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CAPITAL RESERVE POLICY, REGULATION AND CREDIBILITY IN INSURANCE

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ABSTRACT

The aim of this paper is to analyze the need for capital and default regulation in insurance. Proponents of deregulation argue that these requirements are useless as insurers would hold enough capital as soon as the insured are fully informed about their default probability. Adding to the purpose the relationship between an insurer and her security holders (that is the issuance and dividend policy) we show that the second best capital reserve decided by the security holders is suboptimal whenever the return on cash inside the firm is smaller than outside. Because of limited commitment on recapitalization, disclosure of information may not be enough. Given these characteristics, State commitment to recapitalize could be an alternative regulation policy.

Subject headings: insurance, capital reserve, regulation, recapitalization

1. Introduction

Is capital regulation necessary, and, if so, which kind is the most accurate? In all countries that have insurance markets, regulation of insurance companies exists. The main motivation of such a regulation seems to be the protection of insurance buyers against the risk of insolvency of their insurers. This regulation generally takes the form of "technical" or "mathematical" reserves that insurers should at least carry in sufficiently liquid capital. These regulatory reserves are expressed as ratios of premium income and claims expenses. We want to focus in this paper on the necessity and the economic motivations of such regulation rules.

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The first rationale for capital regulation seems to be the potential asymmetry of information between insurers and policyholders. The buyer of insurance pays a premium against the promise that she will receive a payment if specified random events occur. If the insurer does not hold enough reserves to fulfill this promise, the consumer is being cheated ex ante. This potential failure, itself, undermines the confidence on which the market is based. If the policyholders does not observe this default risk, the insurance market faces a typical "lemons problem" : uncertainty about product quality – here solvency – may drive high-quality firms out of the market. Regulation is then intended to make sure that only "good" firms (with low risk of insolvency) are in the market.

Proponents of deregulation however argue that capital requirement is not an appropriate tool to mitigate this adverse selection problem. As the problem arises because insured are not conveniently informed about the risk of insolvency, "disclosure of information" policy, that is the public provision of information about insurers' risk of insolvency, is sufficient to solve the adverse selection issue. Private incentives are then sufficiently high to induce companies to hold enough liquid capital to optimally reduce the risk of insolvency.

Such a reasoning is however silent on other key actors of insolvency: shareholders or debt owners. Indeed, for a company to become insolvent, not only capital has to be insufficient to meet claims but it also has to be suboptimal to recapitalize (or impossible to issue debt). In the present paper we therefore want to focus on possible other reasons of capital regulation, beside the relation "insurer/insured" evoked above. Another bilateral relationship – between the insurer and her security holders – indeed seems to be of interest. It is now well established that agency problems may arise from the asymmetry of information between managers and shareholders. For example, managers can invest in inefficient projects that generate private benefits for them to the detriment of shareholders. Such an issue would therefore give security holders an incentive not to leave capital in the insurance company. The problem becomes clearer in presence of frictional capital market, if we assume that issuing new debt is costly. An interesting trade-off then arise between agency cost and recapitalization cost.

To study these mechanisms we build a dynamic model of insurance and analyze the dynamic of capital through the behavior of security holders. On the one hand, when the company is solvent, that is when assets are sufficient to meet claims, the security holders can either take dividend or issue new debt (or shares). On the other hand, if claims are too large for the current assets to cover it, the security holders choose whether to recapitalize the insurance company or to default.

When the capital market is frictionless, that is when issuing new debt is costless, the optimal strategy consists in taking dividend as long as it is possible – because of agency cost – and to recapitalize each time it is needed (provided the future value of the company is larger than the invested capital). However, if issuing new debt is costly, it can be optimal to leave some capital in the company. The optimal strategy can be, by the way, related to a well-known policy in inventory management: take dividend above a bottom limit, neither take dividend nor issue new debt if the ex-post (capital) stock is positive but below the limit and issue new debt in order to meet claims when the current reserve is insufficient. Taking into account the effect of default on policyholders, we show that the first best policy implies – when recapitalization is costless – no default (shareholders always recapitalize) but no capital reserve. Although this dynamic is a good candidate for a complete information competitive equilibrium, it appears to be hardly implementable. Indeed, it implies an ex-ante commitment to recapitalize which is not credible. An efficient regulation would then consist in making this commitment credible or at least in guaranteeing that the company would always hold enough capital to continue operating. It therefore appears that State commitment to recapitalize can be a more efficient regulation than capital requirement.

Our work fits in the literature on capital reserve and solvency in insurance. Initiated by Borch (1981) in a model where shareholders can only invest in capital during the first period, this literature has then developed in analyzing the optimal dynamic choice of capital. Munch and Smallwood (1981) and Finsinger and Pauly (1984) for example analyze capital choices in a situation where the demand for insurance is elastic with respect to default risk. Both papers however assume that shareholders cannot recapitalize after claims are realized.

In a more recent paper, Rees, Gravelle and Wambach (1999) study a situation in which policyholders are fully informed of the default probability of their insurer. They show that, whereas an unconstrained insurer will optimally choose a corner solution (either zero or maximum), once the insured is informed about the probability of not being indemnified, the insurer's expected value is higher if it holds the maximum amount of capital. They however ignore the possibility of recapitalization when claims exceed assets. They indeed assume that contracts are not fully honored in these cases. Under this assumption insurers can commit on a default probability through their capital reserve. Being informed of the amount of capital their insurer holds, individuals can infer the probability of not being paid. Competition in insurance market then lead the companies to raise the maximum amount of capital. However, as we introduce recapitalization – that is the possibility to reinject capital when claims exceed assets – this mechanism no longer holds. Insurers then cannot commit on a default probability as they cannot commit on the behavior of their security holders. This creates a motive for an internal solution for capital reserve and therefore a room for capital regulation (minimal capital requirement or State guarantee) if this solution is suboptimal.

Blazenko, Parker and Pavlov (2007, 2008) analyze the concept of ”economic ruin” by modeling a situation where new share can be issued in case of capital deficit. They however assume an exogenous dividend policy in the sense that a fixed return (the risk-free interest rate) is paid to shareholders whenever capital is positive and that insurer can continue operating with negative capital (debt). We however want to focus here on optimal (and therefore endogenous) issuance and dividend policy and we assume that shareholders has to recapitalize a company with negative capital if they want the company not to default. Finally, we want to focus on a different regulation scheme than Blazenko, Parker and Pavlov (2007, 2008). They indeed consider a regulation that requires an immediate capital contribution to offset a capital deficit when we model regulation as capital requirement or State guarantee.

