free pricing in a contestable market with a single output technology. Their boundary assumptions slightly differ from ours. They assume that demand functions are positive and that the cost of any output bundle is bounded above by the norm of the output bundle.

5. Two other fixed point approaches are given in Faulhaber and Levinson [6] and Ten Raa [10]. In Faulhaber and Levinson [6], it is proved that an anonymous equitable pricing exists under a cost complementarity property, independent demands and compactness assumption. A closely related approach is Ten Raa [10], the author deduces the existence of an anonymous equity pricing from a condition of supportability. This way of doing is analog to our strategy to deduce subsidy free pricing from fair sharing cost. Furthermore, the author also assumes that there exists consumption threshold $\epsilon > 0$ such that $D(p) \geq \epsilon$. Formally, let us consider the set-valued mapping P:

$$P(d) = \left\{ s \in \mathbb{R}^N \mid s \cdot d = C(d);\ s \cdot d' \leq C(d'),\ \forall d' \leq d \right\}$$

If P has non-empty values, then the cost function is said to be supportable. On suitable compact sets $\mathcal{E}$ and $\mathcal{F}$, a mapping is defined on $\mathcal{E} \times \mathcal{F}$ into itself: $F(p, d) = P(d) \times D(p)$. The mapping admits a fixed point which is exactly the anonymous equitable pricing.

6. As quoted in Sharkey [13], the analysis of cost games to comprehend the stability on natural monopoly market makes abstraction of the preferences of agents. To remedy this drawback, the author defines general cost games where preferences are taken into account in the characteristic function of the games. One is led to the definitions of so called benefit and welfare games. For instance, the characteristic function of a benefit game is given by :

$$v_T = \max_{S \in X} \left\{ \sum_{i \in T} u_i(S) - C(S) \right\}$$

The general formulations of Section 4 also allow to consider that family of games as special cases.

7. In a companion paper, Iehlé [7] uses a different approach based on NTU game theoretical modelling to show the existence of generalized sustainable pricing in a single output contestable markets. It amounts to define an associated NTU game in the price space, the payoffs sets of any coalition being the prices for which the coalition profit is non negative. The sustainable pricing coincides exactly with the core of the NTU game. Then, the existence result follows from Scarf's theorem, indeed the game shall turn out to be balanced under the fair sharing market assumption. It is worth pointing out that in our paper the fair sharing market is also central to get the existence, it guarantees the non-emptiness of the cores of the TU games, parameterized by the prices.

6 Appendix

Proof of Theorem 3.1. The theorem is a direct corollary of Proposition 4.1. Consider the mappings $(\psi_{AB})_{AB\in\mathcal{NL}}$ and $(\phi_a)_{a\in N}$ defined as follows:

$$\psi_{AB}(p) = C(\sum_{a\in A} D_a(p)^B) \text{ and } \phi_a(p) = D_a(p)$$

We show that Assumptions of Proposition 4.1 are all fulfilled. Assumption 1 follows from the continuity of the mapping C and Assumption (D). Continuity Assumption follows from the continuity of the mappings C and $(D_a)_{a\in N}$. Finally, the fair sharing property of the cost function implies that $\mathcal{M}^p$ is non empty for all $p \in \mathbb{R}_+^{LN}$. □

Proof of Lemma 4.1. Assumption (PH1) is satisfied from the continuity of the mappings ψ_A for all $A \in \mathcal{N}$. The game $(v_S(\theta), S \in \mathcal{X})$ is balanced for each $\theta \in \Theta$, from Assumption 3 of Proposition 4.1. We need to check now that Assumption $(PH0)$ is fulfilled. It suffices to show that G is an upper semi-continuous set-valued mapping with non-empty and convex values. Firstly, it is straightforward that the equation in u: $-(u_i\phi_i(\theta))_{i\in NL} \in I(\theta, q)$ has a solution in $\mathbb{R}_+^{NL}$ since I has also non-empty values. The latter stems from the fact that the partition of the singletons forms a balanced family, therefore we deduce that $\sum_{i\in NL} v_{\{i\}}(\theta) \leq v_N(\theta)$ for all $\theta \in \Theta$ from the balancedness condition. Thus, the set of individual rational allocations is non-empty.

Let us check that for all $u \in G(\theta, q)$, $u \in \Theta$. Indeed, for all $i \in NL$, $-u_i\phi_i(\theta) = t_i$, for some $t \in I(\theta, q)$, i.e. $u_i\phi_i(\theta) = -t_i \leq -v_{\{i\}}(\theta) = \psi_{\{i\}}(\theta) \leq B$ since $t \in I(\theta, q)$. Furthermore, $u_i\phi_i(\theta) \geq \epsilon u_i$. Therefore, $u \in \Theta$ and G has non-empty values into Θ.

G has convex values: let $u^1, u^2 \in G(\theta, q)$ and consider $v = \lambda u^1 + (1-\lambda)u^2$ for $\lambda \in [0, 1]$. Since $(u_i^1\phi_i(\theta))_{i\in NL} \in I(\theta, q)$, and, $(u_i^2\phi_i(\theta))_{i\in NL} \in I(\theta, q)$, $(v_i\phi_i(\theta))_{i\in NL} \in \mathrm{co}I(\theta, q)$. It is straightforward to verify that I has convex values, hence we have proved that $v \in G(\theta, q)$.

For upper-semi continuity of G, we need to prove a closed graph assumption, since G is valued into a compact set. It follows directly from the fact that I is upper-semi continuous since p_V is a continuous mapping. Consider the following sequences $(\theta^\nu, q^\nu) \in \Theta \times \mathbb{R}_+^{NL}$ and $u^\nu \in \Theta$ converging respectively to (θ, q) and u such that $u^\nu \in G(\theta^\nu, q^\nu)$ for all ν. Note also, from the continuity of the mappings $(\phi_a)_{a\in N}$, that the vector $(u_i^\nu\phi_i(\theta^\nu))_{i\in NL}$ converges to $(u_i \cdot \phi_i(\theta))_{i\in NL}$. For all ν, $(u_i^\nu\phi_i(\theta^\nu))_{i\in NL} \in I(\theta^\nu, q^\nu)$, since I has a closed graph, we deduce that $(u_i \cdot \phi_i(\theta))_{i\in NL} \in I(\theta, q)$, equivalently $u \in G(\theta, q)$. □

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