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GENERAL EQUILIBRIUM WITH ASYMMETRIC INFORMATION AND DEFAULT PENALTIES

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Abstract. We introduce a two-period general equilibrium model with uncertainty and incomplete financial markets, where default is allowed and agents face in case they do default an utility penalty, which is their own private information. In this setting, if agents have heterogeneous characteristics they will generally pay different returns on any given asset, and thus the same promise made by different agents is in fact not equivalent. If asset trading is anonymous, then the same price is paid for promises whose value can be in fact quite different, and very severe adverse selection problems may arise as consequence. We thus incorporate in the above model an alternative way to negotiate the financial assets, under which an equilibrium exists and the adverse selection problem is mitigated. Succinctly, consumers trade assets non-anonymously with a set of financial intermediaries not allowed to default.

Keywords: Asymmetric information, adverse selection, default penalties, bilateral negotiation, equilibrium.

JEL Classification: D52.

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1. Introduction

1.1. **Motivation.** In the literature on general equilibrium theory under uncertainty, a growing attention has been given in recent years to issues such as the possibility of default in the payment of asset returns, or the existence of asymmetric information about the value of asset returns. These two distinct lines of research have been dissociated one from another, but one can conceive situations where a borrower is allowed to default and knows better than the lender what will be his or her future default level. Concretely, when default is allowed, we need some sort of mechanism to guarantee that borrowers will optimally choose to repay some positive amount in equilibrium, since otherwise nobody would buy assets. In the literature there are three basic types of assumptions made to enforce a positive level of payment: (i) the existence in the economy of some durable good that can serve as collateral and that can be seized if the debtor defaults, (see Dubey, Geanakoplos and Zame [9], for example), (ii) the existence of a default penalty in each debtor's utility function (usually proportional to the level of default), which would reflect a pain of conscience from not keeping his or her promises, or the cost of social disapproval, or still the loss in utility caused by the enforcement of legal sanctions such as a sentence convicting the defaulter to some time in jail

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(see for example Dubey, Geanakoplos and Shubik [8]), or (iii) an hybrid combination of the last two assumptions, in which debtors suffer a penalty relative to the value of default not covered by the collateral (see again for example Dubey, Geanakoplos and Shubik [8]). The introduction of collateral, for instance, can be very advantageous, since it allows for example to guarantee existence of equilibrium in an economy with incomplete markets and infinite horizon (see Araújo, Páscoa and Torres-Martínez [6]).

When we introduce default penalties, it seems reasonable to assume that their level is private information of each borrower, and that lenders do not observe them. In the existing models with asymmetric information relative to future asset returns, the usual approach is to consider that assets are traded anonymously and/or pooled together. The drawback with this approach is that the asset price will reflect an expected average repayment rate, so it will be relatively low for sellers with high repayment rates, and vice-versa. This may possibly lead to the vanishing of asset markets, exactly in the same fashion as in Akerlof's [1] lemons model.

To make the introduction of non-anonymous bilateral negotiation worthwhile[1], it must incorporate some sort of signaling mechanism. In order to get an equilibrium existence result, this signaling mechanism cannot be too complex. The signaling mechanism introduced in our model will be the simplest one could imagine: the buyer asks the sellers how much they want to borrow, before fixing a price for the asset. This will be revealing because the lower an agent's default penalties are, the more assets he or she will want to sell at any given price. Although he or she will be making the announcement before the buyer names a price, he or she does so with a given expectation he or she forms about this price, which must be correct in equilibrium. The buyer in turn, will have a strategy such that the more an agent announces he or she wants to borrow, the lower the price he or she will set. If this strategy function is sufficiently steep, it is possible that even the types with lower default penalties will prefer to borrow a small amount. In equilibrium the asset price and the expected repayment rate will be higher when the asset is negotiated according to this simple signaling mechanism[2] [3].

One may wonder which type of incentive compatibility constraints will the buyers have to satisfy such that the sellers' announcements disclose some useful information about their type. In the particular setting we will adopt we will not need such constraints. Suppose that the number of consumers is very large (but finite, to keep things relatively simple), and that they can be separated in a relatively small number of groups with similar characteristics, including the default penalties they face. Then, a buyer can perfectly choose the same price for the assets sold by all these homogeneous consumers, and since each one is relatively small in comparison with the size of the group with characteristics alike, the particular announcement he or she makes can be assumed to have no impact in the price he or she will receive, such that each and every one of them will act as a price-taker. Then, the announcement they will make will simply be the optimal choice given the (constant) price they expect to pay. To have a term of comparison, we will also analyze what happens when buyers name prices before and then sellers choose quantities. In this case there is no signal to the buyers until the moment they have to form a price (or a return), and so their decision will be the same to every type. As it is easy to guess, this negotiation process will yield very similar results to those of a situation where assets are traded anonymously or pooled together. Also, in this case we do not seem to be able to find an equilibrium existence result, due to a non-convexity in the asset buyers' problem.

The introduction of a monopolistic component in the model is aimed precisely at counter weighting the negative effect of the information asymmetry. I drew this idea from partial equilibrium literature, where very often the solution to improve on the gains from a bilateral negotiation passes

[1]We assume that the costs of bilateral negotiation are null.

[2]Also because when the types with lower penalties get less indebted their optimal repayment rates go up.

[3]If the default penalties are state-dependent the announcement sellers make will never fully uncover their true type, since the signal is a one-dimensional variable and their type is multi-dimensional.

by giving to the part which can be hurt the most the negotiation power. In this kind of literature the message is that an "anomaly", so to speak, can be compensated by another 'anomaly' in order to drive the equilibrium to a superior outcome. And I think that if this is idea holds in many partial equilibrium models, why shouldn't it also hold in general equilibrium models; this papers' main messages has two parts that cannot be dissociated: the introduction of bilateral negotiation as opposed to anonymous negotiation can improve on the outcome, but only if sufficient negotiation power is given to the part which has no private information.

The present model will have some resemblance with the models in either Araújo, Orrillo e Páscoa [4] and Araújo, Fajardo e Páscoa [2]. The main difference between those models and the one presented in this paper is that there we have the presence of some spread functionals that allow the consumers to compute the price they will receive from their sales of assets depending on the amount of collateral they put up to back the promises they make, while here they will face different prices for their short sales simply in function of their identity. This paper has also in this aspect a close similitude with Bisin and Gottardi [7], where they first introduced financial intermediaries who charge bid-ask spreads, and show that this limits the gains that traders can obtain using their informational advantage over the agents in the other side of the market, thus leading to an existence of equilibrium result.

1.2. **Summary.** The economic structure is fully described in detail in section 2. In section 3 we study the equilibrium properties regarding adverse selection problems, and argue that if indexed assets are negotiated in a certain manner that may lead agents to disclose some of their private information (that is, with the signaling mechanism already discussed above), these problems can be lessened and economic efficiency can thus be increased. In section 4 the existence of such equilibrium is established. Finally, in the appendix, the optimization problem of the consumers and the financial intermediaries is given close attention.

2. Model Structure

2.1. **Basic Framework.** In the real world, individuals do not generally trade assets directly with each other. Most often they trade assets with a relatively small number of financial institutions and financial intermediaries. Also very often the assets they can purchase are different from the ones they can sell. The returns in each side of the market will not be in general equal, although correlated. Because there is great concern about bank failures in modern societies, it seems natural that the financial intermediaries may suffer a significantly higher default sanction in case of default than a consumer. We will take this to an extreme by assuming that the financial intermediaries in our model face an infinite default penalty in every state of nature. So, we will consider a two period economy with uncertainty, where there are two types of assets and two types of traders. The traders are divided between consumers (or agents) and financial intermediaries, which we will call bankers. The assets are divided between primitive assets, which promise to pay a given non-negative, state contingent, amount in the second period, and indexed assets whose promised returns in each state are a weighted average of the primitive assets promised returns in that state. Only bankers are allowed to sell the primitive assets, and only consumers are allowed to sell the indexed assets; bankers are also allowed to buy both types of assets, but consumers can only buy primitives. Consumers thus cannot trade directly between them in the financial markets. Indexed assets will be sold at a discount price: their price will be assumed to be equal to the weighted average of the primitive asset prices, minus a certain spread. Primitive assets are standardized and non-exclusive; indexed assets are non-standardized, as the spread may be different for different pairs of buyers and sellers. Agents are allowed to default, but they suffer a subjective default penalty in their utility; bankers are not allowed to do so. Agents have private information about their subjective default penalties.

Our notation and assumptions are the following:

$t = \{0, 1\}$ periods in time.

$S = \{1, ..., S\}$ the set of states of nature in period 1. We take $\overline{S} = \{0\} \cup S$.
$H = \{1, ..., H\}$ the set of consumers.
$B = \{1, ..., B\}$ the set of bankers.
$L = \{1, ..., L\}$ the set of consumption goods.
$w_s^h \in \mathbb{R}_{++}^L$ the endowment of consumer $h \in H$ in state $s \in \overline{S}$.
$w_s^b \in \mathbb{R}_{++}^L$ the endowment of banker $b \in B$ in state $s \in \overline{S}$.
$x_s^h \in \mathbb{R}_{+}^L$ the consumption bundle of consumer $h \in H$ in state $s \in \overline{S}$.
$x_s^b \in \mathbb{R}_{+}$ the consumption bundle of banker $b \in B$ in state $s \in \overline{S}$.
$p_{sl} \in \mathbb{R}_{+}$ the price in units of account of commodity $l \in L$ in state $s \in \overline{S}$
$P = \{1, ..., P\}$ the set of (linearly independent) primitive assets, $P \leqslant S$.
$q \in \mathbb{R}_{+}$ the price in units of account of primitive asset $p \in P$.
$a_s^p \in \mathbb{R}_{+}$ the return of primitive asset p in state $s \in S$.
$\phi_p^h \in \mathbb{R}_{+}$ the purchases of primitive asset $p \in P$ by agent $h \in H$.
$\varphi_p^b \in \mathbb{R}$ the purchases of primitive asset $p \in P$ by banker $b \in B$.
$\psi_h^b \in \mathbb{R}_{+}$ the quantity of indexed asset sold by agent $h \in H$ banker $b \in B$.
$\delta^b \in int\Delta^{B-1}$ the indexant used by banker $b \in B$.
$r_b^s = \delta^b A_s^{'}$ the return promised by the indexed asset b, where $A_s = \left(a_s^p\right)_{p \in P}$.
γ_h^b the spread asked by banker $b \in B$ to agent $h \in H$ for the purchase of one unit of indexed asset $b \in B$.
$\left(\delta^b q - \gamma_h^b\right)$ the price in units of account of each unit of indexed asset $b \in B$.
$\theta_s^h \in [0, 1]$ the repayment rate of consumer $h \in H$ in state $s \in S$.[4]
$\lambda_s^h \in \Lambda_s^h = \left[\underline{\lambda}_s^h, \overline{\lambda}_s^h\right]$ the default utility penalty faced by agent $h \in H$ in state $s \in S$.
$\sigma_s^h(\lambda_s^h) : \Lambda_s^h \to [0, 1]$ the probability distribution of λ_s^h,

The weights vectors δ^b are exogenous: each different banker b is allowed to trade only assets indexed to the returns of a specific portfolio with weights vector δ^b, where we assume that $\delta^b \neq \delta^{b'}$, for every $b \neq b'$.[5] [6]

We assume that bankers consume only one commodity, which we can take, without loss of generality, to be good $l = 1$. One interpretation is that bankers are art lovers whose utility only depends on the number of rare paintings they purchase, or that they only care about the amount of gold they accumulate. The objective is to model bankers as real profit maximizers, the profit being measured by the quantity they can purchase of a specific commodity.

