GREQAM

Groupement de Recherche en Economie
Quantitative d'Aix-Marseille - UMR-CNRS 6579
Ecole des Hautes Etudes en Sciences Sociales
Universités d'Aix-Marseille II et III

Document de Travail
n°2008-43

A FRACTIONALLY INTEGRATED EXPONENTIAL STAR MODEL APPLIED TO THE US REAL EFFECTIVE EXCHANGE RATE

Mohamed BOUTAHAR
Imène MOOTAMRI
Anne PEGUIN-FEISSOLLE

October 2008

DT-GREQAM

A fractionally integrated exponential STAR model applied to the US real effective exchange rate*

Mohamed Boutahar[†] **Imène Mootamri**[‡]
Anne Péguin-Feissolle[§]

July 07, 2008

Abstract

The aim of this paper is to study the dynamics of the US real effective exchange rate by capturing nonlinearity and long memory features. In this context, we use the family of fractionally integrated STAR (FISTAR) models proposed by van Dijk, Franses and Paap (2002) in the case when the transition function is an exponential function and we develop an estimation procedure. Indeed, these models can take into account processes characterized by several distinct dynamic regimes and persistence phenomena.

Keywords: Fractional integration, Nonlinearity, STAR models, Long memory, Real effective exchange rate, Forecasting.

JEL classification: C22, C51, C52, F31

* *Corresponding author:* Imène MOOTAMRI, GREQAM, Centre de la Charité, 2 rue de la Charité, 13236 Marseille cedex 02, FRANCE, tel: +33.4.91.14.07.70, fax: +33.4.91.90.02.27, Email: imene.mootamri@etumel.univmed.fr

[†]GREQAM, Université de la Méditerranée, France

[‡]GREQAM, Université de la Méditerranée, France

[§]GREQAM, CNRS, France

1 Introduction

Long memory processes have received considerable attention by researchers from very diverse fields. The seminal work of Beran (1995), Doukhan, Oppenheim and Taqqu (2003) and Robinson (2003) overview the recent developments on this topic. The long memory processes are characterized by a long-term dependence and the presence of cycles and level changes. They were detected in economics in many fields, for example in the dynamics of exchange rates or the volatility of financial time series. In addition, we assist for the few latest years to a significant development of nonlinear modelling. For instance, in economics and finance, multiple regimes modelling becomes more and more important in order to take into account phenomena characterized, for instance, by recession or expansion periods, or high or low volatility periods. Consequently, a number of different models have been proposed in the literature to account for this behaviour, among which Markov switching models or smooth transition autoregressive (STAR) models. The nonlinearity property of economic time series can also be justified by the existence of asymmetry in variable's dynamics; for instance, favorable shocks have a more important and persistent effect than the unfavorable shocks. In order to consider these possible nonlinearities, it is necessary to have econometric models able to generate different dynamics according to the cycle phase.

Therefore, this paper belongs to a literature exploring simultaneously these two key properties of economic and financial time series, namely the long-memory and nonlinear properties. Indeed, a line of papers has recently proposed that we can call "nonlinear long-memory" models. For instance, some authors provide a joint evidence of mean reversion over long horizons and nonlinear dynamics on exchange rate markets, by generalizing to the nonlinear framework the Beveridge-Nelson decomposition (see, Clarida and Taylor (2003), Sarno and Taylor (2001)). Others propose new classes of long-memory models. For instance, Franses and Paap (2002), Franses, Van der Leij and Paap (2002) introduce CLEAR (Censored Latent Effects Autoregressive) and Switching CLEAR processes, which show autocorrelation at high lags with an ACF that decays at a faster rate in the beginning in comparison to the ACF of an ARFIMA model.

Along this line of research, the fractionally integrated smooth transition autoregressive (FISTAR) models have also been proposed, that offer another potential application to economic and financial data (see van Dijk et al. (2002), Caporale and Gil-Alana (2007) and Smallwood (2005)). van Dijk et al. (2002) present the modelling cycle for specification of these models, such as testing for nonlinearity, parameter estimation and adequacy tests, in the case where the transition function is the logistic function; they study the dynamics of monthly US unemployment rates. Smallwood (2005) extends these results to the FISTAR model with an exponential transition function, and applies this model to the purchasing power parity puzzle by considering the real exchange rate processes for twenty countries against the United States.

In this paper, we study this class of models because these FISTAR models, indeed, make it possible to generate nonlinearity, since they are defined by several distinct modes in dynamics, and to take into account the persistence phenomenon. We consider the case of an exponential transition function and propose a two-step estimation method: in the first step, we estimate the long memory parameter, then, in the second step, the STAR model parameters via nonlinear least squares estimation.

The remainder of this paper is organized as follows. In Section 2, we present the FISTAR model with an exponential transition function and the two-step estimation procedure; we describe also the out-of-sample forecasting. In Section 3, we analyze the monthly US real effective exchange rate in order to illustrate the various elements of the modelling cycle. Finally, Section 4 concludes.

2 The Econometric Specification

2.1 The model

Let us consider a process y_t that satisfies the following long memory scheme:

$$(1-L)^d y_t = x_t \tag{1}$$

where L is the lag operator, d is the long memory parameter and x_t is a covariance-stationary $I(0)$ process. The parameter d is possibly non-integer, in which case the time series y_t is called fractionally integrated (FI) (see, among others, Granger and Joyeux (1980) and Hosking (1981)). If $-0.5 < d < 0.5$, y_t is covariance stationary and invertible process. For $0 < d < 0.5$, y_t is a stationary long memory process in the sense that autocorrelations are not absolutely summable and decay hyperbolically to zero. Finally, if $0.5 \leq d < 1$, y_t is nonstationary and the shocks do not have permanent effects.

