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Securities Index

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Dynamic Analysis of the Insurance Linked Securities Index

Mathieu Gatumel^{*} Dominique Guegan[†]

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Abstract: This paper aims to provide a dynamic analysis of the insurance linked securities index. We are discussing the behaviour of the index for three years and pointing out the consequences of some major events like Katrina or the last and current financial crisis. Some stylized facts of the index, like the non-Gaussianity, the asymmetry or the clusters of volatility, are highlighted. We are using some GARCH-type models and the generalized hyperbolic distributions in order to capture these elements. The GARCH in Mean model with a Normal Inverse Gaussian distribution seems to be very efficient to fit the log-returns of the insurance linked securities index.

Keywords: Insurance Linked Securities, Garch-type models, Normal Inverse Gaussian Distribution.

JEL classification: G12, G14, C16, C22.

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1 Introduction

In Gatumel and Guégan (2008) we built an index summarizing the evolution of the insurance linked securities spreads in the secondary market. Initially we did not include the weight of each issue in the index. Considering that it could be useful to differentiate between the very big issue and the small one we decided to modify the index in order to consider the issued amount. Thus, the value of the index at time t for the issue i, I_{it} , is given by the following formula:

$$I_{it} = I_{it-1} \times \frac{y_{it}}{y_{it-1}},\tag{1.1}$$

where y_{it} is the spread of the issue *i* at time *t* and $I_{i0} = 100$ (0 stands for 1st January 2004).

The value of the market index I_t at time t is:

$$I_t = \frac{1}{w_t} \sum_{i=1}^{n_t} w_i \times I_{it},$$
(1.2)

where w_i corresponds to the issued amount for the issued *i*, w_t the global size of the market and n_t the number of issues in the secondary market at time *t*.

Figure 1 shows the evolution of the index I_t between January, 1st 2004 and March, 31st 2008.

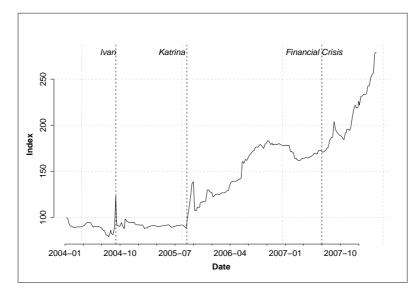


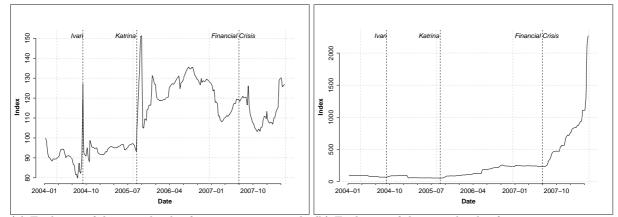
Figure 1: ILS Spread index since 1st January 2004.

We distinguish four periods during the four last years. The first one starts January 1st, 2004 and ends in August 2005. It is characterized by a decrease of the spreads, close to 10%, the index going from 100 to 88.21, with a minimum at 78.95 on August, 6th 2004. The period countains also an explosion in September 2004, the spread index is equal to 123.16 on September 10th, 2004. The second sub-period starts by an explosion of spreads in September 2005, just after Katrina. The index is equal to 88.21 before the hurricane and to 138.55 five weeks after. Then the sub-period is characterized by a highly increasing trend between October 2005 and October 2006, with the index going from 107.37 in October 7th, 2005 to 183.27 in October 6th, 2006 (+70%). The third sub-period starts at the end of the 2006 hurricane season. Firstly, the spreads decrease from 183.27 to 161.79 in March 16th, 2007. Secondly they rise until 184.46 in October 19th, 2007 with a peak at 204.13 in August 2007. Lastly, the fourth sub-period starts in October 2007 and is characterized by an increasing trend of the ILS spreads, the spreads rising from 184.28 to 279.21 at the end of March 2008.

