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Hubert KEMPF, Leopold von THADDEN

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On policy interactions among nations: when do cooperation and commitment matter ?*

Hubert Kempf[†] Leopold von Thadden[‡]

Abstract

This paper offers a framework to study commitment and cooperation issues in games with multiple policymakers. To reconcile some puzzles in the recent literature on the nature of policy interactions among nations, we prove that games characterized by different commitment and cooperation schemes can admit the same equilibrium outcome if certain spillover effects vanish at the common solution of these games. We provide a detailed discussion of these spillovers, showing that, in general, commitment and cooperation are non-trivial issues. Yet, in linear-quadratic models with multiple policymakers commitment and cooperation schemes are shown to become irrelevant under certain assumptions. The framework is sufficiently general to cover a broad range of results from the recent literature on policy interactions as special cases, both within monetary unions and among fully sovereign nations.

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JEL classification numbers: E52, E63.

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[†]Direction de la recherche, Banque de France, Paris School of Economics and Université Paris-1 Panthéon-Sorbonne. e-mail: hubert.kempf@banque-france.fr.

[‡]European Central Bank, Kaiserstrasse 29, D-60311 Frankfurt/Main, Germany. e-mail: leopold.von_thadden@ecb.int. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Banque de France and the European Central Bank.

1 Introduction

The literature on the nature of policy interactions among nations often leads to rather puzzling results. Paradoxes abound, and there exists an impressive range of different views on possible gains and costs from cooperation and commitment schemes. In a political context, these diverse views are a source of constant debate. Examples of controversially discussed cooperation and commitment schemes are, just to name a few among many others, the Stability and Growth Pact of the European Monetary Union, international agreements on exchange rates or the adoption of currency boards.
These debates have clear counterparts in the academic literature. A particularly startling example of the unsettling state of discussion on policy interactions is provided by two recent contributions on policymaking in monetary unions. On the one hand, Chari and Kehoe (2002, 2007) consider a monetary union model which abstracts from any direct fiscal spillovers between countries and which nevertheless has the feature that equilibrium outcomes depend sensitively on the (non)-availability of cooperation schemes and the sequencing of actions of policymakers. In particular, equilibrium outcomes depend sensitively on whether the central bank in a monetary union can move prior to national fiscal authorities, as this device helps to prevent pressures to monetize national deficits, related to private sector coordination failures within countries and their relationship to the common monetary policy. In striking contrast to this finding, Dixit and Lambertini (2003) consider a monetary union model which allows for direct fiscal spillovers between countries and which nevertheless has the feature that policymakers can always attain the same equilibrium outcome, irrespective of whether policymakers cooperate or not and irrespective of the order in which they choose their actions.
Similarly rich analytical results, leading to distinctly different conclusions, are offered by the literature on international monetary policy cooperation. Obstfeld and Rogoff (2000, 2002), for example, using a fully micro-founded open-economy model, derive exact conditions under which cooperative and self-oriented (Nash) policies of monetary policymakers yield the same outcome. This finding is at odds with earlier contributions to this literature like Rogoff (1985) and Canzoneri and Henderson (1988, 1991), who stressed not only the scope for gains from international policy cooperation, but also showed that attempts to internalize such gains could become counterproductive under a particular sequencing of actions (related to private sector activities). Also in most of the very recent contributions the benchmark result of Obstfeld and Rogoff is not uncontested. Canzoneri et al (2005), for example, argue that in micro-founded general equilibrium settings the scope for gains from cooperation, if anything, has increased compared with the older literature which was based on ad-hoc welfare objectives.
These conflicting views, all based on tractable theoretical models, indicate that there is a need of a comprehensive framework of policy interactions which could be used to evaluate and compare various commitment and cooperation assumptions from a unified perspective. Against this background, the goal of the present paper is more modest, namely to provide a clear taxonomy which can be used to understand why some of the above mentioned studies obtain irrelevance results with respect to cooperation and commitment schemes,

while others do not.[1]

To this end, we set up a simple, generic framework for the analysis of strategic interactions among independent but interdependent players. In particular, we use the concept of a 'coalition structure' to characterize cooperative behavior between a particular group of players and the concept of a 'commitment pattern' to characterize a particular order of moves of players.[2] Using this two-dimensional characterization of games, we provide a number of propositions which develop conditions under which games characterized by different commitment patterns and coalition structures can admit the same equilibrium outcome. For this to happen it is crucial that certain spillover effects vanish at the common solution of these games. We provide a detailed discussion of these spillovers, showing that, in general, commitment and cooperation are non-trivial issues. Yet, assuming consensus on the target values of all players, we show that commitment patterns and coalition structures become entirely irrelevant if i) the framework has a certain linear-quadratic structure, ii) the players have access to sufficiently many independent instruments (relative to the number of squared gaps which appear in their payoff functions) and iii) if the economy reaches a social optimum when all gaps are closed.

As we show, this taxonomy is sufficiently general to account for the above mentioned broad range of findings on the (ir)relevance of cooperation and commitment in the recent literature, both within monetary unions and among fully sovereign nations. In particular, the framework of Dixit and Lambertini (2003) and the benchmark model of Obstfeld and Rogoff (2000, 2002) have representations which satisfy all three criteria. Chari and Kehoe (2002, 2007) is an example which does not satisfy the first criterion, since it is not based on a linear-quadratic set-up. Rogoff (1985), Canzoneri and Henderson (1988, 1991), and Canzoneri et al (2005) are all examples which do not satisfy the second criterion, i.e. the relevance of strategic interactions is driven by the shortage of policy instruments within linear-quadratic set-ups. Finally, the paper by Obstfeld and Rogoff (2002) allows for an extension which leads to a qualification of their benchmark result. This extension does not satisfy the third criterion, i.e. it is typically no longer socially optimal to stabilize the economy at the level at which all squared gaps are closed if this level itself suffers from further distortions, related, for example, to incomplete risk sharing.

The remainder of the paper is structured as follows. Section 2 develops a general framework to study commitment and cooperation issues in games with multiple players. It then offers a number of general propositions on the (ir)relevance of commitment patterns and coalition structures. In Section 3, we apply these propositions to discuss recent contributions on policy interactions in monetary unions. In Section 4, we apply these propositions to discuss recent contributions on international policy coordination. Section 5 concludes. Proofs and some technical issues are delegated to the Appendix.

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[1] We offer at this stage no further discussion of the related literature, since our taxonomy was initially motivated to cover exactly the papers cited so far, all of them being widely cited benchmark studies in their fields. However, related literature in either of the two fields is discussed in more depth below when we address the two areas in detail.

[2] As discussed below in some applications, if some of the players belong to the private sector this broad concept of a commitment pattern naturally relates to time inconsistency issues, which typically occur for certain (but not all) timing structures of private and public sector moves.

2 A unifying framework for policy analysis

2.1 Players

We consider a world economy, consisting of N nations with index i. In each nation, there coexist private agents and national policymakers. Moreover, there exist international policymakers. We refer to a generic player in this world economy, be it a private agent, a national policymaker, or an international policymaker, as ξ, and the set of players as $\Xi = \{1, .., \xi, .., X\}$. A particular action of player ξ is denoted by $x_\xi \in \mathcal{X}_\xi$, with $\mathcal{X}_\xi$ being the set of actions available to player ξ. The payoff function of player ξ is given by

$$V_\xi = V_\xi(\mathbf{x}),$$

where the vector $\mathbf{x}$ summarizes the actions of all players, i.e. $\mathbf{x} = (x_\xi, \mathbf{x}_{-\xi})$, and $V_\xi(\mathbf{x})$ is assumed to be continuously differentiable in its arguments, $\forall \xi \in \Xi$. In Sections 3 and 4, we will refine this notation in order to distinguish explicitly between private agents, national policymakers, and international policymakers. However, to establish some general results on cooperation and commitment such a differentiated notation is not needed.

2.2 Commitment

We denote by Γ an extensive form game. There are T^Γ stages in this game, and we denote by $\mathcal{T}^\Gamma$ the set of stages: $\{1, .., t, .., T^\Gamma\}$. We assume that each player is allocated to act at a particular stage and he plays only once in the entire game, at this particular stage. To define the order of moves of players (in the following for short: 'commitment pattern'), determining at which stage every player acts, we use the following:

Definition 1 *A commitment pattern $\mathfrak{C}$ specifies an assignment for each player $\xi \in \Xi$ to act at one particular stage $t \in \mathcal{T}^\Gamma$, denoted by $t(\xi)$.*

2.3 Coalitions

Players may form coalitions. Coalitions can only be formed between players who are allocated to act at the same stage. This is the standard assumption made in macroeconomic games, excluding repeated games. A coalition is a subset of players who cooperate. Any coalition C_θ is defined by three characteristics: i) it decides jointly over the actions chosen by all its members, ii) its members play simultaneously: $\forall \xi, \xi' \in C_\theta, t(\xi) = t(\xi')$ and iii) it maximizes the welfare W_θ of its members, with

$$W_\theta = \sum_{\xi \in C_\theta} \omega_\xi V_\xi(\mathbf{x}),$$

where ω_ξ denotes the weight attached by the coalition members to the welfare of player ξ. Notice that a membership to a coalition is different from the usual definition of membership, in the sense that it is assumed that all agents belong to one coalition only. Moreover, to simplify notation, we define coalitions in a broad sense so that they also include singletons (i.e. players acting in isolation) as special cases. We denote by Θ the number of coalitions and define a coalition structure as a partition of Ξ, that is:

Definition 2 *A coalition structure* $\mathcal{C} = \{C_1, ..., C_\theta, ..., C_\Theta\}$ *is a partition of* Ξ, *that is: i)* $C_\theta \cap C_{\theta'} = \emptyset$ *for all* $\theta \neq \theta'$, *ii)* $\overset{\Theta}{\underset{\theta=1}{\cup}} C_\theta = \Xi$.

We denote by C_Ξ the 'grand coalition' formed by all players. In sum, a game is characterized by a commitment pattern $\mathfrak{C}$ and a coalition structure C. Games are solved by backward induction. Later on we will compare equilibrium outcomes of games characterized by different coalitions structures and commitment patterns. To facilitate such comparisons, we assume throughout that for any player ξ equilibrium actions can be deduced from decision rules which are continuously differentiable in the actions of players acting at the same stage or at previous stages, i.e. $x_\xi = x_\xi(\mathbf{x}'_{-\xi})$, where $\mathbf{x}'_{-\xi}$ contains only actions of players ξ' satisfying $t(\xi') \leq t(\xi)$.

2.4 Spillovers

Given the existence of coalitions, spillover effects between agents will play a crucial role in the rest of our analysis. Generally speaking, the welfare effects of a particular action of a player can be decomposed into three distinct effects, namely the effects on his own welfare, the effects on the welfare of his coalition members (*within-coalition spillover effects*), and the effects on the welfare of players belonging to different coalitions (*between-coalition spillover effects*). In the context of multi-stage games these effects do not only include direct effects, but also indirect effects which are related to anticipated actions of players acting at subsequent stages. To capture these different effects, we use the following characterizations of spillovers.

Definition 3 ***Direct spillovers***
For a given commitment pattern and coalition structure $(\mathfrak{C}, \mathcal{C})$ *and a given vector of actions* $\mathbf{x}$, *consider a representative player* $\xi \in C_\theta$. *Consider a second player* ξ'. *We refer to* $\frac{\partial V_{\xi'}(\mathbf{x})}{\partial x_\xi}$ *as a direct within-coalition (between-coalition) spillover effect if* ξ' *belongs (does not belong) to* C_θ.

Definition 4 ***Indirect spillovers***
For a given commitment pattern and coalition structure $(\mathfrak{C}, \mathcal{C})$ *and a given vector of actions* $\mathbf{x}$, *consider a representative player* $\xi \in C_\theta$. *Consider a second player* ξ' *and a third player* ξ'' *playing at the subsequent stage. Then,* $\frac{\partial V_{\xi'}(\mathbf{x})}{\partial x_{\xi''}} \frac{\partial x_{\xi''}}{\partial x_\xi}$ *denotes an indirect within-coalition (between-coalition) spillover effect between* ξ *and* ξ' *if* ξ' *belongs (does not belong) to* C_θ.

Notice that for the indirect within-coalition spillover effect described in Definition 4 to exist, it is necessary that the term $\partial V_{\xi'}(\mathbf{x})/\partial x_{\xi''}$ is non-zero. The latter term, according to Definition 3, is a direct between-coalition spillover effect which links two players acting at different stages. Later on we will frequently exploit this particular relationship between indirect within-coalitions spillovers and direct between-coalitions spillovers.

2.5 Comparing games

A large number of different games can be played in this economy, varying in terms of commitment patterns and coalition structures. In the following we establish conditions which can be used to compare equilibrium outcomes of two different games Γ and Γ'. We denote by $\mathcal{Z}(\Gamma)$ ($\mathcal{Z}(\Gamma')$) the set of interior subgame perfect Nash equilibrium (SPNE) outcomes associated with Γ (Γ') and by $\mathbf{z}$ an element of $\mathcal{Z}(\Gamma)$. A sufficient (but rather restrictive) condition for a second game Γ' to admit the same SPNE outcome is the following:[3]

Proposition 1 *Consider a game Γ, characterized by $(\mathcal{C}, \mathfrak{C})$, and a game Γ', characterized by $(\mathcal{C}', \mathfrak{C}')$. Then, an element $\mathbf{z}$ belongs to $\mathcal{Z}(\Gamma)$ and $\mathcal{Z}(\Gamma')$ if at $\mathbf{z}$*
i) for any (ξ, ξ'), $\xi \in C_\theta$, $\xi' \in C_{\theta'}$, $C_\theta \in \mathcal{C}$, $C_{\theta'} \in \mathcal{C}$, $t(\xi) \neq t(\xi')$,
and for any (ξ, ξ'), $\xi \in C'_\theta$, $\xi' \in C'_{\theta'}$, $C'_\theta \in \mathcal{C}'$, $C'_{\theta'} \in \mathcal{C}'$, $t(\xi) \neq t(\xi')$

$$\frac{\partial V_{\xi'}(\mathbf{z})}{\partial x_\xi} = 0,$$

ii) for any (ξ, ξ'), $\xi \in C'_\theta$, $\xi' \in C'_\theta$, $C'_\theta \in \mathcal{C}', C'_\theta \notin \mathcal{C}$,

$$\frac{\partial V_{\xi'}(\mathbf{z})}{\partial x_\xi} = 0.$$

Proof: see appendix.

