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Résumé: Ce papier réexamine la question de la forme optimale des incitations en termes de concurrence et coopération dans un contexte d'aléa moral avec plusieurs agents. Une analyse détaillée de la connaissance imparfaite sur la technologie permet d'obtenir de nouveaux résultats. En général, le schéma optimal est collectif quand la production est complémentaire, et compétitif quand les efforts sont substituts. En conséquence, l'idée très répandue que le principal doit utiliser un schéma d'autant plus concurrentiel que les performances sont corrélées à l'équilibre n'est pas valide quand les agents sont neutres au risque. En effet, des niveaux de corrélation croissant avec les efforts des agents créent une forme de complémentarité informationnelle, qui plaide pour des schémas collectifs. Quand les agents sont averses au risque, cet effet informationnel est à mettre en balance avec la demande d'assurance des agents. Le schéma optimal est alors soit concurrentiel, soit mixte, ce qui est une nouveauté dans la littérature. Les schémas mixtes peuvent être interprétés comme l'utilisation conjointe de plans de partage des gains et de clauses de licenciement ou de promotion.

Abstract: This paper reexamines the issue of competitive versus collective incentives in a multiagent moral hazard framework. A detailed analysis of imperfect knowledge on the technology - or uncertainty - is the key to new results. The baseline fact is that under complementarity the optimal scheme is collective, while it is competitive under substitutability. As a consequence, the widespread idea that a principal should use all the more competitive scheme that the equilibrium outcomes are more correlated is shown not to hold under risk-neutrality. This is so because equilibrium correlation levels increasing with effort create a form of informational complementarities, and therefore pleads for cooperative schemes. When the agents are risk-averse, that informational effect has to be traded off against the agents' insurance concerns. The optimal scheme is then either competitive or mixed, a novelty in the contracting literature. Mixed schemes can be interpreted as the use of aggregate profit sharing in combination with selective firing or promotion.

Mots clés : Aléa moral multi-agent; corrélation; incertitude; Evaluation jointe versus Evaluation relative

Key Words : Multi-agent moral hazard; Correlation; Uncertainty; Relative vs Joint Performance Evaluation.

Classification JEL: D8, L2, O3.

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1 Introduction

When the performances of many agents are correlated, agency theory traditionally indicates that competition between them allows to reduce incentive costs. This paper offers a reconsideration of this view by giving a closer look at the informational structure of multiagent settings. In particular, the analysis unveils a new informational effect of correlation that sometimes runs counter to the traditional perspective. The common view says the following: if performances are positively correlated, a good result of one agents indicates a favorable environment, and therefore one agent's compensation should depend negatively on the other's performance in order not to reward luck. An alternative view says in turn that when performances get more correlated when both agents exert more effort, a good performance of one agent is a good indication that the other also worked hard, and therefore high performance of one agent should positively influence the other's reward. The analysis shows that the first view is indeed incomplete in that it does not take properly into account how the correlation varies with the choice of action.

With multiple agents, the literature has since the beginning emphasized the role of relative performance evaluation, prominently the early tournament literature, e.g. Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983). Those authors have shown in particular that competition between agents is all the more valuable that the common risk associated with individual production increases[1]. This is rooted in the multiagent application of Holmström (1979) sufficient statistics result, by Holmström (1982) and Mookherjee (1984). However, this key result only asserts that the reward for one agent should depend on the performance of the other agent, but by no means commands that the type of evaluation should be relative. It is often mistakenly taken as an argument for competitive provision of incentives. From a broad theoretical point of view, results are not that clear-cut. On the one hand works on general stochastic structures in multiagent problems - e.g. Mookherjee (1984); Ma (1988); Brusco (1997) and d'Aspremont and Gérard-Varet (1998) for the case of partnerships - can not provide answer on the desirability of competition in teams, and on the other hand, specific works on that topic consider special informational structures or restricted contract forms (e.g. Holmström and Milgrom, 1990; Ramakrishnan and Takor, 1991; Itoh, 1992; Che and Yoo, 2001). The setting analyzed in the following helps contributes to filling this gap by pro-

[1]This is also the case with correlated private information. Competition allows to crosscheck messages and reduce rents, see Demski and Sappington (1984) for an early contribution.

viding a framework that generalizes many models on the topic, while still allowing for a full characterization of the optimal incentive scheme. In particular, it is possible to derive results also for the case of non-uniform correlation of the results[2] which happens to be crucial. A broader interpretation of the information structure including technological uncertainty and imperfect knowledge, not only observation noise, is then possible.

Following Che and Yoo (2001), we will use the terminology relative performance evaluation (RPE) and joint performance evaluation (JPE) to distinguish the two usual ways of paying the agents[3]. A central issue is when one or the other kind of scheme is the optimal one. We consider both the cases of risk-neutral agents subject to limited liability, and the case of risk-averse agents under a standard participation constraint. Those two cases constitute in fact the two sides of the same coin. This allows to decompose the problem between the two main effects at work: the informational effect, by which the principal gets information on the technology and infers effort, and the insurance effect associated with relative performance evaluation.

Under this general information structure, the scope for relative performance evaluation is reduced compared to what is usually believed. In particular, correlation of performances is not per se an argument in favor of RPE. On the contrary, more correlation of equilibrium performance sometimes pleads for less competitive schemes: The informational effect speaks in favor of management through induced cooperation rather than competition. When including insurance concerns (with risk-aversion), this effect is weakened, but a collective bonus is still optimal in some circumstances. In those cases, the optimal scheme has the following appealing interpretation: a basic incentive tool is aggregate profit sharing (provided by stock-options, shares distribution and the like), used in combination with selective promotions or failures. To our knowledge, that type of schemes

[2]By construction, the linear-exponential-normal model has a uniform correlation over effort pairs. In Mookherjee (1984) and Ramakrishnan and Takor (1991), the papers closest to our setting regarding the information structure, the level of correlation is in fact the same irrespective of the action chosen, see Mookherjee (1984, pp. 441-442) and Ramakrishnan and Takor (1991, pp. 259-260, in particular the beginning of section 4.2)

[3]We abstract from (potentially beneficial) cooperative agreements between the agents. Side-transfers and better observation among the agents may by themselves be reasons for cooperative schemes, see for example Holmström and Milgrom (1990) and Itoh (1992). See also Baliga and Sjöström (1998) for a model of sequential efforts in a limited liability framework close to the one developed here. Their focus is however on collusion, a topic not treated here.

has not been theoretically studied, nor has its (second-best) efficiency been demonstrated.