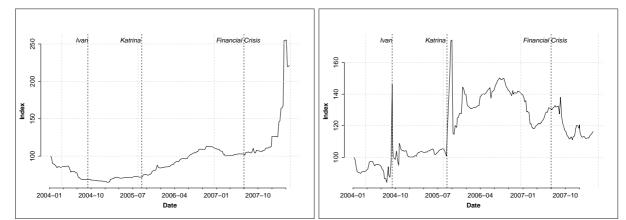
The evolution of the insurance linked securities' spreads is driven by the underlying risk, both the real risk of the issues and the way it is perceived by the investors. For example, the real risk increases and decreases every year because of the seasonality of some catastrophic events. Due to the seasonality of the hurricanes, the market is characterized by the increasing of spreads of the issues covering the US windstorm between March and August. At the middle of the season (31st August) the spreads decrease (as seen in the figure 2(a)). Moreover the increasing trend of the spreads at the beginning of 2006 can be related to the changes of the actuarial risk models. Indeed after Katrina, Risk Management Services (RMS) changed its actuarial models and decreased the return period of such an event, from 100 years to 70 years. Consistently, the investors revised the underlying risk of the traded issues and the spreads increased. On the contrary Katrina caused some panic in the market, mainly due to its severity and the first default of a cat. bond, Kamp Re. That may explain the explosion of spreads in September 2005 and their high volatility in Autumn 2005. Similarly, the behaviour of spreads in October 2004 may be explained by the hurricanes which hit the USA. For a few days the investors expected that some issues defaulted. Thus they required a higher return for some bonds (like Residential Re). Similarly the increasing trend of the spreads since Autumn 2007 may be linked with the subprime crisis which troubles the financial markets from August 2007. In each market the required return of the asset increases. It is also the case for the insurance linked securities. Nevertheless, the evolutions depend on the type of risk which is considered.

In order to specify the consequences of the subprime crisis in the insurance linked securities market, Figure 2 provides an evolution of the index for both some different risks and some different ratings. Thus the figures 2(a) and 2(b) present the evolution of the index respectively for issues covering only catastrophe risk (like US hurricanes, European Wind, Californian earthquakes, etc.) and for issues covering other types of risks (like Mortality or Embedded Value)¹.

¹With the distinction of both the catastrophe and the mortality risks, it it possible to widen the time interval of the different Figures. Indeed, the evolution of the mortality bonds' spreads hides the other movements of the spreads. That explain why we considered previously a time period from 2004-01-01 to 2008-03-31 and now from 2004-01-01 to 2008-04-31, without loss of generality.



(a) Evolution of the spread index for issues covering only (b) Evolution of the spread index for issues covering not catastrophe risk.



(c) Evolution of the spread index for the issues covering (d) Evolution of the spread index for the issues covercatastrophe risk and having a rating between AAA and ing catastrophe risk and having a rating under BBB or BBB+ beeing unrated

Figure 2: Decomposition of the ILS spread index.

It appears that those types of risk react very differently to the subprime crisis. Indeed, the spreads of the catastrophe bonds decrease after the hurricane season before increasing slightly from October 2007. If the insurance linked securities market as a whole cannot be considered as purely independant of the financial market, the catastrophe bonds market is still orthogonal. On the contrary the spreads of the mortality bonds rise strongly from July 2007 (see Figure 2(b)): the spreads are multiplied by five. This figure allows also to highlight the fact that the main driver of the mortality bonds' spreads is the financial factor. Thus, the spread evolution may be explained by the increasing investors' risk aversion for the financial risks. Another reason is the doubt about the accuracy of the actuarial models underlying to the mortality bonds like the subprime crisis casts doubt on the risk models of some financial institutions.

It appears that the catastrophe bonds are not so concerned by the financial crisis. Nevertheless we have to point out a peak in October 2007 and an increasing trend from December 2007. Both may be explained by the financial crisis. But the reversal of the spreads after October illustrates

the fact that financial factors are not the main drivers of the catastrophe bonds' spreads. In order to explain the increasing trend of the spreads from December 2007, Figures 2(c) and 2(d)respectively provide the evolution of the spreads for the catastrophe bonds having the best and the worst rating. We can highlight that the spreads of the bonds with a rating lower than BBB+ are not so characterized by the consequences of the financial crisis. The increase of the spread in March 2008 can be linked with the 2008 hurricane season. But we can also point out that the bonds with the best rating have a return less volatile than the bonds with a worse one. However, only the bonds with the best rating are characterized by a strong increasing trend from December 2007. According to us, the behaviour is not due to the financial crisis but rather to some particular issues and to some collateral losses. Indeed, only ten bonds, having a rating between AAA and BBB+, are traded in March 2008. Moreover, in November 2007 the Merna transaction, the largest issue of 2007, at \$1,265 billion, was launched. The spreads of the three Merna layers rise strongly and are multiplied firstly by two between November 2007 and March 2008 and secondly by two between 14th March and 20th March. That is the main factor explaining the explosion of spreads of almost all the bonds having the best ratings. Some secure bonds are characterized by mispricing issues. As a result almost all the secure bond spreads rise strongly. On the contrary, the bonds having a rating lower than BBB have a decreasing trend due to some new investors who require risk and return.