We consider two possible negotiation rules for the spreads γ_h^b: (i) γ_h^b is chosen by the bankers and then agents choose ψ_h^b (hereafter *rule I*)[7] [8], or (ii) bankers first ask each agent to choose ψ_h^b, before

[4]One might wonder if we do not need to impose some rule determining how an agent who owes two different bankers should split his payments between them in case he defaults. We will assume that the agent has to pay to every banker in proportion to the value of their claims. In this way he or she would be legally forbidden to favor any banker in prejudice of the rest. This hypothesis also allows us to assume that the default penalties are independent from b.

[5]Thus, if $B \leqslant S$ each banker demands to its debtors a vector of returns linearly independent from those of all his competitors. This means that if $B < S$ we are thus introducing an element of monopolistic competition in the model. This hypothesis will be in many cases paradoxically convenient in terms of efficiency, since it gives some degree of market power to the less informed players in the economy.

[6]We are thus introducing an element of monopolistic competition in the model. This hypothesis will be in many cases paradoxically convenient in terms of efficiency, since it gives some degree of market power to the less informed players in the economy.

[7]The first rule is intended to parallel models with anonymous negotiation. In the later an asset buyers' best guess about the future repayment rate is an average across all individuals in the economy, in this model and under rule I the asset buyers best guess is an average across all types an agent can have.

[8]Note that this negotiation rule is equivalent to one in which bankers and agents announce their respective choices about spreads and borrowings simultaneously, since agents will be able to predict exactly each banker's optimal choice in a Nash equilibrium.

setting the spreads (hereafter *rule II*).[9] In this paper we establish the existence of equilibrium when indexed assets are negotiated according to rule II, but we are (apparently) unable to do the same regarding rule I (see lemma 4.2 and footnote 31 on page 14). But in section 3 we argue and provide an example showing that rule II can be anyway superior to rule I in terms of efficiency.[10]

The basic set of relationships can be depicted in the following two figures, the first one for period 0 and the second one for period 1.[11] The thick white arrows represent unit of account flows, the thick grey arrows represent asset flows, and the thin black lines represent commodity flows.

Insert figure 1 here.

Insert figure 2 here.

Note that no assumption is made about if markets are complete. We may either have $P = S$ or $P < S$. But the important thing to have in mind is that since agents face a difference in the price of primitive and indexed assets, even if we have $P = S$ it may be very costly for them to fully span $\mathbb{R}^S$. We can imagine situations in which the spreads are such that $\left(\delta^b q - \gamma_h^b\right)$ is very close (or even equal) to zero.

2.2. **Agent Types.** Agents' choice will depend on their S marginal disutilities λ_s^h. An agent's type must thus be characterized by the vector $\lambda^h := \left[\begin{array}{ccccccc} \lambda_1^h & \lambda_2^h & \cdots & \lambda_s^h & \cdots & \lambda_S^h \end{array}\right]$. This vector is, a random variable with domain in $\Lambda^h = \Lambda_1^h \times ... \times \Lambda_S^h$. We assume that $\sigma_s^h(\lambda_s^h)$ may be correlated across different agents for each $s \in S$.

2.3. **Uncertainty.** In this model there is not only uncertainty about the state of nature in period 1, but also doubt about the entire matrix $\left(\lambda^h\right)_{h\in H}$, and their induced future deliveries. Since bankers only care about consumption of good 1 while agents mind about the L commodities available, then in state s the lower the entries on $\left(\lambda_s^h\right)_{h\in H}$ are, the higher the equilibrium relative prices of goods $l = 2, .., L$ will be in terms of good 1. In this model, prediction also involves trying to figure out what will be the agents' default rates, and by this means, what will be the relative price vector in each state. Moreover, this reasoning goes a little deeper: default rates are themselves influenced by the relative prices since these determine the real value of consumers endowments, asset revenues and debts, and all these factors influence consumers' optimal level of deliveries. Default penalties and future expected relative prices also influence how much consumers will want to consume, borrow, and lend in period 0 for any given period 0 price vector, implying that first period equilibrium variables depend also on default penalties.

Since equilibrium prices depend on λ, their observation can disclose some information about the true matrix λ. We can make different assumptions about the way in which the bankers take information out of prices. We may assume that they do not have the necessary sophistication and skills to perform such heavy computations[12]. Or we may consider, at the extreme opposite, that they can at no cost restrain their beliefs about the true value of λ to the set $\Lambda(y_0) = \{\lambda : y_0 \in \mathcal{Y}_0(\lambda)\}$, where $y_0 \equiv (p_0, q)$ and $\mathcal{Y}_0(\lambda)$ is the set of period 0 equilibrium price vectors y_0 consistent with the matrix λ.[13] The first assumption leads us to the concept of a *Walrasian Equilibrium*, while the second one is in line with the notion of a *Rational Expectations Equilibrium,* as first introduced by

[9]For a discussion on the different inplications of these two rules, refer to section 3.

[10]Although it is conceivable that in some situations the worst payers would be better of under rule I, expecially when they have a low probability.

[11]Again, note that we are allowing bankers to both buy and sell primitive assets, while agents are forbidden from selling them short. Hence when a banker buys a primitive, it must be buying it from another banker. This can be seen as the functioning of a inter-banking monetary market, where financial intermediaries with lack of liquidity borrow from fellows with excess liquidity.

[12]Or that they do have but the cost is too high, or even that they are simply too time consuming and cannot be performed in useful time.

[13]Note that we do not assume that they extract information from second period price vectors. In period 1 there is nothing bankers can do to influence consumers' decisions, so new information is worthless.

Radner [11]. Fortunately, our setting will be general enough to embrace these two different notions of equilibrium.

The matrix λ has a probability distribution $\sigma : \Lambda \to [0,1]$, where $\Lambda = \prod_{s\in S}\prod_{h\in H}\Lambda_s^h$. Bankers' expectations about λ after observing prices will be denoted by $\sigma(\lambda|y_0)$, and if we assume that they extract no information from prices, this will simply be independent from y_0. In the case of agents, they already know their own type, so we replace $\sigma(\lambda|y_0)$ by $\sigma(\lambda|y_0,\lambda^h)$.

2.4. **Optimization Problems.**

2.4.1. *Agents.*

2.4.2. *Utility Functions.*

We assume that each agent has a von-Neumman-Morgenstern utility function, in which time 0 utility is given by

$$U_0^h(x_0^h) = u_0^h(x_0^h)$$

In each state of period 1 his or her utility will be given by

$$U_s^h(x_s^h, D_s^h) = u_s^h(x_s^h) - \lambda_s^h D_s^h$$

$\forall s \in S; \forall \lambda_s \in \Lambda_s = \prod_{h=1}^H \Lambda_s^h$ [14], and where $D_s^h \in \mathbb{R}_+$ is his or her level of default in state s, in units of account. Each agent h gives a subjective probability $\alpha_s^h \geqslant 0$ to state $s \in S$, with $\sum_{s=1}^S \alpha_s^h = 1$. An agent h with type λ^h has an expected utility given by

$$U^h(x^h, D^h, \lambda^h, y_0) = U_0^h(x_0^h) + \sum_{s\in S}\alpha_s^h U_s^h(x_s^h, D_s^h) =$$

$$= u_0^h(x_0^h) + \sum_{s\in S}\alpha_s^h\left[u_s(x_s^h) - \lambda_s^h D_s^h\right]$$

2.4.3. *Budget Constraints.*

The set of budget constraints in period 0 of agent h is given by

$$p_0 x_0^h + q\phi^h \leqslant p_0 w_0^h + \sum_{b\in B}(\delta^b q - \gamma_h^b)\psi_h^b \tag{2.1}$$

$$\psi_h^b \in \mathbb{R}_+^B. \tag{2.2}$$

From now on we will assume that $(p_0, q) \in \Delta^{L+P-1}$. [15]

In state s of period 1, and for every λ, his budget constraints are

$$p_s(\lambda)x_s^h(\lambda) + p_{s1}(\lambda)\theta_s^h(\lambda)\sum_{b\in B} r_b^s\psi_h^b \leqslant p_s(\lambda)w_s^h + p_{s1}(\lambda)A_s\phi^h \tag{2.3}$$

$$\theta_s^h(\lambda) \in [0,1] \tag{2.4}$$

$\sum_{b\in B} r_b^s\psi_h^b$ is agent h's total debt in state s, so $\theta_s^h(\lambda)\sum_{b\in B} r_b^s\psi_h^b$ is the amount of resources that he or she dedicates to debt service. His or her default will be equal to $D_s^h(\lambda) = (1-\theta_s^h(\lambda))\sum_{b\in B} r_b^s\psi_h^b$. From now on we assume that $p_s(\lambda) \in \Delta^{L-1}$, $\forall s \in S$, $\forall \lambda \in \Lambda$. The constraints (2.3) and (2.4) are written in terms of repayment rates, but can alternatively be written in terms of default levels:

$$p_s(\lambda)x_s^h(\lambda) + p_{s1}(\lambda)\left[\sum_{b\in B} r_b^s\psi_h^b - D_s^h(\lambda)\right] \leqslant p_s(\lambda)w_s^h + p_{s1}(\lambda)A_s\phi^h \tag{2.5}$$

[14] Here, consumption bundles and default levels are not presented as functions of agent types and spreads proposed by bankers; this dependency results only from utility maximization.

[15] Note that the assumption that $\delta^b \in int\Delta^{P-1}$, together with $w_0^h \gg 0$, guarantees that 2.1 has an interior point, by choosing $\psi_h^b > 0$ for some b.

$$0 \leqslant D_s^h(\lambda) \leqslant \sum_{b\in B} r_b^s \psi_h^b \tag{2.6}$$

Although the first representation is more elegant, the second is technically more convenient to establish the upper semicontinuity of the agents' best response correspondences. But after that we simply take $\theta_s^h(\lambda) = 1 - \frac{D_s^h(\lambda)}{\sum_{b\in B} r_b^s \psi_h^b}$ and proceed with the first representation.

2.4.4. *Bankers.*

2.4.5. *Utility Functions.* Bankers have von-Neumann-Morgenstern utility functions depending only on consumption of commodity 1, and have infinite default utility penalties. Each banker b gives a subjective probability $\beta_s^b \geqslant 0$ to state s, with $\sum_{s\in S} \beta_s^b = 1$. Thus, banker b expected utility function will be given by

$$V^b(x^b, D^b) = \begin{cases} V_0^b(x_0^b) + \sum_{s\in S} \beta_s^b v_s^b(x_{s1}^b) \text{ if } D^{be} = 0 \\ -\infty \text{ if } D^{be} > 0 \end{cases}$$

where $D^{be} \equiv \sum_{s\in S} \beta_s^b D_s^b$ is the banker's expected default. Notice that if a given banker b attributes a subjective probability equal to zero to some state s, then we could have this banker planning to default in that state. To be consistent with the hypothesis of an infinite default penalty we must impose that $\beta_s^b > 0$, $\forall s \in S$, $\forall b \in B$. This is simply a prudential assumption.[16]

2.4.6. *Budget Constraints.* In the first period, banker's b budget constraint is

$$p_{01} x_{01}^b + q\varphi^b + (\delta^b q - \gamma_h^b) \sum_{h\in H} \psi_h^b \leqslant p_0 w_0^b. \tag{2.7}$$

His constraint for a given state $s \in S$ and matrix λ is[17]

$$p_{s1}(\lambda) x_{s1}^b(\lambda) \leqslant p_s(\lambda) w_s^b + A_s \varphi^b + \sum_{h\in H} \theta_s^h(\lambda) r_b^s \psi_h^b. \tag{2.8}$$

2.5. **Strategies.**

2.5.1. *Agents.* If we consider rule I agents make their decisions only after observing the spreads set by the bankers, and always given the relative price vectors he or she expects to face. So, a strategy under rule I for type λ^h must be a function of the vectors $\gamma_h = (\gamma_{bh})_{b\in B}$, y_0, and $p_1 \equiv \{p_{1s}(\lambda_N)\}_{s\in S}^{\lambda_N \in \Lambda_N}$. It will depend on y_0 not only because y_0 affects directly his optimal plan, but also its observation may change the expectation about p_1, by disclosing some information about the matrix λ^{-h}.