Part *i*) requires that at the vector $\mathbf{z}$ there exist in either game no direct between-coalition spillover effects between players belonging to coalitions playing at different stages. Part *ii*) requires that at the vector $\mathbf{z}$ there exist no direct within-coalition spillover effects between players belonging to a coalition which does not belong simultaneously to C and C'.

Proposition 1 follows from backward induction. It gives us conditions such that two different games can have the same SPNE outcome despite differences in terms of commitment patterns and coalition structures. These conditions are related to the absence of certain spillover effects at the equilibrium outcome $\mathbf{z}$. Notice that Proposition 1 does not require the absence of all spillover effects at $\mathbf{z}$. Such non-vanishing spillover effects can be of two varieties: they can be *i*) direct between-coalition spillover effects between coalitions acting at the same stage, or *ii*) direct within-coalition spillover effects in coalitions which exist in both games. In other words, a common equilibrium outcome $\mathbf{z}$ is not necessarily a solution of the simultaneous Nash game, obtained when all players act as singletons.

2.6 The simultaneous game Γ^{Nash} : a special benchmark

In order to establish an important benchmark, let Γ^{Nash} denote the reference game which is played by all players simultaneously (i.e. no commitment as there exist no sequential

[3] Throughout, the second-order conditions for a maximum are assumed to be satisfied. Notice that in the linear-quadratic applications discussed below this assumption will always be satisfied.

stages) and without any coalitions. We denote by $\mathbf{z}^{Nash}$ an equilibrium outcome of this 'no-commitment and no-cooperation' game. Moreover, let $\mathcal{Z}_{\Xi}$ denote the set of equilibrium outcomes corresponding to the grand coalition. Proposition 1 can then be extended as follows:

Proposition 2 *Consider the 'no-commitment and no-cooperation' game* Γ^{Nash}*, admitting the Nash equilibrium outcome* $\mathbf{z}^{Nash}$*. Then* $\mathbf{z}^{Nash}$ *belongs to* $\mathcal{Z}_{\Xi}$ *and more generally to* $\mathcal{Z}(\Gamma)$ *for any extensive-form game* Γ*, characterized by arbitrary commitment patterns and coalition structures, if*

$$\frac{\partial V_{\xi'}(\mathbf{z}^{Nash})}{\partial x_{\xi}} = 0, \forall \xi, \xi^{'} \in \Xi.$$

This condition requires that there are no direct spillover effects between any pair of players (ξ, ξ') at z^{Nash}. Proposition 2 follows directly from the proof of Proposition 1 and it uses the well-known result that a Nash equilibrium belongs to the set of equilibria corresponding to the grand coalition if there are no direct spillover effects between any pair of players. Exploiting this feature, it states conditions under which cooperation and commitment are entirely irrelevant. These conditions are quite stringent but they cannot be ruled out.[4]

2.7 The linear-quadratic model for policy analysis

The results presented in the previous section can be used to shed some light on the nature of policy interactions in linear-quadratic models. This approach to policy analysis has a long established tradition, dating back to Theil (1964), and our discussion of key policy applications will show that this approach is, indeed, still very much in use. Let us write such a model as follows, using our setting. Consider an economy with X players indexed by ξ. The economy is described by a linear model, that is there exists a $P \times 1-$vector $\mathbf{y}$ which summarizes the state of the economy. This vector depends linearly on the $X \times 1-$vector of actions of all players $\mathbf{x}$

$$\mathbf{y} = \overline{\mathbf{y}} + \mathbf{B}\mathbf{x}, \tag{1}$$

with $\overline{\mathbf{y}}$ being a vector of constants. The $p-th$ element of $\mathbf{y}$, y_p, characterizes the aggregate variable p, with $p = 1, 2, ..., P$. Let $\mathbf{y}^*$ denote a $P \times 1-$vector of target values of these variables, with $p-th$ element y_p^*. It is assumed that the target values are shared by all agents. Moreover, assume $P = X$ and let the $X \times X-$matrix $\mathbf{B}$ being invertible. The payoff function corresponding to player ξ is a weighted sum of squared deviations of the elements of $\mathbf{y}$ from their target values, such that:

$$V_{\xi} = \frac{1}{2}\left[\omega_1^{\xi}(y_1^* - y_1)^2 + ... + \omega_p^{\xi}(y_p^* - y_p)^2 + ... + \omega_X^{\xi}(y_X^* - y_X)^2\right]. \tag{2}$$

Notice that individual payoffs depend on the actions of other players through the model itself (i.e. the $\mathbf{B}-$matrix) and the player-specific weights $\boldsymbol{\omega}^{\xi} \geqslant \mathbf{0}$ in the payoff functions.

[4] In many applications differences in commitment patterns or coalition structures are restricted to subgames, while early stages are identical for the games to be compared. It is straightforward to adapt the reasoning of Propositions 1 and 2 to such a special constellation by applying the conditions specified in the two propositions to the subgames which make the games under comparison different.

To restrict the analysis to non-degenerate cases, we assume that for every variable p there exists a pair of values $\omega_p^\xi > 0$ and $b_{p\xi} \neq 0$ (where the representative entry $b_{p\xi} \in \mathbf{B}$ denotes the marginal effect of player ξ on the variable p) for at least one player $\xi = 1, 2, ..., X$.

Proposition 3 *For an economy described by (1) and (2), the unique Nash equilibrium outcome $\mathbf{z}^{Nash} = \mathbf{B}^{-1}[\mathbf{y}^* - \overline{\mathbf{y}}]$ of the simultaneous Nash game Γ^{Nash} belongs to $\mathcal{Z}(\Gamma)$ for any extensive-form game Γ characterized by arbitrary commitment patterns and coalition structures.*

Proof: see appendix.

Proposition 3 states that in a linear-quadratic model under the assumptions made above neither commitment nor cooperation matter. This result follows directly from Proposition 2 since in the linear-quadratic model all direct spillover effects between any pair of players (ξ, ξ') vanish at the unique Nash equilibrium. Proposition 3 is reminiscent of the analysis offered by Tinbergen (1952). In fact, it may be seen as a generalized Tinbergen rule in a game-theoretical environment, assuming that there is no disagreement about the target values of all players. It is central to stress that this result relies not only on the linear-quadratic nature of the problem, but also on the assumption that each player disposes of an instrument and that the number of independent instruments matches the number of squared gaps in the payoff functions of all players (i.e. $P = X$).[5]

2.8 Stochastic extension of the linear-quadratic model

There exists an obvious extension of Proposition 3 to a particular stochastic environment. Assume the economy is subject to S shocks, summarized by the $S \times 1-$vector $\boldsymbol{\varepsilon}$, with mean zero and variance-covariance matrix Ω_ε. The economy is described by a linear model, i.e. there exists a $P \times 1-$vector $\mathbf{y}$ of aggregate variables which depend linearly on the vector $\boldsymbol{\varepsilon}$ as well as on the $X \times 1-$vector $\mathbf{x}$ of actions of all players

$$\mathbf{y} = \overline{\mathbf{y}} + \mathbf{B}_x\mathbf{x} + \mathbf{B}_\varepsilon\boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} \sim (\mathbf{0}, \boldsymbol{\Omega}_\varepsilon), \tag{3}$$

with $\overline{\mathbf{y}}$ being a vector of constants. Again, we impose $P = X$ such that $\mathbf{B}_x$ denotes an $X \times X-$matrix, which is assumed to be invertible, while $\mathbf{B}_\varepsilon$ denotes a $X \times S-$matrix. All players choose ex ante (i.e. before the realization of $\boldsymbol{\varepsilon}$) non-cooperatively policy rules which are linear in $\boldsymbol{\varepsilon}$, i.e.

$$\mathbf{x} = \mathbf{r} + \mathbf{R}_\varepsilon\boldsymbol{\varepsilon}, \tag{4}$$

where $\mathbf{r}$ is a $(X \times 1)$-vector and $\mathbf{R}_{\boldsymbol{\varepsilon}}$ is a $(X \times S)$-matrix.[6] Let the stacked matrix $\mathbf{R} = [\mathbf{r}, \mathbf{R}_{\boldsymbol{\varepsilon}}]$ summarize the actions of all players, with $\mathbf{R}$ being a $X \times (S+1)-$matrix. Suppose that

[5] For a recent discussion of linear-quadratic frameworks for policy purposes see, in particular, Woodford (2003). Yet, in his applications the Tinbergen criterion (of assuming an identical number of objectives and independent instruments) is typically not satisfied.

[6] We deliberately use this loose wording (rather than to say that players 'commit' via rules) in order to avoid misunderstandings with our usage of the term commitment (i.e. the 'order of moves of players'), as described in Definition 1.

the expected payoff of player ξ can be represented as

$$E(V_\xi) = \frac{1}{2}E\left[\omega_1^\xi(y_1^* - y_1)^2 + ... + \omega_p^\xi(y_p^* - y_p)^2 + ... + \omega_X^\xi(y_X^* - y_X)^2\right], \tag{5}$$

Assume that for every variable p there exists a pair of values $\omega_p^\xi > 0$ and $b_{p\xi}^x \neq 0$ (where the representative entry $b_{p\xi}^x \in \mathbf{B}_x$ denotes the marginal effect of player ξ on the variable p) for at least one player $\xi = 1, 2, ..., X$.

Proposition 4 *For an economy described by (3)-(5) the unique Nash equilibrium outcome* $\mathbf{R}^{Nash} = [\mathbf{r}^{Nash}, \mathbf{R}_\varepsilon^{Nash}]$, *with* $\mathbf{r}^{Nash} = \mathbf{B}_x^{-1}(\mathbf{y}^* - \overline{\mathbf{y}})$ *and* $\mathbf{R}_\varepsilon^{Nash} = -\mathbf{B}_x^{-1} \cdot \mathbf{B}_\varepsilon$, *of the simultaneous Nash game* Γ^{Nash} *belongs to* $\mathcal{Z}(\Gamma)$ *for any extensive-form game* Γ *characterized by arbitrary commitment patterns and coalition structures.*

Proof: see appendix.

Finally, for further reference, we consider a closely related variant of Proposition 4 which gives the entire variance-covariance matrix of $\mathbf{y}$, denoted by $\Omega_\mathbf{y}$, a role in the expected payoffs of players. Specifically, with (3) being unchanged, we replace, ceteris paribus, the policy rule (4) and the specification of expected payoffs *(5)* by

$$\mathbf{x} = \overline{\mathbf{r}} + \mathbf{R}_\varepsilon \boldsymbol{\varepsilon} \tag{6}$$

$$E(V_\xi) = E(\widetilde{V_\xi}) + \boldsymbol{\omega}_\xi' \Omega_\mathbf{y} \boldsymbol{\omega}_\xi, \tag{7}$$

where (7) assumes that $E(V_\xi)$ can be decomposed into an autonomous component $E(\widetilde{V_\xi})$ and a quadratic form $\boldsymbol{\omega}_\xi' \Omega_\mathbf{y} \boldsymbol{\omega}_\xi$, describing a player-specific weighted sum of the variance and covariance terms associated with $\mathbf{y}$. Because of these features, the $(X \times S)-$matrix $\mathbf{R}_{\boldsymbol{\varepsilon}}$ summarizes in (6) the relevant strategic components of $\mathbf{x}$, i.e. $\mathbf{r}$ can be kept fixed at $\overline{\mathbf{r}}$, and one can show:

Corollary to Proposition 4 : *For an economy described by (3), (6), and (7) the unique Nash equilibrium outcome* $\mathbf{R}_\varepsilon^{Nash} = -\mathbf{B}_x^{-1} \cdot \mathbf{B}_\varepsilon$ *of the simultaneous Nash game* Γ^{Nash} *belongs to* $\mathcal{Z}(\Gamma)$ *for any extensive-form game* Γ *characterized by arbitrary commitment patterns and coalition structures.*

Proof: see appendix.

3 Monetary Unions

This section uses the broad framework developed above to address the question under which circumstances cooperation and commitment matter in a monetary union. In general, the possible existence of spillovers within countries (related to private actors), of spillovers between countries (related to fiscal and private actors) and of a common monetary policy (affecting players in all countries) creates a number of channels which make this question non-trivial, i.e. it is clear that, in general, commitment and cooperation (i.e. coalition structures) do matter, within countries and between countries.

Against this general insight two recently established findings seem particularly puzzling.[7] On the one hand, Dixit and Lambertini (2003) consider a model which allows for direct spillovers between players acting in different countries and which nevertheless has the feature that fiscal and monetary policymakers attain the same equilibrium outcome, irrespective of the commitment pattern and of whether policies are coordinated between countries or not. By contrast, Chari and Kehoe (2002) consider a model which abstracts from any direct spillovers between players acting in different countries and which nevertheless has the feature that equilibrium outcomes depend sensitively on commitment patterns and on whether policies are coordinated between countries or not.[8]

Within the framework of Section 2, however, it is straightforward to resolve this puzzle. To this end, let us consider a monetary union with N member countries, indexed by $i = 1, 2, .., N$. Let $\mathcal{M}_i$ denote the set of all private agents in country i. Let a_{ij} denote an action of private agent j in country i and let $\mathbf{a}_i = (a_{ij}, \mathbf{a}_{i,-j})$. For each country there exists a single fiscal policymaker (with action τ_i). Moreover, there exists a single monetary policymaker operating for the monetary union as a whole (with action π). In sum, a profile of actions of all players is given by $\mathbf{x} = (\mathbf{a}, \boldsymbol{\tau}, \pi)$, with $\boldsymbol{\tau} = (\tau_i, \boldsymbol{\tau}_{-\mathbf{i}})$ and $\mathbf{a} = (\mathbf{a_i}, \mathbf{a_{-i}})$. We consider the following payoff functions:

- Payoff function of a representative private agent j in country i:

$$U_{ij} = U_{ij}(\mathbf{a}, \boldsymbol{\tau}, \pi). \tag{8}$$

- Payoff function of fiscal policymaker in country i :

$$V_i = V_i(\mathbf{a}, \boldsymbol{\tau}, \pi) \tag{9}$$

- Payoff function of cooperating fiscal policymakers:

$$V^{FC} = \sum_{i=1}^{n} \omega_i^F V_i = \sum_{i=1}^{n} \omega_i^F V_i(\mathbf{a}, \boldsymbol{\tau}, \pi) \tag{10}$$

  where ω_i^F denotes the fiscal weight of country i in the collective fiscal payoff function.

