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Why will the Kuznets Curve always reverse ?[*]

Patricia CRIFO-TILLET [†] and Etienne LEHMANN[‡]

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Abstract

In this paper, we develop a model of innovation-based growth to address the issue of skill-biased technical change over the long run. We show that innovations fluctuate endogenously from skilled to unskilled sectors, thereby generating periods of increasing and decreasing wage inequality. This could contribute to explain that technological progress exerts a non monotonic pressure on wage inequality over the long run.

Keywords: Innovation-Driven Growth, Wage Inequality, Kuznets Curve, Cycles.

JEL Classification: J31, O31, O41.

Résumé

Dans cet article, nous developpons un modèle de croissance avec innovations en vue de rendre endogène le sens du biais de progrès technique. Nous montrons que les innovations ne se produisent pas systématiquement dans le secteur qualifié et non qualifié. Au contraire, la nature des innovations change, entraînant des périodes d'accroissements et de reductions des inégalités de salaires. Ceci suggèrerait que le progrès technique exerce une pression non monotone sur les inégalités salariales à long terme.

Mots clefs: Croissance fondée sur l'innovation, Inégalités de salaires, Courbe de Kuznets, Cycles.

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1 Introduction

Many have seen in the recent coincidence of computerization and widening wage inequality a skill-biased revolution, in the form of technology-skill or technology-ability complementarity. This technological bias has shed a contradicting light on the Kuznets curve, according to which along the process of development, and particularly along the transition from a rural to an urban and industrialized economy, income inequality initially increases but then declines. One can wonder however whether both stories are really competing. Kuznest's picture of a decline in wage inequality in the first half of the twentieth century in the United States, Britain and Germany and the skill-biased technical change's picture of a dramatic rise in wage inequality since the late 1970s in these countries, may in fact not be contradictory. When considering the joint evolution of technological progress and the wage structure over the long run, wage inequality has evolved in a cyclical fashion, and technological progress is not only characterized by adoptions of innovations permanently raising the returns to skills. Innovations fluctuate between sectors with different skill intensities, explaining why the Kuznets curve has reversed in the second half of the twentieth century, and suggesting that this evolution is likely to happen again.

This paper argues that the Kuznets curve is in fact bound to, and will always, reverse, because researchers are incited to adopt of technologies which may complement skilled and unskilled workers alternatively. Discussions of skill-biased technological change often focus on the relative substitutability or complementarity between skilled and unskilled labor. In this paper, we propose a model that focuses on the decisions to adopt radical technologies over time, and do not take as given the nature of innovations. The choice of the sector in which researchers develop projects in each period is then endogenized. What we are interested in, is therefore to build a model that explains the alternation of innovations from skill-intensive to unskill-intensive technologies, rather than simply comparing the impact of both (exogenous) types of technological advances.

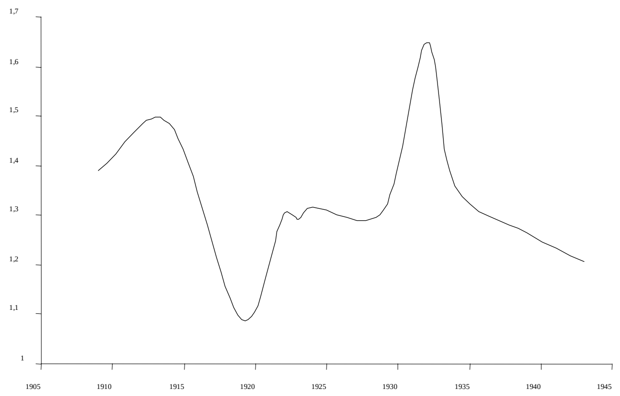
In our model, skill-biased (unskill-biased) technologies are radical innovations which increase the productivity of skilled (unskilled) workers. Creative destruction implies that innovations increase the quality of products (by increasing the productivity of labor) and renders previous ones obsolete [1]. The rents generated by each innovation last therefore only until the next innovation occurs, that is only one period if we define a period as the time interval between two successive innovations. However, when considering two types of sectors in which innovations can occur, the negative effect of creative destruction can

[1] In a sense, our vision of radical technologies is close to that of General Purpose Technologies, that is "enabling technologies" opening up new opportunities and with wide productivity gains (see Bresnahan and Trajtenberg, 1995). We would focus however only on the productivity gains associated with such technologies.

be reduced if the next innovation does not occur in the same sector as the preceding one. In this case indeed, the previous product is not rendered obsolete and the corresponding rents are not destroyed, at least for one more period. In the patent race, researchers are then incited to differentiate the nature of innovation, that is to develop project not always in the same sector. This mechanism introduces another effect in the dynamics of technological adoption, which we call a *business-continuing effect,* in contrast to the traditional business-stealing effect (emphasizing the negative impact of future innovations on current research). A new innovation appearing in a different sector does not make the previous product obsolete. In turn, because there is an incentive to adopt innovations in a different sector, technological progress will be characterized by alternation from one sector to another, thereby exerting a cyclical pressure on wage inequality, rather than a permanent skill bias. The simplest kind of cycle, and the only one which is analytical tractable, is the *two-cycle*, where alternation of innovations between both sectors occurs in each period. With such a dynamics, wage inequality is cyclical.

There is empirical evidence in line with our results. Our first key result is that researchers are incited to alternate the sector in which they develop projects, and growth is not driven only by skill or ability biased innovations. For Caselli (1999), technological revolutions may in fact either be skill-biased, like the dynamo, the steam engine and the information technology, or de-skilling, like the assembly lines. Goldin and Katz (1999) observe that in the history of many innovations, like automobile production, the first technological advances reduced the relative demand for skilled labor and later advances increased it. An interpretation of such an observation is to consider that technical progress is cyclical and exhibits periods of increasing skilled labor productivity and increasing unskilled labor productivity.

As a consequence, this fluctuation in innovation translates into cyclical wage inequality, which also finds support in the literature. The evolution of wage inequality in the first half of the twentieth century in the United States has been documented by Goldin and Katz (1999). The following figure reproduces the evolution of the wage ratio of starting engineers to average low-skilled workers in the beginning of the twentieth century in the U.S., using their data.



Wage ratio of starting engineers to average low-skilled workers in the U.S.

Source: our own computations, from Goldin and Katz (1999).

We see that the wage differential between skilled and unskilled labor declined in the 1910s, increased in the 1920s, and decreased again in the 1930s and the 1940s.

