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Environment factor, private information and the controllability principle

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Environment factor, private information and the controllability principle[1]

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Résumé:

Abstract: Should a manager be held accountable for uncontrollable environment factors like foreign exchange rate or oil price? To address this question, we develop a multi-task agency model where the agent has a limited liability. The profit of the firm is impacted by a stochastic environmental factor which changes the relative productivity of tasks, and whose realization is public ex-post. The agent-manager observes an early private signal of the environment and thus knows better than the principal which task it is optimal to undertake. For the principal, there is therefore a trade-off between congruity (induce the agent to use her information as the principal would use it) and agency costs (in particular to induce the agent to reveal this information).

In our model, it is costly to make the agent responsible for the environment, not because the environment is uncertain, but because the agent has a better knowledge of the environment when the task is chosen. The optimal contract depends on the informativeness of the signal. For an highly informative signal the environment is not filtered from the performance measure to encourage a congruent action from the agent. For a poorly informative one, the environment is eliminated and the agent never reacts to it: congruity is given up to decrease the agency cost. We use this model to discuss the application of the controllability principle in the case of external environment factor.

Mots clés :

Key Words : controllability principle, congruity of performance measure, variance analysis

Classification JEL: D82;_J33;_M41_

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1 Introduction

Should a manager be held accountable for ”uncontrollable” economic factors, like foreign exchange rate or oil price, or should one try to filter them out of the performance measure? There is no consensus in the literature, even among agency theorists. On the one hand Roberts (2004)**?** recommends to follow the example of British Petroleum which removes the impact of the oil price from the performance measure of its managers. For him, it is a direct consequence of the “informativeness principle” of the agency theory. By shielding the agent from this external factor, one can improve the power of incentives because there is less ”noise” in the performance measure. On the other hand, Jensen and Murphy (2004)**?** recommend just the opposite. They give also an example based on the oil price. Consider a fleet manager. Even if he does not control the oil price, he should know how to reduce the impact of an high oil price on the overall fleet cost. If the external factor is removed from the performance measure, the manager would have no incentives to react[1].

The later example suggests to differentiate between a *direct* effect of oil price and a possible *indirect* one. For a given volume of oil used, an higher price will increase the fuel bill; this is the direct effect. There is also a possible indirect effect: it becomes profitable to devote more time and efforts to find ways to use less oil: even if manager's decisions have no impact on the oil price, the oil price should impact the decisions of the manager. Nevertheless, as the realization of the external factor is publicly observed ex post (everybody knows the oil price), one may wonder if it is possible to filter out only the *direct* effect of the oil price while making the manager responsible for the *indirect* effect. In the fleet example the manager would not be responsible for the whole oil cost, but only for the volume of oil used (thus the manager is shielded from the direct effect of oil price). However, the performance assessment would take into account that the optimal quantity of oil which should have been used is inversely correlated with the price (the manager is responsible for the indirect effect). Then it would be possbile to achieve the best of both worlds: low noise of the performance measure and incentives to react.

Our paper intends to analyse these issues related to the application of the controllability principle. One of the main ideas defended in this paper is that a key factor lies not much in the existence of this indirect effect per se, but in the extent of the manager's informational advantage at the time decisions were made.

We develop a multi-task principal(he, the owner of the firm)-agent(she, the manager) model where the agent has a limited liability. The profit of the firm is impacted by a stochastic economic factor (called the environment), the price of oil for example. The agent can work on two tasks. First she can try to influence the impact of this factor; for example she can work to implement a production process less reliant on oil. The second possible task is unrelated to the environment; it allows for instance to decrease some fixed cost. Our model has two special

[1]In a similar way, Lambert (2001)**?** argues that a relative performance evaluation, which is a way to filter the environment, reduces incentives to forecast the environment and adapt the strategy accordingly.

features besides. First the agent observes a private signal of the environment before choosing her action. Second, we suppose it is worthwhile to try to influence the impact of the environment only if the signal predicts it will be high. Thus the optimal action depends on the early private signal of the agent. Ex-post the exact realization of the environment is publicly observed but, as the early signal received by the agent is not perfect, the agent knows better than the principal what was the optimal action (from the principal's point of view) to undertake. In other words it is not enough to observe the realization of the environment ex-post to know what the agent knew when she chose her action.

In an agency model with limited liability, the agent earns rents. All other things being equal, the agent will prefer to work on the task that yields the highest rent. If the principal wants the agent to truthfully report her signal, he may have to compensate the agent when the signal shows it is optimal to work on the task which offers the lowest rent. For the principal, there is therefore a trade-off between congruity or congruence (induce the agent to use her information as he would use it by working on the more relevant task) and incentive cost (on top of the usual agency cost related to each task, there is a new one to induce the agent to reveal her private information). We do not restrict in anyway the contract that the principal can offer. In particular communication is possible during the game, a menu of contracts may be offered, all the observable variables can be used in a contract.

The cost to induce the agent to reveal her information decreases with the signal informativeness. The principal has to infer from the actual realization of the environment the signal observed by the agent. If the informativeness is perfect, then the principal can infer perfectly what was the signal and reward the agent only if she chooses to work on the relevant task. This inference is more and more difficult as the informativeness worsens. If informativeness is low, the probability that the direction of the signal and the actual realization of the environment differ is greater. It becomes more profitable for the agent to gamble and work on the task that yields the higher rent whatever the signal received.

We show that the optimal contract takes the following form. If the informativeness of the signal is good, then a menu of contracts is offered to the agent, one for each possible value of the signal. The environment is not filtered out of the performance measure in order to induce a truthful reporting of the signal received; in other words to induce the agent to undertake the optimal action depending on her knowledge of the environment. Otherwise the agent would always choose to perform the task that yield the higher rent, irrespective of the signal received. If the informativeness of the signal is intermediate then a menu of contract is still offered but the agency cost increases. When the informativeness is too low, the agency cost to induce the agent to use her knowledge would become too high. A pooling equilibrium is then optimal; the principal tells the agent to always work on the task unrelated to the environment, even when the agent knows it would be optimal to react to it. The menu of contract may be interpreted as an interractive process like a flexible budgeting process or a flexible design of responsibility structure.