- Payoff function of the central bank:

$$V^{M} = \sum_{i=1}^{n} \omega_i^M V_i = \sum_{i=1}^{n} \omega_i^M V_i(\mathbf{a}, \boldsymbol{\tau}, \pi). \tag{11}$$

  where ω_i^M denotes the monetary weight attached to country i by the central bank.

[7] Evidently, there exists a broad literature on strategic policy interactions in monetary unions going back at least to Mundell (1961). For recent contributions, using reduced-form one shot games, see, in particular, Beetsma and Bovenberg (1998, 2001), Calmfors (2001), Cukierman and Lippi (2001), and Uhlig (2003). For examples of 'second-generation' models, as discussed in Section 4.2. below, see Lombardo and Sutherland (2004), Beetsma and Jensen (2005), Ferrero (2007), and Gali and Monacelli (2007).

[8] For a closely related, but slightly less general analysis, see also Chari and Kehoe (2007).

Equations (9)-(11) rule out disagreement about the targets between policymakers, i.e. we restrict possible differences between monetary and fiscal policy objectives to the weighting factors ω_i^F and ω_i^M.[9] This general set-up can be used to analyze a large number of different policy constellations. To give the analysis a clear focus, we make a number of simplifying assumptions, in line with the cited literature. First, we consider only games in which all fiscal policymakers act at the same stage. Similarly, all private players act at the same stage. Second, we consider only fully symmetric set-ups, characterized by identical payoff functions within each group of players. Because of this assumption, all equilibrium outcomes are symmetric, satisfying $a_{ij} = a$ for all i, j and $\tau_i = \tau$ for all i. Third, we rule out coalitions between private agents and policymakers, implying that spillovers between these groups of players are always between-coalition spillovers. Finally, it is worth emphasizing that in this general set-up there is scope for four different types of direct within-coalition spillovers: i) direct fiscal spillovers between countries ($\frac{\partial V_i}{\partial \tau_l} \neq 0$, $i \neq l$), ii) direct private spillovers between countries ($\frac{\partial U_{ij}}{\partial a_{lj}} \neq 0$, $i \neq l$), iii) direct private spillovers within countries ($\frac{\partial U_{ij}}{\partial a_{ik}} \neq 0$, $j \neq k$), and iv) direct spillovers between fiscal policy-makers and the central bank. The latter type of spillover can often be neglected, however, since it can only occur if there exist direct fiscal spillovers between countries.[10]

3.1 A simple irrelevance result for cooperation and commitment

To establish a clear link to the set-up of Section 2, consider first a set of benchmark assumptions under which cooperation and commitment for all players become entirely irrelevant. To this end, replace (8) and (9) against

$$U_{ij} = U_{ij}(a_{ij}, \tau_i, \pi) \quad (12)$$

$$V_i = \sum_{j \in \mathcal{M}_i} \omega_{ij} U_{ij}(a_{ij}, \tau_i, \pi) \quad (13)$$

and consider the three payoff functions U_{ij}, V_i, and V^M, belonging to the three types of players which need to be considered in the special game Γ^{Nash}. In generic terms these particular payoff functions satisfy two strong assumptions:

A1: *Absence of any direct private and fiscal spillovers:* $\frac{\partial V_i}{\partial \tau_l} = \frac{\partial U_{ij}}{\partial a_{lj}} = \frac{\partial U_{ij}}{\partial a_{ik}} \equiv 0, i \neq l, j \neq k$.
A2: *Congruence of payoff functions of private agents and policymakers:* $V_i = \sum_{j \in \mathcal{M}_i} \omega_{ij} U_{ij}$.

These two assumptions imply that any symmetric equilibrium of the special game Γ^{Nash} satisfies the requirements of Proposition 2:

[9] Implications of disagreement about the targets between policymakers are discussed, in particular, in Beetsma and Uhlig (1999) and Dixit and Lambertini (2001, 2003b).

[10] In other words, in any symmetric equilibrium $\frac{\partial V_i}{\partial \pi} = 0$ will always be ensured by $\frac{\partial V^M}{\partial \pi} = 0$, while $\frac{\partial V^M}{\partial \tau_i} = \omega_i^M \frac{\partial V_i}{\partial \tau_i} + \sum_{l \neq i} \omega_l^M \frac{\partial V_l}{\partial \tau_i} = 0$ will be ensured by $\frac{\partial V_i}{\partial \tau_i} = 0$ if there are no direct fiscal spillovers between countries.

Proposition 5 ***(Irrelevance of coalitions structures and commitment patterns)*** *Assume that A1 and A2 are satisfied and that the game Γ^{Nash} admits a symmetric Nash equilibrium, with outcome $\mathbf{z}^{Nash}$. Then, any extensive-form game Γ', characterized by arbitrary commitment patterns and coalition structures, admits this outcome since there are no direct spillovers between any pair of players at $\mathbf{z}^{Nash}$.*

Proof: A1 and A2 ensure that in any symmetric equilibrium the Nash requirement $\frac{\partial U_{ij}}{\partial a_{ij}} = 0$ implies $\frac{\partial V_i}{\partial a_{ij}} = 0$, $\frac{\partial V^M}{\partial a_{ij}} = 0$. Similarly, $\frac{\partial V_i}{\partial \tau_i} = 0$ implies $\frac{\partial U_{ij}}{\partial \tau_i} = 0$, $\frac{\partial V^M}{\partial \tau_i} = 0$, and $\frac{\partial V^M}{\partial \pi} = 0$ implies $\frac{\partial V_i}{\partial \pi} = 0$, $\frac{\partial U_{ij}}{\partial \pi} = 0$, for all i, j. Hence, Proposition 2 applies.

Evidently, for Proposition 5 to prevail at this level of generality, both assumptions stressed above are crucial.[11] Against this background, it is straightforward to motivate the particular contributions of Chari and Kehoe (2002) and of Dixit and Lambertini (2003). The key contribution of Chari and Kehoe (2002) is to show that generically the broad irrelevance result of Proposition 5 disappears if one relaxes at least one of the two assumptions A1 or A2. By contrast, the analysis by Dixit and Lambertini (2003) can be used to see that, even if assumptions A1 and A2 are not satisfied, the irrelevance result can reappear if the economy satisfies the additional constraints of a linear-quadratic framework in line with Proposition 3. These additional restrictions make it possible that *in equilibrium* all direct spillovers between all players vanish at $\mathbf{z}^{Nash}$, which is sufficient for Proposition 2 to apply.

3.2 The Chari-Kehoe model

The model of Chari and Kehoe (2002) leads to conclusions which are in spirit very different from the irrelevance result of Proposition 5. To this end, the model introduces, ceteris paribus, one subtle variation into the model of Section 3.1 by replacing (12) against

$$U_{ij} = U_{ij}(a_{ij}, \mathbf{a}_{i,-j}, \tau_i, \pi). \tag{14}$$

The key property of (14) is that it generically allows for direct private spillovers within countries ($\frac{\partial U_{ij}}{\partial a_{ik}} \neq 0$, $j \neq k$) and the model has no channel which makes these spillovers vanish in equilibrium. Consequently, $\frac{\partial U_{ij}}{\partial a_{ij}} = 0 \nRightarrow \frac{\partial V_i}{\partial a_{ij}} = 0$, $\frac{\partial V^M}{\partial a_{ij}} = 0$. This feature is sufficient to make the result of Proposition 5 not applicable. In short, the core assumptions of Chari and Kehoe can be summarized as:

A1': *Absence of direct private and fiscal spillovers between countries:* $\frac{\partial V_i}{\partial \tau_l} = \frac{\partial U_{ij}}{\partial a_{lj}} \equiv 0$, $i \neq l$.
A2: *Congruence of payoff functions of private agents and policymakers:* $V_i = \sum_{j \in \mathcal{M}_i} \omega_{ij} U_{ij}$.

Assuming non-cooperative private sector behavior, Chari and Kehoe show that in a monetary union the strong assumption of zero direct private and fiscal spillovers *between* countries is not sufficient to make fiscal cooperation between countries irrelevant. Instead,

[11] Notice that because there are no direct spillovers *between any pair of players,* the irrelevance result, in fact, covers also mixed coalitions between private agents and policymakers.

non-internalized private spillovers within countries are enough to ensure that fiscal cooperation becomes relevant at least for some commitment patterns. In other words, for some commitment patterns non-internalized private spillovers within countries can create *indirect fiscal spillovers* between countries which make fiscal cooperation desirable. To make the implications of this feature precise, Chari and Kehoe compare fiscal cooperation and non-cooperation under two different commitment patterns:

$\mathfrak{C}_I$ (*monetary policy moves last*): *i/* τ_i, *ii/* a_{ij}, *iii/* π.
$\mathfrak{C}_{II}$ (*monetary policy moves first*): *i/* π, *ii/* τ_i, *iii/* a_{ij}.

Comparing fiscal cooperation and non-cooperation under $\mathfrak{C}_I$ and $\mathfrak{C}_{II}$, the two main propositions of Chari and Kehoe (2002), adopted to our framework, can be summarized as follows:[12]

Chari-Kehoe (2002): *Assume there are no direct spillovers between any players acting in different countries. Then, fiscal cooperation between countries is nevertheless relevant under certain commitment patterns. Specifically, under* $\mathfrak{C}_{II}$*, with monetary policy moving first, the equilibrium outcomes of fiscal cooperation vs. non-cooperation are identical. However, under* $\mathfrak{C}_I$*, with monetary policy moving last, the equilibrium outcomes of fiscal cooperation vs. non-cooperation differ because of indirect fiscal spillovers related to a time inconsistency problem of monetary policy.*

The irrelevance of fiscal cooperation under $\mathfrak{C}_{II}$ is rather obvious: at stage iii), private agents in country i take as given τ_i and π. Hence, when fiscal policy is decided at stage ii), there exist, for a given value of π, neither direct nor indirect fiscal spillovers between countries. By contrast, under the commitment pattern $\mathfrak{C}_I$ this same reasoning does not apply because of indirect fiscal spillovers induced by the interaction of private agents and union-wide monetary policy.[13]

The main contribution of Chari and Kehoe is to discuss thoroughly the subtle role of private sector behavior in this context. In general, it is well-known that monetary policy, if it cannot credibly move prior to the other actors, may be a source of indirect fiscal spillovers in a monetary union, reflecting the logic of a last-round bailout motive of monetary policy. For this argument to prevail under the particularly stringent assumptions A1' and A2 it is crucial that non-cooperative private sector behavior reinforces these spillovers such that monetary policy cannot undo them at the margin by means of a simple envelope theorem argument. To put it differently, if private sector agents expect a monetary reaction to earlier fiscal decisions and if the private sector itself suffers within each country from a (plausible) coordination problem, then this latter feature creates a fiscal cooperation problem in the first place which cannot be undone by monetary policy at a later stage.

[12] See Propositions 1 and 2, Chari and Kehoe (p. 9-12, 2002).

[13] To rephrase this finding in the more detailed language of our Proposition 1 (which, anyway, does not apply in full because of the neglect of private sector cooperation): indirect fiscal within-coalition spillovers exist under $\mathfrak{C}_I$ (but not $\mathfrak{C}_{II}$), because of between-coalition spillovers between fiscal players and private players, i.e. these latter spillovers are themselves a function of the timing of the monetary policy action.

In sum, by invoking the special assumptions A1' and A2, Chari and Kehoe emphasize a source of (indirect) fiscal spillovers which is specific to monetary unions.
To conclude this subsection it is worth making three comments. First, as summarized in the Appendix, one can show that under the special assumptions A1' and A2 the commitment pattern C_I is, in fact, the only one which makes fiscal cooperation relevant. Second, it is well understood that in the broad class of economies studied by Chari and Kehoe fiscal cooperation becomes under all commitment patterns generically relevant if one relaxes A1' and allows for direct fiscal spillover effects between countries, irrespective of whether the lack of commitment of monetary policy may induce an additional fiscal cooperation problem. Third, given the discussion underlying Proposition 3, it is clear that linear-quadratic specifications of the Chari-Kehoe economy exist which lead to additional constraints that make coalition structures and commitment patterns irrelevant. In particular, in order to ensure that the crucial derivative $\frac{\partial U_{ij}}{\partial a_{ik}}$ vanishes in equilibrium, these specifications need to satisfy that the number of squared gaps in the payoff functions of all players matches the number of available independent instruments, without sacrificing the overall structure imposed by (11), (13) and (14).

3.3 The Dixit-Lambertini model

The model of Dixit and Lambertini (2003) can be rewritten as a closely related variant of (8)-(11) which replaces (8) and (9) against

$$U_{ij} = U_i = U = U(a, \pi) \tag{15}$$

$$V_i = V_i(a, \tau_i, \boldsymbol{\tau}_{-i}, \pi). \tag{16}$$

Considering (15) and (16), two deviations from the benchmark model are worth stressing. First, there exists a uniform private sector throughout the monetary union such that private sector behavior reduces to $a_{ij} = a$ for all i, j, implying that there are no direct private spillovers, be they within countries or between countries. However, the payoff function V_i allows, in general, for direct fiscal spillover effects between countries. Second, payoff functions of private agents and policymakers are not congruent. In short, key features of this set-up can be summarized as:

A1": *Existence of direct fiscal spillovers between countries.*
A2": *Non-congruence of payoff functions of private agents and policymakers.*

Notice that because of these assumptions, unless further restrictions are introduced, there exist direct benefits from fiscal cooperation between countries. Moreover, the framework allows, in principle, for direct between-coalition spillover effects (as captured by $\partial U/\partial \pi$ and $\partial V_i/\partial a$), making also commitment patterns non-trivial. Notwithstanding these two properties, the analysis of Dixit and Lambertini gives rise to a general irrelevance proposition of coalitions structures and commitment patterns. The driving force behind this strong result is easily identified if one recognizes that the analysis is conducted within a linear-quadratic framework in line with Section 2.7. Specifically, the scalar a, summarizing union-wide private sector actions, denotes private sector inflation expectations, i.e.