To capture the nonlinear feature of time series, a wide variety of models can be used (see Franses and van Dijk (2000)). In this paper, we consider the fractionally integrated STAR (FISTAR)[1] model introduced by van Dijk

[1] See also Smallwood (2005).

et al. (2002) given by:

$$\begin{cases} (1-L)^d y_t = x_t \\ x_t = \left(\varphi_{10} + \sum_{i=1}^{p} \varphi_{1i} x_{t-i}\right) + \left(\varphi_{20} + \sum_{i=1}^{p} \varphi_{2i} x_{t-i}\right) F\left(s_t, \gamma, c\right) + \varepsilon_t \end{cases} \tag{2}$$

where ε_t is a martingale difference sequence with

$$E\left[\varepsilon_t | \Omega_{t-1}\right] = 0$$

and

$$E\left[\varepsilon_t^2 \middle| \Omega_{t-1}\right] = \sigma^2$$

and Ω_t is the information set available at time t. γ is the transition parameter ($\gamma > 0$) and c is the threshold parameter;s_t,the transition variable[2], is generally the lagged endogenous variable, *i.e.* $s_t = y_{t-m}$ for certain integer $m > 0$ where m is the delay parameter. In most applications, the transition function $F\left(s_t, \gamma, c\right)$ is an exponential function or a logistic function. The FISTAR model can be also be written as follows:

$$\begin{cases} (1-L)^d y_t = x_t \\ x_t = \pi_1' w_t + \pi_2' w_t F\left(s_t, \gamma, c\right) + \varepsilon_t \end{cases} \tag{3}$$

where $w_t = \left(1, x_{t-1}, ..., x_{t-p}\right)'$, $\pi_i = \left(\pi_{i0}, \pi_{i1}, ..., \pi_{ip}\right)'$ and

$$\pi_i\left(L\right) = \varphi_i\left(L\right)\left(1-L\right)^d$$

for $i = 1, 2$. The fractional parameter d and the autoregressive parameters make the FISTAR model potentially useful for capturing both nonlinear and long memory features of the time series y_t. Indeed, the long-run properties of y_t are restricted to be constant and these are determined by the fractional differencing parameter, however, the short-run dynamics are determined by autoregressive parameters.

Our empirical results show that the fractionally integrated exponential STAR ($FIESTAR$) model is more appropriate for modelling real exchange rate dynamics than the FISTAR model with the logistic function ($FILSTAR$). Then, the simple transition function suggested by Teräsvirta et al. (1992) and Teräsvirta (1994), which is particularly attractive in the present context, is the exponential function[3] that takes the following form:

$$F(s_t, \gamma, c) = 1 - \exp\left(-\frac{\gamma}{\sigma_{s_t}^2}\left(s_t - c\right)^2\right) \tag{4}$$

[2]The transition variable can also be assumed an exogenous variable, or a possibly nonlinear function of lagged endogenous variables. See Teräsvirta (1994) for more details.

[3]Paya and Peel (2006), Michael, Nobay and Peel (1997), Taylor, Peel and Sarno (2001), and Sarantis (1999) applied the ESTAR models to exchange rates for differents countries.

where σ_{s_t} is the standard deviation of s_t.

We present the main steps of the specification procedure for FISTAR models, such as it is proposed by van Dijk et al. (2002):

- Specify a linear ARFI(p) model by selecting the autoregressive order p by means of information criteria[4] (Akaike (1974) or Schwarz (1978)).
- Test the null hypothesis of linearity against the alternative of a FISTAR model. If linearity is rejected, select the appropriate transition variable.
- Estimate the parameters in the FISTAR model.
- Evaluate the estimated model using misspecification tests (no remaining nonlinearity, parameter constancy, no residual autocorrelation, among others).

2.2 Linearity tests

Teräsvirta (1994) developed the procedure of testing linearity against STAR models; he pointed out that this procedure is complicated by the presence of unidentified nuisance parameters under the null hypothesis. To overcome this problem, Luukkonen et al. (1988) propose to replace the transition function $F(s_t, \gamma, c)$ with a suitable Taylor series approximation about $\gamma = 0$. In the reparametrized equation, the identification problem is no longer present, and linearity can be tested by means of a Lagrange multiplier (LM) statistic. For an extensive presentation of the test when the alternative is a FISTAR model, the reader is referred to van Dijk et al. (2002) and Smallwood (2005). The, we consider the model given by (3) and (4); the LM-type test statistic can be computed in a few steps as follows:

- Estimate an $ARFI(p)$, obtain the set of residuals $\widehat{\varepsilon}_t$. The sum of squared errors, denoted SSR_0, is then constructed from the residuals $\widehat{\varepsilon}_t$, $SSR_0 = \sum_{t=1}^{T} \widehat{\varepsilon}_t^2$.
- Regress $\widehat{\varepsilon}_t$ on w_t, $-\sum_{j=1}^{t-1} \frac{\widehat{\varepsilon}_{t-j}}{j}$ and $w_t s_t^i$, $i = 1, 2$, and compute the sum of squared residuals SSR_1 under the alternative hypothesis.
- The χ^2 version of the LM test statistic is calculated as:

$$LM_{\chi^2} = \frac{T\,(SSR_0 - SSR_1)}{SSR_0} \tag{5}$$

[4] Beran et al (1998) proposed versions of the AIC, BIC and the HQ (Hannan and Quinn, 1979) which are suitable for fractional autoregressions, but do not consider moving average components.

and is distributed as $\chi^2(2(p+1))$ under the null hypothesis of linearity (T denotes the sample size).

2.3 Estimation of the FISTAR model

It is important to obtain a consistent estimate of the long memory parameter d because the test statistics for the FISTAR model depend on this estimated value. In this section, we present two approaches to estimate the parameters in the FISTAR model: in the first one, we estimate all the parameters simultaneously (as proposed by van Dijk et al. (2002)), while the second method consists in performing the estimation in two steps.

2.3.1 Simultaneous estimation

To estimate the parameters of the FISTAR model, van Dijk et al. (2002) modify Beran's (1995) approximate maximum likelihood (AML) estimator for invertible and possibly nonstationary ARFIMA models to allow for regime switching autoregressive dynamics. This estimator minimizes the sum of squared residuals of the FISTAR model as follows:

$$S(\lambda) = \sum_{t=1}^{T} \varepsilon_t^2(\lambda), \tag{6}$$

where $\lambda = (\pi_1', \pi_2', d, \gamma, c)$ denotes the parameters of the FISTAR model (3). The residuals $\varepsilon_t(\lambda)$ are calculated as follows:

$$\begin{aligned} \varepsilon_t(\lambda) = \;& (1-L)^d y_t - \left(\pi_{10} + \textstyle\sum_{j=1}^{t+p-1} \pi_{1,j} y_{t-j}\right) \\ & - \left(\pi_{20} + \textstyle\sum_{j=1}^{t+p-1} \pi_{2,j} y_{t-j}\right) F(s_t, \gamma, c) \end{aligned} \tag{7}$$

where $F(s_t, \gamma, c)$ is given by (4). Thus, conditional upon d, γ and c, van Dijk et al. (2002) remark that the FISTAR model is linear in the remaining parameters; estimates of π_1 and π_2 can be thus obtained by ordinary least squares as:

$$\widehat{\mu}(d,\gamma,c)' = \left(\sum_{t=1}^{T} w_t(d,\gamma,c)\, w_t(d,\gamma,c)'\right)^{-1} \left(\sum_{t=1}^{T} w_t(d,\gamma,c)\, y_t\right), \tag{8}$$

where $w_t(d,\gamma,c) = (w_t', w_t' F(s_t,\gamma,c))'$. Therefore, the sum of squares function can be obtained by :

$$S(d,\gamma,c) = \sum_{t=1}^{T} \left(y_t - \widehat{\mu}(d,\gamma,c)' w_t(d,\gamma,c)\right)^2. \tag{9}$$