The paper gives the following answer to our opening questions. When the agent is better informed at the time of her choice, it is not possible to neutralize totally the direct effect of the environment while making the agent responsible for the indirect effect, which is to work on the more relevant task depending on the signal received. If the principal wants the agent to react, he has to make her (at least partially) responsible for the direct effect of the environment. On the one hand, this improves the congruence of the incentive scheme. The agent will use optimally her knowledge of the environment, by allocating her time and attention to the more relevant task. But the agency cost increases in most cases because of her private information. On the other hand, when the asymmetry of information is too high it is optimal to filter out the environment even if the manager can positively influence it, because the agency cost would be too high. Congruity in then given up with full knowledge of the facts: the agent will never react to the environment even if it would have been optimal to do so.

What happens if we relax the private information assumption? If there were no asymmetry of information between the principal and the agent and if the signal were contractible, it would always be possible to filter out "smartly" the environment, by neutralizing totally its direct impact without altering the incentive to react to it. Then the principal would achieve perfect congruity without bearing any related agency cost. However if the early signal is known ex-post by the principal but is not contractible ex-ante, there is a possible hold-up from the principal if he tells the manager to "do her best" and rely on an ex-post subjective performance evaluation[2]. In this case the same contracts as in the private information case will be offered and our previous conclusions remain valid.

We investigate further what happens if the agent is privately informed but the principal is unable to commit not to renegotiate the incentive scheme once the agent has revealed her information. Then the revelation principle does not apply and it becomes irrelevant to offer a menu of contract...

Our model extends the study of multi-task incentives in two ways. First, when the agent knows better than the principal the productivity of tasks, the optimal weights placed on the different tasks should be an interactive process where the agent is induced to reveal her knowledge of the environment. The weights thus cannot be fixed at the outset. Second, we prove that the optimal weights vary in a non monotonic way regarding to the informativeness of the signal. There is even a discontinuity when the informativeness falls below a level. Thus, the calibration of a compensation function over several performance measures, though solvable in theory, is quite sensitive to the parameters involved. It may explain the difficulties encountered in practice (see Ittner, Larcker and Meyer, (2003)**?**).

The paper is organized as follows. Related literature is discussed in the next section. Section 3 introduces the model. Section 4 gives some preliminary results: the first best and the perfect signal case. Section 5 resolves the general model.

[2]This hold-up is a real issue in practice. It can be related to the Hindsight effect pointed out by Merchant (2003) and coined by Hawkins and Hastie (1981): evaluators with knowledge of the results tend to assume information about the pre-result circumstances that was not available to those being evaluated.

Section 6 analyses the controllability principle in the light of our model. Major proofs are in the appendix.

2 Related literature

This paper is related to four different streams of literature. The first one studies variance analysis as a motivational device following the seminal paper of Baiman and Demski (1980).**?** As papers in this stream use a pure moral hazard framework, without any adverse selection issue, the informativeness principle applies, and it is always optimal to eliminate the impact of the environment through a variance analysis. Lambert (1985)**?** introduces a friction: there is a direct cost to perform this variance investigation, you have to pay someone to perform this task. The issue is then to know when it is worth the cost: when results are very bad, very good or in line? This early literature does not point out the possible congruity drawbacks of variance analysis as we do.

The second stream analyses the choice of performance measures as a trade-off between congruity (of the second best action the principal wants to induce compared to the first best) and controllability (incentive cost). This is a more general question that includes the traditional one: power of incentives versus agency cost. And it is a powerful tool to analyze actual compensation practices, as in Larmande and Ponssard (2006)**?**. The multi-tasking modelling, Holmström and Milgrom (1991)**?** and Feltham and Xie (1994)**?** allows to have a better understanding of this trade-off. In particular the principal may prefer to add a less congruent measure, like for instance accounting profit compared to stock price, because the controllability improves, even if that distorts the choice of the agent towards the short term. In our paper we introduce two new ideas. First, the congruity may not be definable ex-ante: it is stochastic because it depends on the realization of the signal. Second, the agent may know better than the principal which is the congruent action.

The third stream investigates the relation between noise and incentive strength. The Linear-Exponential-Normal model (LEN) introduced by Holmström and Milgrom (1987) implies a negative relationship. Yet, empirically, there is no clear relation between both. Prendergast (2002)**?** argues that when noise increases, delegation may increase and so incentives strength too in a LEN framework. Here, like in Baker and Jorgensen (2003)**?**, we argue that environmental factors are not completely external because the agent can mitigate their impact. “Noise” may be kept in the performance measure to induce an optimal allocation of efforts across tasks. However, the criterion to decide to keep the noise or remove it is not the volatility of the noise, as in Baker and Jorgensen (2003), but the degree of informativeness of the signal received by the agent before choosing her action.

Finally, from a technical point of view, there is a fourth stream of literature, the one that studies agency relationship with both moral hazard and adverse selection, because the agent has some private pre-decision information. It is a huge literature, and there is not a lot of general results. For instance, Christensen

(1981)**?** shows that the value of this private pre-decision information may be positive or negative for the principal. Here this value is always positive but we want to address another issue: the shape of the optimal contract. In this regard, the work by Laffont and Tirole (1986)**?** about procurement is very close is spirit: when the agent has prior information, her action space becomes very large and it is hard to control the agent. Then simple contracts as cost plus or fixed price are optimal.

3 The Model

A principal (he) hires a manager (she) to run a firm, like a subsidiary or a business unit of a group. Both are risk neutral but the manager has a limited liability: in all states of the world, her wage must be positive.

The profit function of the firm is defined as:

$$\pi = A + \Theta\,(B - b_0)$$

A and B are two random variables whose probability distributions depend on an effort made by the manager. They will be called task A and task B. Θ represents the environment: a random variable whose probability distribution is not influenced by the agent. b_0 is a constant.

The fact that the environment comes in a multiplicative way, $\Theta.B$, is essential in the sense that it affects the relative productivity of tasks, thus impacts the optimal action of the agent. Task A may be considered as a "business as usual" task. Task B as a task to influence the impact of the environment. For instance, if Θ represents the price of oil, then a successful task B allows to reduce the quantity of oil used in the production process. As we will see below, it is worthwhile to work on task B only if one forecasts that Θ will be high.

