Innovation, nature of investment and divergent growth paths: an explanatory model

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Abstract

The principle of conditional convergence, in growth theory, fails to explain growth paths that are durably divergent among countries having similar structural characteristics (same rates of investment, of capital depreciation, of demographic growth, and similar access to technologies and resources...). Our research models the reasons of these divergences by making the assumption that the nature of technical progress is not the same one according to the type of investment that is realized. We first deduce from this assumption relations between investment, production and employment. Then, by introducing the optimizing behavior of the firms, we show the existence of two balanced and durable growth regimes. The "fast growth regime" is characterized by the importance given to the capacity investments (and thus to product innovation). The "slow growth regime" is characterized by the importance given to process investments. The nature of investments thus constitutes a crucial condition of convergence of the economies. (JEL: E22, E24, O33, O40).

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1 Introduction

Usual growth models (both neoclassical and endogenous with technical progress diffusion) are characterized by the principle of conditional convergence. This principle aims at explaining the phenomena of capital per capita convergence between countries having similar structural characteristics. In Solow's model [Solow, 1956], countries having similar exogeneous characteristics (technical progress and saving rates, capital depreciation and demographic growth) tend towards the same level of per capita GDP. Moreover, the rythm of the evolution towards this level is even faster since the country is far from reaching this theoretical long term per capita GDP. Within the first models of endogenous growth [Romer, 1986, Romer, 1987, Romer, 1990, Lucas, 1988, Rebelo, 1991, Aghion and Howitt, 1992, Grossman and Helpman, 1991], this principle was temporarily abandoned and replaced by characteristics of human capital and learning effects. These works have justified that marginal capital productivity is not necessarily decreasing and thus a perpetual growth of per capita production. However, as underlined in [Barro, 1997], the first endogeneous models cannot be used for empirical studies of growth because of the absence of the conditional convergence principle. For this reason, endogenous models integrating technical diffusion were built [Barro and Sala-I-Martin, 1997]. Indeed,

these models have shown that the progessive diffusion of the techniques (by a mechanism of imitation) supports the convergence of the economies, "following" economies tending to catch up with "leading" economies.

However, these researches are not sufficient to completely explain gaps of growth between countries having access to similar material, human and technological resources and having very close investment, capital depreciation and demographical growth rates. How can we justify the existence, through history, of national trajectories of growth (and employment) which are durably distant from each other? Why did England open the way of industrialization during the 18th century? Why did the United States become the "leaders" during the 19th century? How can we explain the persistent gaps between American and European performances since the 1990s? Obviously, technical progress plays a major role [Lorenzi and Bourlès, 1995], as well as the evolution of consumption structures [Flacher, 2003].

In this article, we support the idea suggested by [Villemeur, 2004]: the nature of technical progress is not the same according to the implemented type of investment. In Section 2, we introduce the distinction, based on OECD definitions, between capacity investment and process investment. The first one is associated to product innovation while the second one is related to process innovation. These two types of investment, which are not exclusive from each other, do not have the same impact on economic growth and on employment: the capacity investment promotes production and employment growth whereas process investment aims at a reduction of costs, in particular by reducing the amount of jobs that are necessarily for a given production. Based on this assumption, a production function and an employment function are defined in order to model the relation between nature of investment, growth and employment as in [Villemeur, 2004].

We then show, in Sections 3 et 4, why firms' optimizing behaviors determine two balanced and durable growth regimes with very different performances: the "fast growth regime" and the "slow growth regime" differ, for the same rate of investment, by the implemented type of investments realized. We conclude this article by providing in Section 5 an interpretation and a discussion of the model, leading to a proposal for future work.

2   Investments, production and employment: fundamental relations

In Section 2.1, we present our assumptions concerning the relation between investment, innovation, growth and employment. Under these assumptions, we introduce a production function (Section 2.2) and a employment function (Section 2.3). We also derive some of the main properties (Sections 2.4 and 2.5).

2.1   Investments and innovation

Investment is traditionally divided into two main categories: capacity investment and process investment. These two categories of investment are linked to two different types of innovation [1]: capacity investments are linked to the production of new goods ("product innovation") and to higher outputs. Process investments aim at cutting costs, by introducing process innovation. Of course, this distinction is not always that clear and several investments concern at the same time these two logics.

Therefore, we propose the following definitions and conjecture:

Definition 1 *An investment is called "capacity investment" (I_c) when it generates both production and employment growth.*

Definition 2 *An investment is called "process investment" (I_p) when it causes a reduction of employment rate for a given level of production.*

Conjecture 3 *Any investment can be divided into capacity investment and process investment.*

Indeed, we consider that the investments can be of four types (I_p, I_c, I', I'', cf. Table 1) depending on whether they generate or not an increase in the production and on whether they create, do not create or destroy jobs ("labor saving" investments).

This conjecture is equivalent to assuming that I' and I'' are hybrid investments and that part of them can be considered as capacity investment and the other part as process investments. Thus, $I' = I'_c + I'_p$ and $I'' = I''_c + I''_p$, where $I'_c = \alpha I'$ and $I''_c = \beta I''$ are the capacity investments included contained in I'

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[1] See, for instance [Van Duijn, 1983].

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	Decreasing of employment	No impact on employment	Increase of employment
Increase of production	I'	I''	I_c
No impact on production	I_p	-	-

Table 1: The types of investments and their impact on production and employment.

and I'', and where $I'_p = (1 - \alpha) I'$ and $I''_p = (1 - \beta) I''$ are the process investments contained in I' and I'', with $(\alpha, \beta) \in [0, 1]^2$. This conjecture enables us to simplify the model by considering, thereafter, only "pure" capacity and process investments.

