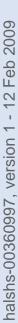# GREQAM

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SHOULD WE REALLOCATE PATENT FEES TO THE UNIVERSITIES ?

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Should we reallocate patent fees to the Universities?

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Abstract

In knowledge economies, patent agencies are often viewed as a relevant instrument of an efficient innovation policy. This paper brings a new support to that idea. We claim that these agencies should play an increasing role in the regulation of the relation between heterogeneous private R&D labs and public fundamental research units, especially concerning the question of the appropriation of free basic research results. Since these two institutions work with opposite institutional arrangements (see Dasgupta and David [9]), we essentially argue that there is, on the one hand, an over-appropriation of these results while, on the other hand, there is also an under-provision of free usable results issued from more fundamental research. We show how a public patent office can restore efficiency.

Keywords: Science and technology, patent agency, innovation policy

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1 Introduction

Should the patents fees only cover the patents offices examination costs or should these fees be a major lever of an innovation policy in a Knowledge based economy? In order to answer this question, several economists suggest, in the line of Scotchmer [23], that these fees, especially their modulation, render the innovation process more efficient[1]. However most of these arguments are worked out in a technological world where Knowledge satisfies the patentability criteria of utility, novelty and non-obviousness. In this paper, we suggest a new argument in favor of the use of patents fees as a policy instrument by looking upstream, at the interaction between Science and Technology.

In fact, Dasgupta and David [9] proposed to define Science and Technology neither according to the types of Knowledge (general principles versus applied Knowledge) they produce nor on the methods of inquiry (focus versus broader perspective) they adopt but as distinct institutional arrangements, broadly corresponding to non-market and market allocation mechanisms[2]. But concerning these institution, it is also largely recognized, especially by several empirical studies, that there are real effects of academic research on corporate patents. According to Jaffe's [15] seminal contribution, the elasticity of economically useful Knowledge - measured by the amount of induced corporate patents - with respect to academic research is quite important[3], even if this contribution is relatively scattered. For instance, Klevorick and al. [17] observed that the direct impact of recent University research in most industries is small when assessed relatively to other sources of scientific Knowledge, while Cohen and al. [7] argue, by making use of the Carnegie Mellon database, that the basic research strongly affects industrials R&D but that the channels are not those who are expected in the sense that there is not a direct causal link from fundamental research to new applied projects and patents. Their survey however indicates (see table 5 in Cohen and al. [7]) that the key channels of the impact of the university research essentially goes through published papers, reports, public conferences and informal exchanges, that is to say through totally free and open scientific results.

So, even if there is a debate that tries to identify which of these indirect channels is the more efficient[4] or to which firm it is mostly helpful[5], we must recognized that (i) the production

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[1]In the line of this paper which emphasizes the impact of the renewal fees, the reader is also referred to Cornelli and Schankerman [8]. For more general recommendations concerning the innovation policy, see Encaoua, Guellec and Martinez [10].

[2]See also Barba Naveretti and al. [5] or Carraro and Siniscalco [6].

[3]For other empirical studies going in the same direction, see also Adams [2] or Narin and al. [21].

[4]One often opposes informal local exchange to worldwide published results, see for instance, respectively, Audretsch and al. [4] who argue that geographic considerations matter, while Cohen and al. [7] put forward open science channels such as publications.

[5]See for instance, respectively, Acs and al. [1] who emphasize comparative advantages of small startups, versus

of Knowledge which encompasses both Science and Technology involves various actors with different motivations and (ii) the transformation of basic Knowledge into product or process innovations is not as mechanical as predicted in the said linear model of innovation. All the basic ingredients inducing inefficiency are therefore present. Our claim is that:

- patents fees can be viewed as tools which helps to restore efficiency in the process of privatization of free scientific Knowledge,

- and that this fees can be reallocated to the Universities in the form of an incentive scheme which increases the set of useful, patentable, Knowledge.

To be more precise, we consider, in the sense of Dasgupta and David [9], two basic institutional arrangements. One the one hand, we look at the University (in, perhaps, some narrow sense) which produces generic basic Knowledge with its own system of incentives and we assume that this Knowledge is freely available and only more or less useful, i.e. it does not meet the criteria of a patents agency. On the other hand, we introduce what we call Private Labs, i.e. institutions which have the ability to access to this Knowledge (or to a part of it), to use it in order to solve market based R&D puzzles and, by the way, to partially privatize this free Knowledge[6]. In other words, the first institution produced only indirectly - because it is less valuated by its own system of incentives - potential R&D solutions, while the heterogeneous Private Labs take advantage of this free Knowledge in order to solve market driven technical problems and compete for its acquisition.

But these freely available embryonic inventions often require further developments for commercial success[7]: they must be accommodated at some costs. We can even expect that this transformation activity exhibits decreasing returns to scale. The easiest potential R&D solutions will be treated first, while the others will be postponed until later due to increasing accommodation costs. In other words, one can expect that the profitability of these Private Labs is not only decreasing with the amount of free Knowledge they accommodate but also with the stock of free ideas which is appropriated by all these units. In other words, the appropriation of this common induces negative external costs which lead to an excessive patenting behavior.

Link and Rees [19] who put forward the existence of R&D departments in large firms.

[6]By doing so, we implicitly assume that a part of the Academic Knowledge is immediately valuable. We therefore neglect the question of the progressive maturation of an idea and the debate around the relative contribution of the public and the private research sectors (see for instance Aghion-Dewatripont and Stein [3]). This is leaved for a later work.

[7]Even if, in general, patents can be based on either basic or applied research, recent survey evidence suggests that potential R&D solutions must often be adjusted, or even totally rethought, in order to meet the taste of the consummers (Thursby and al. [25]).

This is why we suggest that a cautious choice of the patents fees contributes to an adjustment of the private appropriation costs to the social one[8].

If we go a step further, we even observe that this common which is appropriated by the Private Labs is not as limited as expected: it can be increased by targeted research programs. In fact, if these programs are not too circumscribed to operational Knowledge, we can expect that these results meet, at least partially, the incentives schemes of academic research and therefore increase the set of freely useful Knowledge. But, we must also accept the idea that researchers who contribute to these programs are nevertheless forced to deviate from the optimal strategy dictated by pure research criteria and therefore support a cost which must, at least, be covered by a financial transfer. This therefore suggests that the patents fees, which are levied in order to reduce the appropriation costs, must be allocated to these programs: it does not only increases the positive externality of academic research on technological innovation but it also reduces the social appropriation costs of scientific Knowledge, by extending the common.

Our paper mainly illustrates this prospect. We first introduce, in section 2, our basic assumptions on the behavior of the University and of the Private Labs in order to emphasize, on the one hand, the possibility for the University to deviate from her optimal strategy and, on the other one, the external appropriation costs which are at hand in the private sector. By keeping the stock of freely available Knowledge as given, we characterize in section 3 the strategic Knowledge appropriation behaviors which lead to a Tragedy of the Commons. We then propose in section 4 a patents fiscal scheme which has the property to implement an optimal allocation and to be accepted by all Private Labs. In section 5, we then show that a patents agency has even the ability to couple this fiscal scheme with the sponsorship of targeted research program, in order to implement an optimal provision and allocation of the more or less applied and freely available ideas induced by academic research. Finally, section 5 concludes the paper and an appendix is dedicated the proofs of the different propositions.