In Gatumel and Guégan (2008), we rely the spread behaviours on some economic or actuarial factors like the risk aversion of the investors, the uncertainty or the occurrence of a catastrophic events. Now, considering the spreads index like the other financial indices, we would like to analyse them in terms of dynamic analysis. Following the fact that it is certainly too early to study the spread evolution of the mortality bonds and because the sample of the catastrophe bonds having the best rating is too small, we will only study the spreads of the catastrophe bonds with a rating lower than BBB+.

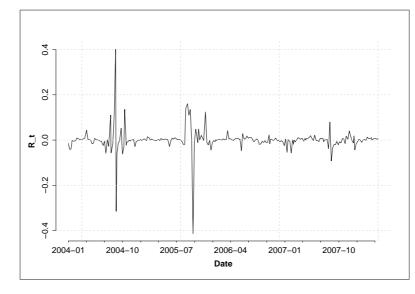
The paper is organized as follows. The section 2 highlights some stylized facts of the series. The section 3 produces an estimation of the market index through GARCH-type models. The section 4 allows to point out some jumps in the serie, jumps related to catastrophic events.

2 Some stylized facts

This section will highlight the main features of the spreads of the catastrophe bonds having a rating lower than BBB+ or being unrated. Figure 2(d) provides the weekly evolution of the Index, I_t , which aggregates them at each date t, t = 1, ..., T, with 1 = 1st January of 2004 and T = 10th April of 2009.

First of all, to make stationary the dataset, we define R_t such that:

$$R_t = \log\left(I_t\right) - \log\left(I_{t-1}\right). \tag{2.1}$$



 R_t is represented on the Figure 3.

Figure 3: Evolution of R_t between 1st, January 2004 and 30th, April 2008.

	Mean	Std. Error	Skewness	Kurtosis
Mean	0.001	0.0525	-0.660	38.382

Table 1: Some statistics relative to the serie R_t .

Table 1 provides some statistics about R_t . The mean of R_t is equal to 0.001. The standard error is equal to 5.25%. Lastly the skewness is equal to -0.660 and the kurtosis to 30.498. In addition to that, the statistic of the Jarque-Bera test is equal to 12275.28. Thus, the dataset is highly non Gaussian. In figure 4(a) we provide the unconditional distribution of the process R_t , which corroborates this conclusion and shows that the tails of the distribution are thicker than those of the normal distribution. This behaviour is due to some exceptional events which occur more often than expected.

Figure 3 shows that the variance of the differentiated serie, R_t , is time-varying. In particular, the time series is more volatile when some catastrophic events, like Ivan or Katrina, occur. Therefore, we can assume that the hypothesis of homoscedasticity is irrelevant in our case. Furthermore, the volatility is characterized by the existence of clusters corresponding to periods of low volatility followed by periods of high volatility.

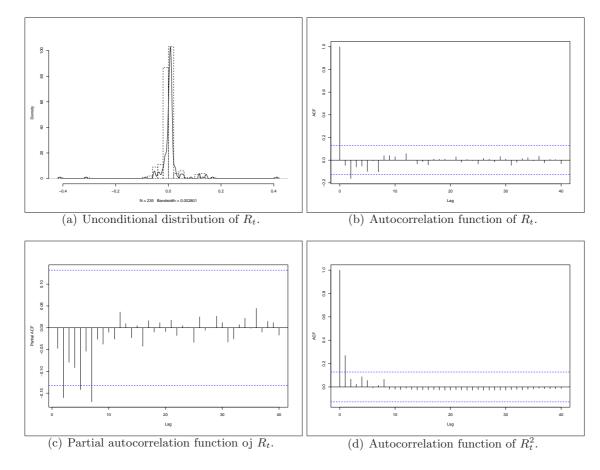


Figure 4: Some representations of the process R_t .

We observe that the volatility rises strongly when R_t rises strongly: bad news, for example a catastrophic event, induce not only an increase of the required return but also some uncertainty in the market and an increase of the volatility. On the contrary when R_t decreases the volatility is rather decreasing, as well. Thus, we decide to link the behaviour of the level of R_t with its volatility.

The Figures 4(b) and 4(d) provide with the autocorrelations of the series R_t and R_t^2 respectively. The two series are quite similar even if $\rho(2)$, i.e. the correlation at order 2, is higher for R_t^2 .