The possible outcome of task A (resp. B) is $\{0, x_a\}$ with $x_a > 0$ (resp. $\{0, x_b\}$ with $x_b > 0$). The manager can exert an effort e_a (resp. e_b) and incurred a private cost C_a (resp. C_b) or no effort at no cost. The outcome x_a on A is obtained with probability p_a if no effort is made on this task and with probability $p_a + e_a$ in case an effort e_a is made. It is assumed at all times that $0 \leq p_a + e_a \leq 1$ and $0 \leq p_b + e_b \leq 1$. The following table summarizes these data:

task $I = A$ or B	cost of effort	Prob $(I = x_i)$	Prob $(I = 0)$
no effort	0	p_i	$1 - p_i$
effort e_i	C_i	$p_i + e_i$	$1 - (p_i + e_i)$

Due to a time constraint, the manager can only work at most on one task: she must choose between an effort on task A, an effort on task B or no effort at all.

The environmental random variable Θ may take two values 0 or 1 with respective probabilities $(1-\theta, \theta)$. The agent receives a private signal of Θ before choosing her effort. This signal is the conditional probability distribution on Θ, denoted $(1-\theta_1, \theta_1)$. More precisely, θ_1 may take two values θ_H and θ_L with $\theta_L < \theta < \theta_H$. For instance, if the signal is perfect, then $\theta_L = 0$ and $\theta_H = 1$.

We do not restrict in any way the contract that the principal can offer. All variables A, B and Θ are contractible. The principal may offer a menu of contracts to the agent in order to induce her to reveal her private information.

The second important feature is that the realization of the environment Θ is observed ex-post, and thus can be used to discipline the agent when she reveals her information. The outcomes of the three random variables A, B are also separately observable ex post but the efforts of the manager are not.

Here is the timing of the game:

1. the principal proposes the bonus scheme which may involve a menu of contracts,
2. the manager gets a private signal θ_1 of the environment,
3. if a menu of contracts has been offered, the manager selects the contract in the menu,
4. the manager makes her effort selectively on one task, either task A or task B (or no effort),
5. the outcomes of both tasks are publicly observed as well as the environment. Manager is paid.

It will be assumed that the parameters are such that:

$$\theta_H e_b x_b - C_b \geq e_a x_a - C_a \geq \theta e_b x_b - C_b \tag{1}$$

$$\theta_H e_b x_b - C_b - C_b p_b/e_b > e_a x_a - C_a - C_a p_a/e_a > \theta e_b x_b - C_b - C_b p_b/e_b \tag{2}$$

$$e_a x_a - C_a - C_a p_a/e_a > 0 \tag{3}$$

As, it will be shown below, assumption 1 ensures that the first best is to work on task A if the signal is not observed or if $\theta_1 = \theta_L$, and to work on task B only if $\theta_1 = \theta_H$. Task A may be seen therefore as "business as usual". Assumption 2 is necessary to have the same results for the second best if the signal is public (observed by the agent and the principal). Assumption 3 ensures that it is never optimal to induce the null effort.

Let $\pi_0(\Theta) = p_a x_a + \Theta p_b x_b - \Theta b_0$ be the expected profit of the principal, if no effort is implemented.

4 Preliminary results: first best and perfect signal

4.1 First best

Proposition 1 *The first best corresponds to the following situation:*

- *if $\theta_1 = \theta_L$ or if the signal is not observed, the principal will choose to work on task A.*
- *if $\theta_1 = \theta_H$, the principal will choose to work on task B*

Proof. Let $\tilde{\theta}$ be the probability that $\Theta = 1$, known by the principal before choosing his action. If there is no signal then $\tilde{\theta} = \theta$; if the signal is observed, $\tilde{\theta} = \theta_1$.

If he works on task A then his expected payoff is $N_a(\tilde{\theta}) = e_a x_a - C_a + \pi_0(\tilde{\theta})$. If he works on task B, $N_b(\tilde{\theta}) = \tilde{\theta} e_b x_b - C_b + \pi_0(\tilde{\theta})$ and if he does not exert any effort, $N_0(\tilde{\theta}) = \pi_0(\tilde{\theta})$.

Assumption 3 ensures that $N_a(\tilde{\theta}) \geq N_0(\tilde{\theta})$ for all $\tilde{\theta}$. It is therefore never optimal to make zero effort.

Assumption 1 ensures that $N_a(\tilde{\theta}) \geq N_b(\tilde{\theta})$ for $\tilde{\theta} = \theta$ and also for $\tilde{\theta} = \theta_L$ because $\theta > \theta_L$. As a result, the principal is better off to work on task A if $\tilde{\theta} = \theta$ or $\tilde{\theta} = \theta_L$.

Finally, assumption 1 ensures also that $N_b(\theta_H) \geq N_a(\theta_H)$. Thus it is optimal to work on task B if $\theta_1 = \theta_H$. ■

4.2 Perfect signal

We derive now the optimal contract when the signal received by the agent is perfect: $\theta_H = 1$ and $\theta_L = 0$. Then the principal knows ex-post exactly what the agent knew when she selected her action. There is therefore no adverse selection in this case, only moral hazard.

Since the agent has limited liability, a compensation scheme S takes the form of a bonus $\omega_a \geq 0$ in case task A is successful, $A = x_a$, and no bonus in case it is a failure, $A = 0$, and likewise a bonus on task B $\omega_b \geq 0$ if $B = x_b$ and no bonus if $B = 0$.

As the agent has only one unit of work, if the principal wants to induce e_a (resp. e_b) there is no point to put a bonus on task B (resp. A). Thus, $\omega_a \omega_b = 0$.

The program of the principal can be broken down in two steps. First to compute the minimal bonus ω_a (resp. ω_b) necessary to have effort e_a (resp. e_b) implemented. Second to compare his net payoff for each action he can induce (e_a, e_b and 0).

Consider first the implementation of e_a. The incentive constraint faced by the agent is:

$$(p_a + e_a)\,\omega_a - C_a \geq p_a \omega_a$$

and the minimum bonus to implement e_a is $\omega_a^* = C_a/e_a$.

