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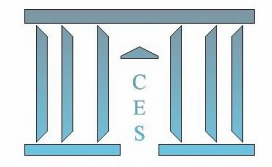

From equilibrium models to mechanism design : On the place and the role of government in the public goods provision analysis in the second part of the twentieth century

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From equilibrium models to mechanism design: On the place and the role of government in the public goods provision analysis in the second part of the twentieth century*

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Abstract

Focussing on their analysis of the optimal public goods provision problem, this paper follows the parallel development of equilibrium models and mechanism design after the accommodation of Samuelson's definition of collective goods to the general equilibrium framework. Both paradigms lead to the negative conclusion of the impossibility of a fully decentralized optimal public goods provision through market or market-like institutions.

Keywords: general equilibrium; Lindahl–Foley equilibrium; Wicksell public competitive equilibrium; private provision equilibrium; mechanism design; free-rider problem; incentive compatibility

1 Introduction

After the second world war, most of European governments were currently supposed and required to coordinate and stimulate the reconstruction effort of national economies devastated by the war, to provide the public goods and services in Health, Education, Research and all kinds of public infrastructures which have made possible the substantial economic growth of this period, and, if possible, to regulate this growth by contra-cyclical fiscal and monetary policies. Sixty years after, as a result of the movement of globalization–regionalization of all economies over the world and the construction of economic areas on the model of the European Union common market, the common representation of governments' tasks has progressively but dramatically changed. Roughly speaking, even if a number of certitudes are nowadays seriously cracked, it is now commonly accepted that policy objectives should be defined at several more decentralized and more or less

*A first draft of this paper was prepared for the Paris HPE Conference (December 2008). This version has benefitted from the comments of the participants, especially of F. Adaman and Y.M. Madra who presented a paper at the same session. Special thanks go to Ü. Zenginobuz for long standing discussions on mechanism design. Conclusions are obviously mine. A theoretical exposition of equilibrium concepts for public goods provision can be found in [11], and general overviews on mechanism design in [28, 46].

coordinated levels. To fulfill these objectives, most of the traditional economic policy instruments, supposed to contradict international agreements, are presented as inadequate, while the others mainly focus on establishing rules that guarantee the fair competition of private firms.

The purpose of this paper is to study how the changes in the *common representation* of governments' tasks are reflected in the evolution of the public goods provision theory, especially at its most formalized level. We obviously do not mean that the theoretical developments we plan to study are only the passive reflect of more ideological representations that they may contribute to shape. Relations are more complex between different levels of discourses which have each one their own internal organization. And, obviously, any assessment on the real evolution of the place and the role of governments in public goods provision would be out of the scope of this paper. Building on the dose of endogenous development that any effort of formalization guarantees, we will maintain our historical analysis at an internal and purely theoretical level.

Specifically, recall that the fifties were the years of creation and development of the general equilibrium theory. In the seventies some general equilibrium models were studying the equilibrium of "second best" economies where the existence of taxes, lump sum transfers and the governmental provision of public goods were explicitly modeled. More or less at the same period, Russian theorists analyzed general equilibrium of "mixed economies" with certain goods provided in fixed quantities and/or at fixed prices and resold at competitive prices on secondary competitive markets. The embedding in general equilibrium models of pricing rules fixed by the government for goods that market may fail to provide was also actively worked out. One could evoke in addition the abundant theoretical elaboration in this time on planning of mixed economies. In all these theoretical works, the rationality of government choices is implicitly assumed to be determined outside the functioning of the competitive system. In counterpart, governmental interventions, whose objectives are taken as given by the agents of the competitive system, are assumed to determine the characteristics of consumers' and producers' behavior and, in particular, to influence the production possibilities of firms. The task of theorists is to investigate the possibility of equilibrium given these public policies and eventually to look for minimizing the distortions introduced by the government interventions.

This is in contrast with two polar equilibrium models, worked out at the same period, where the provision of public goods enters as an argument in consumer's utility function and so determines the equilibrium amount of their provision and their equilibrium price. In the first one, whose likelihood is attested by the development of charities, consumers "provide" public goods, that is, buy them at their market price in order to put them at the disposal of the other consumers. In the second one, consumers pay at personalized prices, called Lindahl prices, their common consumption of public goods produced by competitive producers. Both models, whose consistency is proved under the same standard

assumptions, have as a common feature the fact that public goods are provided without intervention of any public institution. Their respective drawbacks are at the origin of the huge development, since the eighties, of the mechanism design literature called for solving the sub-optimality of the private provision equilibrium and the evasion by free-riding of the Lindahl–Foley equilibrium. Replacing equilibrium of competitive economies by equilibrium of mechanisms whose aim is to implement desirable allocations or to allow for a true revealing of preferences has consequences on the role assigned to government institutions by the public goods provision theory. Roughly speaking, in equilibrium models, the government is supposed to stimulate, coordinate, correct, or plan the competitive functioning of private ownership economies as justified by the welfare theorems. With the today ubiquitous mechanism design in the theoretical literature, the completely decentralized functioning of any system has become an objective *per se* that the role of government is to enforce.

In the sequel, we analyze this by no way linear evolution beginning with the simultaneous publication of the Arrow–Debreu paper [1], birth announcement of general equilibrium theory, and of Samuelson's papers [47, 48] defining public goods. We will see how the theoretical evolution of general equilibrium theory superposes after 1954 two lines of research which overlap, without it be possible to speak of two schools of thought whose only one would accept Samuelson's definition of public goods, since sometimes the same authors use alternatively the two frameworks when tackling ends and means of public goods provision. When elaborating equilibrium concepts for an economy whose definition incorporates explicitly the presence of Samuelson public goods, the relative weight of private provision relative to their financing with Lindahl prices is progressively reversed at the advantage of public provision equilibrium.

During this period, mechanism design theory has difficulties to elaborate a general definition for the design of institutions through which individuals interact. Building on the extension of planning procedures to the provision of public goods, the first ambition of mechanism design is to provide a coherent framework for understanding market functioning in classical and non-classical environments; its first point of application is the resolution of the 'free-rider problem' with the introduction of the notion of incentive compatibility. Since then, mechanism design has become the dominant paradigm for the normative analysis of a wide variety of economic and social issues which, from social choice theory and voting systems to optimal selling of indivisible objects, and analysis of many other regulation institutions designed at macro or micro levels, go far beyond the initial optimal public goods provision problem.

Focussing on this problem and insisting voluntarily on the first significant results in each stream of research, the aim of this paper is to analyze the meaning of these evolutions for a definition of and theoretical foundations for public policy of market economies.

