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Representation and Aggregation of Preferences under Uncertainty ⋆

T. Gajdos [a], J.-M. Tallon [a,∗], J.-C. Vergnaud [a]

[a] *CNRS, Université Paris I, Panthéon-Sorbonne & Paris School of Economics, 106 Bd de l'Hôpital, 75013 Paris, France*

Abstract

We axiomatize in the Anscombe-Aumann setting a wide class of preferences, called rank-dependent additive preferences that includes most known models of decision under uncertainty as well as state-dependent versions of these models. We prove that aggregation is possible and necessarily linear if and only if (society's) preferences are uncertainty neutral. The latter means that society cannot have a non-neutral attitude toward uncertainty on a subclass of acts. A corollary to our theorem is that it is not possible to aggregate multiple prior agents, even when they all have the same set of priors. A number of ways to restore the possibility of aggregation are then discussed.

Key words: Aggregation, uncertainty.
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1 Introduction

Harsanyi (1955)'s celebrated result shows that it is possible to aggregate von

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∗ Corresponding author.
Email addresses: `thibault.gajdos@univ-paris1.fr` (T. Gajdos), `jean-marc.tallon@univ-paris1.fr` (J.-M. Tallon), `jean-christophe.vergnaud@univ-paris1.fr` (J.-C. Vergnaud).

Neumann-Morgenstern (vNM) expected utility maximizers: the social utility is a convex combination of the agents' utilities. Extending this result to more general settings turns out to be difficult. For instance, whenever agents are expected utility maximizers but entertain different beliefs, aggregation becomes impossible.

In this paper, we take up this issue, widening considerably the set of preferences considered, that encompasses many well-known models of decision under uncertainty (subjective expected utility, multiple prior model of Gilboa and Schmeidler (1989), Choquet expected utility of Schmeidler (1989) and more generally c-linear biseparable preferences of Ghirardato and Marinacci (2001), as well as state-dependent versions of these preferences). [1] Our main result takes the following form. Assume agents and society have preferences of this sort on a given set of acts. [2] Assume furthermore that this set of acts is rich enough so that a diversity condition on preferences holds. Then, aggregating (some) agents' preferences is possible if and only if they possess a form of uncertainty neutrality, to be discussed momentarily, and leads to linear aggregation. In particular, if an agent has some kind of non neutral attitude towards uncertainty, then either he is a dictator (society's preferences place a zero weight on all other agents) or he gets a zero weight in the society's preferences. A particular case of interest is when agents conform to the multiple prior model of Gilboa and Schmeidler (1989), in which an agent evaluates an act by taking its minimal expected utility with respect to a *set* of priors. Then, a corollary of our result is that aggregation of such agents is impossible unless they are actually expected utility agents (in which case the set of priors is a singleton.) One crucial point behind the impossibility result is the assumption that there exists a set of acts on which individuals are uncertainty neutral (for instance, constant acts for the multiple prior model) which is also a set of acts on which society is uncertainty neutral. Relaxing this assumption points to a way of restoring aggregation.

The result complements several previous results in the literature. Hylland and Zeckhauser (1979), Seidenfeld et al. (1989), and Mongin (1995) showed that aggregation of subjective expected utility agents' preferences was not possible as soon as they have different beliefs. Mongin (1998) showed that expanding the class of preferences to state-dependent preferences would yield a possibility result but argued against this way of restoring the possibility of aggregating preferences. He showed in particular that as soon as one pins down the beliefs of the agents then state-dependence is of no help. Chambers and Hayashi (2006) showed that eventwise monotonicity (P3) and weak comparative probability (P4) are incompatible with the Pareto axiom. Relaxing these axioms

[1] A limitation is that we adopt Anscombe and Aumann (1963) approach.
[2] We follow Harsanyi's approach by imposing the same rationality requirements on the agents' and the society's preferences.

while keeping the sure-thing principle leads to state-dependent expected utility preferences, for which they show a possibility result in a Savage setting. Our setting allows for state-dependence preferences from the beginning and our impossibility theorem applies to state-dependent preferences as well. Gilboa et al. (2004) showed in a subjective expected utility setting, that imposing the Pareto axiom on issues for which agents are unanimous (have identical beliefs) implies that the society's beliefs have to be an affine combination of agents' beliefs and, similarly, that the society's vNM utility function has to be a linear combination of agents' vNM utility functions (note that this does not imply that society's overall utility function is a convex combination of the agents'.) A corollary to our main result is that in the multiple prior model, aggregation is impossible *even in the case when all agents have the same set of priors*. Thus, restricting the Pareto axiom has no bite here. Finally, Blackorby et al. (2004) showed, in a somewhat different framework (that of ex ante-ex post aggregation), that aggregation was essentially impossible in the rank dependent expected utility model.

As we mentioned, we want to allow for state dependence while at the same time allowing for all kinds of attitudes toward uncertainty. There is no decision model in the literature that achieves this goal. A first contribution of the present paper is hence to develop a fairly general model of preferences under uncertainty, allowing for state-dependence. Then, assuming agents conform to this decision model, we show that it is impossible to aggregate agents' preferences into a well-defined preference relation at the social level that would also conform to this model unless agents have uncertainty neutral preferences, where uncertainty-neutrality is defined in the spirit of Gilboa and Schmeidler (1989) as indifference to mixing indifferent acts.

As argued in Gilboa et al. (2004), the relevance of this literature is partly due to the rhetoric of decision making in a democracy. Quoting these authors,

> "(...) the theoretical conclusion that aggregating tastes and beliefs is impossible, is troubling. If there is, indeed, no way to aggregate preferences of all individuals, then a ruling party or a president may feel exempted from seeking to represent society in its entirety even if elected by an incidental majority. (...) However, we argue that the impossibility results cannot be cited as an indirect justification of ignoring minority views, because they rely on a counterintuitive assumption. By contrast, a more intuitive version of this assumption necessitates aggregation of preferences." Gilboa et al. (2004),p.935.

The counterintuitive assumption they refer to is the Pareto axiom that says that if all individuals in society agree on preferences between two alternatives, so should society. As we mentioned, they replace it by a weaker Pareto condition that applies *only* when all individuals have identical beliefs or, dually, when they have identical tastes. A consequence of our result is to show that their possibility result hinges on the fact that all individuals are Bayesian expected utility maximizers. If, however, some of these individuals do not conform to expected utility, for instance because they do not have precise (subjective) probabilities, but rather a range of probabilities, their result no longer holds. Indeed, the present paper shows that a wide variety of models, allowing for some type of Knightian uncertainty, give rise to the impossibility result even if the Pareto condition is restricted to identical "beliefs". In this sense, we are back to the troubling theoretical conclusion that a ruling party or president might be entitled to act in a dictatorial way, since there is no reasonable way to aggregate preferences when individuals differ both in utilities and in beliefs.

The paper is divided into six sections and three appendices. Section 2 introduces the decision theoretic setup needed, while Section 3 contains a new representation result. The main result of the paper is in Section 4. Section 5 provides a discussion of why known arguments used in the literature to restore the possibility of aggregation fail here, as well as some thoughts on ways to relax some of our assumptions that would allow for some (non linear) aggregation. Section 6 concludes. Appendix A contains three models illustrating the decision theoretic part developed in Sections 2 and 3. Appendix B contains the proof of the representation result given in Section 3, while Appendix C contains the proof of the aggregation result.

2 Setup

We consider a society made of a finite number of agents $N' = \{1, \ldots, n\}$. Let $N = \{0, 1, \ldots, n\}$ where 0 refers to society. Uncertainty is represented by a set S and an algebra of events Σ. We adopt Anscombe and Aumann (1963)'s framework: Let X be a non-empty set of consequences and Y be the set of distributions over X with finite support. Let $\mathcal{A}$ be the set of acts, that is, functions $f : S \to Y$ which are measurable with respect to Σ. Since Y is a mixture space, one can define for any $f, g \in \mathcal{A}$ and $\alpha \in [0, 1]$, the act $\alpha f + (1-\alpha)g$ in $\mathcal{A}$ which yields $\alpha f(s) + (1-\alpha)g(s) \in Y$ for every state $s \in S$.

We model the preferences of an agent $i \in N'$ on $\mathcal{A}$ by a binary relation $\succsim_i$, and, as customary we denote by $\sim_i$ and $\succ_i$ its symmetric and asymmetric components. Society's preferences are denoted $\succsim_0$. The first axiom is usual, will be maintained throughout, and states that preferences are a complete, transitive, and continuous relation.

Axiom 1 *For all $f, g, h \in \mathcal{A}$,*

(1) $f \succsim g$ *or* $g \succsim f$*;*
(2) *if* $f \succsim g$ *and* $g \succsim h$ *then* $f \succsim h$*;*
(3) *if* $f \succ g$ *and* $g \succ h$*, then there exist* $\alpha, \beta \in (0,1)$ *such that* $\alpha f + (1-\alpha)h \succ g$ *and* $g \succ \beta f + (1-\beta)h$*.*

We will almost exclusively be interested in the properties of preferences on a small domain of acts on which they have some structure. We next define the notion of regular acts from which this domain will be constructed, as the relevant domain will consist of binary acts whose components are regular. [3]

For an event E and two acts f and g, denote $f_E g$ the act giving $f(s)$ if $s \in E$ and $g(s)$ if not. For $\mathcal{F} \subset \mathcal{A}$ and $E \in \Sigma$, let $\mathcal{B}(\mathcal{F}, E) = \{f_E g \,|\, f, g \in \mathcal{F}\}$ that is, binary acts on the event E whose components belong to a subset of acts $\mathcal{F}$. When there is no possible confusion about the reference set $\mathcal{F}$, we will simply call such acts *binary acts*.

Definition 1 *Fix* $E \in \Sigma$*. A set of acts* $\mathcal{R} \subset \mathcal{A}$ *is* E-regular *with respect to* $\succsim$ *if it satisfies the following conditions:*

(1) $\mathcal{R}$ *is a mixture set: For all* $f, g \in \mathcal{R}$ *and* $\alpha \in (0,1)$*,* $\alpha f + (1-\alpha)g \in \mathcal{R}$*;*
(2) *Binary* $\mathcal{R}$*-independence: For all* $h \in \mathcal{R}$*, for all* $f, g \in \mathcal{B}(\mathcal{R}, E)$*,* $\alpha \in (0,1]$*,* $f \succsim g \Leftrightarrow \alpha f + (1-\alpha)h \succsim \alpha g + (1-\alpha)h$*;*
(3) *Weak sure-thing principle for* $\mathcal{R}$*-binary acts: For all acts* f*,* g*,* h*,* h' *in* $\mathcal{R}$*,* $f_E h \succ g_E h \Rightarrow f_E h' \succsim g_E h'$*.*

Note that for any event E the whole set $\mathcal{A}$ is E-regular for subjective expected utility (both state-independent and state-dependent). [4] Another instance of E-regular acts is the set of constant acts for the multiple prior model of Gilboa and Schmeidler (1989).

Condition 1 requires that the set of E-regular acts be closed under the mixture operation. Condition 2 is in the spirit of C-independence of Gilboa and Schmeidler (1989) with the difference that it applies only to $\mathcal{R}$-binary acts. It means that E-regular acts cannot be used to hedge against $\mathcal{R}$-binary acts. Condition 2 also entails that the independence axiom holds when restricted to acts in $\mathcal{R}$. Thus, preferences on E-regular acts are uncertainty neutral on

[3] This definition, as well as the definitions and results in the next section are illustrated in Appendix A on three decision models: c-linear biseparable preferences of Ghirardato and Marinacci (2001) and Ghirardato et al. (2005), a state dependent version of the so-called α-MMEU model of Jaffray (1989) and Ghirardato et al. (2004), and the smooth model of ambiguity aversion of Klibanoff et al. (2005), which, although not cast in an Anscombe and Aumann (1963) setup, can be partially linked to results in this paper.
[4] Whenever $\mathcal{A}$ is a E-regular, then condition 3 in the definition can be disposed of since it is implied by condition 2.

the event E[5] and will be of the vNM type. Condition 3 is a weak version of the sure-thing principle, again restricted to $\mathcal{R}$-binary acts. Note that this weak property is not violated in Ellsberg kind of experiments. In the multiple prior model of Gilboa and Schmeidler (1989), this condition is satisfied, being a consequence of monotonicity if we take constant acts as the regular acts.[6]

3 Rank-dependent additive preferences

3.1 Representation result

As explained above, we will be concerned only with the properties of the preference relation on the domain of $\mathcal{R}$-binary acts. We thus define a notion of representation, which is affine with respect to E-regular acts.

Definition 2 *Fix $E \in \Sigma$ and $\mathcal{R} \subset \mathcal{A}$. A function $V : \mathcal{B}(\mathcal{R}, E) \to \mathbb{R}$ is an* $\mathcal{R}$-affine representation *of $\succsim$ on $\mathcal{B}(\mathcal{R}, E)$, if*

(1) for all $f, g \in \mathcal{B}(\mathcal{R}, E)$, $f \succsim g$ if and only if $V(f) \geq V(g)$;
(2) for all $f \in \mathcal{B}(\mathcal{R}, E)$, $h \in \mathcal{R}$, and $\alpha \in (0, 1)$, $V(\alpha f + (1-\alpha)h) = \alpha V(f) + (1-\alpha)V(h)$.

We now characterize preferences that admit an E-regular set of acts for some event E, generalizing results known for the class of c-linear biseparable preferences. This representation will be key to establish under which conditions aggregation is possible.

Proposition 1 *Let $E \in \Sigma$ and $\succsim$ be a binary relation on $\mathcal{A}$ that satisfies Axiom 1. Assume that there exists a set $\mathcal{R} \subset \mathcal{A}$ which is E-regular with respect to $\succsim$ and, furthermore, that $\succsim$ is not degenerate on $\mathcal{R}$ (i.e., there exist $f, g \in \mathcal{R}$ such that $f \succ g$.) Then, there exists an $\mathcal{R}$-affine representation of $\succsim$ on $\mathcal{B}(\mathcal{R}, E)$ $V : \mathcal{B}(\mathcal{R}, E) \to \mathbb{R}$, which is unique up to a positive affine transformation.*

Furthermore, for any $\mathcal{R}$-affine representation V of $\succsim$,

(1) there exist four linear functions $\overline{V}_E, \underline{V}_{E^c}, \underline{V}_E, \overline{V}_{E^c}$ from $\mathcal{R}$ to $\mathbb{R}$ such that for all $f, g \in \mathcal{R}$

$$
\begin{aligned}
V(f_E g) &= \overline{V}_E(f) + \underline{V}_{E^c}(g) \text{ if } f \succsim g \\
&= \underline{V}_E(f) + \overline{V}_{E^c}(g) \text{ if } f \precsim g
\end{aligned}
$$

[5] This notion is formally defined in Definition 3 below.
[6] Monotonicity requires that if $f(s) \succsim g(s)$ for all s, then $f \succsim g$.

(2) there exists $k^E \in \mathbb{R}$ *such that for all* $f, g \in \mathcal{R}$,

$$V(f_E g) + V(g_E f) - V(f) - V(g) = k^E \left| V(f) - V(g) \right|. \qquad (1)$$

Preferences that satisfy the requirements of Proposition 1 will be called *rank-dependent additive* (with respect to $\mathcal{R}$) in the following. Existence of an $\mathcal{R}$-affine representation is straightforward and well-known (it follows from vNM like arguments). The first property establishes that the evaluation of binary acts $f_E g$ with $f, g \in \mathcal{R}$ can be decomposed in a rank-dependent additive manner, the decomposition being dependent on the ranking of the two acts. The second property can be seen as a way to identify the agent's attitude toward uncertainty attached to an event, which we will define and characterize in the following section.