$a \equiv \pi^e$, and all equilibria satisfy the assumption of rational expectations such that $\pi^e = \pi$. This feature can be recovered from writing U as

$$U = U(\pi^e, \pi) = \frac{1}{2}(\pi - \pi^e)^2,$$

i.e. $\pi^e = \pi$ results from a minimization of the squared inflation forecast error. Moreover, the policy objective V_i represents a weighted sum of squared deviations of country-specific output (y_i) and union-wide inflation values from target values, denoted by y_i^* and $\pi^* = 0$, respectively, such that

$$V_i = \frac{1}{2}\left[\omega_i(y_i^* - y_i)^2 + \pi^2\right],$$

while the output levels depend linearly on the vector of actions $\mathbf{x} = (\pi^e, \tau_i, \boldsymbol{\tau}_{-i}, \pi)$[14]:

$$y_i = \overline{y_i} + \sum_{k=1}^{n} b_{ik}\tau_k + b_i(\pi - \pi^e). \tag{17}$$

By construction of U, V_i, and V^M, there is consensus on the target values between all players under all conceivable cooperation and commitment schemes. Hence, as shown in the Appendix, the economy satisfies all the requirements of Proposition 3, i.e. all direct spillovers between all players vanish at the equilibrium outcome of the game Γ^{Nash}. Because of this feature, this outcome is identical to the social optimum (i.e. all players always attain their target values and $U = V_i = 0$, $\forall i$), leading to a broad irrelevance result of cooperation and commitment which, in fact, covers also mixed coalitions between private agents and policymakers. In sum, the main proposition of Dixit and Lambertini (2003), adopted to our framework, can be summarized as follows:

Dixit-Lambertini (2003): *Assume there exist direct fiscal spillover effects between countries. Despite this feature, there are no benefits from fiscal cooperation, as long as there is agreement about all target values of all players in a linear-quadratic framework. In fact, these target values can be attained under arbitrary coalition structures and commitment patterns of all players.*[15]

[14] To facilitate a clear comparison with Chari and Kehoe (2002), our representation abstracts from two features of the original Dixit-Lambertini model which are, however, inconsequential for the key result. First, the original model decomposes inflation into a part controlled by the central bank and a contribution related to fiscal policies. Second, the original model has a certain stochastic flavour, in the sense that the variables $\overline{y_i}$, b_{ik}, and b_i are stochastic. Yet, since policymakers react after the realizations of these variables, the resulting ex post game is in line with the set-up of Section 2.7, where without loss of generality $\overline{\mathbf{y}}$ and $\mathbf{B}$ may also be seen as predetermined rather than as constant variables. This assessment covers also the final scenario in the original paper of so-called 'discretionary monetary leadership' where fiscal policy is strong enough to prevent genuine (ex ante) uncertainty. To see that the second point is inconsequential for the key result, see also our discussion below at the end of Section 4.1.

[15] The exact wording in Proposition 1 in Dixit and Lambertini (2003, p. 245) is as follows: "If the monetary and fiscal authorities in a monetary union have identical output and inflation goals, those goals can be achieved without the need for fiscal coordination, without the need for monetary commitment, irrespective of which authority moves first and despite any disagreement about the relative weights of the two sets of objectives." Under the particular assumption of reducing private sector behaviour to forecasting inflation, the notion of 'arbitrary' timing protocols of private sector activities is not meaningful. Yet, in a refined model with richer private sector strategies this would be different.

This result is refreshing and provocative at the same time since it challenges the conventional wisdom that the existence of spillovers should create meaningful commitment and cooperation problems. Certainly, the model is special in many ways. For example, private sector actions are restricted to the assumption of rational inflation expectations at the union-wide level. Similarly, there is no role for country-specific inflation effects on national output levels, i.e. possible tensions between such effects and policy reactions of the central bank to union-wide inflation developments are ruled out. However, it would be possible to introduce refinements of the model along these lines such that the irrelevance proposition would still be supported.[16]

Hence, the limitations of this proposition are linked to more fundamental concerns. First, compared with the analysis of Chari and Kehoe (2002), it is clear that linear-quadratic frameworks, while being convenient short-cuts, are, by construction, very special. Second, within the above summarized linear-quadratic framework the irrelevance proposition requires that the number of squared gaps matches the number of available independent instruments. Specifically, in the just summarized set-up there are $N+2$ players ($N+1$ policymakers and 1 private sector player) who face $N+2$ gaps and command over $N+2$ independent instruments, as embodied in the vector $\mathbf{x} = (\pi^e, \boldsymbol{\tau}, \pi)$.[17] To see the importance of the assumption that there is no 'instrument shortage' it is constructive to consider the closed-economy counterpart of the model without fiscal policy. Then, the analysis collapses to the standard monetary policy model of Barro and Gordon (1983) where the relevance of monetary commitment is well-known, reflecting the trade-off faced by monetary policy to meet output and inflation objectives with a single instrument. Moreover, it is worth emphasizing that Dixit and Lambertini (2003) assume that monetary and fiscal policymakers differ systematically in their effectiveness vis-à-vis the private sector. Monetary policy suffers from the well-known time inconsistency problem, i.e. in a rational expectations equilibrium ($\pi = \pi^e$) monetary policy cannot close the (structural) output gap $(y_i^* - \overline{y_i})$, while fiscal polices do not face such a restriction. This fundamental difference between the two types of policymakers is remarkably different from the otherwise symmetric treatment of all policymakers.[18]

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[16] There exist hybrid monetary unions models, like Calmfors (2001), which respect for some, but not all reduced form equations, the linear-quadratic structure. Yet, to use them as counterexamples to the reasoning of Dixit and Lambertini is not entirely satisfactory.

[17] The 'independence' assumption can be questioned if one explicitly acknowledges that all policy instruments are tied together by a combined budget constraint of the public sector, as stressed, in particular, by Cooper and Kempf (2004). Otherwise the model of Cooper and Kempf (2004) is very different from Dixit and Lambertini (2003). In particular, it is not of the linear-quadratic variety. However, from a broader perspective, the neglect of the budget constraint could be rationalized if one thinks about policy actions without (direct) budgetary incidence, like reform measures which affect the competitiveness of industries etc. Moreover, in a narrow fiscal context, one could assume that national treasuries have access to balancing items which do not create spillovers between countries.

[18] For an analysis of an economy in which both policymakers face a time inconsistency problem, see, for example, Adam and Billi (2006).

4 International monetary policy cooperation

The purpose of this section is to show that the literature on international monetary policy cooperation among fully sovereign nations offers clear analytical counterparts to our discussion of the (ir)relevance of cooperation and commitment in monetary unions. This assessment holds true for so-called first-generation models (with ad-hoc payoff functions similar to the economies covered so far) as well as for the by now widely used second-generation models (where the payoff functions of policymakers are made fully consistent from first principles with the welfare objectives of private agents).[19] For either type of model 'irrelevance' results obtain under particular assumptions. As we show in the remainder of this section, the relevant features of these assumptions can be reproduced within our general framework. Our key references for first-generation models are Rogoff (1985) and Canzoneri and Henderson (1988, 1991), while our discussion of second-generation models takes the analysis of Obstfeld and Rogoff (2000, 2002) as well as the summary paper by Canzoneri et al (2005) as the main reference points. Given the widespread use of stochastic settings in this literature, we invoke results established in Section 2.8.

4.1 First-generation models

Rogoff (1985) and Canzoneri and Henderson (1988, 1991) offer widely cited contributions of the first-generation type which give clear insights about the nature of cooperation and commitment problems in international monetary policymaking.[20] The Canzoneri-Henderson model is often referred to because it can be used to see the existence of generic benefits from cooperation between policymakers, while the analysis of Rogoff (1985) gives rise to the insight that such benefits can prove elusive for certain commitment patterns (depending, in particular, on the timing of private sector actions).[21]
Both contributions study symmetric two-country set-ups, leading to reduced forms which duplicate the Barro-Gordon trade-offs. These trade-offs, however, are enriched with monetary spillovers between the two countries. Moreover, both studies use linear-quadratic set-ups. We offer a simplified representation which, while capturing the main insights from the two studies, is kept deliberately similar to the exposition of the Dixit-Lambertini model discussed above.[22]

There exist two equally sized and structurally identical countries with two independent currencies. In each country, the monetary policymaker controls domestic inflation (i.e. π_i is the single instrument of the monetary policymaker in country i) and he faces an output and an inflation objective. The inflation objective creates monetary spillover effects between countries. Specifically, the inflation objective is defined in terms of CPI-inflation

[19] These labels are borrowed from Canzoneri et al. (2005).

[20] For further important contributions to this literature, see, among others, Hamada (1985) and Oudiz and Sachs (1984), and the well-structured surveys in Devereux (1990) and Persson and Tabellini (1995).

[21] For a similar insight in a model with international fiscal policy cooperation, see Kehoe (1989).

[22] Similar to our representation, see also the discussion in Walsh (2003, ch. 6.3). In particular, deviating from the original contributions we do not cast monetary policy in terms of money supplies, but directly in terms of inflation outcomes.

which, because of trade linkages, depends on both domestic and foreign inflation. Moreover, the output levels in the two countries also depend on the monetary policy instruments of both countries. To express these interdependencies, exchange rate patterns need to be specified. To this end, let

$$e_r = e_n + \pi_2 - \pi_1,$$

where e_r and e_n denote the change in the real and the nominal exchange rates, respectively. Accordingly, an increase in e_r amounts to a real depreciation of the exchange rate from the perspective of country 1. CPI-inflation of the two countries is given by

$$\pi_1^{CPI} = \beta\pi_1 + (1-\beta)(\pi_2 + e_n) = \pi_1 + (1-\beta)e_r \quad (18)$$
$$\pi_2^{CPI} = \beta\pi_2 + (1-\beta)(\pi_1 - e_n) = \pi_2 - (1-\beta)e_r, \quad (19)$$

with $\beta \in (0,1)$ denoting the openness of the countries in terms of consumption (i.e. a value of β close to 1 represents strong home bias). Output in the two countries is given by

$$y_1 = \overline{y} + b_\pi(\pi_1 - \pi_1^e) - b_e(e_r - e_r^e) + \varepsilon \quad (20)$$
$$y_2 = \overline{y} + b_\pi(\pi_2 - \pi_2^e) + b_e(e_r - e_r^e) + \varepsilon, \quad (21)$$

where $\varepsilon \sim(0, \sigma_\varepsilon^2)$ denotes a common productivity shock, while $b_\pi > 0$ and $b_e < 0$ denote the direct output effects of inflation and real exchange rate surprises.[23] In line with Rogoff (1985), consider the following timing protocol, to be modified below. Private agents in both countries act (i.e. form rational expectations of π_1, π_2, and e_r) prior to the realization of ε. Specifically, private agents, anticipating symmetric policy reactions to the common shock ε, correctly expect that the real exchange rate remains unchanged, i.e. $e_r^e = e_r = 0$. Policymakers in both countries act after the shock ε has been observed. Assuming $\pi_i^{CPI*} = 0$, the expected (ex-ante) payoffs of policymakers are given by

$$E[V_i] = \frac{1}{2}E\left[\omega(y^* - y_i)^2 + (\pi_i^{CPI})^2\right], \quad i = 1, 2. \quad (22)$$

Policymakers are assumed to follow rules which are linear in ε, in line with (4) in Section 2.8. Given the assumed timing protocol, it is instructive to consider first games which involve only the two policymakers. When optimizing ex ante over the reaction coefficients in the policy rules to maximize (22), policymakers take as given the rational expectations of exchange and inflation rates of the private sector (which can be calculated by backward induction). Then, comparing cooperative vs. non-cooperative behavior of policymakers, it is possible to show

$$E[V_i^{nc}] - E[V_i^c] = \frac{1}{2}\sigma_\varepsilon^2\left[\phi_\varepsilon^{nc} - \phi_\varepsilon^c\right] + \frac{1}{2}(y^* - \overline{y})^2\left[\phi_{y^*}^{nc} - \phi_{y^*}^c\right] \quad i = 1, 2 \quad (23)$$
$$\text{with} : \phi_\varepsilon^{nc} - \phi_\varepsilon^c > 0 \text{ and } \phi_{y^*}^{nc} - \phi_{y^*}^c < 0,$$

[23] This assumption is necessary to ensure that the perceived total effect of inflation surprises on home output ($b_\pi + b_e$) is smaller than the direct effect (b_π), reflecting the dampening effect of a real appreciation of the exchange rate on home output. Hence, under non-cooperation, the exchange rate channel has a disciplining effect on inflation incentives. This feature is key for the results summarized below.

where the latter condition requires a mild restriction on b_e and β which, together with the ϕ-terms, is stated in the Appendix. Notice that $E[V]$ is expressed as a 'loss', i.e. low values should be preferred to high ones. From the expressions for the $\phi-$terms one easily verifies that the cooperative and non-cooperative solutions coincide in the special case in which there are no direct monetary spillovers between countries (requiring $\beta = 1$ and $b_2 = 0$). In sum, this stylized representation reproduces the following two well-known results from the literature.

Rogoff (1985) and Canzoneri-Henderson (1988, 1991):
In general, coalition structures and commitment patterns are not irrelevant, notwithstanding agreement about all target values of all players.
i) Assume $y^* = \overline{y}$, i.e. monetary policymakers face no time inconsistency problem. Then there exist benefits from cooperation between policymakers.
ii) Assume $y^* > \overline{y}$, i.e. monetary policymakers face a time inconsistency problem. Then there exist costs and benefits from cooperation between policymakers, and for $y^* - \overline{y}$ being sufficiently large (i.e. the time inconsistency problem being sufficiently severe), cooperation between policymakers is not desirable.