Recall that $\pi_0(\Theta) = p_a x_a + \Theta p_b x_b - \Theta b_0$ is the profit of the principal if no effort is implemented. His net expected payoff after the observation of the signal is therefore:

$N_a(\Theta) = (p_a + e_a)(x_a - C_a/e_a) + \Theta p_b x_b - \Theta b_0 = e_a x_a - C_a - C_a p_a/e_a + \pi_0(\Theta)$

Likewise, the minimum bonus to implement e_b is $\omega_b^* = C_b/e_b$ and the net payoff of the principal is $N_b = \Theta e_b x_b - C_b - C_b p_b/e_b + \pi_0(\Theta)$

The following table summarizes these results:

Action implemented	ω_a^*	ω_b^*	Principal's expected net payoff
e_a	C_a/e_a	0	$[e_a x_a - C_a + \pi_0(\Theta)] - C_a p_a/e_a$
e_b	0	C_b/e_b	$[\Theta e_b x_b - C_b + \pi_0(\Theta)] - C_b p_b/e_b$
0	0	0	$\pi_0(\Theta)$

Compared to the first best, there is one extra term in the principal's payoff: $-C_a p_a/e_a$ or $-C_b p_b/e_b$ which is the rent given to the agent. This agency cost is increasing in $p_i/e_{i.}$ This ratio $p_i/e_{i.}$ can be seen as the "signal to noise ratio": the agency cost is lower when the effort of the agent, e_i $(i = a,b)$, has a larger impact on the probability of success, and it increases with the "noise" p_i. In the traditional risk-averse agent model, the agency cost is also increasing in the "signal to noise ratio". But this agency cost is not a rent paid to the agent but a risk premium to be given to the agent to make her partially bear the risk.

The second step in the principal's program is to select the action that gives the higher net payoff. Assumption 3 ensures that $[e_a x_a - C_a + \pi_0(\Theta)] - C_a p_a/e_a \geq \pi_0(\Theta)$ thus the principal always prefers e_a to zero effort.

If $\Theta = 0$, then $[\Theta e_b x_b - C_b + \pi_0(\Theta)] - C_b p_b/e_b < \pi_0(\Theta)$ and the principal wants the agent to work on task A. If $\Theta = 1$, then assumption 2 ensures that $[\Theta e_b x_b - C_b + \pi_0(\Theta)] - C_b p_b/e_b \geq [e_a x_a - C_a + \pi_0(\Theta)] - C_a p_a/e_a$; the principal wants therefore the agent to work on task B.

Proposition 2 *If the signal received by the agent is perfect then the optimal contract is the following one:*

- *if* $\Theta = 0$, *the principal asks the agent to make an effort* e_a;

  a bonus $\omega_a^* = C_a/e_a$ *is paid if* $A = x_a$ *and no bonus is offered on task* B.

- *if* $\Theta = 1$, *the principal asks the agent to make an effort* e_b;

  a bonus $\omega_b^* = C_b/e_b$ *is paid if* $B = x_b$ *and no bonus is offered on task* A.

When the informativeness of the signal is perfect, there is no asymmetry of information between the principal and the manager regarding to the optimal task to undertake. It is possible to eliminate the direct impact of environment without altering the congruity of the performance measure.

4.3 Noisy but observable (and contractible) signal

The preceding result applies also if the signal is noisy, $0 < \theta_1$ and $\theta_H < 1$, but publicly observed and contractible. There is only one difference: the optimal contract is based on the value of the signal and not on the exact realization of the environment. In the previous proposition, one has to replace $\Theta = 1$ by $\theta_1 = \theta_H$ and $\Theta = 0$ by $\theta_1 = \theta_L$.

This contingent contract may be interpreted as the result of the following variance analysis. If $\theta_1 = \theta_H$, then the manager is responsible for task B and task A is considered to be external to the agent. If $\theta_1 = \theta_L$ the reverse is true: the agent is only responsible for task A[3]. This variance analysis is smart in the sense that the direct effect of the environment is totally neutralized (the agent is not penalized for instance if $B = x_b$ and $\theta_1 = \theta_H$ but $\Theta = 0$), but the agent is responsible for the indirect effect: to work on the most relevant task depending on the realization of the signal.

5 General case

As the agent earns rents in the limited liability model, she will prefer to work on the task that yields the highest rent. As task A represents business as usual, we will suppose that task B is harder to monitor, that is the rent associated with task B is higher than the one associated with task A:

$$p_a C_a / e_a < p_b C_b / e_b \tag{4}$$

Recall that the principal would like to encourage an informed manager to exploit her information i.e., to put her effort on task B if $\theta_1 = \theta_H$ and to put her effort on task A if $\theta_1 = \theta_L$. A possible way to do that is to eliminate the uncertainty from the bonus scheme through a standard variance analysis. This is feasible since the outcome on the environment is observed ex post. But such a scheme may in fact encourage the manager to falsely report that the signal is always good because the rent associated with task B is higher than the one associated with task A.

The principal has therefore to deal with both moral hazard and adverse selection. Using the revelation principle one may conduct the analysis by comparing the principal net payoff between revealing contracts and the pooling contract. A revealing contract determines the bonus of the agent ω_i for each task $I = A$ and B, depending on the signal that she announces, $\widehat{\theta}_1$, and on the outcomes of the random variables I and Θ. Denote $\omega_i(\widehat{\theta}_1, I, \Theta)$ the corresponding bonus. There are sixteen bonuses to be determined. The situation is quite degenerated and there are many optimal contracts.

One key element to find the optimal contract will be the informativeness of the signal θ_1. Define the following parameters:

[3] We obtain this extreme result because of our assumption regarding the private cost function of the agent. If she had more than one unit of time available, then she would also be responsible for task A, albeit with an easier target.

$$\delta(\theta) = \theta_L p_b C_b/\theta_H e_b - p_a C_a/e_a$$

$$s(\theta) = [\theta_H e_b x_b - C_b - p_b C_b/e_b] - [e_a x_a - C_a - p_a C_a/e_a] - [(\theta_H - \theta)/(\theta - \theta_L)]\,\delta(\theta)$$

We will see below the exact meaning of those two parameters. For the moment, let us remark that on the one hand, when θ_H/θ_L is close enough to 1 that is, when the signal is not very informative, $\delta(\theta) > 0$ (because of assumption 4: $p_a C_a/e_a < p_b C_b/e_b$). On the other hand, when θ_H/θ_L is large enough, $\delta(\theta) < 0$.