One should also underline the link with some models of ambiguity. As in traditional in the agency literature, the model stays within the comfortable framework of subjective expected utility and Bayesian probabilities. That is we use the Anscombe and Aumann (1963) framework, and therefore preserve the reduction of compound lotteries property and keep additivity of all probability (i.e. we do not use Gilboa and Schmeidler (1989) or Ghirardato et al. (2004) constructions to introduce uncertainty aversion). But the multiagent nature of the framework is itself the source of a Bayesian ambiguity effect in the problem. In that respect, the closest paper is Halevy and Feltkamp (2005). Essentially, they extend Ellsberg's famous thought experiment by one draw: bets are on two successive draws from the same urn. Their results is that under such circumstances, a purely Bayesian decision-maker will be *uncertainty*-averse as soon as he is *risk*-averse. Roughly[4], the present model has in common shares with that paper the "double draw effect". An interesting aspect is that even under risk-neutrality of the agents, the principal can use uncertainty, and he is in fact better off with ambiguity than without.

In the next section, we set up the model and state the main definitions. Then in the third section we derive the optimal contract under risk-neutrality and limited liability and apply it to various examples. The fourth section deals with risk-aversion and insurance. The last section concludes. Omitted proof are relegated to the appendix.

2 Model under risk-neutrality and preliminary analysis

2.1 Basics

We consider a setting of moral hazard in which two agents, called 1 and 2, work on two different projects. The principal and the agents share the same beliefs on the technology. All players are risk-neutral in the present and the following section, and we relax this assumption in the fourth section by considering risk-averse agents. Following Itoh (1991), Ramakrishnan and Takor (1991) and Che and Yoo (2001), the outcome of each project is either a success or a failure, worth respectively S and F to the principal, and these outcomes are contractible. We denote by $\boldsymbol{R}$ a generic result pair, taking value in $\{SS, SF, FS, FF\}$.

[4]This is a rough parallel in that it abstracts from incentive concerns.

Each agent privately chooses whether he exerts effort or not: agent 1 chooses $e \in \{0, 1\}$ and agent 2 chooses $f \in \{0, 1\}$. Those actions are never observed by the other players. We assume without loss of generality that $c(e) = c.e$ and $c(f) = c.f$. The probability of obtaining outcome $\boldsymbol{R}$ conditional on effort pair (e, f) is $Prob[\boldsymbol{R}|e, f]$. Finally, we consider only projects for which it is worth inducing the agents to work, which amounts to assume that $S - F$ is sufficiently high compared to c. This also renders the assumption of identical payoffs and costs innocuous.

2.2 Incentive Schemes

The performances of the agents are generically related, so that their wages should be tied. The incentive scheme (or wage profile) for agent 1 is thus a collection

$$\boldsymbol{w} = \{w_{SS}, w_{SF}, w_{FS}, w_{FF}\}$$

that represents the wage he receives contingent on his result - the first index - and on the second agent's result - the second index. The wage scheme of agent 2 is denoted by $\boldsymbol{x}$ and follows the same conventions. In practice, the problem is symmetric and separable and we mainly focus on the case of agent 1 only. Given an outcome-contingent wage scheme $\boldsymbol{w}$ and a pair of efforts (e, f), the expected payoffs are:

$$U_1(\boldsymbol{w}|e, f) = \mathbb{E}_{\boldsymbol{R}}\left[w_{\boldsymbol{R}}|e, f\right] - c(e)$$
$$U_2(\boldsymbol{x}|e, f) = \mathbb{E}_{\boldsymbol{R}}\left[x_{\boldsymbol{R}}|e, f\right] - c(f)$$

From each agent's point of view, there are overall three stochastic sources in that payment: first, there is imperfect knowledge on the technology - a specificity of this paper, second, the result of his effort is non-deterministic - as is standard in moral hazard settings, and third his remuneration also depends on the other agent's performance - a feature of the two-agent setting. The expectation operator pertains to the three random elements. Since there is a finite number of outcomes, we can write the expected wage as a weighted average, where the weight on $w_{\boldsymbol{R}}$ is $Prob(\boldsymbol{R}|e, f)$.

A central question of the analysis is: When will the principal use competition as an incentive device, and when will he prefer to induce cooperative behavior between the agents? To give a precise content to this question, we borrow from Che and Yoo (2001) the typology for the incentive systems.

Definition 1 *(Standard incentives schemes)*
An incentive scheme exhibits ***Relative Performance Evaluation*** *(RPE) when:*

$$(w_{SF}, w_{FF}) > (w_{SS}, w_{FS})$$

An incentive scheme exhibits ***Joint Performance Evaluation*** *(JPE) when:*

$$(w_{SS}, w_{FS}) > (w_{SF}, w_{FF})$$

An incentive scheme exhibits ***Independent Performance Evaluation*** *when:*

$$(w_{SS}, w_{FS}) = (w_{SF}, w_{FF})$$

The inequalities represent component-wise comparison with at least one strict inequality. With RPE, an agent is better off when the other fails, while it is the converse with JPE. Therefore, RPE is competitive while JPE gives collaborative incentives[5]. Note that these three types of scheme do not exhaust the possible ordering of wages, and indeed additional configurations appear in the fourth section.

2.3 Implementation and generic results

As already mentioned, it is assumed that the principal wants both agent to exert effort, so that we limit ourselves to minimizing the cost of implementing $(1, 1)$ as a Nash equilibrium (not necessarily a unique one). The incentive constraints for each agent provided the other exerts effort write:

$$U_1(\boldsymbol{w}|1, 1) \geq U_1(\boldsymbol{w}|0, 1) \tag{1}$$

$$U_2(\boldsymbol{x}|1, 1) \geq U_2(\boldsymbol{x}|1, 0) \tag{2}$$

In addition, we assume that the agents are subject to limited liability, so that the following constraints hold:

$$\boldsymbol{w}, \boldsymbol{x} \geq 0 \tag{3}$$

[5]Strictly speaking, there is no collaboration in the case of a JPE scheme, since an agent can not influence the other's result in this model beyond his single effort choice. Accounting for wider possibilities would require to enrich the action space to account for sabotage (Lazear, 1989) or help (Itoh, 1991). Note also that JPE and RPE rises different collusion concerns (See Brusco, 1997, for a broad analysis of collusion).