According to van Dijk et al. (2002), it can be difficult to estimate the model parameters jointly. In particular, accurate estimation of the smoothness parameter γ is quite difficult when this parameter is large. They proposed an algorithm that is based on a grid search over d, γ and c in order to obtain starting values for the nonlinear least squares procedure.

2.3.2 Two steps estimation

The properties of the process y_t depend on the value of the parameter d. Many researchers have proposed methods for estimating the long memory parameter d. These methods can be summarized in three classes: the heuristic methods (Hurst (1951), Higuchi (1988), Lo (1991)...), the semiparametric methods (Geweke and Porter-Hudak (1983), Robinson (1994, 1995a and b), Lobato and Robinson (1996)...) and the maximum likelihood methods (Dahlhaus (1989), Fox and Taqqu (1986), Sowell (1992) ...). In the first two classes, we can estimate only the long memory parameter d, and in the last, we estimate simultaneously all the parameters, see Boutahar et al. (2007) for more details.

The estimation method of the FISTAR model we propose proceeds in two steps:

- In the first step, we estimate the long memory parameter d in the simple model (1) using the heuristic method via the R/S statistic proposed by Hurst (1951) and modified by Lo (1991). The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Specifically, the R/S statistic is defined as:

$$Q_T = \frac{1}{S_T}\left(\max_{1\leq k\leq T}\sum_{j=1}^{k}(y_j-\overline{y}) - \min_{1\leq k\leq T}\sum_{j=1}^{k}(y_j-\overline{y})\right) \tag{10}$$

where $\overline{y} = \frac{1}{T}\sum_{i=1}^{T} y_i$ is the empirical mean and $S_T^2 = \frac{1}{T}\sum_{i=1}^{T}(y_i-\overline{y})^2$ is the empirical variance. Lo (1991) modified the R/S statistic as follows:

$$\widetilde{Q}_T = \frac{1}{Sq(T)}\left(\max_{1\leq k\leq T}\sum_{j=1}^{k}(y_j-\overline{y}) - \min_{1\leq k\leq T}\sum_{j=1}^{k}(y_j-\overline{y})\right) \tag{11}$$

where

$$Sq(T) = \left[S_T^2 + \frac{2}{T}\sum_{j=1}^{q} w_j(q)\left(\sum_{i=j+1}^{T}(y_i-\overline{y})(y_{i-j}-\overline{y})\right)\right]^{1/2};$$

$w_j(q) = 1 - \frac{j}{q+1}$ are the weights proposed by Newey and West (1987), with $j = 1, ..., q$. There is no optimal choice of the parameter q. Lo and MacKinlay (1988) and Andrews (1991) showed by a Monte Carlo study that, when q is relatively large compared to the sample size, then the estimator is skewed and thus q must be relatively small. By default, for obtaining the long run variance, q is chosen to be $\left[4\,(T/100)^{1/4}\right]$, where T is the sample size, and $[x]$ denotes integer part of x. However, when the stationary process y_t has long memory, Mandelbrot (1972) showed that the R/S statistic converges to a random variable at rate T^H, where H is the Hurst coefficient. The link between the parameter H and the ARFI parameter d is that $H = d + \frac{1}{2}$ (Boutahar et al.(2007)).

- Once we obtain $\widehat{d}_{R/S}$, in the second step, we filter out the long memory component and we estimate the STAR model parameters via nonlinear least squares estimation.

2.4 Out-of-sample forecasting performance

Unlike the linear model, forecasting with nonlinear models is more complicated, especially for several steps ahead (see, for instance, Granger and Teräsvirta (1993)). Let us consider the FIESTAR model given by (3) and (4) which can be written as:

$$\begin{cases} (1-L)^d\, y_t = x_t \\ x_t = G\,(w_t, \omega) + \varepsilon_t \\ F(s_t, \gamma, c) = 1 - \exp\left(-\frac{\gamma}{\sigma^2_{s_t}}\,(s_t - c)^2\right) \end{cases} \tag{12}$$

where $G\,(w_t, \omega) = \pi_1' w_t + \pi_2' w_t F\,(s_t, \gamma, c)$ and $\omega = (\pi_1', \pi_2', \gamma, c)'$. The optimal one-step ahead forecast of x_t is given by:

$$x_{t+1|t} = E\,(x_{t+1}|\Omega_t) = G\,(w_{t+1}, \omega)\,; \tag{13}$$

this forecast can be achieved with no difficulty and can be estimated by

$$\widehat{x}_{t+1|t} = G\,(w_{t+1}, \widehat{\omega}) \tag{14}$$

where $\widehat{\omega}$ is the parameter estimate. However, when the forecast horizon is larger than one period, things become more complicated because the dimension of the integral grows with the forecast horizon. For example, the two-step ahead forecast of x_t is given by:

$$\widehat{x}_{t+2|t} = E\left(G\left(\widehat{w}_{t+2|t}, \omega\right)|\Omega_t\right) = \int_{-\infty}^{\infty} G\left(\widehat{w}_{t+2|t}, \widehat{\omega}\right) f(\varepsilon) d\varepsilon \tag{15}$$

with $\widehat{w}_{t+2/t} = \left(1, \widehat{x}_{t+1|t} + \varepsilon_{t+1}, x_t, ..., x_{t+2-p}\right)'$. The analytic expression for the integral (15) is not available. We thus need to approximate it using integration techniques. Several methods obtaining forecasts to avoid numerical integration have been developed (see Granger and Teräsvirta (1993)). In this paper, we use a bootstrap method suggested by Lundberg and Teräsvirta (2001). This approach is based on the approximation of $E\left(G\left(\widehat{w}_{t+2|t}, \omega\right)|\Omega_t\right)$, the optimal point forecast is given by :

$$\widehat{x}_{t+2|t} = \frac{1}{k}\sum_{i=1}^{k} G\left(\widehat{w}_{t+2|t}^{(i)}, \widehat{\omega}\right), \tag{16}$$

where k is some large number and the values of ε_{t+1} in $\widehat{w}_{t+2|t}^{(i)}$ are drawn with replacement from the residuals from the estimated model $\widehat{\varepsilon}_t$.