If rule II is considered instead, agents no longer observe spreads before taking their period 0 decisions, but they do so with a given expectation about what these spreads will be, and this expectation has to be correct in any Nash equilibrium. So, the only difference in the definition of an agent's strategy under rules I and II is that in rule II the observed spread vector γ_h is replaced by an expected spread vector γ_h^e.

[16]The only hypothesis about bankers' preferences towards risk we must postulate is that they cannot be risk-lovers. In the proof of proposition 1 below we need to assume that the bankers preferences are convex, but not necessarily strictly so. We can either assume that a banker is risk-averse or risk-neutral. The fact that the bankers face an infinite default penalty is sufficient to guarantee that they do not take excessive risk even if they are risk-neutrals, because they must be able to fully pay their debts with probability one.

[17]In each period the banker will sell the endowment he or she has of commodities $l = 2, ..., L$, since they do not enter in his or her utility function, and will buy or sell commodity 1. We need to have $w_{sl}^b > 0$, $\forall s \in S$, $\forall l \in L$, to guarantee that constraints (2.7) and (2.8) have always an interior point (note that q can be equal to zero). Otherwise, at zero consumption, an eventualy infinite marginal utility of income would conflict with a infinite default penalty.

With this slight detail in mind, a strategy under rule II for type λ^h is thus a function

$$\Phi_{\lambda^h}(\gamma_h^e, y_0) = \left(x_0^{\lambda^h}(\gamma_h^e, y_0), \left\{ x_s^{\lambda^h}(\gamma_h^e, y_0) \right\}_{s=1}^{S}, \phi^{\lambda^h}(\gamma_h^e, y_0), \psi_{\lambda^h}^b(\gamma_h^e, y_0), \left\{ D_s^{\lambda^h}(\gamma_h^e, y_0) \right\}_{s=1}^{S} \right) :$$

$$: \mathbb{R}^B \times \Delta^{L+P-1} \to \mathbb{R}_+^{L(S+1)} \times \mathbb{R}_+^P \times \mathbb{R}_+^B \times \mathbb{R}_+^S.$$

Let $\Phi_h(\gamma_h^e, y_0)(\lambda^h)$ denote the mapping that assigns to each $\lambda^h \in \Lambda^h$ the corresponding strategy $\Phi_{\lambda^h}(\gamma_h^e, y_0)$. Bankers' expectations about the value of agent h choice variables will be given by the integral $\int_{\Lambda^h} \Phi_h(\gamma_h^e, y_0)(\lambda^h) d\sigma^h(\lambda^h|y_0)$, where $\sigma^h = \Pi_{s\in S}\sigma_s^h$. This includes the expected consumption plan $x^{he} = \int_{\Lambda^h} x^h(\gamma_h^e, y_0)(\lambda^h) d\sigma^h(\lambda^h|y_0)$, the expected primitive asset purchases $\phi^{he} = \int_{\Lambda^h} \phi^h(\gamma_h^e, y_0)(\lambda^h) d\sigma^h(\lambda^h|y_0)$, the expected borrowing $\psi_h^e = \int_{\Lambda^h} \psi_h^e(\gamma_h^e, y_0)(\lambda^h) d\sigma^h(\lambda^h|y_0)$ and his or her expected repayment rates vector $\theta^{he} = \int_{\Lambda^h} \theta^h(\gamma_h^e, y_0)(\lambda^h) d\sigma^h(\lambda^h|y_0)$ (or alternatively, his or her expected default vector $D^{he} = \int_{\Lambda^h} D^h(\gamma_h^e, y_0)(\lambda^h) d\sigma^h(\lambda^h|y_0)$). At equilibrium[18], these functions must maximize the expected utility function of each agent, and in order for a banker to be able to derive these expectations he or she must know, besides the distributions $\sigma_s(\lambda_s)$, the utility function of each agent, their endowments in each period and in each state, and must have the necessary sophistication to solve their problem and thus obtain the above integrals.[19] [20]

Remark 2.1. Although the bankers are modeled as a sort of competitive monopolists, their problem is more complicated than simply taking consumers' demand functions as given; since they depend on parameters which are random from the banker's point of view, the banker has monopoly power on markets where he simply takes as given *expected* demand functions. In this model, the role of rule II will be preciselly to disclose some information about the *true* demand functions.

2.5.2. *Bankers.* Banker's choice variables are his or her consumption in period 0 and in each state of period 1, his or her portfolio of primitive assets, and the spreads to ask to each agent. But if we want to model them as profit maximizers in terms of commodity 1, it makes more sense to take only φ^b and γ_b as his (financial) decision variables, and simply let his consumption in each state $s \in \overline{S}$ be given by the equality in his budget constraints, that is,

$$x_{01}^b = \frac{p_0 w_0^b - q\varphi^b - (\delta^b q - \gamma_h^b) \sum_{h\in H} \psi_h^b}{p_{01}}$$

and

$$x_{s1}^b(\lambda) = \frac{p_s(\lambda) w_s^b + A_s \varphi^b + \sum_{h\in H} \theta_s^h(\lambda) r_b^s \psi_h^b}{p_{s1}(\lambda)}$$

A strategy for banker b must specify the value of his or her decision variables for each vector y_0,[21] for each vector γ_{-b} of spreads chosen by the other bankers, and for each array $\theta \equiv \left\{ \theta_s^h(\lambda) \right\}_{(s,h)\in S\times H}^{\lambda\in\Lambda}$, since these repayment rates enter in the expression of the functions $\left\{ x_{s1}^b(\lambda) \right\}_{s\in S}^{\lambda\in\Lambda}$. If we consider rule II the banker's strategy must depend also on the agents announcements of their borrowing requirements, $\psi^b \in \mathbb{R}_+^H$. So, a strategy for banker b under rule II will be a function

$$\Phi_b(y_0, \gamma_{-b}, \psi^b) = \left(\varphi^b(y_0, \gamma_{-b}, \psi^b), \gamma^b(y_0, \gamma_{-b}, \psi^b) \right) :$$

[18] To be defined below, in section 2.6.

[19] We are assuming that the bankers know the agent's entire utility function, except for a single vector of parameters. This may sound unreasonable, but if the banker's lack of knowledge were deeper the asymmetric information problem would be even more complex, and probably unmanageable. But the consumers' taste for a certain consumption good can many times be accurately anticipated in real life, so this assumption can have some reasonability.

[20] In deriving these functions, it is implicit the use of an expected price vector. If the agents' expected price vector differs from the bankers' expected price vector, the functions derived by the bankers would be different from the true functions Φ_{λ^h}. Since in equilibrium the players' expectations must be correct, this problem will not stand in equilibrium. The same applies for the expected spread vector γ_h^e.

[21] Since it not only can disclose some private information, but also because it influences the agent's optimal choices for whichever their type might be.

$$: \Delta^{L+P-1} \times \mathbb{R}^{(B-1)H} \times \mathbb{R}^{H}_{+} \to \mathbb{R}^{P} \times \mathbb{R}^{H}.$$

2.6. **Equilibrium.** An equilibrium in this model consists in a price vector $p = \left(p_0, q, \{p_s(\lambda)\}_{s\in S}\right)$, a family of functions $\{\Phi_h\}_{h\in H}$, indicating the strategies of all agents, and a family of functions $\{\Phi_b\}_{b\in B}$ indicating the strategies of each one of the B bankers, satisfying

a) the strategies Φ_h maximize agents' utility, given all the bankers' strategies $\{\Phi_b\}_{b\in B}$, the price vector $p = \left(p_0, q, \{p_s(\lambda)\}_{s\in S}\right)$, and their budget constraints.

b) each banker's b strategy maximize Φ_b his or her utility, given all other bankers' strategies, the strategies of all agents $\{\Phi_h\}_{h=1}^{H}$, the price vector $p = \left(p_0, q, \{p_s(\lambda)\}_{s\in S}\right)$, and his or her budget constraints.

c) *market-clearing* in every *spot* market:

$$\sum_{h\in H} x_0^h + \sum_{b\in B} x_0^b = \sum_{h\in H} w_0^h + \sum_{b\in B} w_0^b,$$

$$\sum_{h\in H} x_s^h(\lambda) + \sum_{b\in B} x_s^b(\lambda) = \sum_{h\in H} w_s^h + \sum_{b\in B} w_s^b, \forall s \in S;\ \text{a.e. } \lambda \in \Lambda$$

d) *market-clearing* in primitive asset markets:

$$\sum_{h\in H} \phi^h + \sum_{b\in B} \varphi^b = 0.$$

We do not need to impose market-clearing in the indexed asset market, since bankers do not choose quantities to buy.

3. Adverse Selection and the Second Negotiation Rule

The equilibrium, whose existence under rule II is demonstrated in section 4, exhibits adverse selection symptoms. From the formal analysis in the Appendix, we can see in the agents' first order conditions (equation (5.2)) that the higher the default penalties are the less the agent will borrow. When λ_s^h increases for some state $s \in S$, to re-equate $\frac{\partial \mathcal{L}^h}{\partial \psi_h^b}$ to zero is necessary to increase μ_0, the period 0 income marginal utility, and to decrease μ_s, the state s income marginal utility, for each $s \in S$, and this can only be done by transferring some income from period 0 to period 1, while borrowing does the reverse role. The effective cost of credit can be broken in two components: the deliveries that the agent decide to make, and the penalty he suffers against his default. At the optimal solution the agent will balance these two costs; if the default penalty becomes lower in one state the total cost of borrowing is now smaller if it is optimal to default in that state, and remains unchanged otherwise.

It is also shown in the appendix (remark 5.1) that ψ_h^b is decreasing in γ_h^b, as natural. We also have, again by (5.2) and by the discussion in remark 5.2,

$$\frac{\partial^2 \mathcal{L}^h}{\partial \psi_h^b \partial \gamma_h^b} = -\sum_{s\in S} \alpha_s^h \int_\Lambda \lambda_s^h (1-\theta_s^h) d\sigma(\lambda|y_0, \lambda^h) -$$

$$-\sum_{s\in S} \alpha_s^h \int_\Lambda \mu_s \theta_s^h d\sigma(\lambda|y_0, \lambda^h) = -\sum_{s\in S} \alpha_s^h \int_\Lambda \min\left(\mu_s, \lambda_s^h\right) d\sigma(\lambda|y_0, \lambda^h) \leqslant 0$$

which is higher in absolute value the higher each λ_s^h is, not only because of their direct effect, but also because a higher λ_s^h leads to a higher repayment rate and smaller consumption in state s, and thus a to higher income marginal utility μ_s. The fundamental inference is that agents with higher default penalties are more responsive to changes in γ_h^b. This is natural, since when λ_s^h is lower the agent becomes less worried about paying the future returns, and thus more willing to borrow even at a lower indexed asset price $q - \gamma_h^b$. So we have the following picture:

Insert figure 3 here.

where we are assuming for graphical simplicity that $\lambda_1^h = \lambda_2^h = ... = \lambda_S^h = \lambda^h$. The higher λ^h is, the lower the supply of indexed assets to banker b, for any fixed value of γ_h^b, and the higher the reduction in ψ_h when γ_h^b increases from $\gamma_h^{b\prime}$ to $\gamma_h^{b\prime\prime}$.