The sequel of the paper is organized as follows. In the next section (Section 2) we present the model of dynamic insurance and its implication on the optimal issuance and dividend policy under different setting (private/public optimum, costly/costless capital). Such an approach allows us to capture the need for capital regulation and to analyze in Section 3 the efficiency of two forms of regulation: capital requirement and State guarantee. Conclusions and directions for future research are eventually provided in Section 4.

2. The model

An insurance company has a portfolio of n policyholders. Each policyholder incurs a loss $\widetilde{x}_i$. We suppose that the random variables $\widetilde{x}_i$ are IID and denote by $\widetilde{x}$ the total claim. We note f the distribution of claims, F its CDF and e its expectation.

Policyholders are completely insured. They value wealth through a von Neumann, increasing and strictly concave, utility function u.

We consider a discrete dynamic model with infinite horizon. The company holds a capital $K \geq 0$ so that total assets at the beginning of the first period amounts to $n\pi + K$ where π is the premium payed by insured.

For sake of simplicity, we will assume that claims occur at the beginning of the period. Two cases are possible.

- In the first case, the total claim $\widetilde{x}$ is lower than total assets $A = (n\pi + K)$. Shareholders of the company can the recapitalize or take dividends for the following period. Let $k(x)$ the amount of recapitalization (if negative , $-k(x)$ is the amount of dividends). For the following period, the insurance company begins with a new cash reserve that amounts to : $\rho(A - x + k(x))$, where ρ is the return on the cash left in the company.

- In the second case the total claim $\widetilde{x}$ is larger than $A = (n\pi + K)$ and the company is potentially insolvent. The security holders can either refuse to keep on operating —in that case insured are not fully indemnified and A is simply equally shared between them—or subscribe to an issue of new securities (shares or debt) to meet the claims. We suppose that there is a potential cost of issuance : 1 dollar of fresh capital in the company costs $\gamma \geq 1$ to the security holders. This can be, for example, explained by transaction costs (see Gomes 2001 for a justification and an evaluation of these costs). Notice that there is no possibility of negative balance sheet. If $\widetilde{x}$ is larger than A the company must either stop or issue new shares or debt.

In the paper we describe the optimal policy of the insurer in alternative frameworks (private behavior, perfect information framework, and credible policy). We suppose that shareholders discount future with a discount rate δ. We make the assumption that $\delta\rho < 1$. Inside cash has a lower return ρ than outside opportunities δ^{-1}.There is no direct incentive to leave cash in the company since it has a better return outside. For instance, firms' managers may commit inside cash to inefficient projects that generate only private benefits for them to the detriment of outside security holders (see La Porta et al. 2000 for an overview of these problems).

The trade off between issuance and agency costs can either lead to solutions where capital reserves are optimally positive (if issuance costs are large) or to situations where no cash optimally remain in the insurance company.

2.1. Security holder's private policy

2.1.1. The frictionless case

In a frictionless world, fresh cash has no opportunity cost : $\gamma = 1$. We claim that, because of agency costs, the optimal policy is to maintain liquidity to zero and stop the activity when losses are too large. To show that, we are considering the optimal intertemporal decision of shareholders of a company starting with an initial capital K. In this part we consider that π is fixed. Notice that, in this section, we do not allow policyholders to leave their company.

Let $V^*(K, \pi)$ the intertemporal optimal value of the company.

Using the Bellman principle, we have :

$$V^*(K, \pi) = \max_{I, k(x) \geq x - A} \int_I \left[\delta V^*(\rho\left(A - x + k(x)\right), \pi) - k(x)\right] f(x) dx, \qquad (1)$$

where I is the range of the values of x for which security holders decide to keep on operating.

A simple change of variables gives :

$$V^*(K,\pi) = \max_{I,H(x)\geq 0} \int_I \left[\delta V^*\left(H(x),\pi\right) - \frac{1}{\rho}H(x) + (K+n\pi) - x\right] f(x)dx \qquad (2)$$

In the above equation $H(x)$ is the reserve decided for the next period by the security holders. As we do not allow the company to operate with negative cash reserve (short term debt), $H(x)$has to be positive.

Setting then $\arg\max_{H\geq 0}(\delta V^*(H,\pi) - \frac{1}{\rho}H) = H^*$ and $\max_{H\geq 0}(\delta V^*(H,\pi) - \frac{1}{\rho}H) = \delta W^*$, the equation 2 becomes :

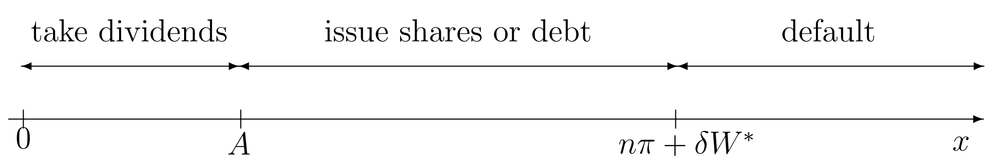
$$V^*(K,\pi) = \max_I \left(\int_I \left(\delta W^* + K + n\pi - x\right) f(x)dx\right) \qquad (3)$$

By the envelop theorem we have, noticing I^* as the optimal range of operating, $V_K^{*'}(K,\pi) = \mu(I^*)$ (where μ is the measure associated with F). This implies, as $\delta\rho < 1$,that $\delta V^*(H,\pi) - \frac{1}{\rho}H$ is a strictly decreasing function of H, which in turn implies that $H^* = 0$.

It is optimal to leave no cash reserve in the company. Even if $\delta\rho = 1$,that is even when cash has the same return inside and outside, $H^* = 0$ is an (now possibly non unique) optimal solution.

We can then deduce the optimal range I^*. It is optimal to keep on operating until the integrand of 3 is positive. We have hence $I^* = (0, b^*(K))$, where $b^*(K) = K + n\pi + .\delta W^*$.is the default threshold.

The optimal policy can therefore be depicted as follows.