Invoking the first-order approach, the principal's objective is to minimize the sum of transfers under the previous constraints:

$$\min_{\boldsymbol{w},\boldsymbol{x}} \mathbb{E}_{\boldsymbol{R}} \left[w_{\boldsymbol{R}} + x_{\boldsymbol{R}} | 1, 1 \right]$$
$$\text{subect to } (1), (2), (3)$$

It is however possible to deal with a simplified problem as the next observation indicates.

Lemma 1 *The program of the principal is separable in two independent optimizations, one for each agent.*

This result is useful for simplifying the exposition. The separability follows from two facts: First, since the principal is risk neutral, the objective function is linear in wages, and second, the constraints feature the wages of only one agent at a time, and they can thus be divided into two independent sets. In the following, we consequently restrict attention to the program pertaining to agent 1. In other words, we consider only one side of the problem, but the other can be dealt with in the exact same way. Since for the remaining of the analysis we stick to agent 1's point of view, the following definitions are implicitly given for agent 1 only. Those definitions will help grasping some economic intuition during the resolution.

Definition 2 *For any pair of results $\boldsymbol{R}$, the **effort informativeness** of $\boldsymbol{R}$ is:*

$$h(\boldsymbol{R}) = \frac{Prob(\boldsymbol{R}|1,1)}{Prob(\boldsymbol{R}|0,1)}$$

To clarify why we call informativeness this ratio, it is illustrative to consider one of them, $h(SS)$. It is the likelihood ratio between effort and shirking for the first agent upon observing two successes, conditional on the other agent exerting effort. The higher $h(SS)$ is, the more likely it is upon observing two successes that the agent has exerted effort and not shirked. The different possible results carry more or less information about the choice of effort, and h is a measure of that information. Note that in this definition we only consider cases in which agent 2 exerts effort, since in the end we are concerned with the implementation of two efforts.

Definition 3 *For any pair of results $\boldsymbol{R}$, the **incentive efficiency** of $w_{\boldsymbol{R}}$ is:*

$$I(\boldsymbol{R}) = 1 - \frac{1}{h(\boldsymbol{R})} = \frac{Prob(\boldsymbol{R}|1,1) - Prob(\boldsymbol{R}|0,1)}{Prob(\boldsymbol{R}|1,1)}$$

The right-hand side is the ratio between the coefficient of the wage $w_{\boldsymbol{R}}$ in the incentive constraint and the probability of paying this wage. This is therefore the (constant) ratio of marginal incentive and marginal costs for that wage, which explains the notion of incentive efficiency. Note that it is at most 1, in which case the wage is fully effective, since the result $\boldsymbol{R}$ then indicates with certainty that the agent has exerted effort.

Two remarks are in order about these two definitions. First, in a setting with one single agent and binary outcomes, these two concepts are trivial, but they make sense when the outcome space in richer, as with multiple agents or additional signals (see Laffont and Martimort, 2002, pp. 167-172). Second, note that a result $\boldsymbol{R}$ is more informative than a result $\boldsymbol{R}'$ if and only if the associated wage $w_{\boldsymbol{R}}$ is more incentive efficient than $w_{\boldsymbol{R}'}$. In other words, it is equivalent to reason in terms of how to best infer the action or in terms of how to spread optimally the incentive weight. The next result will be useful in characterizing the optimal incentive scheme.

Lemma 2 *Under risk-neutrality and limited liability, an optimal incentive scheme entails positive wages only for the result(s) with the highest incentive efficiency.*

The lemma expresses the intuitive idea that the incentive weight should be put on the outcomes that are most efficient at inducing effort. In fact, since the principal's objective is linear, the result is even more extreme, and all the weight will generically be put on one single wage.

We are now in position of grasping a generic flavor of the situations under which the optimal scheme exhibits RPE or JPE. Another definition is needed, once again limiting us to the point of view of agent 1.

Definition 4 *Effort e is a **strong complement** (resp. **substitute**) to effort f if:*

$$\frac{Prob(.S|1,1)}{Prob(.F|1,1)} \geq (\leq) \frac{Prob(.S|0,1)}{Prob(.F|0,1)}$$

That is, when agent 1 exerts effort, this increases the likelihood of a success relative to a failure for agent 2 (for any result of agent 1). Note that this relationship is not symmetric, since it may well be the case that effort f is detrimental to the success of the first project. In any case, we have the following result.

Proposition 1 *The optimal incentive scheme of agent 1 exhibits JPE (resp. RPE) if and only if effort e is a strong complement (resp. substitute) to effort f.*

Keeping as a reference this generic insight that collective schemes are optimal under (some notion of) complementarity[6], and relative performance evaluation optimal under substitutability, we investigate now the information structure in more details.

3 Main result under risk-neutrality

3.1 Informational setting

No restrictions where yet imposed on how efforts interact in the production process. From now on, we will focus exclusively on the informational dimension of the problem. To do so, we assume away cross-effects of efforts: The probability of success for each project depends only on the effort of the corresponding agent. We assume that the probabilities of success conditional on effort are not known with certainty[7]. All players hold the same beliefs on that probabilities. For agent 1, the success probability conditional on effort e is the random variable $\tilde{p}_e$, and it is $\tilde{q}_f$ for agent 2. In the following we use the notations:

$$p_e = \mathbb{E}[\tilde{p}_e], \qquad \sigma_e^2 = var(\tilde{p}_e), \qquad q_f = \mathbb{E}[\tilde{q}_f], \qquad \tau_f^2 = var(\tilde{q}_f)$$

[6] It is worthwhile observing that Aggarwal and Samwick (1999) obtain a related result in the case of competing structures, each with one principal and one agent. When firms compete à la Cournot, the actions of the agents - quantity choices - are strategic substitutes, and RPE is desirable for the principals, while JPE is optimal under Bertrand competition, under which agents' actions are strategic complements. In fact, the present model could also be applied to competing structures, insofar as each principal wants his agent to exert effort. Indeed, in such a case, both agents will exert effort in equilibrium and therefore the incentive constraint in each competing structure is the same as that for each agent in an integrated structure. We do not elaborate here on this issue, but interesting predictions might be derived in the field of top executives compensation.