As $[\theta_H e_b x_b - C_b - p_b C_b/e_b] > [e_a x_a - C_a - p_a C_a/e_a]$ (assumption 2), $\delta(\theta) < 0 \Rightarrow s(\theta) > 0$. But the reverse is not true: if $\delta(\theta) > 0$ but not too far from 0, $s(\theta)$ remains positive.

Definition 3 *The informativeness of the signal is said to be:*

- ***good*** *when θ_H/θ_L is large enough, so that $\delta(\theta) < 0$ and $s(\theta) > 0$,*
- ***bad*** *when θ_H/θ_L is close enough to 1, so that $\delta(\theta) > 0$ and $s(\theta) < 0$,*
- ***intermediate*** *when $\delta(\theta) > 0$ and $s(\theta) > 0$*

For an easy reading, the optimal contracts are set out in three different propositions. Proofs for the three propositions are in the appendix.

Proposition 4 *If the informativeness of the signal is **good**, the principal offers a menu of contracts:*

- *if $\widehat{\theta}_1 = \theta_H$,*
  - *No bonus on task A*
  - *A bonus $\omega_b = C_b/e_b\theta_H$ on task B iff $\Theta = 1$ and $B = x_b$*
- *if $\widehat{\theta}_1 = \theta_L$,*
  - *A bonus $\omega_a = C_a/e_a$ on task A iff $A = x_a$*
  - *No bonus on task B*

If the informativeness is good, there is a separating equilibrium: the principal will tell the agent to work on either task, depending on the signal reported. There is no loss for the principal compared to the perfect signal case. If the signal received is θ_H, then no variance analysis is performed: the agent is responsible for the joint result ΘB. The observation of Θ ex-post is used to discipline the report of the agent: she may be punished if the realization of Θ does not correspond to the signal reported.

Proposition 5 *If the informativeness of the signal is **intermediate**, the principal offers a menu of contracts:*

- *if* $\widehat{\theta}_1 = \theta_H$,
  - *No bonus on task A*
  - *A bonus* $\omega_b = C_b/e_b\theta_H$ *on task B iff* $\Theta = 1$ *and* $B = x_b$
- *if* $\widehat{\theta}_1 = \theta_L$,
  - *A bonus* $\omega_a = C_a/e_a + \delta(\theta)/(p_a + e_a)$ *on task A iff* $A = x_a$
  - *No bonus on task B*

If the contract of the good informativeness case were used, then the agent would always announce $\widehat{\theta}_1 = \theta_H$ even if she observed θ_L. As the signal is less informative, the probability that $\Theta = 1$ remains large enough so that this gamble is worthwhile. θ_L is precisely the probability that $\Theta = 1$ when $\theta_1 = \theta_L$, and $\delta(\theta) = \theta_L p_b C_b/\theta_H e_b - p_a C_a/e_a$ represents the net payoff associated with this "gamble strategy". To undo this incentive to gamble the principal must therefore increase the bonus ω_a placed on task A if the agent announces $\widehat{\theta}_1 = \theta_L$. The bonus increases by $\delta(\theta)/(p_a + e_a)$. As $\delta(\theta)$ increases when θ_L/θ_H increases, the agency cost increases when the informativeness worsens.

In both previous cases, it is not possible to remove totally the direct impact of the environment without altering the incentive of the agent to react to the signal. Contrary to the public and contractible signal case, the agent is also responsible for the *direct effect* of the environment: the award of the bonus w_b depends on the joint result ΘB. However the impact of environment is always partially filtered out: the agent is never responsible for $-b_0\Theta$.

These menus of contracts can be interpreted as an interactive process between the principal and the agent. The principal offers at the beginning of the period a bonus only on task A; the exact value of this bonus depends of the informativeness of the signal: C_a/e_a or $C_a/e_a + \delta(\theta)/(p_a + e_a)$. In the middle of the period, the manager may tell the principal (when she observes $\theta_1 = \theta_H$) that she has the feeling that the environment will be high. Then the contract is changed and the environment is included in the performance measure.

Proposition 6 *If the informativeness of the signal is* ***bad****, there is only one contract*

-
  - *A bonus* $\omega_a = C_a/e_a$ *on task A iff* $A = x_a$
  - *No bonus on task B*

$\delta(\theta)$ increases when the informativeness worsens. At some point, $\delta(\theta)$ is so high that $s(\theta)$ becomes negative. $s(\theta)$ represents the net payoff for the principal of inducing the agent to reveal her information. If the informativeness is bad that is, if $s(\theta) < 0$, then a pooling contract is optimal: tell the agent to always work on task A, whatever the signal received. It would become too costly to induce the agent to reveal her information. The principal is then better off to give up congruity (which is to work on task B if $\theta_1 = \theta_H$) in order to decrease the agency cost. He performs a rough variance analysis and remove totally the environment from the performance measure.

6 Complements

6.1 Signal publicly observed ex-post but non-contractible

Suppose that at the end of the game the principal observes the early signal of the agent but that it is not possible to contract upon it ex-ante. A possible hold-up from the principal prevents him to tell the agent to do her best (that is, to work on the task she thinks is the more valuable) and that she will be rewarded according to the value of the signal observed ex-post by the principal. Imagine for instance that $\theta_1 = \theta_H$, task B is successful but task A fails. Then it would be optimal for the principal to say: I think you observed $\theta_1 = \theta_L$, you should have worked on task A. As task A is not successful, you will not earn any bonus. The optimal reaction of the agent to a "do your best" order is to never make an effort. Thus the principal has to commit ex-ante by offering the same menu of contracts as above.

There is therefore no gain for the principal to know ex-post the early signal observed by the agent if it is not contractible.