In general, forecasts are evaluated using the mean squared prediction error (MSPE) and the root mean squared prediction (RMSE), where m is the number of steps-ahead forecasts. Models with smaller MSPE have a better forecast performance. Further, in order to assess the accuracy of forecasts derived from two different models, the Diebold and Mariano (1995) test is likely to be widely used in empirical evaluation studies, and is considerably more versatile than any alternative test of equality of forecast performance.

Let $y_{t+h/t}^1$ and $y_{t+h/t}^2$ denote two competing forecasts of y_{t+h} from FIESTAR and ARFI models, respectively, based on Ω_t, where $\Omega_t = \{y_t, y_{t-1}, ..\}$ is the information set available at time t. The forecast errors from the two models are given by $e_{t+h/t}^i = y_{t+h} - y_{t+h/t}^i$, $i = 1, 2$. The accuracy of each forecast is measured by a particular loss function:

$$g\left(y_{t+h}, y_{t+h/t}^i\right) = g\left(e_{t+h/t}^i\right), \quad i = 1, 2.$$

To determine if a model predicts better than the other one, we may test the null hypothesis of equality of expected forecast performance:

$$\begin{cases} H_0 : E\left(g\left(e_{t+h/t}^1\right)\right) = E\left(g\left(e_{t+h/t}^2\right)\right) \\ H_1 : E\left(g\left(e_{t+h/t}^1\right)\right) \neq E\left(g\left(e_{t+h/t}^2\right)\right) \end{cases}.$$

The Diebold-Mariano test is based on the loss differential:

$$d_t = g\left(e_{t+h/t}^1\right) - g\left(e_{t+h/t}^2\right). \tag{17}$$

The null of equal predictive accuracy is then: $H_0 : E(d_t) = 0$. The Diebold-Mariano test statistic is:

$$S_1 = \frac{\overline{d}}{\sqrt{\frac{2\pi \widehat{f}_d(0)}{T}}}, \tag{18}$$

where T is the sample size, $\overline{d} = \frac{1}{T}\sum_{t=1}^{T} d_t$ is the sample mean of d_t and $\widehat{f}_d(0) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty}\widehat{\gamma}_d(\tau)$ is a consistent estimate of the spectral density of the loss differential function at frequency zero,

$$\gamma_d(\tau) = E\left[(d_t - \mu)(d_{t-\tau} - \mu)\right]$$

is the autocovariance of the loss differential at rate τ, and μ is the population mean loss differential. Under the null hypothesis of equal forecasts, the statistic S_1 has an asymptotic standard normal distribution.

Harvey et al. (1997) noted that the Diebold-Mariano test statistic could be seriously over-sized as the prediction horizon increases, and therefore provide a modified Diebold-Mariano test statistic. Harvey et al (1997) and Clark and McCracken (2001) show that this modified test statistic performs better than the Diebold-Mariano test statistic, and also that the power of the test is improved when the $p-values$ are computed with a Student distribution with $(T-1)$ degrees of freedom, rather than from the standard normal distribution. Thus, the modified Diebold-Mariano statistic is given by:

$$S_1^* = \left(\frac{T+1-2h+T^{-1}h(h-1)}{T}\right)^{1/2} S_1 \tag{19}$$

where S_1 is the original Diebold and Mariano statistic (18).

3 Empirical results

The fractionally integrated models[5] have been already applied in economics and finance, for instance to exchange rates (Diebold et al. (1991), Cheung and Lai (2001), Baillie and Bollerslev (1994)), inflation (Hassler and Wolters (1995), Baillie et al. (1996)) and unemployment modelling (Diebold and Rudebusch (1989), Tschernig and Zimmermann (1992), Koustas and Veloce (1996), Crato and Rothman (1996)). Therefore, the long memory models, such as the FISTAR, are not only able to study the persistence but also to capture nonlinearity features such as thresholds or asymmetries. They can be applied in various economic and financial fields, in particular the stock indexes, the exchange rates and the interest rates. van Dijk et al. (2002) apply the FISTAR models to US unemployment and Smallwood (2005) to the case of purchasing power parity. In this paper, we study the behaviour of exchange rates and compare the forecast performances of the FIESTAR modelling to some other models.

[5]For a survey on long memory models and their application in economics and finance, see Baillie (1996), Robinson (2003) among others.

3.1 The data

We use monthly data of the seasonally adjusted US real effective exchange rate covering the period June 1978 until April 2002, these data were obtained from the *IMF International Financial Statistics*. The series is expressed in logarithm. The use of monthly data provides us with a reasonably large sample and hence meets the requirement of the linearity tests for many degrees of freedom. The series is shown in Figure 1, which demonstrates a real appreciation of the dollar during the beginning of the 1980's followed by depreciation in 1985. As noted by Smallwood (2005), consistently with the theoretical foundation of Sercu et al. (1995), we observe four periods after 1987 in which the dollar steadily appreciates and then rapidly depreciates after reaching approximately the same value. This provides some support for the use of nonlinear models.

3.2 Linearity tests results

Application of the linearity tests models requires stationary time series. The unit root tests[6] of Phillips and Perron (1988), Kwiatkowski et al. (1992)[7] and Dickey-Fuller Augmented (1979) for the levels and first differences of the real effective exchange rates, measured in logarithms, are shown in Table 1. These results indicate that the time series are integrated of order 1, at both 5% and 1% significance levels.

The selection of the maximum lag p, of the linear ARFI model was made using the AIC and BIC criteria under the non autocorrelation hypothesis. We allow for a maximum autoregressive order of $p = 6$. Both AIC and BIC indicate that an ARFI model with $p = 4$ is adequate.

The linearity tests are displayed in Table 2. In carrying out linearity tests, we have considered values for the delay parameter m over the range $[1, 6]$, and calculated the $p - values$ for the linearity test in each case, the estimate of m is chosen by the lowest $p - value$. Using 5% as a threshold $p - value$, the test classifies the US real effective exchange rates as nonlinear. Although the $p - value$ is slightly higher than 5%, we show thereafter that a nonlinear model describes the features of a time series better than a linear model[8]. Then the lowest $p - value$ corresponds to $m = 4$ $(m \leq p)$.