2 Creation and development of general equilibrium theory

General equilibrium theory is a unified framework for studying, in the Walras tradition, the *general interdependence of economic activities: consumption, production, exchange*. Arrow–Debreu (1954)'s paper is in the same time the seminal definition of a so-called 'private ownership economy' and an equilibrium existence result proving consistency of the model.

The list of data

$$\mathcal{E} = \left(\mathbb{R}^L, (X_i, P_i, e_i)_{i\in I}, (Y_j)_{j\in J}, (\theta_{ij})_{\substack{i\in I\\ j\in J}}\right)$$

is the prototype description of an economy. L is a (finite) set of goods, so that $\mathbb{R}^L$ is the commodity space and the price space of the model. I is a (finite) set of consumers and X_i, P_i and e_i represent respectively the set of possible consumption plans, the preferences and the initial endowment of consumer $i \in I$. J is a (finite) set of producers, and Y_j is the set of possible production plans of firm $j \in J$. For each i and j, θ_{ij} represents the share of consumer i in the profit of firm j. In the above definition, all data of the model may be thought of as historically and socially determined, as the result of past evolutions. The distribution parameters (consumers' initial endowments and profit shares) define the *institutional data* of the economy, result of the current social consensus, but also make precise in some sense what is of *the responsibility of State* in the economy under consideration. The competitive (price-taker) behavior of agents defines the model: a *competitive equilibrium* (the solution concept) is a couple of an allocation and of a price vector such that *markets clear* (the allocation is said to be *feasible*), and, at equilibrium prices, *each producer maximizes its profit*, *each consumer optimizes his preferences in his consumption set under his budget constraint*. In this functioning, the role of firms is purely technical, more suited for 'regulated' than for privately owned firms characterized in the 'real' world by much more complex strategies than the simple profit maximization,[1] and government is absent from the model. Sovereignty of price-taker consumers is the motor of the standard general equilibrium model.

The formalization and the definition adopted by Arrow–Debreu in 1954 for what is often denominated 'Walrasian equilibrium' enabled them to solve the equilibrium existence problem addressed before the second world war by A. Wald [55][2] for the Walras formalization and by J. von Neumann [43] in a somewhat different framework. One time guaranteed the consistency of the model (an equilibrium exists under reasonable assumptions), during the twenty following years, the research program of general equilibrium theory was progressively precised, centered on:

[1] This remark was recurrently used to stress that the Arrow-Debreu model fit better with a planned economy than with a capitalist economy.

[2] A paper which follows a series of papers published by K. Menger in *Ergebnisse eines Mathematischen Kolloquiums, 1935-36.*

- Sufficient conditions for equilibrium existence, a constantly revisited issue with each generalization or extension of the model.[3]

- Optimality properties of equilibrium, in some sense the alpha and omega of general equilibrium theory. According to the first welfare theorem, the equilibrium allocation is optimal from consumers' point of view, a mere tautology for convenient definitions of equilibrium and Pareto optimality. Under continuity, convexity, boundedness assumptions on the economy, local no-satiation of consumers at any component of a feasible consumption allocation, the second welfare theorem states that, given a total amount of available resources, any optimal feasible allocation can be achieved as an equilibrium, that is decentralized by prices, through a convenient redistribution to consumers of endowments and profit shares. Just before 1954, that is before the publication of their joint paper on existence, comparable second welfare theorem results had been separately obtained by Arrow and Debreu.

- For the sake of comparative analysis, study of uniqueness and continuity properties with respect to the initial data of the economy.

After its creation, the general equilibrium model was progressively enriched in order to accomodate, one after the other, most of the different issues successively tackled by neoclassical theory: intertemporal equilibrium, microeconomic foundations of macroeconomics, risk and uncertainty, financial markets, asymetry of information, to quote only important issues among many others. Obviously, the previous research program applies whatever be the framework to which general equilibrium is adapted. The extraordinary plasticity of the general equilibrium paradigm explains its longevity.

Defining public goods, introducing their production, provision, and consumption in the general equilibrium model was a natural objective, a way for explaining the rationale of public expenditure. The publication in 1954-55 of Samuelson's papers was obviously to have an impact on the contribution of general equilibrium theory to public economics.

3 Samuelson's definition of public goods and its impact

A 'collective consumption good' is defined by Samuelson in [47, 48] as a good whose each individual consumption (or using in production) leads to no substraction from any other individual's consumption. The simple definition brought on during more than two decades a host of discussions. Their common characteristic is to call for combining in more flexible or more complex ways (see [29, 38, 41, 49, 57]) the two characteristics of their consumption (non-excludability, non rejectability) assigned by Samuelson to the restrictive definition of (pure) public goods. All these critics call for considering all kinds of "impure" public

[3] With also the objective of better relating equilibrium with optimality and core concepts.

goods. No one[4] questions the main novelty of the paper which is not to recognize the social character of the benefits associated with the provision of public goods (public utilities) but *to extend the domain of consumers' sovereignty to the choice of the amount of public goods to be provided*. In other words, Samuelson's papers cause Public Economics to shift from a political economy approach to public expenditure to a theory of the demand for and the supply of public goods and services.

The problem dealt with by Samuelson, and by each one of the above quoted critics, is the research of (first order) conditions that guarantee optimality of the public goods provision, from the point of view of consumers (Pareto optimality) or from the point of view of a social planner whose utility is depending on individual consumer's utilities. Conclusions are rather negative. Optimum exists, is multiple depending on the particular form of the social utility function. But the externality in consumer's preferences, inherent to the definition of public goods, prevents any implementation by a market mechanism (consumers have no interest to reveal their preferences, their willingness to pay) or by a planning procedure (which would require from an omniscient planner to know all consumer's rate of substitutions between private and public goods).

The impact of Samuelson's papers on the development of general equilibrium was neither immediate (what is rather normal) nor complete. Important contributions of general equilibrium theory to analysis of public interventions have been made independently of Samuelson's public goods provision problem, not necessarily outside the list of other functions that Samuelson concedes to public expenditure at the end of [48]: redistributing income, 'paternalistic policies', provision of goods that market may fail to provide, correction of negative externalities. They are contemporaries with the theoretical elaboration of public goods provision equilibrium concepts which will be reported in the next section. Let us quote:

- **General equilibrium of second best economies** Under this sub-title, we refer to a series of papers [14, 37, 50, 51, 52], published in the seventies. Through different hypotheses specific of each paper, they have as a common feature to study mixed or 'second best' economies where the presence of a public sector is explicitely modelled and to consider public policy decisions on taxes, lump sum transfers, government consumption of private goods, and the (possible) public provision of certain (non marketed) goods as decisions whose analysis should be kept separated from the analysis of the competitive functioning of the resulting economy.[5]

[4] With the notable exception of an unpublished paper of Peaccock and Wiseman (see [44]) where the authors qualify of futile 'the attempts to explain the economic activities of governments by use of *simpliste* conceptions of welfare economics'.