Most models of decision under uncertainty cast in the Anscombe-Aumann framework are rank-dependent additive. Rank-dependent additive preferences can accommodate state-dependence. This is why, contrary to c-linear biseparable preferences, "beliefs" do not appear explicitly in the functional. Indeed, were we to make them explicit, we would get back to the usual problem that, when allowing for state-dependence, beliefs cannot be uniquely pinned down. We will discuss this in more details in Section 5.

3.2 Uncertainty neutral rank-dependent additive preferences

Gilboa and Schmeidler (1989) defined uncertainty aversion as a preference for mixing: for *any* acts f, g, $f \sim g \Rightarrow \alpha f + (1-\alpha) g \succsim f$. We will here limit the domain of application to a smaller set of acts. Furthermore, we define a notion of uncertainty neutrality rather than uncertainty aversion, as the important distinction for us will be between agents that are neutral toward uncertainty and agents that have a non neutral attitude toward uncertainty (uncertainty averse or seeking). Based on the intuition underlying this definition we propose the following definition of uncertainty neutrality *on an event with respect to a set of acts.*[7]

Definition 3 *Let* $E \in \Sigma$ *and* $\mathcal{D} \subset \mathcal{A}$. *Say that* $\succsim$ *is* uncertainty neutral on E with respect to $\mathcal{D}$ *if for all* $f, g \in \mathcal{B}(\mathcal{D}, E)$ *such that* $f \sim g$ *and all* $\alpha \in (0,1)$, $\alpha f + (1-\alpha) g \sim f$.

Obviously, a preference relation that satisfies the independence axiom over the whole set of acts will be uncertainty neutral on any event. The next claim proves that rank-additive preferences with $k^E = 0$ are uncertainty neutral on E.

[7] See also Ghirardato et al. (2004).

Proposition 2 *Under the representation of Proposition 1, $\succsim$ is uncertainty neutral on event E with respect to $\mathcal{R}$ if and only if $k^E = 0$.*[8]

In the following, we argue that in the class of preferences considered, uncertainty neutrality is the crucial property that delimits the frontier between the possibility and impossibility of linear aggregation.

3.3 Betting attitudes

Although it is difficult to define "beliefs" in our model, in particular because it allows for state-dependence, we can define a notion of betting preference and more precisely, give meaning to the idea that two rank-dependent additive decision makers have the same betting preference on an event E.

Definition 4 *Let $E \in \Sigma$ and let $\succsim_i$ and $\succsim_j$ be preferences satisfying the assumptions of Proposition 1. Say that i and j have* the same betting preferences *on E if there exist $\alpha, \beta \in (0,1)$, $f, g \in \mathcal{R}_i$, $f', g' \in \mathcal{R}_j$ such that $f \succ_i g$ and $f' \succ_j g'$ and*

(i) $f_E g \sim_i \alpha f + (1-\alpha) g$ *and* $f'_E g' \sim_j \alpha f' + (1-\alpha) g'$, *or*
(ii) $f \sim_i \alpha f_E g + (1-\alpha) g$ *and* $f' \sim_j \alpha f'_E g' + (1-\alpha) g'$, *or*
(iii) $g \sim_i \alpha f_E g + (1-\alpha) f$ *and* $g' \sim_j \alpha f'_E g' + (1-\alpha) f'$, *or*

and

(i') $g_E f \sim_i \beta g + (1-\beta) f$ *and* $g'_E f' \sim_j \beta g' + (1-\beta) f'$, *or*
(ii') $g \sim_i \beta g_E f + (1-\beta) f$ *and* $g' \sim_j \beta g'_E f' + (1-\beta) f'$, *or*
(iii') $f \sim_i \beta g_E f + (1-\beta) g$ *and* $f' \sim_j \beta g'_E f' + (1-\beta) g'$.

In this definition, we use mixing to calibrate the betting behavior of the decision makers on event E, as is customary in an Anscombe and Aumann (1963) setting. Condition (i) says that both agents evaluate $f_E g$ as if they were placing a weight α on E. The next two conditions would be irrelevant in a state-independent setting. However, in our setting, it is possible for instance that $f \succ_i g$ but $f_E g \succ_i f$. The other three conditions use the same calibrating technique to assess the agents' betting behavior on E^c. The weight β could be greater or smaller than $1-\alpha$.

The next proposition shows that the notion of identical betting preferences is captured, in the representation of Proposition 1, by the fact that the two agents have the same coefficient k^E.

[8] When $k^E = 0$, the representation can be additively decomposed since $\bar{V}_E = \underline{V}_E$.

Proposition 3 *Let $E \in \Sigma$ and let $\succsim_i$ and $\succsim_j$ be preferences satisfying the assumptions of Proposition 1. If i and j have the same betting preferences on E then $k_i^E = k_j^E$.*

This characterization will be useful when we discuss the extension of our main theorem to situations in which agents have identical betting attitudes, in relation to Gilboa et al. (2004) argument.

4 Aggregation of rank-dependent additive preferences: an impossibility result

For the aggregation problem to be interesting, one needs to impose some diversity among the preferences that one seeks to aggregate. The next definition provides one such condition (see Mongin (1998)).

Definition 5 *The n binary relations $\{\succsim_i\}_{i\in N'}$ satisfy the Independent Prospects Property on a set $\mathcal{D} \subset \mathcal{A}$ if for all $i \in N'$, there exist $h_i^\star, h_{\star i} \in \mathcal{D}$ such that:*

$$h_i^\star \succ_i h_{\star i} \text{ and } h_i^\star \sim_j h_{\star i} \, \forall j \in N' \setminus \{i\}.$$

On the other hand, it seems natural to impose for the society's preference to comply with any unanimous agreement among individuals: If everybody agree that some alternative f is strictly better than some other alternative g, so should society. This requirement is formally stated in the following Pareto Axiom.

Axiom 2 (Pareto) *For all $f, g \in \mathcal{A}$, $[\forall i \in N', f \succ_i g \Rightarrow f \succ_0 g]$.*

We can now state our main theorem.

Theorem 1 *Fix $E \in \Sigma$. Let $\{\succsim_i\}_{i\in N}$ be binary relations on $\mathcal{A}$ and $\{\mathcal{R}_i\}_{i\in N}$ be non-empty subsets of $\mathcal{A}$. Assume that*

(1) for all $i \in N$, $\succsim_i$ satisfies Axiom 1;
(2) for all $i \in N$, $\mathcal{R}_i$ is E-regular with respect to $\succsim_i$;
(3) $\{\succsim_i\}_{i\in N'}$ satisfy the Independent Prospects Property on $\cap_{i\in N}\mathcal{R}_i$.

Then, Axiom 2 holds if and only if,

(i) there exist an $\mathcal{R}_i$-affine representation V_i of $\succsim_i$ on $\mathcal{B}(\mathcal{R}_i, E)$ for all $i \in N$, unique weights (given the V_i's) $(\lambda_1, \cdots, \lambda_n) \in \mathbb{R}_+^n \setminus \{0\}$, $\mu \in \mathbb{R}$ such that

$$\forall f \in \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E), V_0(f) = \sum_{i\in N'} \lambda_i V_i(f) + \mu;$$

(ii) $\forall i, j \in N'$, $i \neq j$, $\lambda_i \times \lambda_j \neq 0 \Leftrightarrow k_i^E = k_j^E = 0$.

We next provide an illustrative example and then move on to a discussion of the theorem.

Example 1 *Let* $N' = \{1, 2\}$*,* $S = \{\alpha, \beta\}$ *and* $X = \{x, y\}$*. Assume both agents have multiple prior preferences with the simplex as the set of priors:* $V_i(f) = \min_{s \in S} u_i(f(s))$ *where* u_i *is linear. Assume* $u_1(x) = 0$*,* $u_2(x) = 1$*,* $u_1(y) = 1$*,* $u_2(y) = 0$*, where* x *(resp.* y*) is the degenerate lottery yielding* x *(resp.* y*) for sure. Assume that constant acts are regular for both agents and society. Then, part* (i) *of the theorem yields that* V_0 *is a weighted sum of the individuals' utility:* $V_0 = \lambda V_1 + (1 - \lambda) V_2$.

Then, $V_0(x, x) = \lambda V_1(x, x) + (1 - \lambda) V_2(x, x) = 1 - \lambda$ *and* $V_0(y, y) = \lambda V_1(y, y) + (1 - \lambda) V_2(y, y) = \lambda$.

Assume the society is also of the multiple prior type. Then, $u_0(x) = V_0(x, x) = 1 - \lambda$ *and* $u_0(y) = V_0(y, y) = \lambda$*. Thus,* $V_0(x, y) \geq \min(u_0(x), u_0(y)) = \min(\lambda, 1 - \lambda)$*. But note that* $V_0(x, y) = \lambda V_1(x, y) + (1 - \lambda) V_2(x, y) = 0$.

In words the theorem says that, under the assumptions that individuals and society's preferences are "well behaved" on a subset of acts –and notably satisfy the independence axiom on this subset–, either society's preferences are a linear aggregation of *uncertainty neutral* individuals' preferences or there is a dictator. It cannot be the case that society's preferences are the result of the aggregation of an uncertainty averse agent with any other type (uncertainty averse, loving or neutral) of agent. A consequence is that if society's preferences display a non neutral attitude toward uncertainty (of the limited kind corresponding to the fact that it is not uncertainty neutral on some event with respect to $\mathcal{R}_0$), then it must be dictatorial. Remark that the theorem is in a sense stronger than Harsanyi's since uncertainty neutrality of the preferences is a consequence and not an assumption of the theorem. [9]

While we will discuss in the next section variations around this theorem, it is important to notice here that it applies even if we restrict all agents to have the same betting preferences on E. It is enough to observe that nothing in the assumptions of the theorem prevents the fact that all agents have the same coefficient k^E. Thus, we have the following corollary.

Corollary 1 *Let* $E \in \Sigma$*. Let* $\{\succsim_i\}_{i \in N}$ *be binary relations on* $\mathcal{A}$ *and* $\{\mathcal{R}_i\}_{i \in N}$ *be non-empty subsets of* $\mathcal{A}$*. Assume that the condition of Theorem 1 holds*

[9] This was already the case in Blackorby et al. (2004) study of the aggregation of rank dependent expected utility agents. As they put it "the EU-like conditions are to be found here in the conclusion, whereas Harsanyi put them in the assumption; apparently, he did not realize the logical power of his own framework."

and that all agents have the same betting preferences. Then, the conclusion of Theorem 1 holds as well and it is impossible to aggregate preferences unless they are all uncertainty neutral on event E.

Thus, what's driving the impossibility result *is not heterogeneity in betting attitudes*. To make this point clear in models where a notion of beliefs have been defined, consider the class of c-linear biseparable preferences and let $\mathcal{A}^c$ denote the set of constant acts. If for all $i \in N$, $\succsim_i$ are c-linear biseparable and not uncertainty neutral on E, and the Independent Prospects Property holds on $\mathcal{A}^c$, then Axiom 2 holds if and only if there exists $j \in N'$ such that $\succsim_0 = \succsim_j$. This is a direct consequence of the fact that $\mathcal{A}^c$ is regular for c-linear biseparable preferences and as we establish in the Appendix, that these preferences are not uncertainty neutral with respect to that set.

Two important particular cases covered are when agents and society have multiple prior preferences and when they have Choquet expected utility preferences of Schmeidler (1989). Hence, for instance, *it is not possible to aggregate multiple prior preferences into a multiple prior social preferences*, irrespective of the fact that the sets of priors are identical among agents. Whereas in an expected utility setting it is possible to aggregate agents with the same beliefs, this does not generalize to non-expected utility settings.

The proof of the theorem is divided into two distinct parts. The first one is a direct application of Proposition 2 in De Meyer and Mongin (1995). It states that, given the underlying convex structure (recall we are in an Anscombe-Aumann setting), the Pareto axiom implies that V_0 is a weighted sum of the V_is. Hence, aggregation has to be linear. The second part can itself be divided in two.

First, the Independent Prospects Property on $\cap_{i \in N} \mathcal{R}_i$ states that for any i, there exist $h^\star$, $h_\star$ in $\cap_{i \in N} \mathcal{R}_i$ such that $h^\star \succ_i h_\star$ and $h^\star \sim_j h_\star$, $\forall j \in N' \setminus \{i\}$. Using these acts for any i, one can establish that for any agent i that has a non zero weight λ_i, $k_i^E = k_0^E$ for any event E. Thus, all agents that are taken into account in V_0 must have the same attitude toward uncertainty.

Second, we prove that $k_0^E = 0$ as soon as there are two agents with non zero weights. Assume for simplicity that only agent 1 and 2 have non zero weight. The argument relies on the fact that, using the Independent Prospects Property and mixing acts, one can find two acts $f, g \in \cap_{i \in N} \mathcal{R}_i$ such that $f \succ_1 g$ and $f \prec_2 g$, while $f \sim_0 g$. The uncertainty neutrality of the preferences can then be established by computing $V_0(f_E g) + V_0(g_E f) - V_0(f) - V_0(g)$ in two different ways. The first one is direct and establishes that this quantity is zero since $f \sim_0 g$. The second one is to compute it decomposing V_0 as the sum of $\lambda_1 V_1$ and $\lambda_2 V_2$. Using the fact that $k_1^E = k_2^E = k_0^E$, this last part establishes that $k_0^E = 0$.

5 Restoring possibility

In this Section, we provide a discussion of the assumptions made to obtain our result. We first start by reviewing known arguments to restore possibility in the expected utility setting and show how they fail to apply in our setting. We also show that our impossibility result extends to smooth ambiguity averse decision makers. In the next subsection, we show that dropping some requirement at the society level might restore the possibility of aggregation.

5.1 What does not work...

5.1.1 Weakening the Pareto axiom

Gilboa et al. (2004) suggested to weaken the Pareto principle to acts on which the agents have the same beliefs.[10] They established then that it is possible to aggregate linearly and separately tastes and beliefs. Such a way to restore possibility would not work in our context. As we argued, even if agents have the same betting attitudes (which, under expected utility amounts to same beliefs), aggregation is impossible under uncertainty non neutrality. In the multiple prior model for instance one can identify, for the sake of the argument, “beliefs” with the set of priors. Then, as we have shown, aggregation is not possible even when agents all have the same “beliefs”. Thus weakening the Pareto principle to acts on which agents have the same betting attitudes does not appear to be a solution here.

5.1.2 State dependence

As shown by Mongin (1998) and Chambers and Hayashi (2006) (in a Savage setting) a way to circumvent the impossibility of aggregating subjective expected utility agents when they have different beliefs is to enrich the possible domain for society’s preferences. Specifically, they allowed for state-dependence in society’s preferences (while remaining in the subjective utility class). Since state-dependent preferences are already included in our class of preferences, our result embeds their possibility result. However, it also shows that such a way of restoring the possibility of aggregation will not work when preferences are not uncertainty neutral.

[10] Identical beliefs are defined in their paper in terms of the representation rather than in terms of the preferences.