[6] For other unit root tests see Elliot et al. (1996), among others.

[7] Contrary to ADF test, the KPSS test considers the stationnarity under the null hypothesis, and the alternative hypothesis is the presence of unit root.

[8] This result is also found in Sarantis (1999).

3.3 Estimation results

Estimation results for the ARFI and FIESTAR models are shown in Table 3. The second column gives the ARFI model estimation, the estimate of d is -0.484, showing that the process y_t is stationary and invertible. The results of the second model are based on the specification (3) where y_t is the first difference of the US real effective exchange rates. The third column of Table 3 contains simultaneous estimation results of the parameters. In particular, the estimate of d is equal to -0.169 and belongs thus to the interval $]-0.5, 0.5[$, suggesting that the process is stationary and invertible. The autocorrelation function decreases more quickly than in the case where $0 < d < 0.5$: y_t is an anti-persistent process. It is also interesting to note, in the last column corresponding to the two-step estimation, that the degree of persistence measured by the differentiation parameter increases. The Lo's (1991) estimator using the modified R/S statistic is $\widehat{d}_{R/S} = 0.221$, then, the process is stationary and invertible, the autocorrelation function decays hyperbolically to zero and y_t is a long memory process. The modified R/S statistic 1.896 is significant at 5%. The ratio of the standard errors for the nonlinear and linear models for the simultaneous estimation of the FIESTAR model is equal to 0.840, it's higher than for the two-step estimation 0.670. We can thus confirm that the nonlinear model improves the modelling of the exchange rate process, as shown by both estimation methods. It is worthwhile noting here the relative small value of the estimation of γ for the second estimation (2.547 compared to 12.655 for simultaneous estimation), suggesting that the transition from one regime to the other is rather slow, contrary to first estimation which assumes a slightly sharp switch. The parameter c indicates the halfway point between the different phases of the exchange rate. The value of c is negative for the first case, and not significantly different from zero in the other. These values belong to the neighborhood of the sample mean for the first difference exchange rates. Figures 2 and 3 show the curves of the exponential transition function corresponding to the estimation of the FIESTAR model, the first one using the simultaneous estimation method and the second one the two-step method.

Table 4 gives summary statistics and misspecification tests for ARFI and FIESTAR models. In particular, the hypothesis of no residual autocorrelation, no conditional heteroscedasticity, and normality are not rejected in the residuals for both models at 5% level of significance. From the skewness and kurtosis of the series, it is evident that the US real effective exchange rate is symmetric and the frequency curve is normal, this is confirmed by the Jarque-Bera test for normality. Moreover, the null hypothesis of parameter constancy against the alternative of smoothly changing parameters for

$s_t = t$, and the null of no remaining non linearity are not rejected, following the LM test statistics LM_{NL} and LM_C for the FIESTAR model.

3.4 Forecasting performance of estimated models

The final two years of data from January 2002 to April 2004 for US real effective exchange rate are used to evaluate the forecast performance of the estimated linear ARFI and FIESTAR models. For each point, we compute 1-12- step-ahead forecasts of real exchange rates. To obtain the forecasts from nonlinear model, we use the bootstrap method exposed in section 2.4.

The results of forecasting performance are reported in Table 5. Forecast accuracy is evaluated using mean squared prediction error (MPSE) criterion. The forecasts produced by the FIESTAR are compared to the forecasts generated by a random walk and linear ARFI models. Further, in order to assess the accuracy of forecasts derived from two different models, we employ the modified Diebold and Mariano test statistic proposed by Harvey et al. (1997) discussed in Section 2.4 for which the null hypothesis is the hypothesis of equal accuracy of different predictive methods.

The results successfully provide evidence in favour of the predictive superiority of the FIESTAR model against the random walk and ARFI models using MPSE: the MPSE of the linear model and a random walk is actually greater than the MPSE of the FIESTAR model. Comparing our results to those obtained in the previous literature we can see that the FIESTAR model gives very much more accurate forecasts and outperform random walk and linear ARFI models in out-of-sample forecasting performances for all forecast horizons. The statistical significance of this result is confirmed executing the modified Diebold and Mariano test: there is a statistically significant difference in predictive accuracy for the FIESTAR model over the random walk and ARFI specifications. We can thus conclude that the forecasts of the FIESTAR modelling are significantly better than those of the other models. The same conclusion is given by Chung (2006) who finds clear evidence in favour of the nonlinear long-memory model over some other estimated models for the real exchange rates of Germany, France, Italy, UK, Japan, and Switzerland.

4 Conclusion

The aim of this paper was to study the dynamic modelling of the US real effective exchange rates covering the period June 1978 until April 2002. We considered the FISTAR model, as proposed by van Dijk et al. (2002), that can describe long memory and nonlinearity simultaneously and be used to produce out-of-sample forecasts. We used their model to the case of an exponential transition function. To this end, we employ two modelling approaches corresponding to two different estimations (simultaneous estimation or two-step estimation) of a FIESTAR model. The estimated FIESTAR model seem to provide a satisfactory description of the nonlinearity and persistency found in the US real effective exchange rates. With regards to the out-of-sample forecasting performance for US exchange rate, the tests for comparing the predictive accuracy show that the FIESTAR model seems better that the random walk and linear models.

References

[1] Akaike, H. (1974), "A new look at the statistical model identification", *IEEE Transactions on Automatic Control* 19, 6, 716–723.

[2] Andrews, D.W.K (1991). "Heteroskedasticity and autocorrelation consistent covariance matrix estimation," *Econometrica*, 59, 3, 817-58.

[3] Baillie, R.T. (1996), "Long memory processes and fractional integration in econometrics", *Journal of Econometrics* 73, 5-59.

[4] Baillie, R.T., and T. Bollerslev (1994), "Cointegration, fractional cointegration, and exchange rate dynamics", *Journal of Finance* 49, 737-45.

[5] Baillie, R., Chung, C.F., and M. Tieslau (1996), "Analysing inflation by the fractionally integrated ARFIMA-GARCH model," *Journal of Applied Econometrics* 11, 23-40.

[6] Beran, J. (1995), "Maximum likelihood estimation of the differencing parameter for invertible short and long memory autoregressive integrated moving average models"*, Journal of the Royal Statistical Society,* Series B, 57, N°.4, 659–672.