This indexed asset supply behavior has very important implications. A banker would like to have some way to limit the value of ψ_h^b chosen by the worst types, but the only mechanism available in our setting is the choice of γ_h^b. He could achieve this goal by asking a sufficiently high spread. But this would decrease more than proportionally the value of ψ_h^b for the types with higher default penalties, which is already comparatively low. So, by asking higher spreads the banker will be driving the best types away from dealing with him, constraining himself to trade indexed assets only with the "not so good" types. And he or she could react to this by increasing γ_h^b even further, with the consequence that only the worst amongst the worst types would still be interested in selling assets. The result of this process could be, in the limit, and if such a process is unchained for all pairs banker-consumer, the vanishing of the indexed asset markets. And if the indexed assets market disappears, the level of trade in the primitive asset markets will also be reduced.

This adverse selection problem is also likely to arise in models where asset markets are anonymous. The reason is that the asset price will reflect an average default rate, so that agents that intend to have a high repayment rate may consider the price too low and decide not to sell the asset. That would worsen the expected default rate, leading to an increase in the asset price, unchaining the same kind of market vanishing process.

In this particular model, asymmetric information has a negative effect, even if asset markets do not vanish. Since agents cannot short-sell primitive assets, even if the number of primitive assets satisfies $P = S$, a consumer seen by bankers as being to risky may not be able to choose an asset portfolio that fully spans $\mathbb{R}^S$, because he could face spreads very close to q (or even equal), rendering high transfers of income from period 1 to period 0 unfeasible or non-optimal for him.[22]

So, a main issue that should be addressed when asymmetric information is present is to devise some sort of mechanism that could mitigate these adverse selection distortions. In our setting, if bankers could impose a limit κ_h^b on ψ_h, figure 3 would change to

Insert figure 4 here.

κ_h^b would serve to explicitly limit the short sales of the types with lower default penalties, while at the same time a lower γ_h^b could induce the types with higher default penalties to increase ψ_h^b. But we are unable to guarantee that such an equilibrium exists, since bankers now would have the ability to simultaneously determine the price and explicitly influence the quantity traded. This would introduce a non-convexity in each banker's strategy set, and the only way we could try to go around this would be introducing a continuum of bankers (which would not make much economic sense), but without success. The intuitive reason is that agents do not care about the expected value of the spreads that each banker propose to them; what matters to agents is which banker represents the less costly financing opportunity. Our hope would be to use a purification technique similar to the one used in Araújo, Orrillo and Páscoa [4] or Araújo and Páscoa [5], but the agents' problems in this case will not depend on the bankers' mixed strategies profile only through a finite number of expected values, so no purification technique seems feasible.

Nonetheless, bankers can influence the quantity traded, but only implicitly, if they use rule II. If the banker's strategy where to ask higher spreads when the announced values of ψ_h^b are also

[22]This could be seen as an argument in favor of less pungent default penalties, the fact that default may help agents to obtain full insurance more easily, but at the same time one of the reasons for the spread being too high may be the fact that the agent has a low default penalty with high probability. The other reason may be the imperfect competition that exist between the bankers in the choice of spreads. But if the indexants for distinct bankers are not very different, that is, if $\delta^b \approx \delta^{b'}$ for each $b \neq b'$, then the *spread* will tend to be close to zero if the bankers look at the agents as being reliable debtors, because competition will then be relatively intense (the gain in market share from a certain decrease in the asked spread will be significant) leading the demanded price to be closer to the bankers' marginal cost, in this setting the opportunity cost of funds, that is, q.

higher, then the agents, anticipating the optimal strategy for the bankers might prefer to choose a lower ψ_h^b to benefit from a lower spread. A main difference between the two negotiation rules is that in the first one spreads are independent from the quantities traded, since they are chosen before, while in the second rule they are not. Rule II displays yet another interesting feature, which is concomitant with the previous one and reinforces it: the announcement of ψ_h^b may disclose some new information about the type of agent h. Since bankers know the agents' entire utility functions and endowments except the default penalties, and are able to derive their optimal solution for each vector $\lambda^h \in \Lambda^h$, bankers could infer from the observation of the optimal choices ψ_h^b something about the true value of λ^h.

Since no incentive compatibility constraints are introduced in the bankers' problem, agents could try to mislead them by choosing a ψ_h^b below the optimal, in order to take advantage of a lower spread and then default in period 1 more than bankers were expecting. But if the set H can be partitioned in J subsets H_j, each one with a sufficiently high cardinality, and such that the agents belonging to the same partition have highly correlated types,[23] this problem can be avoided. The reason is that since each individual is small relative to the size of his group, the banker's strategy can perfectly consist in setting a spread, which can be different for each agent, based not on the announcement of each agent taken separately, but on the profile of announcements across all agents in the same group. Then each agent's announcement will have very little impact on the spread, and we can assume that in equilibrium every agent will choose their optimal ψ_h^b, with no strategic consideration whatsoever.[24]

The main message is that under rule II a nonlinear pricing schedule arises implicitly in each banker's strategy, which is not necessarily incentive compatible, but agents choose the "right" announcement for their type due to the price-taking hypothesis. In a remarkable paper, Monteiro and Page [10] established a competitive analogue to the revelation principle, the implementation principle, and showed that a game between oligopolistic firms facing a consumer possessing private information, and where the firms' strategy space is the space of all implementable nonlinear pricing schedules can be reduced to a strategically equivalent game played over product-price catalogs. They also showed that a Nash equilibrium exists for the mixed extension of the later class of games, but argue that, since the space of product-price catalogs is not a vector space, no equilibrium can be shown to exist in pure strategies.

Finally, notice that the assumption that $\delta^{b'} \neq \delta^{b''}$ for $b' \neq b''$ permits the bankers to have higher flexibility in spread choice, since they have some degree of market power over the agents, so they can choose steeper spread functions in their optimal strategy than they would otherwise be able to do.[25] [26] [27]

3.1. **A "Simple" Example.** Consider an economy with just one commodity, one state of nature in period 1, one primitive asset, with price q and return $r = 1$, one banker, and one consumer with only two equally likely possible types, $\underline{\lambda}^1 = \frac{1}{4}$ and $\overline{\lambda}^1 = \frac{3}{4}$. The endowments are $\left(w_0^1, w_1^1\right) = (1, 2)$ for the agent and $\left(w_0^b, w_1^b\right) = (3, 1)$ for the banker.

[23] Suppose for example that all academics will face similar default penalties across all states of nature.

[24] This assumption is by no means different than asssuming that agents act as price takers in a simple finite dimensional pure exchange economy.

[25] Monteiro and Page [10] assume that each type utility depends on the identity of the the firm with which he or she contracts.

[26] Without this assumption, the spreads in equilibrium do not have to be the same for all bankers, since different bankers may give different subjective probabilities to different states of nature, thus regarding the same agent differently.

[27] If, under rule II, the agents of a certain group H_J announce high ψ_s^b's, then all bankers will expect to suffer a high level of default in case they lend to these agents, and will choose high spreads even if $\delta^{b'} \neq \delta^{b''}$ for all $b' \neq b''$. In fact bankers may end up competing between them to **not** lend to these agents, leading to $\gamma_h^b = q$, for all $b \in B$ and $h \in H_j$.

Since there is only one market in period 1, we can take the normalization $p_1 = 1$. In period 0 there are two markets open, but by Walras law only one is independent, and by homogeneity we can normalize $p_0 = 1$, leaving q totally free. The agent's problem is

$$\underset{\left\{x_0^1, x_1^1, \phi^1, \psi_1, \theta^1\right\}}{Max} U^1 = \ln x_0^1 + \ln x_1^1 - \lambda^1 \left(1 - \theta^1\right) \psi_1$$

$$s.t. \qquad \begin{cases} \left(x_0^1 - w_0^1\right) + q\phi^1 - (q - \gamma_1)\,\psi_1 \leqslant 0 \\ \left(x_1^1 - w_1^1\right) + \theta^1\psi_1 - \phi^1 \leqslant 0,\ \theta^1 \in [0,1] \end{cases}$$

I totally abstain from presenting the consumer's best response function, due to its complexity and size (it would occupy almost an entire page). It is a function with a total of 12 branches, each one for a different combination of q, γ_1 and λ^1. Although it is continuous, it exhibits kinks in the passage between different branches.

Due to the (relative) simplicity of this example, it must be assumed that the banker does not extract information from the observation of q, because otherwise this would be all he needs to determine the value of λ^1. Assume that his objective function is $U^b = \ln x_0^b + \ln x_1^b$.

It can be shown after some equally messy computations, which have to be made branch by branch due to the non-differentiability of the above reaction function, that the equilibrium under negotiation rule I, independently of the true realization of λ^1, is such that $q = 3$ and $\gamma_1 \geqslant \frac{11}{4}$, implying $q - \gamma_1 \leqslant \frac{1}{4}$. At such a low indexed asset price neither type wants to sell the indexed asset, and they are also not interested in buying the primitive asset at such high price. Thus in this equilibrium financial markets vanish and both individuals have to content themselves with consuming their endowments.

Under rule II, if the agent does not try to behave strategically,[28] there are two possible equilibriums, but both such that type $\underline{\lambda}^1$ prefers not to borrow and the banker chooses in response to an announcement $\psi_1 = 0$ a spread $\gamma_1 \geqslant q - \frac{1}{4}$. Type $\overline{\lambda}^1$ in turn announces an intention to sell $\psi_1 = \frac{7}{8}$ units of indexed asset, and the banker's reaction to such an announcement is to demand a spread $\gamma_1 = 0$. In period 1 type $\overline{\lambda}^1$ will choose $\theta^1 = \frac{4}{5}$, for a total repayment equal to $\frac{7}{10}$. We have two possible equilibrium values for the price q: when the true realization of the agent's type is $\underline{\lambda}^1$, we have $q = 3$, and when its true realization is $\overline{\lambda}^1$ we have $q = \frac{4}{3}$. So, when the agent is of type $\underline{\lambda}^1$, both him and the banker have the same utility under both rules, simply because the banker does not want to trade with him. Under rule II, the banker and the consumer are better off when the consumer is of type $\overline{\lambda}^1$, since he can signal that he is a relatively good payer. The consumers utility is improved from $\ln 1 + \ln 2 = 0.693\,15$ under rule I to $\ln\left(1 + \frac{4}{3}\frac{7}{8}\right) + \ln\left(2 - \frac{7}{10}\right) - \frac{3}{4}\left(\frac{7}{8} - \frac{7}{10}\right) = 0.904\,3$, while the banker's payoff is improved from $\ln 3 + \ln 1 = 1.0986$ to $\ln\left(3 - \frac{4}{3}\frac{7}{8}\right) + \ln\left(1 + \frac{7}{10}\right) = 1.136\,8$.

4. Existence of Equilibrium

In this section we prove the following theorem:[29]

Theorem 4.1. *If all agents' utility functions are concave and strictly monotone in consumption, if all bankers' utility functions are concave and monotone, if rank $A = P$, if $\underline{\lambda}_s^h > 0$ for all $(s,h) \in S \times H$, and if $\beta_s^b > 0\ \forall b \in B$ and $\forall s \in S$, then a pure strategies equilibrium exists for each realization of $\lambda \in \Lambda$, when indexed assets are negotiated according to rule II.*

4.1. **Truncated Economy.** Our proof will be done through finite dimensional approximations. We start by truncating the economy, in terms of consumption bundles, asset portfolios, spreads, and also agent types.