Lastly, we consider replacement investment $(I_r(t))$ as a process investment. It is defined classically by:

$$\dot{K}(t) = I(t) - I_r(t) = I(t) - \delta I(t) = (1 - \delta) I(t) \tag{1}$$

where $K(t), I(t)$ and δ are respectively the accumulated capital, the global investment and the investment replacement rate $(\delta > 0)^2$.

2.2 Investments and production

According to the previous assumptions on investment, we define the production function $(Y(t))$ as follows:

$$\dot{Y}(t) = p_c x(t) I(t) \tag{2}$$

where $I(t)$, $x(t) = \frac{Ic(t)}{I(t)}$, p_c denote respectively the total investment, the share of capacity investments[3] and the productivity of capacity investments. We asssume that p_c is a strictly positive parameter of the model and that it characterizes the development level of the country. Its slow evolution allows us

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[2]In the whole article, when a variable (say $X(t)$) depends of time, its derivative $(\frac{dX(t)}{dt})$ is denoted $\dot{X}(t)$.

[3]The variation in capacities use rate has an impact on the growth regime. Depending on whether this rate increases or decreases, the impact is equivalent to an increase or a decrease of the capacity investments level. We will consider that $I_c(t)$ integrates these variations. We will then say that it represents "equivalent" capacity investments ($x(t)$ representing their share in the total investment).

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to consider this parameter as a constant of the model. The $x(t)$ variable will be called "productive effectiveness". We assume that $x(t) \in]0,1]$ because we want to identify positive and sustainable growth regimes.

The production growth rate can be simply written as:

$$\frac{\dot{Y}}{Y}(t) = p_c x(t) i(t) \tag{3}$$

where $i(t) = \frac{I(t)}{Y(t)}$ is the investment rate. In an equivalent way, taking t_0 as the initial instant (and writing $Y(t_0) = Y_0$), we have:

$$Y(t) = Y_0 e^{p_c \int_{t_0}^{t} x(t)i(t)dt} \tag{4}$$

Note that the production function can also be written in order to put to the fore its relation with "AK" endogenous growth models since we derive from Equations 1 and 3 that $\dot{Y} = \left(\frac{p_c}{1-\delta}x\right)\dot{K}$. Here, A depends on the productive effectiveness (x) and on the parameters of the model (p_c and δ). We can see that for a given productive effectiveness and a given investment rate, the marginal capital productivity is constant. Production per capita can keep growing indefinitely, according to the predictions of endogenous growth models.

2.3   Investments and employment

By assuming that the nature of technical progress is not the same according to the chosen type of investment, our model allows us to establish a relation of "creative destruction" between investments (and thus capital) and employment. On the one hand capital and labour appear to be substitutes when the investment is a process investment (due to job losses for a given production). On the other hand capital and labour are complements when the investment is a capacity investment (since these investments generate at the same time production and employment growth). Finally, the evolution of employment $(L(t))$ can be analysed as the result of a double movement of created $(L_c(t))$ and suppressed $(L_s(t))$ jobs:

$$\dot{L}(t) = \dot{L}_c(t) - \dot{L}_s(t) \tag{5}$$

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We model this phenomenon of "creative destruction" by the following functions:

$$\frac{\dot{L_c}}{L}(t) = \varepsilon_c(t) \, x(t) \, i(t) \tag{6}$$

$$\frac{\dot{L_s}}{L}(t) = \varepsilon_s(t) \, (1 - x(t)) \, i(t) \tag{7}$$

where $\varepsilon_c(t)$ et $\varepsilon_s(t)$ are strictly positive variables, respectively called created jobs and supressed jobs coefficients. They can be interpreted as synthetic indicators of the policies of employment effectiveness. To simplify the model, we suppose that these two coefficients vary in a symmetrical way: a policy which is favorable to employment contributes to develop jobs creations and to reduce employment destruction. We thus assume that $\varepsilon_c(t) = \varepsilon_c^{mx} - \varepsilon_s(t)$ where ε_c^{mx} is the maximum coefficient of created jobs ε_c^{mx} is a constant which characterizes the considered economy. The employment function can thus be written as:

$$\begin{aligned}
\frac{\dot{L}}{L}(t) &= \varepsilon_c(t) \, x(t) \, i(t) - (\varepsilon_c^{mx} - \varepsilon_c(t)) \, (1 - x(t)) \, i(t) \tag{8} \\
&= \varepsilon_c^{mx} x(t) \, i(t) - (\varepsilon_c^{mx} - \varepsilon_c(t)) \, i(t) \tag{9}
\end{aligned}$$

That is to say, taking t_0 as the initial instant (and writing $L(t_0) = L_0$):

$$L(t) = L_0 e^{\varepsilon_c^{mx} \int_{t_0}^{t} x(t)i(t)dt - \int_{t_0}^{t} (\varepsilon_c^{mx} - \varepsilon_c(t))i(t)dt} \tag{10}$$