[7] The reader will notice the link with the classical two-armed bandit problem. Two-armed bandits are classically equipped with one safe and one risky arm, see for example Bolton and Harris (1999). DeGroot (1970, pp. 399-405) contains a model of two-armed-bandit with dependent arms which is the closest setting up to our knowledge. However it consists of an example in which the two Bernoulli parameters have two points supports.

We allow any correlation between the results in case of high effort and low effort, but the following correlation parameters are central:

$$\rho_{e,f} = \frac{cov(\tilde{p}_e, \tilde{q}_f)}{\sigma_e \tau_f}$$

This is the most general form of imperfect knowledge one can introduce in the present setting. Note that the correlation coefficient pertains to the beliefs on the technology, and is therefore not directly representing the correlation of outcomes as is often the case.

Although intuitive lemma 2 can be, it still leaves the door open to much less intuitive indications regarding optimal schemes. The next example illustrates this point.

Example 1 *Extreme innovation.*

Consider symmetric agents using identical technologies. Assume that an old well-known technology yields at no cost a success with probability $p_0 (= q_0)$. In turn, the new technology might either perfectly fit, with probability $p_1 > p_0$, in which case it would yield a success with probability 1, or be completely useless, yielding a failure for sure. Using the new technology requires a learning cost c. Here $\tilde{p}_1 (= \tilde{q}_1)$ is a binomial distribution with parameter p_1 - and consequently variance $\sigma_1^2 = p_1(1 - p_1)$. Simple calculations yield $h(SS) = \frac{1}{p_0}$, $h(SF) = h(FS) = 0$ and $h(FF) = \frac{1}{1-p_0}$. Thus if $p_0 > \frac{1}{2}$, the highest likelihood ratio is that of a double failure: agents should be compensated only in that state.

There may thus be counter-intuitive situations in which agents are rewarded only upon obtaining two failures, such as that described in example 1. Note however that in example 1 the Nash equilibrium that the principal would like to implement may be Pareto-dominated from the agents point of view. Indeed, for small enough p_0 relative to p_1, they are better off in the shirk-shirk equilibrium (for any w_{FF}). Tacit coordination of the agents on their Pareto-optimal equilibrium[8] could be by itself a reason not to use a scheme with $w_{FF} > 0$, since this would encourage the worst behavior from the principal's point of view. We will simply rule out such situations by the following assumption on the technology, and we will not impose here unique implementation.

[8]The topic of unique and/or undominated implementation is of interest in itself. It has been thoroughly studied since the early contributions of Ma (1988) and Ma et al. (1988). For the closest settings, see in particular Arya and Glover (1995) and Arya et al. (1997).

Assumption 1 *(Effective Effort)* $\quad Prob(\tilde{p}_1 \geq \tilde{p}_0) = 1$

This quite natural assumption says that any draw for the technology in case of effort is better than any possible draw in absence of effort. Therefore, exerting effort always (weakly) increases the probability of success. This does not mean that effort is always efficient, since efficiency considerations should also take into account the cost c. Example 1 is ruled out because in some states success could be less likely when the agents exerted effort. The assumption is stronger than needed for our results, but it has the advantage of being perfectly transparent. A useful consequence of assumption 1 is the following.

Lemma 3 *Under assumption 1, an optimal scheme entails* $w_{FF} = w_{FS} = 0$.

In fact, for any result of agent 2, the assumption implies that a failure of agent 1 is always ranked below a success in terms of incentive efficiency.

3.2 Main result

We are now in position to fully characterize the optimal incentive scheme. This is done in the next proposition, using the definition:

$$\gamma = \frac{\sigma_0}{p_0}\frac{p_1}{\sigma_1} \tag{4}$$

Proposition 2 *Under assumption 1, the optimal wage profile is:*

if $\frac{\rho_{11}}{\rho_{01}} < \gamma$, *a RPE scheme with*

$$w_{SF} = \frac{c}{(1-q_1)(p_1-p_0) - \tau_1(\rho_{11}\sigma_1 - \rho_{01}\sigma_0)}, \quad w_{SS} = w_{FS} = w_{FF} = 0$$

if $\frac{\rho_{11}}{\rho_{01}} > \gamma$, *a JPE scheme with*

$$w_{SS} = \frac{c}{q_1(p_1-p_0) + \tau_1(\rho_{11}\sigma_1 - \rho_{01}\sigma_0)}, \quad w_{SF} = w_{FS} = w_{FF} = 0$$

if $\frac{\rho_{11}}{\rho_{01}} = \gamma$, *any scheme (possibly IPE) with*

$$(q_1 + \frac{\tau_1}{p_0})w_{SS} + (1 - q_1 - \frac{\tau_1}{p_0})w_{SF} = \frac{c}{p_1 - p_0}, \quad w_{FS} = w_{FF} = 0$$

The criterion for relative vs joint performance evaluation, though it looks very simple, has a rich economic content. On the technical side, it is completely generic in that it does not depend on any assumption on the shape of the underlying distributions[9], nor on whether they are discrete or continuous, and so on. Also, it depends only on the properties of the distributions, so that it is purely informational, while proposition 1 included the technological dimension. We can however relate this result to the link between effort complementarity and JPE demonstrated in the first proposition. We have on the one hand:

$$\frac{Prob(SS|1,1)}{Prob(SF|1,1)} = \frac{p_1 q_1 + \rho_{11}\sigma_1\tau_1}{p_1(1-q_1) - \rho_{11}\sigma_1\tau_1}$$

which is increasing in ρ_{11} and on the other hand:

$$\frac{Prob(SS|0,1)}{Prob(SF|0,1)} = \frac{p_0 q_1 + \rho_{01}\sigma_0\tau_1}{p_0(1-q_1) - \rho_{01}\sigma_1\tau_1}$$

which is increasing in ρ_{01}. Following definition 4, the effort of the first agent is all the more complementary (for successes) to that of the second agent that ρ_{11} is relatively higher than ρ_{01}. According to proposition 1, such complementarity calls for JPE. In that sense, proposition 2 emphasizes that correlation creates partial *informational* complementarities.

