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2008.08

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Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://ces.univ-paris1.fr/cesdp/CES-docs.htm
ISSN : 1955-611X

Effect of noise filtering on predictions : on the routes of chaos

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Abstract

The detection of chaotic behaviors in commodities, stock markets and weather data is usually complicated by large noise perturbation inherent to the underlying system. It is well known, that predictions, from pure deterministic chaotic systems can be accurate mainly in the short term. Thus, it will be important to be able to reconstruct in a robust way the attractor in which evolves the data, if this attractor exists. In chaotic theory, the deconvolution methods have been largely studied and there exist different approaches which are competitive and complementary. In this work, we apply two methods : the singular value method and the wavelet approach. This last one has not been investigated a lot for filtering chaotic systems. Using very large Monte Carlo simulations, we show the ability of this last deconvolution method. Then, we use the de-noised data set to do forecast, and we discuss deeply the possibility to do long term forecasts with chaotic systems.

1 Introduction

The nonlinear modelling and forecasting of time series has a very important history. The statistic community has proposed a lot of parametric models, and independently the dynamical systems community has constructed also a lot of deterministic non linear models. Applications to experimental data are now numerous. Indeed, to model complex phenomena is a very interesting challenge and the way to do it is not unique. In this paper we consider a deterministic approach in order to put out the complexity of the data on which we work, and we link it with classical forecasting approaches for statisticians.

In order to investigate nonlinear time series using dynamical systems, noise reduction methods are necessary. The purpose is to separate into "‘noise"’

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and "'signal"' the data sets to examine. This means that we are interested in a time series produced by a system whose underlying behavior can be characterized as low-dimensional chaos. Different approaches have been suggested for noise reduction in order to find this low dimensional chaos, if it exists. For a review, we refer to Kostelitch and Yorke (1990). Most classical methods have important similarities: they are based on state space method, kernel method and predictions. Another way concerns the wavelet approach and has not been used a lot in order to get noise reduction in presence of chaos. In the following, we compare a classical method with this wavelet approach. We use Monte Carlo simulations to show their accuracy in reconstructing the true attractor. As wavelets method performs, we use it on real data sets.

In a second step, we discuss the forecasting problem inside chaotic systems. It is clearly admitted that, if a process is governed by a deterministic chaotic system, then only short term predictions are available for such a system. We discuss this fact and propose a possible way to obtain long term prediction inside the attractor. The question of the mid-term prediction is always opened, but recent new routes using Lyapunov exponents seem to be able to correct this gap, see Guégan and Leroux (2007).

When adding noise to an otherwise deterministic system, we have to distinguish between dynamic and measurement noises. Assume that the noise-free dynamics would be:

$$X_{t+1} = f(X_t). \tag{1}$$

We speak of measurement noise if there exists a trajectory satisfying this exact dynamics (1) and if the measured trajectory $(Y_t)_t$ is corrupted by additive noise:

$$Y_t = X_t + \varepsilon_t. \tag{2}$$

Dynamic noise in contrast is added during the evolution:

$$Y_{t+1} = f(Y_t) + \varepsilon_t. \tag{3}$$

In this paper we work with the former representation.

In order to reconstruct an attractor, we follow the Takens time-delay embedding method . Given a time series $(X_t)_t$, the reconstructed attractor consists of the m-vectors $(\bar{X}_i)_i$, where $\bar{X} = (X_i, X_{i-\tau}, \cdots, X_{i-(m-1)\tau})$. The constant τ is the time delay and m is the embedding dimension. Takens (1981) shows that under mild conditions both on the function f and the measurement noise, there is a one-to-one mapping between the points on the original attractor and the reconstructed set that preserves information about the original system, provided the embedding dimension is sufficiently large.

In the following we begin to de-noise the observation $(Y_t)_t$ and apply the Takens approach on the de-noised data set for which we discusss forecasting methodology.

The paper is organized as followed. In Section two we briefly recalled the different methods of deconvolution and describe more precisely the two methods used in this paper. In Section three, we provide a Monte carlo experiment permitting to calibrate the de-noising method. It permits to show that the wavelets approach is efficient. In Section four we apply it to real data set. Section five is devoted to a discussion on forecasts for chaotic systems, when long memory behavior is detected. Section six concludes.