2.4   Link between production and employment functions

We derive from Equations 3 and 8 the relation between the production and employment growth rates:

$$\frac{\dot{Y}}{Y}(t) = \frac{p_c}{\varepsilon_c^{mx}} \frac{\dot{L}}{L}(t) + \frac{p_c}{\varepsilon_c^{mx}} (\varepsilon_c^{mx} - \varepsilon_c(t)) \, i(t) \tag{11}$$
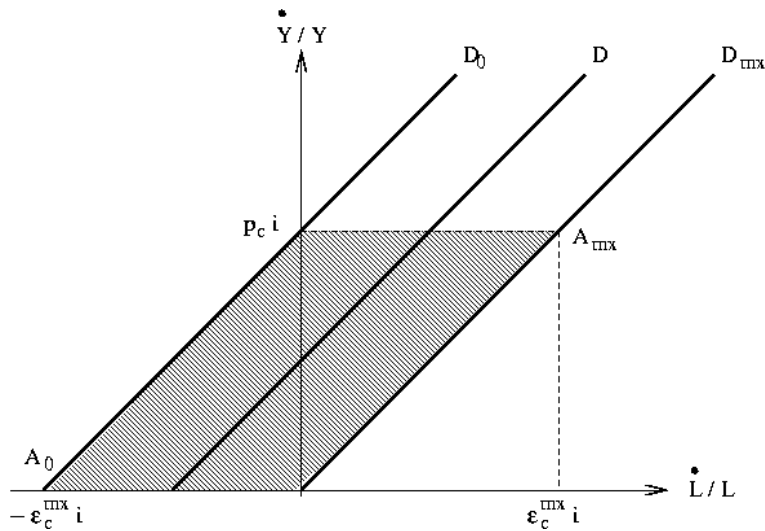
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Figure 1: Relation between production and employment growth rates.

This shows that the employment growth rate depends on the production growth rate, on the investment rate, on the coefficients of created jobs and on the parameters of the model. It is thus possible to define the range in which the economy can operate using a phase diagram (Figure 1).

The lines D_0 and D_{mx} represent the polar cases. They are characterized repectively by the absence of created jobs $(\varepsilon_c(t) = 0)$ and the maximum level of jobs creation $(\varepsilon_c(t) = \varepsilon_c^{mx})$. Since $x(t) \in \,]0, 1]$, we have $\frac{\dot{Y}}{Y}(t) \in [0, p_c i(t)]$ (Equation 2). The area of operation of the economy in the diagram of phases is thus a rhombus. Its extension depends on the investment rate. If both the investment rate and created jobs coefficients are constant, the economy evolves along a line segment D, parallel to D_{mx}.

2.5   Fundamental variables

In addition to production $(Y(t))$, employment $(L(t))$ and capital $(K(t))$, we define two other fundamental variables: the wage rate $(\omega(t) = \frac{Y(t)}{(1-c(t))L(t)}$ where $c(t) \in \,]0, 1[$ represents the profit share into total income) and the capital profitability $(q(t) = \frac{c(t)Y(t)}{K(t)})$. We then show the following result.

Proposition 4 *The fundamental variables of the model (production $(Y(t))$, employment $(L(t))$, capital $(K(t))$, wage rate$(\omega(t))$ and profitability $(q(t))$)*

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depend on:

- *the productive effectiveness ($x(t)$), the created jobs coefficient ($\varepsilon_c(t)$), the investment rate ($i(t)$) and the profit share ($c(t)$));*

- *the time (t);*

- *the parameters of the model (capacity investments productivity (p_c), maximal created jobs coefficient (ε_c^{mx}) and the replacement rate of investments (δ));*

- *and the initial conditions (Y_0, L_0, K_0).*

This can be summarized as:

$$[Y(t), L(t), K(t), \omega(t), q(t)] = f(x(t), \varepsilon_c(t), i(t), c(t), t \mid p_c, \varepsilon_c^{mx}, \delta, Y_0, L_0, K_0)$$

Proof. We already verified this proposition in Sections 2.2 and 2.3 for $Y(t)$ and $L(t)$. The result for $K(t)$ is derived from Equation 1; for $\omega(t)$ using equations 4 and 10; and for $q(t)$ using the previous results. ∎

Thus, the fundamental variables (production, employment, capital and factors prices) depend, at each instant t, on only four other variables.

3   Firm behavior and optimal growth paths

In a traditional way, in our model, the firms make their decisions by minimizing a cost function under constraints. The decisions are made at each instant. They concern two variables: $x(t)$, that mainly describes the investment structure and $\varepsilon_c(t)$ that represents the effectiveness of the employment policies that are applied in the considered country. These decisions are made with imperfect information: 1) on the one hand producers only take into account available data at instant t (optimization is not intertemporal) and thus have a short term approach; 2) on the other hand, producers consider that the investment rate ($i(t)$), the profits share ($c(t)$), the wages rate growth ($\frac{\dot{\omega}}{\omega}(t)$) as well as the anticipated cost of created jobs per unit of capital (taking into account required profitability $q_r(t)$[4]) are exogeneous. Thus, while making their decisions, the producers consider that they do not have significant impact on these variables within the period considered. The firms minimize the increase of

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[4]According to Equation 6, the anticipated cost of created jobs per unit of capital can be written $\frac{\omega(t)\dot{L}_c(t)}{K(t)} = \frac{\omega(t)L(t)\varepsilon_c(t)x(t)i(t)q_r(t)}{c(t)Y(t)}$.

the production cost $(\dot{Cost}(t))$ per unit produced under these constraints. The optimization is thus scheme the following:

Conjecture 5 *Every instant t, producers' decisions are deduced from:*

$Min \left\{ \frac{\dot{Cost}}{Y}(t) \right\}$

under constraints $\begin{cases} i(t) = c_1(t) & c(t) = c_2(t) \\ \frac{\dot{\omega}}{\omega}(t) = c_3(t) & \frac{\omega(t)L(t)\varepsilon_c(t)x(t)i(t)q_r(t)}{c(t)Y(t)} = c_4(t) \\ x(t) \in]0,1] & \varepsilon_c(t) \in]0, \varepsilon_c^{mx}] \end{cases}$

where

- the increase of the production cost[5] is defined by

$$\dot{Cost}(t) = \omega(t)\dot{L}(t) + \dot{\omega}(t)L(t) + q_r(t)I(t) \qquad (12)$$

- $c_i(t), i \in \{1,2,3,4\}$ are exogeneous variables (i.e. they only depend on time).

