A NOTE ON INDETERMINACY IN OVERLAPPING GENERATIONS ECONOMIES WITH ENVIRONMENT AND ENDOGENOUS LABOR SUPPLY[1]

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[1]We would like to thank an associate editor and two anonymous referees for helpful comments and suggestions. This paper also benefited from comments of participants to CEPET workshop at Udine June 2004, XIXth EEA Annual Congress August 2004, Journée à Générations Imbriquées at La Rochelle November 2004, and T2M Conference at Lyon January 2005. All remaining errors are of our own responsibility.

INDETERMINACY IN OLG WITH ENVIRONMENT

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Abstract

We consider an overlapping generations model with environment and an elastic labor supply. In this framework, consumers have to choose between consumption, environmental quality, and leisure. We show the existence of both deterministic cycles and indeterminacy. In contrast to previous results, the emergence of endogenous fluctuations does not require a high emission rate of pollution.

JEL classification: C62, E32, Q50.

Keywords: Environment, labor supply, overlapping generations, indeterminacy, endogenous cycles.

1 INTRODUCTION

The link between environment and economic dynamics has been studied in a lot of contributions.[1] In this literature, overlapping generations models have been used in order to clearly analyze intergenerational problems (John and Pecchenino (1994), John et al. (1995), Pezzey and Toman (2002)). However, most of these papers deal with the question of sustainability. This is why they are only interested in steady state analysis and monotonic convergence.

Recently, some papers have enlightened that more complex dynamics can emerge when one considers the interaction between environment and economic activity. Indeed, considering overlapping generations models where consumers have to choose between consumption and abatement, Seegmuller and Verchère (2004) and Zhang (1999) have shown the existence of deterministic cycles and chaos.

However, in these contributions, labor supply is inelastic and indeterminacy is ruled out. Nevertheless, elastic labor supply and the consumer choice between leisure and consumption often play key roles on the emergence of endogenous fluctuations.[2] In this paper, we introduce an elastic labor supply in an overlapping generations economy with environment. Then, consumers have to choose between consumption, environmental quality, and leisure. Moreover, they share their labor income between savings and abatement, because pollution negatively affects their preferences.

Studying local dynamics, we prove that a flip bifurcation as well as local indeterminacy can occur. It means that not only deterministic cycles can appear, but also endogenous fluctuations due to self-fulfilling expectations. Moreover, these fluctuations occur under weaker conditions than previous existing works. Indeed, contrary to several contributions, the existence of indeterminacy and cycles needs neither a weak substitution between production factors (Grandmont et al. (1998), Reichlin (1986)), nor increasing returns or imperfect competition (see Benhabib and Farmer (1999) for a survey). We can further notice that, in contrast to Seegmuller and Verchère (2004) and Zhang (1999), the occurrence of such fluctuations does not require a high level of the pollution emission rate. Consequently, this paper shows more generally that non monotonic dynamics can easily occur in economies where an environmental dimension is taking into account.

The paper is organized as follows. In the next section, we present the model. In section 3, we establish the existence of a steady state. Finally, in section 4, we study local dynamics and provide interpretations of our findings.

2 THE MODEL

We consider a perfectly competitive overlapping generations model with discrete time $t = 1, 2, ..., \infty$ and a constant population size normalized to one. A generation of consumers is born in each period and households live two periods. When young, the representative consumer supplies labor l_t, which is remunerated at the real wage w_t. He shares his wage earnings between savings, through the purchase of aggregate capital k_t, and environmental maintenance d_t.[3] When old, he rents capital to the firms, earns the real interest factor r_{t+1} and consumes the final good c_{t+1}.[4] The two budget constraints faced by the consumer can be written:

$$k_t + d_t = w_t l_t \tag{1}$$

$$c_{t+1} = r_{t+1} k_t \tag{2}$$

In this economy, the environmental quality decreases with respect to the pollution. At period $t+1$, the level of pollution is given by:

$$P_{t+1} = \alpha k_{t-1} - d_t \equiv P(k_{t-1}, d_t) \text{ , with } \alpha > 0 \tag{3}$$

The pollution, which is always strictly positive, linearly increases with respect to the capital stock inherited from the previous period and is a decreasing function of the environmental maintenance.[5]

Consumers derive utility from consumption, leisure and environmental quality. Assuming additively separable preferences, the utility function of the representative household is given by:

$$U(c_{t+1}/B) - vl_t - P(k_{t-1}, d_t)^{1+\phi}/(1+\phi) \tag{4}$$

where $B > 0$ and $v > 0$ are two scaling parameters, and $\phi > 0$. We can notice that as usually in dynamic macroeconomic models, the disutility of labor is linear (see Hansen (1985)), while the disutility of pollution is increasing and convex. Furthermore, we assume:

Assumption 1 *The function $U(x)$ is continuous for all $x \geq 0$, $\mathcal{C}^n$ for $x > 0$ and n large enough, with $U'(x) > 0$, $U''(x) \leq 0$ and $U'(x) + xU''(x) > 0$.*

The representative consumer maximizes his utility function (4) under the constraints (1), (2) and (3). We deduce the two following equations:

$$U'(c_{t+1}/B) r_{t+1} w_t / B = v \tag{5}$$

$$(\alpha k_{t-1} - d_t)^{\phi} w_t = v \tag{6}$$

These two expressions define the consumer choice between leisure, environmental maintenance and future consumption.

