MEASURES OF THE GEOGRAPHIC CONCENTRATION OF INDUSTRIES:

IMPROVING DISTANCE-BASED METHODS

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ABSTRACT

This study introduces two new measures of spatial concentration. The proposed *M functions* constitute an extension to Ripley's functions (Ripley, 1976, 1977). They allow the evaluation of the relative geographic concentration and co-location of industries in a non-homogeneous spatial framework. Some rigorous comparisons with similar recently developed tools prove the relevance of the *M functions* in the field of spatial economics.

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I Introduction

“Step back and ask, what is the most striking feature of the geography of economic activity? The short answer is surely *concentration*”. Krugman (1991, p.5).

This citation echoes a long list of studies analyzing the geographic distribution of production since Marshall (1890) and Weber (1909). As underlined by Paul Krugman in the first pages of his book *Geography and Trade*, economic activities are definitely not homogeneously distributed. A great number of studies in the economic literature have already attempted to explain theoretically and estimate empirically the determinants of industrial location (Fujita and Thisse, 2002 ; Rosenthal and Strange 2003, 2004). During the last decade, the appraisal of the degree of spatial concentration economic activities has received increasing attention. Economists improved the measurement of the geographic concentration of economic activities in order to evaluate more accurately the observed agglomeration. For instance, Duranton and Overman (2005) suggest some fundamental properties for a “good concentration index”. It should (i) compare the geographic concentration results across industries, (ii) control for industrial concentration, (iii) control for the overall aggregation patterns of industries, (iv) test the significance of the results and, (v) keep the empirical results unbiased across geographic scales. Some additional properties have been recommended by other studies like Combes and Overman (2004) who mention that the industrial classification may constitute another bias (by aggregating sub-sectors) and recommend a theoretical foundation for the index in order to facilitate an economic explanation of the results. To the best of our knowledge, no existing measure respects all of the above criteria.

In this article, we propose new statistical tools for the evaluation of spatial concentration which respect as many as possible properties listed above. More precisely, two measures are given to improve the evaluation of intra- and inter-industrial geographic concentration. We called them the *M functions* because they constitute an extension of existing distance-based methods, namely Ripley’s *K function* (1976, 1977), Besag’s *L function* (1977), and their various extensions based on the second-order property of point patterns. They are now widely applied in other scientific fields such as forestry and ecology[1] but they suffer from important drawbacks for applications in economics (see Marcon et Puech (2003) for a critical analysis). In what follows, we provide a theoretical proof of the usefulness of the *M functions*. A comparison with other similar distance-based methods demonstrates the effectiveness of these tools in quantifying the spatial concentration of economic activities. The *M functions* obey a number of the fundamental statistical properties listed in the previous paragraph that existing measures do not integrate.

In order, to evaluate the geographic distribution of establishments, economists have traditionally employed cluster-based methods, *i.e.,* they measure the spatial concentration of economic activity according to pre-defined geographic limits (regions, counties…). It is now widely admitted that these methods such as the Gini and the Ellison and Glaeser (EG) indices[2] introduce a statistical bias resulting from the chosen notion of space. Cluster-based methods resort to a specific zoning of the territory: dividing space into a set of geographical units raises the well-known issue of the Modifiable Areal Unit Problem (MAUP) which can be summarized as follows: *"the result will be sensitive to the shape, size, and position of the areal units chosen"* (Morphet 1997, p.1039). This problem is a violation of property (v) and the use of such measures is problematic.[3] The solution to this problem was found by using a continuous approach to space (Arbia and Espa 1996; Marcon and Puech, 2003; Duranton and Overman 2005, 2008). In this case, the statistician does not resort to any zoning and only Euclidean distances between plants are considered (each plant of the sample is localized by its coordinates (*x*,*y*)). Unlike measures that only describe the location of economic activity at a single scale, distance-based methods detect the spatial structures at all scales. The geographic concentration phenomenon is evaluated by counting the average number of neighbors of plants in a circle of a given radius (*r*). This operation is then repeated for all the possible radii. The main advantage of these methods is the fact that the particular distance at which a significant geographic concentration or dispersion of establishments exists is detected.[4] We provide two significant improvements to Ripley's function in this paper.

[1] A survey of empirical studies in ecology or forestry using the *K* or *L function* is given in Puech (2003, p.324).

[2] For instance, see Fratesi (2008) for a review of the main spatial concentration indices.

[3] An empirical estimation of the shape and size bias resulting from different French territory zonings could be found in Briant *et al*. (2008).

[4] Cluster-based methods only disclose whether the distribution is concentrated or dispersed at a particular geographic level.

First, Ripley's function measures absolute concentration: it rests on the null hypothesis of a completely random spatial distribution of establishments (*i.e.*, plants are distributed uniformly and independently). Although the debate for implementing absolute or relative measures is not new (see Haaland *et al.*, 1999), relative measures are more widely used. Comparing a sector distribution to that of the whole industry is even one of the theoretical criteria (property iii) defined by Duranton and Overman (2005) or Combes and Overman (2004). Relative measures detect for each industry if an overrepresentation or an underrepresentation exists (with respect to a reference distribution which could be the overall location pattern of industries). In other words, statistical tools based on relative concentrations effectively measure the existence of specialized territories.[5]

Secondly, Ripley's function does not control for industrial concentration that is the productive concentration within an industry among plants belonging to this sector (property ii): every establishment is considered to be a plot, regardless of its size. Recently, Duranton and Overman (2005) proposed the *K-density* (denoted *Kd*) function. However, even if their function has some valuable properties because it fulfills the five previously mentioned fundamental criteria, it still has some mathematical problems (Marcon and Puech, 2003). The *M functions* constitute a significant improvement on existing distance-based measures and overcome those mathematical difficulties. Our measures fulfill all of the five fundamental properties of Duranton and Overman and also integrate other appreciable qualities such as being easily applied to any underlying geography (in contrast to Ripley's function) or the interpretation of the results (more easily than Duranton and Overman's function).

