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Wavelets Unit Root test vs DF test:

A further investigation based on monte carlo experiments

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Abstract

Test for unit root based in wavelets theory is recently defined(Gençay and Fan, 2007). While the new test is supposed to be robust to the initial value, we bring out by contrast the significant effetcs of the initial value in the size and the power. We found also that both the wavelets unit root test and ADF test give the same efficiency if the data are corrected of the initial value. Our approach is based in monte carlo experiment.

Keywords: Unit root tests, Wavelets, Monte carlo experiments, Size-Power curve.

JEL: C12, C15, C16, C22.

Résumé:Des tests de non stationnarité(racines unité) basés sur la théorie des ondelettes ont récemment fait leur apparition dans la littérature(2007). Ces tests sont supposés être robustes à la valeur initiale. Nous prouvons au contraire que l' ignorance de l'importance de la valeur initiale entraîne une distortion de taille pour ces nouveaux tests. Nous montrons enfin qu' une fois prise en compte les effets de la valeur initiale, il n' y a plus de différences significatives en terme de puissance entre les nouveaux tests et les tests classiques ADF.

Mots clés: Test racine unité, Ondelettes, Simulation monte carlo, Courbe taille-puissance.

Classification: C12, C15, C16, C22.

1 Introduction

It is well known that for an I(1) process , the spectral density is infinite for zero frequency. The spectral distribution is concentrated on the low frequencies. So, the variance of the process is primarily explained by the frequencies close to zero. These properties of the spectral density are recently exploited by Fan and Gençay (2007) to define unit root tests using wavelets framework. A wavelets basis (by contrast with traditional Fourier's basis) provides a simultaneous analysis in both time and frequency domain. In a wavelets basis the concept of multiresolution allows to bring out the time varying impacts of each frequency in the variance of the process. So the variance can be decomposed as a function of time and frequency. It is then possible to detect the time varying impact of low frequencies. If the average of such effetcs are significantly high, the null of unit root is not rejected. For traditional spectral analysis, the process must be covariance stationary. Wavelets framework allows processes with time varying stochastics properties(abrupt or continuously changes in the variance or covariance of the process) as some financial data. In this paper we compare the performances of traditional ADF test with one of the new tests suggested by Fan and Gançay(2007). While ADF tests are based in time domain, the wavelets tests are defined simultaneously in the time and frequency domain. In this paper we consider only the case of no drift in the traditional ADF models. While the distribution of ADF test in the case of no drift depends on the initial value, the wavelets based tests are supposed to be robust with the initial value. We show that the effects of initial value in the new tests must be considered. More concretely, we found that the new wavelets tests can appear significantly more powerful than traditional ADF tests if the influence of the initial value is ignored . But if the effects of the initial value are considered, the two approaches give almost the same efficiency. This paper is organized as follows: the second section presents the wavelets test. In the third section, the results of Monte Carlo simulation are analysed. The last section presents remarks and conclusion.

2 Fan and Gançay tests (2007)

Let us consider a time series $\{X_t\}_{t=1}^{T}$ generated by the following $AR(1)$ process:

$$X_t = \rho X_{t-1} + u_t \tag{1}$$

where $\{u_t\}$ is weakly stationary, $E(u_t) = 0$, $\omega^2 = \gamma_0 + 2\sum_{j=1}^{\infty} \gamma_j$ with $\gamma_j = E(u_t u_{t-j})$. The size T of the sample is supposed being an even number. We consider the null $H_0 : \rho = 1$ against the alternative $H_1 : |\rho| < 1$. The statistic defined by Fan and Gançay(2007) using Haar basis, is the following:

$$\widehat{S}_{T,1} = \frac{\sum_{t=1}^{T/2} V_{t,1}^2}{\sum_{t=1}^{T/2} V_{t,1}^2 + \sum_{t=1}^{T/2} W_{t,1}^2} \tag{2}$$