2 Deconvolution methods

There exists a lot of works concerning noise reduction methods for chaotic time series. Without being exhaustive, we can refer to the works of Farmer and Sidorowich (1988), Casdagli (1989) and Guégan and Tchernig (2001) for predictive approach. Kostelich and Yorke (1990) use local polynomial maps. Schreiber and Grassberger (1991) use a local method, close to the moving average approach. A modified version of this approach has been proposed by Sauer (1992). Cawley and Hsu (1992) suggest that noise in the observations can be reduced by projecting, on the attractor, the observations onto the subspace spanned by a suitable collection of singular vectors at each point. We will consider this last approach in the following. Thus, for de-noising we consider the classical singular value method and a new approach based on wavelets method.

2.1 Singular value method

The deconvolution method based on singular value decomposition (SVD) assumes that we observe a discrete time series $(Y_t)_t$ with a measurement additive noise like in equation (2), such that the underlying signal $(X_t)_t$ and the noise $(\varepsilon_t)_t$ are separable. We consider the Hankel matrix associated to the signal $(Y_t)_t$, denoted H_t and we decompose it as :

$$H_t = H_t^X + H_t^\epsilon. \tag{4}$$

Then the so-called singular value decomposition provides:

$$H_t = U \; \Sigma \; V,$$

where U is a unitary matrix whose vectors represent the directions of the biggest variations and Σ is a diagonal matrix consisting of the singular values in decreasing order. This property permits to build an empirical model using only the first more important terms. Cancelling the singular values of Σ on the diagonal provides filtering data from a certain threshold. The unitary matrix V is called analysis matrix. Thus, we get the following decomposition on X and ε:

$$H_t = [U_X \quad U_\varepsilon] \begin{pmatrix} \Sigma_X & 0 \\ 0 & \Sigma_\varepsilon \end{pmatrix} [V_X \quad V_\varepsilon] .$$

To eliminate the noise, we proceed steps by steps cancelling the singular values of H_t^ϵ. Then, we obtain $H_t^X = U_X \ \Sigma_X \ V_X$ and by the way we reconstruct the true matrix associated to X. To choose the treshold from which we decide to cancel the terms on the diagonal of Σ is a complex problem. For examples we refer to Pesola and Olkkonen (1997).

2.2 Wavelet method

The wavelet method is a time-scale analysis. To extract the signal which pollutes a system the wavelets method uses two different but complementary projections: one on an approximation space and the other one on the detail space. Since the wavelet transformation is an orthogonal operation, it preserves the probabilistic property of the underlying system and all the useful information which characterizes a chaotic system when this one is highly polluted. The wavelets method is now well known and has been a lot applied, Meyer (1994). The wavelet method is a multi-resolution approach. Daubechies (1992) provides an inspiring method based on the use of explicit orthonormal bases on multi-resolution analysis. Wavelets bases offer a degree of localization in space as well as in frequency that enables the decomposition of a signal into compactly supported oscillating components. The coefficients associated with each of the components are called wavelet coefficients. A remarkable property of wavelet coefficients is to reflect the local regularity of the original function, being large when the function is irregular and small when the function is smooth. The property is very useful to detect discontinuities or sharp changes in a noisy signal. Wavelet coefficients are discrete transformations of a so-called mother wavelet ψ. First a doubly indexed family of wavelets is generated, by dilating and translating ψ,

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \tag{5}$$

$j, k \in \mathbb{Z}$. The wavelet coefficient of a process $(Y_t)_t$ is a function of the scale parameter j and translation parameter k and is defined by:

$$W(Y) = w_{j,k} = 2^{j/2} \int Y_t \psi(2^j t - k) dt. \tag{6}$$

The operator W which associates wavelet to a given signal Y is called the Discrete Wavelet Transform. We can calculate the Discrete Wavelet Transform of any N-sampled signal $Y_1, \cdots, Y_N$:

$$\hat{w}_{j,k} = \hat{W}(Y), \tag{7}$$

and $\hat{w}_{j,k}$ are N empirical wavelet coefficients and W is an orthogonal transformation which depends on the choice of the wavelet family. For a level j, the details of the signal $(Y_t)_t$ are equal to:

$$D_j = \sum_{k \in \mathbb{Z}} w_{j,k} \psi_{j,k}(t), \tag{8}$$

and then the global reconstruction of the signal is given by:

$$Y_t = \sum_j \sum_{k \in \mathbb{Z}} w_{j,k} \psi_{j,k}(t). \tag{9}$$

Now, for a given level J, we get the approximation of the signal at this level:

$$A_J = \sum_{j>J} D_j, \tag{10}$$

and the reconstruction of the true signal $(X_t)_t$ denoted $(\tilde{X}_t)_t$ is obtained as the sum of its approximation at level J and its finer details.