[7] Beran, J., Bhansali, R.J, and Ocker, D. (1998), "On unified model selection for stationary and nonstationary short- and long-memory autoregressive processes", *Biometrika* 85, 4, 921-934.

[8] Boutahar, M. Marimoutou, V. and Nouira, L. (2007), " Estimation Methods of The Long Memory Parameter: Monte Carlo Analysis and Application", *Journal of Applied Statistics*, vol 10, No 3, 261-301.

[9] Caporale, G.M. and L.A. Gil-Alana (2007), "Nonlinearities and fractional integration in the US unemployment rate", *Oxford Bulletin of Economics and Statistics* 69, 521-544.

[10] Cheung, Y.W., and K.S. Lai (2001), "Long memory and nonlinear mean reversion in Japanese yen-based real exchange rates", *Journal of International Money and Finance* 20, 115-132.

[11] Chung, S.K. (2006), "The out-of-sample forecasts of nonlinear long-memory models of the real exchange rate", *International Journal of Finance and Economics*, 11, 4 , 355-370.

[12] Clarida, R. H. and M. P. Taylor (2003), "Nonlinear permanent-temporary decomposition, with applications in macroeconomics and finance", *The Economic Journal* 113*,* 125-139.

[13] Clark, T.E. and M.W McCracken (2001), "Tests of equal forecast accuracy and encompassing for nested models", *Journal of Econometrics*, 105, 1, 85-110.

[14] Crato, N. and P. Rothman (1996), "Measuring hysteresis in unemployment rates with long memory models", unpublished manuscript, East Carolina University.

[15] Dahlhaus, R. (1989), "Efficient parameter estimation for self-similar processes", *The Annals of Statistics* 17, 1749-1766.

[16] Dickey, D. A. and W. A. Fuller (1979), "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association* 74, 27-431.

[17] Diebold, F.X., Husted, S., and M. Rush (1991), "Real exchange rates under the Gold Standard," *Journal of Political Economy* 99:06, 1252-71.

[18] Diebold, F.X., and R.S. Mariano (1995), "Comparing predictive accuracy", *Journal of Business and Economic Statistics* 13, 253–263.

[19] Diebold, F.X., and G. Rudebusch (1989), "Long memory and persistence in aggregate output," *Journal of Monetary Economics* 24, 189-209.

[20] Doukhan, P., Oppenheim, G., and M.S. Taqqu (2003), *Theory and applications of long-range dependence.* Birkhäuser, Basel.

[21] Elliot, G., J.H. Stock and T. Rothenberg (1996), "Efficient tets of an autoregressive unit root", *Econometrica* 64, 813-836.

[22] Fox, R. and M.S. Taqqu (1986), "Large sample properties of parameter estimates for strongly dependent stationary time series", *The Annals of Statistics* 14, 517-532.

[23] Franses, P. H. and R. Paap (2002), "Censored latent effects autoregression, with an application to US unemployment", *Journal of Applied Econometrics* 17, 347-366.

[24] Franses, P.H.,van Der Leij, M., and R. Paap (2002), "Modelling and forecasting level shifts in absolute returns", *Journal of Applied Econometrics* 17, 601-616.

[25] Franses, P.H., and D. van Dijk (2000), *Nonlinear Time Series Models in Empirical Finance*, Cambridge University Press: Cambridge.

[26] Geweke, J., and S. Porter-Hudak (1983), "The estimation and application of long-memory time series models", *Journal of Time Series Analysis* 4, 221-238.

[27] Granger, C.W.J., and R. Joyeux (1980), "An introduction to long-range time series models and fractional differencing", *Journal of Time Series Analysis* 1: 15–29.

[28] Granger, C.W.J., and T. Teräsvirta (1993), *Modelling Nonlinear Economic Relationships*, Oxford University Press: Oxford.

[29] Harvey, D.I., S.J. Leybourne and P. Newbold (1997), "Testing the equality of prediction mean squared errors", *International Journal of Forecasting* 13, 281-291.

[30] Hassler, U., and J. Wolters (1995), "Long memory in inflation rates: international evidence," *Journal of Business and Economic Statistics* 13, 37-45.

[31] Higuchi, T. (1988), "Approach to an irregular time series on the basis of the fractal theory", *Physica D* 31, 277-283.

[32] Hosking, J.R.M. (1981), "Fractional differencing", *Biometrika* 68, 165–176.

[33] Hurst, H.E. (1951), "Long-term storage capacity of reservoirs", *Transactions of the American Society of Civil Engineers* 116, 770-799.

[34] Koustas, Z., and W. Veloce (1996), "Unemployment hysteresis in Canada: an approach based on long-memory time series models", *Applied Economics* 28, 823-831.

[35] Kwiatkowski, D., P. C. B. Phillips, P. Schmidt, and Y. Shin (1992), "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root", *Journal of Econometrics*, 54, 159-178.

[36] Lardic, S., and V. Mignon (2002), "Econométrie des séries temporelles macroéconomiques et financières", Economica, Paris.

[37] Lo, A. (1991), "Long-term memory in stock market prices", *Econometrica* 59, 1279-1313.

[38] Lo, A., and C. MacKinlay (1988), "Stock Market Prices do not Follow Random Walks: Evidence from a Simple Specification Test", *The Review of Financial Studies*, 1, 1, 41-66.

[39] Lobato, I. and P. M. Robinson (1996), "Averaged periodogram estimation of long memory", *Journal of Econometrics* 73, 1, 303-324.

[40] Lundbergh, S., and T. Teräsvirta (2001) "Forecasting with smooth transition autoregressive models", *In Companion to Economic Forecasting*, Clements M.P., Hendry D.F. (eds). Basil Blackwell: Oxford.

[41] Luukkonen R, Saikkonen P, Teräsvirta T. (1988), "Testing linearity against smooth transition autoregressive models", *Biometrika* 75, 491-499.

[42] Mandelbrot, B. (1972), "Statistical methodology for non-periodic cycles: from the covariance to R/S analysis", *Annals of Economic and Social Measurement*, 1.

[43] Michael, P., Nobay, A.R. and D.A. Peel (1997), "Transaction costs and nonlinear adjustment in real exchange rates: an empirical investigation", *Journal of Political Economy* 105, 803-879.

[44] Newey, W.K., and K.D. West (1987). "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55, 3, 703-08.

[45] Paya, I., and D.A. Peel (2006), "Temporal aggregation of an ESTAR process: some implications for purchasing power parity adjustment", *Journal of Applied Econometrics* 21, 5, 655-668.