The minimization of production cost increase per produced unit is justified in an imperfect economy in which the current choices depend on choices that have been made in the past. Indeed, production growth generates additional costs in a context characterized by many rigidities (concerning labour, technology, skills, already accumulated capital...). Thus producers do not search the best solution (absolute optimum) but the best possible one taking into account the path of the economy and the current situation.

Theorem 6 *The previous optimization problem has a solution if and only if $q_r \leq (1-c)\varepsilon_c^{mx}$ and, when an optimum is reached:*

$$x(t) = \frac{q_r(t)}{(1-c(t))\varepsilon_c^{mx}} \qquad (13)$$

$$\varepsilon_c(t) = \varepsilon_c^{mx}x(t) \qquad (14)$$

When the optimum exists, we will say the the economy follows an "optimal growth path".

Proof. Cf. Appendix 1. ∎

We can first note that profitability required by the shareholders must be lower than a certain threshold so that the economy can reach an optimal

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[5]This cost takes into account the replacement investment.

growth path. In this case productive effectiveness $(x(t))$ and the created jobs coefficient$(\varepsilon_c(t))$ are entirely determined, on such a trajectory, by the required profitability $(q_r(t))$, the profits share $(c(t))$ and the parameters of the model.

Graphically, for a given investment rate, the optimal growth path is represented by the segment $]A_0, A_{mx}]$ of the diagram representing production and employment growth rate (Figure 2), where $A_0 = \left(\frac{\dot{Y}}{Y} = 0; \frac{\dot{L}}{L} = -\varepsilon_c^{mx} i \right)$ and $A_{mx} = \left(\frac{\dot{Y}}{Y} = p_c i; \frac{\dot{L}}{L} = \varepsilon_c^{mx} i \right)$. The support of this segment is defined by the line $\frac{\dot{Y}}{Y}(t) = \frac{p_c}{2\varepsilon_c^{mx}} \frac{\dot{L}}{L}(t) + \frac{p_c i(t)}{2}$. A_{mx} is also characterized by the triplet $(x, \varepsilon_c, q_r)_{mx} = (1, \varepsilon_c^{mx}, (1-c)\varepsilon_c^{mx})$ which represents a situation of maximum production and employment growth, for given investment rate and parameters.

Proposition 7 *Along an optimal growth path, production, employment, capital, wage rate, and profitability levels, but also production and employment growth rates, only depend on $i(t)$, $c(t)$, and of the parameters of the model:*
$$[Y(t), L(t), K(t), \omega(t), q(t)] = f(i(t), c(t), q_r(t), t \mid p_c, \varepsilon_c^{mx}, \delta, Y_0, L_0, K_0)$$
$$\left[\frac{\dot{Y}}{Y}(t), \frac{\dot{L}}{L}(t)\right] = f(i(t), c(t), q_r(t), t \mid p_c, \varepsilon_c^{mx}, \delta)$$

Proof. The result is straightforward by Proposition 4 and Theorem 6. ∎

4   Fast and slow growth regimes

We first deal with balanced growth regime of our model. By definition, a growth regime is balanced when capital and production growth rates are identical, i.e. $\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K}$. Under this condition, we can show that there is a relation between the productive effectiveness $(x(t))$, the average profitability $(q(t))$, the profits share $(c(t))$ and the parameters of the model.

Lemma 8 *If a growth regime is balanced then $x(t) = \frac{(1-\delta)q(t)}{p_c c(t)}$.*

Proof. We have, from Equations 1 and 2, $\frac{\dot{K}}{K} = \frac{(1-\delta)I}{K} = p_c x i = \frac{\dot{Y}}{Y}$. Finally, since q is defined by $q = \frac{cY}{K}$, we obtain the desired relation. ∎

We then identify the balanced and durable growth regimes. For that, we start by introducing the concept of "capacity investments profitability" (of which some properties are provided in appendix 2). We also introduce the notion of "durable growth regime".

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Definition 9 *The capacity investments profitability is defined by*

$q_c(t) = \frac{\dot{Y}(t) - \omega(t)\dot{L}_c(t) - \dot{\omega}(t)L(t)}{(1-\delta)x(t)I(t)}$.

Definition 10 *A growth regime is durable if and only if $q(t) = q_c(t) = q_r(t)$ and $\frac{\partial q(t)}{\partial t} = 0$.*

This definition expresses the fact that capacity investments cannot be durably more profitable than the other investments and that required profitability cannot be durably higher than the average profitability over long period. Using this definition and the previous results, we can finally introduce the following theorem.