Why is cooperation between policymakers, generically, relevant under the assumed commitment pattern? Generally speaking, the equation system (18)-(22) fits the framework discussed in Section 2.8. However, to undo the direct within-coalition spillover effects between policymakers in line with Proposition 4 would require that the number of objectives matches the number of independent instruments. This is not the case, since in (22) there are altogether four gaps to be closed (two output gaps and two CPI-inflation gaps), while there are only two instruments (π_1, π_2) available to the policymakers. This mismatch between instruments and gaps rules out that the direct spillovers vanish at the no-cooperation and no-commitment Nash game played by the two policymakers. Moreover, with Proposition 4 being *not* satisfied, it is clear that these spillovers give rise to strategic conflicts over the choice of the policy instruments also under alternative commitment patterns.
We conclude this subsection by making three comments. First, the assumed timing protocol (i.e. the assumption that the private sector acts prior to the realization of ε, while policymakers act afterwards) helps to separate in the cooperation problem of policymakers stabilization aspects (captured by σ_ε^2) from systematic inflation incentives (captured by $(y^* - \overline{y})^2$). However, this protocol can also be used to see why first-generations models have been criticized for their lack of convincing microfoundations. Specifically, under this timing protocol it is clear that inflation expectations, while being on average correct, will differ from realized inflation if $\varepsilon \neq 0$. But how to account for this feature from a welfare perspective? Assume, for example, that the ex-ante payoffs of the representative private agents in the two countries are given by

$$E[U_i] = \frac{1}{2}E[(\pi_i - \pi_i^e)^2], \quad i = 1, 2.$$

Then, as shown in the appendix, it is straightforward to establish

$$E[U_i^{nc}] - E[U_i^c] = \frac{1}{2}\sigma_\varepsilon^2\,[\psi_\varepsilon^{nc} - \psi_\varepsilon^c] < 0 \quad i = 1, 2, \tag{24}$$

indicating that under this particular measure private agents would *always* prefer non-cooperation of policymakers.[24] This finding is not in line with the comparison based on the ex ante payoffs of policymakers summarized above. Given the ad-hoc specification of the model there is no metric which could be used to address this discrepancy, indicating why the profession has recently shifted to models with explicit microfoundations, as discussed in Section 4.2.
Second, the instrument shortage does not disappear if one considers the alternative timing protocol under which all players, policymakers and private agents, follow rules which call for action after the realization of ε. Compared with the previous discussion, this modification ensures that ex post realized inflation and inflation expectations of the private sector will always be identical. However, it is easy to check that policymakers would still lack independent instruments to close the gaps related to output and inflation.
Third, given the close relationship of the reduced forms with the Dixit-Lambertini framework, these results may be at first sight somewhat surprising. Yet, the discrepancy simply reflects that Rogoff (1985) and Canzoneri and Henderson (1988, 1991) deliberately allow for a mismatch between instruments and objectives which is absent in the Dixit-Lambertini analysis. It would be straightforward to overcome this mismatch within the above framework if one introduced in each country one additional (fiscal) player with an additional and independent instrument that relates linearly to output and CPI-inflation. To this end, assume, ceteris paribus, instead of (20)-(21) and in line with (17)

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \overline{y} \\ \overline{y} \end{pmatrix} + \begin{pmatrix} b_\pi(\pi_1 - \pi_1^e) - b_e(e_r - e_r^e) \\ b_\pi(\pi_2 - \pi_2^e) + b_e(e_r - e_r^e) \end{pmatrix} + B_\tau \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} + \begin{pmatrix} \varepsilon \\ \varepsilon \end{pmatrix},$$

where τ_1 and τ_2 denote the two additional fiscal instruments and the $2 \times 2-$matrix B_τ is assumed to be invertible. Assume private sector expectations are formed prior to the realization of ε, while the four policymakers move afterwards. Then, the policymakers, when acting non-cooperatively, will be able to achieve $y_i = y^*$ and $\pi_i^{CPI} = \pi_i^{CPI*} = 0$ for all realizations of ε and consistent with rational private sector expectations ($\pi_i = \pi_i^e$ and $e_r = e_r^e = 0$, $i = 1, 2$) by setting their four instruments according to the rules

$$\begin{aligned} \pi_1 &= \pi_2 = 0 \\ \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} &= \begin{pmatrix} \widetilde{b}_{11} + \widetilde{b}_{12} \\ \widetilde{b}_{21} + \widetilde{b}_{22} \end{pmatrix} (y^* - \overline{y}) + \begin{pmatrix} \widetilde{b}_{11} + \widetilde{b}_{12} \\ \widetilde{b}_{21} + \widetilde{b}_{22} \end{pmatrix} \varepsilon, \end{aligned}$$

where $\widetilde{b}_{ij}$, $i, j = 1, 2$ denotes the representative entry of the matrix B_τ^{-1}. Consistent with rational private sector expectations, the same outcome can be non-cooperatively implemented by policymakers if the private sector forms expectations after the realization of ε. In sum, it is easy to see that, by adding two additional instruments in the spirit of Dixit and Lambertini, Proposition 4 applies within the augmented framework, leading to a broad irrelevance result of coalitions structures and commitment patterns among international policymakers, covering both monetary and fiscal policymakers. Alternatively,

[24] Intuitively, the private sector does not bear costs associated with the inflation bias (i.e. the equilibrium level of inflation), but only with the forecast error around this level. This error is larger under cooperation, because policymakers use the inflation instrument more actively for any given realization of ε.

rather than increasing the number of instruments, with the same effect a reduction of objectives could be considered: for example, if the two countries were to focus solely on the inflation objective (i.e. $\omega_i = 0$), the desirability of cooperation, once more, would disappear.

4.2 Second-generation models

Cooperation issues are also of central importance in second-generation models. In these models the sources of strategic conflict between countries are, by and large, similar to first-generation models. Yet, the welfare measure of policymakers is consistently derived from optimizing private sector behavior in a fully specified general equilibrium setting. The latter feature implies that the range of different policy games that can be studied within such a set-up is typically large, reflecting the richness of the underlying general equilibrium specification. Moreover, the strategic interaction between policymakers is typically addressed from an ex ante perspective in rule setting games. This feature, combined with the microfounded welfare objective, leads to a genuine importance of risk aspects (which could not be adequately addressed by first-generation models). However, despite these differences, it can be shown that also in second-generation models gains from cooperation can entirely disappear if policymakers have access to sufficiently many instruments and the economy has a suitable linear-quadratic representation.[25]

The framework of Obstfeld-Rogoff (2000, 2002) has been particularly influential for this class of models. It considers a two-country extension of a New Keynesian framework with imperfect competition and sticky wages, thereby ensuring that monetary policy has an effective stabilization role. While the private sector sets nominal wages before the uncertainty has been resolved (which is captured by random productivity shocks), monetary policymakers act after these shocks have been realized, subject to ex ante chosen state contingent rules. To establish a natural benchmark for the assessment of the welfare effects of monetary policy, Obstfeld and Rogoff decompose the national welfare objectives of the two policymakers, which coincide with the expected welfare of representative private agents in the two countries (and will be denoted below by $E(V)$ and $E(V^*)$), into flexible-wage components and residual components which capture the additional effects coming from the existence of sticky wages. Based on this decomposition, a two-step procedure can be invoked to check the (ir)relevance of monetary policy cooperation between the two countries. First, it needs to be checked whether the flexible wage solution around which the monetary stabilization takes place is 'constrained Pareto efficient ex ante'. Broadly speaking, this criterion will be satisfied if the sticky wage distortion is the only general equilibrium distortion which is affected by monetary policy. By contrast, if the set-up allows for further (and genuine open-economy) imperfections that can be affected by monetary policy, like risk-sharing concerns under imperfect capital markets, this criterion is

[25] In general, the papers cited in this Secton focus on the (ir)relevance of policy cooperation. However, for the particular cases in which the irrelevance of policy cooperation can be established by means of the Corollary to Proposition 4, this also implies the irrelevance of commitment patterns, i.e. policymakers may be called to implement their actions under arbitrary orders of moves.

typically no longer satisfied.[26] Second, assuming that the first criterion is satisfied, it needs to be established whether the flexible wage solution can be implemented by monetary policy. Obstfeld and Rogoff offer a fully tractable framework which can be used to address these features with closed-form solutions. Reproducing the model in all its details goes beyond the scope of this paper. Instead, by drawing also on the representation of the model by Canzoneri et al (2005), we offer a sketch that shows how the principles which drive the (ir)relevance of policy cooperation can be linked to the Corollary to Proposition 4 of Section 2.8.

Consider the following sketch of a symmetric model of two countries, home and foreign. Each of these countries produces three types of monopolistically competitive goods (with the foreign produced goods denoted by a star): an aggregate non-traded good (N, N^*), an aggregate tradeable good for the domestic market (D, D^*), and an aggregate tradeable good for the export market (E, E^*). The latter type of goods creates the key channel of policy interaction between the two countries, i.e. the good E (E^*) is produced in the home (foreign) country and consumed in the foreign (home) country, with the terms of trade being determined by monetary policy. In all sectors, production technologies are linear and labor is the only factor of production. Preferences of the representative consumer-producer (h) in the home country are given by[27]

$$\begin{aligned} U(h) &= \frac{C(h)^{1-\rho}}{1-\rho} - [Y_N(h)/E_N + Y_D(h)/E_D + Y_E(h)/E_E] \\ C(h) &= C_N^{\frac{1}{3}}(h) C_D^{\frac{1}{3}}(h) C_{E^*}^{\frac{1}{3}}(h), \end{aligned}$$

where C denotes a Cobb-Douglas consumption aggregator of the three types of goods, ρ denotes the constant coefficient of relative risk aversion, $Y_j(h)$ the output levels of the three types of goods produced by producer (h), and E_j the corresponding random sector-specific productivity levels. Under flexible wages, wage setters in the underlying Dixit-Stiglitz economy can respond to the productivity shocks, while under sticky wages productivity levels are realized after nominal wages have been set.[28] Each policymaker has one instrument, the nominal money supply (M, M^*), which can respond to the realized productivity shocks, subject to an ex-ante chosen rule. In equilibrium, markets for all goods clear and the current account is assumed to be balanced every period, requiring appropriate adjustments of equilibrium prices and the (flexible) exchange rate for given money supplies.

Importantly, under the particular assumption of logarithmic utility ($\rho = 1$), the model exhibits for all conceivable patterns of shocks perfect international risk sharing of con-

[26] For systematic discussions of interactions between closed-economy and open-economy distortions in closely related models, see, among others, Corsetti and Pesenti (2001) and Benigno (2002).

[27] The neglect of monetary balances reflects the assumption of the cashless limit. This assumption is important since it contributes to the central feature of the model that any direct monetary spillovers between countries enter the welfare objective only through the consumption channel.

[28] Canzoneri et al. compare directly an environment of flexible and sticky output prices (measured in terms of producer currency prices), while Obstfeld and Rogoff specify the analysis in terms of flexible and sticky wages. However, with labour being the only production input and constant and identical elasticities of demand across goods, this different representation is inconsequential.

sumption risks in tradeable goods, reflecting the separability of utility in tradeable and non-tradeable goods.[29] This feature ensures that the flexible wage solution becomes constrained Pareto efficient ex ante. In other words, taking as given the non-monetary distortions of the economy related to monopolistic competition, monetary policy cannot Pareto-improve upon replicating the flexible wage equilibrium. Assuming $\rho = 1$, the welfare objectives faced by the two policymakers have a representation of the form

$$E(V) = E(\widetilde{V}) + \boldsymbol{\omega}'\Omega_{\mathbf{y}}\boldsymbol{\omega} \quad \text{and} \quad E(V^*) = E(\widetilde{V^*}) + \boldsymbol{\omega}^{*\prime}\Omega_{\mathbf{y}}\boldsymbol{\omega}^*$$

where $E(\widetilde{V})$ and $E(\widetilde{V^*})$ denote the flexible-wage components in the two countries and where the vector $\mathbf{y}$ summarizes the sources of volatility (with associated variance-covariance matrix $\Omega_{\mathbf{y}}$ and country-specific weighting vectors $\boldsymbol{\omega}$ and $\boldsymbol{\omega}^*$), which make the sticky wage economies depart from the flexible wage economies. As stressed by Canzoneri et al (2005), the vector $\mathbf{y}$, in general, summarizes two distinct effects: i) the effects of the altogether six sector-specific productivity shocks and ii) the demand effects of monetary policy which operate not sector-specific, but proportional to aggregate demand in the two countries. Using the notation $\varepsilon_j = \log(E_j)$ and $\mathbf{x} = (m, m^*)$, with $m = \log(M)$ and $m^* = \log(M^*)$, it can be shown that the 6×1-vector $\mathbf{y}$ can be represented as

$$\mathbf{y} = \overline{\mathbf{y}} + \mathbf{B}_x\mathbf{x} + \mathbf{B}_{\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon},$$

where $\mathbf{y}' = (y_N, y_D, y_{E^*}, y_{N^*}, y_{D^*}, y_E)$, $\boldsymbol{\varepsilon}' = (\varepsilon_N, \varepsilon_D, \varepsilon_{E^*}, \varepsilon_{N^*}, \varepsilon_{D^*}, \varepsilon_E)$ and $\mathbf{B}_x$ and $\mathbf{B}_{\boldsymbol{\varepsilon}}$ denote 6×2 and $6 \times 6-$matrices, respectively.[30] Monetary policy actions are restricted to satisfy

$$\mathbf{x} = \overline{\mathbf{r}} + \mathbf{R}_{\varepsilon}\boldsymbol{\varepsilon}.$$

Within this representation it is easy to see that for arbitrary realizations of $\boldsymbol{\varepsilon}$ the two monetary policymakers do not have sufficient instruments to ensure that in the non-cooperative game ex-post all entries of $\mathbf{y}$ (and, hence, of $\Omega_{\mathbf{y}}$) will be set to zero, since $\mathbf{B}_x$ is not invertible. Exploiting this feature, Canzoneri et al. conclude that, despite assuming $\rho = 1$, cooperative and non-cooperative behavior of monetary policymaker leads in general to different outcomes.[31] However, as a special case within this representation, it is straightforward to establish the benchmark irrelevance result of monetary policy cooperation obtained by Obstfeld and Rogoff if one abstracts from sector-specific shocks and imposes instead that there only exist two country-specific shocks, i.e. $\varepsilon_N = \varepsilon_D = \varepsilon_E = \varepsilon$ and $\varepsilon_{N^*} = \varepsilon_{D^*} = \varepsilon_{E^*} = \varepsilon_*$. Ceteris paribus, this ensures that there remain only two sources of volatility which lead to deviations of the sticky wage economy from the flexible wage economy. Hence, there exists a representation

$$\begin{pmatrix} y \\ y^* \end{pmatrix} = \begin{pmatrix} \overline{y} \\ \overline{y}^* \end{pmatrix} + \mathbf{B}_x \begin{pmatrix} m \\ m^* \end{pmatrix} + \mathbf{B}_{\boldsymbol{\varepsilon}} \begin{pmatrix} \varepsilon \\ \varepsilon_* \end{pmatrix}.$$

[29] For a closely related analysis, see Proposition 2 in Clarida et al. (2002, p. 897).