6.2 Value of information

The value of the private information of the agent on the signal is positive both for the manager and for the principal. For the agent, because if there were no signal the principal would tell to work on task A which yields the minimum possible rent. For the principal because if he offered the "pooling" contract, even a privately informed agent would choose the "pooling" effort, which is to work on task A. But their preferences regarding the informativeness of the signal are not the same.

Corollary 7 *The agent prefers a signal of intermediate informativeness, then good finally bad. The principal values the signal according to its strength (good preferred to intermediate preferred to bad).*

Incentives to have the agent gather information about the environment are not aligned: the principal prefers a very informative signal, the agent an intermediate one. If the informativeness resulted from an effort or an action of the agent but is a hard information[4], it would be in her interest to generate a signal of intermediate informativeness. Not too low to be allowed to work on task B, but not too high to keep an informational advantage over the principal.

[4]Which means that once the agent has revealed it, the principal can audit the truthfulness of this information.

6.3 Optimal weights in multi-task incentives

As in our model the agent has only one "unit of time" – she can work only on one task, there is no real weights in the performance measure, aggregating results of both tasks. However we can use the bonuses themselves as a proxy for the weights we would found in a more sophisticated model with the possibility to work on both tasks simultaneously.

At first when the informativeness is good, the bonus related to task B, ω_B, increases as the informativeness decreases (θ_H decreases), whereas the bonus ω_A remains constant. When the informativeness is intermediate, both ω_B and ω_A increases as the informativeness decreases. There is a discontinuity when the informativeness hits the intermediate-bad frontier: ω_B becomes equal to zero and ω_A returns to its initial level.

The weights varies therefore in a non monotonic way regarding to the informativeness of the signal and that a discontinuity even exists. A similar point was made by Datar, Lambert and Kulp (2001)**?** in the multi-task LEN model: the weight placed on a signal may decrease as the accuracy of this signal increases, contrary to the common sense. Both models suggests that the calibration of the compensation function over several performance measures, though solvable in theory, is quite sensitive to the parameters involved.

7 Just one contract

We now investigate in the section what happens if it is not possible to offer a menu of contract. For instance the principal may be unable to commit not to renegotiate the contract once the agent has revealed her private information. Then the revelation principle does not apply: the agent is better off not to reveal her information[5].

The principal has to fix at the beginning of the relationship, that is before the agent observed her signal, which task(s) would be under the responsibility of the agent. It is not possible to specify a contingent responsibility. There are four possibilities: no task at all, A, B, $\{AandB\}$. By $\{AandB\}$ we mean that the principal wants the agent to use her knowledge of the environment. Recall that the optimal pooling contract is to induce the agent to work on task A; thus B and no task at all are always dominated ex-ante by A. It remains to compare A and $\{AandB\}$. The A-case has already been resolved, it is the pooling contract derived in the bad informativeness case.

In the $\{AandB\}$ case, it would not be optimal to reward the agent on a a performance measure like $M = \alpha A + \beta\Theta B$. As the agent can work on only one task at once, if both tasks are successful, at least one of them is only the result of luck. If both tasks are successful, it is most likely optimal to ask the agent on which one she wants to be rewarded, thus to offer a bonus like $\max(\alpha A, \beta\Theta B)$.

[5]Or the principal may prefer an arm's length relationship to the interactivity associated with the contingent contract. In a dynamic moral hazard setting, the principal may want to commit not to renegiotate the contract to improve incentives of the first period and it may be easier to do so with a single contract than with a contingent one.

Formally, there are eight possible states of the world at the end of the game depending on the realization of A, B and Θ.Let $w(A, B, \Theta)$ be the bonus earned by the agent if the state of the world is (A, B, Θ). As there is no point to reward the agent if both tasks fail, $w(0,0,1) = w(0,0,0) = 0$. There remains six non trivial bonuses to determine.

There are four incentive constraints: make an effort e_b instead of zero effort and instead of e_a if $\theta_1 = \theta_H$, make an effort e_a instead of zero effort and instead of e_b if $\theta_1 = \theta_L$.

....

Conjecture: there is no point to make the bonus on A conditional to Θ. Thus $w(1,0,1) = w(1,0,0)$

Conjecture: $w(0,1,0) = 0$

Remains 4 non trivial bonuses.

There is still a windfall gain in the $\{AandB\}$ contract if, for instance, the agent observes $\theta_1 = \theta_H$, decides to undertake e_b, is not successful $(B = 0)$ but by good luck $A = x_a$.

(Two remarks: 1) there is no point to include $-\Theta b_0$. $-\Theta b_0$ represents the performance standard. 2) There is no reason to have a 1 for 1 aggregation that is, to have a performance measure identical to the gross profit of the firm).

8 Controllability principle and variance analysis

The traditional presentation of the controllability principle stipulates that *a manager should be evaluated only on what he or she can control.* But, as Lambert (2001)**?** points out, in that definition the meaning of "control" is not well specified. Antle and Demski (1988)**?** show that this definition must be understood in a very broad sense. For instance, when a relative performance evaluation is used, there is something the manager does not control in the performance measure: peer actions, peer luck... Yet it seems natural and fair to use such relative evaluation. The controllability principle should then be stated as: all information that can reduce the noise of the performance measure should be used to evaluate the manager. This is the informativeness principle of the agency theory (Holmström (1979)**?**, Gjesdal (1982)**?**).

Here, even this formulation is violated. The manager does not "control" the environment, that is, her actions have no impact on the probability distribution of the environment. And to remove this uncertainty decreases the incentive cost indeed. Yet in some circumstances it is optimal to let this uncertainty in the performance measure to induce a truthful reporting of the private information of the manager, or equivalently, to induce an optimal use of the private information observed by the manager. Our result may explain why the controllability principle is not always applied by firms for economic factors, as documented by Merchant (1989)**?** and recommended by Jensen and Murphy (2004).

We cannot neither state an "influenceability principle", that all noises the manager can partially influence should remain in the performance measure. The criterion to decide whether to include an environment factor or to exclude it is the informativeness of the signal, that is how easy is it ex post to know if the agent had enough information to make the right choice at first. For that reason, variance analysis may decrease the congruity of the incentive scheme, because the principal does not know exactly what the agent knew when she undertook her action. The full neutralization of the environment is a second best tool to be used only if it is too costly to induce the agent to use optimally her private information.