[5] In all quoted papers, the devices of taxes and lump sum transfers are exogenous data of the equilibrium model, determined by historical considerations or more voluntary objectives in the redistribution of income. They are the main distorsion relative to first best equilibrium [52, 50]. The role assigned to government

- **Equilibrium of 'mixed economies' with certain goods provided in fixed quantities and/or at fixed prices** This literature (see [53] and its Russian references) investigates existence and (constrained and unconstrained) efficiency of equilibrium in the context of mixed economies characterized by the the possible presence and interplay of dual markets for each commodity. On the first one, market prices are fixed and the allocation of goods is determined by rationing schemes and governmental orders. On the second market, flexible prices resulting from the market mechanism coordinate demand and supply. This model, first elaborated in the eighties by Russian economists, is supposed to be a still valid approximation to the main features of the government intervention in the transition economies of the NIS and some other countries.

- **Embedding in general equilibrium model of pricing rules fixed by government for goods that market could fail to provide** This is one of the deepest achievements of general equilibrium theory for the last thirty years. Re-visiting a long and controversial debate in public economics [4, 6, 23, 32, 45] on pricing, regulating and financing public utilities (like public transportations, electric power plants and many other examples) produced by firms with increasing returns (to scale) technologies, this literature[6] extends equilibrium definition, existence and optimality properties to economies which satisfy neither the differentiability assumptions made in all above quoted papers nor the convexity assumptions of standard general equilibrium. The mathematical tool for this extension is the notion of normal cone whose (not unique) definition always captures in the same time the ideas of profit maximization in the convex case and of 'normal' in the smooth case. Several statements of the second welfare theorem for a production economy have been provided, beginning with Guesnerie [22]. Whatever be the chosen notion of normal cone in the subsequent papers,[7] in order to decentralize Pareto optimal allocations of an economy, *firms must be instructed to behave in conformity with the (necessary) first order conditions of Pareto optimality*, that is to choose prices in the normal cone to their component of the Pareto optimal feasible allocation.

  In the corresponding equilibrium definition, firms are described by the *pricing rule*

goes from a simple tax collecting and revenue dispersing agency [51] to its explicit modelling either as an additional agent, submitted to budgetary equilibrium, and whose exogenous preferences are defined on its provision of public goods [14] or, more generally, on the current allocation of private and public goods [37], or as an additional producer with an exogenous set of possible input combinations for an exogenous output of goods to be publicly provided [50]. In all cases, public goods, if any, are defined by the public character of their provision; the amount of their provision shapes consumers' preferences and the production possibilities of the other agents.

[6] One will find a first excellent account in the JME special issue [8].

[7] Depending on the assumptions made on the production sets, each definition has a different implication for the economic sugnificance of the price decentralization result. Clarke's normal cone is considered as the full generalization of the marginal cost pricing rule.

they are instructed to follow. In addition, a *wealth structure* defines, in the list of data of the economy, the revenue of consumers as a function of current prices and of the current production allocation, defined so as to guarantee that, at equilibrium, consumers can finance the possible losses of firms. Under appropriate assumptions, equilibrium exists but one should notice that, for nonconvex production economies, the first welfare theorem does not hold: an equilibrium allocation needs not be Pareto optimal. The effect of particular wealth structures, in terms of efficiency of the equilibrium allocation or of alteration of the income distribution, is the translation in the general equilibrium model of the above quoted and still controversial debate on entry fees versus lump sum transfers for financing the deficit of regulated firms.

4 Equilibrium concepts for Samuelson's public goods provision

Let now L be the set of private goods and, according to Samuelson's definition, K be the set of pure public goods, so that $\mathbb{R}^L \times \mathbb{R}^K$ is the commodity space and the price space of the equilibrium model.

The list of data

$$\mathcal{E} = \left(\mathbb{R}^L \times \mathbb{R}^K, (X_i, P_i, e_i)_{i\in I}, (Y_j)_{j\in J}, (\theta_{ij})_{\substack{i\in I \\ j\in J}}\right)$$

is the prototype description of a *private ownership economy with public goods*, with now

- For each consumer $i \in I$, $X_I \subset \mathbb{R}^L \times \mathbb{R}^K$ and $e_i \in \mathbb{R}^L \times \mathbb{R}^K$,
- For each producer $j \in J$, $Y_j \subset \mathbb{R}^L \times \mathbb{R}^K$, that is, producers (firms) jointly produce private goods and public goods,
- As far as production technology sets Y_j are assumed to be convex, there is no need to reconsider the role in revenue distribution of the profit shares θ_{ij} of consumers on profits of firms.

For defining equilibrium concepts, the difficulty begins with the interpretation of the strategy set X_i of each consumer i and thus of his preferences P_i.

- Either for consumer i, $(x_i, x_i^g) \in X_i$ represents the couple of a consumption of private goods and of a **private provision** of public goods.

  Then, the utility for consumer i of (x_i, x_i^g) depends on its own provision of public goods and on the private provisions of the other agents. But in 1954, there is no equilibrium existence theorem for dependent preferences.

- Or $(x_i, G_i) \in X_i$ represents the couple of a consumption of private goods and of a claim for an amount of public goods.

  Then, at equilibrium, all consumers have to agree on a same provision of public goods, and the definition of feasibility for an allocation ("markets clear") has to be reformulated.

These preliminary considerations explain the multiplicity of equilibrium concepts and the order in which they appear in the literature.

4.1 Lindahl–Foley equilibrium (1967-1970)

Lindahl–Foley equilibrium corresponds to what is called by Samuelson the **Lindahl solution**. At equilibrium, consumers consume a same amount $\overline{G}$ of public goods and face personalized prices for public goods, so that an equilibrium is a couple

$$\Big(\big((\overline{x}_i)_{i\in I}, \overline{G}\big), (\overline{y}_j, \overline{y}_j^g)_{j\in J}\big), \big((\overline{p}, (\overline{p}_i^g)_{i\in I})\Big) \in \big(\prod_{i\in I} X_i \times \prod_{j\in J} Y_j\big) \times (R^{L+|I|K} \setminus \{0\})$$

of an allocation and of a non null price vector such that

- markets clear for private goods; for public goods, total supply of firms = consumers' *common demand* (**Lindahl–Foley feasibility**),

  and, at equilibrium prices,

- each producer maximizes his profit $\overline{p} \cdot y_j, +(\sum_{i\in I} \overline{p}_i^g) \cdot y_j^g$, using the common private goods price vector and the sum of personalized price vectors for public goods as production price vector for public goods,

- each consumer, using the common private goods price vector and **his personalized public goods price vector**, optimizes his preferences in his consumption set under his budget constraint.