where $W_{t,1} = \frac{1}{\sqrt{2}}(X_{2t} - X_{2t-1})$ and $V_{t,1} = \frac{1}{\sqrt{2}}(X_{2t} + X_{2t-1})$ for $t = 1, 2, ..., T/2$. The set of the coefficients $\{V_{t,1}\}$ allows to detect the time varying effects of the low frequencies in the variance of the process. The low frequencies are localised in frequencies band $[0, 1/2]$. While the coefficients $\{W_{t,1}\}$ detect the effects of the high frequencies band $[1/2, 1]$. More concretely, the coefficients $\{V_{t,1}\}$ and $\{W_{t,1}\}$ are respectively the coordinates of the series $\{X_t\}_{t=1}^{T}$ in the time-low frequencies components and the time-high frequencies components of the Haar wavelet basis. Since the Haar wavelets basis is an orthonormal one, the variance of X_t can be decomposed as follows: $\sum_{t=1}^{T} X_t^2 = \sum_{t=1}^{T/2} V_{t,1}^2 + \sum_{t=1}^{T/2} W_{t,1}^2$. The quantity $\sum_{t=1}^{T/2} V_{t,1}^2$ measures the spectral mass explained by the low frequencies whereas $\sum_{t=1}^{T/2} W_{t,1}^2$ gives the spectral mass of high frequencies. Since the spectrum of an I(1) process is concentrated in the low frequencies and remains negligible elsewhere($\sum_{t=1}^{T/2} W_{t,1}^2 \simeq 0$) then we have $\sum_{t=1}^{T/2} V_{t,1}^2 + \sum_{t=1}^{T/2} W_{t,1}^2 \simeq \sum_{t=1}^{T/2} V_{t,1}^2$ if the process contains an unit root. Under the null, the values taken by the statistic $\widehat{S}_{T,1}$ are around one. In practice one uses a standardized version of $\widehat{S}_{T,1}$ as follows:

$$FG_1 = \frac{T\widehat{\lambda}_v^2}{\widehat{\gamma_0}} \left[\widehat{S}_{T,1} - 1\right] \tag{3}$$

with $\widehat{\lambda}_v^2 = 4\widehat{\omega^2}$. Under the null of unit root(and the validity of usual conditions), the limiting distribution of $FG1$ is given by

$$FG_1 \overset{\mathcal{LF}}{\to} \frac{-1}{\int_0^1 [W(r)]^2 \, dr}$$

where $W(r)$ is a standard Brownian motion. For more details on wavelet theory, one can consult Gençay and al.(2002), Percival and Waden(1999).

3 Monte Carlo experiments

In this section, a comparison study between the new test and traditional ADF test(case of no drift) is proposed . Our approach is based on monte carlo experiments.

3.1 Size of the tests

In *ADF* test , the asymptotic distribution of the statistics under the null assumption is diferent according to whether the initial value X_0 is null or not for the model without drift. For example, the table.1 provides the simulated critical values of ADF statistic whenever $X_0 = 0$ (column $ADF1$) and $X_0 \neq 0$(column $ADF2$) for two diferent sizes ($n = 128$ and $n = 512$). The simulated critical values come from the DGP given by the model(1) with $\rho = 1$. The $FG1$ statistic does not make this distinction about the initial value. Its theoretical asymptotic distribution is supposed to be invariant for any initial value. We show in this section that, on the contrary, the distribution of the new test is clearly sensitive to the initial value, X_0. In the column named WAVEFG (table 1) we present some simulated critical values of the statistic $FG1$ if $X_0 = 0$ for various sizes ($n = 128$ and $n = 512$). We notice that the simulated critical values of $FG1$ coincident well with the asymptotic critical values (column " Asympt WAVEFG " table 1). So The empirical distributions of $FG1$ under H_0 coincide well with the theoretical distribution if $X_0 = 0$. One should note the same behavior for any value of X_0 if the distribution of $FG1$ under H_0 does not realy depend on the initial value. In the table.2, simulated critical values of the statistic $FG1$ are given for two diferents initial values: $X_0 = 5$ and $X_0 = 10$.We can see in table.2 that the obtained critical values of $FG1$ deviate significantly from the theoretical asymptotic distribution (the theoretical asymptotic distribution is given in the column: "Asympt WAVEFG ", table.1). For example, if $X_0 = 0$, $X_0 = 5$ and $X_0 = 10$, the empirical critical values of $FG1$ for $\alpha = 0,05$ are respectively $-17,829$, $-12,719$ and $-5,84$ (see table.2) whereas the value given by the asymptotic distribution is $-17,66$ (column named "Asympt WAVEFG " table.1). The deviation of the distribution increases with the initial value. The empirical and asymptotic distributions coincide in the only case where $X_0 = 0$. The Statistic $FG1$ is significantly sensitive to the initial value. The new test shows a size distortion if the effects of the initial value are ignored.