2   The model

In order to illustrate this feature, we need two kinds of actors: the University and the Private R&D Labs. We mainly look at their strategic interactions. This is why we do not directly address the question of the production of basic and applied Knowledge, which tackles the question of the

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[8]This is not in contradiction with the Anticommons problem suggested by Heller and Eisenberg [14]. We simply suggest that an excessive patenting behavior which induces in a second step an Anticommons Tragedy can be initiated by a Tragedy of the Commons (Hardin [13]).

scientists' decision[9] in both the University and the Private R&D Laboratories. We rather take some macroscopic view which takes these decisions as granted and which gives us the opportunity to focus on the problems induced by their interactions. So let us now present the assumptions we made on the University and the Private R&D Labs.

2.1 The University

We assume that there is a public institution - a University, for instance - which conducts fundamental research on the basis of public funds. This institution operates by its own rules, typically a peer validation one, which provide incentives to produce basic research, and, by the way, to produce a set of free useful ideas for the industry. Since we concentrate our attention on the interactions between the University and the Private Labs, we however do not explicitly model the behavior of the researchers within the University. We simply assume that a given stock of public funds generates both fundamental Knowledge and a stock y of applicable ideas and that the allocation of the research activity between both fields corresponds to an optimal running of the institution in the sense that each researcher has chosen her optimal strategy given the said peer validation rule.

This also implies that these optimal behaviors can only be changed at some costs. This may happen, for instance, if the University wants to increase of Δ the stock of applicable ideas relatively to its optimal level y. This is why we introduce a function $C(\Delta)$ which measures the cost supported by the University if she decides to encourage her members to increase the set of applicable ideas of an amount of Δ. This last function captures, in other words, the cost induced by the deviation from a standard research activity, based on pure academic research, to a more applied one. But this does not necessarily imply that these activities act as substitutes. If they are, for instance, complementary[10], and if an optimal mixture is chosen under a specific institutional arrangement, we simply claim that any change in this mixture leads to a worse solution and hence induces a cost of change.

Moreover if this cost comes from a deviation from an optimal solution of a concave program, it is immediate that $C(\Delta)$ is increasing and convex, i.e. $\frac{dC}{d\Delta} > 0$, $\frac{d^2C}{d\Delta^2} > 0$ and that $C(0) = 0$. We also assume, in the same vein, that very small deviations are not too costly while very large ones are prohibitive, i.e. $\lim_{\Delta \to 0} \frac{dC}{d\Delta} = 0$ and $\lim_{\Delta \to +\infty} \frac{dC}{d\Delta} = +\infty$.

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[9] The reader is refered, for instance, to Levin and Stephan's [18] early theoretical contribution or, more recently, to Jensen and al. [16] or Thursby and al. [26].

[10] See for instance Zucker and al. [27], Murray [20] or Stephan and al. [24] who recently claimed that basic and applied researches are complementary.

2.2 The Private R&D Labs

Let us now move to the behavior of the $j = 1, ..., m$ Private R&D Labs. Each Lab transforms a certain amount x_j of freely available Knowledge y into private patented Knowledge. Since this Knowledge becomes by this transformation mutually exclusive, we denote by $X := \sum_{j=1}^{m} x_j$ the total amount of patented ideas.

This patenting activity, even if it is done at some R&D costs (in order to adapt the free ideas), is assumed to increase the profitability of firm j. Several standard Industrial Organization arguments can be recalled in order to justify such an assumption, going from the construction of a dominant position on existing market to the development of new profit opportunities by creating new products. We take all these arguments for granted and simply assume that the profit π_j is increasing in the number of patented ideas x_j.

But, one of the assumption of the paper is that the set y of free ideas is heterogeneous in the sense that some ideas are less costly to accommodate than some others. The different Labs will of course compete for the ideas which are easier to accommodate, but are, *ex ante,* uncertain on the result of this competition. Since the accommodation cost increases with the total number of adopted ideas, we argue that the expected individual accommodation costs must be at least related to the number of unused free ideas. This is why we decided to use the amount of remaining free ideas $S := y - X$ as an indicator of the level of these accommodation costs. We, of course, expect that these costs are decreasing with S, seeing that a lower residual stock of free ideas indicates a greater adoption of less productive ideas. In order to spare notations, we nevertheless do not explicitly introduce this cost function, we simply say that the profit $\pi_j(x_j, S)$ of each private Lab is a function of both the quantity of free Knowledge she patents and the remaining stock S of free ideas. We also try to keep this function as general as possible.

However, due to this negative externality, it is a matter of fact to observe that these m Labs play a game in which they simultaneously decide on the amount x_j of public Knowledge they transform into a private one by having in mind that they receive a payoff of $\pi_j(x_j, S)$. So, in order to make sure that the competition between firms is not too severe, we introduce some strategic substitutability by controlling the effect of the residual stock of Knowledge on the marginal productivity of the patents x_j. Moreover, we also need more technical assumptions which guarantee the interiority of our solutions. We first say that a firm who patents some free Knowledge as the remaining stock becomes null has always an incentive to decrease her patenting activity x_j in order to benefit from the induced increase with the residual stock of free Knowledge. Secondly, we assume that every firm is active, even from the point of view of the social planner: if the patenting level of one firm j is low, the marginal gain of an increase of

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patents x_j covers the aggregate costs due to the negative externality.

To summarize, we assume that each Private R&D Lab j is characterized by a profit function $\pi_j(x_j, S)$ satisfying the following restrictions:

1. This function is increasing, i.e. $\partial_{x_j}\pi_j > 0$ and $\partial_S\pi_j > 0$, globally strictly concave (which implies that $\partial^2_{S,S}\pi_j$ and $\partial^2_{x_j,x_j}\pi_j < 0$) and satisfies $\partial^2_{x_j,S}\pi_j \geq 0$ (i.e. the best responses are strategic substitutes[11]).

2. Inactivity is allowed, i.e. $\forall S$, $\pi_j(0, S) = 0$.

3. The following boundary conditions[12] are met: (i) $\forall x_j > 0$, $\lim_{S \to 0} \partial_{x_j}\pi_j / \partial_S\pi_j < 1$ (ii) $\forall S > 0$, $\lim_{x_j \to 0} \partial_{x_j}\pi_j / \sum_{k=1}^{m} \partial_S\pi_k > 1$ and (iii) $\forall S > 0$, $\lim_{x_j \to \infty} \partial_{x_j}\pi_j / \partial_S\pi_j < 1$.

3   The inefficient patents allocation

The existence of external costs between the different users of this stock y of free Knowledge leads immediately to the following definition of a patents allocation.