5.1.3 Impossibility with smooth preferences: an example

We provide here an example in which the class of preferences considered is of the "smooth ambiguity averse" type à la Klibanoff et al. (2005) and in which aggregation is not possible. Consider two agents, 1 and 2, and denote society as above by 0. Let $V_i(f_E g) = \varphi_i^{-1}\left[p_i(E)\varphi_i\left(U_i(f)\right) + (1-p_i(E))\varphi_i\left(U_i(g)\right)\right]$ for $i = 0, 1, 2$, where p_i is a unique subjective probability distribution, U_i is a vNM utility function on $\mathcal{A}^c$ and φ_i is the second level utility function which captures attitude toward ambiguity. Assume φ_i, $i = 1, 2$ is strictly concave, reflecting ambiguity aversion. Assume furthermore that the Independent Prospects Property holds on $\mathcal{A}^c$. Since Harsanyi's conditions are satisfied on these acts, it has to be the case that $U_0 = \alpha_1 U_1 + \alpha_2 U_2$, for some $\alpha_1, \alpha_2 \in \mathbb{R}_+$. Assume without loss of generality that $\alpha_1 \geq \alpha_2$. Let E be an event and assume for the sake of simplicity that $p_1(E) = p_2(E) = 1/2$.

¿From the Independent Prospects Property, there exist three constant acts x, y, and z such that 1 prefers x to y and 2 prefers y to x, while z is indifferent to y for 1 and to x for 2.

	U_1	U_2	U_0
x	1	0	α_1
y	0	1	α_2
z	0	0	0

Construct now the three constant acts h, k, and ℓ as follows:

$$h = \frac{1}{8}\left(1 - \frac{\alpha_2}{\alpha_1}\right)x + \frac{1}{4}y + \left(\frac{5}{8} + \frac{1}{8}\frac{\alpha_2}{\alpha_1}\right)z,$$
$$k = \frac{1}{8}\left(1 + \frac{\alpha_2}{\alpha_1}\right)x + \left(\frac{7}{8} - \frac{1}{8}\frac{\alpha_2}{\alpha_1}\right)z;$$
$$\ell = \frac{1}{8}x + \frac{1}{8}y + \frac{3}{4}z.$$

These three acts are constructed so that society is indifferent among them, while 1 and 2 have opposite preferences on them, namely $k \succ_1 \ell \succ_1 h$ and $h \succ_2 \ell \succ_2 k$ Indeed, direct computation yields the following table:

	U_1	U_2	U_0
h	$\frac{1}{8}(1-\frac{\alpha_2}{\alpha_1})$	$\frac{1}{4}$	$\frac{\alpha_1+\alpha_2}{8}$
k	$\frac{1}{8}(1+\frac{\alpha_2}{\alpha_1})$	0	$\frac{\alpha_1+\alpha_2}{8}$
ℓ	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{\alpha_1+\alpha_2}{8}$

Since society is indifferent among these three constant acts, it cannot exhibit any uncertainty attitude with respect to, for instance, the binary act of the kind $h_E k$. In particular, $h_E k$ is indifferent from the society view point to ℓ. Hence, it will fail to capture the uncertainty non neutrality of agents 1 and 2. We now make this more precise.

Observe that $V_0(h_E k) = \varphi_0^{-1}\left[p_0(E)\varphi_0\left(U_0(h)\right) + (1-p_0(E))\varphi_0\left(U_0(k)\right)\right] = \frac{\alpha_1+\alpha_2}{8} = V_0(\ell)$. Hence, society is indifferent between ℓ and $h_E k$. Let's now show that $V_i(\ell) > V_i(h_E k)$ for $i = 1, 2$ thus establishing a violation of the Pareto axiom

By construction $V_1(\ell) = \varphi_1^{-1}\left[1/2\varphi_1\left(\frac{1}{8}\right) + 1/2\varphi_1\left(\frac{1}{8}\right)\right] = \frac{1}{8}$, and similarly, $V_2(\ell) = \frac{1}{8}$.

Furthermore, $V_1(h_E k) = \varphi_1^{-1}\left[1/2\varphi_1\left(\frac{1}{8}(1-\frac{\alpha_2}{\alpha_1})\right) + 1/2\varphi_1\left(\frac{1}{8}(1+\frac{\alpha_2}{\alpha_1})\right)\right]$. Given that φ_1 is assumed to be strictly concave, one has

$$V_1(h_E k) < \varphi_1^{-1}\left[\varphi_1\left(1/2\left(\frac{1}{8}(1-\frac{\alpha_2}{\alpha_1})\right) + 1/2\left(\frac{1}{8}(1+\frac{\alpha_2}{\alpha_1})\right)\right)\right] = \frac{1}{8}.$$

In a similar fashion, it is easy to establish that

$$V_2(h_E k) < \varphi_1^{-1}\left[\varphi_1\left(1/2\left(\frac{1}{4}\right) + 1/2\,(0)\right)\right] = \frac{1}{8}.$$

We then get the contradiction to the Pareto axiom we were after, namely $V_1(h_E k) < V_1(\ell)$ and $V_2(h_E k) < V_2(\ell)$ while $V_0(h_E k) = V_0(\ell)$. Observe that the argument in the example follows closely the one of the proof of Theorem 1 (see the intuition given page 11). Note also that we do not need to specify the distortion function φ_i and hence that this example shows that aggregation fails even when agents have the same attitude toward ambiguity.

This example, which is not pathological, shows that it is not possible to prove an aggregation result concerning ambiguity averse agents à la Klibanoff et al. (2005). As we show in Appendix A, these preferences fail to satisfy $\mathcal{A}^c$-independence, but do satisfy the weak sure-thing principle, while they obviously satisfy the independence axiom on $\mathcal{A}^c$ (being of the vNM type on this domain). This, we conjecture, might be enough to show a more general impossibility result, namely that if preferences satisfy the independence axiom on $\{\mathcal{R}_i\}_{i\in N}$ and the weak sure-thing principle for $\{\mathcal{R}_i\}_{i\in N}$-binary acts, then the conclusion of Theorem 1 holds, that is, it is not possible to aggregate uncertainty non neutral agents. This however would require to have a more general decision theoretic model in which binary independence does not hold. Klibanoff et al. (2005) is one such model but a rather specific one and a general characterization remains to be done.

5.2 What might work...

We now explore what type of result is achievable relaxing the independent prospects property, binary independence and the weak sure-thing principle.

5.2.1 Same risk preferences

Embedded in the assumptions of the Theorem is the fact that social preferences has a rich set of regular acts. Conditions 2 and 3 indeed entail that there exists a set on which binary $\mathcal{R}$-independence holds for all preferences and such that, on this set, the Independent Prospects Property hold. One might wonder what would happen, were one to relax this assumption. Assume that there is a set $\mathcal{R}$ which is E-regular for the society and for all individuals. Assume that all individuals have the same preferences on $\mathcal{R}$, which means that Independent Prospects fails to hold. Then, Axiom 2 holds if and only if there exists an affine representation V_0 on $\mathcal{B}(\mathcal{R}, E)$ of $\succsim_0$ which is a linear aggregation of the individuals' affine V_i's.

Hence, if all agents have the same risk preferences (i.e., their preferences on $\mathcal{R}$) but different betting preferences, then aggregation is possible and amounts to linear aggregation.

Note also that if all individuals have the same preferences on a set $\mathcal{R}$ which is E-regular for them, then Axiom 2 implies that society's preferences are the same as the individuals' on $\mathcal{R}$ and therefore satisfy the independence axiom on this set. In that case, V_0 coincides with the V_i's on $\mathcal{R}$. On the other hand, $\mathcal{R}$ is not necessarily E-regular for the society and $\mathcal{R}$-independence might fail.

5.2.2 Diversity and $\mathcal{R}$-independence

The preceding subsection shows that the diversity condition is critical to obtain our result. This condition is a joint condition on individuals' and society's preferences. One might wonder if diversity restricted to individuals' preferences alone implies a form of impossibility theorem. As we have explained above, if the diversity condition is imposed on individuals' preferences, then Axiom

2 imposes only that the society's preferences satisfy the independence axiom on subsets of the intersection of the individuals' E-regular sets where all individuals have the same preferences. Besides this fact, there is nothing in our approach that constrains $\cap_{i\in N'}\mathcal{R}_i$ to be E-regular for the society.[11] In this case, our theorem does not apply and aggregation might be possible. Take for instance, $V_0(f) = \min_i V_i(f)$. This represents a preference for the society, that respects the Pareto axiom. But it is not clear what axioms this preference obeys besides Axiom 1.

5.2.3 Dropping the weak sure-thing principle

The next result shows that linear aggregation is possible if one is willing to drop the weak sure-thing principle at the society's level. In other words, defining V_0 as $\sum_i \lambda_i V_i$ is an aggregation procedure that satisfies the Pareto axiom. Furthermore, V_0 hence defined satisfies the independence axiom with respect to mixing with acts in $\cap_{i\in N}\mathcal{R}_i$. However, it violates the weak sure-thing principle.

Proposition 4 *Let $E \in \Sigma$. Let $\{\succsim_i\}_{i\in N}$ be binary relations on $\mathcal{A}$ and $\{\mathcal{R}_i\}_{i\in N}$ be non-empty subsets of $\mathcal{A}$. Assume that*

(1) for all $i \in N$, $\succsim_i$ satisfies Axiom 1;
(2) for all $i \in N'$, $\mathcal{R}_i$ is E-regular with respect to $\succsim_i$;
(3) $\{\succsim_i\}_{i\in N'}$ satisfy the Independent Prospects Property on $\cap_{i\in N}\mathcal{R}_i$;
(4) For all $f \in \cap_{i\in N}\mathcal{R}_i$, for all $g, h \in \mathcal{A}$, $\alpha \in (0, 1]$, $g \succsim_0 h \Leftrightarrow \alpha g+(1-\alpha)f \succsim_0 \alpha h + (1-\alpha)f$.

Then, Axiom 2 holds if and only if, there exists an $\mathcal{R}_i$-affine representation V_i of $\succsim_i$ for all $i \in N$, unique weights $(\lambda_1, \cdots, \lambda_n) \in \mathbb{R}^n_+ \setminus \{0\}$, $\mu \in \mathbb{R}$ such that

$$\forall f \in \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E), V_0(f) = \sum_{i\in N'} \lambda_i V_i(f) + \mu.$$

This way of aggregating preferences has the same pros and cons as those identified in the discussion of Harsanyi's theorem (see Weymark (1991) and Mongin (2002) for instance). In particular, if one wants to use this theorem as an operational means to identify society's preferences, then one is forced to adopt some extra assumptions bearing on inter-personal welfare comparisons (via cardinalization of the preferences for instance).

What are the consequences of abandoning the weak sure-thing principle for binary acts for the society? When it is relaxed, it is not possible to define

[11] Arguments along the line of Diamond (1967), Sen (1970), Epstein and Segal (1992) might give a justification for the fact that $\cap_{i\in N'}\mathcal{R}_i$ is not E-regular for society.

conditional preferences any longer. Thus, it is not possible to construct *ex ante* preferences from a notion of conditional preferences. At best, an incomplete notion of conditional preferences ($f \trianglerighteq_0^E g$ if and only if $f_E h \succsim_0 g_E h$ for all $h \in \mathcal{R}$) can be defined.

The important consequence of this technical remark is that dominance reasoning is hence not possible at the society's level. In the multiple prior example, even if one obtains a vNM utility function u_0 for the society, it is not possible to conclude that an act that yields higher utility to another act state by state is preferred to that latter act. If one considers that dominance is a property that individuals' preferences should satisfy, then linear aggregation is here possible at the cost of assuming that the society's preferences do not satisfy the same "rationality" criteria as individuals.

6 Concluding remarks

We have explored in detail the (im)possibility of aggregating preferences under uncertainty and have established that under rather weak requirements, expected utility over the entire domain is actually a necessary condition to obtain a possibility result. For decision makers who have expected utility over lotteries, there is no scope for any non neutral attitude toward uncertainty if one wants to be able to construct a social preference (that also respect expected utility over lotteries). Thus, for instance, the notion of a representative multiple prior agent does not make much sense (even in the particular case in which all agents have the same set of priors).

We also identified the conditions that are at the heart of the impossibility results. Dropping a monotonicity requirement (the weak sure-thing principle on binary acts) at the society's level restores the possibility of aggregation à la Harsanyi. As we argued in the previous section, this comes at a cost since a consequence of dropping this requirement is that society's conditional preferences are not well-defined, in the sense that they do not form a complete order, and therefore dominance arguments become irrelevant. However, one could proceed with this incompleteness, and also assume that society's ex ante preferences be incomplete. We leave this exploration for further research. Another research avenue would be to conduct a systematic study of what kind of aggregation result is allowed when one drops the requirement that social preferences match the individuals' indifference to mixing on a rich set of acts, as alluded to in section 5.2.2.

Appendix A: Examples

In this Appendix, we provide three illustrations of our decision theoretic constructs.

A1. c-linear biseparable preferences

For any subset $\mathcal{F}$ of $\mathcal{A}$, let $\mathcal{B}(\mathcal{F}) = \{f_E g \,|\, f, g \in \mathcal{F} \text{ and } E \in \Sigma\}$.

$\succsim$ is *c-linear biseparable* if it can be represented by a function $V : \mathcal{A} \to \mathbb{R}$, such that:

(1) for $f, g \in \mathcal{A}^c$, $f \succeq g$, $V(f_E g) = \rho(E)u(f) + (1 - \rho(E))u(g)$ where ρ is a capacity;
(2) $V(\alpha f + (1-\alpha)g) = \alpha V(f) + (1-\alpha)V(g)$ for all $f \in \mathcal{B}(\mathcal{A}^c)$ and $g \in \mathcal{A}^c$.

Claim 3 *$\mathcal{A}^c$ is E-regular for c-linear biseparable preferences for any event E.*

Proof. Let $E \in \Sigma$ be an event. That such preferences satisfy binary $\mathcal{A}^c$-independence is a direct implication of the fact that $V(\alpha f + (1-\alpha)g) = \alpha V(f) + (1-\alpha)V(g)$ for all $f \in \mathcal{B}(\mathcal{A}^c, E)$ and $g \in \mathcal{A}^c$.

Let's now check that these preferences satisfy the weak sure-thing principle for $\mathcal{A}^c$-binary acts as well. Let f, g, h, h' be constant acts. Assume for instance that $f \succsim h$ and $h \succsim g$, (other cases can be dealt with in a similar manner). Note that this implies that $f \succsim g$. Assume finally that $f_E h \succ g_E h$. This implies:

$$\rho(E)u(f) + (1 - \rho(E))u(h) > (1 - \rho(E^c))u(g) + \rho(E^c)u(h).$$

We now check that $f_E h' \succsim g_E h'$ for any $h' \in \mathcal{A}^c$. Three cases must be considered.

Case 1: $f \succsim h'$ and $g \succsim h'$.

In this case,

$$\begin{aligned} f_E h' \succsim g_E h' &\Leftrightarrow \rho(E)u(f) + (1 - \rho(E))u(h') \geq \rho(E)u(g) + (1 - \rho(E))u(h') \\ &\Leftrightarrow u(f) \geq u(g), \end{aligned}$$

which is the case by assumption.

Case 2: $f \succsim h'$ and $h' \succsim g$.