3.2 Power of the tests

In this section the powers of the two tests are investigated by using size-power curves. These curves are built by choosing beforehand a DGP under the nulle H_0 and another one under the alternative H_1(see appendix for the details and for more details see Davidson and MacKinnon 1998). For the more powerful test, the size-power curve converges quickly to the axis $y = 1$. Three DGP under H_1 are considered in our simulations, they correspond to ($\rho = 0,90$; $n = 128$), ($\rho = 0,95$; $n = 128$) and ($\rho = 0,90$, $n = 512$) in model(1). The initial values are selected in a random way. On the one hand, we construct the size-power curves the statistic $FG1$ and $ADF2$ in the case of no null initial values (we note $ADF2$ the ADF statistic if the initial value is not null). We also built the size-power curves of the statistics $FG1$ and $ADF1$ by using the corrected series of the initial value, $\{X_t - X_0\}$(we note $ADF1$ the ADF statistic if the initial value is null). We notice in all the cases(see figure.1, figure.2 and figure.3) that the sizes-power curves of $FG1$ converge more quickly towards the axis $y = 1$ than those of the $ADF2$ statistic. The Convergence is nearly immediate when one moves away from the null, H_0 ($\rho = 0,90$). These results let to think that the $FG1$ test is more powerful than the one of the ADF as that was found by Fan and Gançay(2007). But this conclusion is hasty for two reasons. Initially because we saw that $FG1$ statistic moved away from its theoretical asymptotic distribution under H_0 if the initial value were not equal to zero. So, the sizes-power curves of $FG1$ can be slightly biased. For the second raison, when the comparison is made with the corrected data $\{X_t - X_0\}$ (i.e. by using the $ADF1$ statistic), the size-power curves of the two tests are almost confused in all the three cases(figure.1, figure.2 and figure.3). Both tests have almost the same power if the comparison is carried out using the $ADF1$ statistic. These two reasons lead us to confiem that the comparison of $FG1$ and ADF must be carried out on corrected data of the initial value. So it is extremely probable that the two tests have comparable powers when using the corrected data and the $ADF1$ statistic.

4 Conclusion

In this paper a new unit root test based in wavelets framework(Fan and Gançay,2007) is presented. We see that this test is built by exploiting the properties of the spectral density of an I(1) process. The test appears more powerful than the ADF if the role of the initile value is ignored.We showed by simulations experiments that the new test is not robust with the initial value. Size distortion appears if the influence of the initial value is ignored. But both tests seem to be of the same efficiency if the used data are corrected of initial value.

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Size-Power Curve

These graphics are constructed as follows; we realize 100.000 simulations of the statistic under the null hypothesis and order them. We take for instance the 10.000 th value . This is the 10% p-value under H_0. On the other hand we realize 100.000 simulations under the alternative and order them. This is the distribution under H_1. In this distribution we look for the place of the 10% p-value under H_0. If it corresponds for example to the 90.000 th value this gives us a power of 90%. So the size-power curve contains the point (0.1; 0.9). We do that for all the p-values under H_0 to obtain the size-power curve(See Davidson and MacKinnon,1998, for more details about sizes-power curves when the distributions of the statistc can be exactly computed).