Our study is organized as follows. In the next section, the *M functions* are introduced and their mathematical properties discussed. Next, a comparison with other similar existing distance-based methods is given to prove the importance of these two new measures (section III). The last section concludes.

[5] A discussion on the limits of relative indices can be found in the appendix A of Mori *et al.* (2005).

II Improving Ripley's functions: introduction of the *M functions*

In this section, we propose two versions of a new statistical tool, the *M function*, for the measurement of intra- and inter-industrial geographic concentration. We first give an intuitive presentation of the common framework. We then successively define the functions and discuss their properties.

1 Intuition

Our relative measure compares the location patterns of an economic sector to that of aggregate activity (represented by all sectors). For this, we develop a cumulative function counting neighboring points up to a chosen distance denoted *r*. Consider points as plants, located on a map. We choose:

- a reference point type, say a specific sector,
- a target neighbor type called *T*: the same sector for intra-industrial concentration or another one for inter-industrial concentration.

The average number of target neighbors is compared to a benchmark to detect whether they are more or less frequent than if plants were distributed randomly and independently from each other. To control for variations in the local density of points, each number of target neighbor (T_i around a point *i*) is normalized by the number of all neighbors in the same area (N_i). Around each reference point: we obtain a ratio of target neighbors (T_i/N_i) within the distance *r* from each point *i*. The average of this ratio ($\overline{T_i/N_i}$) is compared to the global ratio of the target type (T/N) calculated on the entire territory. If $\overline{T_i/N_i}$ is greater than T/N, we conclude that more plants of the target type are observed within a distance *r* around points of the reference type than on average, if we drew circles of radius *r* anywhere. In other words, target points are concentrated around reference points. The ratio $M = \dfrac{\overline{T_i/N_i}}{T/N}$ will be used in our analysis for convenience because the benchmark is equal to one. *M* values are computed on a large distance range and presented as a continuous function of *r* on a graph including confidence intervals for the null hypothesis of independence of plant locations (significance is controlled by appropriate statistical tests). Any value of *M*, thanks to these successive normalizations, can be interpreted immediately and compared across sectors and distances. Finally, points can also be weighted, counting, for example, the number of employees instead of the number of plants. Finally, tests of significance must properly control for the non-independence of their distribution (*i.e.* industrial concentration).

2 Evaluating the geographic intra-industrial concentration

In mathematical terms, let us consider an area *A* containing a total number of *N* plants of a variety of industries. We focus on a particular industry *S* where N_S is the total number of establishments of that sector in the territory *A*. The description of the N_S plants' neighborhood follows. Consider a dummy variable $c_S(i, j, r)$ which is equal to 1 if the Euclidian distance between the two plants *i* and *j* of the sector *S* is less than the radius *r* ($c_S(i, j, r) = 0$ otherwise). The number of neighboring establishments of the plant *i*, belonging to the same sector and located within a distance *r* from it, is thus $\sum_{j=1, i\neq j}^{N_S} c_S(i, j, r)$. In the same way, we define the dummy $c(i, j, r)$ to be equal to 1 if the plant *j* (whatever its industry) is located at a distance inferior or equal to *r* from the establishment *i* (the dummy's value is 0 otherwise). Consequently, the number of establishments located at most at a distance *r* from the unit business *i* is: $\sum_{j=1, i\neq j}^{N} c(i, j, r)$. Plants' size may now be included in our analysis. For each dummy, the weight associated is that of the neighboring plant *j*, we denote it w_j. Plants' weight may be their number of employees. The average proportion of employees of the industry *S* at a given radius *r* is clearly:

$$\frac{1}{N_S}\sum_{i=1}^{N_S}\frac{\sum_{j=1, i\neq j}^{N_S} c_S(i, j, r) w_j}{\sum_{j=1, i\neq j}^{N} c(i, j, r) w_j}$$

In the same way, we can define the ratio of employees of the industry *S* on the entire territory *A* compared to the whole industry by: $\frac{1}{N_S}\sum_{i=1}^{N_S}\frac{W_S - w_i}{W - w_i}$ where W_S is the total number of *S* employees on the territory *A*; *W*, that of the aggregate activity and w_i, the weight of the plant *i*.[6] The ratio of the above quantities, averaged on all establishments of sector *S*, defines the *M function* for the intra-industrial geographic concentration of the *S* sector as:

$$M_S(r) = \sum_{i=1}^{N_S} \frac{\sum_{j=1, i\neq j}^{N_S} c_S(i,j,r) w_j}{\sum_{j=1, i\neq j}^{N} c(i,j,r) w_j} \Bigg/ \sum_{i=1}^{N_S} \frac{W_S - w_i}{W - w_i} \qquad (1)$$