In this case,

$$\begin{aligned} f_E h' \succsim g_E h' &\Leftrightarrow \rho(E)u(f) + (1-\rho(E))u(h') \geq (1-\rho(E^c))u(g) + \rho(E^c)u(h') \\ &\Leftrightarrow \rho(E)u(f) + (1-\rho(E^c))u(h') \geq \rho(E)u(h') + (1-\rho(E^c))u(g), \end{aligned}$$

which is the case since $u(f) \geq u(h') \geq u(g)$.

Case 3: $h' \succsim f$ and $h' \succsim g$.

In this case,

$$\begin{aligned} f_E h' \succsim g_E h' &\Leftrightarrow (1-\rho(E^c))u(f) + \rho(E^c)u(h') \geq (1-\rho(E^c))u(g) + \rho(E^c)u(h') \\ &\Leftrightarrow u(f) \geq u(g), \end{aligned}$$

which is the case by assumption. ■

Claim 4 *c-linear bi-separable preferences are rank-dependent additive with respect to $\mathcal{A}^c$ for any event E.*

Proof. Define $\overline{V}_E(f) = \rho(E)u(f)$ and $\underline{V}_{E^c} = (1-\rho(E))u(g)$, and observe that, when $f \succ g$, $V(f_E g) = \rho(E)u(f)+(1-\rho(E))u(g)$. The decomposition for $g \succsim f$ is done in a similar fashion. Note finally that if $f \succsim g$, $V(f_E g) + V(g_E f) - V(f) - V(g) = \rho(E)u(f) + (1-\rho(E))u(g) + \rho(E^c)u(f) + (1-\rho(E^c))u(g) - u(f) - u(g) = (\rho(E)+\rho(E^c)-1)(u(f)-u(g))$. Defining $k^E = \rho(E)+\rho(E^c)-1$ yields the desired result (property 2 in the proposition). ■

Claim 5 *c-linear bi-separable preferences fail in general to be uncertainty neutral on an event E with respect to $\mathcal{A}^c$.*

Proof. Let $f, g, h, \ell \in \mathcal{A}^c$ be such that $f \succ g$ and $\ell \succ h$. Let $\alpha \in (0,1)$ and assume w.l.o.g. $(\alpha f + (1-\alpha)h) \succ (\alpha g + (1-\alpha)\ell)$. Then,

$$\begin{aligned} V(\alpha f_E g + (1-\alpha)h_E \ell) &= V((\lambda f + (1-\lambda)h)_E(\lambda g + (1-\lambda)\ell)) \\ &= \alpha V(f_E g) + (1-\alpha)[\rho(E)u(h) + (1-\rho^c(E))u(\ell)]. \end{aligned}$$

Therefore, these preferences are uncertainty neutral on an event E with respect to $\mathcal{A}^c$ if, and only if $\rho(E) = 1 - \rho(E^c)$, which does not hold in general. ■

A2. State dependent α-MMEU

$\succsim$ is *a state-dependent $\alpha-$MMEU preference* if it can be represented by

$$V(f) = \alpha \min_{p\in\mathcal{C}} E_p u_s(f(s)) + (1-\alpha) \max_{p\in\mathcal{C}} E_p u_s(f(s)),$$

where u_s is an affine function on Y for all $s \in S$.

Claim 6 *Let $E \in \Sigma$. The set $\mathcal{A}^{cv} = \{f \in \mathcal{A}$ s.th. $\forall s,t\ u_s(f(s)) = u_t(f(t))\}$ of constant utility acts is E-regular for state-dependent $\alpha-$MMEU preferences.*

Proof. Notice first that $\mathcal{A}^{cv}$ is a mixture set. Second, it is also easy to establish that $V(\alpha f+(1-\alpha)g) = \alpha V(f)+(1-\alpha)V(g)$ for all $f \in \mathcal{B}(\mathcal{A}^{cv}, E)$ and $g \in \mathcal{A}^{cv}$. Third, we check that condition 3 of Definition 1 holds as well. Remark that for all $f, h \in \mathcal{A}^{cv}$, one has:

$$\begin{aligned} V(f_E h) &= \alpha \min_{p\in\mathcal{C}} (p(E)V(f) + (1-p(E))V(h)) \\ &\quad + (1-\alpha) \max_{p\in\mathcal{C}} (p(E)V(f) + (1-p(E))V(h)) \\ &= \begin{cases} \left(\alpha \underline{p}(E) + (1-\alpha)\overline{p}(E)\right) V(f) \\ \qquad + \left(\alpha(1-\underline{p}(E)) + (1-\alpha)(1-\overline{p}(E))\right) V(h) \text{ if } V(f) \geq V(h) \\ \left(\alpha \overline{p}(E) + (1-\alpha)\underline{p}(E)\right) V(f) \\ \qquad + \left(\alpha(1-\overline{p}(E)) + (1-\alpha)(1-\underline{p}(E))\right) V(h) \text{ if } V(f) \leq V(h) \end{cases} \end{aligned}$$

where $\underline{p}(E) = \min_{p\in\mathcal{C}} p(E)$ and $\overline{p}(E) = \min_{p\in\mathcal{C}} p(E)$.

Now, for all $f, g, h \in \mathcal{A}^{cv}$, it is straightforward, using the expression obtained for $V(f_E h)$ and looking at all the possible ranking of $V(f), V(g), V(h)$, to check that $V(f_E h) \geq V(g_E h)$ if and only if $V(f) \geq V(g)$, thus establishing that property 2 holds. ■

Claim 7 *State-dependent α-MMEU are rank-dependent additive for event E with respect to $\mathcal{A}^{cv}$*

Proof. Recall that

$$V(f_E h) = \begin{cases} \left(\alpha \underline{p}(E) + (1-\alpha)\overline{p}(E)\right) V(f) \\ \qquad + \left(\alpha(1-\underline{p}(E)) + (1-\alpha)(1-\overline{p}(E))\right) V(h) \text{ if } V(f) \geq V(h) \\ \left(\alpha \overline{p}(E) + (1-\alpha)\underline{p}(E)\right) V(f) \\ \qquad + \left(\alpha(1-\overline{p}(E)) + (1-\alpha)(1-\underline{p}(E))\right) V(h) \text{ if } V(f) \leq V(h) \end{cases}$$

where $\underline{p}(E) = \min_{p\in\mathcal{C}} p(E)$ and $\overline{p}(E) = \min_{p\in\mathcal{C}} p(E)$.

To conclude that state-dependent α-MMEU are rank-dependent additive with respect to $\mathcal{A}^{cv}$, it is enough to identify the functions $\overline{V}_E$, $\underline{V}_{E^c}$, $\underline{V}_E$, and $\overline{V}_{E^c}$ by looking at the expression obtained for $V(f_E h)$. ■

Claim 8 *Let $E \in \Sigma$. State-dependent $\alpha-$MMEU fail in general to be uncertainty neutral for E with respect to the set of constant utility acts $\mathcal{A}^{cv} = \{f \in \mathcal{A}$ s.th. $\forall s,t\ u_s(f(s)) = u_t(f(t))\}$.*

Proof. Let $f,g,h,\ell \in \mathcal{A}^c$ be such that $f \succ g$ and $\ell \succ h$. Let $\lambda \in (0,1)$ and assume wlog $(\lambda f + (1-\lambda)h) \succ (\lambda g + (1-\lambda)\ell)$. Let $\bar{p}(E)$ (resp. $\underline{p}(E)$) be the upper (resp. lower) probability of E in $\mathcal{C}$. Then,

$$\begin{aligned} V(\lambda f_E g + (1-\lambda)h_E \ell) &= V((\lambda f + (1-\lambda)h)_E(\lambda g + (1-\lambda)\ell)) \\ &= \alpha \min_{p\in\mathcal{C}} E_p u_s(\lambda f(s) + (1-\lambda)h(s)) \\ &\qquad +(1-\alpha)\max_{p\in\mathcal{C}} E_p u_s(\lambda g(s) + (1-\lambda\ell(s)) \\ &= \alpha\underline{p}(E)[\lambda V(f) + (1-\lambda)V(h)] \\ &\qquad +(1-\alpha)\bar{p}(E)[\lambda V(g) + (1-\lambda)V(\ell)] \\ &= \lambda V(f_E g) + (1-\lambda)[\underline{p}(E)V(h) + \bar{p}(E)V(\ell)]. \end{aligned}$$

Therefore, V is uncertainty neutral on E with respect to $\mathcal{A}^{cv}$ only if $\bar{p}(E) = \underline{p}(E)$, which does not hold in general. ■

A3. Smooth ambiguity aversion

$\succsim$ is *smoothly ambiguity averse* on $\mathcal{B}(\mathcal{A}^c, E)$ if it can be represented by:

$$V(f_E g) = \varphi^{-1}\left[p(E)\varphi\left(U(f)\right) + (1-p(E))\varphi\left(U(g)\right)\right],$$

on $\mathcal{B}(\mathcal{A}^c, E)$, where p is a unique subjective probability distribution, U is a vNM utility function on $\mathcal{A}^c$ and φ is the second level utility function which captures attitude toward ambiguity.

Claim 9 *Smooth ambiguity averse preferences violate binary $\mathcal{A}^c$ independence.*

Proof. Note that for all $f,g,h \in \mathcal{A}^c$ and $\lambda \in (0,1)$:

$$\begin{aligned} V(\lambda f_E g + (1-\lambda)h) &= \varphi^{-1}[p(E)\varphi(\lambda U(f) + (1-\lambda)U(h)) \\ &\qquad +(1-p(E))\varphi(\lambda U(g) + (1-\lambda)U(l))] \\ &\neq \lambda V(f_E g) + (1-\lambda)V(h) \text{ unless } \varphi \text{ is linear.} \end{aligned}$$

In other words, this functional is not linear with respect to probabilistic combination between $\mathcal{A}^c$-binary acts and constant acts. ■

Claim 10 *Smooth ambiguity averse preferences satisfy the weak sure-thing principle and independence on $\mathcal{A}^c$.*

Proof. Since $V(f_E h) = \varphi^{-1}\left[p(E)V(f) + (1-p(E))V(h)\right]$, $V(f_E h) > V(g_E h)$ implies $V(f) > V(g)$, and $V(f_E h') > V(g_E h')$ for any $h' \in \mathcal{A}^c$.

Independence holds given that preferences on constant acts are vNM. ∎

Observe that smooth ambiguity averse preferences are *not* rank-dependent additive.

Claim 11 *Let $E \in \Sigma$. Smooth ambiguity averse preferences in general not uncertainty neutral on E with respect to the set of constant acts.*

Proof. Since smooth ambiguity averse preferences reduce to expected utility on constant acts, they are uncertainty neutral with respect to $\mathcal{A}^c$.

Now, let $f, g, h, \ell \in \mathcal{A}^c$ and $\lambda \in (0,1)$. Then,

$$\begin{aligned} V(\lambda f_E g + (1-\lambda)h_E \ell) &= V((\lambda f + (1-\lambda)h)_E(\lambda g + (1-\lambda)\ell)) \\ &= \varphi^{-1}[p(E)\varphi(\lambda U(f) + (1-\lambda)U(h)) \\ &\qquad + (1-p(E))\varphi(\lambda U(g) + (1-\lambda)U(l))]. \end{aligned}$$

On the other hand,

$$\begin{aligned} \lambda V(f_E g) + (1-\lambda)V(h_E \ell) &= \lambda\varphi^{-1}\left[p(E)\varphi\left(U(f)\right) + (1-p(E))\varphi\left(U(g)\right)\right] \\ &\quad + (1-\lambda)\varphi^{-1}\left[p(E)\varphi\left(U(h)\right) + (1-p(E))\varphi\left(U(l)\right)\right]. \end{aligned}$$

Therefore, these preferences are uncertainty neutral with respect to $\mathcal{A}^c$ only if φ is linear, i.e., when these preferences satisfy the reduction of compound lottery axiom, in which case they are ambiguity neutral according to Klibanoff et al. (2005). ∎

Appendix B

B1. Proof of Proposition 1

Existence of an $\mathcal{R}$-affine representation follows from a usual vNM kind of proof and is omitted here.

Next, for sake of simplicity we prove the properties 1 & 2 at the same time. Let V be any $\mathcal{R}$-affine representation of $\succsim$.

For any event E and acts $f, g \in \mathcal{R}$, say that $f \trianglerighteq_E g$ if for all act $h \in \mathcal{R}$, $f_E h \succsim g_E h$. This relation is well-defined since $\succsim$ satisfies the weak sure-thing principle for binary acts. Denote $\triangleright_E$ and $\approx_E$ strict preference and indifference respectively. It can be checked that by definition of $\mathcal{R}$, $\trianglerighteq_E$ satisfies the vNM axioms.

The proof is decomposed in two depending on whether there exist $f^\star, f_\star \in \mathcal{R}$ such that $V(f^\star_E f_\star) + V(f_{\star E} f^\star) \neq V(f^\star) + V(f_\star)$ or not.

Case 1.

There exist $f^\star, f_\star \in \mathcal{R}$ such that $V(f^\star_E f_\star) + V(f_{\star E} f^\star) \neq V(f^\star) + V(f_\star)$

As a first step, we show that either $\trianglerighteq_E = \trianglerighteq_{E^c}$ or $\trianglerighteq_{E^c}$ is a reverse order of $\trianglerighteq_E$, in the sense that $f \trianglerighteq_{E^c} g$ if and only if $g \trianglerighteq_E f$, for all $f, g \in \mathcal{R}$. In step 2, we complete the proof of properties 1 and 2.

Step 1. $\trianglerighteq_E = \trianglerighteq_{E^c}$ or $\trianglerighteq_{E^c}$ is a reverse order of $\trianglerighteq_E$

Suppose that $f^\star \succsim f_\star$. Then, we necessarily must be in one of the following cases:

- $f^\star \triangleright_E f_\star$ and $f^\star \triangleright_{E^c} f_\star$,
- $f^\star \triangleright_E f_\star$ and $f_\star \trianglerighteq_{E^c} f^\star$,
- $f_\star \trianglerighteq_E f^\star$ and $f^\star \triangleright_{E^c} f_\star$
- $f_\star \trianglerighteq_E f^\star$ and $f_\star \trianglerighteq_{E^c} f^\star$.

This last case is not possible. Indeed, $f_\star \trianglerighteq_E f^\star$ implies that $f_\star \succsim f^\star_E f_\star$ and $f_{\star E} f^\star \succsim f^\star$ while $f_\star \trianglerighteq_{E^c} f^\star$ implies that $f_\star \succsim f_{\star E} f^\star$ and $f^\star_E f_\star \succsim f^\star$. Thus $f_\star \succsim f^\star_E f_\star, f_{\star E} f^\star \succsim f^\star$ while by assumption $f^\star \succsim f_\star$ and therefore $f^\star_E f_\star \sim f_{\star E} f^\star \sim f_\star \sim f^\star$ and thus $V(f^\star_E f_\star) + V(f_{\star E} f^\star) = V(f^\star) + V(f_\star)$ which leads to a contradiction.

Therefore, we essentially have only two cases to consider : (a) $f^\star \triangleright_E f_\star$ and $f^\star \triangleright_{E^c} f_\star$, and (b) $f^\star \triangleright_E f_\star$ and $f_\star \trianglerighteq_{E^c} f^\star$ (the third case being the symmetric of case (b)).

Case (a): $f^\star \triangleright_E f_\star$ and $f^\star \triangleright_{E^c} f_\star$.