If we note $e_K = \mathbb{E}(x/x \leq b^*(K))$, we can write the complete solution :

$$\delta V^*(0,\pi) = \delta W^* = \delta \frac{F(b^*(0))(n\pi - e_0)}{1 - \delta F(b^*(0))}$$

The value of the company holding K and pricing π is hence :

$$V^*(K,\pi) = F(b^*(K)) \left(K + (n\pi - e_K) + \frac{\delta F(b^*(0))(n\pi - e_0)}{1 - \delta F(b^*(0))} \right) \quad (4)$$

with $e_K, b^*(K)$,solutions of :

$$\begin{cases} (1 - \delta F(b^*(0)))\, b^*(0) + \delta F(b^*(0)) e_0 = n\pi \\ F(b^*(K)) e_K = \int_0^{b^*(K)} x f(x) dx \\ b^*(K) = b^*(0) + K \end{cases} \quad (5)$$

On the following picture we represent in the plane $(b, W)$the functions $W = \int_0^b (b - x) f(x) dx$, $b = n\pi + \delta W$ and $W = b - e$. The intersection between the first two curves is the point $b^*(0), W^*$, the intersection between the two lasts gives $W = \frac{n\pi - e}{1-\delta}$ which is the present value of the firm when it is systematically recapitalizes (never default). Obviously this value is smaller than W^*.

[Figure 1 about here.]

As $V_K^{*\prime}(K,\pi) = F(b^*(K))$ and $V_{KK}^{*\prime\prime}(K,\pi) = f(b^*(K)) \geq 0$, The value function is increasing and convex. The asymptote for large K is $K + (n\pi - e) + \delta V^*(0,\pi) = K + (n\pi - e) + \dfrac{\delta F(b^*(0))(n\pi - e_0)}{1 - \delta F(b^*(0))}$. The following picture gives the shape of V as a function of K.

[Figure 2 about here.]

The optimal strategy is the following : at the first period, shareholders take away all the money left if any, or reinject enough capital to meet claims. Then the liquidity is maintained to zero except when the total claim exceeds $\dfrac{\delta F(b^*(0))(n\pi - e_0)}{1-\delta F(b^*(0))} + (K + n\pi)$. This value is exactly the present value of the firm computed with the "modified" discount factor $\delta F(\overline{x}^*(0))$ that takes into account the probability of default. This simply means that security holders refuse to put more money than the present value of future returns.

It is worth noticing that the value of a firm holding K and pricing π, $V^*(K, \pi)$ is larger than the one obtained for a company that would adopt the non optimal strategy $I = [0, +\infty)$, for which the value is simply equal to the NPV : $K + \dfrac{n\pi - e}{1-\delta}$.

With this optimal strategy from the point of view of security holders, the level of utility achieved by the consumer as long as he stays in the company is :

$$B(K, \pi) = (u(w-\pi) + \delta B(0, \pi))\, F(b^*(K)) + \int_{b^*(K)}^{+\infty} u\left(w + \frac{K-x}{n}\right) f(x)dx$$

The above result must be contrasted with the ones obtained by Rees, Gravelle and Wambach (1999) In their model default is exogenous : if assets A are at least enough to meet claims, then the insurer remains in business and receives a continuation value V that is the expected present value of being in the insurance business at the end of the first period. If claims costs turn out to be greater than assets,the insurer pays out his assets and defaults on the remaining claims, losing the right to the continuation value. In this framework, they find that it can be optimal, for the insurer, to put enough initial capital to avoid default, provided that insurance claims distributions belong to the class of "increasing failure rate" distributions on a bounded support. This result seems to be questionable since, in particular, there is no reason to assume that cash or assets must be put ex-ante and that ex-post recapitalization is impossible. This feature can, by the way, lead to accumulate ex ante a huge (and potentially infinite) level of capital up to the maximal value of total claims. It is as if there was an infinite cost of recapitalization and a zero cost of initial capital.

We obtain here a more nuanced result : default is endogenous and optimally decided when new capital needed is too large compared with the expected returns. This leads to a policy where permanent capital is useless. The only reason for permanent cash to be useful would be the case where recapitalization is costly.

In the following section we will introduce a (finite) cost of issuing new debt or shares.

2.1.2. Opportunity cost of recapitalization

In this second part we assume that capital market imperfections make issues of new shares (or new debt) costly : $\gamma > 1$. In particular, when cash reserve becomes negative security holders have to choose between issuing new shares and to stop operating. The important consequence is that this cost creates an ex ante incentive to some precautionary policy which takes the form of capital reserves. Intuitively, when γ is low, reserves can be maintained to zero . But when γ becomes larger, it turns to be optimal to hold some permanent strictly positive capital.

Suppose that shareholders can recapitalize when needed, and inject $k(x)$ in the firm at a cost $\gamma > 1$. When k is negative, that is when shareholders take dividends, there is no opportunity cost.

The optimal value of the firm holding $K$and pricing π becomes :

$$V(K,\pi) = \max_{k(x)\geq x-A,\ I}\ \min_{\alpha(x)\in\{1,\gamma\}} \int_I \left[\delta V(\rho\,(A-x+k(x)),\pi) - \alpha(x)k(x)\right] f(x)dx, \tag{6}$$

In the above equation, $\alpha(x)$ is optimally fixed to γ for x such that $k(x) \geq 0$ and to 1 elsewhere.

Setting, as previously, $H(x) = \rho\,(A - x + k(x))$ this equation becomes :

$$V(K,\pi) = \max_{H(x)\geq 0, I}\ \min_{\alpha(x)\in\{1,\gamma\}} \int_I \left[\delta V(H(x),\pi) - \alpha(x)\frac{1}{\rho}H(x) + \alpha(x)\,(K + n\pi - x)\right] f(x)dx \tag{7}$$

Therefore $\alpha^*(x) = 1$ if $H(x) \leq (A-x)$and $\alpha^*(x) = \gamma$ if $H(x) \geq (A-x)$.

Let set $H^* = \arg\max_{H\geq 0}\left(\delta V(H,\pi) - \frac{1}{\rho}H\right)$ and $H^*_\gamma = \arg\max_{H\geq 0}\delta V(H,\pi) - \frac{\gamma}{\rho}H$.

The two levels of capital H^* and H^*_γ have the following meanings : the optimal strategy consists in taking all cash reserves above H^* if any, and to recapitalize up to H^*_γ when needed.

As previously, set : $\max\left(\delta V(H,\pi) - \frac{1}{\rho}H\right) = \delta W^*$ and $\max\left(\delta V(H,\pi) - \frac{\gamma}{\rho}H\right) = \delta W^*_\gamma$.

Lemma 1 *:* $0 \leq H^*_\gamma \leq H^*$ *and* $W^*_\gamma \leq W^*$

Proof. see appendix ■

It is now easy to state the following proposition which gives the optimal strategy and the value of the firm.