In the following, we choose wavelet functions which are well located around zero (Haar or Daubechies functions for instance), decreasing rapidly to zero as $t \to \infty$ and oscillating such that $\int \psi(t)dt = 0$. We strengthen these conditions imposing a lot of vanishing moments P. The wavelet coefficient $w_{j,k}$ represents how much information is lost (gained), if the series $(Y_t)_t$ is sampled less (more) often. The index j is called the resolution level and corresponds to a frequency 2^{-j}, the index k is called the time (or space) parameter and corresponds to the dyadic position $\frac{k}{2^j}$.

Now, when we separate the noise from the signal using the wavelets' theory, the noise appears in the details' coefficients. Thus, to remove this noise, we proceed in two steps: we need to determine a certain threshold λ which permits to keep the details which are interesting for the reconstruction. The choice of this threshold is very difficult and it does not exist, until now, a specific method to choose it. When the choice of the threshold is done, we have to determine the thresholding function to threshold the wavelet coefficients. We refer to Guégan and Hoummiya (2005) for examples of thresholds and thresholding functions.

3 Monte Carlo experiments

In order to compare the two previous methods and their accuracy in terms of noise reduction, we have simulated a well known chaotic system: the Lorenz system. We added a measurement noise to the system. Thus, if $(X_t)_t$ represents the original chaotic system, $(\varepsilon_t)_t$ any noise, this means that we observe a process $(Y_t)_t$ obtained from the following recursive scheme defined by:

$$\begin{cases} Y_t = X_t + \varepsilon_t \\ X_t = f(X_{t-1}), \end{cases} \tag{11}$$

and here f is the the Lorenz system:

$$\begin{cases} X_t = a(Y_{t-1} - X_{t-1}) \\ Y_t = X_{t-1}(b - Z_{t-1}) - Y_{t-1} \\ Z_t = X_{t-1}Y_{t-1} - cZ_{t-1}. \end{cases} \tag{12}$$

We have simulated 5 000 points from (12), adding a Gaussian measurement white noise, mean 15 and variance 10. On figure 1 we provide attractor in dimension three.

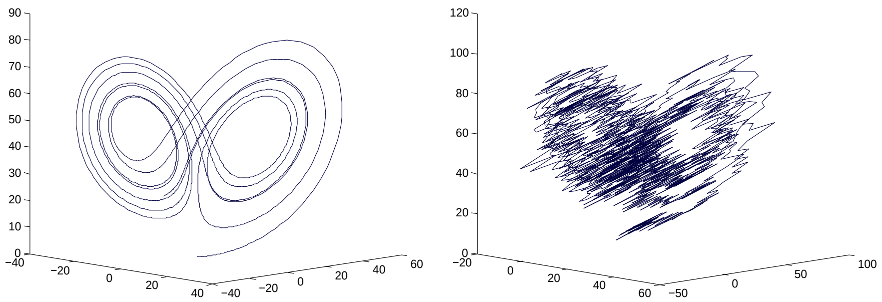

Figure 1: Original Lorenz attractor (left graph) and polluted Lorenz attractor with Gaussian noise, mean 15 and variance 10 (right graph).

Using the two previous methods, the singular value decomposition and the wavelets approach, we reconstruct the Lorenz attractor. On figure (2) we provide the reconstruction of the attractor using SVD method, in dimension three. The reconstruction of the attractor using Daubechies wavelets with J=4 and P=16 is given in Figure (3) also in dimension three. We observe that in both cases we get the classical form of the attractor. The shape is more accurate with the wavelets reconstruction than with the SVD method. In particular we observe a better regularity for the orbits using the wavelets approach. As we know the "true" attractor for this system, we have computed the RMSE, R, for the two reconstructions. We get $R_W = 0.0662$ using the wavelets method and $R_{SWD} = 0.1180$ with the SVD method. Thus, the de-noising using wavelet method is better and we focus on it in the following.