Let us prove that $\trianglerighteq_E = \trianglerighteq_{E^c}$. Assume to the contrary that there exist $f, g \in \mathcal{R}$ such that $f \triangleright_E g$ while $g \trianglerighteq_{E^c} f$. W.l.o.g, we can take these acts such that $f^\star \triangleright_E f \triangleright_E g \triangleright_E f_\star$ and $f^\star \trianglerighteq_{E^c} g \trianglerighteq_{E^c} f \trianglerighteq_{E^c} f_\star$. Indeed, we can always exhibit two acts satisfying our conditions by mixing f and g with either $f^\star$or $f_\star$. Then there exist a, $a^c, b, b^c \in (0,1)$ such that $1 \geq a > b \geq 0$ and $1 \geq b^c \geq$

$a^c \geq 0$ and

$$
\begin{aligned}
&f \approx_E af^\star + (1-a)f_\star;\\
&f \approx_{E^c} a^c f^\star + (1-a^c)f_\star;\\
&g \approx_E bf^\star + (1-b)f_\star;\\
&g \approx_{E^c} b^c f^\star + (1-b^c)f_\star.
\end{aligned}
$$

Assume $a > a^c$. By definition of $\mathcal{R}$, $f \sim (af^\star + (1-a)f_\star)_E\,(a^c f^\star + (1-a^c)f_\star)$. Hence,

$$
\begin{aligned}
V(f) &= V\left((af^\star + (1-a)f_\star)_E\,(a^c f^\star + (1-a^c)f_\star)\right)\\
&= V\left(\frac{a-a^c}{1-a^c} f^\star_E\,(a^c f^\star + (1-a^c)f_\star) + \frac{1-a}{1-a^c}\,(a^c f^\star + (1-a^c f_\star)\right)\\
&= \frac{a-a^c}{1-a^c} V\left(f^\star_E\,(a^c f^\star + (1-a^c)f_\star)\right) + \frac{1-a}{1-a^c} V\left(a^c f^\star + (1-a^c)f_\star\right)\\
&= \frac{a-a^c}{1-a^c}\left(a^c V(f^\star) + (1-a^c)V(f^\star_E f_\star)\right)\\
&\qquad\qquad + \frac{1-a}{1-a^c}(a^c V(f^\star) + (1-a^c)V(f_\star))\\
&= a^c V(f^\star) + (a-a^c)\,V(f^\star_E f_\star) + (1-a)V(f_\star)\,.
\end{aligned}
$$

Since $f \in \mathcal{R}$,

$$
\begin{aligned}
V\left(\frac{1}{1+a-a^c} f \right.&\left. + \frac{a-a^c}{1+a-a^c} f_{\star E} f^\star\right)\\
&= \frac{1}{1+a-a^c} V(f) + \frac{a-a^c}{1+a-a^c} V(f_{\star E} f^\star)\\
&= \frac{1}{1+a-a^c}\left(a^c V(f^\star) + (a-a^c)\,V(f^\star_E f_\star) + (1-a)V(f_\star)\right)\\
&\qquad + \frac{a-a^c}{1+a-a^c} V(f_{\star E} f^\star)\,.
\end{aligned}
$$

But we also have by definition of $\mathcal{R}$,

$$
\begin{aligned}
& V\left(\frac{1}{1+a-a^c} f+\frac{a-a^c}{1+a-a^c} f_{\star E} f^{\star}\right) \\
& =V\left(\left(\frac{1}{1+a-a^c} f+\frac{a-a^c}{1+a-a^c} f_{\star}\right)_E\left(\frac{1}{1+a-a^c} f+\frac{a-a^c}{1+a-a^c} f^{\star}\right)\right) \\
& =V\left(\left(\frac{1}{1+a-a^c} f+\frac{a-a^c}{1+a-a^c} f_{\star}\right)_E\left(\frac{1}{1+a-a^c} f+\frac{a-a^c}{1+a-a^c} f^{\star}\right)\right) \\
& =V\left(\left(\frac{1}{1+a-a^c}\left(a f^{\star}+(1-a) f_{\star}\right)+\frac{a-a^c}{1+a-a^c} f_{\star}\right)_E\right. \\
& \qquad\qquad \left.\left(\frac{1}{1+a-a^c}\left(a^c f^{\star}+\left(1-a^c\right) f_{\star}\right)+\frac{a-a^c}{1+a-a^c} f^{\star}\right)\right) \\
& =V\left(\left(\frac{1}{1+a-a^c}\left(a f^{\star}+(1-a) f_{\star}\right)+\frac{a-a^c}{1+a-a^c} f_{\star}\right)_E\right. \\
& \qquad\qquad \left.\left(\frac{1}{1+a-a^c}\left(a^c f^{\star}+\left(1-a^c\right) f_{\star}\right)+\frac{a-a^c}{1+a-a^c} f^{\star}\right)\right) \\
& =V\left(\left(\frac{a}{1+a-a^c} f^{\star}+\frac{1-a^c}{1+a-a^c} f_{\star}\right)_E\left(\frac{a}{1+a-a^c} f^{\star}+\frac{1-a^c}{1+a-a^c} f_{\star}\right)\right) \\
& =V\left(\left(\frac{a}{1+a-a^c} f^{\star}+\frac{1-a^c}{1+a-a^c} f_{\star}\right)_E\left(\frac{a}{1+a-a^c} f^{\star}+\frac{1-a^c}{1+a-a^c} f_{\star}\right)\right) \\
& =\frac{a}{1+a-a^c} V\left(f^{\star}\right)+\frac{1-a^c}{1+a-a^c} V\left(f_{\star}\right) \\
& =\frac{a}{1+a-a^c} V\left(f^{\star}\right)+\frac{1-a^c}{1+a-a^c} V\left(f_{\star}\right).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\left(a^c V\left(f^{\star}\right)+\left(a-a^c\right) V\left(f_E^{\star} f_{\star}\right)+(1-a) V\left(f_{\star}\right)\right)+\left(a-a^c\right) V\left(f_{\star E} f^{\star}\right) & \\
=a V\left(f^{\star}\right)+\left(1-a^c\right) V\left(f_{\star}\right), &
\end{aligned}
$$

which is equivalent to $\left(a-a^c\right)\left(V\left(f_E^{\star} f_{\star}\right)+V\left(f_{\star E} f^{\star}\right)\right)=\left(a-a^c\right)\left(V\left(f^{\star}\right)+V\left(f_{\star}\right)\right)$. This contradicts the fact that $V(f_E^{\star} f_{\star})+V\left(f_{\star E} f^{\star}\right) \neq V(f^{\star})+V(f_{\star})$ and $a>a^c$. In the case where $a \leq a^c$, then either $a<a^c$ or $a=a^c$ but in this last event, $b<b^c$ and the proof can be easily adapted in both cases. Hence, $\trianglerighteq_E=\trianglerighteq_{E^c}$.

Case (b) : $f^{\star} \triangleright_E f_{\star}$ and $f_{\star} \trianglerighteq_{E^c} f^{\star}$.

In this case, we show that $\trianglerighteq_{E^c}$ is a reverse order of $\trianglerighteq_E$, that is, for all $f, g \in \mathcal{R}$, $f \trianglerighteq_E g$ if and only if $g \trianglerighteq_{E^c} f$.

Observe first that it has to be the case that $f_{\star} \triangleright_{E^c} f^{\star}$. Indeed, if $f_{\star} \approx_{E^c} f^{\star}$, then by definition of $\mathcal{R}$, $f^{\star} \sim f_E^{\star} f_{\star}$ and $f_{\star} \sim f_{\star E} f^{\star}$ and thus $V(f_E^{\star} f_{\star})+V(f_{\star E} f^{\star})=V(f^{\star})+V(f_{\star})$.

Suppose $\trianglerighteq_{E^c}$ is not a reverse order of $\trianglerighteq_E$, that is, there exist $f, g \in \mathcal{R}$, such that $f \rhd_E g$ while $f \unrhd_{E^c} g$. As in case (a), we can assume w.l.o.g that $f^\star \rhd_E f \rhd_E g \rhd_E f_\star$ and $f_\star \unrhd_{E^c} f \unrhd_{E^c} g \unrhd_{E^c} f^\star$. Then, there exist a, $a^c, b, b^c \in (0,1)$ with $a > b$ and $a^c \leq b^c$ such that

$$
\begin{aligned}
&f \approx_E af^\star + (1-a)f_\star;\\
&f \approx_{E^c} a^c f^\star + (1-a^c)f_\star;\\
&g \approx_E bf^\star + (1-b)f_\star;\\
&g \approx_{E^c} b^c f^\star + (1-b^c)f_\star.
\end{aligned}
$$

Either $a > a^c$, or $a < a^c$, or $a = a^c$ and $b < b^c$. In the case $a > a^c$, we can replicate the argument for case (a) to show that $(a-a^c)\left(V\left(f^\star_E f_\star\right) + V\left(f_{\star E} f^\star\right)\right) = (a-a^c)\left(V\left(f^\star\right) + V\left(f_\star\right)\right)$. The proof can be adapted to the other cases to show a similar contradiction.

Step2. Properties 1 and 2 hold when there exist $f, g \in \mathcal{R}$ such that $V(f_E g) + V\left(g_E f\right) \neq V(f) + V(g)$

Case (a) $\unrhd_E = \unrhd_{E^c}$.

Given that $\succsim$ is not degenerate on $\mathcal{R}$, there exist $f^\star, f_\star \in \mathcal{R}$ such that $f^\star \succ f_\star$.

Thus, define for any f

$$
\begin{aligned}
\overline{V}_E(f) &= \frac{V(f^\star_E f_\star) - V(f_\star)}{V(f^\star) - V(f_\star)} V(f);\\
\underline{V}_E(f) &= \frac{V(f^\star) - V(f_{\star E} f^\star)}{V(f^\star) - V(f_\star)} V(f);\\
\overline{V}_{E^c}(f) &= \frac{V(f_{\star E} f^\star) - V(f_\star)}{V(f^\star) - V(f_\star)} V(f);\\
\underline{V}_{E^c}(f) &= \frac{V(f^\star) - V(f^\star_E f_\star)}{V(f^\star) - V(f_\star)} V(f).
\end{aligned}
$$

Let us prove that for all $f, g \in \mathcal{R}$,

$$
\begin{aligned}
V(f_E g) &= \overline{V}_E(f) + \underline{V}_{E^c}(g) \text{ if } f \succsim g\\
&= \underline{V}_E(f) + \overline{V}_{E^c}(g) \text{ if } f \precsim g.
\end{aligned}
$$

Consider $f, g \in \mathcal{R}$ such that $f \succsim g$ and consider the case where $V(f^\star) \geq$

$V(f) \geq V(g) \geq V(f_\star)$. We have that

$$f \approx_E \frac{V(f)-V(f_\star)}{V(f^\star)-V(f_\star)} f^\star + \left(1 - \frac{V(f)-V(f_\star)}{V(f^\star)-V(f_\star)}\right) f_\star,$$

and

$$g \approx_{E^c} \frac{V(g)-V(f_\star)}{V(f^\star)-V(f_\star)} f^\star + \left(1 - \frac{V(g)-V(f_\star)}{V(f^\star)-V(f_\star)}\right) f_\star.$$

By definition of $\mathcal{R}$, $f_E g \sim (af^\star + (1-a)f_\star)_E \, (bf^\star + (1-b)f_\star)$ where $a = \frac{V(f)-V(f_\star)}{V(f^\star)-V(f_\star)}$ and $b = \frac{V(g)-V(f_\star))}{V(f^\star)-V(f_\star)}$. Thus

$$\begin{aligned} V(f_E g) &= V\left((af^\star + (1-a)f_\star)_E \, (bf^\star + (1-b)f_\star)\right) \\ &= bV\left(f^\star\right) + (a-b)\, V\left(f^\star_E f_\star\right) + (1-a)V\left(f_\star\right) \\ &= \frac{\left(V(g)-V(f_\star)\right) V\left(f^\star\right) + \left(V(f)-V(g)\right) V\left(f^\star_E f_\star\right)}{V(f^\star)-V(f_\star)} \\ &\quad + \frac{(V\left(f^\star\right) - V(f))V\left(f_\star\right)}{V(f^\star)-V(f_\star)} \\ &= \frac{\left(V\left(f^\star_E f_\star\right) - V\left(f_\star\right)\right) V(f) + \left(V\left(f^\star\right) - V\left(f^\star_E f_\star\right)\right) V(g)}{V(f^\star)-V(f_\star)} \\ &= \overline{V}_E(f) + \underline{V}_{E^c}(g). \end{aligned}$$

In the case where $V(f^\star) \geq V(g) \geq V(f) \geq V(f_\star)$, a similar computation shows that $V(f_E g) = \underline{V}_E(f) + \overline{V}_{E^c}(g)$.

In the other cases, the proof can be easily adapted to show that

$$\begin{aligned} V(f_E g) &= \overline{V}_E(f) + \underline{V}_{E^c}(g) \text{ if } f \succsim g \\ &= \underline{V}_E(f) + \overline{V}_{E^c}(g) \text{ if } f \precsim g. \end{aligned}$$

Define $k^E = \frac{V(f^\star_E f_\star) + V(f_{\star E} f^\star) - V(f^\star) - V(f_\star)}{V(f^\star)-V(f_\star)}$.

If $f \succsim g$,

$$\begin{aligned}
&V(f_E g) + V(g_E f) - V(f) - V(g) \\
&= \overline{V}_E(f) + \underline{V}_{E^c}(g) + \underline{V}_E(g) + \overline{V}_{E^c}(f) \\
&\qquad - \overline{V}_E(f) - \underline{V}_{E^c}(f) - \underline{V}_E(g) - \overline{V}_{E^c}(g) \\
&= \overline{V}_{E^c}(f) - \underline{V}_{E^c}(f) + \underline{V}_{E^c}(g) - \overline{V}_{E^c}(g) \\
&= \left(\frac{V(f_{\star E} f^\star) - V(f_\star)}{V(f^\star) - V(f_\star)} - \frac{V(f^\star) - V(f^\star_E f_\star)}{V(f^\star) - V(f_\star)} \right) V(f) \\
&\qquad - \left(\frac{V(f_{\star E} f^\star) - V(f_\star)}{V(f^\star) - V(f_\star)} - \frac{V(f^\star) - V(f^\star_E f_\star)}{V(f^\star) - V(f_\star)} \right) V(g) \\
&= k^E \left(V(f) - V(g) \right).
\end{aligned}$$

If $f \precsim g$,

$$\begin{aligned}
&V(f_E g) + V(g_E f) - V(f) - V(g) \\
&= \underline{V}_E(f) + \overline{V}_{E^c}(g) + \overline{V}_E(g) + \underline{V}_{E^c}(f) \\
&\qquad - \underline{V}_E(f) - \overline{V}_{E^c}(f) - \overline{V}_E(g) - \underline{V}_{E^c}(g) \\
&= \underline{V}_{E^c}(f) - \overline{V}_{E^c}(f) + \overline{V}_{E^c}(g) - \underline{V}_{E^c}(g) \\
&= k^E \left(V(g) - V(f) \right).
\end{aligned}$$

Case (b): $\trianglerighteq_{E^c}$ is a reverse order of $\trianglerighteq_E$.