Proposition 1 *The optimal cash policy is given by two optimal thresholds* $a^*(K) \leq A \leq b^*(K)$ *such that :*

if $x \leq a^*(K)$, $H(x) = H^*$ *: take money above* H^*

if $a^*(K) \leq x \leq A, H(x) = \rho(A - x)$ *: neither take money nor issue shares*

if $A \leq x \leq b^*(K)$, $H(x) = H^*_\gamma = 0$ *: issue shares up to* $H^*_\gamma = 0$

if $x \geq b^*(K)$ *the company defaults.*

where $a^*(K)$, $b^*(K)$, *and* $V(K, \pi)$, *are solutions of :*

$$\begin{aligned} V(K,\pi) &= \max_{a \leq A \leq b} \int_{-\infty}^{a} \left[\delta W^* + (K + n\pi - x)\right] f(x)dx \\ &\quad + \int_{a}^{A} \delta V(\rho(K + n\pi - x), \pi) f(x)dx \\ &\quad + \int_{A}^{b} \left[\delta V(0,\pi) + \gamma (K + n\pi - x)\right] f(x)dx \end{aligned}$$

Proof. see appendix ■

Here, when issuing debt or shares is necessary, the security holders put just enough to meet claims : $H^*_\gamma = 0$. When the profit is large, security holders take away dividends and leave some "precautionary reserve" $H^* \geq 0$. Intuitively, this capital reserve is larger as γ is large. Conversely, when γ is sufficiently low, this level can be maintained to 0. Indeed, the derivative of V w.r.t. K for $K = 0$ can be computed (thanks to the envelop theorem) :

$$\begin{aligned} V'_K(0,\pi) &= F(a^*(0)) + \int_{a^*(0)}^{n\pi} \delta\rho V'(\rho(n\pi - x), \pi) f(x)dx \\ &\quad + \gamma \left(F(b^*(0)) - F(n\pi)\right) \end{aligned}$$

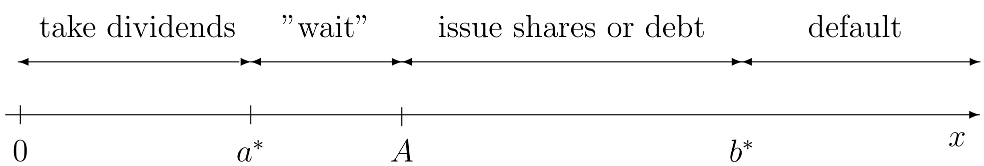
Suppose now that the optimal value is $H^* = 0$. This implies that $a^*(0) = n\pi$ and $b^*(0) = B$ is defined by :

$$\begin{cases} V(0,\pi)\left[1-\delta F(B)\right] = \int_{-\infty}^{B}\left[(n\pi - x)\right] f(x)dx + (\gamma - 1)\int_{n\pi}^{B}\left[(n\pi - x)\right] f(x)dx \\ \gamma B = \delta V(0,\pi) + n\pi + (\gamma - 1)\, n\pi \end{cases}$$

The optimal reserve will be $H^* > 0$,if the previous expression of $V_K'(0,\pi)$ in which we take $a^*(0) = n\pi$ is greater than $\frac{1}{\delta\rho}$. That is :

$$F(n\pi) + \gamma\left(F(B) - F(n\pi)\right) > \frac{1}{\delta\rho}$$

In this case the optimal policy can be depicted as follows.



This issuance and dividend policy can be related to the one defined in Décamps and al. (2008) on banking market. The main difference is that, in their continuous framework, default never optimally occurs.

When γ is large enough the value function as a function of K has a concave part for small values of K.

Another interesting question concerns the variation of the default threshold $b^*(K)$when γ increases. We know that :

$b^*(K) = \frac{\delta}{\gamma}V(0,\pi) + K + n\pi$. It is easy to show that $V(0,\pi)$ is decreasing with γ,so that we have the following proposition :

Proposition 2 *When the opportunity cost γ increases, both the precautionary reserve H^* and the probability of default increase.*

Proof. see appendix ■

A simple consequence of this proposition is that the level of permanent capital is not a signal of better solvency risk of the insurer. The precautionary reserve is not aimed at diminishing the risk of insolvency but at diminishing the cost of recapitalization.

2.2. Optimal First Best reserve policy

In the previous section, when deciding the reserve policy, shareholders don't take into account the impact of this policy on the welfare of policyholders. A natural question would then be to find the optimal policy, (premium, dividends, reserves) under the constraint that the insured achieve a given per-period level of expected utility $\underline{u}$. As a consequence, the premium is now endogenous. This provides the company a new instrument and introduces a trade off between capital reserve and premiums as in Bourlès (2007). In a perfect information framework, this situation (optimal policy under some reservation level for the insureds), can be considered as a good candidate for a competitive equilibrium. We examine here the case $\gamma = 1$.

The problem now is hence the following :

$$\begin{aligned} V^{FB}(K, \underline{u}) &= \max_{I, k(.), \pi} \int_I - k(x) f(x) dx + \int_I \delta V^{FB}(\rho\,(A - x + k(x))) f(x) dx \\ \text{s.c.} &\int_I u(w - \pi) f(x) dx + \int_{I^C} u\left(w + \frac{K - x}{n}\right) f(x) dx \geq \underline{u} \end{aligned} \tag{8}$$

That is :

$$\begin{aligned} V^{FB}(K, \underline{u}) = \max_{I, H(.) \geq 0, \pi} &\int_I (K + n\pi - x)\, f(x) dx + \int_I \left(\delta V^{FB}(H(x), u_0) - \frac{1}{\rho} H(x)\right) f(x) dx \\ \text{s.c.} &\int_I u(w - \pi) f(x) dx + \int_{I^C} u\left(w + \frac{K - x}{n}\right) f(x) dx \geq \underline{u} \end{aligned} \tag{9}$$

As previously set $H^{FB} = \arg\max \delta V^{FB}(H, \underline{u}) - \frac{1}{\rho} H$ and $\delta W^{FB} = \max \delta V^{FB}(H, u_0) - \frac{1}{\rho} H$