Definition 1 *Given any stock y of free Knowledge produced by the University, a Nash equilibrium allocation of patents, also called a decentralized or an inefficient one, is a vector $(x_j^d(y))_{j=1}^{m} \in \mathbb{R}_+^m$ given by:*

$$\forall j = 1, \ldots, m \quad x_j^d(y) \in \operatorname*{arg\,max}_{x_j \in \mathbb{R}_+} \pi_j \left(x_j, y - x_j - \sum_{\substack{k=1 \\ k \neq j}}^{m} x_k^d \right) \quad s.t \quad y - x_j - \sum_{\substack{k=1 \\ k \neq j}}^{m} x_k^d \geq 0$$

And in order to check consistency, we can prove, under our assumptions, that:

Proposition 1 *For any strictly positive free Knowledge stock y, this game admits **a unique interior Nash equilibrium** which has the property that the marginal private benefit of patenting, $\partial_{x_j}\pi_j$, is equal to the marginal private costs of patenting, $\partial_S\pi_j$, which is captured by the*

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[11] Note also that we implicitly assume that $\partial^2_{S,S}\pi_j \partial^2_{x_j,x_j}\pi_j - \left(\partial^2_{x_j,S}\pi_j\right)^2 > 0$, otherwise the Hessian of $\pi_j(x_j, S)$ is not negative definite.

[12] The reader can be surprised by this asymmetrical treatment, but it is a matter of fact to observe that the first boundary assumption implies that $\forall x_j > 0$, $\lim_{S \to 0} \partial_{x_j}\pi_j / \left(\sum_{k=1}^{m} \partial_S\pi_j\right) < 1$ while the second induces that $\forall S > 0$, $\lim_{x_j \to 0} \partial_{x_j}\pi_j / \partial_S\pi_k > 1$. Interiority is therefore guaranteed in both the centralized and the decentralized problems.

decrease of the remaining stock of free idea. More formally we verify that:

$$\forall j = 1, \ldots, m \quad \partial_{x_j} \pi_j \left(x_j^d, y - x_j^d - \sum_{\substack{k=1 \\ k \neq j}}^{m} x_k^d \right) - \partial_S \pi_j \left(x_j^d, y - x_j^d - \sum_{\substack{k=1 \\ k \neq j}}^{m} x_k^d \right) = 0 \qquad (1)$$

It now remains to understand how the stock y of free Knowledge impacts the patenting strategies $\left(x_j^d(y) \right)_{j=1}^{m}$ and the equilibrium payoffs $\left(\pi_j^d(y) \right)_{j=1}^{m} := \left(\pi_j(x_j^d(y), y - \sum_{j=1}^{m} x_j^d(y)) \right)_{j=1}^{m}$. Since the University exerts a positive externality on the Private Labs, we expect that both function are increasing. This is immediate when all Labs are symmetric. In this case, condition (1) can be reduced to a single equation given by:

$$\partial_x \pi \left(x^d, y - m x^d \right) - \partial_S \pi \left(x^d, y - m x^d \right) = 0 \qquad (2)$$

and a standard computation leads to:

$$\frac{dx^d}{dy} = \frac{\partial_{x,S}^2 \pi - \partial_{S,S}^2 \pi}{\left(\partial_{S,x}^2 \pi - \partial_{x,x}^2 \pi \right) + m \left(\partial_{x,S}^2 \pi - \partial_{S,S}^2 \pi \right)} \leq \frac{\partial_{x,S}^2 \pi - \partial_{S,S}^2 \pi}{m \left(\partial_{x,S}^2 \pi - \partial_{S,S}^2 \pi \right)}$$

which implies that $\frac{dx^d}{dy} \in \left[0, \frac{1}{m} \right]$. It follows that the equilibrium profits $\pi_j^d(y)$ accruing to each firm are increasing with the stock y of useful ideas:

$$
\begin{aligned}
\frac{d\pi^d}{dy} &= (\partial_x \pi - m \partial_S \pi) \frac{dx^d}{dy} + \partial_S \pi \\
&= \left[1 - (m-1) \frac{dx^d}{dy} \right] \partial_S \pi \text{ (by the first order equilibrium conditions)} \\
&\geq 0 \text{ (since } \frac{dx^d}{dy} < \frac{1}{m})
\end{aligned}
$$

Using again symmetry, we also obtain that the aggregate equilibrium profits accruing to the private sector,

$$\Pi^d(y) := \sum_{j=1}^{m} \pi_j^d(y) = m \pi^d(y)$$

and the total patents allocation,

$$X^d(y) = \sum_{j=1}^{m} x_j^d(y) = m x^d(y)$$

are also increasing with y.

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By making use of a more elaborated argument presented in the appendix and based on the proof of proposition 1, we can even extend these results to the non-symmetric case and state that:

Proposition 2 *For any strictly positive free Knowledge stock y, the individual allocation of patents $x_j^d(y)$ and the profits accruing to each firm $\pi_j^d(y)$ are increasing with the stock y of useful ideas. And so are the aggregate allocation $X^d(y)$ and profits $\Pi^d(y)$.*

4   Overuse of free Knowledge and optimal patents taxation

It is first a matter of fact to observe that the Private Labs, at a Nash equilibrium, does not care about the external accommodation costs. In fact, when a Lab increases her patents, she knows that she modifies her own costs of exploiting an additional free idea, but she does not realize that she also increases the adoption costs for all the other Labs. This Tragedy of the Commons usually leads to an inefficient patents allocation characterized by a global over-investment in the patenting activity.

In order to present this problem, let us, as in the previous section, take as given the stock y of free Knowledge and let us look at a central planer allocation of the patents. Such an allocation is characterized in definition 2.

Definition 2 *Given any stock y of free Knowledge produced by the University, a centralized, also said decentralized, patents allocation is a vector $\left(x_j^c(y)\right)_{j=1}^m \in \mathbb{R}_+^m$ with the property that:*

$$\left(x_j^c(y)\right)_{j=1}^m \in \underset{(x_j)_{j=1}^m \in \mathbb{R}_+^m}{\arg\max} \sum_{j=1}^m \pi_j\left(x_j, y - X\right) \quad s.t. \quad y - X \geq 0$$

The reader immediately observes what is stated in the following remark.

Remark 1 *This problem admits a unique solution since:*
(i) the objective function is strictly concave because $\forall j$, $\pi_j\left(x_j, S\right)$ has this property,
(ii) as y is given, the domain of the problem is compact and convex.