Theorem 11 *There are two optimal, balanced and durable growth regimes. They are defined by:*

$$A_m \left(\begin{array}{c} x = \frac{1}{2-\delta}, \varepsilon_c = \frac{\varepsilon_c^{mx}}{2-\delta} \\ q = q_c = q_r = \frac{p_c \varepsilon_c^{mx}}{(2-\delta)[p_c+(1-\delta)\varepsilon_c^{mx}]}, c = \frac{(1-\delta)\varepsilon_c^{mx}}{p_c+(1-\delta)\varepsilon_c^{mx}} \end{array} \right) \qquad (15)$$

and

$$A_{mx} \left(\begin{array}{c} x = 1, \varepsilon_c = \varepsilon_c^{mx} \\ q = q_c = q_r = \frac{p_c \varepsilon_c^{mx}}{p_c+(1-\delta)\varepsilon_c^{mx}}, c = \frac{(1-\delta)\varepsilon_c^{mx}}{p_c+(1-\delta)\varepsilon_c^{mx}} \end{array} \right) \qquad (16)$$

These growth regimes are called respectively "fast growth regime" and "slow growth regime".

For these two regimes, the fundamental variables of the model (production $(Y(t))$, employment $(L(t))$, capital $(K(t))$ et wages $(\omega(t))$) only depend on the investment rate, on the parameters of the model and of the initial conditions:

$[Y(t), L(t), K(t), \omega(t)] = f(i(t), t \mid p_c, \varepsilon_c^{mx}, \delta, Y_0, L_0, K_0)$

Proof. The assumption that growth is durable amounts to $q(t) = q_c(t) = q_r(t)$ and $\frac{\partial q(t)}{\partial t} = 0$. From Lemma 12 (Appendix 2), we have:

$q_c(t) = q(t) + \frac{\varepsilon_c^{mx}(1-c(t))(1-x(t))[(2-\delta)x(t)-1]}{(1-\delta)x(t)}$.

Since $q = q_c$, the solutions of the equation are $\{x(t) = \frac{1}{2-\delta}$ or $x(t) = 1\}$. Using Theorem 6, we know that the corresponding created jobs coefficients are given by $\varepsilon_c(t) = \varepsilon_c^{mx}x(t)$.

Since growth is balanced, we also have, from Lemma 8, $x(t) = \frac{(1-\delta)q(t)}{p_c c(t)}$. Moreover, in a durable growth regime, $q(t) = q_r(t) = x(t)(1-c(t))\varepsilon_c^{mx}$

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because of Theorem 6. It is then easy to deduce the profitability and profits share for the two balanced and durable growth regimes.

Lastly, according to Proposition 7, $q_r(t)$ and $c(t)$ being determined by the parameters of the model for the two growth regimes (slow and fast), the fundamental variables of the models do not depend on any other variables than $i(t)$. ∎

In our model, there are two optimal, balanced and durable growth regimes (Figure 2). These two regimes are characterized in particular by constant growth rates of the production and of employment. These rates only depend on the parameters of the model and of the investment rate. The two growth regimes are characterized by very different performances. As shown in Table 2, the slow growth regime is characterized by a production and employment growth rates definitely lower than those which characterize the fast growth regime. Thus, within economies having comparable techniques (same p_c and ε_c^{mx}) and investment rate, we obtain (taking $\delta = 0.3$) that the production (resp. employment) growth rate is 1.7 (resp. 5.7) times weaker in the slow growth regime than in the fast growth regime.

	Slow growth regime	Fast growth regime
$\frac{\dot{Y}}{Y}(t)$	$\frac{p_c i(t)}{2-\delta}$	$p_c i(t)$
$\frac{\dot{L}}{L}(t)$	$\frac{\delta \varepsilon_c^{mx} i(t)}{2-\delta}$	$\varepsilon_c^{mx} i(t)$
$c(t)$	$\frac{(1-\delta)\varepsilon_c^{mx}}{p_c+(1-\delta)\varepsilon_c^{mx}}$	$\frac{(1-\delta)\varepsilon_c^{mx}}{p_c+(1-\delta)\varepsilon_c^{mx}}$
$q(t)$	$\frac{p_c \varepsilon_c^{mx}}{(2-\delta)[p_c+(1-\delta)\varepsilon_c^{mx}]}$	$\frac{p_c \varepsilon_c^{mx}}{p_c+(1-\delta)\varepsilon_c^{mx}}$
$\frac{\dot{\omega}}{\omega}(t)$	$\frac{p_c-\delta\varepsilon_c^{mx}}{2-\delta} i(t)$	$(p_c - \varepsilon_c^{mx}) i(t)$
$x(t)$	$\frac{1}{2-\delta}$	1
$\varepsilon_c(t)$	$\frac{\varepsilon_c^{mx}}{2-\delta}$	ε_c^{mx}
$I(t)$	$I_c + (I_p + I_r)$	$I_c + I_r$

Table 2: Main characteristics of the slow and fast growth regimes.