And then :

$$\begin{aligned} V^{FB}(K, \underline{u}) &= \max_{I, \pi} \int_I (K + n\pi - x)\, f(x) dx + \int_I \delta W^{FB} f(x) dx \\ \text{s.c.} &\int_I u(w - \pi) f(x) dx + \int_{I^C} u\left(w + \frac{K - x}{n}\right) f(x) dx \geq \underline{u} \end{aligned} \tag{10}$$

Let λ be the lagrange multiplier associated with the constraint. Optimization with respect to π gives :

$$(n - \lambda u'(w - \pi))\, \mu(I) = 0$$

Set $I = (0, \overline{b}]$, when total loss is larger than $\overline{b}$,then the security holders provoke default. The derivative of the Lagrangian w.r.t.$\overline{b}$ is :

$$L(\overline{b}) \equiv \left(K + n\pi - \overline{b}\right) + \delta W^{FB} + \lambda \left(u(w-\pi) - u\left(w + \frac{K - \overline{b}}{n}\right)\right) \qquad (11)$$

It is easy to see that $L(\overline{b})$ is a convex function with a minimum for $\overline{b}$ such that $\frac{\lambda}{n}u'\left(w + \frac{K-\bar{b}}{n}\right) = 1$, that is precisely $\overline{b} = K + n\pi$. This minimum is exactly δW^{FB}. This value being essentially positive we can conclude that $L(\overline{b})$is always positive, and hence :

$$\begin{aligned} V^{FB}(K, u_0) &= K + n\underline{\pi} - e + \delta W^{FB} \\ \text{with} &: u(w - \underline{\pi}) = \underline{u} \end{aligned} \qquad (12)$$

As $\delta\rho < 1$, $\arg\max_{H \geq 0} \delta V^{FB}(H, u_0) - \frac{1}{\rho}H = 0$: the optimal policy is again to maintain cash reserves to zero, and hence :

$$\begin{aligned} V^{FB}(K, u_0) &= K + \frac{(n\underline{\pi} - e)}{1-\delta} \\ \text{with} &: u(w - \underline{\pi}) = \underline{u} \end{aligned}$$

As $\dfrac{F(b^*(0))(n\underline{\pi} - e_0)}{1 - \delta F(b^*(0))} \geq \dfrac{(n\underline{\pi} - e)}{1-\delta}$, the next proposition holds.

Proposition 3 *The First Best policy implies recapitalization in situations where it is not the (private) optimal behavior for security holders. Therefore, an insurer has a short term incentive to deviate from this First Best policy by provoking default. This makes the First Best allocation unstable even if the policyholders perfectly observe the level of reserve capital.*

2.3. Credibility : second best policy

The optimal First Best policy cannot be non cooperatively implemented since there is no way for the shareholders to commit to recapitalize when losses are larger than the expected present value of future income. An interesting question would be to find a second best credible policy. In this second best world, capital allows to commit to some default probability.

Endogenously, shareholders provoke default when the present value of future income is less than the claims, that is when x is larger than $K+n\pi+\delta W^{SB}$, where W^{SB} is the optimal second best value of the firm.

We hence have :

$$\begin{aligned} V^{SB}(K,\underline{u}) &= \int_0^{K+n\pi+\delta W^{SB}} \left(K+n\pi+\delta W^{SB}-x\right) f(x)dx \quad (13)\\ &\text{with } \pi \text{ the largest value of the premium } p \text{ such that,}\\ U(K,p) &= \int_0^{K+np+\delta W^{SB}} u(w-p)f(x)dx \\ &+ \int_{K+np+\delta W^{SB}}^{+\infty} u\left(w+\frac{K-x}{n}\right) f(x)dx \geq \underline{u} \quad (14)\\ \text{with} &: \delta W^{SB} = \max \delta V^{SB}(H,\underline{u}) - \frac{1}{\rho}H \quad (15) \end{aligned}$$

The difference with the First Best situation is that I is constrained to be the interval $(0, K+n\pi+\delta W^{SB}]$.This is due to the fact that the decision not to default must be ex ante credible.

Intuitively increasing K allows to decrease the probability of default which in turn increases the value of the contract for the policyholders and hence allows an increase of $n\pi$. A natural question is to compare the return of this "investment" with its cost.

As π is implicitly defined by 14, we have :.

$$n\frac{\partial \pi}{\partial K} = -n\frac{U_K}{U_\pi} = \frac{\int_b^{+\infty} u'\left(w+\frac{K-x}{n}\right) f(x)dx + n\left[u(w-\pi) - u\left(w-\pi-\frac{\delta W^{SB}}{n}\right)\right] f(b)}{u'(w-\pi)F(b) - n\left[u(w-\pi) - u\left(w-\pi-\frac{\delta W^{SB}}{n}\right)\right] f(b)},$$

with $b = K+n\pi+\delta W^{SB}$.

As π is the largest value providing the level $\underline{u}$, U_π is necessarily negative, and then $-n\frac{U_K}{U_\pi} \geq 0$.

We Have hence :

$$n\frac{\partial \pi}{\partial K} \geq \frac{(1-F(b))}{F(b)} \frac{\mathrm{E}\left[u'\left(w+\frac{K-x}{n}\right)/x \geq b\right]}{u'(w-\pi)}.$$

As u is strictly concave and $w + \frac{K-x}{n} \leq w - \pi$ when $x \geq b$, we have :

$$n\frac{\partial \pi}{\partial K} \geq \frac{(1-F(b))}{F(b)}.$$

Now differentiating 13 gives :

$$\frac{\partial V^{SB}}{\partial K}(K, \underline{u}) = F(b) + F(b)n\frac{\partial \pi}{\partial K} \geq 1.$$

This investment is profitable if this derivative is greater than $\frac{1}{\rho\delta}$.