Moreover, our restrictions on the marginal rates of substitution between x_j and S on the boundaries ensure that this solution is an interior one. The following first order conditions are therefore necessary and sufficient:

$$\forall j = 1, ..., m \quad \partial_{x_j} \pi_j\left(x_j^c, y - X^c\right) - \sum_{k=1}^m \partial_S \pi_k\left(x_j^c, y - X^c\right) = 0 \text{ with } X^c := \sum_{j=1}^m x_j^c \quad (3)$$

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By comparing equations (1) and (3), we observe that, in the centralized case, the congestion effects induced by the patenting activity is internalized by the central planner, i.e. the negative effect of a new patent on the stock of free Knowledge and so on the profits is considered in aggregate. In other words, it is the social accommodation costs of an idea which matter here, instead of the private costs like in a decentralized allocation. This suggests, if we have in mind the symmetric case, that each Lab should optimally invest less in patents. But this intuition does not extend to the asymmetrical case since one Lab should perhaps increase her activity while another should be refrained. We nevertheless prove that there is, at the aggregate level, an overuse of the free resource y. In fact we say that:

Proposition 3 *For a given stock of useful Knowledge y, the centralized allocation problem induces a lower aggregate level of patents than the decentralized one, i.e.*

$$X^c(y) := \sum_{j=1}^{m} x_j^c(y) < X^d(y) := \sum_{j=1}^{m} x_j^d(y)$$

while the aggregate profit is higher in the centralized case than in the decentralized one, i.e.

$$\Pi^c(y) := \sum_{j=1}^{m} \pi_j \left(x_j^c(y), y - X^c(y) \right) \geq \Pi^d(y) := \sum_{j=1}^{m} \pi_j \left(x_j^d(y), y - X^d(y) \right)$$

This last proposition also suggests that there is some room for a public policy since there is a gain in aggregate from the implementation of the centralized solution and it is immediate that this policy must be implemented by a patents public agency which has the ability to modulate the patents fees.

Let us first illustrate this argument in the symmetric case before moving to the more general case. In this simplified set-up, equation (3) becomes:

$$\partial_x \pi \left(x^c, y - mx^c \right) - m \partial_S \pi \left(x^c, y - mx^c \right) = 0 \tag{4}$$

By comparing equation (4) to (2), we observe that a patents agency which introduce a *pigovian tax* on each patent given by $t = (m-1) \partial_S \pi \left(x^c, y^d - mx^c \right)$, hence slows down the patents race by implementing an optimal allocation. But this also increases the gross profits (before taxation) accruing to each Lab. The patents agency has therefore even the opportunity to associate a lump-sum transfer, which leaves each firm at the same profit level as in the decentralized case and which is given by:

$$F = \pi \left(x^d, y - mx^d \right) - \pi \left(x^c, y - mx^c \right) + \mathbb{I}_{x>0} t x^c$$

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where $\mathbb{I}_{x>0} = 1$ if $x > 0$ and 0 otherwise. Moreover, by acting in this way, the patents agency will be sure that (i) the unique outcome of the decentralized patents allocation coincides to the efficient solution, (ii) the firms are willing to participate since they maintain their profits at their initial level, and (iii) all additional profits are collected for, as we will see later, further use.

This intuition straightforwardly extends to the asymmetrical case. We simply need to check that the set of Nash Equilibria (NE) of the game modified by the fiscal scheme coincides with the set of optimal allocations. Since we also know from the remark 1 that every centralized solution is unique, we can therefore conclude that the decentralized equilibrium of the modified game exists and is unique.

Proposition 4 *Let the stock of useful Knowledge y produced by the University be taken as given, if we introduce, in a decentralized patents allocation mechanism (see definition 1), a distortionary tax system on patents and a lump-sum transfer scheme, given respectively by:*

$$t_j(y) = \sum_{\substack{k=1 \\ k \neq j}}^{m} \partial_S \pi_k \left(x_k^c(y), y - X^c(y) \right)$$

and:

$$F_j(y) = \left[\pi_j \left(x_j^d, y - X^d \right) - \pi_j \left(x_j^c, y - X^c \right) \right] + \mathbb{I}_{x_j>0} t_j x_j^c$$

where $\mathbb{I}_{x_j>0} = 0$ if $x_j = 0$ and 1 if $x_j > 0$, we can assert that the unique decentralized patents allocation of the modified game corresponds to the efficient allocation introduced in definition 2.

Let us, finally, spell out some properties of the centralized solution. These results will help to conduct the discussion in our next section.

Proposition 5 *We observe that:*
(i) The implementation of this fiscal scheme leaves to the patents agency an amount of money $\Pi^c(y) - \Pi^d(y) \geq 0$, this inequality holding strictly when the decentralized allocation introduced in definition 1 is inefficient.
(ii) The aggregate profit $\Pi^c(y)$ at a centralized solution is increasing and strictly concave with respect to the amount y of freely available Knowledge.
(iii) At an efficient solution, the total amount $X^c(y)$ of free Knowledge which is applied in order to solve market puzzles as well as the residual stock $S^c(y)$ of unused free Knowledge are increasing with the stock y of freely available Knowledge.

5   Toward a reallocation of the patents fees to the University

The preceding section tells us, loosely speaking, that independently of the stock of free available Knowledge, the patents agency has always an incentive to collect fees in order to limit the over-appropriation of the free and potentially applicable Knowledge produced by the University. By doing so, this public agency implements an optimal allocation which leaves her some money back since she collects all the additional gains of the Private Labs with respect to a situation without regulation.

Now let us bring to mind that:

- the total amount $X^c(y)$ of innovations increases with the amount of freely available Knowledge (see (iii) of proposition 5). We can therefore expect, even if this is not a part of our model, that the welfare also increases with y since a larger amount $X^c(y)$ of scientific Knowledge can then be transformed into new demand driven technical solutions.

- The residual stock $S^c(y)$ of unused free Knowledge is also increasing with y (see again (iii) of proposition 5). This suggests, when the optimal patents allocation is implemented, that the social accommodation costs of new ideas decrease as y growths. This will again be welfare improving.

- The profits left to the Private Labs after taxation, and which correspond to the one they obtain at a Nash equilibrium, are increasing with the stock of free available Knowledge (see proposition 2). Private Labs are also better off when y increases.

These three observations therefore suggest that a patents public agency must not only collect fees in order to limit the over-appropriation of free Knowledge. She also ought to support more or less focused research programs whose results meet, at least partially, the peer validation criterion, so that these results stay in the public domain and increase the set y of potentially applicable Knowledge.

We know that the realization of such programs requires a deviation from the optimal research strategy dictated by the institutional arrangement being at work in the academic sector: the University supports a cost $C(\Delta)$ when an increase Δ of the amount y of free useful ideas is requested. So, by taking the position of a central planner or, here, of our patents public agency, the question becomes the following one: what is the optimal increase Δ that can be implemented by targeted research programs knowing that the patents allocation is an efficient one?

A natural way to answer this question consists in comparing the marginal cost supported by the University to the marginal gains of an increase of y when an optimal patents allocation is

implemented. In other words, we should set Δ to Δ^* given by:

$$\Delta^* \in \arg\max_{\Delta \geq 0} \Pi^c(y + \Delta) - C(\Delta)$$

And it is a matter of fact to observe what is explained in the next remark.