[30] For this representation to be exact under $\rho = 1$, it requires that all shocks are log-normally distributed. Moreover, to preserve consistency with our Section 2.8, notice that, slightly different from the notation in Canzoneri et al (2005, p. 373), the elements of the vector $\mathbf{y}$ do not stand for the logarithms of sectoral output levels per se, but they rather capture the differences between the logarithms of sectoral output and productivity levels, i.e., using *their* notation, they denote $y_j - z_j$.

[31] For this conclusion, see Proposition 3 in Canzoneri et al. (2005, page 376).

This representation, with the 2×2-matrix $\mathbf{B}_x$ being invertible, satisfies all the requirements for the Corollary to Proposition 4 to apply. In other words, non-cooperative and cooperative outcomes of monetary policy are identical and the flexible wage solution can be ex post implemented for arbitrary realizations of ε and ε_*.
The analytical strength of the Obstfeld-Rogoff framework is reflected by the fact that clear insights can also be obtained for environments characterized by $\rho \neq 1$. Under this assumption, the availability of sufficiently many stabilization tools (as required for the implementation of the flexible wage outcome) does no longer suffice to ensure that non-cooperative ('self-oriented') Nash policies coincide with the cooperative outcome. Intuitively, the risk-sharing criterion comes in as an additional welfare objective which changes the trade-offs under cooperative and non-cooperative policies, implying that, in general, $E(\widetilde{V})$ and $E(\widetilde{V^*})$ are no longer independent of $\mathbf{x}$. However, for the special case in which the country-specific shocks are identical in the two countries (i.e. caused by the same global shock such that $\varepsilon = \varepsilon_*$) this latter concern is no longer relevant. In other words, assuming $\rho \neq 1$, it can be shown that under the particular assumption that all shocks are only of global nature the irrelevance proposition will be restored.
In sum, adapted to our framework, this reasoning can be summarized as follows:

Obstfeld-Rogoff (2000, 2002) and Canzoneri et al (2005):
Assume there exist direct monetary spillover effects between countries. Then, coalition structures and commitment patterns between policymakers, in general, are not irrelevant. However, i) if the flexible wage solution is constrained Pareto efficient ex ante and ii) if there exist sufficiently many instruments in a linear-quadratic framework to stabilize the economies at this solution, coalition structures and commitment patterns between policymakers become irrelevant.

In order to obtain an irrelevance result along these lines, both criteria need to be satisfied. As indicated by the discussion of the risk-sharing criterion, often the two criteria cannot be independently assessed. Because of these features, the related literature typically concludes that the *theoretical* requirements for a complete absence of gains from cooperation in second-generations models are very restrictive. This can be seen from the following and not exhaustive list of variations of the Obstfeld-Rogoff benchmark model, stressing channels which are different from Canzoneri et al. (2005). For example, Devereux and Engel (2003) and Corsetti and Pesenti (2005) relax the assumption of producer currency pricing and re-establish gains from cooperation under different pricing regimes. In particular, gains from coordination arise for 'intermediate pricing cases' in which there is neither zero nor complete pass-through from exchange rate changes to consumer currency prices. Benigno and Benigno (2006) drop the assumption of a unit elasticity of substitution between home and foreign goods and argue that, in general, the terms-of-trade channel becomes strategically relevant if the intertemporal and intratemporal elasticities of substitution in consumption are different from each other. Moreover, they allow for alternative types of shocks (like mark-up shocks) and argue that in optimal cooperative outcomes the exchange rate regime itself might, in fact, be different for different types of shocks. Like Benigno and Benigno (2006), Sutherland (2004) as well as Lombardo and Sutherland

(2004) also drop the assumption of a unit elasticity of substitution between home and foreign goods. Sutherland (2004) finds that gains from cooperation depend significantly on the assumed asset market structures, in particular with respect to their ability to facilitate consumption risk sharing. Lombardo and Sutherland (2004) consider an extension of the Obstfeld-Rogoff framework in which both monetary and fiscal policies can be used as stabilization tools and identify conditions under which fiscal cooperation improves welfare, both under flexible and sticky prices and conditional on the (non)-cooperation between monetary policies.[32]

It should be emphasized, however, that despite this range of rather unambiguous theoretical predictions the verdict on the *quantitative* relevance of spillovers in second-generation models is still very much open. In particular, Obstfeld and Rogoff (2002, p. 504) argue that "...our attempts to parameterize our model suggest that even when cooperation is beneficial in theory, it may be relatively unimportant empirically". By contrast, Canzoneri et al (2005, p. 364) "...conclude - based on theoretical considerations and a first pass calibration of our model - that second generation models may have more scope for policy coordination than did the first."[33] It seems safe to conclude that this debate will not be solved soon, given the inherently complex relationship between policy instruments and model-specific welfare objectives in second-generations models. At the same time, this insight points at the importance of tractable benchmark models, as discussed in this section, which give rise to unambiguous analytical results and which therefore can give some structure to the quantitative findings from larger models.

5 Conclusion

In this paper, we set up a general framework to address the importance of commitment patterns and cooperation schemes in policy games between various policymakers. We prove that the nature of spillover effects between agents is of key relevance to answer this issue. To this end, we offer a simple classification of spillover effects between agents which distinguishes between within-coalition and between-coalition spillover effects. Based on this classification, we provide general propositions which prove that under some conditions, linked to these spillover effects, commitment and cooperation schemes do not matter. In particular, frameworks with a certain linear-quadratic structure lead to the conclusion that commitment and cooperation issues are entirely irrelevant. Yet, the conditions which are responsible for this puzzling result are shown to be rather restrictive and, more importantly, they have no longer any bite in a generic, non-linear-quadratic environment.

Having established the general principles for this classification in the first part of the paper, we then apply these principles to explain the driving forces behind seemingly contradictory results from a number of recent contributions on the nature of policy interactions both within monetary unions and among fully sovereign nations.

[32] The paper also discusses these findings in a monetary union version of the model, related to the analysis by Beetsma and Jensen (2005).

[33] For a more comprehensive discussion of quantitative findings in this context, see Coenen et al. (2007).

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6 Appendix

6.1 Proof of Proposition 1

We consider two-stage games Γ and Γ', allowing for coalitions among subsets of agents. It is easy to generalize the proof to games with more stages. Consider first a game Γ. We partition the set of players into two subsets Ξ_1 and Ξ_2. Ξ_1 (Ξ_2) is formed of players making their decision at stage 1 (2). At each stage, coalitions may be active. There are K (L) coalitions at stage 1 (2), denoted by C_k (C_l). We denote by $\mathcal{C}_1$ ($\mathcal{C}_2$) the set of coalitions formed in stage 1 (2). For a given structure of coalitions, the game is solved by subgame perfection. An interior SPNE outcome $\mathbf{z}$ of the game satisfies the following conditions:
At stage 2,

$$\omega_\xi \frac{dV_\xi(\mathbf{z})}{dx_\xi} + \sum_{\xi' \in C_l,\ \xi' \neq \xi} \omega_{\xi'} \frac{dV_{\xi'}(\mathbf{z})}{dx_\xi} = 0, \qquad \forall \xi \in C_l, \forall C_l \in \mathcal{C}_2 \tag{25}$$

where the first term captures the effect of the action of player ξ on his own welfare, while the second term describes the direct within-coalition spillover effects on the coalition members in the coalition C_l. Since stage 2 is the final stage of the game, there are by construction no indirect within-coalition spillover effects.
At stage 1,

$$\begin{aligned} & \omega_\xi \frac{\partial V_\xi(\mathbf{z})}{\partial x_\xi} + \omega_\xi \left[\sum_{C_l \in \mathcal{C}_2} \sum_{\xi'' \in C_l} \frac{\partial V_\xi(\mathbf{z})}{\partial x_{\xi''}} \frac{\partial x_{\xi''}}{\partial x_\xi} \right] \\ & + \sum_{\xi' \in C_k,\ \xi' \neq \xi} \omega_{\xi'} \frac{\partial V_{\xi'}(\mathbf{z})}{\partial x_\xi} + \sum_{\xi' \in C_k,\ \xi' \neq \xi} \omega_{\xi'} \sum_{C_l \in \mathcal{C}_2} \sum_{\xi'' \in C_l} \frac{\partial V_{\xi'}(\mathbf{z})}{\partial x_{\xi''}} \frac{\partial x_{\xi''}}{\partial x_\xi} \\ = \ & 0,\ \forall \xi \in C_k, \forall C_k \in \mathcal{C}_1. \end{aligned} \tag{26}$$

To describe stage 1 interactions, four effects can be distinguished. The first term captures the direct effect of the action of player ξ on his own welfare, while the second term describes the indirect effect on his own welfare through actions taken by players in coalitions formed in the second period. For this second term to be non-zero it is necessary that there exist direct between-coalition spillover effects between ξ and at least one player ξ'' acting at stage 2, i.e. $\frac{\partial V_\xi(\mathbf{z})}{\partial x_{\xi''}}$ must be non-zero for at least one pair ξ and ξ''. The third term describes the direct within-coalition spillover effects of the action x_ξ on the coalition members in the coalition C_k. Finally, the fourth term captures the indirect within-coalition spillover effects on the coalition members in the coalition C_k through actions taken by players in coalitions formed at stage 2. For this fourth term to be non-zero it is necessary that there exist direct between-coalition spillover effects between at least one other member of C_k and at least one player ξ'' acting at stage 2.

Correspondingly, one can derive the set of conditions applying to Γ'.

To ensure that (25) and (26) admit the same equilibrium outcome for two games Γ and Γ', the set of sufficient conditions summarized in Proposition 1 are derived from the following two-step procedure. First, to undo the effects of different commitment patterns in Γ and Γ', all direct between-coalition spillover effects between players acting at different stages are required to be zero at $\mathbf{z}$. This requirement ensures that the second and fourth term discussed in (26) vanish in equilibrium. Second, a condition is needed which addresses the effects of different coalitions structures in Γ and Γ'. Certainly, a sufficient condition would be to require that the direct within-coalition spillover effects for all coalitions formed in Γ and Γ' vanish at $\mathbf{z}$, implying that (25) and (26) reduce for both games to $\omega_\xi \frac{\partial V_\xi(\mathbf{z})}{\partial x_\xi} = 0$, $\forall \xi \in \Xi$. Yet, having controlled for possible differences in commitment patterns already in the first step, (25) and (26) admit for Γ and Γ' the same equilibrium outcome also under the weaker condition that the direct within-coalition spillover effects need to vanish at $\mathbf{z}$ only for those coalitions which are formed in Γ', but not in Γ, and vice versa.
This reasoning can be generalized to games with more than 2 stages, as any $h - stage$ extensive form game can be restated as a sequence of 2-stage extensive-form games. □

Remark at Proposition 2: If at $\mathbf{z}^{Nash}$ all direct spillovers between any pair of players vanish it is clear from (25) and (26) that $\mathbf{z}^{Nash}$ is a subgame perfect Nash equilibrium outcome for any possible extensive-form game characterized by arbitrary commitment patterns and coalition structures.