Proposition 2 makes an important connection between two dimensions: First, the correlation conditional on effort and second the effect of effort on the variability of the result. The first dimension is a pure multiagent effect, while the second is a pure single agent effect (γ depends only on the agent's choice). The coefficient of variation $\frac{\sigma_e}{p_e}$ is a measure of how noisy the success signal is depending on effort. If γ is smaller than 1, effort increases this noise[10], while it decreases noise if γ is higher than 1. Regarding correlations, if the ratio $\frac{\rho_{11}}{\rho_{01}}$ is higher than 1, the results of the agents are more correlated when they choose the same actions than when they choose different actions. This ratio is new in the multiagent analysis since previous papers consider uniform correlation[11]. What matters in the choice between RPE and JPE is the relative quality of information between the two possible actions for one agent, and how the correlation between results varies with the

[9] In the multiagent models mentioned in the introduction, almost all distributions are either multivariate normal representing additive noise or two-point distributions.

[10] Whether exerting more effort increases noise or not and what are the consequences for the career concern model is a point discussed in Dewatripont et al. (1999).

[11] Ramakrishnan and Takor (1991) insist on the role of *conditional* correlation, but as they mention p. 260, the agents take the value of the correlation as exogenous in their model. It seems reasonable to assume that sophisticated agents take into account the fact that the correlation varies with their own choice of action.

different actions.

We finish this discussion by two corollaries to proposition 2.

Corollary 1 *An increase in the equilibrium correlation of the results favors JPE.*

Proof. The equilibrium covariance of the two results is $cov(\tilde{p}_1 S + (1 - \tilde{p}_1)F, \tilde{q}_1 S + (1 - \tilde{q}_1)F) = (S - F)^2 \rho_{11} \sigma_1 \tau_1$, while the variances of the results are $var(\tilde{p}_1 S + (1 - \tilde{p}_1)F) = (S - F)^2 \sigma_1^2$ and $var(\tilde{q}_1 S + (1 - \tilde{q}_1)F) = (S - F)^2 \tau_1^2$. The equilibrium correlation is thus exactly ρ_{11}, and an increase ρ_{11} increases the desirability of JPE from the criterion of proposition 2. ■

This observation runs counter to usual results. This is the most striking consequence of the informational complementarity effect. Other models focus on the idea that good performances indicate a favorable environment and that this favorable noise should be filtered by RPE, while a previously ignored effect is here at work: A good result of the other agent might also be a good signal of effort under high equilibrium correlation.

Corollary 2 *The principal always benefits from uncertainty on the technology.*

Proof. The expected gains of the principal do not depend on uncertainty. While with perfect knowledge of the technology, the principal would use independent schemes, with imperfect knowledge, he can still use a pair of independent contract (IPE) but it is not optimal. Therefore he earns more with RPE or JPE and thus benefits from the uncertainty. ■

It has already been remarked a number of times in the literature that correlation helps reducing rents, smoothes information revelation and so on. Here, the same positive conclusion can be drawn with respect to uncertainty in general, provided all the players are risk-neutral.

3.3 Examples

To illustrate further the main result, we briefly apply it to usual forms of uncertainty that have been considered in the literature. The optimal scheme is RPE in the first three examples, while it is IPE or JPE in the last two ones.

Example 2 *The additive model.*

The most classical way of introducing technological uncertainty amounts in our discrete model to the following form of uncertainty:

$$\tilde{p}_e = p_e + \varepsilon$$
$$\tilde{q}_f = q_f + \eta$$

where ε and η are random variables with zero means, variances σ^2 and correlation coefficient ρ. What matters here is that the noise is additively separable from the influence of the action. Note that the variance of the probability of success depends only on ε, which implies $\sigma_0 = \sigma_1 = \tau_0 = \tau_1 = \sigma$. All pairs $(\tilde{p}_e, \tilde{q}_f)$ have the same correlation ρ, so that $\frac{\rho_{11}}{\rho_{01}} = 1$. Also, $\gamma = \frac{\sigma_0}{p_0}\frac{p_1}{\sigma_1} = \frac{p_1}{p_0} > 1$. Therefore, according to proposition 2, RPE is always optimal with additive uncertainty. That formulation of additive uncertainty parallels that of Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983). We have shown that this setting favors competition between agents, even abstracting from risk-sharing concerns.

Example 3 *Effort is sometimes irrelevant.*

Che and Yoo (2001) use an original information structure. They assume that with some probability ν, the technology is such that a project is a success regardless of effort[12], and with probability $(1-\nu)$, the technology is such that the outcome depends on the effort. Let us denote the probability of success in that case by r_e. Overall, this corresponds to the situation:

$$\{\tilde{p}_0, \tilde{p}_1\} = \{\tilde{q}_0, \tilde{q}_1\} = \begin{cases} \{1, 1\} & \text{with probability } \nu \\ \{r_0, r_1\} & \text{with probability } (1-\nu) \end{cases}$$

The relationships with our notations are simply:

$$r_e = \frac{p_e - \nu}{1-\nu}, \qquad \sigma_e^2 = \frac{\nu}{1-\nu}(1-p_e)^2 \qquad \text{and} \qquad \rho_{ef} = 1$$

Therefore, since we have here $\gamma = \frac{p_1(1-p_0)^2}{p_0(1-p_1)^2} > 1$, this setting generates RPE as optimal incentive scheme[13].

[12]It is unimportant that the probabilities are exactly 1, what matters is that they are the same in some state of the world, making effort irrelevant in that state.