The final good is supplied by a representative firm using a constant returns to scale technology. The production is given by $y_t = f(a_t)l_t$, where $a_t = k_{t-1}/l_t$ denotes the capital-labor ratio and $f(a_t)$ the intensive production function. In what follows, we assume:

Assumption 2 *The intensive production function $f(a)$ is continuous for $a \geq 0$, positively valued and differentiable as many times as needed for $a > 0$, with $f'(a) > 0$ and $f''(a) < 0$.*

The producers maximize their profits. Since the economy is perfectly competitive, we obtain:

$$r_t = f'(a_t) \equiv r(a_t) \tag{7}$$

$$w_t = f(a_t) - a_t f'(a_t) \equiv w(a_t) \tag{8}$$

Substituting equations (1), (2), (7) and (8) into (5) and (6), we can define an intertemporal equilibrium as follows. An intertemporal equilibrium is a sequence $(a_t, k_{t-1})_{t\geq 1}$ which satisfies:

$$U'(r(a_{t+1})k_t/B)r(a_{t+1})w(a_t)/B = v \tag{9}$$

$$k_t = w(a_t)\frac{k_{t-1}}{a_t} - \alpha k_{t-1} + \left(\frac{v}{w(a_t)}\right)^{1/\phi} \tag{10}$$

$$\alpha k_{t-1} > w(a_t)k_{t-1}/a_t - k_t \geq 0 \tag{11}$$

where $k_0 > 0$ is given.

Note that inequalities (11) ensure a strictly positive pollution and a positive environmental maintenance. Taking as given these conditions, equations (9) and (10) define a two-dimensional dynamic system with one predetermined variable, the capital. One can further notice that equation (10) means in fact that $k_t = w(a_t)l_t - d_t$ at equilibrium, where $d_t = \alpha k_{t-1} - P_{t+1}$ and $P_{t+1} = (v/w(a_t))^{1/\phi}$. Note that in the limit case without any pollution ($\alpha = 0$), the trade-off between investments in productive capital and environmental maintenance disappears ($d_t = 0$), and equation (10) becomes $k_t = w(a_t)l_t$.

Before studying steady states, we define the following relationships. We note $s(a) \equiv r(a)a/f(a) \in (0, 1)$ the capital share in income and furthermore, if $\sigma(a)$ represents the elasticity of capital-labor substitution, $1/\sigma(a) = dlnw(a)/dlna - dlnr(a)/dlna$. Since $w'(a) = -ar'(a)$, we obtain the two following expressions:

$$w'(a)a/w(a) = s(a)/\sigma(a) \quad \text{and} \quad r'(a)a/r(a) = -(1-s(a))/\sigma(a) \qquad (12)$$

3 EXISTENCE OF A STEADY STATE

A stationary solution of the dynamic system defined by (9), (10) and (11) is given by (a, k) such that:

$$U'(r(a)k/B)r(a)w(a)/B = v \qquad (13)$$

$$k\left(\alpha + 1 - \frac{w(a)}{a}\right) = \left(\frac{v}{w(a)}\right)^{1/\phi} \qquad (14)$$

$$\alpha + 1 > w(a)/a > 1 \qquad (15)$$

We can notice that in section 4 we are interested in fluctuations around a steady state. Since environmental maintenance has to be positive at each period, we assume a strictly positive environmental maintenance at the steady state, i.e. $w(a)/a > 1$.

Following Cazzavillan et al. (1998), we ensure in what follows the existence of a normalized steady state $(a, k) = (1, 1)$ by choosing appropriate values of the two scaling parameters $B > 0$ and $v > 0$.[6] In order to do that, we assume:

Assumption 3 $\alpha + 1 > w(1) > 1$.

Under Assumptions 1-3, there exists a unique solution $v^* > 0$ to the equation:

$$v^* = (\alpha + 1 - w(1))^{\phi} w(1) \qquad (16)$$

Assume that $lim_{x\to 0} xU'(x) < v^*/w(1) < lim_{x\to +\infty} xU'(x)$. Then, under Assumption 1 and taking v^* as given, there is a unique B^* which satisfies:

$$U'(r(1)/B^*)r(1)/B^* = v^*/w(1) \qquad (17)$$

Proposition 1 *Assuming that* $lim_{x\to 0} xU'(x) < v^*/w(1) < lim_{x\to +\infty} xU'(x)$ *and Assumptions 1-3 are satisfied, then* $(a, k) = (1, 1)$ *is a steady state of the dynamic system (9)-(10) if and only if* v^* *and* B^* *are the unique solutions of (16) and (17).*

4 LOCAL DYNAMICS

In this section, we analyze the occurrence of local indeterminacy and endogenous cycles. Furthermore, interpreting our results, we put in evidence that the consumer choice between leisure, environmental maintenance and future consumption has a key role on the occurrence of endogenous fluctuations.