The evolution of the wage structure in the U.S. in the second half of the century, as documented by Autor, Katz and Krueger (1998), Goldin and Margo (1992), and Katz and Murphy (1992) is also consistent with a cyclical picture. These studies reveal that the wage differential between skilled and unskilled workers widened in the 1950s and the 1960s, narrowed until the 1970s, and then widened until the 1990s.

Our model is related to the recent theoretical literature on skill-biased technical change supported by Galor and Moav (2000), Caselli (1999), Acemoglu (1998), Aghion and Howitt (1998), and Galor and Tsiddon (1997) among others. Caselli (1999) develops a model of technological revolutions that explains the evolution of the wage structure and the absolute decline in unskilled wages since the 1970s. A technological revolution is skill-biased if the skills required to use a new and more productive machine are more costly than the skills needed for a preexisting machine, and it is de-skilling if the new skills required can be acquired at a lower cost. Skill-biased revolutions raise the skill premium, and workers staying on the old machines experience a fall in both their relative and absolute wage. By symmetry, a de-skilling technological revolution reduces wage inequality. However, his model does not endogenize the sequential nature of both types of technological adoption (skill-biased or de-skilling) and does not generate endogenous cyclical fluctuations of wage inequality.

Closer to this issue, Galor and Tsiddon (1997) propose a model in which the life cycle of technology governs the evolution of wage inequality. In periods of major technological progress (inventions) the return to ability increase, driving wage inequality upward. Once existing technologies become more accessible, that is in periods of technological innovations, the role of ability in individual earnings declines, and so does wage inequality. The transition from inventions to innovations is however not the result of a trade-off in the research sector between different technologies. The occurrence of inventions is an increasing function of the average level of human capital in technologically advanced sectors, while we assume that researchers face a trade-off between improving skill-intensive or unskill-intensive technologies in each period. In addition, we are solely concerned by radical innovations, i.e. inventions.

Considering two sectors of intermediate goods into a productivity-augmenting growth model *à la* Aghion and Howitt (1992), the closest model to ours seems to be that of Acemoglu (1998). However, innovations are radical in our model, whereas they are incremental in Acemoglu's framework. Besides, Acemoglu endogenizes the skill bias by a market size effect according to which an increase in the supply of skills increases the market size for technologies complementary to skilled labor, thereby driving the skill premium upward. Here, we endogenize the sectorial choice of technological adoptions and show that researchers are incited to alternate from one sector to another, thereby exerting a non monotonic pressure on wage inequality.

The reminder of the article is organized as follows. Section 2 presents the basic structure of the model. Section 3 defines the competitive equilibrium concept used and shows what determines alternation of innovations between sectors. Section 4 analyzes the simple case of a two-cycle dynamics, and its impact on wage inequality. Section 5 concludes.

2 The model

2.1 Description of the environment

The framework considered is a closed economy where growth is driven by productivity gains associated with successive radical innovations. The economy is composed of a final good sector, two intermediate good sectors ($i = H, L$) and a research sector. The final good is produced in a perfectly competitive environment, using both types of intermediate goods as inputs. Each intermediate good sector is composed of a continuum of firms that monopolistically produce imperfectly substitutable goods. The outcome of the research process is a radical innovation. Each successful innovator obtains a patent on its radical innovation which enables him to sell without any additional cost a continuum of licences to the intermediate good producers. A radical innovation has different consequences on the productivity of intermediate good sectors, and for simplicity, we restrict our attention to radical innovations which are specific to a single intermediate good sector. Intermediate good sectors also differ in their skill requirement. We assume that firms in the L sector rely only on unskilled workers, whereas firms in the H sector rely only on skilled workers. We do this for simplicity, although smaller differentiation could be more realistic. Hence, we consider that any radical innovation specifically increases the productivity of a single type of workers. For instance, the diffusion of electricity was a radical innovation which induced many incremental innovations (new machines) that tended to decrease inequality during the 1909-1929 period. On the contrary, electronics is a radical innovation which is leading to many incremental innovations (new software) that tend to increase wage inequality nowadays. The research sector is characterized by a patent race, and innovations are both radical and uncertain, occurring according to a Poisson process.

Time is continuous. Subscript t denotes the number of innovations until the current period [2]. The intermediate good sectors are indexed by $i = H, L$. Let A_t^i be the productivity of workers employed in sector i after t innovations. Innovations improve the production processes of intermediate goods that replace the old ones, and raise the technology level (i.e. the labor productivity parameter) in the corresponding intermediate good sector by a factor $1/(1-\gamma)$, with $0 < \gamma < 1$. If the $(t+1)^{st}$ innovation occurs in sector i, then $A_{t+1}^i = \frac{1}{1-\gamma} \cdot A_t^i$ and $A_{t+1}^{-i} = A_t^{-i}$. Individuals are risk-neutral and can perfectly substitute consumption over time at the exogenous discount rate r.

2.2 Labor market and resources constraints

The economy is populated by a fixed mass of individuals. Three continuum of workers are considered: skilled workers employed in the skilled intermediate good sector, unskilled workers employed in the unskilled intermediate good sector and researchers. Let N_t^i be the mass of workers employed in the intermediate good sector i, and R_t^i be the mass of skilled workers engaged in the research for an innovation in sector i. We assume that workers can be skilled workers, unskilled workers, or researchers, and that there is no mobility between the research and the skilled intermediate good sector [3]. In other words, this is equivalent to assume that the allocation of the labor force is constant across sectors, and the three categories of workers are immobile. Normalizing the size of each category of workers to

[2] From now on, what we call a period is the time interval between two successive innovations, which is of random duration.