Remark 2 *There always exists a unique strictly positive solution to the previous program because (i) we know by proposition 5 that Π^c is increasing and strictly concave and (ii) we have assumed that $C(\Delta)$ is concave and verifies $\lim_{\Delta \to 0} \frac{dC}{d\Delta} = 0$ and $\lim_{\Delta \to +\infty} \frac{dC}{d\Delta} = +\infty$.*

But this answer gives us no indication on the implementation of this optimal solution in a way that meets the agreement of the University and the Private Labs. To be more precise, up to now, we know that a patents public agency has the ability, whatever the stock of free Knowledge is, to implement an efficient allocation which leaves each private Lab at least at the same level of profit as initially. But nothing insures that the maximum amount of money that can be collected when the stock of free available Knowledge is $(y + \Delta^*)$ covers the cost supported by the University in order to attain this level.

In order to check this, it becomes important to identify the outside options of the Private Labs and the University. So let us assume that the patents agency makes a take or leave offer including simultaneously a fiscal scheme for the first and a bundle of research contracts for the second. In this case, each private Lab compare her new profit level to the one she obtains in a decentralized patents allocation when no additional free Knowledge is produced while the University strikes a balance between a pure research strategy inducing no cost of change and returns induced by research contracts.

This suggests that the optimal increase Δ^* is implementable only if the sum of the deviation cost $C(\Delta^*)$ supported by the University and the aggregate profits of the Labs when no policy is implemented is smaller than the aggregate profit level obtained at the centralized solution.

In order to check that point, let us first consider the symmetric case. Under this configuration, let us denote by x^{c*} the symmetric centralized patents allocation for a free Knowledge stock being equal to $(y + \Delta^*)$, i.e. the quantity which solves[13]:

$$\partial_x \pi \left(x^{c*}, y + \Delta^* - mx^{c*} \right) - m\partial_S \pi \left(x^{c*}, y + \Delta^* - mx^{c*} \right) = 0$$

---

[13] This quantity, of course, exists and is unique because we have, in the last section, worked out our argument whatever y is. So it is especially true for $y = y_0 + \Delta^*$.

Let us bring to mind that x^d is the symmetric Nash patents allocation when no additional effort is asked to the University, i.e. the quantity which solves:

$$\partial_x \pi \left(x^d, y - mx^d \right) - \partial_S \pi \left(x^d, y - mx^d \right) = 0$$

Since the public patents agency is willing to implement x^{c*}, she must set her pigovian tax rate at:

$$t = (m - 1) \, \partial_S \pi \left(x^{c*}, y + \Delta^* - mx^{c*} \right)$$

But she knows that each private Lab compares her new situation to the one in which no policy is implemented at all, i.e. to the level $\pi \left(x^d, y - mx^d \right)$. This means that any lump-sum transfer associated to the pigovian patents tax must leave at least $\pi \left(x^d, y - mx^d \right)$ to each Lab. This transfer F is therefore such that:

$$F = \pi \left(x^d, y - mx^d \right) - \pi \left(x^{c*}, y + \Delta^* - mx^{c*} \right) + \mathbb{I}_{x>0} t x^{c*}$$

and the global maximal amount of money that can be extracted by the patents agency is given by:

$$T = \underbrace{m\pi \left(x^{c*}, y + \Delta^* - mx^{c*} \right)}_{\Pi^c(y+\Delta^*)} - \underbrace{m\pi \left(x^d, y - mx^d \right)}_{\Pi^d(y)}$$

From that point of view, the optimal increase Δ^* of the set of free Knowledge is implementable, if the maximum amount of tax that can be collected covers the cost $C(\Delta^*)$ of the research contracts that must be signed with the University.

In order to check this last point, let us remember (see proposition 5) that the aggregate profit level $\Pi^c(y)$ of the Private Labs at the centralized solution is strictly concave. By a standard concavity argument, we can therefore say that:

$$\Pi^c(y + \Delta^*) - \Pi^c(y) > (\Pi^c)' \big|_{y+\Delta^*} \times \Delta^*$$

By the definition of an optimal allocation, we know that $\Pi^c(y) \geq \Pi^d(y)$, so that:

$$T = \Pi^c(y + \Delta^*) - \Pi^d(y) \geq \Pi^c(y + \Delta^*) - \Pi^c(y)$$

Now let us remember that Δ^* is obtained by $(\Pi^c)' \big|_{y+\Delta^*} = C' \big|_{\Delta^*}$. By putting together the two previous equations, we obtain that:

$$T > C' \big|_{\Delta^*} \times \Delta^*$$

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Finally, let us recall that the cost function $C(\Delta)$ is convex and has the property that $C(0) = 0$. By a standard convexity argument, we then know that the marginal cost is always greater then the average cost. We can therefore conclude that:

$$T > C'\big|_{\Delta^*} \times \Delta^* \geq C(\Delta^*)$$

But the reader has surely observed that this last argument does not rely on symmetry. This is why we can generalize to what is stated in proposition 6.

Proposition 6 *Within our setting, a patents public agency has the ability, on the one hand, to control by a suitable fiscal scheme the over-appropriation of the free and potentially useful academic Knowledge in order to reduce the social appropriation costs and, on the other hand, to finance, with these fees, some research projects in order to reach an optimal transfer of Knowledge from the Universities to the Private Labs. In our specific model, a budget balanced fiscal scheme associated to an optimal transfer of $C(\Delta^*)$ to the University, which reaches the agreement of all actors, is given by:*

$$
\begin{aligned}
t_j &= \sum_{\substack{k=1 \\ k \neq j}}^{m} \partial_S \pi_k \left(x_k^{c*}, y + \Delta^* - X^{c*} \right) \\
\text{and } F_j &= \left[\pi_j \left(x_j^d, y - X^d \right) - \pi_j \left(x_j^{c*}, y + \Delta^* - X^{c*} \right) \right] + \mathbb{I}_{x_j > 0} t_j x_j^{c*}
\end{aligned}
$$

6 Concluding remarks

This last proposition concludes our paper. It tells us that the patents agencies are not only registration chambers which protect the innovators but are also a relevant instrument of an innovation policy. We essentially looked at this question by considering the relation between Science and Technology and more precisely the relation between public fundamental research and Private heterogeneous R&D Labs. Following Dasgupta and David [9], we assumed that these two institutions work with opposite institutional arrangements so that there is, on the one hand, an over-appropriation of the free applicable results produced by the University while there is, on the other hand, a relative scarcity of the amount of such applicable ideas since their evaluation by academic standards are less efficient. This over-appropriation issue was related to the existence of accommodation costs: an idea issued from the academic sector must be transformed in order to meet a social demand but all ideas do not have the same transformation costs.

This brings us to the conclusion that a patents public agency can be a suitable instrument

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of this relation regulation. This agency has the ability to control the over-appropriation of this Knowledge by setting suitable patents fees and therefore to contribute to the reduction of the accommodation costs. But she can also contribute to an optimal provision of free useful ideas by sponsoring, thanks to the returns of the patents fees, more or less targeted research programs, i.e. research programs whose results solve more applied puzzles but nevertheless meet the academic standards and which therefore increase the set of freely available ideas.