[13]To be clear, the contribution of Che and Yoo (2001) is precisely to show that, while in this static setting RPE is optimal, JPE becomes optimal in the infinitely repeated version of the problem. One contribution of this paper is to give a definite answer to their footnote 16, p. 530-531 regarding how the form of the common shocks affects the choice RPE vs JPE in the static setting.

The next example focuses on negative correlation.

Example 4 *Umbrellas versus sunglasses.*

Consider a shop with two divisions, the first responsible for sunglasses and the other for umbrellas. Before the weather is known to be rainy or sunny, they have to decide whether to buy additional products, at the small cost c of a phone call. The story is represented by the following information structure:

$$(\tilde{p}_1, \tilde{q}_1, \tilde{p}_0, \tilde{q}_0) = \begin{cases} \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{2}\} & \text{with probability } \frac{1}{2} \\ \{\frac{1}{2}, 1, \frac{1}{2}, \frac{1}{4}\} & \text{with probability } \frac{1}{2} \end{cases}$$

The first case is that of sun: in that case, the optimal choice is to buy sunglasses, since this would yield a success for sure ($\tilde{p}_1 = 1$), while not having bought sunglasses makes clients unhappy ($\tilde{p}_0 = \frac{1}{4}$). Also, in that case, buying umbrellas is not crucial ($\tilde{q}_1 = \tilde{q}_0 = \frac{1}{2}$). The case of rain is symmetric. Overall, if c is sufficiently small, it is better to buy both goods. Simple calculations show that $\rho_{11} = -1$ and $\rho_{01} = \rho_{10} = 1$. Thus $\frac{\rho_{11}}{\rho_{10}} < 0$, and RPE is optimal.

While with negative correlation insurance can be provided by joint evaluation, this example illustrate that in fact it is quite intuitive with negative correlation to pay agents relatively. A double success is indeed more indicative of different efforts than an asymmetric outcome, which is very likely when both agents exert efforts. This illustrates the case of informational substitutability.

Example 5 *The multiplicative model.*

Consider the following setup where the probability of success of an agent is given by:

$$\tilde{p}_e = \varepsilon p_e$$

$$\tilde{q}_f = \eta q_f$$

where ε and η are random variable with means 1, variances σ^2 and correlation ρ. This functional form of uncertainty is used as an example in Holmström (1982) and appears also in career concerns model (e.g. Dewatripont et al., 1999). Note that all pairs $(\tilde{p}_e, \tilde{q}_f)$ have the same correlation ρ, so that $\frac{\rho_{11}}{\rho_{01}} = 1$. In addition we have $\sigma_0 = \tau_0 = r_0\sigma_\varepsilon$ and $\sigma_1 = \tau_1 = r_1\sigma_\varepsilon$, so that $\gamma = 1$. Thus IPE is an optimal scheme in that case (but not the unique one as seen in the proposition). Note that this is true for any level of equilibrium correlation ρ.

Example 6 *Technological change.*

As a last application, consider the problem of implementing technological change. A status quo solution consists in using an old, known technology, which generates a success with fixed probabilities p_0 and q_0 (so that $\sigma_0 = \tau_0 = 0$). In turn, the agents can learn a new technique, at a cost c, in order to use a new, imperfectly known technology, characterized by the random probability of success $\tilde{p}_1 = \tilde{q}_1$. In that case, $\gamma = 0$, while $\frac{\rho_{11}}{\rho_{01}}$ tends to infinity, thus proposition 2 indicates that the optimal way of inducing technological change is to use a JPE scheme.

4 Risk-averse agents and mixed schemes

Now that we have identified in isolation the effect of pure uncertainty on the optimal shape of contracts, we turn to the issue of risk-sharing. Indeed, one of the arguments put forward concerning relative performance evaluation is its risk-filtering property. This property is better understood when comparing relative performance evaluation to independent contracts or individual piece-rates (e.g. Lazear and Rosen, 1981) in a context when agents are risk-averse. When a common noise influences the performance of the two agents, the principal can use the output of one agents to at least partially correct for the common noise in the other agent's incentive scheme, which reduces the risk-premium to be conceded. The trade-off between incentives and insurance is then solved at smaller costs.

To treat those aspects, we consider a variant of the model which has the following features:

- Agents are risk averse with utility separable between money and effort, the monetary part is evaluated according to the concave function u.
- Agents are subject to a participation constraint with reservation utility v, instead of limited liability.

Thus the payoff of agent 1 now writes:

$$U_1(\boldsymbol{w}|e, f) = \mathbb{E}_{\boldsymbol{R}}\left[u(w_{\boldsymbol{R}})|e, f\right] - c(e) \tag{5}$$

while the incentive constraints remains formally the same as (1) and (2).

$$U_1(\boldsymbol{w}|1, 1) \geq U_1(\boldsymbol{w}|0, 1) \tag{6}$$

In turn, the limited liability constraint for agent 1 is replaced by the following participation constraint:

$$U_1(\boldsymbol{w}|1,1) \geq v \tag{7}$$

We are in position to solve the principal's problem, and to obtain a picture parallelling the results of the preceding sections. As a first step, the next lemma is a smooth equivalent to lemma 2:

Lemma 4 *Under risk-aversion, the optimal wages are ranked according to their incentive efficiency.*

Now, the optimal incentive scheme can be fully characterized, using the definition:

$$\delta = \frac{\sigma_0}{1-p_0}\frac{1-p_1}{\sigma_1} \tag{8}$$

Proposition 3 *When assumption 1 holds and the agents are risk-averse, the optimal wage profile is:*

if $\frac{\rho_{11}}{\rho_{01}} < \delta$*, a scheme with profit sharing at the bottom, relative evaluation at the top with*

$$w_{SF} > w_{SS} > w_{FS} > w_{FF}$$

if $\delta < \frac{\rho_{11}}{\rho_{01}} < \gamma$*, a pure RPE scheme with*

$$w_{SF} > w_{SS} > w_{FF} > w_{FS}$$

if $\frac{\rho_{11}}{\rho_{01}} > \gamma$*, a scheme with profit sharing at the top and relative evaluation at the bottom with*

$$w_{SS} > w_{SF} > w_{FF} > w_{FS}$$

The interpretation of the result follows directly the lines of the discussion of proposition 2. In addition, the case of a failure of the agent under consideration (i.e. the cases FF and FS) matters here, since no wages are constrained by limited liability, and all four wages have to be chosen. Similarly to the case of a success, the relative informativeness ($\frac{\sigma_e}{1-p_e}$) of a failure is crucial. A simple observation is that under independent productions and assumption 1, strong complementarity is not possible for any kinds of correlation. Thus pure JPE can not be optimal given insurance concerns. It is always optimal either to filter at least part of the noise through an element of RPE, or to use informational substitutability, which is also done through an element of RPE.