[3] Relaxing this assumption to allow mobility between research and skilled intermediate manufacturing does not change the qualitative results. However, in this case, the model is no more analytically tractable, and only numerical simulations, available upon request, can be conducted.

one, the labor market clearing conditions write:

$$N_t^H = 1 \qquad N_t^L = 1 \qquad \text{and} \qquad R_t^H + R_t^L = 1 \tag{1}$$

2.3 Final consumption good sector

The final good is the numeraire in this economy. It is produced competitively under a standard constant return to scale technology using both types of intermediate goods as inputs. The production function is given by:

$$Y_t = F\left(C_t^H, C_t^L\right) \qquad F_i' > 0 \qquad F_{12}'' > 0 \qquad F_{ii}'' < 0 \quad , \quad i = H, L \tag{2}$$

where $F_i'(.,.)$ denotes partial derivative of F with respect to C_t^i. C_t^i is an aggregate for the intermediate good i used, and is defined by:

$$C_t^i = \left(\int_0^1 \left[C_t^i(s)\right]^\beta ds\right)^{\frac{1}{\beta}} \qquad 0 < \beta < 1$$

where $C_t^i(s)$ is the quantity of the s^{th} type-i intermediate good used. Let $p_t^i(s)$ be the price of intermediate good i used to produce final goods. Profit maximization by a representative firm in the final good sector yields the following inverse demand function:

$$p_t^i(s) = F_i'\left(C_t^H, C_t^L\right) \cdot \left(\int_0^1 \left[C_t^i(s)\right]^\beta ds\right)^{\frac{1}{\beta}-1} \cdot \left[C_t^i(s)\right]^{\beta-1} \tag{3}$$

2.4 Intermediate goods sector

In each sector, we assume that there is a continuum (of mass 1) of intermediate good producers, with a technology featuring constant returns to scale as follows:

$$C_t^i(s) = A_t^i(s) \cdot n_t^i(s), \qquad i = H, L \tag{4}$$

where $i = H, L$, $n^i(s)$ is the number of workers employed by firm s in sector i, and $A_t^i(s)$ is the productivity of firm s in sector i. We consider that innovations are radical. Hence, the patent owner of the latest technology required to produce the s^{th} type-i intermediate good is in a monopoly position. Each intermediate good producer maximizes her profit stream $\pi_t^i(s) = p_t^i(s) \cdot C_t^i(s) - w_t^i(s) \cdot n_t^i(s)$, given equations (3) and (4). The first order condition of this program yields[4]:

[4] If innovations are non-drastic, the price set by the owner of the last technology is the highest that excludes the owner of the before the last technology from the market of s^{th} type$-i$ intermediate good. This price limit strategy leads to

$$p_t^i(s) = \frac{w_t^i(s)}{(1-\gamma) \cdot A_t^i(s)}$$

and nothing in the rest of the article is changed provided $1 - \beta = \gamma$. The condition for innovations to be drastic is thefore $\gamma > 1 - \beta$.

$$\beta \cdot p_t^i(s) = \frac{w_t^i(s)}{A_t^i(s)} \tag{5}$$

In turn, individual equilibrium profits write $\pi_t^i(s) = (1-\beta) \cdot p_t^i(s) \cdot C_t^i(s)$. By the way, symmetry in the intermediate good sector implies $p_t^i(s) = p_t^i = F_i'\left(C_t^H, C_t^L\right)$, $C_t^i(s) = C_t^i$ and $n_t^i(s) = n_t^i$. Since firms in sector H employ only skilled labor, and firms in sector L employ only unskilled labor, labor market clearing condition implies that $\int n^i(s)ds \equiv N^i$, $i = H, L$. Recall that there is no mobility between research and intermediate good production, therefore $N^i \equiv 1$. Taking into account equations (3), (4), (5), and the labor resources constraint $N_t^i \equiv 1$, aggregate intermediate profits in each sector are then given by:

$$\pi_t^i = \Pi^i\left(A_t^H, A_t^L\right) = (1-\beta) \cdot A_t^i \cdot F_i'\left(A_t^H, A_t^L\right) \tag{6}$$

2.5 Research sector

Innovations are radical and consist in a *continuum* of improvements in the production processes of intermediate goods of type i which replaces the old ones and raise*s* the technological parameter (that is the labor productivity) in sector i, A_t^i, by a constant factor $\frac{1}{1-\gamma}$ [5].

Innovation is portrayed as a patent race with a probability of success following a Poisson process. Research for the $(t+1)^{st}$ innovation lasts one period, and takes place in period t. The duration of a period between two successive innovations (i.e. between t and $t+1$) is stochastic, and follows a Poisson process of parameter $\lambda \cdot \left(R_t^H + R_t^L\right)$, where λ is the individual Poisson probability of innovating and R_t^i is the amount of labor used in research in sector $i = H, L$. Because time is continuous, the probability that two innovations occur at the same time in both sectors is null. This does not imply of course that researchers cannot develop projects in both sectors.

We assume indeed that there is a continuum of researchers normalized to one indexed by $j \in [0,1]$, and that each researcher devotes a fraction of his time to the development of project that may lead to higher product quality (i.e. higher labor productivity) in sector L or H. Each researcher is endowed with one unit of time. Let $T_t^i(j)$ denotes the fraction of time devoted by researcher j to sector i. With unit time endowment, we have $T^H(j) + T^L(j) = 1$. Besides, summing over all researchers yields the aggregate amount of research devoted to sector i: $R_t^i \equiv \int_0^1 T_t^i(j)dj$, $i = H, L$.

[5] Recall that if the t^{th} innovation occurs in sector i, then $A_t^i = \frac{1}{1-\gamma} \cdot A_{t-1}^i$ whereas $A_t^{-i} = A_{t-1}^{-i}$, and *vice versa*.

The research technology is a one-for-one relationship. The individual rate of successfully innovating in sector i for researcher j is given by:

$$y_t^i(j) = \lambda \cdot T_t^i(j)$$

Such a technology implies constant returns to scale in the research sector on the one hand, and on the other hand, it relies on the assumption that the productivity of the time devoted to research in sector i is not affected by the time devoted to sector $-i$.

We define by V_t^i the sum of prices of licences in sector i for the t^{th} innovation. Researchers allocate their time between both sectors by maximizing their expected profits, according to the following program, taking as given the price of licences:

$$\max_{T_t^H(j), T_t^L(j)} \lambda \cdot T_t^H(j) \cdot V_t^H + \lambda \cdot T_t^L(j) \cdot V_t^L \quad s.t. \quad T_t^H(j) + T_t^L(j) = 1$$

The solution of this program is straightforward: whenever $V_t^H > V_t^L$, researchers are incited to allocate their entire time endowment to sector H, that is $V_t^H > V_t^L \Leftrightarrow T_t^H(j) = 1, \quad T_t^L(j) = 0$; and *vice versa*. The case $V_t^H = V_t^L$ implies extreme assumptions on initial conditions regarding the value of productivity and profits in the intermediate sectors, which are not robust to small variations, and is therefore ruled out.