We must nevertheless concede that this paper can be too caricatural in several aspects. First of all, we essentially concentrates on patents which are induced by free academic Knowledge, as we said we only looked at upstream. We therefore neglected (i) the patents induced by innovations generated by the applied Knowledge developed by the Private Labs or (ii) the question of the renewal, breadth or length of a patent. The first point is linked to the sequential aspect of the innovation process and the problem related to the anti-common tragedy introduces by Heller and Eisenberg [14]. The second point refers to the kind of market protection from a private and a social point of view. This point has induced a lot of works since Nordhaus' [22] seminal argument in favor of the finiteness of the patents length[14]. But many of these studies often detect other sources of inefficiency that can, from our point of view, be corrected by an appropriate patents fees policy.

Furthermore, the reader surely observed that our argument is worked out in a complete information setting. From that point of view neither the University nor the Private Labs are able to capture an information rent. In other words, the introduction of imperfect information would lead to a second best solution since the patents agency has, on the one hand, to control a Bayesian game between the Private Labs and, on the other hand, to manage a standard Principal-Agent problem concerning her relation with the University.

Finally, we have also observed that the regulation introduced by the patents agency is globally improving. We can therefore, especially in the context of incomplete information, ask the question of the existence of another institutional arrangement that also render the system more efficient. However and at least in the short term, we hope that we have convinced the reader that we should see patents public offices not as profit centers, but as agencies in charge of the different aspects of the innovation policy.

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[14]In order to reach such a conclusion, Nordhaus considered decreasing returns to scale concerning R&D activities. Moreover, by defining the breadth of a patent according to the monopoly power it gives to its owner, Gilbert and Shapiro [12] concluded that a long and narrow patent is more likely to insure a given level of incentives. On the contrary, Gallini [11] proposed to also measure a patent breadth with the R&D costs needed to imitate a patented innovation outside the patent domain and proved that, within such a context, a short and large patent is better.

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APPENDIX

A   Proof of proposition 1

Step 1: $(x_j^d)_{j=1}^n$ *is a NE if and only if* $\forall j = 1, \ldots, n$

$$\partial_{x_j} \pi_j \left(x_j^d, y - x_j^d - \sum_{\substack{k=1 \\ k \neq j}}^m x_k^d \right) - \partial_S \pi_j \left(x_j^d, y - x_j^d - \sum_{\substack{k=1 \\ k \neq j}}^m x_k^d \right) = 0$$

Since the profit functions $\pi_j(x_j, S)$ are concave, standard concavity inequalities allow us to check that the profit function of each player is concave with respect to her own strategic variable. It therefore remains to verify that the constrains are not binding. If at equilibrium $S = y - x_j^d - \sum_{\substack{k=1 \\ k \neq j}}^m x_k^d = 0$, each player j has an incentive to decrease x_j^d since we have assumed that $\lim_{S \to 0} \partial_{x_j} \pi_j / \partial_S \pi_j < 1$. Now let us assume that $\exists j$ at equilibrium such that $x_j^d = 0$. In this case this player has an incentive to increase x_j^d because one of our boundary conditions says that $\forall S > 0$, $\lim_{x_j \to 0} \partial_{x_j} \pi_j / \partial_S \pi_j > 1$.

Step 2: *An application of the Implicit Function Theorem (IFT).*

Let us define $H_j(x_j, X, y) := \partial_{x_j} \pi_j (x_j, y - X) - \partial_S \pi_j (x_j, y - X)$ and let us look at $H_j(x_j, X, y) = 0$ for any fixed y (it will be omitted for the moment in order to spare notations). Since we have assumed that $\forall S > 0$, $\lim_{x_j \to 0} \partial_{x_j} \pi_j / \partial_S \pi_j > 1$ and $\forall S > 0$, $\lim_{x_j \to +\infty} \partial_{x_j} \pi_j / \partial_S \pi_j < 1$, we respectively observe that $\forall X < y$, $\lim_{x_j \to 0} H_j(x_j, X) > 0$ and $\lim_{x_j \to +\infty} H_j(x_j, X) < 0$. But we also know that $\partial_{x_j} H_j(x_j, X) = \partial_{x_j, x_j} \pi_j - \partial_{S, x_j} \pi_j < 0$. The IFT therefore says that:

$$\exists \phi_j :]0, y[\to \mathbb{R}_{++} \mid H_j(\phi_j(X), X) = 0$$

and that:

$$\partial_X \phi_j = -\frac{\partial_X H_j}{\partial_{x_j} H_j} = -\frac{-\partial^2_{x_j, S} \pi_j + \partial^2_{S, S} \pi_j}{+\partial^2_{x_j, x_j} \pi_j - \partial^2_{S, x_j} \pi_j} < 0,$$

We can even go a step further and observe that:

$$\forall X < y, \quad \phi_j(X) > 0 \text{ and } \lim_{X \to y} \phi_j(X) = 0$$

The last point is immediate. We have observed that $\forall X < y$, $\lim_{x_j \to 0} H_j(x_j, X) > 0$ and that $\partial_{x_j} H_j < 0$ so that any solution $\phi_j(X)$ must be strictly positive for $X < y$. Moreover, we have assumed that $\forall x_j > 0$, $\lim_{S \to 0} \partial_{x_j} \pi_j / \partial_S \pi_j < 1$, so that $\forall x_j > 0$, $\lim_{X \to y} H_j(x_j, X) < 0$. If we have in mind that $\partial_{x_j} H_j(x_j, X) < 0$, the equation $H_j(x_j, X) = 0$ cannot admit a strictly positive solution as X goes to y. But we just check that $x_j = \phi_j(X) > 0$ for all $X < y$. We can therefore conclude that $\lim_{X \to y} \phi_j(X; y) = 0$.

Step 3: *The computation of an equilibrium.*

Let us now construct $\Phi(X) := \sum_{j=1}^m \phi_j(X) - X$. By step 2, we know that $\lim_{X \to y} \phi_j(X) = 0$, and that $\forall X < y$, $\phi_j(X) > 0$, we deduce respectively that $\lim_{X \to y} \Phi(X) < 0$ and that $\lim_{X \to 0} \Phi(X) > 0$. By computation, we also observe that $\partial_X \Phi = \sum_{j=1}^m \partial_X \phi_j - 1 < 0$. We can therefore assert that $\exists X^d \in]0, y[$ a unique scalar satisfying $\Phi(X^d) = 0$. So by moving back to step 2, we can say that there exists a unique vector $(x_j^d)_{j=1}^m = (\phi_j(X^d))_{j=1}^m$

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which verifies the condition stated in equation (1). Moreover it is immediate, by construction, that $x_j^d > 0$ and that $\sum_{j=1}^m x_j^d = X^d < y$ for all $y > 0$.