The capital profitability is higher in the fast growth regime. The profits share is identical for both growth regimes. Concerning the wage rate, we can note that it can grow more quickly in one or the other of the growth regimes depending on the parameters values[6]. Indeed, Table 2 shows that wage rate

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[6]In both balanced and durable growth regimres, the profit share is a constant determined by the parameters of the model. This result allows us to deduce that the wage rate growth is equal to the labour productivity growth rate $\frac{\dot{\omega}}{\omega}(t) = \frac{L}{Y}\frac{\partial \frac{Y}{L}}{\partial t} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$.
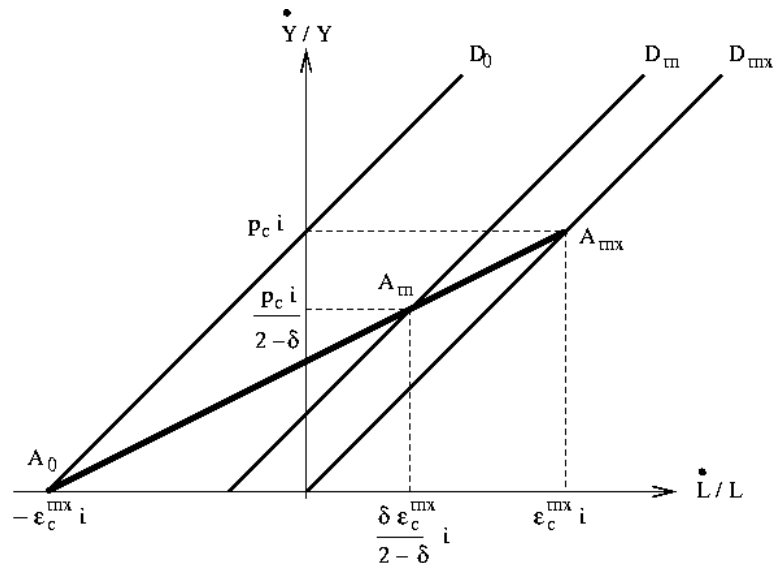
Figure 2: The two balanced and durable growth regimes.

grows more quickly in the fast growth regime than in the slow one if $\frac{p_c}{\varepsilon_c^{mx}} > 2$. The situation is reversed if $\frac{p_c}{\varepsilon_c^{mx}} < 2$. In these two growth regimes, the capital productivity growth rate is null by definition of the balanced growth. Lastly, the investment is primarily turned towards the capacity investment in the fast growth regime[7] and more towards process investments in the slow growth regime.

We can stress that the two growth regimes are also characterized by two very distinct forms of investments. On the one hand, the fast growth regime is characterized by an investment turned towards the production capacities: In this regime, investments are used to increase the production and to develop new products. By massively creating jobs, capacity investments appear also favorable to the development of the demand, which is essential for producers. On the other hand, the slow growth regime is characterized by an investment structure less favorable to the capacities of production and to product innovation. The investment structure is more turned towards cost reductions induced by process investments. This situation appears less favorable to the demand side because it creates less jobs.

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[7]The only process investments made in the fast growth regime are thus the replacement investments.

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5   Interpretation and discussion

By considering the existence of a link between the nature of technical progress and the type of investment carried out, this article provides a significant contribution to the endogenous theories of growth. Indeed it provides an explanation of the existence of divergent growth paths between countries having initially the same technical, physical and human resources.

The fundamental idea underlying this model is that, on the one hand capacity investment is associated mainly with the product innovation, which implies production but also employment growth. Process investment, on the other hand, implies cost reductions by reducing the number of jobs that are necessary to produce a given quantity. This models the existence of persistent disparities of production and employment growth between initially similar countries.

This orginal contribution allows us to draw a possible explanation of the divergence between United States and Europe since the 1990's[8]. We can also derive explanations, in terms of divergence (but also of convergence), over more remote historical periods like the ones of industrial revolutions[9].

Nevertheless, beyond these contributions, the model has also limits at least because it does not provide the determinants explaining the choice of one or the other of the two balanced and durable growth regimes[10]. In the same way, it does not provide (except a few indications) the explanation of transition dynamics from the situation in which the economy is out of an equilibrium to an equilibrium situation. Lastly, the model does not allow at this stage to endogeneize the investment rate which remains the only exogeneous variable in the two balanced and durable growth regimes.

Among the possible tracks to enrich this model and to answer the open questions always outstanding, it would be interesting to model the demand of the households and the way in which consumption is structured. This will allows us to provide a macroeconomic loop that does not exist for the moment in the model and an explanation of the growth regime choice. Indeed, it seems relevant to consider that a final consumption structure favorable to the new products could be a reason of the adoption of the fast growth regime. On the

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[8]Cf. [Villemeur, 2004].

[9]Cf. [Flacher, 2003].

[10]About the choice between product and process innovation, only two articles directly model this question as stressed in [Bonanno and Haworth, 1998]. Their model, on the one hand, highlights the role of the competitive regime (Cournot or Bertrand) and the one of the quality of the goods that are produced. The article of [Rosenkrantz, 1995], on the other hand, underlines the role of the consumers as a key issue of firms decisions.

contrary, if demand is less dynamic and less favorable to the new products, it would seem reasonable to note that the economy chooses to invest more in the processes. Such an approach could enable us to introduce, for example, the relative prices of the various products. It could also integrate a model of R&D or the innovation or imitation costs. Finally the robustness of the results could be tested using elementary empirical work.

6   Conclusion

Whereas many models of growth (including endogenous growth models) show the existence of conditional convergence process, we wondered in this article about the reasons of persistent divergences between production and employment growth within countries having of comparable investment rate and capital depreciation but also similar resources (human, technical and financial ones).

We proposed an original answer based on the relation between the type of investment realized and the nature of technical progress (the incorporation of this progress into the economy) which results from it. From the modeling of original production and employment functions on the one hand and of the behavior of the firms on the other hand, we have shown the existence of two durable and balanced growth regimes: the "fast growth regime" which is characterized by the significant place that it grants to capacity investments (and thus to product innovation) and the "slow growth regime" which supports the investment of process.