Aggregate research decisions consist then in the following rule:

$$V_t^H < V_t^L \Leftrightarrow R_t^H = 0 \quad and \quad R_t^L = 1; \qquad V_t^L < V_t^H \Leftrightarrow R_t^L = 0 \quad and \quad R_t^H = 1 \tag{7}$$

According to this rule, in each period research occurs only in a single sector. When innovating in sector i, the $(t+1)^{st}$ innovator sells his patent to the corresponding intermediate good firms at a price V_t^i. This patent enables to produce the intermediate good i at a lower cost, thereby driving out of the market the old intermediate good producers. Assuming perfect financial markets, V_t^i must equals the intertemporal expected profit earned monopolistically until replacement by the next innovation in sector i.

Unlike Aghion and Howitt (1992) 's basic model, if the t^{th} innovation occurred in sector i, the $(t+1)^{st}$ innovation is not bound to occur in sector i. Basically, a patent may be of finite or infinite duration, depending on whether the $(t+2)^{nd}$, $(t+3)^{rd}$ ... innovations take place in sector i or in sector $-i$. This is a fundamental difference between our approach and the previous literature. Hence, V_t^i is conditional upon the expected dynamics of R_t^H and R_t^L for the next periods, and therefore upon the expected duration of patents. For instance, when the t^{th} innovation occurs in sector H, if the $(t+1)^{st}$ occurs also in sector

H the t^{th} innovator is replaced, and the patent lasts only one period. In contrast, if the $(t+1)^{st}$ innovation occurs in sector L, then the t^{th} innovator is not replaced at the beginning of period $t+1$, and the patent lasts at least two periods.

Let $W_t^i(s)$ denotes the value of firm s in sector i during period t. The successful innovator is in a monopoly position which allows him to sell the continuum of licences at the highest price an intermediate good producer is willing to pay. Hence patent price and firms' values are related according to the appropriation equation:

$$V_t^H = \int_0^1 W_{t+1}^H(s)ds \qquad \text{and} \qquad V_t^L = \int_0^1 W_{t+1}^L(s)ds \tag{8}$$

The incentives to undertake research projects are such that the patent price equals the expected value of an intermediate firm. The value $W_t^i(s)$ is defined by the following asset equation:

$$r \cdot W_t^i(s) = \pi_t^i(s) - \underbrace{\lambda \cdot R_t^i(s) \cdot W_t^i(s)}_{\text{“business-stealing”}} + \underbrace{\lambda \cdot R_t^{-i}(s) \cdot \left[W_{t+1}^i(s) - W_t^i(s)\right]}_{\text{“business-continuing”}} \tag{9}$$

where r is the (exogenous) discount rate.

Because of the symmetry property, $A_t^i(s) = A_t^i$ and all indexes s can be dropped. The interpretation of equation (9) is the following. The expected income generated by a licence on the t^{th} innovation in sector i is composed of three elements (appearing in the right hand side).

- The first one is the flow of aggregate profits of intermediate good producers π_t^i during period t in sector i. With probability $\lambda \cdot R_t^i$, the $(t+1)^{st}$ innovation occurs in sector i, which implies that incumbents are replaced by new innovators in sector i.

- The second element in the right hand side of (9) is therefore the expected "capital loss" incurred by incumbent s when its monopoly position disappears because a new radical innovation appeared in sector i. This loss corresponds to the standard Aghion and Howitt's "business-stealing" effect, whereby the next innovator destroys the surplus attributable to the previous generation of intermediate good i, by making it obsolete. However, with probability $\lambda \cdot R_t^{-i}$ the $(t+1)^{st}$ innovation occurs in sector $-i$, and the incumbent is not replaced. Licences on the t^{th} innovation are then still valid when the $(t+1)^{st}$ innovation occurs.

- The third element in the right hand side of (9) is thus the expected "capital gain" obtained by incumbent s when the next radical innovation occurs in the other sector so that its patent lasts more than one period. By symmetry with the business-stealing effect, we call this gain a “business-continuing” effect, whereby the next innovator does not make the preceding generation of intermediate good i obsolete.

3 Equilibrium dynamics

An equilibrium in this economy is defined as follows.

Definition 1: *An equilibrium is a sequence of* A_t^H, A_t^L, R_t^H, R_t^L, V_t^H, V_t^L, $W_t^H(.)$ *and* $W_t^L(.)$ *defined by equations (6) to (8), where*

1. *when* $V_t^L < V_t^H$, *the economy's stock of researchers between innovations* t *and* $t+1$ *is allocated to sector* H *only*: $R_t^H = 1$, $R_t^L = 0$,
2. *when* $V_t^L > V_t^H$, *the economy's stock of researchers between innovations* t *and* $t+1$ *is allocated to sector* L *only*: $R_t^H = 0$, $R_t^L = 1$.

Hence, the equilibrium consists in a fixed point between 3 relations. First, the dynamics of the number of researchers $\{R_t^L, R_t^H\}_{t\geq 0}$ defines $\{W_{t+1}^L, W_{t+1}^H\}_{t\geq 0}$ according to the asset equations (9). Second, equations (8) define $\{V_t^L, V_t^H\}_{t\geq 0}$ as functions of $\{W_{t+1}^L, W_{t+1}^H\}_{t\geq 0}$. Lastly, patents prices $\{V_t^L, V_t^H\}_{t\geq 0}$ simply give $\{R_t^L, R_t^H\}_{t\geq 0}$ at any point in time depending on which intermediate good yields the highest profits (H or L) according to the researchers' decision rule (7). The corner stone of this definition is that all researchers are atomistic and ignore their impact on the invention probability of other researchers in the same sector. Thus, they take as given the patent prices $\{V_t^L, V_t^H\}_{t\geq 0}$ when choosing which technology (H or L) they want to improve. They don't take into account the influence of their decisions on the dynamics of patent prices, which is indeed a standard assumption in competitive equilibria.

This paper questions the relevance of the hypothesis of a permanent adoption of skill-biased technologies. We have therefore to look at conditions under which researchers permanently improve skilled intermediate goods. In our economy this is equivalent to establish the conditions under which in each period innovations occur in the skilled sector. The case where innovation appears permanently in the unskilled sector is perfectly symmetric and therefore omitted. The equilibrium condition is given by:

$$\text{for all } t, \qquad R_t^H = 1,\ R_t^L = 0 \quad \text{and} \quad V_t^L < V_t^H \tag{10}$$

The present value of innovations in the skilled sector, V_t^H, is easily determined by substituting condition (10) in the asset equation (9):

$$V_t^H = W_{t+1}^H = \frac{1}{r+\lambda}\Pi^H\left(\frac{A_t^H}{1-\gamma}, A_t^L\right)$$

Since innovation is assumed to occur systematically in the skilled sector, each patent in this sector lasts only one period, and we obtain Aghion and Howitt's asset equation as a particular case of our model.