The numerator corresponds to the relative weight of sector *S* in comparison with the whole industrial activity in the circles of radius *r*. The denominator represents the relative weight of the considered sector in comparison with all activities in the territory *A*. The benchmark of the *M function* is 1 and defines the same location pattern for the specific sector as for the aggregate activity. This means that whatever the considered radius, there are proportionally as many employees who belong to sector *S* as there are in the global area *A*. Thus, *M* values superior to 1 $(M_S(r) > 1)$ indicate that there are proportionally more employees close to plants in sector *S* (within a distance *r*) relative to the whole area. This corresponds to the relative geographic concentration definition of the sector *S* at a distance *r*. In contrast, the geographic relative dispersion of the sector *S* at a distance *r* is defined by $M_S(r) < 1$, indicating that there are relatively fewer employees in sector *S* within a distance *r* around the establishments than in the whole area. One can see that the *M* values can be easily interpreted. For instance, $M_S(r) = 2$ indicates that, within a particular distance *r*, the relative density of employees in the sector *S* is double that in the whole area. In the same way, $M_S(r) = 0.5$ indicates that within a given distance *r* around *S* plants the density of employees in this sector is on average half that of the whole area.

How can the significance of the results be tested and how can the industrial concentration be controlled? Two types of confidence intervals of the null hypothesis are generated: local and global ones. The null hypothesis is that establishments belonging to sector *S* are located following the same pattern as the others. To test this, we generate a series of random and independent distributions of the plants dataset by conserving: the actual set of possible locations and the industry/size plants couples (*i.e.*, the industrial concentration as given). The local confidence interval is determined using the Monte-Carlo method. Practically, we generate a large number of simulations and a confidence level, say 5%, is chosen. The 95% confidence interval of

[6] w_i must be subtracted from W_S and W since we count the number of establishments around the plant *i* within a radius *r*: the reference establishment *i* itself should not be counted.

M function for each value of *r* is delimited by the outer 5% of the randomly generated values. Nearly all empirical studies resorting to Ripley's *K function* (or one of its extensions) only compute the local confidence intervals to test the significance of the results. However, Duranton and Overman (2005) recently criticized the single computation of the local confidence intervals, considering them as too "optimistic", and they highlight the need for the global confidence intervals of the null hypothesis as well. Supposing the values of the *M function* at different radii are independently distributed, a proportion of them equal to the confidence threshold is expected to be outside the confidence interval even though the point process corresponds to the null hypothesis. For instance at a 5% threshold, complete spatial randomness should not be rejected when 5 points of a curve made of 100 points leave the confidence interval. Successive values of Ripley's functions are actually highly correlated: the risk of erroneous rejection of the null hypothesis is consequently reduced but cannot be quantified. A global confidence is defined such that the confidence threshold is the risk that the curve of a function generated by the null hypothesis exceeds the interval at least once. It may be chosen in many ways but it should have an equal weight at all distances. A simple method to compute global confidence is by generating local confidence intervals at increasing confidence levels until the ratio of simulated curves leaving them reaches the fixed threshold. As an example, suppose that 1,000 curves have been generated, and the confidence level is fixed to 5%. The outer values at each distance are eliminated, defining a local confidence interval at 2/1,000 = 0.2%. The curves leaving this interval are counted. Suppose 10 curves are concerned, the global confidence level is then 1%. The process is repeated until the threshold is reached. If it is not reached exactly, interpolation is used.

Five fundamental criteria characterizing a "good" measure of geographic concentration in economics were presented in the introduction, it should be noted that the *M function* respects all of them. Moreover, as we underlined above, an appreciable property of this index is that values of the *M function* may be interpreted. Additionally, the *M function* can be calculated for any topology. Ripley's function and its developments (see Goreaud and Pélissier, 1999) require edge-effect correction for points which are close to borders, so complex geographical shapes are intractable hence the domain is always a polygon or a disc.[7] The *M function* presents an answer to this problem: comparing the number of neighbors of a certain industry to the number of neighbors of all establishments *in the same area* avoids any need for correction. Working on complex geographical limits, such as a country's boundaries, is now possible. The last remark justifies the somewhat complex computation of the *M function*. Software downloaded from the authors' website[8] is available in order to facilitate its implementation.

3 Evaluating the co-location of industries

Evaluating the co-location of industries may be interesting if the researcher suspects some interactions between them. The geographic concentration between industries can be investigated using the inter-industrial version of the *M function* which possesses the same properties as the intra-industrial one. In what follows, we consider the co-location between two sectors denoted S_1 and S_2. A complete description of the spatial distribution of the co-location patterns of these industries leads not to one but to two definitions of the *M functions*. The first one M_{S_1,S_2} depicts the spatial structure in non-homogenous space of plants belonging to sector S_2 around those of sector S_1. The second function M_{S_2,S_1} describes the spatial structure of plants belonging to sector S_1 around those of sector S_2. The meaning of the co-location *M functions* is thus simple: we test whether the relative density of employees of one sector located around establishments of another sector is on average greater or lesser than that on the whole territory.