Specifically, we define a truncated economy $\mathcal{E}_N$ as one in which:

[28] You can consider that instead of one agent there are 1000 exact copies of the same consummer, with types perfectly positivelly correlated, and multiply also the banker's endowment by 1000.

[29] Which holds wheather banker extract information from period zero prices or not.

a) for each pair (h,s) the domain $\Lambda_s^h = \left[\underline{\lambda}_s^h, \overline{\lambda}_s^h\right]$ is partitioned into N sub-intervals of equal length. In subinterval $\Lambda_{snN}^h = \left[\underline{\lambda}_s^h + \frac{n-1}{N}\left(\overline{\lambda}_s^h - \underline{\lambda}_s^h\right), \underline{\lambda}_s^h + \frac{n}{N}\left(\overline{\lambda}_s^h - \underline{\lambda}_s^h\right)\right]$, $1 \leqslant n \leqslant N$, the value of the default utility penalty of agent h in state s will be given by

$$\lambda_{snN}^h = \int_{\underline{\lambda}_s^h + \frac{n-1}{N}\left(\overline{\lambda}_s^h - \underline{\lambda}_s^h\right)}^{\underline{\lambda}_s^h + \frac{n}{N}\left(\overline{\lambda}_s^h - \underline{\lambda}_s^h\right)} \lambda_s^h d\sigma_s^h(\lambda_s^h),$$

b) agent h's admissible consumption space is

$$X_N^h = \left\{x^h \in \mathbb{R}_+^{L(SN^{SH}+1)} : x_{0l}^h \leqslant N \ \forall l \in L \wedge x_{sl}^h(\lambda) \leqslant N \ \forall (s,l,\lambda) \in S \times L \times \Lambda\right\}$$

and his admissible portfolio space is

$$Y_N^h = \left\{\left(\phi^h, \psi_h\right) \in \mathbb{R}_+^P \times \mathbb{R}_+^B : \phi_p^h \leqslant N \ \forall p \in P \wedge \psi_h^b \leqslant N \ \forall b \in B\right\}$$

c) banker b's admissible primitive asset portfolio space is

$$Y_N^b = \left\{\varphi^b \in \mathbb{R}^P : \left|\varphi_p^b\right| \leqslant N \ \forall p \in P\right\}$$

and his admissible spread space is

$$\Gamma_N^b = \left\{\gamma^b \in \mathbb{R}^H : \left|\gamma_h^b\right| \leqslant N \ \forall h \in H\right\}.$$

In the truncated economy $\mathcal{E}_N$, λ_{sN}^h is a random variable with N possible realizations λ_{snN}^h, $1 \leqslant n \leqslant N$. Every player in the truncated economy $\mathcal{E}_N$ will expect λ_{sN}^h to assume the value λ_{snN}^h with probability $\sigma_s^h(\lambda_{snN}^h) = \sigma_s^h\left(\left[\underline{\lambda}_s^h + \frac{n-1}{N}\left(\overline{\lambda}_s^h - \underline{\lambda}_s^h\right), \underline{\lambda}_s^h + \frac{n}{N}\left(\overline{\lambda}_s^h - \underline{\lambda}_s^h\right)\right]\right)$. Let $\lambda_N = (\lambda_{sN}^h)_{s\in S}^{h\in H}$, let Λ_N be the set of all possible realizations of the matrix λ_N, and let $\sigma_N = \sigma\left(\left[\underline{\lambda}_s^h + \frac{n-1}{N}\left(\overline{\lambda}_s^h - \underline{\lambda}_s^h\right), \underline{\lambda}_s^h + \frac{n}{N}\left(\overline{\lambda}_s^h - \underline{\lambda}_s^h\right)\right]_{s\in S}^{h\in H}\right)$. Then the problem of each consumer in the truncated economy $\mathcal{E}_N$ is to maximize the following function

$$u_0^h(x_0^h) + \sum_{s\in S}\alpha_s^h \sum_{\lambda_N \in \Lambda_N}\left[u_s(x_s^h(\lambda_N)) - \lambda_{sN}^h D_s^h(\lambda_N)\right]\sigma_N(\lambda_N|y_0,\lambda_N^h)$$

subject to the set

$$B_N^h(\lambda_N,\gamma_h,y_0) = \left\{\left(x^h,\phi^h,\psi_h\right) \in X_N^h \times Y_N^h : \text{constraints 2.1, 2.5 and 2.6 are satisfied}\right\}.$$

Note that in this truncated economy we have a finite number of agents each one with a finite number of types. Thus we can look at each type of the same agent in the truncated economy as a different player. As N increases, the number of types tends to a continuum.

The problem of each banker becomes that of maximizing

$$\begin{cases} v_0^b\left(\frac{p_0 w_0^b - q\varphi^b - (\delta^b q - \gamma_h^b)\sum_{h\in H}\psi_h^b}{p_{01}}\right) + \\ + \sum_{s\in S}\sum_{\lambda_N\in\Lambda_N}\beta_s^b v_s^b\left(\frac{p_s(\lambda_N)w_s^b + A_s\varphi^b + \sum_{h\in H}\theta_s^h(\lambda_N)r_b^s\psi_h^b}{p_{s1}(\lambda_N)}\right)\sigma_N(\lambda_N|y_0) \text{ if } D^{be} = 0 \\ -\infty \text{ if } D^{be} > 0 \end{cases}$$

where we redefine $D^{be} = \sum_{s\in S}\sum_{\lambda_N\in\Lambda_N}\beta_s^b D_s^b(\lambda_N)\sigma_N(\lambda_N|y_0)$,[30] over his feasible space $Y_N^b \times \Gamma_N^b$.

Now, let $\Pi_0 = \left\{(p_0,q) \in \mathbb{R}_+^{L+P} : (p_0,q) \in \Delta^{L+P-1}\right\}$ be the set of admissible period 0 prices, and let $\Pi_s(\lambda_N) = \left\{(p_s(\lambda_N)) \in \mathbb{R}_+^L : p_S(\lambda_N) \in \Delta^{L-1}\right\}$ be the set of state $s \in S$ and matrix $\lambda \in \Lambda$ admissible prices.

[30] $D^{be} > 0$ will be equivalent to having $\frac{p_s(\lambda_N)w_s^b + A_s\varphi^b + \sum_{h\in H}\theta_s^h(\lambda_N)r_b^s\psi_h^b}{p_{s1}(\lambda_N)} < 0$ for λ_N in positive measure subset of Λ_N.

We now consider a generalized game $\mathcal{J}_N$ for the truncated economy $\mathcal{E}_N$, where, in addition to the above players, there is a fictitious player who choose $y_0 = (p_0, q) \in \Pi_0$ in order to maximize

$$p_0 \left(\sum_{h \in H} x_0^h + \sum_{b \in B} x_0^b - \sum_{h \in H} w_0^h - \sum_{b \in B} w_0^b \right) + q \left(\sum_{h \in H} \phi^h + \sum_{b \in B} \varphi^b \right)$$

and another SN^{SH} fictitious players, one for each state of nature and each realization λ_{nN} of the matrix λ_N, who choose a price vector $p_s(\lambda_{nN}) \in \Pi_s(\lambda_{nN})$ in order to maximize

$$p_s(\lambda_{nN}) \left(\sum_{h \in H} x_s^h(\lambda_{nN}) + \sum_{b \in B} x_s^b(\lambda_{nN}) - \sum_{h \in H} w_s^h - \sum_{b \in B} w_s^b \right).$$

We now state and prove several auxiliary lemmas.

Lemma 4.2. *Each banker's best response correspondence is upper semicontinuous, compact and convex valued.*

Proof of Lemma 4.2. The function

$$x_{01N}^b = \frac{p_0 w_0^b - q\varphi^b - \sum_{h \in H} (\delta^b q - \gamma_h^b) \psi_h^b}{p_{01}}$$

is monotone in each γ_h^b, $h \in H$, for each fixed ψ_h^b (and under rule II the banker takes ψ_h^b as given) thus is simultaneously quasiconcave and quasiconvex in γ_h^b. The above function is also monotone, thus quasiconcave and quasiconvex, in each φ_p^b, $p \in P$. Since a function $h(x)$ defined as $h(x) = g(f(x))$ is quasiconcave if $f(x)$ is quasiconcave and $g(\cdot)$ is increasing, $v_0^b \left(\frac{p_0 w_0^b - q\varphi^b - (\delta^b q - \gamma_h^b) \sum_{h \in H} \psi_h^b}{p_{01}} \right)$ is quasiconcave in the banker's decision variables.[31]

The function

$$x_{s1N}^b = \frac{p_s(\lambda_N) w_s^b + A_s \varphi^b + \sum_{h \in H} \theta_s^h(\lambda_N) r_b^s \psi_h^b}{p_{s1}(\lambda_N)}$$

is monotonic in each φ_p^b, $p \in P$, and by remark 5.3 in the Appendix, $\theta_s^h(\lambda_N)$ is decreasing in γ_h^b. Hence, by the same argument above, the functions v_s^b are all quasiconcave $\left(\varphi^b, \gamma^b\right)$. Then the result follows immediately from the compacity and convexity of $Y_N^b \times \Gamma_N^b$.[32] □

Lemma 4.3. *The generalized game $\mathcal{J}_N$ has an equilibrium in pure strategies.*

Proof of Lemma 4.3. The agents' budget sets $B_N^h(\lambda_N, \gamma_h, y_0)$ are compact and convex valued and their objective functions are continuous in all their choice variables. Hence, their best response correspondences $\pi_N^h(\gamma_h, y_0, p_1)$ are upper semicontinuous and compact valued. By quasiconcavity of their objective functions, $\pi_N^h(\gamma_h, y_0, p_1)$ is also convex valued. Similarly, each banker's best response correspondence $\pi_N^b(y_0, \gamma_{-b}, \psi^b)$ is upper semicontinuous, compact and convex valued by lemma 4.2. The same applies to the best response correspondence $\pi_N^0(x_0, \phi, \varphi)$ of the period 0 auctioneer and the best response correspondences $\pi_N^{s,\lambda_{nN}}(x_s(\lambda_{nN}))$ of all the SN^{SH} second period auctioneers. Then the product correspondence $\pi_N : M \to M$, where $M = [0,N]^{HL(SN^{SH}+1)} \times [0,N]^{HP} \times [0,N]^{HB} \times [0,1]^{HSN^{SH}} \times [-N,N]^{BP} \times [-N,N]^{BH} \times \Delta^{L+P-1} \times \left(\Delta^{L-1}\right)^{SN^{SH}}$, is also

[31] Under rule 1 ψ_h^b would not be taken as fixed by the banker, but as a decreasing function of γ_h^b, and thus the product $\gamma_h^b \psi_h^b$ could be not monotone, rendering us unable to guarantee the quasiconcavity of the banker's objective function in the spreads.