6.2 Proof of Proposition 3

Using (1) in (2), we can express V_ξ as:

$$\begin{aligned} V_\xi &= \frac{1}{2}[\omega_1^\xi(y_1^* - \overline{y_1} - \sum_{j=1}^{X} b_{1j}x_j)^2 + ... + \omega_p^\xi(y_p^* - \overline{y_p} - \sum_{j=1}^{X} b_{pj}x_j)^2 + ... \\ &\quad + \omega_X^\xi(y_X^* - \overline{y_X} - \sum_{j=1}^{X} b_{Xj}x_j)^2] \end{aligned}$$

In general, any Nash equilibrium outcome $\mathbf{z}^{Nash}$ of the simultaneous Nash game Γ^{Nash} played by the X players satisfies the set of conditions:

$$\frac{\partial V_\xi(\mathbf{z}^{Nash})}{\partial x_\xi} = 0, \forall \xi \in \Xi.$$

Because of the linear-quadratic structure, this set of equations can be expressed as follows:

$$\frac{\partial V_\xi(\mathbf{z}^{Nash})}{\partial x_\xi} = \sum_{p=1}^{X} \omega_p^\xi b_{p\xi} \left[y_p^* - \overline{y_p} - \sum_{j=1}^{X} b_{pj}x_j \right] = 0, \forall \xi \in \Xi \tag{27}$$

The direct spillover effect of ξ' on agent ξ's payoff is given by the expression:

$$\frac{\partial V_\xi(\mathbf{z}^{Nash})}{\partial x_{\xi'}} = \sum_{p=1}^{X} \omega_p^\xi b_{p\xi'} \left[y_p^* - \overline{y_p} - \sum_{j=1}^{X} b_{pj}x_j \right] \tag{28}$$

Let $\mathbf{z}^{Nash} = \mathbf{B}^{-1}\left[\mathbf{y}^{*} - \overline{\mathbf{y}}\right]$. This vector satisfies (27), as required for a Nash-equilibrium. Moreover, this solution is unique since it has been assumed that for every p the product $\omega_p^{\xi} b_{p\xi}^{x}$ is non-zero for at least one $\xi = 1, 2, ..., X$. Finally, at $\mathbf{z}^{Nash}$ for any pair (ξ, ξ') equation (28) is zero. Hence, the linear-quadratic case satisfies Proposition 2. □

6.3 Proof of Proposition 4

The proof is analogous to the proof of Proposition 3. Combine (3) and (4) and define $\mathbf{M} = (\mathbf{B}_x \mathbf{R}_\varepsilon + \mathbf{B}_\varepsilon)$ to obtain

$$\mathbf{y}^{*} - \mathbf{y} = \mathbf{y}^{*} - \overline{\mathbf{y}} - \mathbf{B}_x \mathbf{r} - \left(\mathbf{B}_x \mathbf{R}_\varepsilon + \mathbf{B}_\varepsilon\right) \boldsymbol{\varepsilon} = \mathbf{y}^{*} - \overline{\mathbf{y}} - \mathbf{B}_x \mathbf{r} - \mathbf{M}\boldsymbol{\varepsilon}$$

Use this equation in (5) to obtain for player ξ

$$E(V_\xi) = \frac{1}{2} E\left[\sum_{p=1}^{X} \omega_p^{\xi} \left[y_p^{*} - \overline{y_p} - \sum_{j=1}^{X} b_{pj}^{x} r_j - \sum_{s=1}^{S} m_{ps}\varepsilon_s\right]^2\right], \tag{29}$$

where b_{pj}^{x} and m_{ps} denote representative entries of the $X \times X-$matrix $\mathbf{B}_x$ and of the $X \times S-$matrix $\mathbf{M}$, respectively. Any Nash equilibrium outcome $\mathbf{R}^{Nash} = [\mathbf{r}^{Nash}, \mathbf{R}_{\boldsymbol{\varepsilon}}^{Nash}]$ of the simultaneous Nash game Γ^{Nash} played by the X players satisfies the set of conditions

$$\begin{aligned} \frac{\partial E[V_\xi(\mathbf{R}^{Nash})]}{\partial r_\xi} &= 0, \forall \xi \in \Xi. \\ \frac{\partial E[V_\xi(\mathbf{R}^{Nash})]}{\partial r_{\xi s}^{\varepsilon}} &= 0, \forall s = 1, 2, ..., S, \text{ and } \forall \xi \in \Xi, \end{aligned}$$

where r_ξ and $r_{\xi s}^{\varepsilon}$ are representative entries of the $X \times 1-$vector $\mathbf{r}$ and the $X \times S-$matrix $\mathbf{R}_\varepsilon$, respectively. Because of the linear-quadratic structure, this set of equations can be expressed as follows:

$$\frac{\partial E[V_\xi(\mathbf{R}^{Nash})]}{\partial r_\xi} = E\left[\sum_{p=1}^{X} \omega_p^{\xi} b_{p\xi}^{x} \left[y_p^{*} - y_p\right]\right] = 0, \forall \xi \in \Xi. \tag{30}$$

$$\frac{\partial E[V_\xi(\mathbf{R}^{Nash})]}{\partial r_{\xi s}^{\varepsilon}} = E\left[\sum_{p=1}^{X} \omega_p^{\xi} \frac{\partial m_{ps}}{\partial r_{\xi s}^{\varepsilon}} \varepsilon_s \left[y_p^{*} - y_p\right]\right] = 0, \forall s = 1, 2, ..., S, \text{ and } \forall \xi \in \Xi \tag{31}$$

The direct spillover effects of ξ' on agent ξ's expected payoffs are given by the expressions:

$$\frac{\partial E[V_\xi(\mathbf{R}^{Nash})]}{\partial r_{\xi'}} = E\left[\sum_{p=1}^{X} \omega_p^{\xi} b_{p\xi'}^{x} \left[y_p^{*} - y_p\right]\right] = 0 \tag{32}$$

$$\frac{\partial E[V_\xi(\mathbf{R}^{Nash})]}{\partial r_{\xi' s}^{\varepsilon}} = E\left[\sum_{p=1}^{X} \omega_p^{\xi} \frac{\partial m_{ps}}{\partial r_{\xi' s}^{\varepsilon}} \varepsilon_s \left[y_p^{*} - y_p\right]\right] = 0, \forall s = 1, 2, ..., S. \tag{33}$$

Let $\mathbf{R}^{Nash} = [\mathbf{r}^{Nash}, \mathbf{R}_\varepsilon^{Nash}]$, with $\mathbf{r}^{Nash} = \mathbf{B}_x^{-1}(\mathbf{y}^{*} - \overline{\mathbf{y}})$ and $\mathbf{R}_{\boldsymbol{\varepsilon}}^{Nash} = -\mathbf{B}_x^{-1} \cdot \mathbf{B}_{\boldsymbol{\varepsilon}}$, implying $\mathbf{y}^{*} = \mathbf{y}$. Hence, $\mathbf{R}^{Nash}$ satisfies (30) and (31), as required for a Nash-equilibrium. Moreover, this solution is unique since it has been assumed that for every p the product $\omega_p^{\xi} b_{p\xi}$ is non-zero for at least one $\xi = 1, 2, ..., X$. Finally, at $\mathbf{R}^{Nash}$ for any pair (ξ, ξ') equations (32) and (33) are zero. Hence, this stochastic extension of the linear-quadratic case satisfies Proposition 2. □

Proof of the Corollary to Proposition 4 :
Notice that (29) can be alternatively expressed as

$$E(V_\xi) = V_\xi^{Det} - \frac{1}{2}E\left[\sum_{p=1}^{X}\omega_p^\xi\left[\sum_{s=1}^{S}m_{ps}\varepsilon_s\right]^2\right] = V_\xi^{Det} - \frac{1}{2}\sum_{p=1}^{X}\omega_p^\xi Var(y_p),$$

where V_ξ^{Det} denotes a deterministic component. Based on this decomposition consider now the extended problem (which ignores, for simplicity, all constant terms) of the form

$$E(V_\xi) = E\left[\sum_{p=1}^{X}\sum_{q=1}^{X}\omega_{pq}^\xi\left[\sum_{s=1}^{S}m_{ps}\varepsilon_s\right]\left[\sum_{s=1}^{S}m_{qs}\varepsilon_s\right]\right] = \sum_{p=1}^{X}\sum_{q=1}^{X}\omega_{pq}^\xi Cov(y_p, y_q), \tag{34}$$

where the indices of the ω-terms have been extended to account for the additional covariance terms. Because of the additively multiplicative structure within (34) it is clear that $\mathbf{M} = \mathbf{0}$, as implied by $\mathbf{R}_\varepsilon^{Nash} = -\mathbf{B}_x^{-1}\cdot\mathbf{B}_\varepsilon$, i) satisfies the set of first-order conditions characterizing the simultaneous Nash game Γ^{Nash} and ii) ensures that all direct spillovers between all pairs of players vanish at $\mathbf{R}_\varepsilon^{Nash}$. □

6.4 The model of Dixit and Lambertini: a special case of Proposition 3

The representation of the model of Dixit and Lambertini described in Section 3.3 can be rewritten as follows such that it satisfies (1) and (2). First, define the inflation forecast error such that $\pi^{fe} \equiv \pi - \pi^e$. Then, introduce a new vector $\widetilde{\mathbf{y}} = (\pi^{fe}, \mathbf{y}, \pi)$, with $\widetilde{\mathbf{y}}$ relating to the states of the three groups of agents: single private sector actor, country-specific fiscal policymakers, single monetary policymaker. Since

$$\begin{aligned}\pi^{fe} &= \pi - \pi^e \\ y_i &= \overline{y_i} + \sum_{k=1}^{n} b_{ik}\tau_k + b_i(\pi - \pi^e) \\ \pi &= \pi,\end{aligned}$$

$\widetilde{\mathbf{y}}$ can be linearly linked to the instruments $\mathbf{x} = (\pi^e, \boldsymbol{\tau}, \pi)$ in line with (1), i.e.

$$\widetilde{\mathbf{y}} = \overline{\widetilde{\mathbf{y}}} + \widetilde{\mathbf{B}}\mathbf{x},$$

with $\overline{\widetilde{\mathbf{y}}} = (0, \overline{\mathbf{y}}, 0)$. Moreover, the target value of the inflation forecast error satisfies $\pi^{fe^*} = 0$. Hence, the payoff functions of all three types of (non-cooperative) players

$$\begin{aligned}U &= \frac{1}{2}(\pi^{fe^*} - \pi^{fe})^2 = \frac{1}{2}(\pi - \pi^e)^2 \\ V_i &= \frac{1}{2}\left[\omega_i(y_i^* - y_i)^2 + (\pi^* - \pi)^2\right] = \frac{1}{2}\left[\omega_i(y_i^* - y_i)^2 + \pi^2\right] \\ V^M &= \sum_{i=1}^{n}\omega_i^M V_i = \frac{1}{2}\left[\sum_{i=1}^{n}\left[\omega_i^M\omega_i(y_i^* - y_i)^2\right] + \pi^2\right]\end{aligned}$$

are in line with (2). $\square$

6.5 The model of Chari and Kehoe: main results

Consider first the four games CK1 - CK4 studied by Chari and Kehoe and summarized in the main text, with:

1. Game CK1 under $\mathfrak{C}_I$ (*no fiscal cooperation, monetary policy moves last*): *i/* τ_i (non-cooperatively), *ii/* a_{ij} (non-cooperatively), *iii/* π.
2. Game CK2 *under* $\mathfrak{C}_I$ *(fiscal cooperation, monetary policy moves last):* *i/* τ_i (cooperatively), *ii/* a_{ij} (non-cooperatively), *iii/* π.
3. Game CK3 under $\mathfrak{C}_{II}$ *(no fiscal cooperation, monetary policy moves first):* *i/* π, *ii/* τ_i (non-cooperatively), *iii/* a_{ij} (non-cooperatively).
4. Game CK4 under $\mathfrak{C}_{II}$ *(fiscal cooperation, monetary policy moves first):* *i/* π, *ii/* τ_i (cooperatively), *iii/* a_{ij} (non-cooperatively).

Let $\mathbf{z}_g = (\mathbf{a}, \boldsymbol{\tau}, \pi)$, $g = 1, 2, 3, 4$ denote the solution vectors of the four games. For simplicity (and without loss of generality), let $\omega_{ij} = \omega_i^F = \omega_i^M = 1$.

I) Comparison of CK1 and CK2 under $\mathfrak{C}_I$
Consider CK1:
Stage 3 gives rise to the first-order condition

$$\frac{\partial V^M(\mathbf{z}_1)}{\partial \pi} = \frac{\partial \left[\sum_{i=1}^{n} \sum_{j \in \mathcal{M}_i} U_{ij}(\mathbf{z}_1) \right]}{\partial \pi} = 0, \tag{35}$$

leading (at least implicitly) to a solution for π such that $\pi = \pi(\mathbf{a}, \boldsymbol{\tau})$.
Stage 2 gives rise to a system of first-order conditions, with

$$\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ij}} + \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial a_{ij}} = 0 \tag{36}$$

to be calculated for all i and all j, leading to a solution $\mathbf{a} = \mathbf{a}(\boldsymbol{\tau})$.
Stage 1 simplifies if one combines the preceding steps to obtain $\pi = \pi(\mathbf{a}(\boldsymbol{\tau}), \boldsymbol{\tau})$. Then, stage 1 gives rise to a system of first-order conditions, with

$$\begin{aligned} \frac{\partial V_i(\mathbf{z}_1)}{\partial \tau_i} &= \sum_{j \in \mathcal{M}_i} \left[\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \tau_i} + \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial \tau_i} + \left(\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ij}} + \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial a_{ij}} \right) \frac{\partial a_{ij}}{\partial \tau_i} \right] \quad (37) \\ &\quad + \sum_{j \in \mathcal{M}_i} \left[\sum_{k \in \mathcal{M}_i, k \neq j} \left(\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ik}} + \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial a_{ik}} \right) \frac{\partial a_{ik}}{\partial \tau_i} \right] \\ &= 0 \end{aligned}$$

to be calculated for all i.
These first-order conditions can be further simplified by using stepwise the envelope theorem. Notice that a symmetric equilibrium across all countries and players (with $a_{ij} = a$ $\forall i, j$ and $\tau_i = \tau$ $\forall i$) requires $\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} = 0$ in (35). Hence, (36) simplifies to $\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ij}} = 0$ and (37) simplifies to

$$\frac{\partial V_i(\mathbf{z}_1)}{\partial \tau_i} = \sum_{j\in\mathcal{M}_i} \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \tau_i} + \sum_{j\in\mathcal{M}_i} \sum_{k\in\mathcal{M}_i, k\neq j} \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ik}} \frac{\partial a_{ik}}{\partial \tau_i} = 0 \tag{38}$$