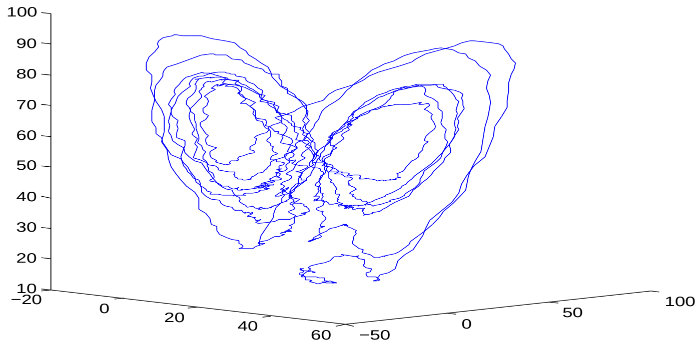

Figure 2: De-noised Lorenz'attractor using SVD method

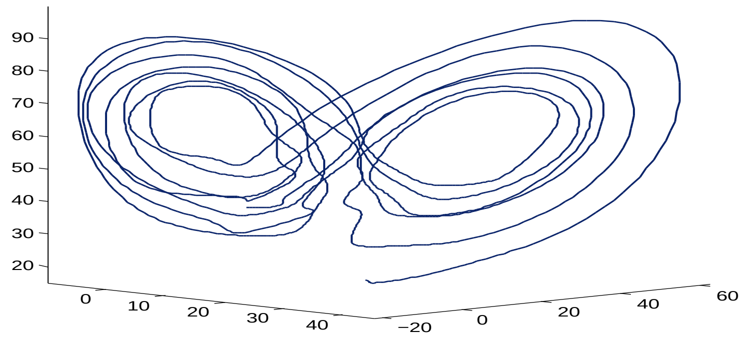

Figure 3: De-noised Lorenz'attractor using Daubechies functions with J=4 and P=16.

4 Applications to wind speed data set

Now, we apply the previous SVD method and wavelet de-noisising method to a wind speed data set. For insurance companies, it is interesting to model such kind of data sets.

Here the data are collected every 3 hours UTC in m/s at Montsouris Park in Paris, France, observed from the 17th of December 1996 at 9:00 a.m to the 30th November 2004 at 9:00 p.m. The sample size is $N = 23245$. The data appear second order stationary.

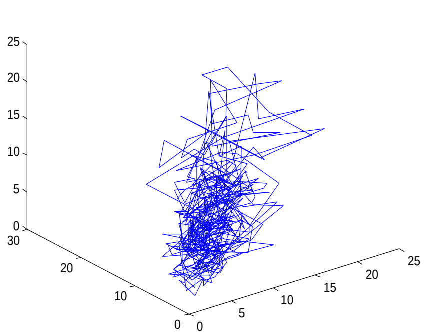

Figure 4: De-noising of the wind's speed data recorded from the 17th of December 1996 at 9:00 a.m to the 30th November 2004 at 9:00 p.m, each day, with the SVD method, $\tau = 1$ on the left, $\tau = 4$ on the right

When we embed the data set in dimension three, with $\tau = 1$ or 4, we distinguish patterns and the orbits belong to different ellipses whose diameters change all the time. The two de-noising methods give results which are significantly different. The reconstructions obtained using the SVD method for these embeddings ($\tau = 1$ and 4) are provided on Figure 4. Using wavelet method, the reconstruction is given in Figure 5, for $\tau = 4$ and the representation is given in dimension two, to make the graph clearer. On the right part of the Figure 5, we exhibit a zoom inside the speudo-attractor. We have computed the RMSE for the two methods. Using the SVD method, $R_{SVD} = 0.1167$ and using the wavelet method, $R_W = 0.0451$. The last one appears better in the sense of the RMSE.