[32] Strictly speaking, the banker's objective function is discontinuous, but a banker can prevent his or her utility from being equal to $-\infty$ by simply not trading assets at all. For this, he or she only needs to demand sufficiently high spreads such that no agent will want to borrow from him or her. If $\gamma_{bh} = \delta^b q \leqslant 1$, we are guaranteed of this. So, we can ignore the positive default branch of a banker's utility function for every N, and we can treat his utility function as if it were continuous.

upper semicontinuous and compact convex valued. Thus, by Kakutani's Fixed point Theorem π_N has a fixed point $\left(x^*, \phi^*, \theta^*, \psi^*, \gamma^*, \widehat{\delta}^*, p_0^*, q^*, \{p_s^*(\lambda_{nN})\}_{s\in S}^{\lambda_{nN}\in\Lambda_N}\right)$. □

Lemma 4.4. *The equilibrium of the generalized game $\mathcal{J}_N$ constitutes an equilibrium of the truncated economy $\mathcal{E}_N$, for N large enough.*

Proof of Lemma 4.4. Each banker's period 0 budget constraint holds with equality by construction. Aggregating all bankers' period 0 budget constraints, we have

$$p_{0N}^* \sum_{b\in B} x_{0N}^{b^*} + q_N^* \sum_{b\in B} \varphi_N^{b^*} + \left(\delta^b q_N^* - \gamma_{hN}^{b*}\right) \sum_{b\in B}\sum_{h\in H} \psi_{hN}^{b^*} = p_{0N}^* \sum_{b\in B} w_0^b \tag{4.1}$$

Also, agents's period 0 budget constraints must hold with equality, since their utility is strictly increasing, and aggregating over all agents, we have

$$p_{0N}^* \sum_{h\in H} x_{0N}^{h^*} + q_N^* \sum_{h\in H} \phi_N^{h^*} = p_{0N}^* \sum_{h\in H} w_0^h + \sum_{b\in B}\sum_{h\in H} \left(\delta^b q_N^* - \gamma_{hN}^{b^*}\right) \psi_{hN}^{b^*} \tag{4.2}$$

and period 0 auctioneer's optimality conditions imply

$$\left(\sum_{h\in H} x_{0N}^{h^*} + \sum_{b\in B} x_{0N}^{b^*} - \sum_{h\in H} w_0^h - \sum_{b\in B} w_0^b\right) + \left(\sum_{h\in H} \phi_N^{h^*} + \sum_{b\in B} \varphi_N^{b^*}\right) \leqslant 0 \tag{4.3}$$

Combining 4.1 and 4.2, we get

$$p_{0N}^* \left(\sum_{h\in H} x_{0N}^{h^*} + \sum_{b\in B} x_{0N}^{b^*} - \sum_{h\in H} w_0^h - \sum_{b\in B} w_0^b\right) + q_N^* \left(\sum_{h\in H} \phi_N^{h^*} + \sum_{b\in B} \varphi_N^{b^*}\right) = 0.$$

Now, for N large enough, we must have $p_{0lN}^* > 0$, $\forall l \in L$, since otherwise every agent would choose $x_{0lN}^{h^*} = N$, contradicting (4.3). For any N we must also have $q_{pN}^* > 0$, $\forall p \in P$, because if we had $q_{pN}^* = 0$ for any p then every agent and banker would choose $\phi_{pN}^{h^*} = N$, so we would have $\sum_{h\in H} \phi_N^{h^*} + \sum_{b\in B} \varphi_N^{b^*} > 0$, implying that the auctioneer would want to choose $q_{pN}^* = 1$, a contradiction. Also, by a classical non-arbitrage argument, no vector $z \in \mathbb{R}^P$ such that $(-q_N^* z, Az)' \geqslant 0$ with strict inequality for at least one coordinate can exist, since bankers would demand N units of the primitive assets p such that $z_P > 0$ and supply N units of those with $z_p < 0$, contradicting again the optimality for the auctioneer. Then, we must have

$$\sum_{h\in H} x_{0N}^{h^*} + \sum_{b\in B} x_{0N}^{b^*} - \sum_{h\in H} w_0^h - \sum_{b\in B} w_0^b = 0.$$

and

$$\sum_{h\in H} \phi_N^{h^*} + \sum_{b\in B} \varphi_N^{b^*} = 0. \tag{4.4}$$

In every state s of period 1, and for every realization of the matrix λ_N, the budget constraints each agent must hold with equality, again because their utility is strictly increasing, and aggregating all agent's budget constrains, we have

$$p_{1sN}^*(\lambda_N) \sum_{h\in H} x_{1sN}^{h^*}(\lambda_N) + \sum_{h\in H} \left(\theta_{sN}^{h^*}(\lambda_N) \sum_{b\in B} r_b^s \psi_{hN}^{b^*}\right) = p_{1sN}^*(\lambda_N) \sum_{h\in H} w_s^h + A_s \sum_{h\in H} \phi_N^{h^*}, \tag{4.5}$$

$\forall\, s \in S$, $\forall \lambda_N \in \Lambda_N$.

Since bankers cannot default in equilibrium, because this is incompatible with their utility maximization, we must have $D_{sN}^{b^*}(\lambda_N) = 0$ for every b, every s, and every $\lambda_N \in \Lambda_N$. Their

budget constraints must also hold with equality in period 1 for every s and every $\lambda_N \in \Lambda_N$, thus aggregating we have

$$p^*_{1sN}(\lambda_N)\sum_{b\in B} x^{b^*}_{1sN}(\lambda_N) = A_s \sum_{b\in B}\varphi^{b^*}_N + \tag{4.6}$$
$$+\sum_{b\in B}\sum_{h\in H}\left[r^s_b\psi^{b^*}_{hN} - D^{h^*}_{sN}(\lambda_N)\right] + p^*_{1sN}(\lambda_N)\sum_{b\in B} w^b_s,\ \forall s\in S,\ \forall\ \lambda_N\in\Lambda_N$$

Now, state s and matrix λ_N auctioneer's optimality conditions imply that,

$$\left(\sum_{h\in H} x^{h^*}_{1sN}(\lambda_{nN}) + \sum_{b\in B} x^{b^*}_{1sN}(\lambda_{nN}) - \sum_{h\in H} w^h_s - \sum_{b\in B} w^b_s\right) \leqslant 0,\ \forall s\in S,\ \forall\ \lambda_N\in\Lambda_N \tag{4.7}$$

By definition, we have that

$$\sum_{h\in H}\left(\theta^{h^*}_{sN}(\lambda_N)\sum_{b\in B} r^s_b\psi^{b^*}_{hN}\right) = \sum_{b\in B}\sum_{h\in H}\left[r^s_b\psi^{b^*}_{hN} - D^{h^*}_{sN}(\lambda_N)\right],\ \forall s\in S,\ \forall\ \lambda_N\in\Lambda_N$$

and using this in combination with (4.5), (4.6), and (4.4), we get,

$$p^*_{1sN}(\lambda_N)\left(\sum_{h\in H} x^{h^*}_{1sN}(\lambda_{nN}) + \sum_{b\in B} x^{b^*}_{1sN}(\lambda_{nN}) - \sum_{h\in H} w^h_s - \sum_{b\in B} w^b_s\right) = 0,\ \forall s\in S,\ \forall\ \lambda_N\in\Lambda_N$$

We cannot have $p^*_{1slN} = 0$ for N large enough, since every agent would choose $x^{h^*}_{1slN} = N$, and that would contradict (4.7). Then we have

$$\left(\sum_{h\in H} x^{h^*}_{1sN}(\lambda_{nN}) + \sum_{b\in B} x^{b^*}_{1sN}(\lambda_{nN}) - \sum_{h\in H} w^h_s - \sum_{b\in B} w^b_s\right) = 0,\ \forall s\in S,\ \forall\ \lambda_N\in\Lambda_N$$

as we wanted to show. □

For what follows, when we use the expression "uniformly bounded", it should be understood as uniformly bounded with respect to $\lambda \in \Lambda$.

Lemma 4.5. *$(x^*_N)_{N=1,2,\ldots}$ is a uniformly bounded sequence.*

Proof of Lemma 4.5. For period every state $s \in \overline{S}$ and good l, $x^{h^*}_{slN} \leqslant \sum_{h\in H} w^h_{sl} + \sum_{b\in B} w^b_{sl}$, $\forall N$. Also $x^{b^*}_{s1N} \leqslant \sum_{h\in H} w^h_{s1} + \sum_{b\in B} w^b_{s1}$ for each $s \in \overline{S}$ and $\forall N$. □

Lemma 4.6. *$(\gamma^*_N)_{N=1,2,\ldots}$ is a bounded sequence (from above and from below).*

Proof of Lemma 4.6. If $\gamma^{b^*}_{hN} > \delta^b q^*_N$ for some N, then agent h would face a negative price on his or her sale of indexed assets to banker b, so he or she would choose $\psi^{b^*}_{hN} = 0$. If $\gamma^{b^*}_{hN} < 0$ for some N agent b could profit in period zero, at the expense of banker b, by selling him or her N units of indexed asset at unit price $\delta^b q^*_N - \gamma^{b^*}_{hN}$ and purchasing a portfolio of primitive assets with weights exactly equal to δ^b, at unit price $\delta^b q^*_N$. In state s the net promised return would be simply $\delta^b r^s_b - \delta^b r^s_b = 0$. Then $\left(\gamma^{b^*}_{hN}\right)_{N=1,2,\ldots} \in \left[0, \max_{q\in\Delta^{P-1}}\left(\delta^b q\right)\right]$. □

Hence $(\gamma^*_N)_{N=1,2,\ldots}$ has a cluster point γ^*.

Lemma 4.7. *$(\phi^*_N, \varphi^*_N, \psi^*_N)_{N=1,2,\ldots}$ is a bounded sequence.*

Proof of Lemma 4.7. For each agent h and each N, let $\delta^{h^*}_N \in \Delta^{P-1}$ denote the relative weights vector of his or her primitive assets portfolio $\phi^{h^*}_N$, that is, $\delta^{h^*}_{pN} = \phi^{h^*}_{pN} / \sum_{p\in P}\phi^{h^*}_{pN}$. $\left(\delta^{h^*}_N\right)_{N=1,2,\ldots}$ has a cluster point δ^{h^*}. Let $\xi \in \Delta^{B-1}$, and let $c^\xi_{hN} = \max_s\left(\sum_{b\in B}\xi_b\delta^b R^{s'}/\delta^{h^*}_N R^{s'}\right)$. Then

(i) If ξ is such that $\sum_{b\in B}\xi_b\delta^b R^{s'}/\delta^{h^*}_N R^{s'}$ is not constant across s, then $\gamma^{b^*}_{hN}$ must be such that $\sum_{b\in B}\xi_b\left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right)/q^*_N\delta^{h^*}_N < c^\xi_{hN}$. Otherwise, agent h could sell ξ_b units of indexed asset to

banker b, buy c^{ξ}_{hN} units of portfolio $\delta^{h^*}_N$, and in every state s would receive $c^{\xi}_{hN}\delta^{h^*}_N R^{s'} \geqslant \sum_{b\in B}\xi_b\,\delta^b R^{s'}$, with strict inequality for at least one s, a sure profit.