Two elements are crucial for the understanding of CK1: i) Since the common monetary policy moves last, this makes private sector actions in stage 2 depend on the entire vector of fiscal actions $\boldsymbol{\tau}$. ii) The non-cooperative behavior of private agents within countries creates indirect fiscal spillovers which become relevant at stage 1 and which depend on the particular commitment pattern $\mathfrak{C}_I$. Under the assumption of non-cooperative fiscal policy in CK1, these spillovers are not internalized.
Consider CK2: By backward induction, stage 3 and 2 are identical to CK1. Stage 1, because of fiscal cooperation between countries, gives rise to a different first-order condition

$$\frac{\partial V_i(\mathbf{z}_2)}{\partial \tau_i} + \sum_{l=1,\ l\neq i}^{n} \frac{\partial V_l(\mathbf{z}_2)}{\partial \tau_i} = 0,$$

implying

$$\begin{aligned}
& \sum_{j\in\mathcal{M}_i} \left[\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \tau_i} + \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial \tau_i} + \left(\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ij}} + \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial a_{ij}} \right) \frac{\partial a_{ij}}{\partial \tau_i} \right] \\
& + \sum_{j\in\mathcal{M}_i} \sum_{k\in\mathcal{M}_i, k\neq j} \left(\frac{\partial U_{ij}(\mathbf{z}_1)}{\partial a_{ik}} + \frac{\partial U_{ij}(\mathbf{z}_1)}{\partial \pi} \frac{\partial \pi}{\partial a_{ik}} \right) \frac{\partial a_{ik}}{\partial \tau_i} \\
& + \sum_{l=1,\ l\neq i}^{n} \sum_{j\in\mathcal{M}_l} \left[\frac{\partial U_{lj}(\mathbf{z}_2)}{\partial \pi} \frac{\partial \pi}{\partial \tau_i} + \left(\frac{\partial U_{lj}(\mathbf{z}_2)}{\partial a_{lj}} + \frac{\partial U_{lj}(\mathbf{z}_2)}{\partial \pi} \frac{\partial \pi}{\partial a_{lj}} \right) \frac{\partial a_{lj}}{\partial \tau_i} \right] \\
& + \sum_{l=1,\ l\neq i}^{n} \sum_{j\in\mathcal{M}_l} \sum_{k\in\mathcal{M}_l, k\neq j} \left(\frac{\partial U_{lj}(\mathbf{z}_2)}{\partial a_{lk}} + \frac{\partial U_{lj}(\mathbf{z}_2)}{\partial \pi} \frac{\partial \pi}{\partial a_{lk}} \right) \frac{\partial a_{lk}}{\partial \tau_i} \\
= \ & 0
\end{aligned} \tag{39}$$

Consider again a symmetric equilibrium, implying $\frac{\partial U_{ij}(\mathbf{z}_2)}{\partial \pi} = \frac{\partial U_{lj}(\mathbf{z}_2)}{\partial \pi} = 0$ and $\frac{\partial U_{ij}(\mathbf{z}_2)}{\partial a_{ij}} = \frac{\partial U_{lj}(\mathbf{z}_2)}{\partial a_{lj}} = 0$. Then, (39) simplifies to

$$\frac{\partial V_i(\mathbf{z}_2)}{\partial \tau_i} + \sum_{l=1,\ l\neq i}^{n} \frac{\partial V_l(\mathbf{z}_2)}{\partial \tau_i} = \sum_{j\in\mathcal{M}_i} \frac{\partial U_{ij}(\mathbf{z}_2)}{\partial \tau_i} + \sum_{l=1}^{n} \sum_{j\in\mathcal{M}_l} \sum_{k\in\mathcal{M}_l, k\neq j} \frac{\partial U_{lj}(\mathbf{z}_2)}{\partial a_{lk}} \frac{\partial a_{lk}}{\partial \tau_i} = 0, \tag{40}$$

i.e. the indirect fiscal spillover effects are internalized. Hence, by comparing (38) and (40) one infers that the SPNE of CK 1 and CK 2 are generically different.

II) Comparison of CK3 and CK4 under $\mathfrak{C}_{II}$

Consider CK 3. By backward induction, stage 3 gives rise to a system of first-order conditions $\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ij}} = 0$, leading to a solution $\mathbf{a}_i = \mathbf{a}_i(\tau_i, \pi)$. Stage 2 gives rise to a system of first-order conditions

$$\sum_{j\in\mathcal{M}_i}\left[\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial \tau_i} + \frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ij}}\frac{\partial a_{ij}}{\partial \tau_i} + \sum_{k\in\mathcal{M}_i,k\neq j}\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ik}}\frac{\partial a_{ik}}{\partial \tau_i}\right] = 0, \tag{41}$$

leading to a solution for τ_i such that $\tau_i = \tau_i(\pi)$. Stage 1, using $\mathbf{a}_i = \mathbf{a}_i(\tau_i(\pi), \pi)$, gives rise to a first-order condition

$$\begin{aligned} &\sum_{i=1}^{n}\sum_{j\in\mathcal{M}_i}\left[\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial \pi} + \frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ij}}\frac{\partial a_{ij}}{\partial \pi} + \sum_{k\in\mathcal{M}_i,k\neq j}\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ik}}\frac{\partial a_{ik}}{\partial \pi}\right] + \\ &\sum_{i=1}^{n}\sum_{j\in\mathcal{M}_i}\left[\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial \tau_i} + \frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ij}}\frac{\partial a_{ij}}{\partial \tau_i} + \sum_{k\in\mathcal{M}_i,k\neq j}\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ik}}\frac{\partial a_{ik}}{\partial \tau_i}\right]\frac{\partial \tau_i}{\partial \pi} \\ = \ & 0 \end{aligned} \tag{42}$$

These first-order conditions can be simplified by using stepwise the envelope theorem. Using $\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ij}} = 0$, (41) reduces to

$$\sum_{j\in\mathcal{M}_i}\left[\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial \tau_i} + \sum_{k\in\mathcal{M}_i,k\neq j}\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ik}}\frac{\partial a_{ik}}{\partial \tau_i}\right] = 0,$$

while (42) reduces to

$$\sum_{i=1}^{n}\sum_{j\in\mathcal{M}_i}\left[\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial \pi} + \sum_{k\in\mathcal{M}_i,k\neq j}\frac{\partial U_{ij}(\mathbf{z}_3)}{\partial a_{ik}}\frac{\partial a_{ik}}{\partial \pi}\right] = 0$$

Consider CK 4: By backward induction, the solution of CK 4 satisfies the same first-order conditions as CK 3, since the solution of stage 3, namely $\mathbf{a}_i = \mathbf{a}_i(\tau_i, \pi)$, implies that there are no indirect fiscal spillover effects between fiscal players at stage 2.

III) Irrelevance of fiscal cooperation under all other commitment patterns

There are four more commitment patterns $\mathfrak{C}_{III} - \mathfrak{C}_{VI}$:
$\mathfrak{C}_{III}$: *i/* π, *ii/* a_{ij}(non-cooperatively), *iii/* τ_i (cooperatively or non-cooperatively).
$\mathfrak{C}_{IV}$: *i/* a_{ij}(non-cooperatively), *ii/* π, *iii/* τ_i (cooperatively or non-cooperatively).
By backward induction, under $\mathfrak{C}_{III}$ and $\mathfrak{C}_{IV}$ fiscal cooperation evidently is irrelevant,

$\mathfrak{C}_{V}$: *i/* a_{ij}(non-cooperatively), *ii/* τ_i (cooperatively or non-cooperatively), *iii/* π.
To see that fiscal cooperation is irrelevant, consider the two stage-game:
$\mathfrak{C}\prime$: *i/* a_{ij}(non-cooperatively), *ii/* τ_i (non-cooperatively), π.

Notice that under $\mathfrak{C}_V$ there are no direct spillovers between the players of the Nash game of stage ii). Hence, applying Proposition 2 to this subgame, fiscal cooperation is irrelevant under $\mathfrak{C}_V$.

$\mathfrak{C}_{VI}$: $i/$ τ_i (cooperatively or non-cooperatively), $ii/$ π, $iii/$ a_{ij}(non-cooperatively):
Fiscal cooperation is irrelevant, since, differently from $\mathfrak{C}_I$, private agents take π as given, i.e. direct private spillovers within countries cannot generate indirect fiscal spillovers between countries. □

6.6 Canzoneri and Henderson (1988, 1991) and Rogoff (1985): main results

This appendix summarizes how the comparison of payoffs under cooperative and non-cooperative behavior in (23) and (24) has been derived, using:

$$\begin{aligned}
E[V_i] &= \frac{1}{2}E\left[\omega(y^* - y_i)^2 + (\pi_i^{CPI})^2\right] \\
\pi_1^{CPI} &= \pi_1 + (1-\beta)e_r = \pi_1 + (1-\beta)(e_n + \pi_2 - \pi_1) \\
\pi_2^{CPI} &= \pi_2 - (1-\beta)e_r = \pi_2 - (1-\beta)(e_n + \pi_2 - \pi_1) \\
y^* - y_1 &= y^* - \overline{y} - b_\pi(\pi_1 - \pi_1^e) + b_e(e_r - e_r^e) - \varepsilon \\
&= y^* - \overline{y} - b_\pi(\pi_1 - \pi_1^e) + b_e(e_n + \pi_2 - \pi_1 - e_r^e) - \varepsilon \\
y^* - y_2 &= y^* - \overline{y} - b_\pi(\pi_2 - \pi_2^e) - b_e(e_r - e_r^e) - \varepsilon \\
&= y^* - \overline{y} - b_\pi(\pi_2 - \pi_2^e) - b_e(e_n + \pi_2 - \pi_1 - e_r^e) - \varepsilon.
\end{aligned}$$

Let country 1 be the representative country.
Non-cooperation:
Policymaker 1 maximizes his payoff over π_1 after ε has been observed, taking as given π_2 and private sector expectations, leading to the first-order condition

$$-\omega\left[y^* - \overline{y} - b_\pi(\pi_1 - \pi_1^e) + b_e(e_r - e_r^e) - \varepsilon\right](b_\pi + b_e) + \left[\pi_1 + (1-\beta)e_r\right]\beta = 0$$

Private agents, anticipating this reaction, form rational expectations, where we exploit $e_r^e = e_r = 0$. Hence, in any symmetric equilibrium

$$\pi_1^{e,nc} = \pi_2^{e,nc} = \pi^{e,nc} = \frac{\omega}{\beta}(b_\pi + b_e)(y^* - \overline{y}).$$

Using this expression within the first-order condition one obtains

$$\begin{aligned}
\pi^{nc} &= \frac{\omega}{\beta}(b_\pi + b_e)(y^* - \overline{y}) - \omega\frac{b_\pi + b_e}{\omega b_\pi(b_\pi + b_e) + \beta}\varepsilon \\
y^* - y^{nc} &= y^* - \overline{y} - \frac{\beta}{\omega b_\pi(b_\pi + b_e) + \beta}\varepsilon,
\end{aligned}$$

which leads to

$$
\begin{aligned}
E[V^{nc}] &= \frac{1}{2}\left[\omega\left[1+\omega(\frac{b_\pi+b_e}{\beta})^2\right](y^*-\overline{y})^2+\omega\frac{\beta^2+\omega(b_\pi+b_e)^2}{[\omega b_\pi(b_\pi+b_e)+\beta]^2}\sigma_\varepsilon^2\right] \\
\phi_{y^*}^{nc} &= \omega\left[1+\omega(\frac{b_\pi+b_e}{\beta})^2\right],\ \ \phi_\varepsilon^{nc}=\omega\frac{\beta^2+\omega(b_\pi+b_e)^2}{[\omega b_\pi(b_\pi+b_e)+\beta]^2} \\
E[U^{nc}] &= \frac{1}{2}\psi_\varepsilon^{nc}\sigma_\varepsilon^2=\frac{1}{2}\left[\omega\frac{b_\pi+b_e}{\omega b_\pi(b_\pi+b_e)+\beta}\right]^2\sigma_\varepsilon^2.
\end{aligned}
$$

Cooperation:
Policymakers 1 and 2 jointly maximize the sum of their payoffs over π_1 and π_2 after ε has been observed, taking as given private sector expectations, leading to the (representative) first-order condition of policymaker 1:

$$
\begin{aligned}
0 &= -\omega\left[y^*-\overline{y}-b_\pi(\pi_1-\pi_1^e)+b_e(e_r-e_r^e)-\varepsilon\right](b_\pi+b_e)+\left[\pi_1+(1-\beta)e_r\right]\beta \\
&\quad +\omega\left[y^*-\overline{y}-b_\pi(\pi_2-\pi_2^e)-b_e(e_r-e_r^e)-\varepsilon\right]b_e+\left[\pi_2-(1-\beta)e_r\right](1-\beta)
\end{aligned}
$$

Private agents, anticipating this reaction, form rational expectations, where we exploit $e_r^e=e_r=0$. Hence, in any symmetric equilibrium

$$
\begin{aligned}
\pi_1^{e,c} &= \pi_2^{e,c}=\pi^{e,c}=\omega b_\pi(y^*-\overline{y}) \\
\pi^c &= \omega b_\pi(y^*-\overline{y})-\omega\frac{b_\pi}{1+\omega b_\pi^2}\varepsilon \\
y^*-y^c &= y^*-\overline{y}-\frac{1}{1+\omega b_\pi^2}\varepsilon,
\end{aligned}
$$

implying

$$
\begin{aligned}
E[V^c] &= \frac{1}{2}\left[\omega\left[1+\omega b_\pi^2\right](y^*-\overline{y})^2+\omega\frac{1}{1+\omega b_\pi^2}\sigma_\varepsilon^2\right] \\
\phi_{y^*}^c &= \omega\left[1+\omega b_\pi^2\right],\ \ \phi_\varepsilon^c=\omega\frac{1}{1+\omega b_\pi^2} \\
E[U^c] &= \frac{1}{2}\psi_\varepsilon^c\sigma_\varepsilon^2=\frac{1}{2}\left[\frac{\omega b_\pi}{1+\omega b_\pi^2}\right]^2\sigma_\varepsilon^2.
\end{aligned}
$$

Comparison of cooperation vs. non-cooperation:
Comparing coefficients, one obtains (i) $\phi_\varepsilon^{nc}-\phi_\varepsilon^c>0$ if $[(1-\beta)b_\pi+b_e]^2>0$, (ii) $\phi_{y^*}^{nc}-\phi_{y^*}^c<0$ if $b_\pi^2>(\frac{b_\pi+b_e}{\beta})^2$, and (iii) $\psi_\varepsilon^{nc}-\psi_\varepsilon^c<0$ if $(\frac{\beta}{\omega(b_\pi+b_e)}+b_\pi)^2>(\frac{1}{\omega b_\pi}+b_\pi)^2$. Notice that (i) will always be satisfied. By assumption $b_\pi>0$, $b_e<0$, $\beta\in(0,1)$. Assume $0<b_\pi+b_e<\beta b_\pi$. Then, (ii) and (iii) will also be satisfied. Hence, for the logic of Rogoff to obtain, there exists, for any given direct output effect of inflation (b_π) a certain trade-off between the real exchange rate effect of inflation (b_e) and the the openness of the economy (β).