However, it is particularly interesting that *mixed* schemes are often optimal. Those schemes mixing an element of RPE and an element of JPE have clear economic interpretations and are indeed used in practice. For example, they correspond to the combination of profit sharing with selective promotions (in the first case) or selective firing (in the third case). Finally, this suggests that an analysis in which the level of relative evaluation is constant over results pairs[14] such as in the LEN model (Holmström and Milgrom, 1990; Itoh, 1992), is unsatisfactory in that it does not allow for such mixed schemes.

5 Conclusion

The main message of this paper is twofold. First, relative performance evaluation is not necessarily the best informational tool. The nature and shape of uncertainty may matter even with risk-neutral agents, and a principal generically benefits from uncertainty in that case. Regarding the desirability of relative evaluation, the model demonstrates that standard results in multiagent moral hazard problems are not robust, and identifies the specificity of previous analyses. In particular, under risk-neutrality, high equilibrium correlation of the agents' performances pleads for joint performance evaluation. Second, under risk-aversion, correlated risks call for noise filtering and two opposite effects have thus to be traded off in the wage schedule. Optimal mixed schemes balancing those two effects typically exhibit real-life features that had not been theoretically studied before. Interestingly, (relative) sticks and carrots are not equivalent in those schemes. This calls for more thorough inquiry on whether the level of relative evaluation should be higher at higher performance levels or at lower performance levels.

[14]While Holmström and Milgrom (1987) obtain conditions under which an optimal incentive scheme is linear in aggregate profit, there is no result stating that in a model with multiple observables - possibly from different agents - should be linear in those performances.

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A Appendix: Omitted Proofs

A.1 Proof of lemma 2

In the principal's program, let $\lambda > 0$ be the Lagrange multiplier associated with the incentive constraint, and $\mu_{\mathbf{R}} \geq 0$ that associated with the limited liability constraint $w_{\mathbf{R}} \geq 0$. The first-order conditions for each $w_{\mathbf{R}}$ is:

$$-Prob(\mathbf{R}|1,1) + \lambda(Prob(\mathbf{R}|1,1) - Prob(\mathbf{R}|0,1)) + \mu_{\mathbf{R}} = 0$$

If a wage $w_{\mathbf{R}}$ is positive then $\mu_{\mathbf{R}} = 0$ and necessarily:

$$I(\mathbf{R}) = \frac{1}{\lambda}$$

For a wage equal to zero, say $w_{\mathbf{R'}}$, one has $I(\mathbf{R'}) = \frac{1}{\lambda}(1 - \frac{\mu_{\mathbf{R'}}}{Prob(\mathbf{R'}|1,1)}) < \frac{1}{\lambda}$, hence the conclusion.

A.2 Proof of proposition 1

Consider the case of complementary effort. By definition, we have:

$$\begin{aligned}\frac{Prob(SS|1,1)}{Prob(SF|1,1)} \geq \frac{Prob(SS|0,1)}{Prob(SF|0,1)} &\Leftrightarrow \frac{Prob(SS|1,1)}{Prob(SS|0,1)} \geq \frac{Prob(SF|1,1))}{Prob(SF|0,1)}\\ &\Leftrightarrow h(SS) \geq h(SF)\end{aligned}$$

and

$$\begin{aligned}\frac{Prob(FS|1,1)}{Prob(FF|1,1)} \geq \frac{Prob(FS|0,1)}{Prob(FF|0,1)} &\Leftrightarrow \frac{Prob(FS|1,1)}{Prob(FS|0,1)} \geq \frac{Prob(FF|1,1))}{Prob(FF|0,1)}\\ &\Leftrightarrow h(FS) \geq h(FF)\end{aligned}$$

From lemma 2, the only wages that can be positive are thus w_{SS} and w_{FS}. Note that we used equivalences, hence the conclusion. The case of substitute is dealt with similarly.

A.3 Proof of lemma 3

By complementary probabilities and independent productions, we have the identities:

$$\begin{aligned}Prob(SS|1,1) - Prob(SS|0,1) &= -\left(Prob(FS|1,1) - Prob(FS|0,1)\right)\\ Prob(SF|1,1) - Prob(SF|0,1) &= -\left(Prob(FF|1,1) - Prob(FF|0,1)\right)\end{aligned}$$

so that the incentive constraint can be written as:

$$(Prob(SS|1,1) - Prob(SS|0,1))(w_{SS} - w_{FS}) \\ + (Prob(SF|1,1) - Prob(SF|0,1))(w_{SF} - w_{FF}) \geq c$$

Now, we have:

$$Prob(SS|1,1) - Prob(SS|0,1) = \mathbb{E}[\tilde{p}_1\tilde{q}_1] - \mathbb{E}[\tilde{p}_0\tilde{q}_1] = \mathbb{E}[\tilde{q}_1(\tilde{p}_1 - \tilde{p}_0)]$$

From assumption 1, $(\tilde{p}_1 - \tilde{p}_0)$ is a positive random variable, as is $\tilde{q}_1$. Thus the coefficient of w_{FS} in the incentive constraint is negative, which implies that this wage should be 0. Similarly, one has:

$$\begin{aligned} Prob(SF|1,1) - Prob(SF|0,1) &= \mathbb{E}[\tilde{p}_1(1-\tilde{q}_1)] - \mathbb{E}[\tilde{p}_0(1-\tilde{q}_1)] \\ &= \mathbb{E}[(1-\tilde{q}_1)(\tilde{p}_1 - \tilde{p}_0)] \end{aligned}$$

which is also positive from the assumption.