After some tentatives [2, 30] calling for applying Lindahl's ideas (see [33, 34]) definition of Lindahl equilibrium and a proof of its existence emerged in two papers of Foley [12, 13] in 1967 and 1970. The now classical equilibrium existence proof consists in building an economy with only private goods defined on a commodity space of an increased dimension, by considering each consumer's bundle of public goods as a separate group of commodities, and applying to the corresponding economy some known equilibrium existence result which will imply equilibrium existence in the original public goods economy.

Besides the usual convexity, continuity, boundedness assumptions for the economy and local no-satiation of consumer's preferences at components of feasible allocations, conditions obtained by Foley for equilibrium existence were quite general with four exceptions which can be partially or totally removed:

1. Consumers have **no initial endowment in public goods**: for each $i \in I$, $e_i = (\omega_i, 0) \in \mathbb{R}^L \times \mathbb{R}^K$
2. Consumer's utility is increasing in public goods (**no public bads**)
3. Public goods **are not production inputs**
4. There is only one producer who produces with constant returns.

More important, the first and second welfare theorems hold: an equilibrium allocation is Pareto optimal and any Pareto optimal Lindahl feasible allocation can be decentralized *using convenient lump sum transfers and consumers' personalized prices for public goods*. This is true even without the second and third assumptions and if the last assumption is replaced by a decreasing returns productive system. Such a result confirms the role generally given in general equilibrium to government for achieving Pareto optimality through redistribution policy.

4.2 Wicksell–Foley public competitive equilibrium (1967-1970)

The definition of Wicksell–Foley public competitive equilibrium is an interesting variation on optimality properties of Lindahl–Foley equilibrium. A Wicksell–Foley public competitive equilibrium is a triple

$$\left((\overline{x}_i)_{i\in I}, \overline{G}), (\overline{y}_j, \overline{y}^g_j)_{j\in J}, (\overline{p}, \overline{p}^g), (\overline{t}_i)_{i\in I}\right) \in \prod_{i\in I} X_i \times \prod_{j\in J} Y_j \times R^{L+K} \setminus \{0\} \times \mathbb{R}^I$$

of a Lindahl–Foley feasible allocation, a price vector, and a vector of consumers' taxes such that for equilibrium prices $(\overline{p}, \overline{p}^g)$,

- the sum of equilibrium taxes $\sum_{i\in I} \overline{t}_i$ finances the cost of production of the equilibrium public good provision $\overline{G}$,
- each producer maximizes his profit,
- given the equilibrium provision of public goods, each consumer chooses a consumption of private goods so as to optimize his preferences under his after taxes budget constraint $\overline{p} \cdot x_i + \overline{t}_i \leq \overline{p} \cdot \overline{\omega}_i$, and
- There is no other **government proposal** $\left(G, (t_i)_{i\in I}\right)$ such that the sum of taxes together with the sum of equilibrium profits finances the provision of public goods and that appears to every consumer to leave him better off.

In other words, the grand coalition cannot block with another government proposal the equilibrium government proposal $\left(\overline{G}, (\overline{t}_i)_{i\in I}\right)$. Even if the realized consensus is rather

negative, 'a kind of last resort, or worst case' (Foley [12]), this condition can be thought of as translating the Wicksell idea (see [58]) of an unanimous consent on the couple of a public goods provison together with the set of financing taxes.

Two easy and not surprising properties:

1. A Lindahl–Foley equilibrium allocation is a Wicksell–Foley public competitive equilibrium allocation which is, in turn, Pareto optimal;

2. For an economy $\mathcal{E}$, under the usual conditions for their decentralization with prices, the set of Lindahl–Foley Pareto optimal feasible allocations and the set of Wicksell–Foley public competitive equilibrium allocations coincide.

A more interesting property, stated and proved by Foley in his PhD dissertation [12] under restrictive assumptions, but not reported in his Econometrica paper [13] and today forgotten, shows that Pareto optimality of the allocation is compatible with an equitable repartition ot the tax burden: there exists a Wicksell–Foley public competitive equilibrium where the tax paid by each consumer is proportional to his equilibrium revenue. Foley even conjectures the existence of a public competitive equilibrium for an arbitrary progressive income tax.

4.3 Public goods private provision equilibrium (1976-1986)

The idea of private provisions of public goods corresponds to the idea of private donations to charity, campaign funds of political parties and so on. And papers abound during the period on theoretical (and experimental) analysis of voluntary contributions to schools, churches, etc., in relation with the free-rider problem.

The equilibrium definition in the public goods private provision model differs from the standard general equilibrium definition only in the fact that the utility for an agent of a couple (x_i, x_i^g) of a private goods consumption and a public goods private provision depends on his own provision of public goods but also on the public goods private provisions of the other agents. On the same economy $\mathcal{E}$ as above, an equilibrium is a couple

$$\Big(\big((\overline{x}_i, \overline{x}_i^g)_{i\in I}, (\overline{y}_j, \overline{y}_j^g)_{j\in J}\big), \big(\overline{p}, \overline{p}^g\big)\Big) \in \big(\prod_{i\in I} X_i \times \prod_{j\in J} Y_j\big) \times (R^{L+K} \setminus \{0\})$$

of an allocation and of a non null price vector such that

- markets clear for private goods and private provisions of public goods (total supply of firms = consumers' demand),

  and, at equilibrium prices,

- each producer maximizes his profit $\overline{p} \cdot y_j, + \overline{p}^g \cdot y_j^g$,

- each consumer, taking as given the equilibrium public goods provisons of other consumers, optimizes (from the point of view of his preferences and under his budget constraint) his consumption of private goods and his own provision of public goods.

As for the standard general equiibrium model, equilibrium exists under the same standard conditions, but it has no reason to be optimal. It is only 'constrained' optimal, that is, it is optimal for each agent given the public goods provisions of the other agents. The intellectual tool for the equilibrium existence result is an equilibrium existence result for an *abstract economy*[8] where preferences of players formally depend on the current strategies of the other players, to be found in a (1976) Shafer–Sonnenschein paper [50] for which public goods private provision was not the concern. Actually, the S-S paper was belonging to and following a series of equilibrium existence results for second best economies. Public goods, evoked at the very end of the paper, were, in the ideas of Shafer–Sonnenschein, to be publicly provided. For this reason, the interest of the paper for public goods private provision equilbrium was ignored, as was and is still ignored its potential interest for "warm glow" and any other form of benevolent, malevolent, or 'concerned for face' public goods provision.