B   Proof of proposition 2

Let us now remark that the result of proposition 1 is true for each y and let us remember that the function ϕ_j introduced in step 2 of the previous proof also takes y as an argument. By applying the IFT, we even observe that $\partial_y \phi_j(X, y) = -\partial_X \phi_j(X, y)$. So if we move to step 3 of the previous proof, we can, by using again the IFT, observe that:

$$\frac{dX^d}{dy} = -\frac{\sum\limits_{j=1}^m \partial_y \phi_j(X, y)}{\left(\sum\limits_{j=1}^m \partial_X \phi_j(X, y) \right) - 1} = \frac{\sum\limits_{j=1}^m \partial_X \phi_j(X, y)}{\left(\sum\limits_{j=1}^m \partial_X \phi_j(X, y) \right) - 1} > 0 \text{ since } \forall j, \, \partial_X \phi_j(X, y) < 0$$

Moreover,

$$
\begin{aligned}
\frac{dx_j^d}{dy} &= \partial_X \phi_j(X^d(y), y)\frac{dX^d}{dy} + \partial_y \phi_j(X^d(y), y) = \partial_X \phi_j(X^d(y), y)\left[\frac{\sum\limits_{j=1}^m \partial_X \phi_j(X^d(y), y)}{\left(\sum\limits_{j=1}^m \partial_X \phi_j(X^d(y), y) \right) - 1} - 1 \right] \\
&= \frac{\partial_X \phi_j(X^d(y), y)}{\left(\sum\limits_{j=1}^m \partial_X \phi_j(X^d(y), y) \right) - 1} > 0
\end{aligned}
$$

It follows that:

$$\frac{d\pi_j^d}{dy} = \partial_x \pi_j \frac{dx_j^d}{dy} + \left(1 - \frac{dX^d}{dy} \right) \partial_S \pi_j = \partial_x \pi_j \frac{dx_j^d}{dy} + \frac{\partial_S \pi_j}{1 - \left(\sum\limits_{j=1}^m \partial_X \phi_j \right)} > 0$$

which implies that:

$$\frac{d\Pi^d}{dy} = \sum_{j=1}^m \frac{d\pi_j^d}{dy} > 0$$

C   Proof of proposition 3

This proof is going to be very closed to the one of proposition 1 and we will directly use some of its elements, contained in steps 2 and 3.

Step 1: *A preliminary observation.*

Let us define, for a fixed y, $H_j^d(x_j, X, y) := \partial_{x_j} \pi_j (x_j, y - X) - \partial_S \pi_j (x_j, y - X)$ and $H_j^c(x_j, X, y) := \partial_{x_j} \pi_j (x_j, y - X) - \sum_{k=1}^m \partial_S \pi_k (x_k, y - X)$. We know from the proof of proposition 1 that $\exists \phi_j^d(X, y)$ with the property that $H_j^d(\phi_j^d(X, y), X, y) = 0$. Since we have assumed that $\forall S > 0$, $\lim_{x_j \to 0} \partial_{x_j} \pi_j / \sum_{j=1}^m \partial_S \pi_j > 1$ and $\lim_{x_j \to +\infty} \partial_{x_j} \pi_j / \partial_S \pi_j < 1$, we can, by a similar argument, also prove that $\exists \phi_j^c(X, y)$ such that $H_j^c(\phi_j^c(X, y), X, y) = 0$ and that $\partial_X \phi_j^c(X, y) < 0$. Now remember that $\forall j, \, \partial_S \pi_j > 0$. It follows that $\forall(x_j, X, y), \, H_j^c(x_j, X; y) < H_j^d(x_j, X; y)$. But we also know that $\partial_{x_j} H_j^d < 0$ and $\partial_{x_j} H_j^c < 0$. By combining these two observations, we obtain:

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$$\forall j, \forall\, (X, y)\,,\ \phi_j^c(X, y) < \phi_j^d(X, y)$$

Step 2: $\forall y,\ X^c(y) < X^d(y)$

Let us now construct $\Phi^c(X, y) := \sum_{j=1}^m \phi_j^c(X, y) - X$ and $\Phi^d(X, y) := \sum_{j=1}^m \phi_j^d(X, y) - X$. The proof of proposition 1 tells us that $\exists X^d(y) \in\,]0, y[$ verifying $\Phi^d(X^d, y) = 0$. With a similar argument requiring the boundary condition associated to the centralized problem, it is also easy to prove that $\exists X^c(y) \in\,]0, y[,\ \Phi^c(X^c, y) = 0$. Now remember, from step 1, that $\forall\, (X, y)\,,\ \phi_j^c(X, y) < \phi_j^d(X, y)$. It follows that $\forall\, (X, y)\,,\ \Phi^c(X, y) < \Phi^d(X, y)$. But we know that $\partial_X \Phi^c < 0$ and $\partial_X \Phi^d < 0$. We can therefore conclude that $X^c(y) < X^d(y)$.

Step 3: $\forall y,\ \Pi^c(y) \geq \Pi^d(y)$

This follows directly from the definition of a maximum.

D Proof of proposition 4

Let us first recall that a decentralized patent equilibrium associated to the fiscal scheme:

$$(t_j, F_j)_{j=1}^m = \left(\sum_{\substack{k=1 \\ k \neq j}}^m \partial_S \pi_k \left(x_k^c, y - X^c \right),\ \left[\pi_j \left(x_j^d, y - X^d \right) - \pi_j \left(x_j^c, y - X^c \right) \right] + \mathbb{I}_{x_j > 0} t_j x_j^c \right)$$

is a vector $(\tilde{x}_j)_{j=1}^m$ of patents allocation with the property that:

$$\forall j = 1, \ldots, m,\ \tilde{x}_j \in \operatorname*{arg\,max}_{x_j \in \mathbb{R}_+^m} \pi_j \left(x_j, y - x_j - \sum_{\substack{k=1 \\ k \neq j}}^m \tilde{x}_k \right) - t_j x_j + F_j \quad s.t. \quad y - x_j - \sum_{\substack{k=1 \\ k \neq j}}^m \tilde{x}_k \geq 0 \qquad (5)$$

Step 1: *Every equilibrium of the modified game is an interior one.*

Let us first verify that $\forall j,\ \tilde{x}_j > 0$. The definition of the fiscal scheme (t_j, F_j) matters. In fact, we know that $t_j x_j^c > 0$ since (i) $x_j^c > 0$ i.e. efficient allocations are interior solutions and (ii) $\forall j,\ \partial_S \pi_k (x_k, S) > 0$. So, if $\exists j_0,\ \tilde{x}_{j_0} = 0$, this player obtains a strictly smaller transfer F_j than in a situation where she plays $\varepsilon > 0$. Because of this jump and since $\pi_{j_0}(x_{j_0}, S)$ is continuous, we can choose an ε_0 with the property that j_0 is better off when $x_{j_0} = \varepsilon_0$ is played.

Now assume that $\exists j_0,\ y - \tilde{x}_{j_0} - \sum_{\substack{k=1 \\ k \neq j}}^m \tilde{x}_k = 0$. Since $y > 0$, we can choose j_0 such that $\tilde{x}_{j_0} > 0$. But we have assumed that $\forall x_j > 0,\ \lim_{s \to 0} \partial_{x_j} \pi_j / \partial_S \pi_j < 1$ and we know that $t_j > 0$. Player j_0 has therefore an incentive to decrease x_j^d because he increases her gross profit $\pi_{j_0}(x_{j_0}, S)$ and decreases the tax she pays.