Let us consider the same territory A and the same notations as used in the previous section. We now examine the Euclidian distances between plants of two different industries. At first, we consider the definition of M_{S_1,S_2}: the reference plants are establishments of the industry S_1 (*i.e.* those at the centre of the circles). The definition of the co-location $M_{S_1,S_2}(r)$ function is

[7] Sweeney and Feser (1998) fig.1, p.52 or Feser and Sweeney (2000) fig.2, p.361; Pancer-Koteja *et al.* (1998) fig.1, p.757; Rowlingson and Diggle (1993) fig.5, p.634.

[8] http://e.marcon.free.fr/Ripley (English and French versions).

hence:

$$M_{S_1,S_2}(r) = \sum_{i=1}^{NS_1} \frac{\sum_{j=1}^{NS_2} c_{S_2}(i,j,r) w_j}{\sum_{n=1,i\neq n}^{N} c(i,n,r) w_n} \Bigg/ \sum_{i=1}^{NS_1} \frac{W_{S_2}}{W - w_i} \qquad (2)$$

According to the value of the expression (2), we can observe whether the relative density of plants S_2 located around those of sector S_1 is greater ($M_{S_1,S_2}(r) > 1$) or less ($M_{S_1,S_2}(r) < 1$) than that observed on the entire area *A*. In the same manner, we can define the function $M_{S_2,S_1}(r)$ which describes the spatial structure of plants S_1 located around those of sector S_2. The alterations to the function are obvious:

$$M_{S_2,S_1}(r) = \sum_{i=1}^{NS_2} \frac{\sum_{j=1}^{NS_1} c_{S_1}(i,j,r) w_j}{\sum_{n=1,i\neq n}^{N} c(i,n,r) w_n} \Bigg/ \sum_{i=1}^{NS_2} \frac{W_{S_1}}{W - w_i} \qquad (3)$$

Concerning the significance of the results, both local and global confidence intervals to the null hypothesis will be computed but we have to draw particular attention to the null hypothesis. The Monte Carlo techniques are retained for the generation of simulated distributions (the threshold, the number of simulations are exogenous). Nevertheless, the null hypothesis has to eliminate the sector specific patterns in order to detect only interactions between the two industries. For instance, if S_1 is highly aggregated and S_2 completely randomly distributed, the relative importance of S_2 plants around S_1 establishments is low and an artificial segregation is detected. In these conditions, the null hypothesis must control for both the S_1 and S_2 patterns. The solution is as follows. The null-hypothesis plants set for M_{S_1,S_2} is generated by keeping S_1 establishments fixed and redistributing all other plant size/sector couples amongst all other locations, thus controlling for the S_1 pattern. To be sure that no under or over estimation of the density of plant's employees is caused by the structure of the industry S_2, we also need to control for the structure of S_2: the same process applied to M_{S_2,S_1} controls for the S_2 pattern. Lastly, a significant interaction is accepted if both values are significantly different from their respective null hypothesis. Note that the null hypothesis excludes the detection of a "multi-concentration" phenomenon: a situation in which we could observe a significant co-location which does not result from an interaction between these two industries (this would be the case for instance if both industries locate around the plants of another industry). This is undoubtedly a limit of the inter-industry *M function* shared with all other distance-based functions.[9]

III Towards an unified framework on distance-based methods

The incentive of using distance-based methods rather than cluster-based methods is now well established in the literature. However, the choice of retaining one particular distance-based method is generally weakly motivated. In what follows, we compare the statistical properties of distance-based methods and especially those of the two leading geographic concentration measures namely the *Kd* and the *M function*.[10] Our aim is to disclose in which cases one measure should be preferred to another. The heart of our discussion focuses on the implications of using probability density functions rather than cumulative ones because this remains the main difference between the *Kd* and the *M function*. Surprisingly, the preference of using probability density function or a cumulative function has never been argued in empirical economic papers.[11] The only exception is Duranton and Overman (2005) who claim in the conclusion of their paper that *"K-densities are more informative than [Ripley's] K-functions with respect to the scale of localization"* without any demonstration. Their idea will be more deeply analyzed hereinafter. The main conclusion of the advantages/drawbacks comparison is that relative-distance based measures are generally more complements than substitutes.

1 Common statistical framework

Historically, methods characterizing the structure of point processes as a function of bilateral distances between pairs of points have been developed by Ripley (1976, 1977).

[9] The term *co-localisation* is employed by Duranton and Overman (2005) if the co-agglomeration indeed results from an attractive pattern of both industries whereas they prefer employing *joint-localisation* in case of a significant co-agglomeration of both industries occurring for another reason. However, they mention that they can not disentangle empirically from these location patterns.

[10] As we underlined in the introduction, the debate for implementing absolute, topographic or relative measures for evaluating the geographic concentration has been recently settled since one of the criteria of a "good" concentration measure states that relative indices must be preferred (Duranton and Overman, 2005; Combes and Overman, 2004). We take this result for granted in our paper.

[11] See for instance Fratesi (2008) for an empirical application of the *Kd* or Arbia and Espa (1996), Barff (1987), Ó hUallacháin and Leslie (2007) and Arbia *et al.* (2008) for different applications of cumulative distance-based methods.