9 Conclusion

In this paper we develop a limited liability framework to investigate the controllability principle and the use of variance analysis in compensation schemes when the agent is better informed than the principal of the environment at the time of her choice. The full elimination of the environment from the agent's compensation lowers congruity but reduces also the agency cost. The optimal incentive contract may keep the environment in the performance measure to induce an optimal allocation of efforts across tasks. If the agent had no informational advantage over the principal then it would always be possible and optimal to remove the direct effect of the environment from the performance measure without altering the incentives to react to it.

As the model is intentionally simple, the reader may question the generality of the result. First, it seems possible to introduce different levels of efforts, more than one "unit of time" for the manager and a possible balanced distribution of effort across tasks (possibility to work a little on task A and little on task B instead of only on task A or on task B) without changing the main conclusions. Second, it would be worthwhile to investigate what happens if the rent associated with task A is higher than the one associated with task B, and more generally if productivity of both tasks are unknown. Finally, one could investigate how to shape incentives if the signal is not free but requires an effort of the agent to be generated as in Lambert (1986)**?**.

Appendix: proof of propositions 4, 5 and 6

Recall that $\omega_i\left(\hat{\theta}, I, \Theta\right)$ is the wage earned by the agent depending on what she announces, $\hat{\theta}$, and on the realization of the random variables I and Θ.

We are going to derive first the optimal revealing contracts. According to the revelation principle we can restrict the search of optimal contracts to direct truthful mechanisms.

If the agent observes θ_H, then because of assumption 2, the principal wants to implement task e_b.There is no point to reward the agent if the task fails ($X_b = 0$). Thus $\omega_b\left(\theta_H, 0, 1\right) = \omega_b\left(\theta_H, 0, 0\right) = 0$

The first incentive constraint is thus:

$$e_b \left[\theta_H \omega_b \left(\theta_H, x_b, 1\right) + \left(1 - \theta_H\right) \omega_b \left(\theta_H, x_b, 0\right)\right] - C_b \geq 0 \tag{5}$$

On the other hand, if the agent observes $\theta_{L,}$ then because of assumption 2, the principal wants to implement task e_a. Here also $\omega_a \left(\theta_L, 0, 1\right) = \omega_a \left(\theta_L, 0, 0\right) = 0$

The second incentive constraint is therefore:

$$e_a \left[\theta_L \omega_a \left(\theta_L, x_a, 1\right) + \left(1 - \theta_L\right) \omega_a \left(\theta_L, x_a, 0\right)\right] - C_a \geq 0 \tag{6}$$

We are going now to find the revelation constraints. If the agent lies and announces θ_L when she really observes θ_H, what will be her optimal action?

If she chooses a *zero* effort, her net payoff would be
$p_a \left[\theta_H \omega_a \left(\theta_L, x_a, 1\right) + \left(1 - \theta_H\right) \omega_a \left(\theta_L, x_a, 0\right)\right]$. And if she chooses e_a,
$\left(p_a + e_a\right) \left[\theta_H \omega_a \left(\theta_L, x_a, 1\right) + \left(1 - \theta_H\right) \omega_a \left(\theta_L, x_a, 0\right)\right] - C_a$.

Let $G\left(\hat{\theta} = \theta_L, \theta_1 = \theta_H\right)$ be the maximum of those two payoffs.

If the agent lies and announces θ_H when she really observes θ_L, then her net payoff would be:

$$G\left(\hat{\theta} = \theta_H, \theta_1 = \theta_L\right) = \max \left[\begin{array}{c} p_b \left[\theta_L \omega_b \left(\theta_H, x_b, 1\right) + \left(1 - \theta_L\right) \omega_b \left(\theta_H, x_b, 0\right)\right]; \\ \left(p_b + e_b\right) \left[\theta_L \omega_b \left(\theta_H, x_b, 1\right) + \left(1 - \theta_L\right) \omega_b \left(\theta_H, x_b, 0\right)\right] - C_b \end{array}\right]$$

The two revelations constraints are thus:

$$\left(p_b + e_b\right) \left[\theta_H \omega_b \left(\theta_H, x_b, 1\right) + \left(1 - \theta_H\right) \omega_b \left(\theta_H, x_b, 0\right)\right] - C_b \geq G\left(\hat{\theta} = \theta_L, \theta_1 = \theta_H\right) \tag{7}$$

$$\left(p_a + e_a\right) \left[\theta_L \omega_a \left(\theta_L, x_a, 1\right) + \left(1 - \theta_L\right) \omega_a \left(\theta_L, x_a, 0\right)\right] - C_a \geq G\left(\hat{\theta} = \theta_H, \theta_1 = \theta_L\right) \tag{8}$$

The principal minimizes the expected wage bill under those four constraints, $(5), (6), (7)$ and (8).

It is always possible to set $\omega_b \left(\theta_H, x_b, 0\right) = 0$: for a given level of expected wage in the state of the world $\theta_1 = \theta_H$, that is for $\theta_H \omega_b \left(\theta_H, x_b, 1\right) + \left(1 - \theta_H\right) \omega_b \left(\theta_H, x_b, 0\right)$ remaining constant, by setting $\omega_b \left(\theta_H, x_b, 0\right) = 0$, the principal does not change (5) nor (7) but (8) becomes more slack, because $\theta_H > \theta_L$.

Thus (5) becomes $\omega_b \left(\theta_H, x_b, 1\right) \geq C_b / \left(\theta_H e_b\right)$, and

and

$$G\left(\hat{\theta} = \theta_H, \theta_1 = \theta_L\right) = \max \left[\begin{array}{c} p_b \theta_L \omega_b \left(\theta_H, x_b, 1\right); \\ \left(p_b + e_b\right) \theta_L \omega_b \left(\theta_H, x_b, 1\right) - C_b \end{array}\right]$$

and (7) : $(p_b + e_b)\,\theta_H \omega_b(\theta_H, x_b, 1) - C_b \geq G\left(\hat{\theta} = \theta_L, \theta_1 = \theta_H\right)$

Let $\omega_a = \theta_L \omega_a(\theta_L, x_a, 1) + (1 - \theta_L)\,\omega_a(\theta_L, x_a, 0)$

We are going now to suppose that (5) is saturated, that is to set

$\omega_b(\theta_H, x_b, 1) = C_b / (\theta_H e_b)$. We will show that 1) it is always feasible and 2) it is not possible for the principal to improve his welfare by setting $\omega_b(\theta_H, x_b, 1) > C_b / (\theta_H e_b)$.