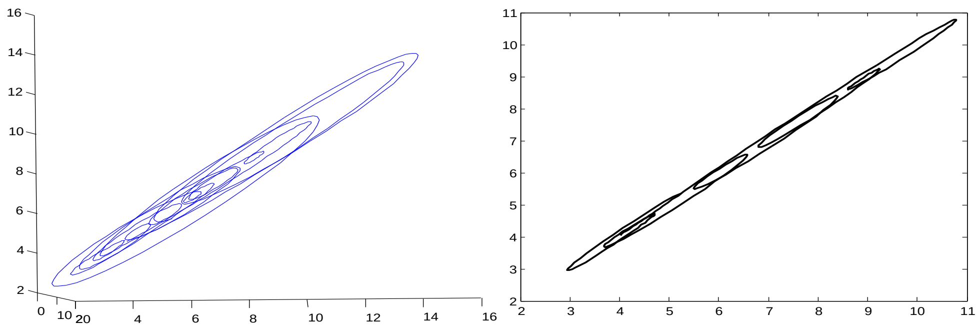

Figure 5: De-noising of the wind's speed data recorded from the 17th of December 1996 at 9:00 a.m to the 30th November 2004 at 9:00 p.m, each day, using daubechies wavelets by hard tresholding with J=4 ; P=13. Details of the attractor is given on the right graph. Reconstruction is given using $\tau = 4$ in dimension two.

5 Forecasts and Long memory behavior

A variety of techniques has been developed to tackle the problem of forecasting chaotic time series. All these methods show a characteristic behavior when they are used to forecast complex time series generated by some deterministic mechanism with better results than the standard statistical models. Most of the works concern the short term predictions and even in that case, performances are better with deterministic dynamical models, Sugihara and May (1990).

It is well known that short term predictions perform using this approach when we compare them with short memory time series like ARMA or GARCH processes for instance. The properties are similar but the confidence intervals are generally smaller when we re-built them using boostrap method inside the attractor. Nevertheless, if we are able to reconstruct the chaotic map in a consistent way then robust short term predictions can be obtained, see for instance Guégan and Lisi (1997) and Guégan and Mercier (1997). Indeed predictions with chaotic systems are better than those obtained with complex stochastic systems, because in the latter case we mainly predict noise whereas with the former approach it is the complex structure of the system which provides the predictions. In that latter case, as soon as the orbits have been correctly estimated, we can predict inside the attractor and the existence of noise is negligeable.

Doing long term predictions has not been a lot discussed in the literature, Guégan (2003). We conjecture that this kind of predictions appears reasonable as soon as we work inside an attractor, if the noise is negligeable and the orbits correctly reconstructed and if a long memory behavior characterizes the dynamical system.

Assuming that the orbits have been correctly de-noised and rebuilt, if we want to make long term forecasting, first we need to detect kind of persiistence or long memory behavior inside the attractor. Here, we exhibit examples for which a long memory behavior is detected using the adjustment with a FARMA process, Granger and Joyeux (1980), Hosking (1981) or a k-factor Gegenbauer process, Woodword, Cheng and Gray (1998).

5.1 Lorenz system

For the Lorenz system, existence of long memory behavior, in the covariance sense, Guégan (2005), has been already detected and details are provided in Guégan (2003). Thus, we want to verify if this long memory behavior is kept when the system had been de-noised. We have adjusted a FARMA process and two k-factor Gegenbauer processes on the three data sets obtained from equation (12), without noise, with noise and de-noising with wavelets method. For the FARMA process we estimate the long memory parameter using the Whittle method (Dalhaus, 1989) and the wavelet approach (Whitcher and Nelsen, 2000). For the k-factor Gegenbauer models, we use the Whittle method, Ferrara and Guégan (2000). The results are provided in Table below for the three axis of Lorenz system. With respect to the stability of the topological properties of the system, the best results are obtained using the wavelet method in case of the FARMA model. We observe also that the estimated value using the original system and the denoised system are very close. Now in terms of noise adjustement, the 1-factor Gegenbauer process provides the best result and this model has to be used to do forecasts. We observe the stability of the estimated parameter for the "'true"' model and the "'denoised"' system using this last model. De-noising using wavelet approach permits to keep the persistence property.

Lorenz system: Estimation of 'd'

	FARMA(Whittle)	FARMA(Wavelets)	GG_1 factor	GG_2 factor
sys.Lorenz	x: 1.7423	0.5604	x: 0.9514	[1.2903 -0.339]
	y: 1.9797	0.4542	y: 1.1056	[-0.624 1.7235]
	z: 1.4597	0.3293	z: 0.7700	[2.4343 -1.664]
Noisy sys.	x: 0.5186	0.3697	x: 0.2900	[0.1703 0.1165]
	y: 0.6216	0.3331	y: 0.3479	[0.1737 0.1732]
	z: 0.6455	0.2275	z: 0.3267	[0.5482 -0.222]
Denoised	x: 1.7793	0.5558	x: 0.9884	[1.1538 -0.165]
	y: 1.9827	0.4561	y: 1.1098	[-0.637 1.7411]
	z: 1.4629	0.3397	z: 0.7732	[2.5124 -1.739]

5.2 Hénon and Rossler systems

This notion of persistence can be detected in a lot of well known chaotic systems. We provide now two other examples with the Hénon and the Rossler systems. Concerning the generalized maps on $[0, 1]$, this kind of behavior has also been done, see Guégan and Ladoucette (2001). Here, we only provide the long memory adjustement on the free-noise systems. Indeed, the results are identical with the free-noise systems and with the de-noised systems using the wavelets approach.