[46] Phillips, P. C. B. and P. Perron (1988), "Testing for a unit root in time series regression", *Biometrika* 75, 2, 335–346.

[47] Robinson, P.M. (1994), "Semiparametric analysis of long-memory time series", *Annals of Statistics* 22, 515-539.

[48] Robinson, P.M. (1995a), "Log-periodogram regression of time series with long range dependence", *The Annals of Statistics* 23, 1048-1072.

[49] Robinson, P.M. (1995b), "Gaussian semiparametric estimation of long range dependence", *The Annals of Statistics* 23, 1630-1661.

[50] Robinson, P.M. (2003), *Time series with long memory,* Oxford University Press, Oxford.

[51] Sarantis, N. (1999), "Modelling non-linearities in real effective exchange rates", *Journal of International Money and Finance* 18: 27-45.

[52] Sarno, L., and M. Taylor (2001), "Real exchange rate dynamics in transition economies: a nonlinear analysis", *Studies in Nonlinear Dynamics and Econometrics* 5, 153-177.

[53] Schwarz, G. (1978), "Estimating the dimension of a model", *Annals of Statistics* 6, 461-464.

[54] Sercu, P., Uppal, R., and C. Van Hulle (1995), "The exchange rate in the presence of transactions costs: implications for tests of purchasing power parity," *Journal of Finance* 50, 1309-1319.

[55] Smallwood, A.D. (2005), "Joint tests for non-linearity and long memory: the case of purchasing power parity", *Studies in Nonlinear Dynamics and Econometrics* 9, 2, 1-28.

[56] Sowell, F. (1992), "Maximum likelihood estimation of stationary univariate fractionally integrated time series models", *Journal of Econometrics* 53, 165–188.

[57] Taylor, M.P., Peel, D.A. and L. Sarno (2001), "Nonlinear mean-reversion in real exchange rates : toward a solution to the purchasing power parity puzzles", *International Economic Review* 4, 1015-1041.

[58] Teräsvirta, T. (1994), "Specification, estimation, and evaluation of smooth transition autoregressive models", *Journal of the American Statistical Association* 89: 208–218.

[59] Teräsvirta, T., and H.M. Anderson (1992), "Characterizing nonlinearities in business cycles using smooth transition autoregressive models", *Journal of Applied Econometrics* 119-136.

[60] Tschernig, R., and K.F. Zimmermann (1992), "Illusive persistence in German unemployment", *Recherches Economiques de Louvain* 58, 441-453.

[61] van Dijk, D., Franses, P. H., and R. Paap (2002) "A nonlinear long-memory model with an application to US unemployment", *Journal of Econometrics* 110, 135-165.

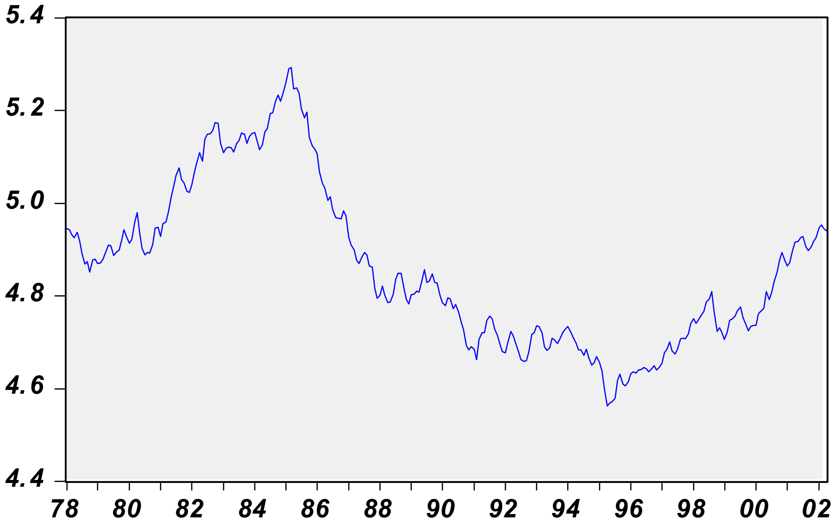

Figure 1: Monthly US real effective exchange rate (Log)

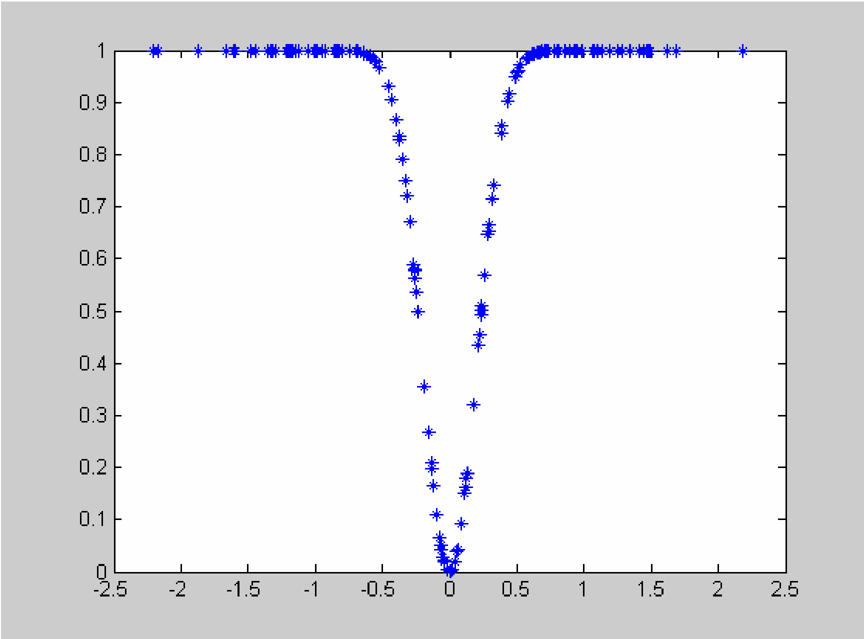

Figure 2: Exponential transition function (simultaneous estimation)