This result, which explains a possible major and durable divergence between two countries having an identical investment rate, offers a perspective for future interesting research on technical progress assimilation. Further research could consider the determinants of the adoption of the one of the two growth regimes and the transition between these two regimes. An idea would be to take into account the role played by final consumption structure or the one of R&D.

Appendix 1 - Proof of theorem 6

In order to simplify the notations, we remove t in the demonstration.

From Equations 12 and 8, $\frac{\dot{Cost}}{Y}$ can be written as:

$$\frac{\dot{Cost}}{Y} = \left[\frac{\omega LI}{Y^2} \left[\varepsilon_c^{mx} x - \varepsilon_c^{mx} + \varepsilon_c \right] \right] + \frac{\dot{\omega}L}{Y} + \frac{q_r I}{Y} \qquad (17)$$

According to proposition 4, $\frac{\dot{Cost}}{Y}$ depends, at each instant t, on the variables $x, \varepsilon_c, i, c, q_r, \omega, \dot{\omega}$, on the parameters $p_c, \varepsilon_c^{mx}, \delta$ and on the initial conditions Y_0, L_0, K_0.

Using the constraints $c_i, i \in \{1, 2, 3, 4\}$, the function to minimize can be expressed as:

$$\frac{\dot{Cost}}{Y} = (1 - c_2) c_1 \left[\varepsilon_c^{mx} x - \varepsilon_c^{mx} + \varepsilon_c - \frac{c_3}{c_1} + \frac{q_r}{1 - c_2} \right]$$

Moreover, we have $c_4 = \frac{\omega L \varepsilon_c x i q_r}{cY} = \frac{(1-c_2)c_1}{c_2} x \varepsilon_c q_r$. The function that we have to minimize becomes $f\left(x, \varepsilon_c \mid c, \varepsilon_c^{mx}, c_4'\right) = \varepsilon_c^{mx} x + \varepsilon_c + \frac{c_4'}{(1-c)x\varepsilon_c}$ where $(x, \varepsilon_c) \in \,]0, 1] \times \,]0, \varepsilon_c^{mx}]$ and where c and $c_4' = \varepsilon_c x q_r > 0$ are exogeneous.

Since $f\left(x, \varepsilon_c\right)$ is positive and C^∞ on his support, it has a lower bound. It also has a critical point defined by:

$$\frac{\partial f}{\partial x} = \varepsilon_c^{mx} - \frac{c_4'}{(1-c)x^2\varepsilon_c} \qquad \text{and} \qquad \frac{\partial f}{\partial \varepsilon_c} = 1 - \frac{c_4'}{(1-c)x\varepsilon_c^2}$$

i.e.

$$(x, \varepsilon_c) = \left(\left[\frac{c_4'}{(1-c)\left(\varepsilon_c^{mx}\right)^2} \right]^{\frac{1}{3}}, \left[\frac{\varepsilon_c^{mx} c_4'}{1 - c} \right]^{\frac{1}{3}} \right) \qquad (18)$$

To check that it is indeed a minimum, we compute the Hessian matrix of this function at this point:

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x^2}\left(x, \varepsilon_c\right) & \frac{\partial^2 f}{\partial \varepsilon_c \partial x}\left(x, \varepsilon_c\right) \\ \frac{\partial^2 f}{\partial x \partial \varepsilon_c}\left(x, \varepsilon_c\right) & \frac{\partial^2 f}{\partial \varepsilon_c^2}\left(x, \varepsilon_c\right) \end{bmatrix} = \begin{bmatrix} \frac{2c_4'}{(1-c)x^3\varepsilon_c} & \frac{c_4'}{(1-c)x^2\varepsilon_c^2} \\ \frac{c_4'}{(1-c)x^2\varepsilon_c^2} & \frac{2c_4'}{(1-c)x\varepsilon_c^3} \end{bmatrix}$$

We have:

$$\frac{\partial^2 f}{\partial x^2}\left(x, \varepsilon_c\right) = \frac{2c_4'}{(1-c)x^3\varepsilon_c} > 0$$

$$\frac{\partial^2 f}{\partial x^2}\left(x, \varepsilon_c\right) \frac{\partial^2 f}{\partial \varepsilon_c^2}\left(x, \varepsilon_c\right) - \left[\frac{\partial^2 f}{\partial x \partial \varepsilon_c}\left(x, \varepsilon_c\right) \right]^2 = \frac{3\left(c_4'\right)^2}{(1-c)x^4\varepsilon_c^4} > 0$$

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The Hessian matrix is therefore definite positive. The critical point is thus a global minimum of the function.

We finally have to check if the constraints $x \in \;]0,1]$ and $\varepsilon_c \in \;]0, \varepsilon_c^{mx}]$ are respected.

From Equation 18, at the critical point, (x, ε_c) verifies the equations:

$$x = \frac{q_r}{(1-c)\varepsilon_c^{mx}} \qquad \text{and} \qquad \varepsilon_c = \varepsilon_c^{mx} x$$

We thus have:

$$(x, \varepsilon_c) \in \;]0,1] \times]0, \varepsilon_c^{mx}] \iff \frac{q_r}{(1-c)\varepsilon_c^{mx}} \in \;]0,1] \iff q_r \leq (1-c)\,\varepsilon_c^{mx}$$

If $q_r > (1-c)\,\varepsilon_c^{mx}$ then we can check easily that there is no minimum that verifies the whole constraints on the border of equation $x = 1$ and $\varepsilon_c = \varepsilon_c^{mx}$.