Step 2: *The modified game has at least an equilibrium.*

By step 1, we knows that an equilibrium verifies:

$$\forall j = 1, \ldots, m \quad \underbrace{\partial_{x_j} \pi_j \left(\tilde{x}_j, y - \tilde{X} \right) - \partial_S \pi_j \left(\tilde{x}_j, y - \tilde{X} \right) - \sum_{\substack{k=1 \\ k \neq j}}^m \partial_S \pi_k \left(\tilde{x}_k^c, y - \tilde{X}^c \right) = 0}_{:= H_j(\tilde{x}_j, \tilde{X})\ \text{with}\ \tilde{X} := \sum_{j=1}^m \tilde{x}_j}$$

and since $\pi_j(x_j, S)$ is concave we can argue as in the proof of proposition 1 that these conditions are not only

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necessary but also sufficient. We can even observe that the centralized solution obtained in definition 2 solves this system of equations. This modified game admits therefore at least one equilibrium given by $\left(\tilde{x}_j^c\right)_{j=1}^m$.

Step 3: *The modified game has a unique equilibrium.*

Assume that $\exists \left(\tilde{x}_j\right)_{j=1}^m \neq \left(\tilde{x}_j^c\right)_{j=1}^m$ another equilibrium. First, let us assume that $\tilde{X} = \tilde{X}^c$, and let us choose j_0 such that $\tilde{x}_{j_0} \neq \tilde{x}_{j_0}^c$. Since $\partial_x H_{j_0}(x_{j_0}, \tilde{X}) < 0$ it is impossible that $H_j(\tilde{x}_j, \tilde{X}) = H_j(\tilde{x}_j^c, \tilde{X}) = 0$. Now assume that $\tilde{X} > \tilde{X}^c$, there exists therefore at least one j_1, such that $\tilde{x}_{j_1} > \tilde{x}_{j_1}^c$. Since $\partial_X H_{j_0}(x_{j_0}, X) > 0$, we have $H_{j_0}(\tilde{x}_{j_0}, \tilde{X}) > H_{j_0}(\tilde{x}_{j_0}, \tilde{X}^c)$ and since $\partial_x H_{j_0}(x_{j_0}, \tilde{X}^c) < 0$ we observe that $H_{j_0}(\tilde{x}_{j_0}, \tilde{X}) > H_{j_0}(\tilde{x}_{j_0}, \tilde{X}) = 0$, a contradiction. Finally, observe that symmetric argument works when $\tilde{X} < \tilde{X}^c$.

E   Proof of proposition 5

(i) This point is obvious.

(ii) By the envelop theorem, we immediately obtain that $\Pi^c(y)$ is increasing since:

$$\frac{d\Pi^c(y)}{dy} = \sum_{j=1}^m \partial_S \pi_j \left(x_j^c(y), y - X^c(y)\right) > 0$$

It remains to check that $\Pi^c(y)$ is concave. So let us choose $y^1, y^2 \in \mathbb{R}$ and $\lambda \in]0, 1[$. Moreover let us denote by $(x_j^1)_{j=1}^m$ and $(x_j^2)_{j=1}^m$ the optimal patents allocation under respectively y^1 and y^2. It is a matter of fact to observe that $\lambda x_j^1 + (1 - \lambda)x_j^2$ is a feasible allocation when the stock is $\lambda y^1 + (1 - \lambda)y^2$:

$$\sum_{j=1}^m \left(\lambda x_j^1 + (1 - \lambda)x_j^2\right) \leq \lambda y^1 + (1 - \lambda)y^2$$

By definition of a maximum which is unique in our case, we have:

$$
\begin{aligned}
\Pi^c\left(\lambda y^1 + (1-\lambda)y^2\right) &= \sum_{j=1}^m \pi_j\left(\lambda x_j^1 + (1-\lambda)x_j^2, \lambda y^1 + (1-\lambda)y^2 - \sum_{j=1}^m \left(\lambda x_j^1 + (1-\lambda)x_j^2\right)\right) \\
&= \sum_{j=1}^m \pi_j\left(\lambda x_j^1 + (1-\lambda)x_j^2, \lambda\left(y^1 - \sum_{j=1}^m x_j^1\right) + (1-\lambda)\left(y^2 - \sum_{j=1}^m x_j^2\right)\right) \\
&> \lambda \sum_{j=1}^m \pi_j\left(x_j^1, y^1 - \sum_{j=1}^m x_j^1\right) + (1-\lambda)\sum_{j=1}^m \pi_j\left(x_j^2, y^2 - \sum_{j=1}^m x_j^2\right)
\end{aligned}
$$

since for $j = 1, \ldots, m$, $\pi_j(x, S)$ is concave in S. Thus, we can conclude that:

$$\Pi^c\left(\lambda y^1 + (1-\lambda)y^2\right) > \lambda \Pi^c(y^1) + (1-\lambda)\Pi^c(y^2)$$

(iii) Let us come back to the proof of proposition 3. We have observed that $\Phi^c(X, y) := \sum_{j=1}^m \phi_j^c(X, y) - X = 0$ where $H_j^c(\phi_j^c(X, y), X, y) = 0$ for all $j = 1, \ldots, m$ and $H_j^c(x_j, X, y) := \partial_{x_j} \pi_j(x_j, y - X) - \sum_{k=1}^m \partial_S \pi_k(x_k, y - X)$. If follows by the IFT that:

$$\frac{dX^c}{dy} = -\frac{\sum_{j=1}^m \partial_y \phi_j^c}{\sum_{j=1}^m \partial_X \phi_j^c - 1}, \ \partial_X \phi_j^c(X, y) = -\frac{-\partial_{x_j, S}^2 \pi_j + \sum_{k=1}^m \partial_{S, S}^2 \pi_k}{\partial_{x_j, x_j}^2 \pi_j - \partial_{S; x_j}^2 \pi_j} \text{ and } \partial_y \phi_j^c(X, y) = -\partial_X \phi_j^c(X, y)$$

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But we have assumed that for all $j = 1, \ldots, m$, $\partial^2_{x_j, S} \pi_j > 0$, $\partial^2_{x_j, x_j} \pi_j < 0$ and $\partial^2_{S, S} \pi_j < 0$. We deduce that $\partial_X \phi^c_j(X, y) < 0$, $\partial_y \phi^c_j(X, y) > 0$ and that $\dfrac{dX^c}{dy} > 0$. It remains to check that $S^c(y) = y - X^c(y)$ is increasing. This is immediate by computation since:

$$\frac{dS^c}{dy} = 1 - \frac{dX^c}{dy} = 1 - \frac{\sum\limits_{j=1}^{m} \partial_X \phi^c_j}{\sum\limits_{j=1}^{m} \partial_X \phi^c_j - 1} = \frac{-1}{\sum\limits_{j=1}^{m} \partial_X \phi^c_j - 1} > 0$$

F Proof of proposition 6

This proof is obvious.