Ripley defined the function $g(r)$ as the ratio of the probabilities of finding two points at a distance *r* from each other to the product of the probabilities of finding each of them. If points are distributed independently, $g(r)$=1; higher values show that point pairs at this distance are more frequent than under the null hypothesis of independence. The integral function $K(r)=\int_{\rho=0}^{r} g(\rho)2\pi\rho\,d\rho$ is easy to estimate.[12] Assuming the point density is uniform on an area *A* and denoting by *N* the total number of points on the domain *A*, we find (Sweeney and Feser, 1998 for example):

$$\hat{K}(r)=\frac{A}{N(N-1)}\sum_{i=1}^{N-1}\sum_{j>i}^{N}c(i,j,r) \tag{4}$$

In a space where edge effects do not occur (say a torus), $c(i,j,r)$ is a dummy: its value is 1 if the distance between points *i* and *j* is less than *r*. In the real world, it is corrected for edge effects: when the point *i* is close to the border of the domain, it has less neighbors because those outside the domain are not observed. After computing $\hat{K}$, $g(r)$ can be estimated by $\hat{g}(r)=\frac{\hat{K}(r+\Delta r)-\hat{K}(r-\Delta r)}{2\Delta r}$, taking Δr arbitrarily small. $\hat{g}$ is proportional to the number of point pairs whose distance is close to *r*.

K is a cumulative function, while *g* is a local function. $\hat{K}$ has been widely used in the literature, but $\hat{g}$ has not. Both are restricted to homogenous point processes. To characterize inhomogenous point sets, to control for the spatial distribution of the whole economy, further mathematical developments are necessary. Duranton and Overman (2005) chose to define the *Kd* function as the probability density function of point-pair distances. *Kd* is also proportional to the number of point pairs whose distance is close to *r*. The differences between *Kd* and $\hat{g}$ are: (i) *Kd* integrates appropriate smoothing, but this is only a technical improvement, and (ii) *Kd* does not correct for any edge effects. Its value is compared to those of point distributions with the same geometry, which have the same edge effects. We chose another way. From equation 4 and retaining the same definitions of T_i and *T* as those given in section II.1., it follows that $\hat{K}$ can be rearranged as:

[12] For more details, see Marcon and Puech (2003) for a concise presentation of this function.

$$\frac{\hat{K}(r)}{\pi r^2} = \frac{\sum_{i=1}^{N-1} \frac{\sum_{j>i}^{N} c(i,j,r)}{\pi r^2}}{N} \bigg/ \frac{N-1}{A} = \frac{\sum_{i=1}^{N} T_i / N_i}{N} \bigg/ T/N \qquad (5)$$

It is thus a particular case of *M*.

To summarize, all these functions are derived from the raw data that is the number of point pairs at a given distance. Ripley's $\hat{g}$ is normalized so that its value is 1 when points are distributed independently. Duranton and Overman's *Kd* is normalized to be a probability density function. Ripley's $\hat{K}$ is the cumulative function of $\hat{g}$. It can be interpreted as the ratio of the observed number of neighbors to what it would be if points where distributed independently. Our *M* function is its generalization in non-homogenous space.

2 Theoretical examples and consequences

Four examples highlight the main properties of *Kd* and *M*. We successively study an absence of spatial structure, a geographically concentrated distribution, and then two spatially concentrated distributions with or without some regularity aspects. In what follows, we consider theoretical examples where there are only two industries on a 10 x 10 territory. The weight of each plant is equal to 1 to simplify the examples and the confidence intervals are computed at a 1% threshold, from 10,000 simulations (only global confidence intervals are shown on the figures). The *Kd* and *M functions* and global intervals are computed at intervals of 0.5 up to a radius of 10.

a Poisson distribution

The first industry A has 50 units of production, the second one (industry I) has 500 plants. Establishments from both industries are randomly distributed over the whole area (two Poisson distributions). The distribution of plants of both industries is shown on figure 1. The evaluation of the geographic concentration of the industry A (triangles on figure 1) is given using the *Kd* measure (figure 2) and the *M function* (figure 3).