The Hénon system is defined by the two following equations:

$$\begin{cases} X_t = Y_{t-1} + 1 - aX_{t-1}^2 \\ Y_t = bX_{t-1}. \end{cases} \tag{13}$$

The periodogram of this system exhibits two peaks, see Guégan (2003), thus we adjust a 2-factor Gegenbauer process defined by:

$$(I - 2\nu_1 B + B^2)^{d_1}(I - 2\nu_2 B + B^2)^{d_2} X_t = \varepsilon_t.$$

We only provided the results for the X component. The estimates are equal to $(\hat{\nu}_1, \hat{d}_1) = (0.65, 0.25)$ and $(\hat{\nu}_2, \hat{d}_2) = 0.96, 0.28)$. This estimation permits to say that speudo-seasonalities exist and are characterized by persistence. These stylized facts can be used to do forecasts in the long term.

The Rossler system is defined by three equations which are:

$$\begin{cases} X_t = -Y_{t-1} - Z_{t-1} \\ Y_t = X_{t-1} + aY_{t-1} \\ Z_t = b + Z_{t-1}(X_{t-1} - c). \end{cases} \tag{14}$$

The periodogram of this system exhibits four peaks, see Guégan (2003), thus we adjust a 4-factor Gegenbauer process defined by:

$$G_1(B).G_2(B).G_3(B).G_4(B)X_t = \varepsilon_t.$$

We only provided the results for the X component. The Gegenbauer filters are respectively equal to:

$$G_1(B) = (1 - 2cos(\lambda_{86})B + B^2)^{0.3010},$$
$$G_2(B) = (1 - 2cos(\lambda_{171})B + B^2)^{0.1807},$$
$$G_3(B) = (1 - 2cos(\lambda_{256})B + B^2)^{0.1748},$$
$$G_4(B) = (1 - 2cos(\lambda_{341})B + B^2)^{4098},$$

All the parameters are significant and thus we can consider that this adjustment permits to take into account the speudo-seasonalities and persistence which are stylized facts for this systemThen we can use this modelling to do forecasts in the long term.

5.3 Applications to wind data set

We consider now the data set studied in Section 3. For this data set, the autocorrelation function does not decrease towards zero very quickly and the periodogram explodes in two frequencies far from zero, for details we refer to Guégan and Hoummiya (2005). Thus, this data set exhibits a long memory behavior in the covariance sense. We have estimated the long memory parameter using a FARMA model and two k-factor Gegenbauer processes. We report the results for the long memory estimation in the Table below. For estimation, we use the same methods as before.

$\hat{d}$	Wind Speed Farma(Whittle)	Farma(wavelets)	GG_1 factor	GG_2 factor
Wind	0.5215	0.1648	0.5727	[0.0844 0.2914]
Denois.wind	2.2679	0.3635	1.3266	[-0.895 2.2191]

The best adjustement for this series is obtained using a 1-factor Gegenbauer (1.3266, 0.7071) process with AR(1) residuals. Here, a great variability between the parameter estimated from the denoised data set and the observational data set is observed. Thus, a deeper investigation is necessary before doing forecasts using this approach.

5.4 Forecasts

The previous study shows two important facts concerning chaotic systems. First, if we know that we are in presence of chaotic system, we can use the wavelet method to de-noise it. In particular, this method preserves the long memory behavior inside the chaotic system when this property exists. Second, we see that, for some deterministic chaotic systems, we can exhibit, in covariance sense, existence of long memory behavior.

Thus, in order to make forecasts using this fact, we can adjust on the original system or on the de-noised system a long memory process like a FARMA process or a Gegenbauer process. These adjustments are possible and we see that they provide accurate estimates for the long memory parameters. Now the question is the coherence between this adjustment and the original deterministic system.