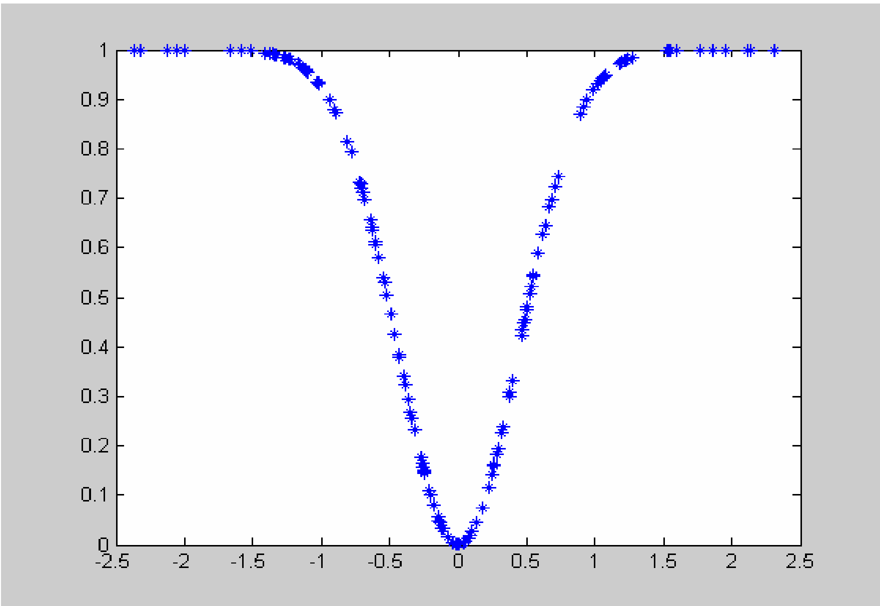

Figure 3: Exponential transition function (two-step estimation)

Table 1. Unit root tests

	Level	First difference
ADF	-1.118	-7.287
PP	-1.106	-12.281
KPSS	3.090	0.251

Note: The unit root tests are Phillips and Perron (PP), Kwiatkowski, Phillips, Schmidt and Shin (KPSS) and Dickey-Fuller Augmented (ADF) tests. For ADF test, the 1%, and 5% critical values are -3.455 and -2.871, respectively. For KPSS test, the 1%, and 5% critical values are 0.739 and 0.463, respectively.

Table 2. Linearity tests ($p - values$)

m	1	2	3	**4**	5	6
LM-test	0.868	0.346	0.087	**0.073**	0.251	0.171

Table 3. Estimation of the different models

	ARFI	FIESTAR (simultaneous estimation)	FIESTAR (two-step estimation)
π_{10}	0.850 (0.284)	-0.063 (0.026)	-0.003 (0.014)
π_{11}	-0.183 (0.125)	0.665 (0.295)	-0.175 (0.142)
π_{12}	0.179 (0.079)	-0.167 (0.335)	0.156 (0.144)
π_{13}	-0.059 (0.083)	0.194 (0.394)	0.345 (0.163)
π_{14}	0.103 (0.065)	-0.683 (0.290)	0.217 (0.126)
π_{20}		-0.001 (0.001)	-0.004 (0.015)
π_{21}		1.256 (0.078)	0.470 (0.115)
π_{22}		-0.458 (0.122)	-0.238 (0.120)
π_{23}		0.172 (0.119)	0.121 (0.103)
π_{24}		0.035 (0.075)	-0.195 (0.121)
d	-0.484 (0.282)	-0.169 (0.007)	0.221* **(1.896)**
γ		12.655 (8.648)	2.574 (1.190)
c		-0.101 (0.003)	0.022 (0.020)
S_E		0.840	0.670

Note: The standard errors are displayed in parentheses. * : Lo's (1991) estimator based on first difference; the value of modified R/S statistic for long memory test is in parentheses. S_E is the ratio of residual variance for the nonlinear and linear models.

Table 4. Diagnostic tests

	ARFI	FISTAR
AIC	-8.195	-8.181
BIC	-7.846	-0.132
SK	-0.166	-0.133
K_r	3.297	3.006
JB	1.313 (0.518)	0.463 (0.793)
ARCH(1)	0.981 (0.321)	0.714 (0.398)
ARCH(2)	1.778 (0.411)	1.292 (0.524)
ARCH(3)	5.634 (0.130)	2.933 (0.402)
ARCH(4)	7.605 (0.107)	4.276 (0.370)
$LM_{SI}(2)$	0.765 (0.467)	1.764 (0.175)
$LM_{SI}(4)$	1.174 (0.325)	2.179 (0.075)
$LM_{SI}(6)$	1.280 (0.271)	2.111 (0.057)
$LM_{SI}(8)$	1.118 (0.355)	1.690 (0.106)
$LM_{SI}(31)$	0.746 (0.817)	0.965 (0.529)
LM_{NL}	-	0.937 (0.521)
LM_C	-	0.701 (0.778)

Note: The table presents selected diagnostic and misspecification tests statistics for the estimated FIESTAR on two step and ARFI models for the US real effective exchange rate; the numbers in parentheses are p-values. SK is skewness, K_r is kurtosis, JB is the Jarque–Bera test of normality of the residuals, ARCH(r) is the LM test of no autoregressive conditional heteroscedasticity up to order r, $LM_{SI}(q)$ denotes the LM test of no serial correlation in the residuals up to order q, LM_{NL} is the LM test of no remaining non linearity, and LM_C is the LM test of parameter constancy.

Table 5. Out-of-sample MPSE and modified Diebold-Mariano statistics from random walk (Rw), ARFI and FIESTAR models

h	Rw	$ARFI$	$FIESTAR$	$ARFI\&FIESTAR$	$Rw\&FIESTAR$
1	0.0079	0.0053	0.0019	8.71 (0.000)	8.98 (0.000)
2	0.0176	0.0098	0.0085	7.52 (0.000)	8.11 (0.000)
3	0.0292	0.0218	0.0192	6.39 (0.000)	7.13 (0.000)
4	0.0495	0.0438	0.0346	6.22 (0.000)	6.87 (0.000)
5	0.0799	0.0748	0.0543	5.49 (0.000)	6.22 (0.000)
6	0.1314	0.1044	0.0775	5.36 (0.000)	5.74 (0.000)
7	0.1670	0.1565	0.1059	5.22 (0.000)	5.49 (0.000)
8	0.2045	0.1926	0.1363	4.54 (0.000)	4.72 (0.000)
9	0.2449	0.2398	0.1728	4.44 (0.000)	4.59 (0.000)
10	0.3298	0.2896	0.2129	3.71 (0.001)	4.10 (0.000)
11	0.3770	0.3595	0.2583	3.68 (0.002)	3.88 (0.001)
12	0.7021	0.6812	0.3087	3.07 (0.007)	3.58 (0.002)

Note: Columns 2–4 report the MPSE for the random walk and ARFI models, and columns 5-6 report the modified DM test statistics with p-values in parentheses.