Let $f^\star, f_\star \in \mathcal{R}$ be such that $V(f^\star_E f_\star) + V\left(f_{\star E} f^\star\right) \neq V(f^\star) + V(f_\star)$. Without loss of generality, suppose that $f^\star \succsim f_\star$, $f^\star \rhd_E f_\star$ and $f_\star \rhd_{E^c} f^\star$.

Consider $\overline{V}_E$, $\underline{V}_E$ the vNM utility functions representing $\trianglerighteq_E$ and $\overline{V}_{E^c}$, $\underline{V}_{E^c}$ the vNM utility functions representing $\trianglerighteq_{E^c}$ such that

- $\overline{V}_E\left(f^\star\right) = \overline{V}_{E^c}\left(f^\star\right) = V\left(f^\star\right)$;
- $\underline{V}_E\left(f^\star\right) = \underline{V}_{E^c}\left(f^\star\right) = 0$;
- $\overline{V}_E\left(f_\star\right) = V\left(f^\star\right) + V\left(f_\star\right) - V\left(f^\star_E f_\star\right)$;
- $\underline{V}_E\left(f_\star\right) = V(f_{\star E} f^\star) - V\left(f^\star\right)$;
- $\overline{V}_{E^c}\left(f_\star\right) = V\left(f^\star\right) + V\left(f_\star\right) - V(f_{\star E} f^\star)$;
- $\underline{V}_{E^c}\left(f_\star\right) = V\left(f^\star_E f_\star\right) - V\left(f^\star\right)$.

Note that it is possible to choose this normalization for these vNM utility functions since $f^\star \rhd_E f_\star$ and $f_\star \rhd_{E^c} f^\star$ and thus

$$V\left(f^\star_E f_\star\right) > V\left(f^\star\right), V\left(f_\star\right) > V(f_{\star E} f^\star),$$

which implies that $\overline{V}_E(f^\star) > \overline{V}_E(f_\star)$, $\underline{V}_E(f^\star) > \underline{V}_E(f_\star)$, $\overline{V}_{E^c}(f_\star) > \overline{V}_{E^c}(f^\star)$ and $\underline{V}_{E^c}(f_\star) > \underline{V}_{E^c}(f^\star)$.

Let us prove that for all $f, g \in \mathcal{R}$,

$$\begin{aligned} V(f_E g) &= \overline{V}_E(f) + \underline{V}_{E^c}(g) \text{ if } f \succsim g \\ &= \underline{V}_E(f) + \overline{V}_{E^c}(g) \text{ if } f \precsim g. \end{aligned}$$

Let $f, g \in \mathcal{R}$ such that $f \succsim g$. Consider a first case where $f^\star \trianglerighteq_E f \trianglerighteq_E f_\star$ and $f^\star \trianglerighteq_E g \trianglerighteq_E f_\star$. Then there exist a, $b \in (0,1)$ such that

$$\begin{aligned} f &\approx_E af^\star + (1-a)f_\star; \\ g &\approx_E bf^\star + (1-b)f_\star. \end{aligned}$$

Since $\trianglerighteq_{E^c}$ is a reverse order of $\trianglerighteq_E$, we also have that

$$\begin{aligned} f &\approx_{E^c} af^\star + (1-a)f_\star; \\ g &\approx_{E^c} bf^\star + (1-b)f_\star. \end{aligned}$$

Then, by definition of $\mathcal{R}$, $f \sim af^\star + (1-a)f_\star$ and $g \sim bf^\star + (1-b)f_\star$. Since $f \succsim g$ and $f^\star \succsim f_\star$, we get that $a \geq b$. Thus,

$$\begin{aligned} V(f_E g) &= V\left((af^\star + (1-a)f_\star)_E\,(bf^\star + (1-b)f_\star)\right) \\ &= bV(f^\star) + (a-b)\,V(f^\star_E f_\star) + (1-a)V(f_\star) \\ &= aV(f^\star) + (1-a)(V(f^\star) + V(f_\star) - V(f^\star_E f_\star)) + 0.b \\ &\qquad +(1-b)(V(f^\star_E f_\star) - V(f^\star)) \\ &= a\overline{V}_E(f^\star) + (1-a)\overline{V}_E(f_\star) + b\underline{V}_{E^c}(f^\star) + (1-b)\underline{V}_{E^c}(f_\star) \\ &= \overline{V}_E(af^\star + (1-a)f_\star) + \underline{V}_{E^c}(bf^\star + (1-b)f_\star) \\ &= \overline{V}_E(f) + \underline{V}_{E^c}(g)\,. \end{aligned}$$

Consider a second case where $f \trianglerighteq_E f^\star$ and $f_\star \trianglerighteq_E g$. Then, there exist a, $b \in (0,1)$ such that

$$f^\star \approx_E af + (1-a)g \text{ and } f_\star \approx_E bf + (1-b)g,$$

and

$$f^\star \approx_{E^c} af + (1-a)g \text{ and } f_\star \approx_{E^c} bf + (1-b)g,$$

and $f^\star \sim af + (1-a)g$ and $f_\star \sim bf + (1-b)g$. Thus $a > b$ and

$$\begin{aligned} V(f^\star_E f_\star) &= V\left((af+(1-a)g)_E\,(bf+(1-b)g)\right) \\ &= bV\,(f)+(a-b)\,V\,(f_E g)+(1-a)V\,(g)\,. \end{aligned}$$

Thus

$$V\,(f_E g) = \frac{V(f^\star_E f_\star) - bV\,(f) - (1-a)V\,(g)}{a-b}.$$

We also have

$$\begin{aligned} \overline{V}_E\,(f) &= \frac{(1-b)\overline{V}_E\,(f^\star) - (1-a)\overline{V}_E\,(f_\star)}{a-b}; \\ \underline{V}_{E^c}\,(g) &= \frac{b\underline{V}_{E^c}\,(f^\star) - a\underline{V}_{E^c}\,(f_\star)}{b-a}. \end{aligned}$$

and thus

$$\begin{aligned} &\overline{V}_E\,(f) + \underline{V}_{E^c}\,(g) \\ &\quad= \frac{(1-b)\overline{V}_E\,(f^\star) - (1-a)\overline{V}_E\,(f_\star) - b\underline{V}_{E^c}\,(f^\star) + a\underline{V}_{E^c}\,(f_\star)}{a-b} \\ &\quad= \frac{(1-b)\overline{V}_E\,(f^\star) - (1-a)\overline{V}_E\,(f_\star) - b\underline{V}_{E^c}\,(f^\star) + a\underline{V}_{E^c}\,(f_\star)}{a-b} \\ &\quad= \frac{(1-b)V\,(f^\star) - (1-a)\left(V\,(f^\star) + V\,(f_\star) - V\,(f^\star_E f_\star)\right)}{a-b} \\ &\qquad\qquad + \frac{a\left(V\,(f^\star_E f_\star) - V\,(f^\star)\right)}{a-b} \\ &\quad= \frac{V\,(f^\star_E f_\star) - bV\,(f^\star) - (1-a)V\,(f_\star)}{a-b} \\ &\quad= \frac{V\,(f^\star_E f_\star) - b\left(aV(f) + (1-a)V(g)\right)}{a-b} \\ &\qquad\qquad - \frac{(1-a)\left(bV(f) + (1-b)V(g)\right)}{a-b} \\ &\quad= \frac{V\,(f^\star_E f_\star) - bV(f) - (1-a)V(g)}{a-b}. \end{aligned}$$

which proves that $V\,(f_E g) = \overline{V}_E\,(f) + \underline{V}_{E^c}\,(g)$.

The proof can be adapted in the cases where $f \trianglerighteq_E f^\star$ and $g \trianglerighteq_E f^\star$ (or $f^\star \trianglerighteq_E g \trianglerighteq_E f_\star$), or $f^\star \trianglerighteq_E g \trianglerighteq_E f_\star$ and $f_\star \trianglerighteq_E g$, or $f_\star \trianglerighteq_E f$ and $f_\star \trianglerighteq_E g$.

Assume now that $f^\star \trianglerighteq_E f \trianglerighteq_E f_\star$ and $g \triangleright_E f^\star$. Then, there exist $a, b \in (0,1)$ such that

$$f \approx_E af^\star + (1-a)f_\star;$$
$$f^\star \approx_E bg + (1-b)f_\star.$$

Then we also have $f \approx_{E^c} af^\star + (1-a)f_\star$ and $f^\star \approx_{E^c} bg + (1-b)f_\star$, and thus, $f \sim af^\star + (1-a)f_\star$ and $f^\star \sim bg + (1-b)f_\star$, which yields a contradiction to the fact that $f \succsim g$.

We can prove that a similar contradiction occurs if we assume $f_\star \trianglerighteq_E f$ and $g \triangleright_E f_\star$.

Since $\overline{V}_E$, $\underline{V}_E$ are vNM representations of $\trianglerighteq_E$, $\overline{V}_{E^c}$, $\underline{V}_{E^c}$ are vNM representations of $\trianglerighteq_{E^c}$ and since they are two reverse orders, the uniqueness conditions imply that

- $\underline{V}_E = \frac{V(f^\star)-V(f_{\star E}f^\star)}{V\left(f^\star_E f_\star\right)-V(f_\star)}\left(\overline{V}_E - V\left(f^\star\right)\right);$
- $\overline{V}_{E^c} = \frac{V(f_\star)-V(f_{\star E}f^\star)}{V(f_\star)-V\left(f^\star_E f_\star\right)}\left(\overline{V}_E - V\left(f^\star\right)\right) + V\left(f^\star\right);$
- $\underline{V}_{E^c} = \frac{V\left(f^\star_E f_\star\right)-V(f^\star)}{V(f_\star)-V\left(f^\star_E f_\star\right)}\left(\overline{V}_E - V\left(f^\star\right)\right).$

Note that for all $f \in \mathcal{R}$, $V(f) = \frac{V(f^\star)-V(f_\star)}{V\left(f^\star_E f_\star\right)-V(f_\star)}\overline{V}_E(f) + \frac{V\left(f^\star_E f_\star\right)-V(f^\star)}{V\left(f^\star_E f_\star\right)-V(f_\star)}V\left(f^\star\right)$. Let's now check that the representation satisfies property 2.

If $f \succsim g$,

$$\begin{aligned}
V(f_E g) + V(g_E f) - V(f) - V(g) \\
&= \overline{V}_{E^c}(f) - \underline{V}_{E^c}(f) + \underline{V}_{E^c}(g) - \overline{V}_{E^c}(g) \\
&= \left(\frac{V\left(f_\star\right) - V\left(f_{\star E}f^\star\right)}{V\left(f_\star\right) - V\left(f^\star_E f_\star\right)} - \frac{V\left(f^\star_E f_\star\right) - V\left(f^\star\right)}{V\left(f_\star\right) - V\left(f^\star_E f_\star\right)}\right)\left(\overline{V}_E(f) - \overline{V}_E(g)\right) \\
&= \frac{V\left(f^\star\right) - V\left(f_\star\right)}{V\left(f^\star_E f_\star\right) - V\left(f_\star\right)}\left(\overline{V}_E(f) - \overline{V}_E(g)\right) \\
&= V(f) - V(g).
\end{aligned}$$

If $f \precsim g$,

$$\begin{aligned}
V(f_E g) + V(g_E f) - V(f) - V(g) &= \underline{V}_{E^c}(f) - \overline{V}_{E^c}(f) + \overline{V}_{E^c}(g) - \underline{V}_{E^c}(g) \\
&= V(g) - V(f).
\end{aligned}$$

Case 2.

For all $f, g \in \mathcal{R}$, $V(f_E g) + V(g_E f) - V(f) - V(g) = 0$.

If for all $f, g \in \mathcal{R}$, $f_E g \sim f$, then for $V_E = V$ and $V_{E^c} = 0$, we have that $V(f_E g) = V_E(f) + V_{E^c}(g)$ which proves that properties 1 and 2 hold.

Suppose now that there exist $f^\star, f_\star \in \mathcal{R}$ such that $f^\star_E f_\star \nsim f^\star$. Since $V(f^\star_E f_\star) + V(f_{\star E} f^\star) = V(f^\star) + V(f_\star)$, we can w.l.o.g restrict our attention to two cases: (a) $V(f^\star) > V(f^\star_E f_\star), V(f_{\star E} f^\star) > V(f_\star)$ and (b) $V(f^\star_E f_\star) > V(f^\star) > V(f_\star) > V(f_{\star E} f^\star)$.

In either case, consider V_E and V_{E^c} the vNM utility functions representing $\trianglerighteq_E$ and $\trianglerighteq_{E^c}$ such that $V_E(f^\star) = V(f^\star)$, $V_{E^c}(f^\star) = 0$, $V_E(f_\star) = V(f_{\star E} f^\star)$, $V_{E^c}(f_\star) = V(f^\star_E f_\star) - V(f^\star)$. Note that it is possible to choose this normalization for these vNM utility functions. Indeed, in case (a), we have $f^\star \vartriangleright_E f_\star$ and $f^\star \vartriangleright_{E^c} f_\star$ and the normalization proposed is such that $V_E(f^\star) > V_E(f_\star)$ and $V_{E^c}(f^\star) > V_{E^c}(f_\star)$, while in case (b), we have $f^\star \vartriangleright_E f_\star$ and $f_\star \vartriangleright_{E^c} f^\star$ and the normalization proposed is such that $V_E(f^\star) > V_E(f_\star)$ and $V_{E^c}(f^\star) < V_{E^c}(f_\star)$.