Figure 1: First theoretical distribution of establishments on a 10x10 area

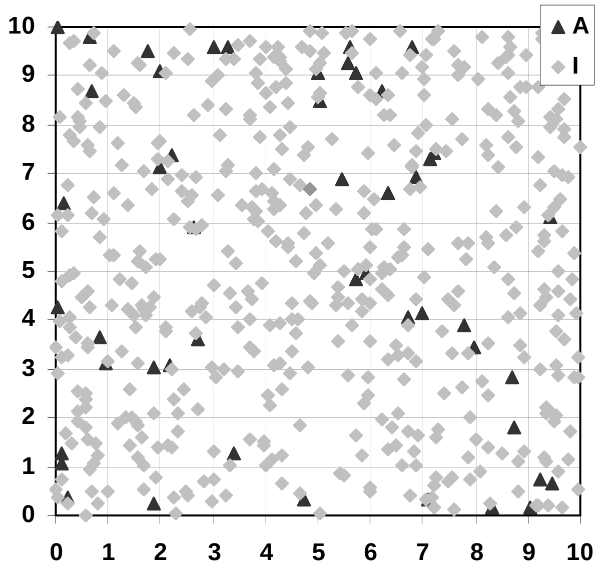


Figure 2: *Kd function* for industry A

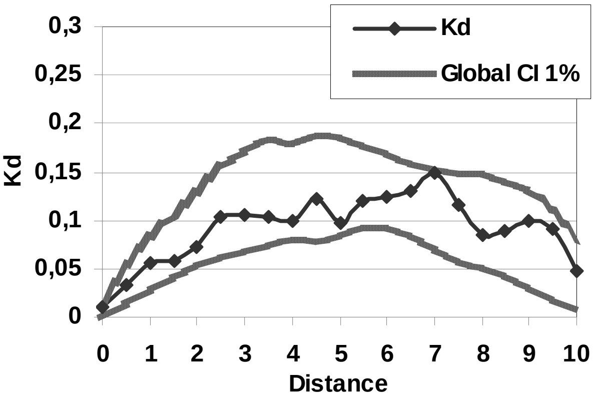
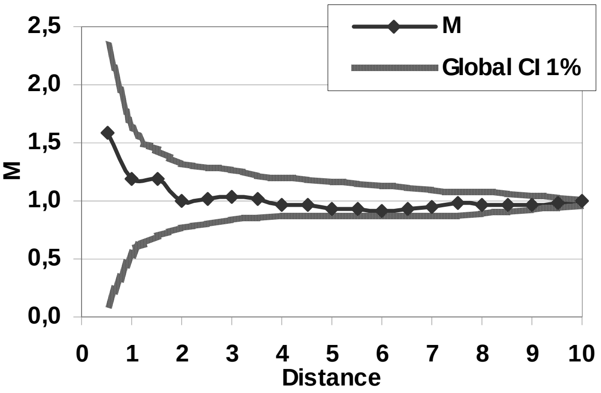


Figure 3: *M function* for industry A



As we see from figures 2 and 3, the curves of *Kd* as well as the *M function* for the randomly distributed sector A never leave the confidence interval of the null hypothesis. The main result in that case is that both measures give similar conclusions: no spatial pattern can be detected for the industry A.

Note that general results can be highlighted on this example. First, consider the *M function* (figure 3). At very small distances (when *r* is lower than the smallest distance between two points), *M* is not defined since no point has any neighbor. Then, neighbors are scarce so the effect of randomness is important: confidence intervals are wide. Finally, at larger distances, all values converge to 1: when *r* is greater than the maximum distance between two points, all points are neighbors. Second, consider the *Kd function* (figure 2). Duranton and Overman (2005) recommend to analyze the spatial pattern up to the median distance between all pairs of plants. However, in what follows, results are given for all possible radii to completely describe the behavior of the functions (even though this should not be done in empirical studies). Moreover, the general inversed U-shape of the confidence interval can be noted. From the first radius to half of the length of the area, the maximum-minimum bands of the confidence intervals grow: more point pairs are found as the bilateral distance increases. Afterwards, they gradually decrease and return to zero: edge effects get more importance with distance until no neighbor can be found because they would have to be outside the domain.

b Aggregated distribution

Let us turn to the second theoretical example where the first industry labelled I has 200 units of production randomly distributed (Poisson distribution) over the whole area. The second industry B is generated by a Matérn process (Matérn, 1960 cited by Stoyan *et al.*, 1987): 50 points are uniformly generated in one cluster of radius 0.5.

Figure 4: Second theoretical distribution of establishments on a 10x10 area

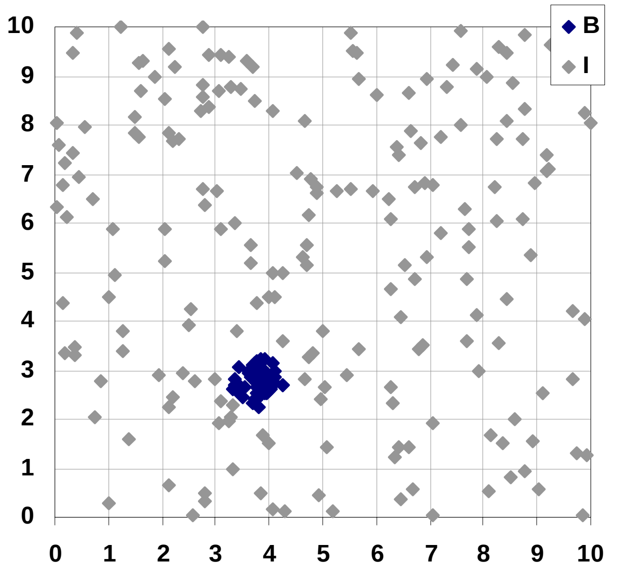


Figure 5: *Kd function* for industry B

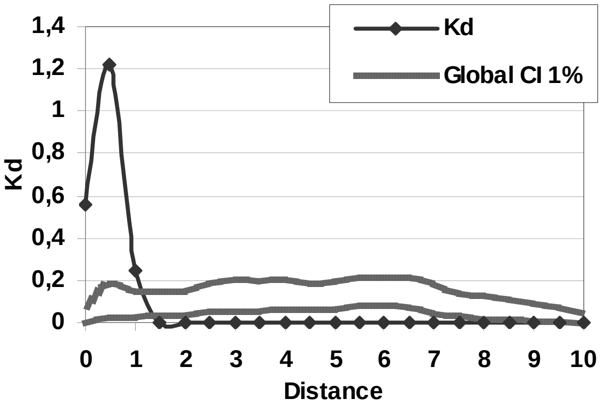
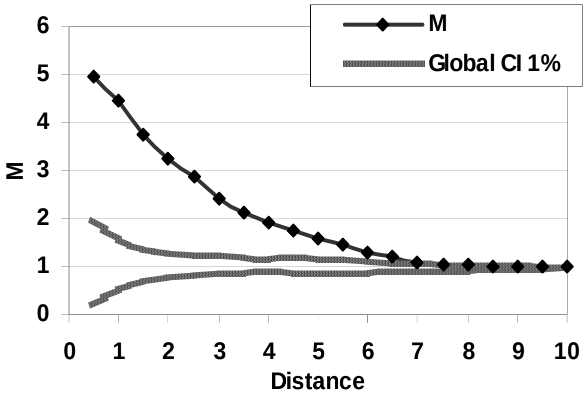