We know that any chaotic system is characterized by an asymptotic invariant measure through the Birkoff theorem, Lasota and Mackey (1987). The stochastic representation of a deterministic process makes the introduction of a noise which corresponds to the "‘physical"’ distribution for the deterministic system. To consider such a distribution is pertinent particularly when we work with experimental systems. For a stochastic process, the noise which intervens in the expression of the model is generally a white noise with an invariante measure μ_ε. The asymptotic measure μ (called sometimes the Kolmogorov measure) associated to the dynamical system can be considered like the limit of μ_ε when ε tends to 0. In another hand, we know that a ergodic dynamical system is characteriezd by several invariant measures. Thus the approach that we have proposed here, introducing another modelling for a chaotic system, permits to illustrate this fact.

For the well known chaotic systems that we previously considered, we know that they are characterized by an invariant measure even if this one is not always known. Adjusting on these systems a Gegenbauer process permits to define for these systems a "‘physical"’ measure that we can use to do forecasts. The interest here is that this adjustment permits to use long memory processes which permit to build long term forecasts.

Thus, if we want to use the previous approach to do forecasts inside the attractor, we need to compute $X_{T+h|T} \equiv E\big[X_{T+h}|\mathcal{I}_T\big]$, where $\mathcal{I}_T$ is the information set, minimizing the quadratic error:

$$E\Big[\big(X_{T+h} - \hat{X}_{T+h|T}\big)^2|\mathcal{I}_T\Big].$$

When we adjust a Gegenbauer process, the predictions will be given by the following equations:

$$\hat{X}_t(h) = -\sum_{j\geq 0}\Big[\sum_{i=0}^{h-1}\psi_i(d,\nu)\ \pi_{h+j-i}(d,\nu)\Big]X_{t-j},$$

where the coefficients ψ_j and π_j are equal to:

$$\prod_{i=1}^{k}\big(1-2\nu_i z+z^2\big)^{-d_i} = \sum_{j\geq 0}\psi_j(d,\nu)z^j \quad \text{and} \quad \prod_{i=1}^{k}\big(1-2\nu_i z+z^2\big)^{d_i} = \sum_{j\geq 0}\pi_j(-d,\nu)z^j,$$

Guégan and Ferrara (2000).

The FARMA process and the Gegenbauer processes will provide accurate long term predictions and generally their short term predictions are very bad. In case of the de-noised Lorenz system we provide the predictions using a FARMA and a Gegenauer process for the X component. We observe that, as soon as the horizon h grows, the Gegenbauer process provides better results. This means that, for this attractor, the Gegenbauer process permits to take into account the seasonality observed inside the attractor and on the autocorrelation function. It seems natural that the FARMA process does not provide good forecasts. Indeed, we have not observed explosions in zero on the periodogram. This means that this system is not characterized by an "‘infinite " cycle.

X Lorenz

RMSE	Farma(wavelets)	GG_1 factor
h=1	0.5215	279.93
h=3	2.2679	1573,4
h=10	2.2679	0.3635
h=50	2.2679	0.3635

6 Conclusion

In this chapter we have discussed methods permitting to denoise chaotic time series. We observe that the wavelets method provides the best results. It is interesting to note that, as soon as we want to make forecasts using long memory parameters, the use of the wavelets method give interesting results when we use the FARMA process in order to estimate the long memory parameter. Nevertheless other examples need to be discussed and this work is in progress. It permits to open new perspectives concerning forecasting inside the attractor of a chaos.

Other perspectives can also be considered for previsions. Indeed, for chaotic systems, it seems interesting to link the minimum error predictions with the main characteristic of the chaos which is the dimension of the attractor. To estimate this dimension, we can compute the standard deviation associated to the data set inside the attractor. Another way is to consider the correlation coefficient which quantifies the link between the predicted value and the true value computing

$$\rho = \frac{E[\hat{X}_{n+h} X_{n+h}]}{E[X_{n+h}^2]}.$$

An accurate prediction will give a value of this coefficient close to 1. Another way consists to choose a small neighor such that the predicted value is inside it, using reconstruction by nearest neighbors for instance.

Diebold and Mariano (1995) could be used in order to compare the different predictions.

Aknowledgements I would like to thank Laurent Ferrara and Kebira Hoummiya who provide me the long memory estimations for some systems.

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