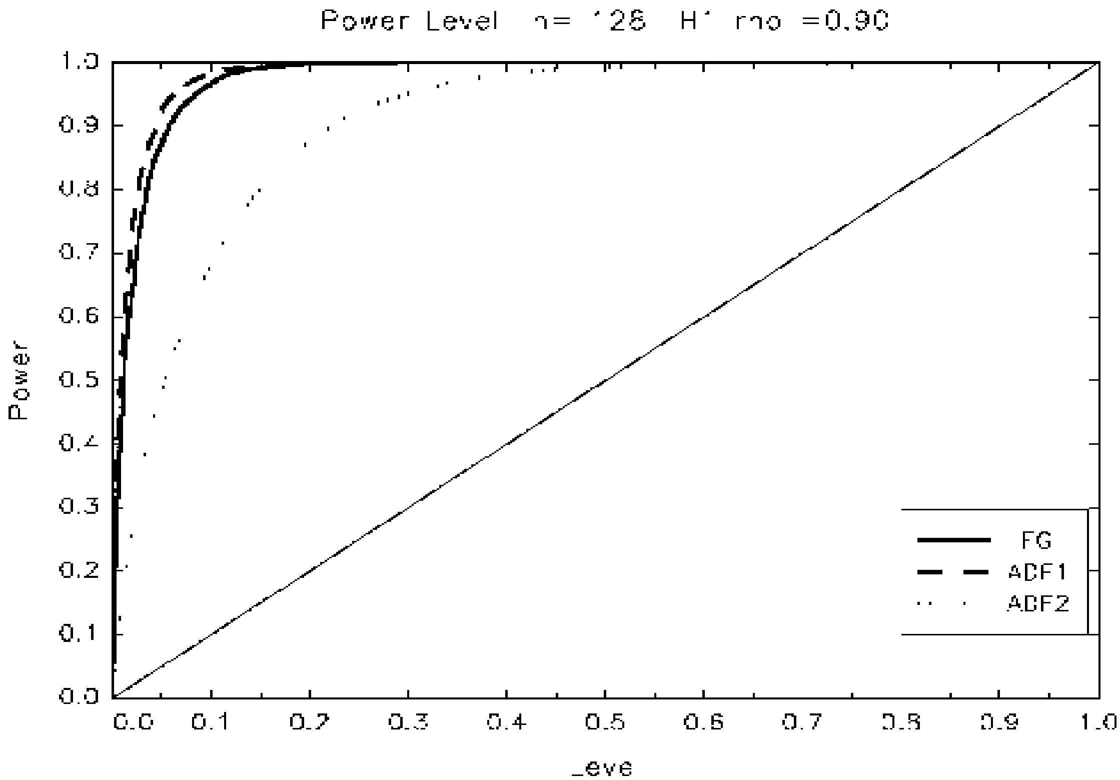

Figure.1. (FG is the statistic $FG1$)