Then, $G\left(\hat{\theta} = \theta_H, \theta_1 = \theta_L\right) = p_b \theta_L \omega_b(\theta_H, x_b, 1)$

The principal minimizes ω_a under the constraints (6) and (8) (provided that (7) is verified).

Thus $\omega_a = \max\left[C_a/e_a; C_a/e_a + \delta(\theta) / (p_a + e_a)\right]$

The condition $C_a/e_a \geq C_a/e_a + \delta(\theta) / (p_a + e_a)$ is equivalent to $\delta(\theta) < 0$.

If $\delta(\theta) < 0$, (6) is binding and $\omega_a = C_a/e_a$;

This situation is optimal for the principal because the two incentive constraints are binding (remember that the principal wants to minimize the expected wage bill which is a combination of the left hand sides of both incentive constraints).

if on the contrary $\delta(\theta) \geq 0$, (8) is binding and $\omega_a = C_a/e_a + \delta(\theta) / (p_a + e_a)$

Can the principal improve his welfare by decreasing ω_a and increasing $\omega_b(\theta_H, x_b, 1)$? No because then the LHS of (8) would decrease and the RHS would increase, which is not possible because (8) is already binding.

It remains to see that (7) is verified in both cases. The principal has one degree of freedom left "inside" ω_a : the distribution between $\omega_a(\theta_L, x_a, 1)$ and $\omega_a(\theta_L, x_a, 0)$. We are going to show that by setting $\omega_a(\theta_L, x_a, 1) = \omega_a(\theta_L, x_a, 0) = \omega_a$, (7) is verified.

$$\begin{aligned} G\left(\hat{\theta} = \theta_L, \theta_1 = \theta_H\right) &= \max\begin{pmatrix} p_a\left[\theta_H \omega_a(\theta_L, x_a, 1) + (1 - \theta_H)\,\omega_a(\theta_L, x_a, 0)\right]; \\ (p_a + e_a)\left[\theta_H \omega_a(\theta_L, x_a, 1) + (1 - \theta_H)\,\omega_a(\theta_L, x_a, 0)\right] - C_a \end{pmatrix} \\ &= \max\left[p_a \omega_a; (p_a + e_a)\,\omega_a - C_a\right] \end{aligned}$$

As $\omega_a = \max\left[C_a/e_a; C_a/e_a + \delta(\theta) / (p_a + e_a)\right]$, we have $\omega_a \geq C_a/e_a$, thus:

$$G\left(\hat{\theta} = \theta_L, \theta_1 = \theta_H\right) = (p_a + e_a)\,\omega_a - C_a$$

The LHS of (7) is equal to $(p_b + e_b)\,C_b/e_b - C_b = p_b C_b/e_b$

If $\delta(\theta) < 0$, $\omega_a = C_a/e_a$ and (7) is equivalent to $p_b C_b/e_b \geq p_a C_a/e_a$ which is true because of assumption (4).

In the other hand, if $\delta(\theta) \geq 0$, that means that (8) is binding, so $(p_a + e_a)\,\omega_a - C_a = p_b \theta_L C_b / (\theta_H e_b)$, which is less than $p_b C_b/e_b$ because $\theta_L/\theta_H < 1$, and (7) is verified.

We have finished to derive the optimal revealing contract. It remains to compare it to the optimal pooling contract.

Assumption (2) entails that the principal wants to implement e_a is the pooling case. The agent will receive a wage C_a/e_a in case of success. And the payoff of the principal in the pooling case is: $e_a x_a - C_a - p_a C_a/e_a$

let λ be the probability that $\theta = \theta_H$.
$\theta = \lambda\theta_H + (1-\lambda)\theta_L \Longrightarrow \theta = \lambda(\theta_H - \theta_L) + \theta_L \Longrightarrow \lambda = (\theta - \theta_L)/(\theta_H - \theta_L)$
Therefore, $(1-\lambda)/\lambda = (\theta_H - \theta_L)/(\theta - \theta_L) - 1 = (\theta_H - \theta)/(\theta - \theta_L)$

The payoff of the principal with the optimal revealing contract is :
If $\delta(\theta) < 0$, $(1-\lambda)[e_a x_a - C_a - p_a C_a/e_a] + \lambda[\theta_H e_b x_b - C_b - p_b C_b/e_b]$ which is superior or equal to $e_a x_a - C_a - p_a C_a/e_a$ because of assumption (2).
If $\delta(\theta) \geq 0$, $(1-\lambda)[e_a x_a - C_a - \delta(\theta) - p_a C_a/e_a] + \lambda[\theta_H e_b x_b - C_b - p_b C_b/e_b]$
because when $\delta(\theta) \geq 0$, (8) is binding, and
$(p_a + e_a)\omega_a = C_a + p_b\theta_L C_b/(\theta_H e_b) = C_a + \delta(\theta) + p_a C_a/e_a$

The payoff is greater in the pooling case iff:

$$(1-\lambda)[e_a x_a - C_a - \delta(\theta) - p_a C_a/e_a] + \lambda[\theta_H e_b x_b - C_b - p_b C_b/e_b] \leq e_a x_a - C_a - p_a C_a/e_a$$

that is, iff:

$$-[e_a x_a - C_a - p_a C_a/e_a] - (1-\lambda)\delta(\theta)/\lambda + [\theta_H e_b x_b - C_b - p_b C_b/e_b] \leq 0$$

that is iff $s(\theta) \leq 0$.

The principal offers a revealing contract if and only if $s(\theta) > 0$. This is the case when the informativeness of the signal is either good or intermediate.