As regards the present discounted value of an innovation in sector L, V_t^L, since all innovations occur in sector H, an innovation in sector L is expected to yield a patent that lasts indefinitely. If innovation t occurs in sector L, this is considered by any atomistic innovator as a deviation from the equilibrium behavior which only happens once. Thus, V_t^L is given by $V_t^L = W_{t+1}^{d,L,t}$, where $W_{t+k}^{d,L,t}$, $k \geq 1$ is defined recursively by:

$$rW_{t+k}^{d,L,t} = \Pi^L\left(\frac{A_t^H}{(1-\gamma)^{k-1}}, \frac{A_t^L}{1-\gamma}\right) + \lambda\left[W_{t+k+1}^{d,L,t} - W_{t+k}^{d,L,t}\right]$$

which implies:

$$V_t^L = W_{t+1}^{d,L,t} = \frac{1}{r+\lambda}\sum_{k=1}^{+\infty}\left(\frac{\lambda}{r+\lambda}\right)^{k-1}\Pi^L\left(\frac{A_t^H}{(1-\gamma)^{k-1}}, \frac{A_t^L}{1-\gamma}\right)$$

Proposition 1 *A dynamic such that for any $t \geq t_0$ innovation permanently appears in the same sector is an equilibrium iff:*

$$\forall t \quad \sum_{k=1}^{+\infty}\left(\frac{\lambda}{r+\lambda}\right)^{k-1}\Pi^L\left(\frac{A_{t_0}^H}{(1-\gamma)^{k-1}}, \frac{A_{t_0}^L}{1-\gamma}\right) < \Pi^H\left(\frac{A_{t_0}^H}{1-\gamma}, A_{t_0}^L\right) \tag{11}$$

$$or \quad \forall t \quad \sum_{k=1}^{+\infty}\left(\frac{\lambda}{r+\lambda}\right)^{k-1}\Pi^H\left(\frac{A_{t_0}^H}{1-\gamma}, \frac{A_{t_0}^L}{(1-\gamma)^{k-1}}\right) < \Pi^L\left(A_{t_0}^H, \frac{A_{t_0}^L}{1-\gamma}\right) \tag{12}$$

Thes inequalities are very unlikely to be satisfied. In particular, as the discount rate r tends to 0, the left hand side of equations (11) and (12) tend to $+\infty$ [6]. This is precisely the reason why we consider this kind of dynamics as very implausible. There is a trade-off between a patent which lasts one period only, and an infinite-lived patent. With no discounting, deviating once leads to a rent that tends to $+\infty$. Thus, permanent innovations in the same sector are very implausible dynamics.

[6] From equation (6) and the fact that $F''_{L,H} > 0$, we get that $\Pi^L\left(A^H, A^L\right)$ increases in A^H, implying that, for any $k \geq 1$, $\Pi^L\left(\frac{A_t^H}{(1-\gamma)^{k-1}}, \frac{A_t^L}{1-\gamma}\right) > \Pi^L\left(A_t^H, \frac{A_t^L}{1-\gamma}\right)$. This implies that, as r tends to 0, the right hand side of equation (11) tends to $+\infty$.

Proposition 2 *If equation (11) and (12) are false, any equilibrium dynamics is characterized by infinitely alternation of periods where innovations occur in sector H and periods where innovations occur in sector L.*

The proof derives from the counter-proposition of proposition 1. Since (11) or (12) are necessary conditions for a permanent bias in the same sector to occur, and since these conditions are very implausible, innovations will never occur infinitely in the same sector.□

In other words, at any point in time, there will be a moment when the direction of the technical bias will change. There is therefore always a time when alternation occurs. Fluctuation in technolgoy adoption is endogenous and stems from the business-continuing effect whereby the rents generated on a particular innovation last as long as future innovations occur in a different sector. The Kuznets curve is then bound to reverse because of this incentive to innovate in different sectors.

4 Two-Cycle dynamics

Our theoretical model generates multiple dynamics with alternation. The simplest one is the two-cycle, where alternation occurs each time an innovation appears. Such a dynamics only constitutes a proxy for the discontinuity in radical innovation adoption. Yet, this kind of dynamics is analytically tractable and provides a simple stylized version of the patterns of cycles of innovation and inequality observed during the twentieth century in the United States. The evolution of the in the U.S. over the twentieth century, appears indeed overall highly cyclical. As mentioned in the introduction, the wage differential between skilled and unskilled labor widened during the 1910s, the 1930s, the 1950s, the 1960s, the 1980s and 1990s, while it declined during the 1920s, the 1940s, and the 1970s. At the same time, the life cycle of technology also appears cyclical. After a period of bias towards skilled workers generating technology-skill complementarity (the very first advances may reduce the relative demand for skills, but later ones increase it, as observed by Goldin and Katz, 1996); increased accessibility of technologies then favors unskilled workers (Galor and Tsiddon, 1997). A two-cycle dynamics seems in line with this picture, even if this is in fact an extreme - but tractable - example.

4.1 Characteristics of a two-cycle

The kind of cycles we study is such that during even periods researchers improve skilled intermediate goods, and during odd periods, researchers improve unskilled intermediate goods. With such a dynamics and using the constant return to scale property of functions $\Pi^i(.,.)$, the variables of the model evolve in the following way:

t	**0**	**1**	**2**	**3**	...	**2 t**	**2 t + 1**
R_t^H	1	0	1	0	...	1	0
R_t^L	0	1	0	1	...	0	1
A_t^H	A^H	$\frac{1}{1-\gamma}A^H$	$\frac{1}{1-\gamma}A^H$	$\left(\frac{1}{1-\gamma}\right)^2 A^H$	...	$\left(\frac{1}{1-\gamma}\right)^t A^H$	$\left(\frac{1}{1-\gamma}\right)^{t+1} A^H$
A_t^L	A^L	A^L	$\frac{1}{1-\gamma}A^L$	$\frac{1}{1-\gamma}A^L$	...	$\left(\frac{1}{1-\gamma}\right)^t A^L$	$\left(\frac{1}{1-\gamma}\right)^t A^L$
π_t^H	π_0^H	π_1^L	$\frac{1}{1-\gamma}\pi_0^H$	$\frac{1}{1-\gamma}\pi_1^H$	...	$\left(\frac{1}{1-\gamma}\right)^t \pi_0^H$	$\left(\frac{1}{1-\gamma}\right)^t \pi_1^H$
π_t^L	π_0^L	π_1^L	$\frac{1}{1-\gamma}\pi_0^L$	$\frac{1}{1-\gamma}\pi_1^L$	...	$\left(\frac{1}{1-\gamma}\right)^t \pi_0^L$	$\left(\frac{1}{1-\gamma}\right)^t \pi_1^L$