As far as equilibrium existence is concerned, the celebrated (1986) Bergstrom–Blume–Varian paper: "On the private provision of public goods" [3] represents a regression with respect to the Shafer–Sonnenshein equilibrium model. Their model allows for one public good (extended to several, at the end of the paper) and **one private good**. Ten years after, equilibrium existence in this model is a consequence of equilibrium existence in S-S's paper. The B-B-V paper studies also uniqueness and sensitivity of equilibrium to wealth distribution. This makes for their epigones,[9] the interest of B-B-V's paper. To know if and to what extent private provision of public goods can be (Pareto) improved by government interventions, is still today an issue and an object of researches.

[8] An abstract economy is a 'generalized game' $\Gamma = \big((X_i, \alpha_i, P_i)_{i \in N}\big)$ where in addition to his strategy set and preferences defined on his strategy set, a constraint correspondence defines for each agent the set of strategies he can choose given the strategies of the other agents. An equilibrium of the generalized game is a t-uple of individual strategies where each agent optimizes his preferences in his constraint set. The idea of deducing equilibrium existence in an economy from an equilibrium existence result in an associated abstract economy where an additional agent, the 'Walrasian auctioneer', chooses prices so as to maximize the total excess demand comes from Arrow–Debreu (1954). The decisive improvement of S-S's paper was to allow preferences in the abstract economy to depend on the strategies chosen by other agents. In the original economy, preferences may thus depend on the current allocation and prices and the interest of the S-S paper goes much farther than its simple application to the public goods provision problem .

[9] Some of them as Villanacci and Zenginobuz [54] have completely generalized the B-B-V model and its conclusions. See also subsequent papers of Villanacci–Zenginobuz and their references.

5 Drawbacks of the different equilibrium concepts

Existence and optimality properties of the previous equilibrium concepts strongly depend on convexity assumptions on consumers'preferences and production. On the consumption side, nonconvexity may come from indivisibilities in the quantities to be consumed, or are related with non risk aversion in preferences. We will neglect these causes of nonconvexity. On the production side, most of so-called collective goods (public utilities) are classical examples of decreasing costs and are goods that market may fail to provide. This is the main drawback of the results reported in Section 4. As we have seen in Section 3, even if such goods are considered as private goods from consumers' point of view, non convexity on the production side requires government intervention for enforcing pricing rules and designing revenue distribution rules allowing consumers to survive and to finance a possible deficit in their production. Regulation of firms and the necessity of this alteration of distribution rules with respect to the institutional definition of private ownership economies provides new foundations for public policy of market economies without providing means for calculating and implementing an appropriated public policy. When goods produced under nonconvex technologies are considered as Samuelson public goods, this adds new difficulties in the definition of market mechanisms or planning procedures for their provision to those which will be reported below.

However, one can rely on [15] for conditions of existence of public goods private provision equilibrium in nonconvex production economies,[10] on [5] for conditions of existence of Lindahl equilibria in the same model, on [31] for the extension of the second welfare theorem in economies with nonconvexities and public goods, on [42] for the extension to the nonconvex case of the Wicksell–Foley public competitive equilibrium concept. As in the convex case, public goods private provision equilibrium exists but is not optimal. Lindahl equilibrium exists but, unlike in the convex case, is not necessarily Pareto optimal. As in the convex case, consumers' Lindahl prices and lump sum transfers exist for decentralization of optimal Lindahl–Foley feasible allocations. It is in this framework that one should re-visit the old controversy, referred to at the end of Section 3, on entry fees versus lump sum transfers for financing the deficit of regulated firms.

Under convexity on the consumption side, whether public goods be produced by a convex or a nonconvex technology, the other drawbacks of the equilibrium concepts for their provision stay today exactly the ones pointed out in 1954 by Samuelson.

On the one hand, improved or not by government interventions, public goods private provision equilibrium fails, by definition, to be Pareto optimal. And this holds true what-

[10] Like Shafer–Sonnenschein [50], when extending equilibrium existence in nonconvex production economies to dependent consumers' preferences, Gourdel does not refer to the externality inherent to the public goods private provision but to more general dependences of preferred sets on the current allocation and prices.

ever be the incentives one may invoke for more Pareto satisfactory individual consumers' donations.

On the other hand, general equilibrium theorists are unanimously ready to stress, like Mas-Colell [39], that "the concept of Lindahl prices is primarily of theoretical interest, as it is not devised to model any existing market, or even (...) potentially existing ones". Revealing his 'willingness to pay" at Lindahl–Foley equilibrium is not, in the spirit of general equilibrium, a realistic equilibrium consumers' behavior, since it is not a best strategy for every consumer. In other words, according to the terminology of mechanism design theory, the main drawback of Lindahl–Foley equilibrium is not to be 'incentive compatible'.

Moreover, implementing with Lindahl prices and lump-sum transfers or taxes a desired Pareto optimal Lindahl–Foley feasible allocation would require, for their calculation by a hypothetical coordinating center, government's perfect information on consumer's preferences. An analoguous difficulty holds true with planning procedures *à la* Dreze–La Vallee Poussin [10] or Malinvaud [36]. Such planning procedures rule the exchange of informations between a 'central board' (central agency, government, whatever) and the economic agents, consumers and (convex) producers,[11] in a process operating in continuous time whose convergence should lead to an equilibrium relative to a system of private goods prices and Lindahl prices for public goods, thus to a Pareto optimal Lindahl–Foley feasible allocation. In a kind of 'Walras tâtonnement', at each stage of such procedures, the indicators issued by the board are an amount of public goods provision together with production prices of all goods and an amount of numeraire left to consumers for their expenditure on private goods. In response, consumers declare their private goods demand and their marginal willingness to pay for the proposed public goods consumption, while producers declare a profit maximizing net supply of private and public goods. Rules of revision of next stage board's indicators in function of consumers and producers' proposals complete the specification of the dynamic procedure. Besides desirable properties of the different processes, in particular their convergence towards a satisfactory allocation, the possiblity of implementing the procedure requires for the board knowledge, at each stage of the process, of consumers' marginal willingness to pay for the proposed public goods provision, an information that consumers may have incentives for under-reporting. The possibility and the design of procedures implying incentives for correct reporting of these marginal rates is thus an important concern for the study of planning procedures in the management of public goods provision, as it is more generally an important concern for mechanism design theory and its contribution to modeling and understanding the provision of public goods.

[11] Mas-Colell[39] specifies an analogous procedure in the more general case of public projects produced with nonconvex technologies.