3 Dynamic Analysis

We assume that R_t is such that $R_t = f(R_{t-1}, \theta) + \epsilon_t$, with θ a vector of parameters and ϵ_t a sequence of i.i.d random variable. Following the form of the autocorrelation and of the partial autocorrelation functions, it may be relevant to use an ARMA-type process, for R_t . Figures 4(b) and 4(c) provide the fact that the autocorrelation coefficients, ρ_k , with $\rho_k = \frac{cov(R_t, R_{t-k})}{V(R_t)}$, are close to zero for k > 2, and that the partial autocorrelation coefficients do not differ from zero after the 6th lag. In order to take into account these elements and to deal with parsimonious, we assume that the level of R_t follows an ARMA(1,1) process and may be written as follows:

$$R_t = \delta + \phi R_{t-1} + \rho \epsilon_{t-1} + \epsilon_t. \tag{3.1}$$

Now in order to take into account the heteroscedasticity, we may assume that the conditional variance depends on the squared errors from previous periods and on its own history. Thus, we get a GARCH(1,1) model:

$$\begin{cases} R_t = \delta + \phi R_{t-1} + \rho \epsilon_{t-1} + \sigma_t \epsilon_t, \\ \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2, + \beta \sigma_{t-1}^2, \end{cases}$$
(3.2)

An important restriction of the GARCH model is its symmetry: only the absolute values of the innovations matter, not their sign. In other words, good news have the same consequences on the volatility than bad news. As R_t is characterized by some asymmetry in terms of volatility, we observe that an unexpected drop in spreads has a larger impact on future volatility than an unexpected decrease in prices, even if the market is not characterized by some jumps of the same magnitude. Thus, we also consider an exponential GARCH model for R_t . Following Nelson (1991), the EGARCH(1,1) model can be expressed as:

$$\begin{cases}
R_t = \delta + \phi R_{t-1} + \rho \epsilon_{t-1} + \sigma_t \epsilon_t, \\
\log \sigma_t^2 = \omega + \alpha \log \sigma_{t-1}^2 + \beta \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \gamma \left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - E \left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}} \right) \right).
\end{cases}$$
(3.3)

Thanks to the term $\frac{\epsilon_{t-1}}{\sigma_{t-1}}$, the EGARCH model is asymmetric as long as β is different of 0. When β is negative, the positive shocks generate less volatility than negative shocks ("bad news"). Moreover, the logarithmic transformation guarantees that the volatility is never negative.

Finally to take into account the fact that the conditional variance may affect the mean of the process, we choose the ARCH-M model of Engle and Robins (1987). This permits to model that a change in volatility of the insurance linked securities spreads reflects a change of the underlying risk. For instance the strong volatility of spreads just after Katrina may be interpretated like a signal of the investors'unability to correctly price the issues. Thus, Risk Management Services modified its model in order to increase the underlying risk. As a consequence, the spreads rose, too. Finally, we retain for R_t an ARMA(1,1)- GARCH-M(1,1) process:

$$\begin{cases} R_t = \delta + \phi R_{t-1} + \rho \epsilon_{t-1} + \lambda \sigma_t^2 + \sigma_t \epsilon_t, \\ \sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2, + \beta \sigma_{t-1}^2. \end{cases}$$
(3.4)

Until now nothing is said about the distribution of ϵ_t . Previously we pointed out the nonnormality of the distribution of R_t , characterized by some asymmetry (the skewness is equal to -0.660) and by tail thicker than the normal distribution. It is due to some events which occur more often than expected. They cause some consequences on the insurance linked spreads which may be considered as outliers. Indeed, in September 2004 or in August 2005 the spreads strongly increase after Ivan and Katrina hurricanes before moving slowly to their initial level. It can be interpreted as a temporary or transient change (see for example Box and Reinsel (1994)) in the serie. Franses and Ghijsels (1999) and McAleer and Verhoeven (2000) point out the fact that neglecting outliers (additive outliers in their case) lead to not skewed residuals and to thick tails. Moreover McAleer and Verhoeven (2000) show that outliers have some consequences on the estimated parameters - they may biaised them, that outliers tend to dominate the quasi maximum likelihood estimates, resulting in larger ARCH and smaller GARCH estimates. These consequences reveal the need to modify the estimation methods in order to take into account the presence of outliers. That explains why Chen and Liu (1993) present a recursive method in the case of an ARMA process. Similarly, Franses and Van Dijk (1999), Park (2002) and McAleer (2004) provide some solutions to deal with the presence of outliers in the case of GARCH processes.