Table 1: Two-cycle dynamics

Where

$$\pi_0^H = (1-\beta)\ A^H\ F_H'\left(A^H, A^L\right) \qquad \pi_1^H = \frac{1-\beta}{1-\gamma}\ A^H\ F_H'\left(\frac{A^H}{1-\gamma}, A^L\right) \tag{13}$$

$$\pi_0^L = (1-\beta)\ A^L\ F_L'\left(A^H, A^L\right) \qquad \pi_1^H = (1-\beta)\ A^L\ F_L'\left(\frac{A^H}{1-\gamma}, A^L\right)$$

Proposition 3 *A two-cycle dynamics is an equilibrium iff:*

$$\pi_1^H + \frac{\lambda}{r+\lambda}\frac{\pi_0^H}{1-\gamma} > \pi_1^L \qquad \text{and} \qquad \pi_0^H < \pi_0^L + \frac{\lambda}{r+\lambda}\pi_1^L \tag{14}$$

Proof: see appendix 6.1. □

Condition (14) means that for a two-cycle to exist, the expected profits of the innovator (evaluated over two periods) must be higher than the expected profits of its competitor in the other intermediate sector. More interestingly, condition (14) can be rewritten as:

$$\frac{1}{r+\lambda}\frac{\pi_0^H}{1-\gamma} > \frac{\pi_1^L - \pi_1^H}{\lambda} \qquad \text{and} \qquad \frac{1}{r+\lambda}\pi_1^L > \frac{\pi_0^H - \pi_0^L}{\lambda} \tag{15}$$

This condition states then that the marginal benefit of innovation in each sector when the next innovation occurs in this sector (discounted at rate $r+\lambda$) must be higher than the difference in marginal cost between both firms (opportunity costs measured in profits), a condition that meets Aghion and Howitt (1992)'s basic condition.

4.2 Application to a CES technology

To study the links between the fluctuations of technological adoptions from one sector to the other and the dynamics of wage inequality, we have to rely on specific functional forms. We consider thus that the final good is produced according to the following CES technology:

$$Y_t = \left[\alpha^H \left(C_t^H\right)^\sigma + \alpha^L \left(C_t^L\right)^\sigma\right]^{\frac{1}{\sigma}} \tag{16}$$

where $\sigma \in]-\infty, 1[$, $\sigma \neq 0$, $0 < \alpha_i < 1$ and $\alpha^H + \alpha^L = 1$. Hence, $1/(1-\sigma) > 0$ is the elasticity of substitution between skill-intensive and unskilled-intensive intermediate goods. With such a production function, the prices of intermediate goods are given by [7]:

$$p_t^i = F_i' = \alpha^i \left(A_t^i\right)^{\sigma-1} \left[\alpha^H \left(A_t^H\right)^\sigma + \alpha^L \left(A_t^L\right)^\sigma\right]^{\frac{1-\sigma}{\sigma}} \tag{17}$$

In equilibrium, the relative inverse demand function writes:

$$p \equiv \frac{p^H}{p^L} = \frac{\alpha^H}{\alpha^L} \left(\frac{C^L}{C^H}\right)^{1-\sigma}$$

Besides, profits in the intermediate good sectors are given by:

$$\pi_t^i = (1-\beta)\, \alpha^i \ \left(A_t^i\right)^\sigma \left[\alpha^H \left(A_t^H\right)^\sigma + \alpha^L \left(A_t^L\right)^\sigma\right]^{\frac{1-\sigma}{\sigma}} \tag{18}$$

To see whether condition (14) is realistic, we can consider a particular case of (16) where the elasticity of substitution between both intermediate goods tends to 1 (i.e. $\sigma \to 0$), when α tends to 1/2 by upper values and A^H tends to A^L. The difference between π_t^H and π_t^L then tends to 0, implying that condition (15) is satisfied. Hence, by continuity, a two-cycle is an equilibrium, at least when the differentiation between intermediate goods is not too wide.

4.3 Growth rates of output and wages

The time interval between two successive innovations is stochastic and given by a Poisson process of rate $\lambda \cdot R_t^i$, given the labor market clearing condition $R_t^H + R_t^L = 1$. Besides, in the two-cycle equilibrium, we have either $R_t^H = 1$ and $R_t^L = 0$, or $R_t^L = 1$ and $R_t^H = 0$. The time interval between two successive innovations is therefore simply equal to λ. The average growth rate is then defined by $g_{t\to t+1} \equiv \lambda \ln \frac{Y_{t+1}}{Y_t}$.

Using equations (5), (16) and (17), and the values in table 1, the approximate growth rates of output and wages are given in the following table (see appendix 6.2 for details):

	$2t \to 2t+1$	$2t+1 \to 2t+2$	$2t \to 2t+2$
g^Y	$\lambda \cdot \gamma \cdot \Gamma$	$\lambda \cdot \gamma \cdot (1-\Gamma)$	$\lambda \cdot \gamma$
g^{w^H}	$\lambda \cdot \sigma \cdot \gamma + \lambda \cdot (1-\sigma) \cdot \gamma \cdot \Gamma$	$\lambda \cdot (1-\sigma) \cdot \gamma \cdot (1-\Gamma)$	$\lambda \cdot \gamma$
g^{w^L}	$\lambda \cdot (1-\sigma) \cdot \gamma \cdot \Gamma$	$\lambda \cdot \sigma \cdot \gamma + (1-\sigma) \cdot \gamma \cdot (1-\Gamma)$	$\lambda \cdot \gamma$
$g^{w^H} - g^{w^L}$	$\lambda \cdot \sigma \cdot \gamma$	$-\lambda \cdot \sigma \cdot \gamma$	0

Table 2: Growth rates of output and wages

[7] Recall that because of the symetry property, we have dropped index s.

where $\Gamma \equiv \frac{\alpha^H (A^H)^\sigma}{\alpha^H (A^H)^\sigma + \alpha^L (A^L)^\sigma}$ corresponds both to the share of expenditures of skilled intermediate goods in the total final good production, and to the share of skilled workers' wages in the total wage bill $w_t^H N_t^H / \left(w_t^H N_t^H + w_t^L N_t^L \right)$. Since $N_t^H = N_t^L = 1$, we get $\Gamma/(1-\Gamma) = w^H/w^L$, therefore it is likely that $\Gamma > 1 - \Gamma$. The inequality $g^Y_{2t \to 2t+1} > g^Y_{2t+1 \to 2t+2}$ is therefore the most plausible. Hence, innovations in the skill-intensive sector make the economy grow faster than when innovations appear in the unskilled-intensive sector. Depending on the value of σ, growth rates along a two-cycle dynamics evolve differently, as shown in table 3.