6 Are mechanisms providing better foundations for public policy of public goods provision in market economies?

Mechanism design theory traces back to discussion papers of Hurwicz at the Cowles Foundation in the first years of development of general equilibrium theory,[12] successively summarized in some seminal papers [24, 25, 26]. A constant reference is done in the beginning to the Barone, Mises, von Hayek, Lange and Lerner debates over the feasibility of a centralized socialist economy, as well as to the different formalizations of Walrasian tatonnement process. When applied to the resource allocation problem in a society, a constant objective of this new formalization of social and economic interaction is to provide a coherent framework encompassing competititive equilibrium as a particular example of mechanism.

What is first meant under different names given for mechanisms in Hurwicz's papers is the design of a *process* leading, through the collect of decentralized information, to a desired resource allocation. The conceptual framework $\left(N, (\mathcal{M})_{i\in N}, Z, \Phi, (f_i)_{i\in N}\right)$ consists of a *set of agents* N with or without a coordination center, a *space of messages* $\mathcal{M} = \prod_{i\in N} \mathcal{M}_i$, a *space of outcomes* Z, for example, the set of all feasible (or non necessarily feasible) resource allocations in some economy, on which agents have individual preferences, a pre-specified function $\Phi\colon \mathcal{M} \to Z$ assigning outcomes for every collection of received messages. With $\mathcal{M}$ and Φ, *rules for revision of individual messages*, $f_i\colon \mathcal{M} \to \mathcal{M}_i$, complete the definition of the process and define a *dynamic adjustment process* which should converge towards an element $\overline{m} \in \mathcal{M}$, the equilibrium of the process, with an image $\Phi(\overline{m})$ belonging to a certain class of desirable outcomes. But there is also a more static definition of the equilibrium of a mechanism given by Hurwicz [26]. The mechanism is the game form $\left(N, (\mathcal{M})_{i\in N}, Z, \Phi\right)$ and, according to such or such game theoretic equilibrium notion, its equilibrium should yield a desirable outcome. In both cases, as written by Hurwicz [24], "unlike in the more traditional approach, the mechanism becomes[13] the unknown of the problem rather than a datum". In addition, processes and mechanisms should be, as much as possible, *informationally decentralized*, which implies an initial dispersion of information among the agents and limited communication. The resulting outcome should have certain *optimality properties* relative to individual agents' preferences on the different outcomes. Finally, the rules prescribed by the mechanism should be *compatible with either individual or group incentives* based on their own interest, and individual participation to the mechanism should be *voluntary*.

According to these definitions and requirements, it is obvious that competitive equilibrium of a private ownership economy is, under classical assumptions, a kind of Nash equilibrium for a static mechanism involving the addition of an artificial player called Walrasian 'auctioneer'. The same is true for Lindahl–Foley equilibrium of an economy with

[12] Avaiiable at cowles.econ.yale.edu/P/ccdp/ccdp1.htm
[13] For its designer.

public goods. The interest of the static mechanisms proposed in [26] is not to require any auctioneer. The various dynamic models of market processes and also the different decentralized planning models, including the planning procedures studied above for an optimal provision of public goods, are examples of dynamic mechanisms. All these processes are informationally decentralized (the message sent by each agent requires only information on its own characteristics and does not allow any agent to infer complete information on other agents' characteristics), satisfy static or dynamic optimality (the first and second welfare theorems are satisfied), participation is voluntary (the resulting oucome is individually rational, that is, preferable to the initial allocation).[14] But they need not satisfy individual *incentive compatibility*, a requirement present in the first papers of Hurwicz but whose formal statement was of difficult elaboration. Roughly speaking, the response function precribed to each player by each of these mechanisms is not a best strategy for agents who may have interest to give false signals on their own characteristics.[15]

Incentive compatibility as a solution to the free rider problem (1977–80)

The research of incentive compatible mechanisms for public goods provision has been one of the first points of application of mechanism design theory, in relation with the free-rider problem.

Let us come back to the classical model of a private ownership with public goods defined in Section 4. The mechanism set by Groves–Ledyard [19] requires the definition of a space of messages $M = \prod_{i\in I} M_i$ for consumers and of an hypothetical agent, the 'government',[16] which sets the provision $G(m, p, p^g)$ of public goods and the individual contributions of consumers $t_i(m, p, p^g)$ in function of the collection of messages $m = (m_i)_{i\in I}$ sent by consumers and the current market prices (p, p^g) for private and public goods. An *equilibrium of the mechanism* is a list $\big(\overline{m}, (\overline{p}, \overline{p}^g), ((\overline{x}_i)_{i\in I}, \overline{G}), (\overline{y}_j, \overline{y}^g_j)_{j\in J}\big)$ of a collection of messages, a non null price vector and an allocation of the public goods economy such that

- markets clear for private goods and the provision $\overline{G} = G(\overline{m}, \overline{p}, \overline{p}^g)$ of public goods
- each producer j maximizes at $(\overline{y}_j, \overline{y}^g_j)$ his profit $(\overline{p}, \overline{p}^g) \cdot (y_j, y^g_j)$
- each consumer i optimizes his preferences relative to private and public goods in the budget set

$$\Big\{(x_i, m_i) \in X_i \times M_i \colon \overline{p} \cdot x_i + t_i(\overline{m}, \overline{p}, \overline{p}^g) \leq \overline{p} \cdot \omega_i + \sum_{j\in J} \theta_{ij}(\overline{p}, \overline{p}^g) \cdot (\overline{y}_j, \overline{y}^g_j)\Big\}.$$

[14] At least under the Foley assumptions, for mechanisms leading to a Lindahl–Foley equilibrium.

[15] As explained by Samuelson, this is obvious for Lindahl equilibrium. But this is also true in some sense for competitive equilibrium of an economy with private goods and a finite number of agents since these agents would be better to manipulate prices rather than being price takers.

[16] In probable addition to the Walrasian auctioneer which should guarantee in the existence proof the feasibility of the equilibrium allocation.