(ii) Similarly, if ξ is such that $\sum_{b\in B}\xi_b\delta^b R^{s'}/\delta^{h^*}_N R^{s'} = c^{\xi}_{hN}$ for all s, then $\gamma^{b^*}_{hN}$ must satisfy $\sum_{b\in B}\xi_b\left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right)/q^*_N\delta^{h^*}_N \leqslant c^b_{hN}$.[33]

The agent's first period budget constraint at the equilibrium of the truncated economy can be written as

$$p^*_{0N}(x^{h^*}_{0N} - w^h_0) + q^*_N\delta^{h^*}_N\sum_{p\in P}\phi^{h^*}_{pN} - \psi^*_{hN}\sum_{b\in B}\xi^*_{bN}\left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right) = 0,$$

where $\psi^*_{hN} = \sum_{b\in B}\psi^{b^*}_{hN}$, and $\xi^*_{bN} = \psi^{b^*}_{hN}/\sum_{b\in B}\psi^{b^*}_{hN}$. $(\xi^*_N)_{N=1,2,\ldots}$ has a cluster point ξ^*. Now

(a) Suppose that $\sum_{b\in B}\xi^*_{bN}\delta^b R^{s'}/\delta^{h^*}_N R^{s'}$ is not constant across s. Then, if $\sum_{p\in P}\phi^{h^*}_{pN} \underset{N}{\to} +\infty$ we have that $\left(\psi^*_{hN}\right)_{N=1,2,\ldots}$ must diverge also to $+\infty$, and at a relative rate superior to $1/c^{\xi^*_N}_{hN}$, by (i) above. Thus $0 > \left(\delta^{h^*}_N R^{s'}\sum_{p\in P}\phi^{h^*}_{pN} - \psi^*_{hN}\sum_{b\in B}\xi^*_{bN}\delta^b R^{s'}\right) \underset{N}{\to} -\infty$ for at least one state s. Even if the consumer sets $x^h_{1sN} = 0$ for every N, his or her default will be equal to $\left(\psi^*_{hN}\sum_{b\in B}\xi^*_{bN}\left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right) - \delta^{h^*}_N R^{s'}\sum_{p\in P}\phi^{h^*}_{pN} - p^*_s(\lambda_N)\,w^h_s\right) \underset{N}{\to} +\infty$, so he or she will suffer a utility penalty equal to at least

$$\min_{\lambda_N\in\Lambda_N}\lambda^h_{s_N}\left(\psi^*_{hN}\sum_{b\in B}\xi^*_{bN}\left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right) - \delta^{h^*}_N R^{s'}\sum_{p\in P}\phi^{h^*}_{pN} - p^*_s(\lambda_N)\,w^h_s\right) \underset{N}{\to} +\infty.$$

Let $U^h(w)$ denote the value of agent h utility if he or she consumes the total aggregate endowment of the economy. $U^h(w)$ is finite, so we conclude that if $\sum_{p\in P}\phi^{h^*}_{pN} \underset{N}{\to} +\infty$ then agent h utility would become negative.

(b) Suppose that $\sum_{b=1}^{B}\xi^*_{bN}\delta^b R^{s'}/\delta^{h^*}_N R^{s'} = c^{\xi^*_N}_{hN}$ for all s, and recall (ii) above. If $\sum_{b=1}^{b}\xi^*_{bN}\left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right)/q^*_N\delta^{h^*}_N < c^{\xi^*_N}_{hN}$ the above argument remains valid. If $\sum_{b=1}^{b}\xi^*_{bN}\left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right)/q^*_N\delta^{h^*}_N = c^{\xi^*_N}_{hN}$, the agent can only profit at the expense of the bankers if he or she defaults. But then to profit boundlessly he or she would have to default boundlessly and his or her utility would become negative. If the agent does not default he or she does not gain or loose anything if he or she lets $\sum_{p\in P}\phi^{h^*}_{pN}$ and ψ^*_{hN} go to infinity. Note that in this case the bankers for which $\lim_N \xi^*_{bN} > 0$ are forced to make their short sales of primitive assets go to infinity. Then we must have $q^*\widehat{\delta}^{b^*}/\left(\delta^b q^* - \gamma^{b^*}_h\right) = \widehat{\delta}^{b^*}R^{s'}/\delta^b R^{s'}$ for all s. Otherwise in the limit the banker would be loosing or gaining boundlessly in either period 0 or in some state of nature of period 1. Then, we can, without loss of generality, replace these sequences by bounded ones, without affecting any other equilibrium variables.

Thus we have shown that $\lim_N \sum_{p\in P}\phi^{h^*}_{pN} < +\infty$, for all h. We then have $\sum_{b=1}^{B}\varphi^{b^*}_N = -\sum_{h\in H}\phi^{h^*}_N \gg -\infty$, by lemma 4.4, thus $\lim_N q^*_N\sum_{b=1}^{B}\varphi^{b^*}_N > -\infty$. This implies that if for some banker $\widehat{b}$ we had $\lim_N q^*_N\varphi^{\widehat{b}^*}_N = -\infty$, then $\lim_N q^*_N\varphi^{\widetilde{b}^*}_N = +\infty$ for some other banker $\widetilde{b}\neq\widehat{b}$. But for each banker $q^*_N\varphi^{b^*}_N$ is bounded from above by $p^*_{0N}w^b_0$, a contradiction. $\left(q^*_N\varphi^{b^*}_N\right)_{N=1,2,\ldots}$ is hence bounded, and since $q^*_N\in\mathbb{R}^P_{++}$ for N high enough, $(\varphi^*_N)_{N=1,2,\ldots}$ is bounded.

Finally, by the bankers' budget constraints, $(\psi^*_N)_{N=1,2,\ldots}$ becomes bounded also. □

Lemma 4.8. $\lim_N\left(p^*_{0N}, q^*_N, \{p^*_{1sN}(\lambda_N)\}_{s\in S}\right)_{N=1,2,\ldots} \gg 0$.

[33] We should expect the spreads to be higher than these lower bounds, due to the possibility of default.

Proof of Lemma 4.8. Suppose $p^*_{0lN} \underset{N}{\to} 0$ for some l. Then, by preferences strict monotonicity, $x^{h^*}_{0lN} \underset{N}{\to} +\infty$ and $x^{b^*}_{0lN} \underset{N}{\to} +\infty$. Similarly, if $p^*_{1slN}(\lambda_N) \underset{N}{\to} 0$, there would be an order above which $\sum_{h\in H} x^{h^*}_{1slN}(\lambda_N) > \sum_{h\in H} w^{h^*}_{1sl} + \sum_{b=1}^{B} w^{b^*}_{1sl}$, contradicting the optimality for the (s,λ_N) auctioneer in the generalized game $\mathcal{J}_N$. Finally, suppose $q^*_{pN} \underset{N}{\to} 0$ for some p. Then each agent can fix $q^*_p \phi^h_{pN} = k_h > 0$, such that $\phi^h_{pN} \underset{N}{\to} +\infty$ without increasing the expenditure in the purchase of primitive asset p. Since $a^p_s \geqslant 0$ with strict inequality for at least one s, their state s budget constraints would explode. □

Let $\mu^{h^*}_N(\lambda_N)$ denote the set of Lagrange multipliers of agent h at his or her optimal solution in the truncated economy $\mathcal{E}_N$. They depend on λ_N because the agents have, in each state s, a different budget constraint for each realization of the matrix λ_N.

Lemma 4.9. *$\left(\mu^{h^*}_N(\lambda_N)\right)_{N=1,2,...}$ is a uniformly bounded sequence.*

Proof of Lemma 4.9. The fact that each $\left(\mu^{h^*}_{sN}(\lambda_N)\right)_{N=1,2,...}$ is bounded can be shown following the same steps as in the proof of lemma 4 in Araújo, Monteiro and Páscoa [3]. As for $\left(\mu^{h^*}_{0N}\right)_{N=1,2,...}$ we have the following inequality:

$$\frac{\partial \mathcal{L}^h}{\partial \psi^b_h} = \mu^{h^*}_0 \left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right) \leqslant \sum_{s\in S} \alpha^h_s \sum_{\lambda_N \in \Lambda_N} \lambda^h_s (1-\theta^{h^*}_{sN}) r^s_b \sigma_N(\lambda_N | y^*_{0N}, \lambda^h_N) +$$

$$+ \sum_{s\in S} \alpha^h_s \sum_{\lambda_N \in \Lambda_N} \mu^{h^*}_{sN}(\lambda_N) \theta^{h^*}_{sN} r^s_b \sigma_N(\lambda_N | y^*_{0N}, \lambda^h_N),\ \forall b \in B.$$

where $\mathcal{L}^h$ is the Lagrangian of agent h optimization problem. Then $\mu^{h^*}_0$ is bounded if $\left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right)$ does not go to zero for at least one b. If $\lim_N \left(\delta^b q^*_N - \gamma^{b^*}_{hN}\right) = 0$ for all b, then the agent cannot borrow anything in the limit. Thus, $\lim_N \mu^{h^*}_0 = +\infty$ would imply that $\lim_N x^{h^*}_{0N} = 0$, and since we have $p^*_0 w^h_0 > 0$, $q^* \gg 0$, and the agent's period zero constraint must be holding with equality for N high enough, we would then have $\lim_N \phi^{h^*}_{pN} > 0$ for at least one p, and then it would follow that

$$\frac{\partial \mathcal{L}^h}{\partial \phi^h_p} = \sum_{s\in S} \alpha^h_s \sum_{\lambda_N \in \Lambda_N} \mu^{h^*}_{sN}(\lambda_N) a^p_s \sigma_N(\lambda_N | y^*_{0N}, \lambda^h_N) - \mu_0 q^*_{pN} = 0,$$

a contradiction, due to lemma 4.8. □

Now define $f^N : \Lambda \to \mathbb{R}^z$ (we do not specify z) :

$$f^N(\lambda) = (\mu^{h^*}_N, x^{h^*}_N, \phi^{h^*}_N, \theta^{h^*}_N, x^{b^*}_N, \varphi^{b^*}_N, \psi^{b^*}_{hN}, \gamma^{b^*}_{hN}, p^*_{0N}, q^*_N, p^*_{sN})^{b\in B}_{h\in H}$$

$\left(f^N\right)_N$ is uniformly bounded, hence uniformly Lebesgue integrable. And the sequence $\int f^N(\lambda)\, d\sigma(\lambda)$ converges, maybe passing to a subsequence. By Fatou's lemma, there is a function $\overline{f} : \Lambda \to \mathbb{R}^z$ such that

(i) $\overline{f}(\lambda) = (\mu^{h^*}, x^{h^*}, \phi^{h^*}, \theta^{h^*}, x^{b^*}, \varphi^{b^*}, \psi^{b^*}_h, \gamma^{b^*}_h, p^*_0, q^*, p^*_s)^{b\in B}_{h\in H} \in cl\left\{f^N(\lambda)\right\}_{N\in\mathbb{N}}$, for almost every $\lambda \in \Lambda$.

(ii) $\int \overline{f}(\lambda)\, d\sigma(\lambda) = \lim_N \int f^N(\lambda)\, d\sigma(\lambda)$.

We finally have to check if $(\mu^{h^*}(\lambda), x^{h^*}(\lambda), x^{b^*}(\lambda), \theta^{h^*}(\lambda), p^*_s(\lambda), \phi^{h^*}, \varphi^{b^*}, \psi^{b^*}_h, \gamma^{b^*}_h, p^*_0, q^*)^{b\in B}_{h\in H}$ is an equilibrium for the economy $\mathcal{E}$. The market clearing conditions hold pointwise, and result from the fact that the truncated economy $\mathcal{E}_N$ is in equilibrium for N high enough and (i) above. To verify the optimality for the agents, we must show that for each h $\nexists (x^h(\lambda), \theta^h(\lambda), \phi^h, \psi^b_h)$ budget feasible such that

$$\tau = u^h_0(x^h_0) + \sum_{s\in S} \alpha^h_s \int_\Lambda \left[u_s(x^h_s(\lambda)) - \lambda^h_s D^h_s(\lambda)\right] d\sigma(\lambda | y_0, \lambda^h) -$$

$$-u_0^h(x_0^{h^*}) - \sum_{s\in S}\alpha_s^h \int_\Lambda \left[u_s(x_s^{h^*}(\lambda)) - \lambda_s^h D_s^{h^*}(\lambda)\right] d\sigma(\lambda|y_0,\lambda^h) > 0.$$

Following the same steps as in the proof of lemma 1 in Araújo, Monteiro and Páscoa [3], we have, in the limit,

$$u_0^h(x_0^h) + \sum_{s\in S}\alpha_s^h \int_\Lambda u_s(x_s^h(\lambda))d\sigma(\lambda|y_0,\lambda^h) - u_0^h(x_0^{h^*}) - \sum_{s\in S}\alpha_s^h \int_\Lambda u_s(x_s^{h^*}(\lambda))d\sigma(\lambda|y_0,\lambda^h) \leqslant$$

$$\leqslant \mu_0^{h^*} p_0^*(x_0^h - x_0^{h^*}) + \sum_{s\in S}\alpha_s^h \int_\Lambda \mu_s^{h^*}(\lambda)\, p_s^*(\lambda)\,(x_s^h(\lambda) - x_s^{h^*}(\lambda))d\sigma(\lambda|y_0,\lambda^h).$$