Proposition 4 *Supposing* W_0*,* b_0 *and* π_0 *are solutions of*

$$\begin{cases} W = \int_0^b (b-x)f(x)dx \\ b = n\pi + \delta W \\ u(w-\pi)F(b) + \int_b^{+\infty} u\left(w - \frac{x}{n}\right) f(x)dx = \underline{u} \end{cases} \quad (16)$$

If

$$F(b_0) + (1-F(b_0))\frac{E\left[u'\left(w - \frac{x}{n}\right)/x \geq b_0\right]}{u'(w - \frac{b_0}{n})} > \frac{1}{\delta\rho} \quad (17)$$

it is optimal to hold positive permanent capital $H^{SB} > 0$

Proof. Easily proven by supposing that $H^{SB} = 0$ and minoring $\frac{\partial V^{SB}}{\partial K}(0, \underline{u})$by $F(b_0) + (1 - F(b_0))\frac{\mathrm{E}\left[u'\left(w - \frac{x}{n}\right)/x \geq b_0\right]}{u'(w - \frac{b_0}{n})}$. ■

On the following picture we represent in the plane (b, W), the curves $W = \displaystyle\int_0^b (b-x)f(x)dx$ and $u\left(w - \dfrac{b}{n} + \dfrac{\delta W}{n}\right) F(b) + \displaystyle\int_b^{+\infty} u\left(w - \dfrac{x}{n}\right) f(x)dx = \underline{u}$. The latter is an increasing curve with a slope smaller than $\dfrac{1}{\delta}$, $\left(= \dfrac{1}{\delta} \text{ for } W = 0\right)$ and an asymptote $b = n\underline{\pi} + \delta W$. The intersection of these two curves give the solution W_0, b_0,and π_0 if $H^{SB} = 0$. Obviously, $W_0 < W^{FB} \leq W^*$ meaning that the credible proof value of the company is lower than under the first best policy.

[Figure 3 about here.]

When the condition of the previous proposition is fulfilled, $V(K, \underline{u})$ is a function of K with a slope greater than $\frac{1}{\delta\rho}$ for $K = 0$, and a slope exactly equal to $\frac{1}{\delta\rho}$ for $K = H^{SB}$. For K going to infinity, the slope tends to 1as π tends to $\underline{\pi}$ (and hence $\dfrac{\partial\pi}{\partial K}$tends to 0). The asymptote being $K + (n\underline{\pi} - e) + \delta W^{SB}$, the optimal credible capital reserve can be represented as follows.

[Figure 4 about here.]

3. Regulation

In this section, we intuitively analyze the possible impact of two possible regulation schemes: capital requirement (that is the minimal amount of capital insurers have to hold to continue operating) and State guarantee to recapitalize. As the lack of credibility on recapitalization has been show to be the main issue in reserve policy, the second option may be preferable.

3.1. The impact of minimal capital requirement

Let us first examine the possible implications of capital requirement on optimal capital reserve. Such a regulation rule (chosen in most countries) constraints the insurance companies to hold a minimal amount of capital $\underline{K}$. In our setting, this implies that recapitalization is needed as soon as claims exceed assets *minus this ceiling* $\underline{K}$ (and therefore, more often than without regulation, for a given amount of initial capital). Moreover – under this scheme – when new shares (or debt) have to be issued, security holders need to build up capital reserve up to the required minimum (and no longer up to zero). As, the amount of capital shareholders are willing to inject are still bounded by the present value of future returns these two mechanisms that (i) increases the need for recapitalization and (ii) potentially reduce the value of company, may lead to a perverse effect of capital requirement through an increase in the probability of default. Lastly, it appears that capital requirement may reduce the potential amount of "precautionary reserve" (by imposing early recapitalization) that could – as shown in Proposition 2 – reduce the cost of recapitalization.

The exact implication of reserve requirement (in particular on the value of capital) however remains to be investigated and calls for further research. The analyze of the precise constrained program would for example allow us to discuss more precisely the impact of capital requirement on the value of the firm and to provide some interesting comparative statics on $\underline{K}$. Our analysis moreover seems to call for the study of an alternative policy that would consist in fixing a minimum amount of capital above which security holders are prevented to take dividend but do not constrain them to recapitalize above zero (when claims exceed assets). Such a policy would create precautionary reserves without increasing the need for recapitalization.

3.2. A solution to the credibility issue: State guarantee to recapitalize

The questionable efficiency of capital requirement and the fact that the first best policy cannot be implemented because of a credibility issue lead us to consider an alternative form of regulation: State guarantee to inject capital. We have shown in this paper that default occurs when security holders are reluctant to inject enough capital for the company to keep on operating. This issue will therefore be solved if the State commits – in these cases – to buy enough shares for the capital reserve not to be negative.

This however creates a typical moral hazard issue as it will then be easy for the shareholders to cheat on there capacity/willingness to add capital. They we then benefit from the capital injected by the government without bearing the costs. This issue however disappears if we assume that government can infer the value of the company and can be eased by assuming that the State conditions its intervention to a takeover of the company. Such a nationalization would lead to a null value of the company (from the point of view of shareholders). Therefore the call for State capital would only be optimal if the amount of needed capital exceed the value of the company.

4. Conclusion

We highlight in the paper to role of the relationship between an insurer and its security holders in the need for capital and default regulation. We show that beside the informational issue between an insurer and its policyholders, regulation can be needed to solve an issue of credibility on recapitalization. It indeed appears in our work that the first best policy is not credible as it induces recapitalization in situation where shareholders have no incentive to inject capital.

Contrarily to existing literature we moreover show that an interior solution for capital reserve can be optimally chosen if recapitalization is costly. When the capital market is frictionless, that is when issuing new debt is costless, the optimal strategy consists in taking dividend as long as it is possible – because of agency cost – and to recapitalize each time it is needed (provided the future value of the company is larger than the invested capital). However, if issuing new debt is costly, it can be optimal to leave some capital in the company. The optimal strategy then consists in (a) taking dividend above a bottom limit, (b) neither take dividend nor issue new debt if the ex-post capital reserve is positive but below the limit and (c) issue new debt in order to meet claims when the current reserve is insufficient.

Taking into account the effect of default on policyholders, we show that the first best policy implies no default but no capital reserve. This policy however appears to be hardly implementable as it implies an ex-ante commitment to recapitalize which is not credible. An efficient regulation would therefore consist in making this commitment credible and therefore State commitment to recapitalize may be a more efficient regulation than capital requirement.

Is left for future research to analyze more precisely the optimal regulation. It would for example be interesting to evaluate optimal capital policy under capital requirement that is if capital reserve are constrained to be above a given level. We would then be able to define more exactly the efficient regulation. An other extension of interest would consist in studying the value of a share (and not of the company). Such a variant of our model may create an incentive for positive reserve (even with costless capital) as recapitalization – that is the issuance of new shares – would reduce the value of an existing share.