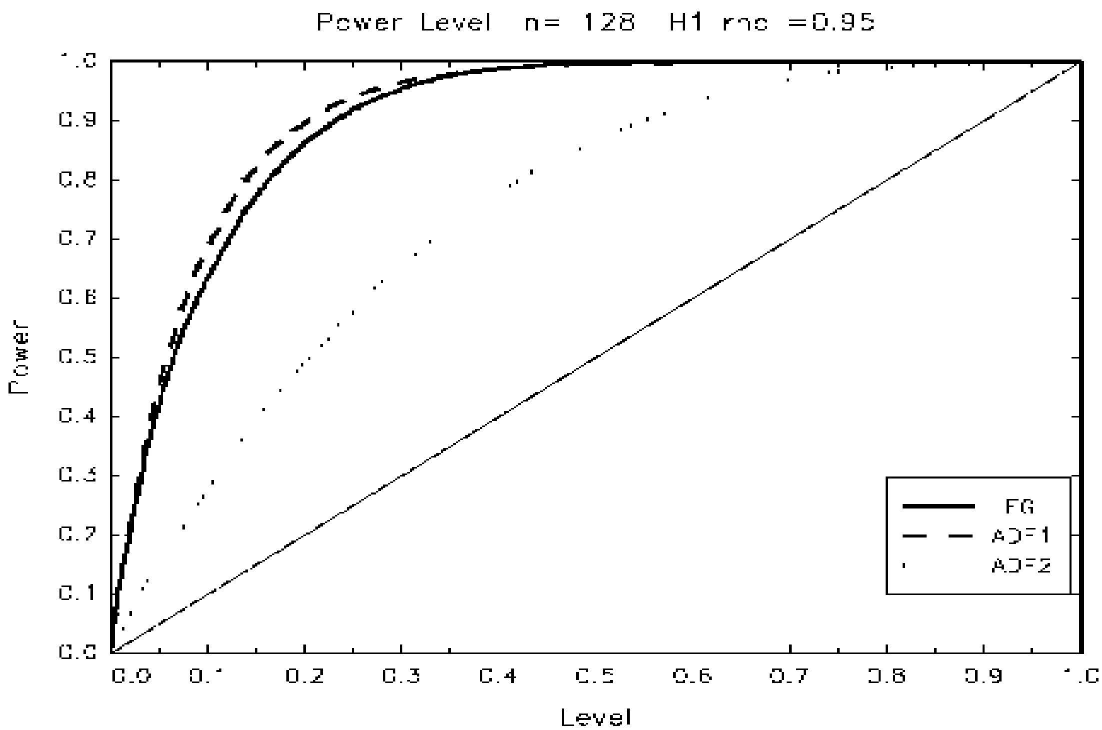

Figure.2. (FG is the statistic $FG1$)