As a conclusion, the minimum exists and satisfies the whole constraints if and only if $q_r \leq (1-c)\,\varepsilon_c^{mx}$. In this case, we have:

$$x = \frac{q_r}{(1-c)\varepsilon_c^{mx}} \qquad \text{and} \qquad \varepsilon_c = \varepsilon_c^{mx} x$$

Appendix 2 - Profitability of capacity investments

In order to specify the long term equilibrium paths, we introduce in our model the profitability of capacity investment (q $q_c(t) = \frac{\dot{Y}(t) - \omega(t)\dot{L}_c(t) - \dot{\omega}(t)L(t)}{(1-\delta)x(t)I(t)}$). This definition means that an investment $I(t)$ corresponds to an equivalent capacity investment of $x(t)I(t)$ and to a capital accumulation of $(1-\delta)\,x(t)\,I(t)$ resulting from this investment and from its depreciation.

From Equations 2 and 6, this capacity investment is linked to an increase in production level ($\dot{Y}(t)$), created jobs ($\dot{L}_c(t)$) and wages rate ($\dot{\omega}(t)$)[11]. The income generated by capacity investments is thus $\dot{Y}(t) - \omega(t)\,\dot{L}_c(t) - \dot{\omega}(t)\,L(t)$.

Lemma 12 *If capital profitability ($q(t)$) is independant of time then $q_c(t) = q(t) + \frac{1-x(t)}{x(t)}\left[q(t) - \frac{1-c(t)}{1-\delta}\left(\varepsilon_c^{mx} - \varepsilon_c(t)\right)\right]$.*

Moreover, if the economy follows an optimal growth path and if required profitability is equal to the average capital profitability ($q_r(t) = q(t)$) then $q_c(t) = q(t) + \frac{\varepsilon_c^{mx}(1-c(t))(1-x(t))[(2-\delta)x(t)-1]}{(1-\delta)x(t)}$.

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[11] In order to simplify, we suppose that costs associated to the increase of wages are due to capacity investments.

Proof. $\frac{\partial q(t)}{\partial t} = 0 \iff q(t) = \frac{\dot{Y}(t) - (\dot{\omega L})(t)}{(1-\delta)I(t)} = \frac{\dot{Y}(t) - \left[\omega(t)\left(\dot{L}_c(t) - \dot{L}_s(t)\right) + \dot{\omega}(t)L(t)\right]}{(1-\delta)I(t)}$. By definition of $q_c(t)$, we thus have:

$q_c(t) = \frac{q(t)(1-\delta)I(t) - \omega(t)\dot{L}_s(t)}{(1-\delta)x(t)I(t)} = \frac{q(t)(1-\delta)I(t) - \omega(t)L(t)(1-x(t))(\varepsilon_c^{mx} - \varepsilon_c(t))i(t)}{(1-\delta)x(t)I(t)}$.

Thus, finally, $q_c(t) = q(t) + \frac{1-x(t)}{x(t)}\left[q(t) - \frac{1-c(t)}{1-\delta}\left(\varepsilon_c^{mx} - \varepsilon_c(t)\right)\right]$.

If required profitability is such that $q_r(t) = q(t)$, then $\varepsilon_c(t) = \frac{q_r(t)}{1-c(t)} = \frac{q(t)}{1-c(t)}$. From the previous equation, we derive the last formula of the theorem. ∎

References

[Aghion and Howitt, 1992] Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60(1):323–351.

[Barro, 1997] Barro, R. J. (1997). *Determinants of Economic Growth. A Cross-Country Empirical Study*. Economica, Paris.

[Barro and Sala-I-Martin, 1997] Barro, R. J. and Sala-I-Martin, X. (1997). Technological diffusion, convergence, and growth. *Journal of Economic Growth*, 2(1):1–27.

[Bonanno and Haworth, 1998] Bonanno, G. and Haworth, B. (1998). Intensity of competition and the choice between product and process innovation. *International Journal of Industrial Organization*, 16(4):495–510.

[Flacher, 2003] Flacher, D. (2003). *Révolutions Industrielles, Croissance et Nouvelles Formes de Consommation*. PhD thesis, Université Paris IX-Dauphine, Paris.

[Grossman and Helpman, 1991] Grossman, G. M. and Helpman, E. (1991). *Innovation and Growth in the Global Economy*. MIT Press, cambridge, MA.

[Lorenzi and Bourlès, 1995] Lorenzi, J.-H. and Bourlès, J. (1995). *Le Choc Du Progrès Technique*. Economica, Paris.

[Lucas, 1988] Lucas, R. E. J. (1988). On the mechanics of development planning. *Journal of Monetary Economics*, 22(1):3–42.

[Rebelo, 1991] Rebelo, S. (1991). Long-run policy analysis and long-run growth. *Journal of Political Economy*, 99(1):500–521.

[Romer, 1986] Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–1037.

[Romer, 1987] Romer, P. M. (1987). Growth based on increasing returns due to specialization. *American Economic Review*, 77(2):56–62.

[Romer, 1990] Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5, part II):S71–S102.

[Rosenkrantz, 1995] Rosenkrantz, S. (1995). *Simultaneous Choice of Process and Product Innovation*. Mimeo, Wissenschaftszentrum, Berlin.

[Solow, 1956] Solow, R. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics*, 70(1):65–94.

[Van Duijn, 1983] Van Duijn, J. J. (1983). *The Long Wave in Economic Growth*. Allen & Unwin, London.

[Villemeur, 2004] Villemeur, A. (2004). *La Divergence Économique Etats-Unis - Europe*. Economica, Paris.