5. Appendix

5.1. Proof of Lemma 1

We have

$$\begin{cases} \delta W^* = \delta V(H^*,\pi) - \frac{1}{\rho}H^* \geq \delta V(H^*_\gamma,\pi) - \frac{1}{\rho}H^*_\gamma \\ \delta W^*_\gamma = \delta V(H^*_\gamma,\pi) - \frac{\gamma}{\rho}H^*_\gamma \geq .\delta V(H^*,\pi) - \frac{\gamma}{\rho}H^* \end{cases}$$

Summing up these two inequalities gives :

$$\frac{\gamma-1}{\rho}H^* \geq \frac{\gamma-1}{\rho}H^*_\gamma$$

Moreover $S(\gamma) = \max_{H\geq 0} \delta V(H,\pi) - \frac{\gamma}{\rho}H$ is the supremum of decreasing affine (and hence convex) functions. It is hence a convex decreasing function, such that $S'(\gamma) = \frac{-H^*_\gamma}{\rho}$ (almost everywhere).

5.2. Proof of Proposition 1

We have :

$$V(K,\pi) = \max_{H(x)\geq 0, I} \int_I \left[\delta V(H(x),\pi) - \alpha(x)\frac{1}{\rho}H(x) + \alpha(x)(A-x)\right] f(x)dx \tag{18}$$

With $\alpha(x) = 1$ if $H(x) < \rho(A-x)$ and $\alpha(x) = \gamma$ if $H(x) > \rho(A-x)$

take $H^*(x)$the optimal policy and define :

$I_1 = \{x, H^*(x) < \rho(A-x)\}$, $I_2 = \{x, H^*(x) = \rho(A-x)\}$, $I_3 = \{x, H^*(x) > \rho(A-x)\}$

We have :

$$\begin{aligned} V(K,\pi) &= \int_{I_1} \left[\delta V(H^*(x),\pi) - \frac{1}{\rho}H^*(x) + (A-x)\right] f(x)dx \qquad (19) \\ &+ \int_{I_2} \delta V(H^*(x),\pi) f(x)dx \qquad (20) \\ &+ \int_{I_1} \left[\delta V(H^*(x),\pi) - \frac{\gamma}{\rho}H^*(x) + \gamma(A-x)\right] f(x)dx \qquad (21) \end{aligned}$$

Clearly in I_1, $H^*(x) = \min(\rho\,(A-x)\,, H^*)$, and in I_3, $H^*(x) = \max(\rho\,(A-x)\,, H^*_\gamma)$.

This in turn implies that :

$$
\begin{aligned}
V(K,\pi) &= \max_{a\leq b\leq A\leq \overline{x}} \int_{-\infty}^{a} \left[\delta W^* + (A-x)\right] f(x)dx && (22)\\
&+ \int_a^b \delta V(\rho\,(A-x)\,,\pi) f(x)dx && (23)\\
&+ \int_b^{\overline{x}} \left[\delta W^*_\gamma + \gamma\,(A-x)\right] f(x)dx && (24)
\end{aligned}
$$

In order to find H^* and H^*_γ it is helpful to compute the first derivative of V Thanks to the envelop theorem we have :

$$V'_K(K,\pi) = \int_{I^*} \alpha^*(x) f(x)dx$$

Where we define $\alpha^*(x)$ by :

$$
\begin{aligned}
\alpha^*(x) &= 1 \text{ if } x \leq a^*\\
\alpha^*(x) &= \gamma \text{ if } x \geq b^*\\
\alpha^*(x) &= \delta\rho V'_K(\rho\,(A-x)\,,\pi) \text{ if } a \leq x \leq b
\end{aligned}
$$

This implies that $V'_K(K,\pi)$ is smaller than γ,and then that $\delta V'_K(K,\pi)$is smaller than $\delta\gamma$.which is smaller than $\frac{\gamma}{\rho}$ This means that $H^*_\gamma = 0$. and hence that $b^* = A$ and $W^*_\gamma = V(0,\pi)$ Under the optimal cash policy the company issues new shares only in case of negative cash, that is only when claims are greater than reserves A.

Then the set of equations characterizing the solution of the problem are :

$$\begin{aligned}
V(K,\pi) &= \int_{-\infty}^{a^*} \left[\delta W^* + (K+n\pi-x)\right] f(x)dx \\
&+ \int_{a^*}^{A} \delta V(\rho\left(K+n\pi-x\right),\pi) f(x)dx \\
&+ \int_{A}^{b^*} \left[\delta V(0,\pi) + \gamma\left(K+n\pi-x\right)\right] f(x)dx \\
\delta V(\rho\left(K+n\pi-a^*\right),\pi) &= \delta W^* + (K+n\pi-a^*) \\
\delta V(0,\pi) + \gamma\left(K+n\pi-b^*\right) &= 0 \\
W^* &= \max_{H\geq 0}\left(\delta V(H,\pi) - \frac{1}{\rho}H\right)
\end{aligned}$$

5.3. Proof of Proposition 2

We know that :

$$\begin{aligned}
V(K,\pi) &= \max_{a\leq A\leq b} \int_{-\infty}^{a} \left[\delta W^* + (K+n\pi-x)\right] f(x)dx \\
&+ \int_{a}^{A} \delta V(\rho\left(K+n\pi-x\right),\pi) f(x)dx \\
&+ \int_{A}^{b} \left[\delta V(0,\pi) + \gamma\left(K+n\pi-x\right)\right] f(x)dx
\end{aligned}$$

Thanks to the envelop theorem the derivative of V with respect to γ is hence :

$$V'_\gamma(K,\pi) = \int_{A}^{b^*(K)} \left[\delta V'_\gamma(0,\pi) + (K+n\pi-x)\right] f(x)dx.$$

In particular :

$$V'_\gamma(0,\pi) = \int_{A}^{b^*(0)} \left[\delta V'_\gamma(0,\pi) + (n\pi-x)\right] f(x)dx,$$

which gives :

$$V'_\gamma(0,\pi)\left(1-\delta\left(F(b^*(0))-F(A)\right)\right) = \int_{A}^{b^*(0)} (n\pi-x) f(x)dx,$$

which implies $V'_\gamma(0,\pi)\leq 0$

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Fig. 1.— On the suboptimality of ”automatical” recapitalization