Figure 6: *M function* for industry B



The first positive significant peak of the *Kd* (figure 5) and the *M functions* (figure 6) appears at a radius of 0.5. At this distance, the circular cluster of the Matérn process is well detected by both measures. However, at larger distances, *Kd* and *M* curves are clearly different. *Kd* is rapidly equal to 0 and between a distance from 1.50 to 10 its values are below the global confidence interval. This *does not* indicate a repulsion of plants *but* a significant lack of industry B's plants. It is a compensatory effect: since the probability density is high at low distances, it is necessary low at larger distances in order to sum to 1. *M* slowly returns to the confidence interval (see figure 6) without detecting repulsion, in line with the point process. Nevertheless, *M* as any cumulative function is less precise to the scale of localisation than any probability density function (including the *Kd* function). Some imprecision on the plants distribution analysis emerge here because *M* returns gradually and not suddenly inside the confidence interval bands (even if there are no industry B neighbors' after a radius of 0.5). Both functions give pertinent and complementary information on the spatial structure.

c More complex spatial patterns

The complementarity of the *Kd* and *M* functions can be underlined in more complex examples. Let us consider several clusters in the same area 10 x 10. In what follows, our purpose is to distinguish an independent distribution of clusters from repulsion.

In the first example, clusters of industry C are generated by a Matérn process: 50 plants are uniformly generated in 9 clusters of radius 0.5 randomly distributed on the domain. 500 establishments of another industry called I are randomly distributed according to a Poisson distribution. The map of the distribution of plants is given in figure 7 and results of the *Kd* and *M* for the industry C are shown in figures 8 and 9.

Figure 7: Third theoretical distribution of establishments on a 10x10 area

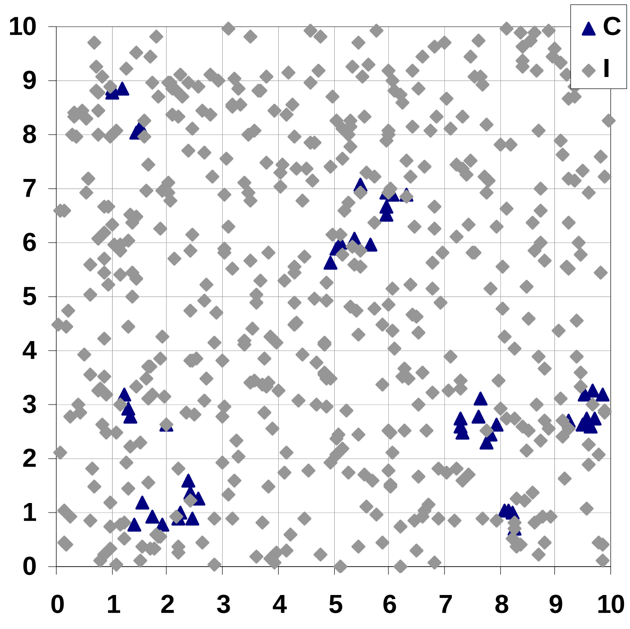


Figure 8: *Kd function* for industry C

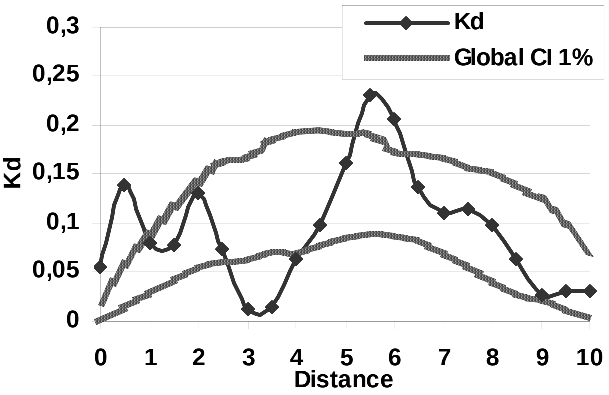
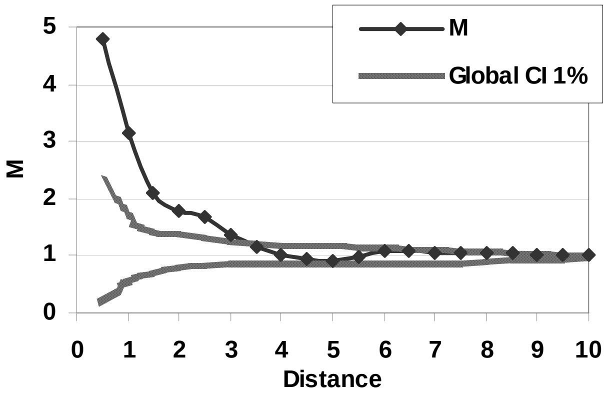


Figure 9: *M function* for industry C



In the second example, the same industry I composed of 500 plants randomly distributed is located on the same territory. However, the 50 establishments of the industry C are now distributed in 9 clusters regularly distributed on a squared grid (figure10).