Groves and Ledyard give first two examples of mechanisms, one with a so-called 'naive' government leading to a public goods private provision equilibrium, a second one with a so-called 'Lindahl' government leading to a Lindahl–Foley equilibrium whose achievement requires from the consumers truthfully reporting of their marginal rate of substitution between each public good and some numeraire private good. Each one of these examples suffers the drawbacks of the two equilibrium concepts: sub-optimality of the resulting allocation in the first case, lack of incentive compatibility in the second one. This motivates the main result of the paper consisting in the definition of two 'optimal' mechanisms [17] proved to lead to Pareto optimal allocations, and such that, *at equilibrium*, it is each consumer's individual self-interest to reveal his true valuation for or demand of the public goods. In addition, the second mechanism allows for a decentralization of Pareto optimal allocations through the redistribution of initial endowments and shares of profits. These results are by no mean immediate and are completed by a mechanism equilibrium existence result which can be found in [20].[18]

Dominant strategy mechanisms for public goods provision (1970–80)

A much more restrictive condition for incentive compatibility can be given when equilibrium in dominant strategies is the solution concept adopted for the mechanism. For an agent, a strategy is *dominant* if it is a best strategy no matter what are the strategies chosen by the other agents. And a mechanism is *dominant strategy incentive compatible* or *strongly individually incentive compatible* if truthfully reporting is for each agent a dominant strategy, that is, a best strategy *whatever be the messages sent by the other agents.*[19]

For public goods provision, dominant strategy incentive compatibility is generally stated in the partial equilibrium setting of a decision on a public project with a (already known) cost in numeraire to be financed collecting contributions (taxes and transfers) from different agents having quasi-linear utility functions on the public goods provision versus their allocation of numeraire.

Namely, X is a set of possible public projects in an economy with a unique private good (the numeraire) and a set N of individuals. Simultaneously with the public goods provision corresponding to the public project, one considers positive or negative monetary transfers $(t_i)_{i\in N}$. Each individual $i \in N$ has preferences on X represented by a *valuation*

[17] In the first one, each message space is a space of differentiable and strictly concave functions $m_i : \mathbb{R}^K_+ \to \mathbb{R}$ representing individual valuation functions for public goods; in the second one, $M_i = \mathbb{R}^K$, each m_i representing an individual claim for public goods provision.

[18] Equilibrium is proved to exist under standard assumptions on the public goods economy, with in addition a condition on the parameters of the mechanism and a technical condition whose role is to prevent consumers to go to bankruptcy under the government taxation when other consumers are in equilibrium.

[19] And not only respective of their truthfully reporting equilibrium strategy.

function $v_i\colon X \to \mathbb{R}$, net of the imputed costs, and a utility function $u_i\colon X \times \mathbb{R} \to \mathbb{R}$:

$$u_i(x, t_i) = v_i(x) + t_i$$

sum of his valuation and of the monetary transfer he receives. A *revelation mechanism* is a mechanism in which each agent $i \in N$ communicates a valuation function, not necessarily the true one, belonging to some set V_i of admissible valuation functions. The message space is thus $\mathcal{M} = \prod_{i\in N} V_i$. The outcome rule, $\Phi\colon \mathcal{M} \to X \times \mathbb{R}^N$, associates with each $w = (w_i) \in \mathcal{M}$, $\Phi(w) = \big(\Phi_x(w), \Phi_t(w)\big)$, where $\Phi_x(w)$ specifies the accepted project and $\Phi_t(w)$ specifies a list of monetary transfers $(t_i(w))_{i\in N}$ between a central agency and the agents $i \in N$. The revelation mechanism is a *direct revelation mechanism* if, for each list of messages $w = (w_i)_{i\in N}$, the corresponding level of public goods $\Phi_x(w)$ maximizes on X the total social value $\sum_{i\in N} w_i(x)$. The mechanism is said to be *strongly incentive compatible* if truth is a dominant strategy for each individual, that is, if for each $i \in N$

$$u_i\big(\Phi(w_{-i}, v_i)\big) \geq u_i\big(\Phi(w_{-i}, w_i)\big)\ \forall w_{-i} \in \prod_{j\neq i} V_j,\ \forall w_i \in V_i\,.$$

Clarke [7] in 1971, Groves [18] in 1973, Groves and Loeb [21] in 1975 demonstrated that a class of direct revelation mechanisms[20] exists in which true revelation is a dominant strategy for each agent.

Unfortunately, maximization of the social surplus does not mean Pareto optimality of the outcome $\Big(\Phi_x\big((v_i)_{i\in N}\big), \Phi_t\big((v_i)_{i\in N}\big)\Big)$. Indeed, recall that individual utility functions have two arguments and express the personal trade off of each agent between the utility (net of cost) of the public project and the individual amount of money t_i he receives ($t_i \geq 0$) from the central agency or pays ($t_i < 0$) to the central agency. Since quasi-linear utility functions are strictly monotone with respect to numeraire, Pareto optimality requires the equilibrium transfers to be *balanced*, that is, $\sum_{i\in N} \Phi_t\big((v_i)_{i\in N}\big) = 0$. The Clarke-Groves mechanisms need not satisfy this condition and thus do not necessarily lead to a Pareto optimal outcome.The decade 70-80 will finish with various impossibility theorems for public goods provision mechanisms [17, 27, 56], deeply connected with the general impossibility theorems in voting and social choice theories. In the partial equilibrium setting of a decision on a public project and the schemes of transfers for the distribution of its cost, a truth-dominant mechanism either fails to be social value maximizing or fails to be budget balancing; under suitable assumptions, this negative result is 'generic', that is holds for an open and dense set of valuation profiles in $\prod_{i\in N} V_i$.

This negative result calls for weaker forms of incentive compatibility and motivated the same study in a model where participants have beliefs concerning each other and are expected-utility maximizers.

[20] A class that Green–Laffont [16] have charaterized in 1977 when the set X of public projects is compact and every continuous function $v\colon X \to \mathbb{R}$ is an admissible valuation function.

Bayesian mechanisms for public goods provision (1979–90)

Let us first come back to the setting of the previous paragraph and denote by $v_i(x, \theta_i)$ the valuation function of agent $i \in N$ for the public project x, depending on the value of a parameter $\theta_i \in \Theta_i$. The space Θ_i represents the set of possible types of agents i; $\widehat{\theta}_i \in \Theta_i$ identifies agent i's tastes for of the agent i. The message space can be rewritten $\mathcal{M} = \prod_{i \in N} \Theta_i$ and the outcome rule, $\Phi \colon \mathcal{M} \to X \times \mathbb{R}^N$, associates with each $\theta = (\theta_i) \in \mathcal{M}$, $\Phi(\theta) = \big(\Phi_x(\theta), \Phi_t(\theta)\big)$ where $\Phi_x(\theta)$ specifies the accepted project and $\Phi_t.(\theta)$ a list of monetary transfers $(t_i(\theta)$ to agents $i \in N$. The incentive compatibility constraint of the previous paragraph can be rewritten:

$$v_i\big(\Phi_x(\theta_{-i}, \widehat{\theta}_i), \widehat{\theta}_i\big) + \Phi_t(\theta_{-i}, \widehat{\theta}_i) \geq v_i\big(\Phi_x(\theta_{-i}, \theta_i), \widehat{\theta}_i\big) + \Phi_t(\theta_{-i}, \theta_i) : \forall i \in N; : \forall \theta \in \prod_{i \in N} \Theta_I.$$