	$0 < \sigma < 1$	$\sigma < 0$
$\left(g^{w^H} - g^{w^L} \right)_{2t \to 2t+1}$	$+$	$-$
$\left(g^{w^H} - g^{w^L} \right)_{2t+1 \to 2t+2}$	$-$	$+$
$g^Y_{2t \to 2t+1} - g^Y_{2t+1 \to 2t+2}$	$+$	$-$
$g^{w^H}_{2t \to 2t+1} - g^{w^H}_{2t+1 \to 2t+2}$	$+$	$-$
$g^{w^L}_{2t \to 2t+1} - g^{w^L}_{2t+1 \to 2t+2}$	$-$	$+$

Table 3: Evolution of growth rates

Thus, if $0 < \sigma < 1$, when innovation appears in the skilled sector (from even to odd period), output and skilled wages grow at a faster pace, and wage inequality increases to the detriment of unskilled workers. When innovation occurs in the unskilled sector (from odd to even period),wage inequality decreases to the benefit of unskilled workers. The results are reversed when $\sigma < 0$.

4.4 Comments

Table 3 shows that wage inequality is cyclical, but its fundamental causes vary according to whether both categories of workers are relatively substitutable or not. In fact, what we call skill-biased technological change depends on whether $\sigma < 0$ or $\sigma > 0$, that is on whether skilled and unskilled intermediate goods are more substitutable than in the Cobb-Douglas case.

- When $0 < \sigma < 1$, that is when $\frac{1}{1-\sigma} > 1$, skilled and unskilled intermediate goods are more substitutable than in the Cobb-Douglas case. In that case, firms are incited to reorganize production and demand more workers who have experienced productivity gains, which translates in our model into an increase in the wage premium granted to this type of labor (skilled workers in odd periods, and unskilled workers in even periods). In this case, a skill-biased technological change corresponds to an increase in A^H/A^L which occurs between odd and even periods.

- When $\sigma < 0$, that is when $\frac{1}{1-\sigma} < 1$, skilled and unskilled intermediate goods are less substitutable than in the Cobb-Douglas case. In that case, the rise in unskilled labor productivity incites firms to employ less of this type of labor because of the weak substitutability between skilled and unskilled labor, and relative unskilled wages rise. Indeed, firms are incited to save workers who have experienced productivity gains, which translates in our model into a decrease in the wage premium granted to this type of labor (skilled workers in odd periods, unskilled workers in even periods). In this case, a skill-biased technological change corresponds to a decrease in A^H/A^L which occurs between even and odd periods.

Most estimates of the elasticity of substitution between skilled and unskilled workers $\left(\frac{1}{1-\sigma}\right)$ correspond to the first case. In particular, the usual estimate is near 1.7, which corresponds to a value of 0.4 for σ (Krusell and al., 2000). Then, our model accounts for the cyclical evolution of wage inequality over the century, namely that wage inequality increases when innovations are adopted in skill-intensive sectors, and decreases when innovations appear in unskilled-intensive sectors. However, taking into account the role of physical capital, most studies show a very high complementarity between skilled labor and physical capital, and an elasticity of substitution between capital and unskilled labor smaller than 1 (see Krusell and al., 2000). This suggests that we can not ruled out the case where $\sigma < 0$.

However, in our framework, in both cases, periods of high and periods of low levels of wage inequality alternate. Skill-biased technological changes always increase skilled workers' wage and poverty in relative terms whereas unskilled-biased technological changes always increase unskilled workers' wage and decrease poverty in both absolute and relative terms. The only difference between the cases $\sigma < 0$ and $\sigma > 0$ relies on the cross effects on wages. When $\sigma < 0$, a skill-biased technological change may decrease unskilled workers' wage and may increase poverty both in relative and in absolute terms [8]. This case is therefore not so irrelevant as generally thought. When $\sigma > 0$, a skill-biased technological change increases unskilled workers' wage and poverty in relative terms but decreases poverty in absolute terms. A symmetric conclusion arises regarding the analysis of an unskilled-biased technological change.

[8] When one looks at the evolution of poverty in relative terms, one looks at the evolution of lowest incomes levels compared to the evolution of average income. In this framework, The ratio w_t^H/w_t^L basically measures the evolution of poverty in relative terms. When one looks at the evolution of poverty in absolute terms, one looks at the evolution of lowest income per se. In this framework, w_t^L basically measures poverty in absolute terms.

5 Conclusion

We have built a model of endogenous growth where innovators are incited to alternate the sector in which they introduce new products, in order to increase the life time duration of patents when there is obsolescence and creative destruction. A permanent skill-bias is therefore not plausible in our framework. Under a two-cycle dynamics, patents last two periods, and the wage differential between skilled and unskilled workers successively increases and decreases, reproducing a stylized feature of the evolution of wage inequality over the twentieth century in the U.S..

We have derived implications which are positive and not normative. Indeed, we do not argue that since alternation in technology adoption is bound to occur, then the recent skill-bias is not problematic. In particular, skill-biased technological change seems to increase wage inequality in the U.S. and unemployment in Europe. The institutional environment therefore matters, implying different reactions of the relative skilled wages to a common change in the relative skilled labor demands. One should therefore incorporate imperfect wage settings and their political determinants to draw normative conclusions on welfare issues regarding the relationship between innovation and wage inequality. This constitutes one direction for our future research.