Let $f, g \in \mathcal{R}$ and consider a first case where $f^\star \trianglerighteq_E f \trianglerighteq_E f_\star$ and g is in between $f^\star$ and $f_\star$ according to $\trianglerighteq_{E^c}$. Then there exist $a, b^c \in (0,1)$ such that

$$\begin{aligned} f &\approx_E a f^\star + (1-a) f_\star; \\ g &\approx_{E^c} b^c f^\star + (1-b^c) f_\star. \end{aligned}$$

If $a \geq b^c$, then by definition of $\mathcal{R}$ and since $V(f^\star_E f_\star) + V(f_{\star E} f^\star) = V(f^\star) + V(f_\star)$,

$$\begin{aligned} V(f_E g) &= V\left((a f^\star + (1-a) f_\star)_E (b^c f^\star + (1-b^c) f_\star)\right) \\ &= b^c V(f^\star) + (a - b^c) V(f^\star_E f_\star) + (1-a) V(f_\star) \\ &= a V(f^\star) + (1-a)(V(f^\star) + V(f_\star) - V(f^\star_E f_\star)) + 0.b^c \\ &\qquad + (1-b^c)(V(f^\star_E f_\star) - V(f^\star)) \\ &= a V(f^\star) + (1-a) V(f_{\star E} f^\star) + 0.b^c + (1-b^c)(V(f^\star_E f_\star) - V(f^\star)) \\ &= a V_E(f^\star) + (1-a) V_E(f_\star) + b^c V_{E^c}(f^\star) + (1-b^c) V_{E^c}(f_\star) \\ &= V_E(a f^\star + (1-a) f_\star) + V_{E^c}(b^c f^\star + (1-b^c) f_\star) \\ &= V_E(f) + V_{E^c}(g). \end{aligned}$$

If $b^c \geq a$, then by definition of $\mathcal{R}$ and since $V(f^\star_E f_\star) + V(f_{\star E} f^\star) = V(f^\star) + V(f_\star)$,

$$
\begin{aligned}
V(f_E g) &= V\left((af^\star + (1-a)f_\star)_E\,(b^c f^\star + (1-b^c)f_\star)\right)\\
&= aV\left(f^\star\right) + \left(b^c - a\right) V(f_{\star E} f^\star) + (1-b^c)V\left(f_\star\right)\\
&= aV\left(f^\star\right) + (1-a)V(f_{\star E} f^\star) + 0.b^c + (1-b^c)(V\left(f_\star\right) - V(f_{\star E} f^\star))\\
&= aV\left(f^\star\right) + (1-a)V(f_{\star E} f^\star) + 0.b^c + (1-b^c)(V\left(f^\star_E f_\star\right) - V\left(f^\star\right))\\
&= aV_E\left(f^\star\right) + (1-a)V_E\left(f_\star\right) + b^c V_{E^c}\left(f^\star\right) + (1-b^c)V_{E^c}\left(f_\star\right)\\
&= V_E\left(f\right) + V_{E^c}\left(g\right).
\end{aligned}
$$

Consider now a second case, where $f \trianglerighteq_E f^\star$ and g is in between $f^\star$ and $f_\star$ according to $\trianglerighteq_{E^c}$. Then there exist $a, b^c \in (0,1)$ such that

$$
\begin{aligned}
f^\star &\approx_E af + (1-a)f_\star;\\
g &\approx_{E^c} b^c f^\star + (1-b^c)f_\star.
\end{aligned}
$$

Therefore by definition of $\mathcal{R}$,

$$
\begin{aligned}
& V(f^\star_E g) = V\left((af + (1-a)f_\star)_E\, g\right)\\
\Leftrightarrow\; & V(f^\star_E\,(b^c f^\star + (1-b^c)f_\star)) = aV(f_E g) + (1-a)V(f_{\star E} g)\\
\Leftrightarrow\; & b^c V(f^\star) + (1-b^c)V\left(f^\star_E f_\star\right) = aV(f_E g)\\
& \qquad + (1-a)\left(b^c V(f_{\star E} f^\star) + (1-b^c)V\left(f_\star\right)\right)\\
\Leftrightarrow\; & V(f_E g) = \frac{b^c V(f^\star) + (1-b^c)V\left(f^\star_E f_\star\right)}{a}\\
& \qquad - \frac{(1-a)\left(b^c V(f_{\star E} f^\star) + (1-b^c)V\left(f_\star\right)\right)}{a}.
\end{aligned}
$$

Using the fact that $V\left(f_\star\right) = V(f_{\star E} f^\star) + V\left(f^\star_E f_\star\right) - V(f^\star)$, we get that

$$
V(f_E g) = \frac{(1-a+ab^c)V\left(f^\star\right) - (1-a)V(f_{\star E} f^\star) + a(1-b^c)V\left(f^\star_E f_\star\right)}{a}.
$$

We also have that

$$
\begin{aligned}
V_E\left(f\right) &= \frac{V_E\left(f^\star\right) - (1-a)V_E\left(f_\star\right)}{a};\\
V_{E^c}\left(g\right) &= b^c V_{E^c}\left(f^\star\right) + (1-b^c)V_{E^c}\left(f_\star\right).
\end{aligned}
$$

Thus

$$
\begin{aligned}
&V_E(f) + V_{E^c}(g) \\
&\quad = \frac{V_E(f^\star) - (1-a)V_E(f_\star) + a\left(b^c V_{E^c}(f^\star) + (1-b^c)V_{E^c}(f_\star)\right)}{a} \\
&\quad = \frac{V(f^\star) - (1-a)V(f_{\star E}f^\star) + a(1-b^c)\left(V(f^\star_E f_\star) - V(f^\star)\right)}{a} \\
&\quad = \frac{(1-a+ab^c)V(f^\star) - (1-a)V(f_{\star E}f^\star) + a(1-b^c)V(f^\star_E f_\star)}{a},
\end{aligned}
$$

and therefore $V(f_E g) = V_E(f) + V_{E^c}(g)$.

In the other cases the proof can be adapted to show that $V(f_E g) = V_E(f) + V_{E^c}(g)$.

Finally, remark that property 2 is satisfied with $k^E = 0$.

B2. Proof of Proposition 2

Suppose $\succsim$ is uncertainty neutral on E with respect to $\mathcal{R}$. Let us prove that for all $f, g \in \mathcal{R}$, $V(f_E g) + V(g_E f) = V(f) + V(g)$ and thus that $k^E = 0$.

Let $f, g \in \mathcal{R}$ and assume first that $f \sim g$.

If $f \trianglerighteq_E g$ and $f \trianglerighteq_{E^c} g$, then $f \succsim f_E g$, $g_E f \succsim g$ and thus $f \sim f_E g \sim g_E f \sim g$. Therefore, $V(f_E g) + V(g_E f) = V(f) + V(g)$.

If $f \trianglerighteq_E g$ and $f \vartriangleleft_{E^c} g$, then $f_E g \succsim f \sim g \succsim g_E f$. If $f \sim f_E g \sim g_E f \sim g$ then $V(f_E g) + V(g_E f) = V(f) + V(g)$. However, w.l.o.g let us suppose that $f_E g \succ f$. Since $\succsim$ is not degenerate on $\mathcal{R}$, there exists $h \in \mathcal{R}$ such that $h \nsim f$. Suppose $h \succ f$ and w.l.o.g, suppose that $f_E g \succ h \succ f \sim g \succsim g_E f$. Then

$$\frac{1}{2}f + \frac{1}{2}h \sim a f_E g + (1-a)f \sim b g_E f + (1-b)h,$$

where $a = \frac{1}{2}\frac{V(h)-V(f)}{V(f_E g)-V(f)}$ and $b = \frac{1}{2}\frac{V(h)-V(f)}{V(h)-V(g_E f)}$. Since

$$\frac{1}{2}f + \frac{1}{2}h \sim f_E\left(ag + (1-a)f\right) \sim \left(bg + (1-b)h\right)_E \left(bf + (1-b)h\right),$$

and $\succsim$ is uncertainty neutral on E,

$$
\begin{aligned}
&\left(\frac{b}{a+b}f + \frac{a}{a+b}\left(bg+(1-b)h\right)\right)_E \\
&\qquad\qquad\left(\frac{b}{a+b}\left(ag+(1-a)f\right) + \frac{a}{a+b}\left(bf+(1-b)h\right)\right) \\
&\sim f_E\left(ag+(1-a)f\right) \\
&\sim \frac{1}{2}f + \frac{1}{2}h.
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\left(\frac{b}{a+b}f + \frac{a}{a+b}\left(bg+(1-b)h\right)\right)_E \\
&\qquad\qquad\left(\frac{b}{a+b}\left(ag+(1-a)f\right) + \frac{a}{a+b}\left(bf+(1-b)h\right)\right) \\
&\sim \frac{(1+a)b}{a+b}\left(\frac{1}{1+a}f + \frac{a}{1+a}g\right) + \frac{a(1-b)}{a+b}h \\
&\sim \frac{(1+a)b}{a+b}f + \frac{a(1-b)}{a+b}h.
\end{aligned}
$$

Thus we have that

$$
\frac{(1+a)b}{a+b}V(f) + \frac{a(1-b)}{a+b}V(h) = \frac{1}{2}V(f) + \frac{1}{2}V(h),
$$

which is equivalent to

$$
\begin{aligned}
&\frac{1}{4}\left(\frac{2V(f_Eg)+V(h)-3V(f)}{V(f_Eg)-V(f)}\right)\left(\frac{V(h)-V(f)}{V(h)-V(g_Ef)}\right)V(f) \\
&\qquad + \frac{1}{4}\left(\frac{V(h)-V(f)}{V(f_Eg)-V(f)}\right)\left(\frac{V(h)+V(f)-2V(g_Ef)}{V(h)-V(g_Ef)}\right)V(h) \\
&\qquad\qquad = \frac{1}{4}\left(\frac{V(h)-V(f)}{V(f_Eg)-V(f)} + \frac{V(h)-V(f)}{V(h)-V(g_Ef)}\right)\left(V(f)+V(h)\right),
\end{aligned}
$$

equivalent to

$$
\begin{aligned}
&\left(2V(f_Eg)+V(h)-3V(f)\right)V(f) + \left(V(h)+V(f)-2V(g_Ef)\right)V(h) \\
&\qquad = \left(V(h)-V(g_Ef)+V(f_Eg)-V(f)\right)\left(V(f)+V(h)\right),
\end{aligned}
$$

and finally to

$$
\left(2V(f)-V(f_Eg)-V(g_Ef)\right)\left(V(h)-V(f)\right) = 0.
$$

Since $V(h) > V(f)$, we must have $V(f_Eg) + V(g_Ef) = 2V(f) = V(f) + V(g)$.

The proof is similar for the other cases ($f \succ h$ or $f \lhd_E g$ and $f \unrhd_{E^c} g$).

Suppose now that $f \succ g$ and consider a first case where $f \unrhd_E g$ and $f \unrhd_{E^c} g$ and thus $f \succsim f_E g$, $g_E f \succsim g$. First note that if $f \sim f_E g$, then $g_E f \sim g$ and thus $V(f_E g) + V(g_E f) = V(f) + V(g)$.

If $f \succ f_E g \succsim g_E f$, then $f_E g \sim (af + (1-a)g)_E f$ where $a = \frac{V(f_E g) - V(g_E f)}{V(f) - V(g_E f)}$. Since $\succsim$ is uncertainty neutral on E,

$$\left(\frac{1-a}{2-a}f + (1 - \frac{1-a}{2-a})\,(af + (1-a)g)\right)_E \left(\frac{1-a}{2-a}g + (1 - \frac{1-a}{2-a})f\right) \sim f_E g.$$

Note that

$$\begin{aligned}\left(\frac{1-a}{2-a}f + (1 - \frac{1-a}{2-a})\,(af + (1-a)g)\right)_E &\left(\frac{1-a}{2-a}g + (1 - \frac{1-a}{2-a})f\right) \\ &= \frac{1}{2-a}f + \frac{1-a}{2-a}g.\end{aligned}$$

We also have $f_E g \sim bf + (1-b)g$ where $b = \frac{V(f_E g) - V(g)}{V(f) - V(g)}$. Since $f \succ g$, $b = \frac{1}{2-a}$; this is equivalent to

$$\begin{aligned}
& 2 - \frac{V(f_E g) - V(g_E f)}{V(f) - V(g_E f)} = \frac{V(f) - V(g)}{V(f_E g) - V(g)} \\
\Leftrightarrow\ & (2V(f) - V(g_E f) - V(f_E g))(V(f_E g) - V(g)) \\
&\qquad = (V(f) - V(g_E f))\,(V(f) - V(g)) \\
\Leftrightarrow\ & -V(f)V(g) + 2V(f)V(f_E g) - V(g_E f)V(f_E g) + V(g_E f)V(f) \\
&\qquad -V(f_E g)V(f_E g) + V(f_E g)V(g) - V(f)V(f) = 0 \\
\Leftrightarrow\ & (V(f) - V(f_E g))(-V(f) - V(g) + V(g_E f) + V(f_E g)) = 0.
\end{aligned}$$

Since $f \succ f_E g$, therefore $V(f_E g) + V(g_E f) = V(f) + V(g)$. The proof is similar in the case where $f \succ g_E f \succsim f_E g$.

Conversely, suppose that $k^E = 0$. Consider the utility functions $\overline{V}_E$, $\underline{V}_E$, $\overline{V}_{E^c}$ and $\underline{V}_{E^c}$. As shown in the proof of Proposition 1 these functions are linear with respect to mixture on $\mathcal{R}$. Note that $k^E = 0$ implies that for all $f, g \in \mathcal{R}$, $\overline{V}_E(f) + \underline{V}_{E^c}(g) = \underline{V}_E(f) + \overline{V}_{E^c}(g)$.

Let consider $f, g, h, \ell \in \mathcal{R}$ such that $f_E g \sim h_E \ell$ and $\alpha \in (0, 1)$.

$$
\begin{aligned}
&V((\alpha f+(1-\alpha)h)_E\,(\alpha g+(1-\alpha)\ell))\\
=&\overline{V}_E(\alpha f+(1-\alpha)h)+\underline{V}_{E^c}(\alpha g+(1-\alpha)\ell)\\
=&\alpha\left(\overline{V}_E(f\right)+\underline{V}_{E^c}(g))+(1-\alpha)\left(\overline{V}_E(h\right)+\underline{V}_{E^c}(\ell))\\
=&\alpha V(f_E g)+(1-\alpha)V(h_E\ell),
\end{aligned}
$$

and thus $(\alpha f+(1-\alpha)h)_E\,(\alpha g+(1-\alpha)\ell)\sim f_E g$.

B3. Proof of Proposition 3

Observe first that, since $\succsim_i$ and $\succsim_j$ satisfy the assumptions of Proposition 1, they have a representation as in that proposition. Hence, for f and g such that $V_i(f)>V_i(g)$, it is the case that $k_i^E=\frac{V_i(f_E g)-V_i(g)}{V_i(f)-V_i(g)}-\frac{V_i(f)-V_i(g_E f)}{V_i(f)-V_i(g)}$, and similarly for k_j^E.

Let $f,g\in\mathcal{R}_i$, $f',g'\in\mathcal{R}_j$ such that $f\succ_i g$ and $f'\succ_j g'$.

Consider first the combination of case (i) and (i'), i.e., $f_E g\sim_i \alpha f+(1-\alpha)g$ and $f'_E g'\sim_j \alpha f'+(1-\alpha)g'$, on the one hand and $g_E f\sim_i \beta g+(1-\beta)f$ and $g'_E f'\sim_j \beta g'+(1-\beta)f'$ on the other hand.

Then, using the representation, (i) implies that

$$\alpha=\frac{V_i(f_E g)-V_i(g)}{V_i(f)-V_i(g)}=\frac{V_j(f'_E g')-V_j(g')}{V_j(f')-V_j(g')},$$

while (i') implies that

$$\beta=\frac{V_i(g_E f)-V_i(f)}{V_i(g)-V_i(f)}=\frac{V_j(g'_E f')-V_j(f')}{V_j(g')-V_j(f')}.$$

Hence, $k_i^E=\alpha-\beta=k_j^E$.

Consider next case (i) and (ii'). Then, $\frac{V_i(g_E f)-V_i(f)}{V_i(g)-V_i(f)}=\frac{V_j(g'_E f')-V_j(f')}{V_j(g')-V_j(f')}=\frac{1}{\beta}$ and hence $k_i^E=\alpha-\frac{1}{\beta}=k_j^E$.

The other cases can be dealt with in a similar fashion.