Consider the *Kd* curves (figures 8 and 11) for the industry C. A first significant peak is detected in the radius equal to 0.5 corresponding to the size of the clusters. The *M function* corroborates the *Kd* finding. Then, both *Kd* functions pointed out alternatively some positive significant peaks (corresponding to the relative position of aggregates[13]) and negative significant ones (corresponding to dispersion). We are unable to differentiate without additional analysis if the significant peaks of *Kd* below (or above) the confidence interval correspond to compensation or a true repulsion (or attraction) of industry's C neighbors. Estimations of the *M function* give the answer to that dilemma: only in the second case, a significant repulsion is detected between 2.5 and 4 and between 7 and 8.5. The *M* curve is below the confidence interval at those distances: the *M function* clearly detects the regular position of the clusters on the grid.

Figure 10: Fourth theoretical distribution of establishments on a 10x10 area

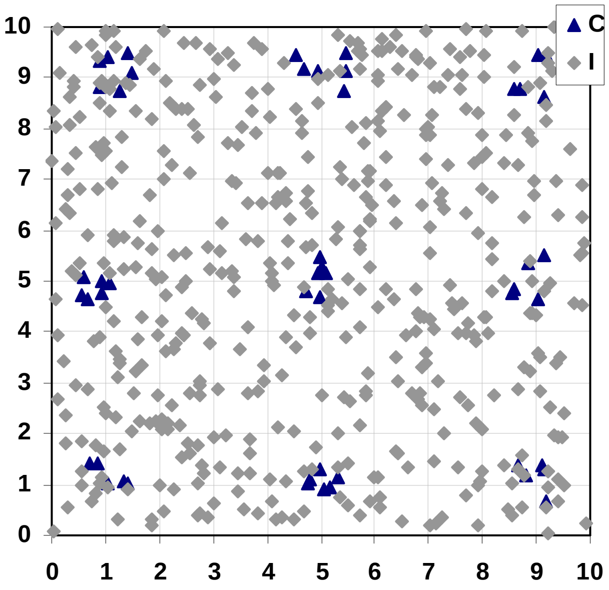


Figure 11: *Kd function* for industry C

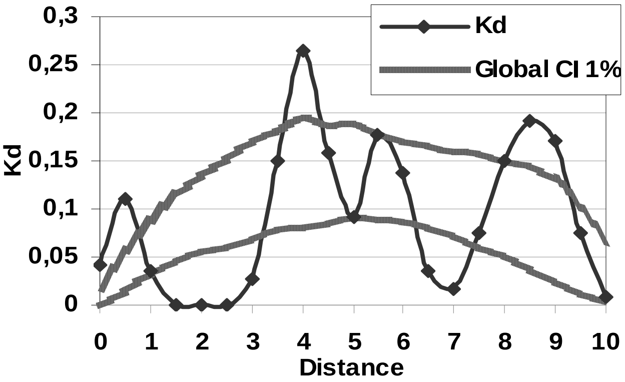
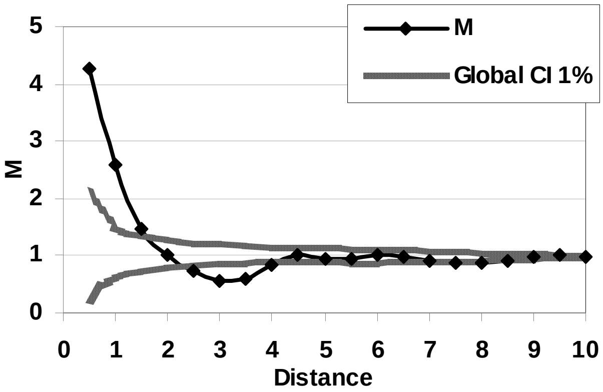


Figure 12: *M function* for industry C



[13] For instance on figure 10, note that the first neighbors of industry C plants outside the cluster appear at a radius approximately equal to 4 (the grid size) and then around 8 (the double of grid size).

What can we learn from those examples? First, the *Kd* and *M* measures present some advantages and drawbacks in the evaluation of spatial structures. Secondly, they seem more complementary than substitutable. On the one hand, the *Kd* function shows the excess or the lack of neighbors that *M* can not systematically detect. On the other hand, the *M function* evaluates the cumulated effects of successive patterns and can distinguish compensation from opposite strength (see the last two examples of the section).

IV Conclusion

The aim of this paper was to improve some existing relative statistical tools to test the spatial concentration of industries. We present evidence that *M functions* constitute first-class instruments to evaluate the intra- or inter-industrial geographic concentration. Nonetheless, some intrinsic limits of these new tools suggest different research agendas to fill the gap between the theoretical and empirical literatures. Despite the considerable recent interest of researchers in an “ideal” concentration index and even if significant progress has been made, work still needs to be done to bridge the last two criteria, the most difficult to respect: the complete integration of the tools to economic theory and the independence of geographic concentration measures to the industrial classification (Combes and Overman, 2004). In this paper, we have enhanced existing distance-based methods but further investigations are still required.

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