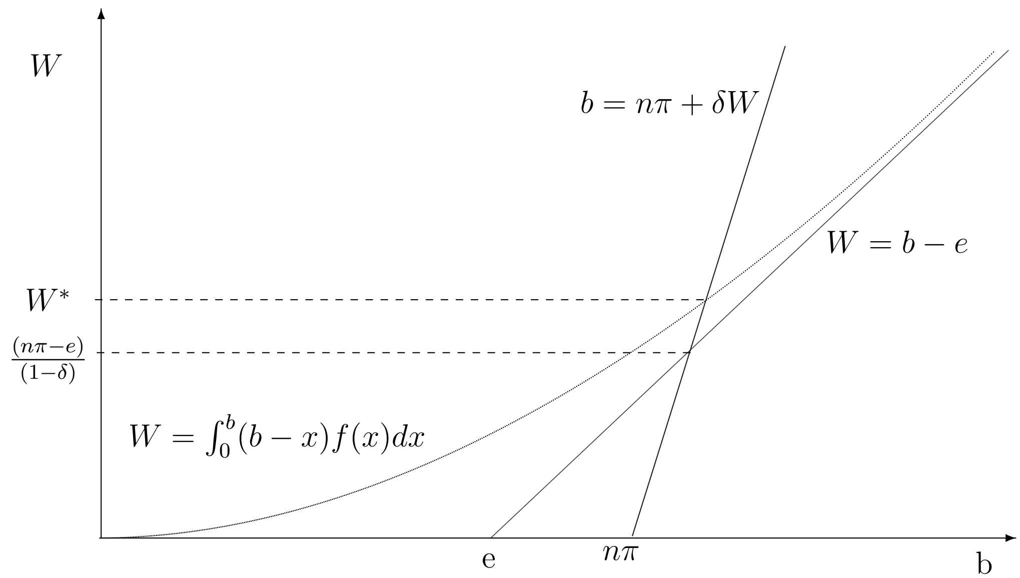

Fig. 2.— The value of a company holding capital K

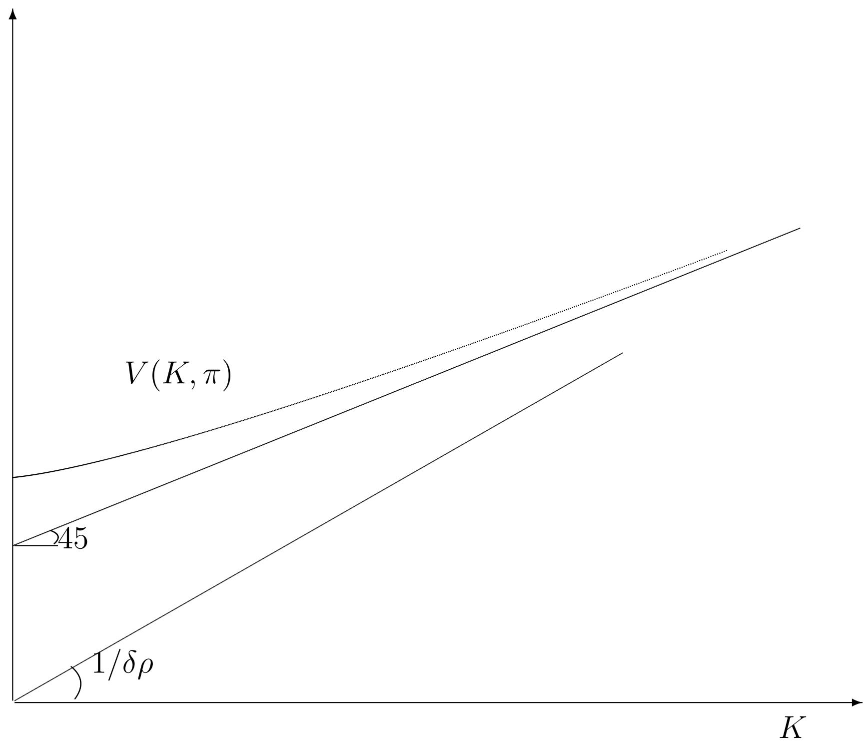

Fig. 3.— The credibility proof value of the firm

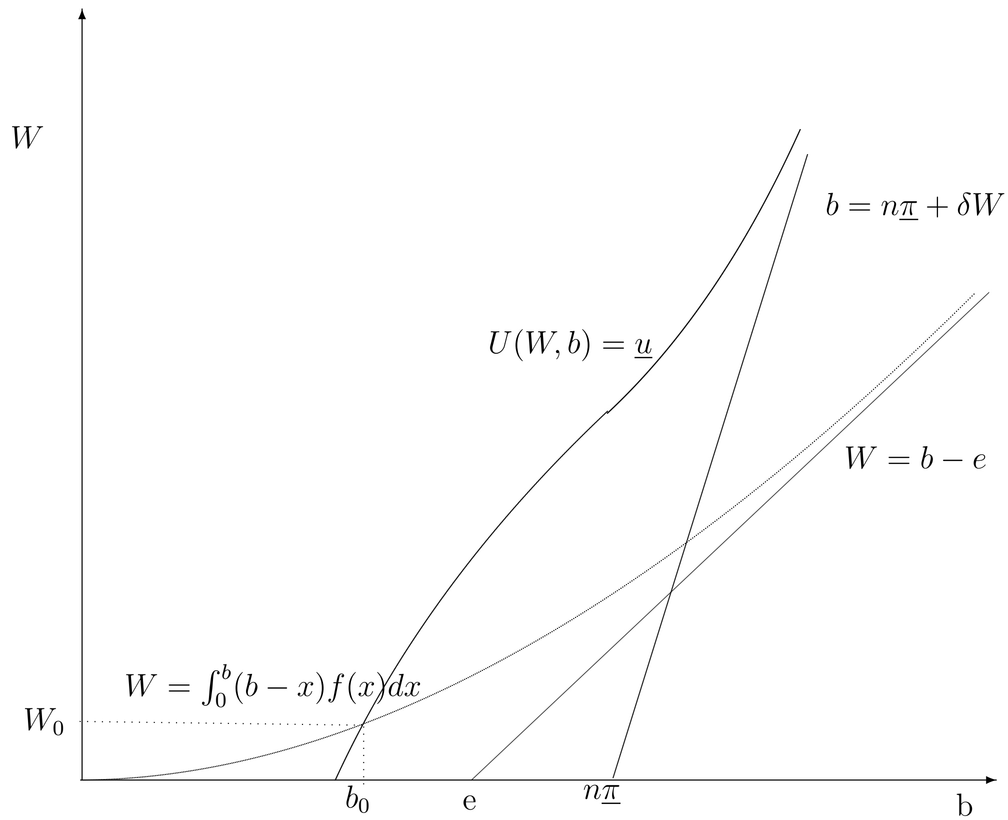

Fig. 4.— The second best capital reserve