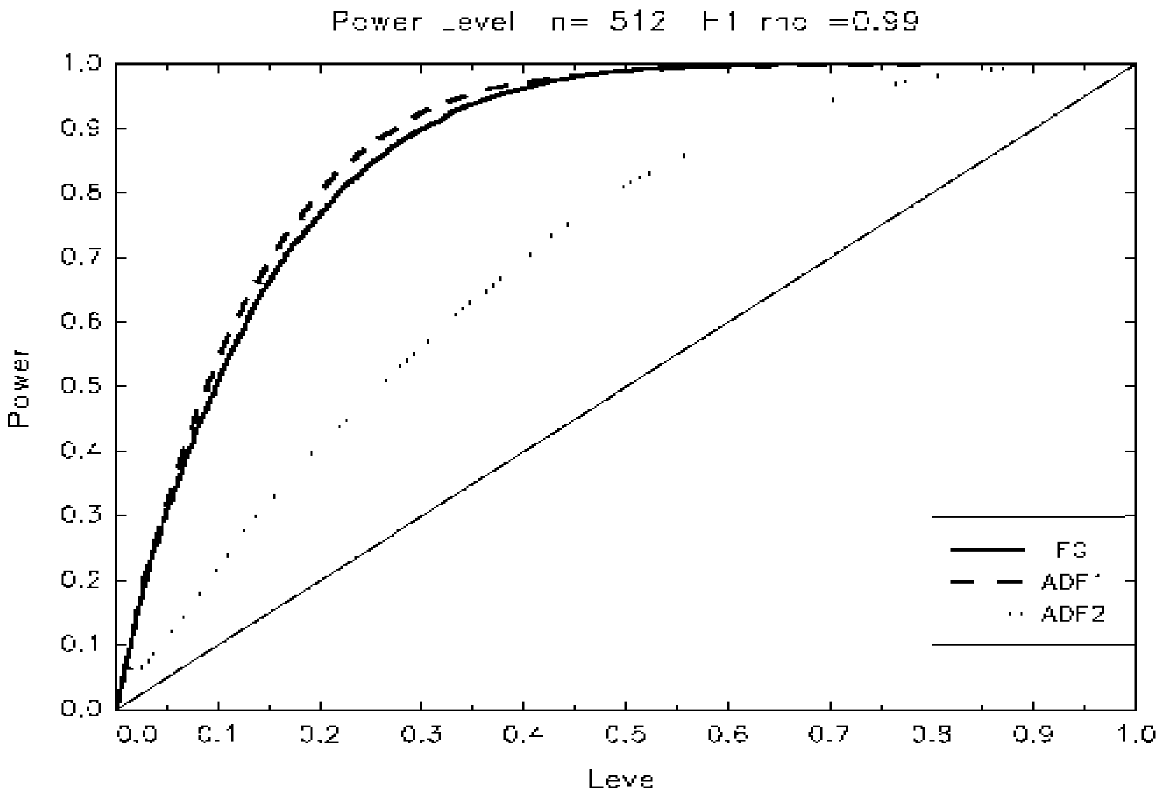

Figure.3. (FG is the statistic $FG1$)

Tables

Distributions for finite sizes are based on 100.000 simulations. The asymptotic values are based on 1 000.000 simulations. We give only values corresponding to common sizes. $ADF1$ and $ADF2$ are the tables usually called B1 and B2 (Hamilton for example). We should notice the relatively stability of the values independently of the size of the sample but it appears necessary to do a large number of simulations given n to stabilize these tables.