Then

$$\tau \leqslant \mu_0^{h^*} p_0^*(x_0^h - x_0^{h^*}) + \sum_{s\in S}\alpha_s^h \int_\Lambda \mu_s^{h^*}(\lambda)\, p_s^*(\lambda)\,(x_s^h(\lambda) - x_s^{h^*}(\lambda))d\sigma(\lambda|y_0,\lambda^h) -$$

$$-\sum_{s\in S}\alpha_s^h \int_\Lambda \lambda_s^h \left(D_s^h(\lambda) - D_s^{h^*}(\lambda)\right) d\sigma(\lambda|y_0,\lambda^h) \leqslant$$

$$\leqslant \mu_0^{h^*} q^* \left(\phi^{h^*} - \phi^h\right) + \mu_0^{h^*} \sum_{b=1}^B \left(q^*\delta^b - \gamma_h^{b^*}\right)(\psi_h^b - \psi_h^{b^*}) +$$

$$+\sum_{s\in S}\alpha_s^h \int_\Lambda \mu_s^{h^*}(\lambda) \left[A_s\left(\phi^h - \phi^{h^*}\right) + \sum_{b=1}^B \left(r_b^s\psi_h^{b^*} - r_b^s\psi_h^b + D_s^h(\lambda) - D_s^{h^*}(\lambda)\right)\right] d\sigma(\lambda|y_0,\lambda^h) -$$

$$-\sum_{s\in S}\alpha_s^h \int_\Lambda \lambda_s^h \left(D_s^h(\lambda) - D_s^{h^*}(\lambda)\right) d\sigma(\lambda|y_0,\lambda^h)$$

$$= \mu_0^{h^*} q^* \left(\phi^{h^*} - \phi^h\right) + \mu_0^{h^*} \sum_{b=1}^B \left(q^*\delta^b - \gamma_h^{b^*}\right)(\psi_h^b - \psi_h^{b^*}) +$$

$$+\sum_{s\in S}\alpha_s^h \int_\Lambda \mu_s^{h^*}(\lambda)\, A_s\left(\phi^h - \phi^{h^*}\right) d\sigma(\lambda|y_0,\lambda^h) -$$

$$-\sum_{s\in S}\alpha_s^h \int_\Lambda \mu_s^{h^*}(\lambda) \sum_{b=1}^B \left(r_b^s\psi_h^b - r_b^s\psi_h^{b^*}\right) d\sigma(\lambda|y_0,\lambda^h) +$$

$$+\sum_{s\in S}\alpha_s^h \int_\Lambda (\mu_s^{h^*}(\lambda) - \lambda_s^h) \left(D_s^h(\lambda) - D_s^{h^*}(\lambda)\right) d\sigma(\lambda|y_0,\lambda^h).$$

Now, $D_s^{h^*}(\lambda) = \sum_{b=1}^B r_b^s\psi_h^{b^*}$ if $\lambda_s^h < \mu_s^{h^*}(\lambda)$ and $D_s^{h^*}(\lambda) = 0$ if $\lambda_s^h > \mu_s^{h^*}(\lambda)$. Using the fact that

$$\min_{0\leqslant D_s^h(\lambda)\leqslant \sum_{b=1}^B r_b^s\psi_h^b} -\mu_s^{h^*}(\lambda)\sum_{b=1}^B r_b^s\psi_h^b + (\mu_s^{h^*}(\lambda) - \lambda_s^h)D_s^h(\lambda) =$$

$$= -\max\left\{\mu_s^{h^*}(\lambda), \lambda_s^h\right\}\sum_{b=1}^B r_b^s\psi_h^b$$

we have

$$\tau \leqslant \mu_0^{h^*} q^* \left(\phi^{h^*} - \phi^h\right) + \mu_0^{h^*} \sum_{b=1}^B \left(q^*\delta^b - \gamma_h^{b^*}\right)(\psi_h^b - \psi_h^{b^*}) +$$

$$+\sum_{s\in S}\alpha_s^h \int_\Lambda \mu_s^{h^*}(\lambda)\, A_s\left(\phi^h - \phi^{h^*}\right) d\sigma(\lambda|y_0,\lambda^h) -$$

$$-\sum_{s\in S}\alpha_s^h \int_\Lambda \max\left\{\mu_s^{h^*}(\lambda), \lambda_s^h\right\} \sum_{b=1}^B \left(r_b^s\psi_h^b - r_b^s\psi_h^{b^*}\right) d\sigma(\lambda|y_0,\lambda^h).$$

By the same steps in the proof of lemma 3 in Araújo, Monteiro and Páscoa [3], with the slight difference that here we also have to make $x_0^h = x_0^{h^*}$, we have

$$0 \leqslant \sum_{s\in S} \alpha_s^h \int_\Lambda \max\left\{\mu_s^{h^*}(\lambda), \lambda_s^h\right\} \sum_{b=1}^{B} \left(r_b^s \psi_h^b - r_b^s \psi_h^{b^*}\right) d\sigma(\lambda|y_0,\lambda^h) -$$

$$- \sum_{s\in S} \alpha_s^h \int_\Lambda \mu_s^{h^*}(\lambda) A_s \left(\phi^h - \phi^{h^*}\right) d\sigma(\lambda|y_0,\lambda^h) +$$

$$-\mu_0^{h^*} q^* \left(\phi^{h^*} - \phi^h\right) - \mu_0^{h^*} \sum_{b=1}^{B} \left(q^* \delta^b - \gamma_h^{b^*}\right)(\psi_h^b - \psi_h^{b^*}) = -\tau.$$

Hence $\tau \leqslant 0$, as we wanted to show.

Finally, the optimality for the bankers results immediately from the upper semicontinuity of their best response correspondence in the limit, since no price tends to zero by lemma 4.8.[34]

5. Appendix

5.1. **Agent's Individual Choice Problem Analysis.** In this Appendix, we derive formally the agents' first order necessary conditions. The set of first order necessary and sufficient Kuhn-Tucker conditions with regard to the asset portfolio and to the repayment rates is (ignoring dependencies on λ to spare some space):

$$\frac{\partial \mathcal{L}^h}{\partial \phi_p^h} = \sum_{s\in S} \alpha_s^h \int_\Lambda \mu_s a_s^p d\sigma(\lambda|y_0,\lambda^h) - \mu_0 q_p \leqslant 0, \;\; \phi_p^h \geqslant 0 \text{ and } \phi_p^h \frac{\partial \mathcal{L}^h}{\partial \phi_p^h} = 0, \forall p \in P. \tag{5.1}$$

$$\frac{\partial \mathcal{L}^h}{\partial \psi_h^b} = \mu_0(\delta^b q - \gamma_h^b) - \sum_{s\in S} \alpha_s^h \int_\Lambda \lambda_s^h (1-\theta_s^h) r_b^s d\sigma(\lambda|y_0,\lambda^h) -$$

$$- \sum_{s\in S} \alpha_s^h \int_\Lambda \mu_s \theta_s^h r_b^s d\sigma(\lambda|y_0,\lambda^h) \leqslant 0, \;\; \psi_h^b \geqslant 0, \text{ and } \psi_h^b \frac{\partial \mathcal{L}^h}{\partial \psi_h} = 0 \tag{5.2}$$

$$\frac{\partial \mathcal{L}^h}{\partial \theta_s^h} = \lambda_s^h r_b^s \psi_h^b - \mu_s r_b^s \psi_h^b - \mu_{2s} \leqslant 0, \;\; \theta_s^h \geqslant 0 \text{ and } \theta_s^h \frac{\partial \mathcal{L}^h}{\partial \theta_s^h} = 0$$

$$\forall s \in S \text{ and a.e. } \lambda \in \Lambda.$$

where μ_s, is the state s budget constraint multiplier, i.e., the state s marginal income utility, $s \in \overline{S}$, and μ_{2s} is the multiplier of the constraint $\theta_s^h \leqslant 1$, $s \in S$.

Remark 5.1. By looking at conditions (5.2) we conclude that when γ_h^b [35] decreases ψ_h^b must increase, except maybe if the agents are in a corner solution with $\psi_h^b = 0$, because $\frac{\partial \mathcal{L}^h}{\partial \psi_h^b}$ would otherwise become positive. When $\frac{\partial \mathcal{L}^h}{\partial \psi_h^b}$ is positive, to re-equate it to zero is necessary to decrease μ_0 and (if $\theta_s^h > 0$) increase μ_s for every $s \in S$, by transferring more income from period 1 to period 0, that is to say, by selling more indexed assets.[36]

Remark 5.2. By monotonicity the budget constraint will always hold with equality ($\mu_s > 0$), and if θ_s^h is different from 0 and 1 (partial default) we will have $\frac{\partial \mathcal{L}^h}{\partial \theta_s^h} = 0$ and $\mu_{2s} = 0$, which implies that $\lambda_s^h = \mu_s$, equal to the nominal income marginal utility. If $\theta_s^h = 0$, we have $\frac{\partial \mathcal{L}^h}{\partial \theta_s^h} < 0$ and $\mu_{2s} = 0$, which implies $\lambda_s^h < \mu_s$. If, on the contrary, $\theta_s^h = 1$, we will then have $\frac{\partial \mathcal{L}^h}{\partial \theta_s^h} = 0$ and $\mu_{2s} > 0$, implying $\lambda_s^h > \mu_s$. By strict monotonicity, and assuming that u_s^h is homothetic just to make the graphical

[34] Remember that none of his or her choices depend on λ.

[35] The efective one under rule I, and the expected under rule II.

[36] Note that througout this analysis we are treating p_0, q and $p_s(\lambda)$ as constants.

illustration simple, the income marginal utility μ_s will strictly monotonically diminish as we move up along the income expansion path. In this path there will exist a single point satisfying $\lambda_s^h = \mu_s$. We will denote it by x^*. The lower λ_s^h is, the further away from the origin the point x^* will be. All points at the income expansion path below x^* are such that $\lambda_s^h < \mu_s$, and vice-versa. Given the agent's asset portfolio and given the price vector p_s, his or her budget constraint position is solely determined by his or her repayment rate. Thus, there are three possible alternatives, represented in figures 5 (λ_s^h low), 6 (λ_s^h intermediate), and 7 (λ_s^h high). All admissible consumption bundles are those below the $\theta_s^h = 0$ budget constraint. The optimal consumption bundle $\overline{x}_s^h$ will always be located on the income expansion path between these two extreme budget constraints.

Insert figure 5 here.

Insert figure 6 here.

Insert figure 7 here.

With the help of this simple figures, several basic comparative analysis can be performed: if the endowment or the portfolio of primitive asset is increased, θ_s^h is increased; if the debt is increased θ_s^h decreases.

Remark 5.3. Combining remarks 5.1 and 5.2, it is immediate that $\theta_s^h(\lambda)$ is decreasing in the effective γ_b^h, for every $s \in S$ and $\lambda \in \Lambda$, under rule I. Under rule II, this conclusion is also true but a bit less obvious. When γ_b^h increases the agent will have less resources available for consumption in period 0, implying an increase in μ_0 and thus, by condition 5.1, the agent will decrease his purchases of primitive assets and, by the statement in the last phrase in remark 5.2, $\theta_s^h(\lambda)$ will decrease. The intuition is very simple: higher spreads represent a contraction in the agent's budget set, making him poorer, and poorer consumers will default more, *ceteris paribus*.

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