In order to do that, we study local dynamics in the neighborhood of the steady state $(a, k) = (1, 1)$. If we note $s = s(1)$, we assume:

Assumption 4 *$s < 1/2$ and $U''(x) = 0$.*

It means that the capital share in income is smaller than one half, which is usually assumed in macroeconomic dynamic models and verified by empirical studies, and the utility for consumption is linear. We introduce this last assumption for simplification.[7] We obtain the following result:

Proposition 2 *Assuming that there exists a steady state (Proposition 1) and Assumptions 1-4 are satisfied, the steady state is locally indeterminate for $\alpha < w(1) + 1$, a flip bifurcation occurs for $\alpha = w(1) + 1$ and the steady state is a saddle for $\alpha > w(1) + 1$.*

Proof. We first differentiate the dynamic system (9)-(10) around $(a, k) = (1, 1)$. Using (12) and after some computations, we obtain the trace T and the determinant D of the associated Jacobian matrix, which respectively represent the sum and the product of the two eigenvalues of the characteristic polynomial $Q(\lambda) \equiv \lambda^2 - T\lambda + D = 0$:

$$T = \frac{s}{1-s} + w(1) - \alpha \tag{18}$$

$$D = \frac{s}{1-s}(w(1) - \alpha) \tag{19}$$

One can easily deduce that the two eigenvalues are defined by $\lambda_1 = s/(1-s)$ and $\lambda_2 = w(1) - \alpha$. Using Assumptions 3 and 4, we have $\lambda_1 \in (0, 1)$ and $\lambda_2 < 1$. Moreover, $\lambda_2 > -1$ for $\alpha < w(1) + 1$, $\lambda_2 = -1$ for $\alpha = w(1) + 1$ and $\lambda_2 < -1$ for $\alpha > w(1) + 1$. This concludes the proof. ∎

This proposition establishes that both deterministic cycles and local indeterminacy, i.e. fluctuations due to self-fulfilling prophecies, can occur. In contrast to previous results analyzing the existence of cycles in dynamic models with environment (see Seegmuller and Verchère (2004), Zhang (1999)), here the emission rate of pollution (α) has not to be too high. Furthermore, endogenous fluctuations can occur when capital and labor are not weak substitutes as it is often required (see among others Grandmont et al. (1998)

or Reichlin (1986)). Our results do not depend any more on the existence of increasing returns or imperfect competition (see Benhabib and Farmer (1999) for a survey).

We now interpret the occurrence of endogenous fluctuations by explaining how non monotonic and cyclical trajectories can emerge due to consumer choices between consumption, environmental quality and leisure.

Assume that one deviates from the steady state by an increase of the capital stock k_{t-1}. Then, taking into account equation (3), young consumers expect that future pollution P_{t+1} will increase and reduce the level of their utility. Since from relation (6) the real wage decreases, they increase their labor supply l_t, and they also reallocate their savings from capital accumulation k_t to environmental maintenance d_t (see equation (1)). Following this decrease of the capital stock, the next generation of consumers will expect a decreased pollution flow P_{t+2} and then, by the reverse mechanism, both l_{t+1} and d_{t+1} will go down and k_{t+1} will raise; and so on successively along the fluctuations.

Finally, one can observe that the greater is α, the more volatile is d_t, which can be a source of instability of the steady state and promote the determinacy of the equilibrium. It explains why fluctuations due to self-fulfilling expectations emerge only if α is not too high.

NOTES

1. For a recent survey see Xepapadeas (2003).

2. See among others Benhabib and Farmer (1994), Cazzavillan (2001) or Grandmont (1985).

3. As in John and Pecchenino (1994), we consider positive environmental maintenance or pollution abatement ($d_t \geq 0$).

4. We assume that capital totally depreciates after one period of use.

5. Note that the pollution P_{t+1} can be interpreted as a flow, or a stock determined by $P_{t+1} = (1-m)P_t + \alpha k_{t-1} - d_t$, with a natural rate of absorption m equal to 1. Since, in overlapping generations models, the length of period is assumed to be long, the assumption $m = 1$ does not seem to be too restrictive.

6. In order to be as short as possible, we do not discuss uniqueness or multiplicity of steady states. For such an analysis, the reader can refer to Seegmuller and Verchère (2005).

7. The reader can refer to Seegmuller and Verchère (2005) who relax this condition.

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