	n=128			n=512			Asympt
p	WAVE FG	ADF1	ADF2	WAVE FG	ADF1	ADF2	WAVE FG
0,005	-33,936	-2,784	-3,706	-34,144	-2,805	-3,654	-33,969
0,010	-29,321	-2,565	-3,476	-29,443	-2,571	-3,433	-28,951
0,015	-26,307	-2,424	-3,340	-26,555	-2,429	-3,302	-26,058
0,020	-24,196	-2,318	-3,238	-24,334	-2,322	-3,206	-24,002
0,025	-22,556	-2,229	-3,158	-22,658	-2,235	-3,125	-22,437
0,030	-21,182	-2,155	-3,088	-21,400	-2,159	-3,064	-21,173
0,035	-20,082	-2,089	-3,027	-20,304	-2,093	-3,009	-20,101
0,040	-19,124	-2,028	-2,977	-19,357	-2,039	-2,958	-19,184
0,045	-18,328	-1,977	-2,930	-18,491	-1,985	-2,910	-18,378
0,050	-17,589	-1,932	-2,885	-17,750	-1,937	-2,868	-17,665
0,055	-16,939	-1,889	-2,846	-17,065	-1,895	-2,832	-17,024
0,060	-16,334	-1,850	-2,809	-16,480	-1,854	-2,795	-16,437
0,065	-15,813	-1,816	-2,772	-15,924	-1,820	-2,760	-15,895
0,070	-15,339	-1,781	-2,741	-15,408	-1,785	-2,730	-15,397
0,075	-14,897	-1,750	-2,709	-14,992	-1,752	-2,699	-14,935
0,080	-14,507	-1,719	-2,681	-14,570	-1,720	-2,673	-14,504
0,085	-14,116	-1,690	-2,655	-14,171	-1,691	-2,647	-14,107
0,090	-13,745	-1,661	-2,630	-13,770	-1,663	-2,622	-13,732
0,095	-13,374	-1,635	-2,604	-13,412	-1,635	-2,599	-13,375
0,100	-13,050	-1,608	-2,579	-13,056	-1,610	-2,575	-13,040
0,105	-12,735	-1,582	-2,555	-12,721	-1,587	-2,554	-12,720
0,110	-12,427	-1,559	-2,533	-12,426	-1,565	-2,531	-12,417
0,115	-12,108	-1,535	-2,511	-12,145	-1,543	-2,510	-12,127
0,120	-11,833	-1,514	-2,491	-11,867	-1,522	-2,490	-11,852
0,125	-11,563	-1,493	-2,471	-11,627	-1,503	-2,469	-11,589
0,130	-11,303	-1,471	-2,452	-11,359	-1,483	-2,450	-11,338
0,135	-11,034	-1,450	-2,433	-11,115	-1,464	-2,430	-11,098
0,140	-10,806	-1,429	-2,415	-10,879	-1,443	-2,412	-10,866
0,145	-10,583	-1,411	-2,399	-10,665	-1,425	-2,395	-10,642
0,150	-10,382	-1,392	-2,381	-10,453	-1,404	-2,376	-10,428
0,155	-10,176	-1,373	-2,363	-10,248	-1,385	-2,359	-10,220
0,160	-9,964	-1,355	-2,347	-10,041	-1,367	-2,342	-10,019
0,165	-9,768	-1,339	-2,329	-9,863	-1,350	-2,325	-9,826
0,170	-9,574	-1,322	-2,313	-9,681	-1,333	-2,309	-9,641
0,175	-9,390	-1,304	-2,297	-9,494	-1,315	-2,292	-9,461
0,180	-9,212	-1,288	-2,282	-9,309	-1,297	-2,276	-9,285
0,185	-9,044	-1,270	-2,267	-9,137	-1,281	-2,261	-9,115
0,190	-8,881	-1,254	-2,252	-8,969	-1,264	-2,247	-8,949
0,195	-8,723	-1,238	-2,237	-8,807	-1,249	-2,234	-8,789
0,200	-8,561	-1,223	-2,222	-8,658	-1,234	-2,220	-8,633
0,205	-8,401	-1,208	-2,208	-8,513	-1,219	-2,208	-8,481
0,210	-8,254	-1,194	-2,194	-8,351	-1,204	-2,194	-8,334
0,215	-8,114	-1,180	-2,179	-8,199	-1,189	-2,180	-8,191
0,220	-7,981	-1,164	-2,165	-8,061	-1,175	-2,165	-8,051
0,225	-7,849	-1,149	-2,152	-7,927	-1,160	-2,152	-7,915
0,230	-7,710	-1,135	-2,139	-7,793	-1,146	-2,140	-7,781
0,235	-7,574	-1,120	-2,125	-7,662	-1,132	-2,128	-7,652
0,240	-7,434	-1,107	-2,112	-7,540	-1,119	-2,114	-7,526
0,245	-7,311	-1,092	-2,099	-7,424	-1,106	-2,103	-7,403
0,250	-7,190	-1,080	-2,087	-7,299	-1,091	-2,091	-7,283

Table.1

N=128	WAVE FG Sous H0		
	Cnst=0	Cnst=5	Cnst=10
0,010	-29,656	-21,334	-9,980
0,020	-24,557	-17,624	-8,154
0,030	-21,553	-15,480	-7,121
0,040	-19,471	-13,886	-6,365
0,050	-17,829	-12,719	-5,844
0,060	-16,515	-11,798	-5,407
0,070	-15,452	-10,964	-5,035
0,080	-14,544	-10,326	-4,722
0,090	-13,750	-9,746	-4,458
0,100	-13,016	-9,212	-4,212
0,110	-12,365	-8,740	-3,996
0,120	-11,803	-8,327	-3,805
0,130	-11,287	-7,952	-3,626
0,140	-10,797	-7,623	-3,467
0,150	-10,346	-7,300	-3,317
0,160	-9,957	-7,028	-3,178
0,170	-9,580	-6,754	-3,046
0,180	-9,231	-6,491	-2,929
0,190	-8,885	-6,245	-2,821
0,200	-8,548	-6,015	-2,720
0,210	-8,249	-5,801	-2,625
0,220	-7,966	-5,599	-2,530
0,230	-7,690	-5,409	-2,441
0,240	-7,430	-5,225	-2,360
0,250	-7,184	-5,046	-2,275

Table.2