Appendix C

C1. Proof of Theorem 1

In this Appendix, we provide the proof of our main result. We decompose the proof into 4 lemmas. Although not always explicitly stated in the lemma, all the assumptions of Theorem 1 are made throughout this Appendix. The following Lemma is adapted from Weymark (1993, Lemma 1):

Lemma 1 *Let $(V_i)_{i\in N}$ be a collection of $\mathcal{R}_i$-affine representation of $\succsim_i$ for all $i \in N$ and assume conditions 1, 2, 3 of Theorem 1 are satisfied. Then, $(V_1, \cdots, V_n)$ are affinely independent on $\cap_{i\in N}\mathcal{R}_i$.*

Proof. Suppose on the contrary that $(V_1, \cdots, V_n)$ are affinely dependent on $\cap_{i\in N}\mathcal{R}_i$, that is, there exists $(\lambda_1, \cdots, \lambda_n) \in \mathbb{R}^n$ and $\mu \in \mathbb{R}$ such that $\sum_{i=1}^n \lambda_i V_i(f) + \mu = 0$ for all $f \in \cap_{i\in N}\mathcal{R}_i$ with at least one $\lambda_j \neq 0$. Without loss of generality, assume that $\lambda_1 = -1$. We then have:

$$V_1(f) = \sum_{i\neq 1} \lambda_i V_i(f) + \mu, \ \forall f \in \cap_{i\in N}\mathcal{R}_i. \tag{2}$$

Let f and g in $\cap_{i\in N}\mathcal{R}_i$ be such that $f \sim_i g$ for all $i \neq 1$ and $f \succ_1 g$ (such acts exist, since $\{\succsim_i\}_{i\in N'}$ satisfy the independent prospects property on $\cap_i\mathcal{R}_i$). But equation (2) implies that $V_1(f) = V_1(g)$, a contradiction. ■

Lemma 2 *There exist $\bar{f}, \underline{f} \in \cap_{i\in N}\mathcal{R}_i$ such that $\bar{f} \succ_i \underline{f}$ for all $i \in N'$.*

Proof. For all $i \in N'$, let $\bar{f}_i, \underline{f}_i \in \cap_{i\in N}\mathcal{R}_i$ be such that $\bar{f}_i \succ_i \underline{f}_i$ and $\bar{f}_i \sim_j \underline{f}_i$ for all $j \neq i$ (such acts exist since $\{\succsim_i\}_{i\in N'}$ satisfy the independent prospects property). Consider $\alpha_j \in]0, 1[$ for $j = 2, .., n$ and define recursively $\bar{f}^j, \underline{f}^j$ by

- $\bar{f}^2 = \alpha_2\bar{f}_1 + (1 - \alpha_2)\bar{f}_2$, $\underline{f}^2 = \alpha_2\underline{f}_1 + (1 - \alpha_2)\underline{f}_2$
- for $j = 3, .., n$, $\bar{f}^j = \alpha_j\bar{f}^{j-1} + (1 - \alpha_j)\bar{f}_j$, $\underline{f}^j = \alpha_j\underline{f}^{j-1} + (1 - \alpha_j)\underline{f}_j$.

Since $\cap_{i\in N}\mathcal{R}_i$ is a mixture space, $\bar{f}^n, \underline{f}^n \in \cap_{i\in N}\mathcal{R}_i$ and it can be checked that $\bar{f}^n \succ_i \underline{f}^n$ for all $i \in N'$. ■

Lemma 3 *Let $E \in \Sigma$. Let $(V_i)_{i\in N}$ be a collection of $\mathcal{R}_i$-affine representation of $\succsim_i$ for all $i \in N$ and assume conditions 1, 2, 3 of Theorem 1 are satisfied. There exist unique weights $(\lambda_1, \cdots, \lambda_n) \in \mathbb{R}^n_+ \setminus \{0\}$, $\mu \in \mathbb{R}$, such that*

$$\forall f \in \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E), V_0(f) = \sum_{i\in N'} \lambda_i V_i(f) + \mu.$$

Proof. Define $F : \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E) \to \mathbb{R}^{n+1}$ by $F(f) = (V_0(f), V_1(f), \cdots, V_n(f))$ and let $K_f = co\{f, \cap_{i\in N}\mathcal{R}_i\}$ for all $f \in \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E)$. Clearly, for all $f \in \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E)$, K_f is a convex set, $\cap_{i\in N}\mathcal{R}_i \subseteq K_f$, and $\bigcup_{f\in\mathcal{B}(\cap_{i\in N}\mathcal{R}_i,E)} K_f = \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E)$.

We first prove that $F(K_f)$ is convex for all $f \in \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E)$. Let f be fixed, and consider $g_1, g_2 \in K_f$, with $g_1 \neq g_2$. Let $\gamma = tF(g_1) + (1-t)F(g_2)$, with $t \in (0,1)$. By definition, there exist $\alpha_1, \alpha_2 \in [0,1]$, and $h_1, h_2 \in \cap_{i\in N'}\mathcal{R}_i$ such that $g_1 = \alpha_1 f + (1-\alpha_1)h_1$ and $g_2 = \alpha_2 f + (1-\alpha_2)h_2$. Let $g_3 = tg_1 + (1-t)g_2$. Let $h_3 = \frac{t(1-\alpha_1)}{t(1-\alpha_1)+(1-t)(1-\alpha_2)}h_1 + \frac{(1-t)(1-\alpha_2)}{t(1-\alpha_1)+(1-t)(1-\alpha_2)}h_2$ [12]. It is easy to see that $g_3 = [t\alpha_1 + (1-t)\alpha_2]f + [1-(t\alpha_1+(1-t)\alpha_2)]h_3$. Note that $\cap_{i\in N}\mathcal{R}_i$ is a mixture set and thus $h_3 \in K_f$.

We hence have, by affinity of the V_i

$$\begin{aligned}
V_i(g_3) &= [t\alpha_1 + (1-t)\alpha_2]V_i(f) + [1-(t\alpha_1+(1-t)\alpha_2)]V_i(h_3) \\
&= [t\alpha_1 + (1-t)\alpha_2]V_i(f) + \\
&\quad [1-(t\alpha_1+(1-t)\alpha_2)]\left[\frac{t(1-\alpha_1)}{t(1-\alpha_1)+(1-t)(1-\alpha_2)}V_i(h_1)\right. \\
&\quad \left. + \frac{(1-t)(1-\alpha_2)}{t(1-\alpha_1)+(1-t)(1-\alpha_2)}V_i(h_2)\right] \\
&= t[\alpha_1 V_i(f) + (1-\alpha_1)V_i(h_1)] + (1-t)[\alpha_2 V_i(f) + (1-\alpha_2)V_i(h_2)] \\
&= tV_i(\alpha_1 f + (1-\alpha_1)h_1) + (1-t)V_i(\alpha_2 f + (1-\alpha_2)h_2) \\
&= tV_i(g_1) + (1-t)V_i(g_2).
\end{aligned}$$

Hence $F(g_3) = \gamma$, which proves that $F(K_f)$ is convex.

By Proposition 2 in De Meyer and Mongin (1995), the convexity of $F(K_f)$, axiom 2 and the existence of two acts f, g such that $f \succ_i g$ for all $i \in N'$ imply that there exist non-negative numbers $\lambda_1(f), \cdots, \lambda_n(f)$, not all equal to zero, and a real number $\mu(f)$ such that, for all $g \in K_f$,

$$V_0(g) = \sum_{i=1}^{n} \lambda_i(f)V_i(g) + \mu(f).$$

Now, consider f_1 and f_2 in $\mathcal{B}(\cap_{i\in N}\mathcal{R}_i)$. Since $\cap_{i\in N}\mathcal{R}_i \subseteq K_{f_1} \cap K_{f_2}$, for all act $h \in \cap_{i\in N'}\mathcal{R}_i$, we have:

$$\begin{cases} V_0(h) = \sum_{i=1}^{n} \lambda_i(f_1)V_i(h) + \mu(f_1) \\ V_0(h) = \sum_{i=1}^{n} \lambda_i(f_2)V_i(h) + \mu(f_2). \end{cases}$$

[12] Since $g_1 \neq g_2$, $\alpha_1 \neq \alpha_2$, and therefore $t(1-\alpha_1) + (1-t)(1-\alpha_2) \neq 0$.

This implies that for all $h \in \cap_{i\in N'}\mathcal{R}_i$, $\sum_{i=1}^n[\lambda_i(f_1) - \lambda_i(f_2)]u_i(h) + [\mu(f_1) - \mu(f_2)] = 0$. Since by lemma 1, the V_i are affinely independent on $\cap_{i\in N'}\mathcal{R}_i$, $\lambda_i(f_1) = \lambda_i(f_2)$ $i \in N'$ and $\mu(f_1) = \mu(f_2)$. Therefore, there exist n non-negative numbers, not all equal to zero, $(\lambda_1, \cdots, \lambda_n)$ and a number μ, such that for all $f \in \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E)$,

$$V_0(f) = \sum_{i=1}^n \lambda_i V_i(f) + \mu.$$

Finally, it remains to show that the weights $(\lambda_1, \cdots, \lambda_n)$ and μ are unique. Since the $\{\succsim_i\}_{i\in N'}$ satisfy the independent prospects property, there exist for all $i \in N'$ $h_i^\star$, $h_{i\star}$ in $\cap_{i\in N}\mathcal{R}_i$ such that

$$\begin{cases} h_i^\star \succ_i h_{i\star} \\ h_i^\star \sim_j h_{i\star}, \forall j \in N' \setminus \{i\}. \end{cases}$$

We have $V_0(h_i^\star) - V_0(h_{i\star}) = \lambda_i\left(V_i(h_i^\star) - V_i(h_{i\star})\right)$ and thus λ_i is unique. This is true for all $i \in N'$. But since $(\lambda_1, \cdots, \lambda_n)$ are unique, so is μ. ■

Lemma 4 *Let $(V_i)_{i\in N}$ be a collection of $\mathcal{R}_i$-affine representation of $\succsim_i$ for all $i \in N$ and assume conditions 1, 2, 3 of Theorem 1 are satisfied. Let the weights $(\lambda_1, \cdots, \lambda_n) \in \mathbb{R}^n_+ \setminus \{0\}$, $\mu \in \mathbb{R}$, be such that*

$$\forall f \in \mathcal{B}(\cap_{i\in N}\mathcal{R}_i, E), V_0(f) = \sum_{i\in N'} \lambda_i V_i(f) + \mu.$$

If there exist $i, j \in N'$ such that $\lambda_i, \lambda_j > 0$, then these two agents have uncertainty neutral preferences on E.

Proof. First, remark that for any $i \in N'$ such that $\lambda_i > 0$, $k_i^E = k_0^E$. Indeed, since the $\{\succsim_i\}_{i\in N'}$ satisfy the independent prospects property, there exist $h^\star$, $h_\star$ in $\cap_{i\in N'}\mathcal{R}_i$ such that

$$\begin{cases} h^\star \succ_i h_\star \\ h^\star \sim_j h_\star, \forall j \in N' \setminus \{i\}. \end{cases}$$

We have that

$$\begin{aligned} V_0(h^\star_E h_\star) + V_0(h_{\star E} h^\star) - (V_0(h^\star) + V_0(h_\star)) &= k_0^E(V_0(h^\star) - V_0(h_\star)) \\ &= k_0^E \lambda_i (V_i(h^\star) - V_i(h_\star)), \end{aligned}$$

but also

$$\begin{aligned} V_0(h^\star_E h_\star) + V_0(h_{\star E} h^\star) - (V_0(h^\star) + V_0(h_\star)) \\ &= \lambda_i (V_i(h^\star_E h_\star) + V_i(h_{\star E} h^\star) - (V_i(h^\star) + V_i(h_\star))) \\ &= k_i^E \lambda_i (V_i(h^\star) - V_i(h_\star)), \end{aligned}$$

and thus $k_0^E = k_i^E$.

Suppose now that there exist $i, j \in N'$ such that $\lambda_i, \lambda_j > 0$. Consider $h^\star_i$, $h_{i\star}, h^\star_j$, $h_{j\star}$ in $\cap_{i \in N'} \mathcal{R}_i$ such that

$$\begin{cases} h^\star_i \succ_i h_{i\star} \\ h^\star_i \sim_h h_{i\star}, \forall h \in N' \setminus \{i\}, \end{cases}$$

and

$$\begin{cases} h^\star_j \succ_j h_{j\star} \\ h^\star_j \sim_h h_{j\star}, \forall h \in N' \setminus \{j\}. \end{cases}$$

Note that for $\alpha = \frac{V_0(h^\star_j) - V_0(h_{j\star})}{V_0(h^\star_i) - V_0(h_{i\star}) + V_0(h^\star_j) - V_0(h_{j\star})} \in [0,1]$, we have

$$V_0\left(\alpha h^\star_i + (1-\alpha) h_{j\star}\right) = V_0\left(\alpha h_{i\star} + (1-\alpha) h^\star_j\right).$$

We also have that

$$V_i\left(\alpha h^\star_i + (1-\alpha) h_{j\star}\right) > V_i\left(\alpha h_{i\star} + (1-\alpha) h^\star_j\right)$$

and

$$V_j\left(\alpha h^\star_i + (1-\alpha) h_{j\star}\right) < V_j\left(\alpha h_{i\star} + (1-\alpha) h^\star_j\right).$$

Thus,

$$\begin{aligned} V_0\left((\alpha h^\star_i + (1-\alpha) h_{j\star})_E \left(\alpha h_{i\star} + (1-\alpha) h^\star_j\right)\right) \\ + V_0\left(\left(\alpha h_{i\star} + (1-\alpha) h^\star_j\right)_E (\alpha h^\star_i + (1-\alpha) h_{j\star})\right) \\ - \left(V_0(\alpha h^\star_i + (1-\alpha) h_{j\star}) + V_0(\alpha h_{i\star} + (1-\alpha) h^\star_j)\right) = 0, \end{aligned}$$

but it must also be the case that

$$
\begin{aligned}
&V_0\left((\alpha h_i^\star+(1-\alpha)h_{j\star})_E\left(\alpha h_{i\star}+(1-\alpha)h_j^\star\right)\right)\\
&\qquad\qquad +V_0\left(\left(\alpha h_{i\star}+(1-\alpha)h_j^\star\right)_E(\alpha h_i^\star+(1-\alpha)h_{j\star})\right)\\
&\qquad\qquad\qquad\qquad \left(V_0(\alpha h_i^\star+(1-\alpha)h_{j\star})+V_0(\alpha h_{i\star}+(1-\alpha)h_j^\star)\right)\\
&=\lambda_i k_i^E\left[V_i\left(\alpha h_i^\star+(1-\alpha)h_{j\star}\right)-V_i\left(\alpha h_{i\star}+(1-\alpha)h_j^\star\right)\right]\\
&\qquad\qquad +\lambda_j k_j^E\left[V_j\left(\alpha h_{i\star}+(1-\alpha)h_j^\star\right)-V_j\left(\alpha h_i^\star+(1-\alpha)h_{j\star}\right)\right]\\
&=k_0^E\left[\lambda_i\left[V_i(\alpha h_i^\star+(1-\alpha)h_{j\star})-V_i(\alpha h_{i\star}+(1-\alpha)h_j^\star)\right]\right.\\
&\qquad\qquad \left.+\lambda_j\left[V_j(\alpha h_{i\star}+(1-\alpha)h_j^\star)-V_j(\alpha h_i^\star+(1-\alpha)h_{j\star})\right]\right].
\end{aligned}
$$

Since

$$
\begin{aligned}
&[\lambda_i\left[V_i\left(\alpha h_i^\star+(1-\alpha)h_{j\star}\right)-V_i\left(\alpha h_{i\star}+(1-\alpha)h_j^\star\right)\right]\\
&\qquad\qquad +\lambda_j\left[V_j\left(\alpha h_{i\star}+(1-\alpha)h_j^\star\right)-V_j\left(\alpha h_i^\star+(1-\alpha)h_{j\star}\right)\right]]>0,
\end{aligned}
$$

we must have $k_0^E=k_i^E=k_j^E=0$. ■

C2. Proof of Proposition 4

Follows from lemma 1 to 3.

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