In the Bayesian model associated with this formalization, the agents have beliefs on the other agent's characteristics conditional to their own type, that is, subjective probabilities $p_i(\cdot|\theta_i)$ on $\prod_{j \neq i} \Theta_j$ known only when their own type θ_i is also known. A strategy is a decision rule associating a unique strategy choice $m_i(\theta_i)$ to each of his possible types. A *Bayesian Nash equilibrium* is a n-tuple of strategies such that, whatever be his type, each agent maximizes the mathematical expectation of the utility of the corresponding outcome, assuming that the other agents will not change their strategy. Finally, a mechanism is *Bayesian incentive compatible* if the n-tuple of strategies $\overline{m} = (\overline{m}_i)_{i \in N}$ defined by for all i, $\overline{m}_i(\theta_i) = \theta_i$ is a Bayesian Nash equilibrium, that is, if

$$E[v_i\big(\Phi_x(\widetilde{\theta}_{-i}, \theta_i), \theta_i\big) + \Phi_t(\widetilde{\theta}_{-i}, \theta_i)|\theta_i] \geq E[v_i\big(\Phi_x(\widetilde{\theta}_{-i}, \theta'_i), \theta_i\big) + \Phi_t(\widetilde{\theta}_{-i}, \theta'_i)|\theta_i]$$

for all i, $\theta_i \in \Theta_i$ and $\theta'_i \in \Theta_i$. In other words, for every agent $i \in N$ and every possible type $\theta_i \in \Theta_i$, sending as a message this information to the center dominates every other possible message whenever the other agents have presumably the same behavior.[21]

The existence of a Bayesian incentive compatible mechanism enabling to solve efficiently a collective decision problem and to ensure budget balancing, thus leading to a Pareto optimal outcome, was proved by D'Aspremont and Gerard-Varet in [9] when the sets Θ_i are finite, under a compatibility condition on individual beliefs and other mild conditions. However, as remarked later (see [35]), their mechanism violates participation constraints. If participation is volontary, the problem of free-riding may become severe, which extends to incentive compatible and individually rational direct revelation mechanisms the negative Samuelson conjecture on the impossibility of a decentralized optimal provision of public goods.

Obviously, the papers reported in the three previous paragraphs are in some sense seminal. Subsequently, the public goods provision problem has been studied in several extended

[21] As in the previous paragraph, a stronger incentive compatibility notion can be defined in term of dominant strategy equilibrium.

settings allowing for large economies, more complex systems of information parameters, the possibility of exclusion or costly access to public goods, and diverse possiblities of renegociation of the allocation corresponding to the government message. Focussing on incentives to be given to individuals for thruthfully revealing their private information, the recurrent theme that comes out of this huge literature is still the difficulty of finding mechanisms that simultaneously result in decisions maximizing the total welfare, the voluntary participation of the individuals, and balanced transfers.

7 Concluding

After this description of the beginnings of their parallel develpment, and despite some lags that we have put in evidence in the respective maturation of their statements and results, let us first stress that from equilibrium theories to mechanism design, the trend is more in the degree of their acceptation by the scientific community of theorists rather than in the progressive replacement of one framework by another one that has become today a dominant paradigm.[22]

Equilibrium theorists and mechanism design theorists have a common scientific background: game theory on the mathematical side, neo-classical theory on the economics side, thus a common definition of public economics as the field in economics that studies the relationship between the state and the market. Both theoretical research communities largely overlap; obtained results in either research stream influence each other. When dealing with public goods provision, in conformity with the general 'normative character' of neo-classical theory, their common objective is to prove the *intellectual possibility* of a functioning of market economies where there is no need for a direct provision of public goods by the government. Both theories point out the importance of wealth distribution for reaching a social optimum and limit the responsability of government to implementing a suitable redistribution of resources. Besides this problematic responsibility of the state in redistribution policies, researchs on implementation of competitive functioning

[22]Some evidence of these differences. General equilibrium was immediately formulated in a setting able to incorporate all ulterior developments by a relatively narrow scientific community with a common scientific programme. During years, the delay between the first circulation of papers and their publication in top journals was relatively short (about three years) for papers in equilibrium theory. In contrast, an analogously small scientific community was existing around Hurwicz. In this community, Arrow, who worked with Hurwicz on convergence of processes, and founded the social choice theory, had a particular place. But, as repeated by Hurwicz, a general formulation of mechanism design was the problem rather than the datum, which maybe explains the relatively long delay at the beginning between the first circulation of papers and their publication in top journals. Even, a paper by Maskin [40] dated from 1977 was not published until 1999. Today, the comparison in delays is largely reversed. While a Nobel prize was awarded in 1972 to Arrow, jointly with J.R. Hicks, for their work on general equilibrium and welfare theory, to Debreu in 1983 for his contributions in the same area, it is in 2007 that the prize was collectively awarded to three important theorists of mechanism design. This prize is the consequence of and makes obvious that the relative weight of both theories is today completely reversed.

belong, for general equilibrium, to applied economics, while for mechanism design, engineering of implementation simply requires an hypothetic government enforcement of the rules designed by the mechanism. The difficulties of such an enforcement may explain the recourse to public production and/or provision of public goods, for example of 'utilities' produced under increasing returns. But even in this case, the elaboration of rules of wealth distribution allowing for financing deficits and the design of mechanisms allowing for the regulation of private provision through agency models are still the object of active researches.

Focussing more and more on information revelation problems and making of incentive compatiblity the main requirement for an appropriate public policy, equilibrium theory and mechanism design have led, for one half of century, to the same negative conclusion on the impossibility of a fully decentralized optimal public goods provision through market or market-like institutions. Paradoxically, at a time of demonstrations in several european countries against the treatment of crisis by public policies, such a negative conclusion, confirming the 1954 Samuelson analysis, is akin to the popular protest that public goods and services as health services, education, research should not be marketed.

Our conclusion will be that Samuelson's definition of collective goods (called later pure public goods) is maybe the original error. Provision of public goods and services is only a component of public policy as a whole. And should this global public policy be understood as an *à la Mas-Colell* public project, the rationality of a collective decision on this political project should not be reduced to its conformity with the result of the interaction of consumers' decision based on their individual trade off between its utility and its cost translated in individual taxes. To paraphrase the general conclusion of a Peacock unpublished paper (with J. Wiseman) [44], economists have much more to contribute in the explicitation of particular specified policies, their consistency, their implications and their effects, than in deducing from over-simple welfare analysis rules for deciding what is the good society.

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