A.4 Proof of proposition 2

From the two preceding lemmata, we know that except in the special case $I(SS) = I(SF)$ only one wage is positive. The criterion for $w_{SS} > 0$ is $I(SS) > I(SF)$. We need the following simple calculation to undertake the comparison:

$$\begin{aligned} Prob(SS|11) &= \mathbb{E}[\tilde{p}_1\tilde{q}_1] = p_1q_1 + \rho_{11}\sigma_1\tau_1 \\ Prob(SS|01) &= \mathbb{E}[\tilde{p}_0\tilde{q}_1] = p_0q_1 + \rho_{01}\sigma_0\tau_1 \\ Prob(SF|11) &= \mathbb{E}[\tilde{p}_1(1-\tilde{q}_1)] = p_1(1-q_1) - \rho_{11}\sigma_1\tau_1 \\ Prob(SF|01) &= \mathbb{E}[\tilde{p}_0(1-\tilde{q}_1)] = p_0(1-q_1) - \rho_{01}\sigma_0\tau_1 \end{aligned}$$

Using those values yields:

$$\frac{Prob(SS|1,1)}{Prob(SS|0,1)} > \frac{Prob(SF|1,1)}{Prob(SF|0,1)} \Leftrightarrow \frac{q_1p_1 + \rho_{11}\sigma_1\tau_1}{p_0q_1 + \rho_{01}\sigma_0\tau_1} > \frac{p_1(1-q_1) - \rho_{11}\sigma_1\tau_1}{p_0(1-q_1) - \rho_{01}\sigma_0\tau_1}$$

which simply boils down to

$$\frac{\rho_{11}}{\rho_{01}} > \frac{p_1}{\sigma_1}\frac{\sigma_0}{p_0}$$

Conversely, one easily obtains that w_{SF} is positive under the reverse inequality. The optimal wages are then straightforwardly obtained by saturating the incentive constraint. In the case of equality, both wages have the same incentive weight, and only their sum matters. The optimal sum is also obtained by saturating the incentive constraint.

A.5 Proof of lemma 4

We associate the positive multipliers $\lambda \geq 0$ and $\mu \geq 0$ to, respectively, the incentive and participation constraints, and form the Lagrangian of the cost minimization problem:

$$\begin{aligned} L(\boldsymbol{w}, \lambda, \mu) &= \Sigma_{\boldsymbol{R}} Prob(\boldsymbol{R}|11) w_{\boldsymbol{R}} + \lambda c + \mu(v+c) \\ &\quad - \Sigma_{\boldsymbol{R}} \left[\lambda \left(Prob(\boldsymbol{R}|11) - Prob(\boldsymbol{R}|01)\right) + \mu Prob(\boldsymbol{R}|11)\right] u(w_{\boldsymbol{R}}) \end{aligned}$$

It is clear that both multipliers have to be positive. The first-order conditions for all $\boldsymbol{R}$ boil down to:

$$\frac{1}{u'(w_{\boldsymbol{R}})} = \mu + \lambda \frac{Prob(\boldsymbol{R}|11) - Prob(\boldsymbol{R}|01)}{Prob(\boldsymbol{R}|11)} = \mu + \lambda I(\boldsymbol{R})$$

Note that $\frac{1}{u'}$ is an increasing function, thus w's are ranked as $\frac{1}{u'(w)}$. This means that the wages are ranked according to their incentive efficiency.

A.6 Proof of proposition 3

The ranking between w_{SS} and w_{SF} corresponds to the criterion of the first proposition, which yields:

$$w_{SS} \geq w_{SF} \qquad \text{if and only if} \qquad \frac{\rho_{11}}{\rho_{01}} > \gamma$$

Also, from assumption 1, we have:

$$\begin{aligned} Prob(FS|11) - Prob(FS|01) &= \mathbb{E}[(1-\tilde{p}_1)\tilde{q}_1] - \mathbb{E}[(1-\tilde{p}_0)\tilde{q}_1] \\ &= \mathbb{E}[\tilde{q}_1(\tilde{p}_0 - \tilde{p}_1)] \leq 0 \end{aligned}$$

and

$$\begin{aligned} Prob(FF|11) - Prob(FF|01) &= \mathbb{E}[(1-\tilde{p}_1)(1-\tilde{q}_1)] - \mathbb{E}[(1-\tilde{p}_0)(1-\tilde{q}_1)] \\ &= \mathbb{E}[(1-\tilde{q}_1)(\tilde{p}_0 - \tilde{p}_1)] \leq 0 \end{aligned}$$

Which indicate that both $I(FS)$ and $I(FF)$ are negative, while we have already seen that $I(SS)$ and $I(SF)$ are positive. This implies that w_{SS} and w_{SF} are always higher than w_{FF} and w_{FS}. To finish the proof, we need the ranking between w_{FF} and w_{FS}, which requires a few additional calculations:

$$\begin{aligned} Prob(FS|11) &= \mathbb{E}[(1-\tilde{p}_1)\tilde{q}_1] = q_1(1-p_1) - \rho_{11}\sigma_1\tau_1 \\ Prob(FS|01) &= \mathbb{E}[(1-\tilde{p}_0)\tilde{q}_1] = q_1(1-p_0) - \rho_{01}\sigma_0\tau_1 \\ Prob(FF|11) &= \mathbb{E}[(1-\tilde{p}_1)(1-\tilde{q}_1)] = (1-p_1)(1-q_1) + \rho_{11}\sigma_1\tau_1 \\ Prob(FF|01) &= \mathbb{E}[(1-\tilde{p}_0)(1-\tilde{q}_1)] = (1-p_0)(1-q_1) + \rho_{01}\sigma_0\tau_1 \end{aligned}$$

Using those values and conducing calculations parallelling that in the other comparison yields:

$$I(FS) - I(FF) > 0 \Leftrightarrow \frac{\rho_{11}}{\rho_{01}} < \frac{\sigma_0}{1-p_0}\frac{1-p_1}{\sigma_1} = \delta$$

Note that $\delta < \gamma$, so that a JPE scheme can never be optimal, since it would require at the same time $w_{SS} \geq w_{SF}$ and $w_{FS} \geq w_{FF}$. All the other combinations are in turn possible, depending on the parameters.