6 Appendix

6.1 Two-cycle

Using table 1 and the properties of functions $F(.;.)$ and $\Pi^i(.;.)$, equation (9) becomes:

$$\begin{aligned} r\ W_{2t}^H &= \left(\frac{1}{1-\gamma}\right)^t \pi_0^H - \lambda\ W_{2t}^H \\ r\ W_{2t+1}^H &= \left(\frac{1}{1-\gamma}\right)^t \pi_1^H + \lambda\ \left[W_{2t+2}^H - W_{2t+1}^H\right] \\ r\ W_{2t}^L &= \left(\frac{1}{1-\gamma}\right)^t \pi_0^L + \lambda\ \left[W_{2t+1}^L - W_{2t}^L\right] \\ r\ W_{2t+1}^L &= \left(\frac{1}{1-\gamma}\right)^t \pi_1^L - \lambda\ W_{2t+1}^L \end{aligned}$$

Hence, we get:

$$W_{2t}^H = \frac{\pi_0^H}{(r+\lambda)(1-\gamma)^t} \qquad W_{2t+1}^H = \frac{\pi_1^H + \frac{\lambda}{(r+\lambda)(1-\gamma)}\pi_0^H}{(r+\lambda)(1-\gamma)^t} \tag{19}$$

$$W_{2t}^L = \frac{\pi_0^L + \frac{\lambda}{r+\lambda}\pi_1^L}{(r+\lambda)(1-\gamma)^t} \qquad W_{2t+1}^L = \frac{\pi_1^L}{(r+\lambda)(1-\gamma)^t} \tag{20}$$

This kind of cyclical dynamics is an equilibrium if and only if, for any t, $V_{2t}^H > V_{2t}^L$ and $V_{2t+1}^H < V_{2t+1}^L$. This implies:

$$\text{for all } t, \qquad W_{2t+1}^H > W_{2t+1}^L \qquad \text{and} \qquad W_{2t}^H < W_{2t}^L$$

Together with equations (19) and (20), this leads to:

$$\pi_1^H + \frac{\lambda}{r+\lambda}\frac{\pi_0^H}{1-\gamma} > \pi_1^L \qquad \text{and} \qquad \pi_0^H < \pi_0^L + \frac{\lambda}{r+\lambda}\pi_1^L$$

6.2 Growth rates

The average growth rate is approximately:

$$g_{t\to t+1} \simeq \lambda \ln \frac{Y_{t+1}}{Y_t}$$

Using (16) and the labor market clearing condition $N_t^i \equiv 1$, we have:

$$\begin{aligned} \lambda \ln \frac{Y_{2t+1}}{Y_{2t}} &= \lambda \ln \left[\frac{\alpha^H \left(A_{2t+1}^H\right)^\sigma + \alpha^L \left(A_{2t+1}^L\right)^\sigma}{\alpha^H \left(A_{2t}^H\right)^\sigma + \alpha^L \left(A_{2t}^L\right)^\sigma}\right]^{\frac{1}{\sigma}} \\ &= \frac{\lambda}{\sigma} \ln \left[1 + \frac{\alpha^H \left(A_{2t+1}^H\right)^\sigma - \alpha^H \left(A_{2t}^H\right)^\sigma}{\alpha^H \left(A_{2t}^H\right)^\sigma + \alpha^L \left(A_{2t}^L\right)^\sigma}\right] \end{aligned}$$

which is then approximately equal to:

$$\begin{aligned}
\lambda \ln \frac{Y_{2t+1}}{Y_{2t}} &\simeq \frac{\lambda}{\sigma} \frac{\alpha^H \left(A^H_{2t+1}\right)^\sigma - \alpha^H \left(A^H_{2t}\right)^\sigma}{\alpha^H \left(A^H_{2t}\right)^\sigma + \alpha^L \left(A^L_{2t}\right)^\sigma} \\
&\simeq \frac{\lambda}{\sigma} \frac{\alpha^H \left(A^H_{2t}\right)^\sigma}{\alpha^H \left(A^H_{2t}\right)^\sigma + \alpha^L \left(A^L_{2t}\right)^\sigma} \frac{\left(A^H_{2t+1}\right)^\sigma - \left(A^H_{2t}\right)^\sigma}{\left(A^H_{2t}\right)^\sigma}
\end{aligned}$$

Using the values of A_t^i in table 1, we have then:

$$\begin{aligned}
g^Y_{2t\to 2t+1} &\simeq \frac{\lambda}{\sigma} \frac{\alpha^H \left(A^H_{2t}\right)^\sigma}{\alpha^H \left(A^H_{2t}\right)^\sigma + \alpha^L \left(A^L_{2t}\right)^\sigma} \left(\left(\frac{1}{1-\gamma} \right)^\sigma - 1 \right) \\
&\simeq \lambda\gamma \frac{\alpha^H \left(A^H\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma}
\end{aligned}$$

Similarly, we obtain

$$\begin{aligned}
g^Y_{2t+1\to 2t+2} &\simeq \lambda\gamma \frac{\alpha^L \left(A^L\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma} \\
g^{w^H}_{2t\to 2t+1} &= \lambda \ln \frac{w^H_{2t+1}}{w^H_{2t}} \simeq \lambda\gamma \left[\sigma + (1-\sigma) \frac{\alpha^H \left(A^H\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma} \right] \\
g^{w^L}_{2t\to 2t+1} &= \lambda \ln \frac{w^L_{2t+1}}{w^L_{2t}} \simeq \lambda\gamma(1-\sigma) \frac{\alpha^H \left(A^H\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma} \\
g^{w^H}_{2t+1\to 2t+2} &= \lambda \ln \frac{w^H_{2t+2}}{w^H_{2t+1}} \simeq \lambda\gamma(1-\sigma) \frac{\alpha^L \left(A^L\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma} \\
g^{w^L}_{2t+1\to 2t+2} &= \lambda \ln \frac{w^L_{2t+2}}{w^{HL}_{2t+1}} \simeq \lambda\gamma \left[\sigma + (1-\sigma) \frac{\alpha^L \left(A^L\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma} \right]
\end{aligned}$$

In turn, we have:

$$\begin{aligned}
g^Y_{2t\to 2t+1} - g^Y_{2t+1\to 2t+2} &= \lambda\gamma \frac{\alpha^H \left(A^H\right)^\sigma - \alpha^L \left(A^L\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma} \\
g^{w^H}_{2t\to 2t+1} - g^{w^H}_{2t+1\to 2t+2} &= \lambda\gamma \left[\sigma + (1-\sigma) \frac{\alpha^H \left(A^H\right)^\sigma - \alpha^L \left(A^L\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma} \right] \\
g^{w^L}_{2t\to 2t+1} - g^{w^L}_{2t+1\to 2t+2} &= \lambda\gamma \left[(1-\sigma) \frac{\alpha^H \left(A^H\right)^\sigma - \alpha^L \left(A^L\right)^\sigma}{\alpha^H \left(A^H\right)^\sigma + \alpha^L \left(A^L\right)^\sigma} - \sigma \right]
\end{aligned}$$

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