Nevertheless, it seems that such adjustments are inappropriate for insurance risks. Indeed, the sudden movements of the insurance linked securities spreads, of the bonds covering the catastrophic events, may not exactly be compared to similar movements which may be observed in the financial markets. They are the result of a pure random and natural event and not the consequences of a kind of financial crisis. Thus, if we want to do some projections of the spreads, an ad-hoc study with, for example, an intervention analysis for the outliers seems to be inappropriate. We have to use a distribution which allows to include such phenomena. Such a distribution is able to create both asymetry and tick tails.

Among the rich world of the distributions besides the normal ones, Eberlein and Keller (1995), after Barndorff-Nielsen (1977), show that the class of hyperbolic distributions are some excellent candidates in order to be an interesting distribution for modelling financial returns. They can be fitted to the empirical distributions with more accuracy than the stable Pareto, the Student and finite discrete mixtures of normals distributions. Moreover, among the hyperbolic distributions,

	ARMA(1,1)-GARCH(1,1)			ARMA(1,1)-EGARCH(1,1)			ARMA(1,1)-GARCHM(1,1)		
δ	0.151		-0.296			10.746			
		(0.077)			NA^*			(4.043)	
ϕ		0.848			0.093			-0.099	
		(0.037)			NA^*			NA^*	
ρ		-0.930			-0.490			3.219	
		(0.020)			(0.073)			(0.185)	
λ								-0.440	
								(0.166)	
ω		52.692			1.752			3.982	
		(12.280)			(0.272)			NA^*	
α		1.000			0.729			392.252	
		(0.231)			(0.041)			(63.066)	
β		0.427			0.167			0.810	
		(0.053)			(0.061)			(0.021)	
γ		· · · ·			1.248			× /	
					(0.112)				
	Gaussian	GH	NIG	Gaussian	GH	NIG	Gaussian	GH	NIG
Mean	-0.007			0.003			0.000		
Std. Error	1.018			0.052			0.031		
λ		-0.639	-0.500		-0.846	-0.500		-0.381	-0.500
Steepness		0.281	0.377		3.277	8.077		10.957	8.165
Asymmetry		-0.012	-0.014		-0.890	-0.886		-2.658	-2.344
Location		0.007	0.008		0.005	0.005		0.003	0.003
Scale		0.468	0.418		0.028	0.021		0.007	0.008
Likelihood	-323.254	-261.046	-261.186	344.059	417.870	416.862	460.646	565.718	565.520

 Table 2: Parameters of the GARCH-type model estimations

 *parameters non significativ

Barndorff-Nielsen (1995) shows that the normal inverse Gaussian distribution can approximate most hyperbolic distributions very closely.

In order to take into account all these elements, we propose to assume that ϵ_t follows a Normal Inverse Gaussian distribution. Nevertheless, we compare also the results with the estimations done assuming that ϵ_t follows a Gaussian and a generalized hyperbolic distributions. The estimations are done following the maximum likelihood methodology². The serie is previously multiplied by 1000. In order to take into account the number of parameters to estimate, the estimations are done in two steps. The first one provides the parameters of the GARCH-type models, whereas the second is relative to the distribution parameter estimations. Table 2 provides the different results.

Table 2 allows to highlight some differences on the three components of the models. First of all, the ARMA-GARCH model is characterized by a strong persistence because ρ is equal to 0.848. This result is very different of the one get for the other models. For both the EGARCH and GARCH in Mean models it appears that the AR component is not significative . Indeed ϕ is equal to 0.093 and to -0.099 whereas the standard errors are respectively equal to 0.102 and 0.103. We may assume that it is due to the consideration of the asymmetry in the GARCH

 $^{^{2}}$ As in Engle and Robins (1987), we do not take into account in this paper the fact that the information matrix is not block diagonal between the parameters of the mean and the parameters of the variance.

side (for the EGARCH model) or the consideration of the consequences of the volatility on the mean (for the GARCH-M model). Moreover, for the ARMA(1,1)-GARCH-M(1,1) model, λ is equal to -0.440. That means that greater is the volatility, smaller is the log return. This result corroborates what was expected. In terms of GARCH components, it appears that all the parameters are significant. The EGARCH model is characterized both by a strong persistence of the volatility (α is equal to 0.729) and by some asymetry (β is equal to 0.167). A similar result has been got with the GARCH-M model. Indeed, β is equal to 0.810. That means that the volatility presents also some persistence. That is not the case for the ARMA-GARCH model. In this case the persistence is taken into account in the ARMA side of the model (with the ϕ parameter).

In terms of distribution, the main conclusions may be summarized as follows:

- The residuals are centered with a location parameter equal to zero for all the distributions and all the ARMA-GARCH type models.
- The ARMA(1,1)-EGARCH(1,1) and ARMA(1,1)-GARCH-M(1,1) models have the less volatil residuals. The standard error of the Gaussian distribution is respectively equal to 0.052 and 0.031 whereas it is equal to 1.018 for the ARMA(1,1)-GARCH(1,1) model. The same conclusion can be done with the scale parameters of the GH and NIG distributions.
- In the case of the ARMA(1,1)-GARCH(1,1) and ARMA(1,1)-EGARCH(1,1) models the steepness of the NIG distribution is higher than the steepness of the GH distribution. It is also the case of the lambda parameter. On the contrary, for the ARMA(1,1)-GARCH-M(1,1) model, the lambda and steepness parameter are lower for the NIG distribution than for the GH distribution. Because the two parameters have an opposite sign on the tails of the distributions, these results are quite difficult to interpret. But the graphical representation (see Figure 5) of the results shows that both distributions provide similar results in terms of tails.
- The asymmetry is higher for the ARMA(1,1)-GARCH-M(1,1) model (the symmetry parameter is equal to -2.658 or -2.344). We may assume that this result is due to the fact that the asymmetry is not taken into account with the GARCH side of the model (as in the case of the EGARCH model). For the GARCH model, this parameter is close to zero.
- Lastly, it appears that the parameters of the GH and NIG distributions are pairwise not so different. Moreover, the log-likelihood function are quite similar, even if the GH allows to improve marginaly the results. Thus, because the NIG requires the estimation of one parameter less than the GH, we prefer to use a NIG distribution in order to capture the distribution of the residuals. Because the higher log-likelihood is obtained with the

ARMA(1,1)-GARCH-M(1,1) model, we think that this model is the better in order to capture the characteristics of the insurance linked securities index log-returns.

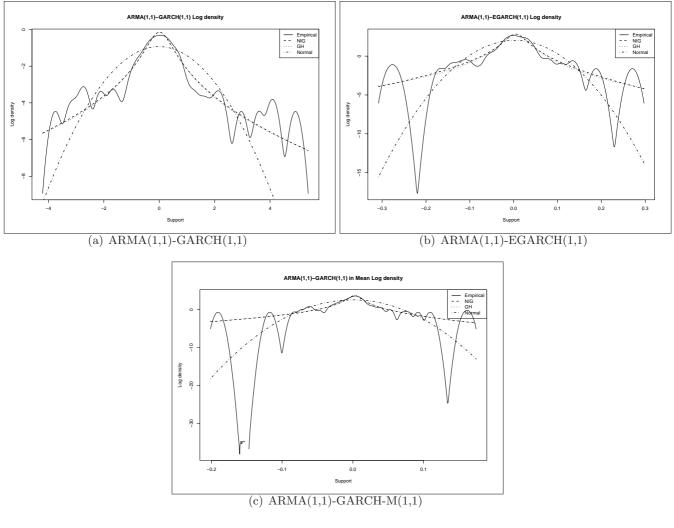


Figure 5: Log-Densities

The figure 5 provides the log-densities of the different distributions. The full line is relative to the empirical distribution whereas the dot-dashed, dotted and dashed are relative to the Gaussian, generalized hyperbolic and normal inverse Gaussian distributions. It appears clearly that the two latters capture better, both the thick tails of the empirical distribution and the behaviour of the empirical distribution around its mean, than the Gaussian distribution. In addition to that, their results are quite comparable. These elements reinforce our choice of the normal inverse Gaussian distribution as distribution for the ARMA(1,1)-GARCH-M(1,1) model.

4 Conclusion

A ARMA(1,1)-GARCH(1,1)-M model, with a Normal Inverse Gaussian distribution for the residuals, is able to capture the main stylized facts of the index of the catastrophe bonds having a rating lower than BBB+. Nevertheless according to the evolution of the index of the bonds covering mortality risk, the same type of model seems to be irrelevant. For instance the strong increase of the spreads from August 2007 causes a break in the time serie whose the consequences are currently unknown. Thus, we have to wait for a wider time interval in order to start a similar study.

An ARMA-ARCH-M model, with a Normal Inverse Gaussian distribution is quite easy to simulate. Thus, such a study gives the opportunity to introduce in the market some derivatives which would cover the underlying risk of a basket of issues.

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